29. Oct. 1940

The Institute for advanced study
Princeton N.J., U.S.A.

Dear Professor Madhava Rao,

Being now visiting Professor in Princeton I got finally yesterday your interesting letter of 3rd May (!).

I propose the following interpretation of your equation (4), which ~~that~~ does not seem me so complicated after all. [I substitute your $F_{\mu\nu}$, $\vec{E}$, $\vec{H}$ by $\frac{1}{2}F_{\mu\nu}$, $\frac{1}{2}\vec{E}$, $\frac{1}{2}\vec{H}$ because then ~~they~~ $\vec{E}$, $\vec{H}$ has the meaning of $\frac{1}{c}\mu$ times field strength, μ being the additional magnetic moment of the particle.] It is convenient to define a vector H^*_μ by $mcH^*_\mu = \hat{F}_{\mu\nu}\,\pi^\nu$ where $\hat{F}_{\mu\nu}$ is dual to $F_{\mu\nu}$ (equal $\vec{H}, -i\vec{E}$) so that $mc\vec{H}^* = \pi_0\vec{H} - [\vec{\pi}\cdot\vec{E}]$; $imcH_0^* = (\vec{\pi}\vec{H})$. Then introduce the four invariants

$$J_1 = -\pi_0^2 + m^2c^2 + \vec{\pi}^2; \quad J_2 = \tfrac{1}{2}(E^2 - H^2); \quad J_3 = (\vec{E}\vec{H})^2.$$

$$J_4 = \vec{H}^{*2} - H_0^{*2} \quad \text{or} \quad m^2c^2J_4 = \pi_0^2H^2 - 2\pi_0(\vec{H}[\vec{\pi}\vec{E}]) + \vec{\pi}^2\vec{E}^2 - (\vec{\pi}\vec{E})^2 - (\vec{\pi}\vec{H})^2.$$

Then your equation (4) writes itself

$$J_1^2 + \tfrac{1}{4}J_2^2 + J_1J_2 + J_3 - m^2c^2J_4 = 0.$$

I want to discuss the equation

$$J_1 = -\frac{1}{2}J_2 \pm \sqrt{m^2c^2J_4 - J_3} \qquad \ldots (*)$$

The discussion is simple only in the case, where $\vec{E}$, $\vec{H}$ are small quantities, so that one can develop the mechanical problem in power series (perturbation theory) of $\vec{E}$, $\vec{H}$. In the first approximation it is then allowed to ~~put~~ interpret

$$\vec{v} = \vec{\pi}/\pi_0$$

as the velocity of the particle, so that $\vec{H}^*$ ~~is~~ in a simple way connected with the magnetic field strength in the rest system of the particle [N.B. the general correct definition of the velocity is $\vec{v} = \frac{\partial \pi_0}{\partial \vec{\pi}}$, where the differentiation is to carry through by fixed values of $\vec{E}, \vec{H}$.] The term $-\frac{1}{2} J_2$ in (*) is then small in comparison to the other terms. ~~The~~ The term $\pm\sqrt{\mu^2 J_4}$ is expected, the double sign corresponding to the 'double refraction' similar to the light rays in a medium with Faraday-effect. The term $-J_3$ under the square root is unexpected and seems to be very characteristic for the theory considered here; so is also the smaller term $-\frac{1}{2} J_2$ outside the square root. Both terms give rise to ~~an~~ additional terms in the scattering of light by ~~light~~ the particle prop. to μ^2 (and higher powers of μ^2). ~~because they make the Maxwell equation non linear~~ [The current vector depends now on $\vec{E}, \vec{H}$ in a more complicated way]. Perhaps you can write down the current-vector explicitly in the approximation of classical orbits you wants to consider – that means as a function of the π's and the field strengths. [If $D(x-q)$ is the density distribution of the particle, the energy is $H = \int \pi_0 D(x-q)\, d^3x$, if π_0 is expressed by π and the field-strengths in space; then $s_\mu = \frac{\delta H}{\delta \phi_\mu}$, the $p_\mu = \frac{\partial S}{\partial x_\mu}$ being fix by the variation.]

What the reprints concerns please write to my assistent in the Institute in Zürich: Dr. J. M. Jauch; I have no reprints here.

Please say my regards to Dr. Bhabha. Does he comes to U.S.A.?

With best regards to yourself

Yours truly

W. Pauli

Pauli's letter

29/10/40
Princeton

Being now visiting Professor in Princeton I got finally yesterday your interesting letter of 3rd May.

I propose the following interpretation of your equation (4) which does not seem ~~was~~ to me so complicated after all. [I substitute your $F_{\mu\nu}$, $\vec{E}$, $\vec{H}$ by $\frac{1}{2}F_{\mu\nu}$, $\frac{1}{2}\vec{E}$, $\frac{1}{2}\vec{H}$ because then my $\vec{E}$, $\vec{H}$ has the meaning of $\frac{1}{c}\mu$ times field strength, μ being the additional magnetic moment of the particle.] It is convenient to define a vector H^*_μ by $mc\,H^*_\mu = \hat{F}_{\mu\nu}\pi^\nu$ where $\hat{F}_{\mu\nu}$ is dual to $F_{\mu\nu}$ [equal $\vec{H}, -i\vec{E}$] so that $mc\,\vec{H}^* = \pi_0\vec{H} - [\vec{\pi}\vec{E}]$, $imc\,H_0^* = (\vec{\pi}\vec{H})$. Then introduce the four invariants

$$J_1 = -\pi_0^2 + m^2c^2 + \vec{\pi}^2 \,;\quad J_2 = \tfrac{1}{2}(\vec{E}^2 - \vec{H}^2)\,;\quad J_3 = (\vec{E}\vec{H})^2$$

$$J_4 = \vec{H}^{*2} - H_0^{*2} \text{ or } m^2c^2J_4 = \pi_0^2H^2 - 2\pi_0(\vec{H}[\vec{\pi}\vec{E}]) + \vec{\pi}^2\vec{E}^2 - (\vec{\pi}\vec{E})^2 - (\vec{\pi}\vec{H})^2.$$

Then your equation (4) writes itself

$$J_1^2 + \tfrac{1}{4}J_2^2 + J_1J_2 + J_3 - m^2c^2J_4 = 0$$

I want to discuss the equation

$$J_1 = -\tfrac{1}{2}J_2 \pm \sqrt{m^2c^2J_4 - J_3} \quad \cdots \quad (*)$$

The discussion is simple only in the case where $\vec{E}$, $\vec{H}$ are small quantities so that one can develop the mechanical problem in power series [perturbation theory] of $\vec{E}$, $\vec{H}_K$. In the first approximation it is then allowed to interpret

$$\vec{v} = \vec{\Pi}/\Pi_0$$

as the velocity of the particle so that $\vec{H}^*$ is, in a simple way, connected with the magnetic field strength in the rest system of the particle [NB: The general correct definition of the velocity is $\vec{v} = \frac{\partial \Pi_0}{\partial \vec{\Pi}}$ where the differentiation is to carry through by fixed values of $\vec{E}, \vec{H}$]. The term $-\frac{1}{2} J_2$ in (*) is then small in comparison with the other terms. The term $\pm \sqrt{m^2 c^2 J_4}$ is expected, the double sign corresponding to the "double refraction" similar to the light rays in a medium with Faraday effect. The term $-J_3$ under the square root is unexpected and seems to be very characteristic for the theory considered here; so is also the smaller term $-\frac{1}{2} J_2$ outside the square root. Both terms give rise to additional terms in the scattering of light by the particle proportional to μ^2 (and higher powers of μ), ~~because they make the Maxwell equation non linear~~. The current vector depends now on $\vec{E}, \vec{H}$ in a more complicated way. Perhaps you can write down the current vector explicitly in the approximation of classical orbits you want to consider, that means as a function of the Π's and the field strengths [If $D(x-q)$ is the density distribution of the particle, the energy is $H = \int \Pi_0 \, D(x-q)\, d^3x$, if Π_0 is expressed by $\vec{\Pi}$ and the field strengths in space; then $S_\mu = \frac{\delta H}{\delta \phi_\mu}$, then $p_\mu = \frac{\partial S}{\partial x_\mu}$ being fixed by the variation.

~~What~~ Concerning the reprints please write to my asst. in the Inst. in Zurich: Dr. J. M. Jauch; I have no reprints here. Please say my regards to Dr. Bhabha. Does he comes to U.S.A.? with ~~regards~~ best regards to yourself. yrs truly W. Pauli