

29. Oct. 1940

The Institute for
Advanced Study
Princeton N.J., U.S.A.

Dear Professor Madhava Rao,

Being now visiting Professor in Princeton
day your interesting letter of ^{3rd} May (!).

I got finally yesterday

I propose the following interpretation of your equation (4), ^{which} ~~is~~ does not seem me so complicated after all. [I substitute for \vec{E}, \vec{H} by $\frac{1}{2} \vec{F}_{\mu\nu}, \frac{1}{2} \vec{E}, \frac{1}{2} \vec{H}$ because then \vec{E}, \vec{H} has the meaning of $\frac{1}{2} \mu$ times field strength, μ being the additional magnetic moment of the particle.] It is convenient to define a vector H_{μ}^* by $m c H_{\mu}^* = \vec{F}_{\mu\nu} \pi^{\nu}$ where $\vec{F}_{\mu\nu}$ is dual to $F_{\mu\nu}$ (equal $\vec{H}, -\vec{E}$) so that $m c \vec{H}^* = \pi_0 \vec{H} - [\vec{\pi} \cdot \vec{E}]$ and $H_0^* = (\vec{\pi} \cdot \vec{H})$. Then introduce the four invariants

$$J_1 = -\pi_0^2 + m^2 c^2 + \vec{\pi}^2; \quad J_2 = \frac{1}{2} (E^2 - H^2); \quad J_3 = (\vec{E} \cdot \vec{H})^2$$

$$J_4 = H^{*2} - H_0^{*2} \text{ or } m^2 c^2 J_4 = \pi_0^2 H^2 - 2\pi_0 (\vec{H} [\vec{\pi} \cdot \vec{E}]) + \vec{\pi}^2 F^2 - (\vec{\pi} \cdot \vec{E})^2 - (\vec{\pi} \cdot \vec{H})^2$$

Then your equation (4) writes itself

$$J_1^2 + \frac{1}{4} J_2^2 + J_3 + J_4 - m^2 c^2 J_4 = 0$$

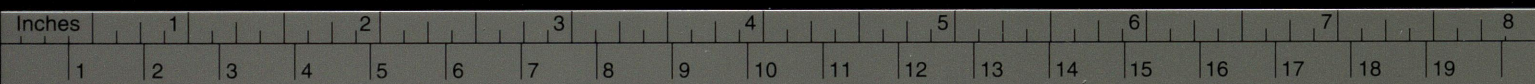
I want to discuss the equation

$$J_1 = -\frac{1}{2} J_2 \pm \sqrt{m^2 c^2 J_4 - J_3}$$

... (*)

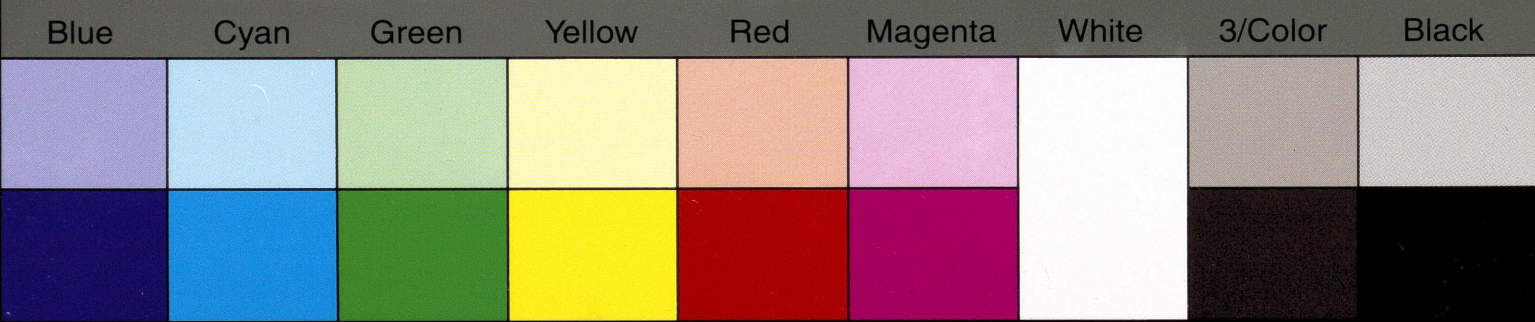
The discussion is simple only in the case, where \vec{E}, \vec{H} are small quantities, so that one can develop the mechanical problem in power series (perturbation theory) of \vec{E}, \vec{H} . In the first approximation it is then allowed to interpret

$$\vec{v} = \vec{\pi} / \pi_0$$



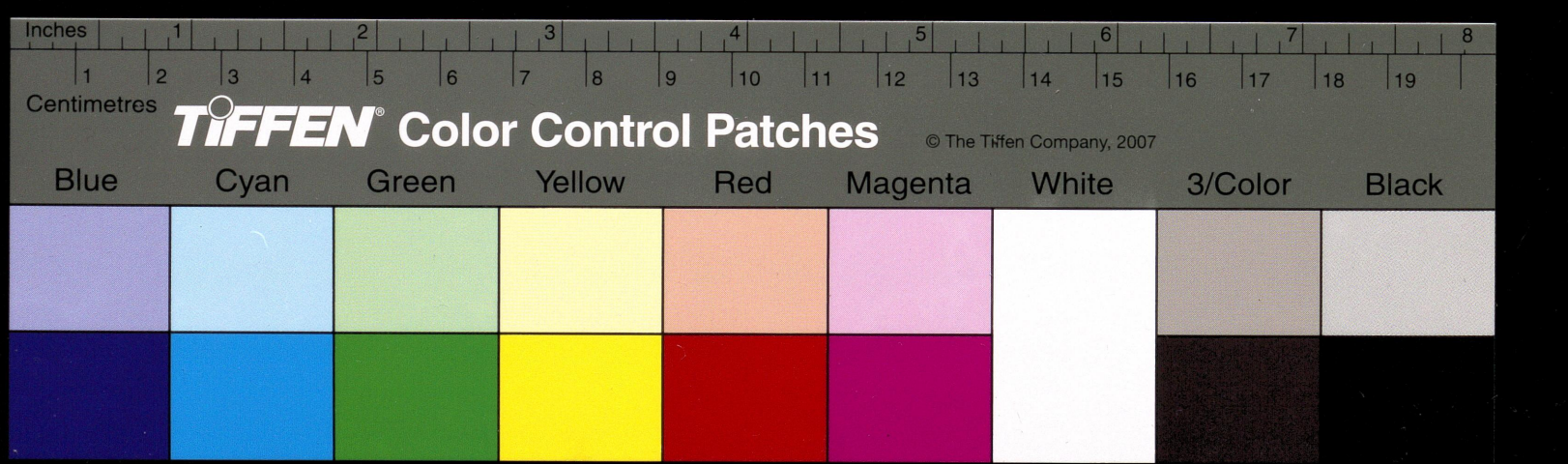
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as the velocity of the particle, so that H^* is ^{in a simple way connected with} the magnetic field strength in the rest system of the particle [L.B. The general correct definition of the velocity is $\vec{v} = \frac{d\vec{\pi}_0}{d\vec{\pi}}$, where the differentiation is to carry through by fixed values of \vec{E}, \vec{H}, J . The term $-\frac{1}{2} J_{\mu\nu} \pi^{\mu\nu}$ is then small in comparison to the other terms. ~~The~~ The term $+\sqrt{\mu^2 J^2}$ is expected, the double sign corresponding to the 'double refraction' similar to the light rays in a medium with Faraday-effect. The term $-J$ under the square root is unexpected and seems to be very characteristic for the theory considered here; as is also the smaller term $-\frac{1}{2} J$ outside the square root. Both terms give rise to ~~an~~ additional terms on the scattering of light by ^{the particle} ~~light~~ prop. to μ^2 (and higher powers of μ^2). ~~As a consequence they make the Maxwell-equation non-linear. The current vector depends now on \vec{E}, \vec{H} in a more complicated way. Perhaps you can write down the current-vector explicitly in the approximation of classical orbits you want to consider, that means as a function of the π 's and the field strengths. If $D(x-q)$ is the density distribution of the particle, the energy is $H = \int \pi_0 D(x-q) d^3x$, if π_0 is expressed by π and the field-strength in space; then $S_{\mu} = \frac{\delta H}{\delta \phi_{\mu}}$, the $p_{\mu} = \frac{\delta S}{\delta x_{\mu}}$ being fixed by the variation.]~~

What the reprint concerns please write to my assistant in the Institute in Zurich: Dr. J. M. Jauch; I have no reprints here. Please say my regards to Dr. Okabe. Does he come to U.S.A.?
 With best regards to yourself
 Yours truly
 W. Pauli



Pauli's letter

29/10/40

Princeton

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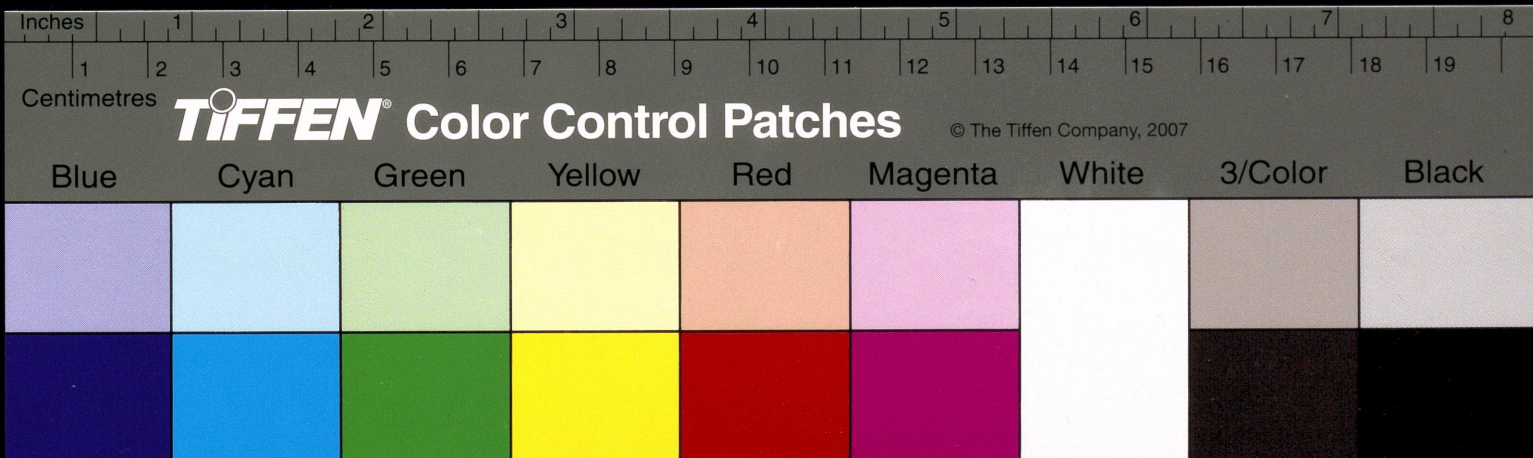
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to additional terms in the scattering of light by the particle proportional

to μ^2 (and higher powers of μ), because they make the Maxwell equation non-

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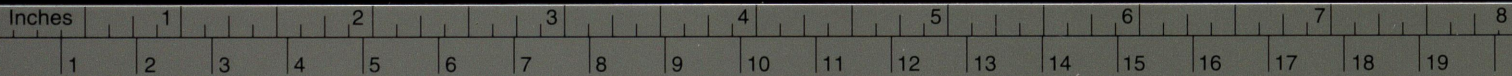
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