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12. Mathematics is what mathematicians do

This is a circular definition since mathematicians are those who do mathematics. However it is the only complete definition, since other definitions leave out significant parts of mathematics. This definition can also ensure that some persons do not define mathematics so as to leave out those parts of the mathematics which they do not like. For ecological health of mathematics, the health of all its components is equally important. The tempering of any component because of fashions or fads or whims can destroy the ecological balance which may not be easy to restore.

Almost all the definitions given above are discussed in greater detail in the following references.

References

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Ramanujan's half a dozen formulae everyday

It seemed ridiculus to worry about how he had found this or that theorem, when he was showing me half a dozen ones almost everyday.

G.H. Hardy

Ramanujan—Hardy number 1729

It was Littlewood who said that every positive integer was one of Ramanujan's personal friends. I remember going to see him once when he was lying ill in Putney. I had ridden in taxi-cab No. 1729 and remarked that the number seemed to be rather a dull one, and that I hoped that it was not an unfavourable omen. 'No', he replied, 'it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways ($1729=12^3+1^3=10^3+9^3$). I asked him, naturally, whether he could tell me the solution of the corresponding problem for 4th power; and he replied; after a moment's thought, that he knew no obvious example, and supposed that the first such a number must be very large . . .

G.H. Hardy

Prof. B.S. Madhava Rao (1900-1987)
A distinguished teacher and an eminent mathematician

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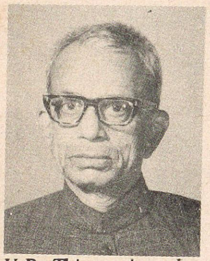

V.R. Thiruvenkatachar

[Prof. B.S. Madhava Rao was one of the most eminent Indian mathematicians in the twentieth century. He was also a great teacher who inspired generations of students. He demonstrated how a group of dedicated teachers could introduce remarkable innovations in both teaching and research. His services to Central College, Bangalore, Indian Mathematical Society and Indian Academy of Sciences will be remembered for a long time. He provided a great encouragement to young researchers. The chief editor of this journal remembers how twenty-five years ago Prof. Madhava Rao, after just hearing him at the IMS conferences, took the initiative to get him elected to the Fellowship of the Indian Academy of Sciences. This is the example of how senior scientists today can encourage young researchers.

The author, Prof. V.R. Thiruvenkatachar is himself a distinguished mathematician, a fellow of the Indian National Science Academy and a former chief scientific officer of the Defence Research and Development Organisation].

Bangalore Srinivasa Rao Madhava Rao was born on 29 May 1900 in Chamarajanagar, a town in the Mysore district of Karnataka State. He had a brilliant educational career throughout from high school to college (gaining the first place at every examination) and this was appropriately concluded at Calcutta University from where he emerged as a medallist at the M.Sc. examination. After completing his studies so admirably, Madhava Rao elected to stay in the academic field rather than the more attractive administration services, which in those days, absorbed many of the most brilliant young men emerging from the Universities. In this, he was following the vocation of his father Srinivas Rao, who chose to serve as a teacher and head master in several high schools in the state. So, after a brief stint as a research scholar, he joined the engineering college of the newly-started Mysore University as Assistant Professor in 1922. During his tenure there, he imparted invaluable training and inspired many young engineers who later attained high distinction in their respective fields. That many of these continued to admire and honour him long after they themselves attained high positions bear eloquent testimony to the quality of his teaching and personality.

In 1938 he was appointed to the newly created chair of Professor of Applied Mathematics in the Central College Bangalore. The Head of the department at that time was (late) Prof. K.S.K. Iyengar, who, soon after his assuming charge of the department a few years earlier, had initiated a programme of revitalising and modernising the mathematical curricula then being taught at the university by introducing post-graduate courses in advanced topics in mathematics and mathematical physics such as Analysis, Modern Algebra, Function Theory, Topology, Analytical Dynamics, Electromagnetic Theory, Relativity Theory etc. In this programme he naturally sought and received a full measure of cooperation from Madhava Rao and they between them shouldered much of the burden of teaching the new courses and training future young colleagues to continue the programme. At the same time, impetus was given to research activity both by their own personal research work as well as by the encouragement and guidance given to younger people. Madhava Rao led this activity by starting to publish his papers on several problems in applied mathematics and theoretical physics. (of this we shall speak in a little more detail later). He was mainly instrumental in establishing firm lines of instruction and research in these fields. It would be no exaggeration to say that the mathematics department of the Central College at that time gained reputation as a vital centre of instruction and research in mathematics, mainly due to the vision and guidance of K.S.K. Iyengar and, more especially of Madhava Rao. The author remembers with gratitude the guidance and encouragement he received from them, both as a student and as a junior colleague. After the death of K.S.K. Iyengar in 1944, the task of directing the activities of the department became the sole responsibility of Prof. Madhava Rao as the head of the department, a task to which he gave full attention, while at the same time not slackening the tempo of his own research work. He became the principal of the Central College in 1953, from where he retired in 1955. Throughout his career as professor and principal, he maintained a close and friendly rapport with other staff members and shared in the teaching work in the department to quite a considerable extent, teaching both the undergraduate and graduate classes. Madhava Rao was a keen sportsman. He captained the Mysore University hockey team and was a champion in tennis, both at Bangalore and Poona. He took keen interest in all sports and was actively associated as President and Vice-President of various State hockey, lawn tennis, table tennis and basket ball associations. Being a good sportsman himself, he had a free and easy manner with successive generations of students, earning their regard and respect thereby. On many occasions, when needed, his personal help was readily available unreservedly.

After retiring from the Mysore University in 1955, he was appointed as Professor of Mathematics and Ballistics at the Institute of Armament Technology, Pune, (1955-1960) and after this served as Tilak Professor of Applied Mathematics in Poona University (1960-1965). After thus completing a period of 45 years of uninterrupted academic activity, he returned to Bangalore to enjoy retired life free from obligatory duties and to pursue intellectual work dictated solely by his personal interests and inclinations. However he remained associated with the Centre of Theoretical Studies of Indian Institute of Science, Bangalore from 1966 till his death, first as a CSIR scholar and later as an Associate. He continued to keep in active touch with current advances in Theoretical Physics and was in demand by many universities as examiner and referee. Apart from these activities, he took up a serious study of the subject of Magic Squares and began writing a comprehensive account of the development of the subject, based on extensive studies of several sources of historical material and incorporating a large collection of examples of magic squares of various types. He aimed to discuss the methods of construction of easy types of magic squares of which reference could be found. He had completed the first draft of the book and was giving it a revision, aiming to add certain new material, when unfortunately his health began to fail and he could not complete the work. To the great sorrow of the large circle of his friends, students and admirers, he passed away on 11th June, 1987.

In addition to his official duties, he served as the secretary of the Indian Academy of Sciences and Editor of its "Proceedings" from its inception in 1934 up to 1955. Thanks to his careful and wise editorship, the "Proceedings" of the Indian Academy of Sciences soon established itself as a premier journal of international standing in the scientific world. He was elected fellow of Indian Academy of Sciences in 1936 and served as its vice-president from 1956 to 1961. He was elected Fellow of the Indian National Science Academy (then the National Institute of Sciences of India) in 1953. In 1945 he received the Ramanujan prize of Madras University.

He was a member of the Council of the Indian Mathematical Society for several years and was also its President for some time. He presided over the section of Mathematics of the Indian Science Congress in 1958 and delivered the presidential address on "Modern Algebra and Theory of Elementary Particles".

In 1935 the late Prof. Max Born was Visiting Professor at the Indian Institute of Science Bangalore. Madhava Rao availed of this opportunity to work on the new electrodynamics of Born and Infeld. On the basis of this work (a brief account of which will be given below) the Calcutta University awarded him the D.Sc. degree. Born himself referred to and used this work in his lectures in Paris and elsewhere. The late Dr. H.J. Bhabha, on returning from Cambridge in 1939 joined the Indian Institute of Science and set up a Cosmic Ray Research Unit there. While at Cambridge he had done work on the theory of cosmic ray showers and on the theory of the meson and other elementary particles. He sought the collaboration of Madhava Rao in continuing this research on the scattering of mesons and related problems. The collaboration ended when Bhabha left for Bombay to head T.I.F.R., but Madhava Rao independently continued the work on elementary particle theory and published a number of important papers on the subject.

I shall now attempt to give a brief account of Madhava Rao's own scientific work. The main topics covered by his research work are (a) Geometry of curves, (b) Analytical Dynamics, (c) Born-Infeld electrodynamics, (d) Elementary Particle Theory. The work under (a) constitutes his first entry into research work. In a number of papers published on this topic, he considered the properties of curves, in particular of cubic curves and proved a theorem known as "Madhava Rao's theorem on a polar cubic". His studies under (b) included investigation of invariant relations of dynamical systems, conditions for a general dynamical system to be of the canonical form, non-holonomic dynamical systems, and separable systems in classical and wave mechanics. We shall consider in some more detail his investigations under (c) and (d) which in fact constitute his major research contribution.

(c) Madhava Rao's work in Born-Infeld theory represents an important contribution to the subject. To appreciate the value of this work it would be perhaps useful to say a few words about what Born-Infeld electrodynamics is about. Every student of physics is familiar with the theory of the electromagnetic field based on the famous Maxwell-Lorentz field equations. This theory gives a very satisfactory explanation of the propagation of electromagnetic waves in space, the field produced by a given charge distribution and also includes a theory of emission of light. It is also well known that these equations are "Lorentz-invariant" i.e. retain their form under a Lorentz transformation which is a transformation from one coordinate system (x,y,z,t) to another (x',y',z',t') moving with constant velocity relative to the first. However difficulties arise in the problem of the inter-action of the field with elementary particles. For a point charge the energy of the field becomes infinite. We are thus forced to attribute a finite size $r_0 \sim \frac{e^2}{mc^2}$ (e, m = charge & mass of electron, c = velocity of light). Also the process of quantization of the field leads to the conclusion that for light of wave length << Compton wave length ($\lambda_0 = \frac{h}{mc}$) electron pairs can be treated even in vacuum. At very high energies the theory does not agree with experiment. The Born-Infeld theory is a modification of the Maxwell electrodynamics based on a different action integral but agreeing with the Maxwell theory for weak fields. The electron appears as a point singularity of the field which however involves no divergencies or other difficulties. Madhava Rao's contribution to this theory can be divided into two parts relating respectively to the physical and the mathematical aspects. In two papers, he investigated the ring-singularity model for the electron (in contrast to the point-singularity model) under certain suitable assumptions. The important conclusion that emerged from this investigation is that all models, except perhaps that of the point-singularity, are unsuitable for the representation of elementary particles. In the more mathematical papers, Madhava Rao deals with questions of Lorentz invariance, several types of representations of the theory (use of biquaternions, complex representation etc) and derives a very general form of the action integral (from which the field equations may be derived in the well-known way) including certain invariants. which had been neglected in previous treatments.

(d) Madhava Rao's interest in the subject of elementary particle theory began when H.J. Bhaba came to work at the Indian Institute of Science. The latter had done important research on meson theory and sought the collaboration of Madhava Rao in continuing work on this and other allied problems. In a joint paper on the scattering of charged mesons, they showed that with Bhaba's idea of allowing elementary particles to exist in all states of integral charge, the scattering of charged masons shows complex correspondence with the classical theory. Madhava Rao soon got interested in Dirac's relativistic wave equation of the electron.

$$\beta_u \partial_u \Psi + x \Psi = 0 \qquad (u = 1,2,3,4)$$

where ∂_u = the differential operator $\partial / \partial_x u$, Ψ is a 4 − component spinor (wave function) and β_u are 4 − rowed matrices which satisfy the "Commutation relations" $\beta_u \beta_v + \beta_v \beta_u = 2 \delta_{uv}$ ($\delta_{uv} = \begin{cases} 0, & u \neq v \\ 1, & u = v \end{cases}$)

the spin of the electron which had been *postulated* in order to explain spectroscopic data came out as a natural *consequence* of the Dirac equation, which was formulated so as to satisfy relativistic invariance. It later turned out that the particles of higher spin (integral or half-odd-integral) could also be described by a similar equation but with the β-matrices satisfying other appropriate commutation relations. The algebra of the original Dirac β-matrices and of those corresponding to higher spins was studied intensively by Madhava Rao. In the algebra of the Dirac β-matrices, he generalised certain algebraic identities of Pauli and also derived certain more generalised types of multi-linear and polynomial identities. In the cases of particles of spin 0 and 1, the commutation relations were given by Duffin and the algebra of the corresponding β-matrices was worked by Schrodinger and Kemmer. The latter also considered the algebra generated by *s* elements instead of the usual 4 only. The question naturally arises of deriving the commutation relations of β's for arbitrary spins and give them in a compact form corresponding to those in the cases of spin (½, 0, +1). Madhava Rao attacked this problem in a number of papers in which he developed a method for deriving the commutation relations for arbitrary spin assuming (i) the relativistic invariance of the wave equation (ii) that the spin operator s_{uv} has the special form $s_{uv} = \beta_u \beta_v - \beta_v \beta_v$ and (iii) that the eigen values of the spin operator are ± ½n (n integer). In the particular cases of spin 3/2 and 2, he was able to give the commutation relations in a single compact equation. The full algebra of the β-matrices in the case of spin 3/2 was worked out explicitly, showing that it is the direct product of the algebra of the Dirac matrices and another algebra. Results of possible physical application were also derived. In a later paper he studied the general case of *s* elements β_u by using the result that the representations of the β-algebra would be the same as those of the real orthogonal group of s + 1 dimensions and derived the number and order of the several irreducible representations of the generalised s-element β-algebra, corresponding to the spin ½n (n = integer)*

I have attempted to give some idea of the most important aspects of Madhava Rao's research. There is no doubt that his researches represent a significant contribution to the subjects dealt with.

* This work was carried out and published between 1942-1947. Yet in his article reviewing the subject published in the Rev. Mod. Phys. (1949), Bhabha omits to give any reference to Madhava Rao's work.

Ramanujan's score on the basis of pure talent

Paul Erdos has passed on to us Hardy's personal ratings of mathematicians. Suppose that we rate mathematicians on the basis of pure talent on a scale from 0 to 100. Hardy gave himself a score of 25, Littlewood 30, Hilbert 80 and Ramanujan 100.

B.C. Berndt

A superlatively great mathematician

Srinivasa Ramanujan was a mathamatician so great that his name transcends jealousies, the one superlatively great mathematician whom India has produced in the last thousand years.

Neville

CLASSROOM NOTES

1. A diophantine system

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Many problems on Diophantine Equations can be tackled by congruence consideration, and use of the well known formula for integer solutions of the Pythagorean equation

$$x^2 + y^2 = z^2$$

As an instance we prove here the following Theorem. The system of equations

$$7x^2 + 5x + 1 = y^2,\ 7x^2 + 8x + 2 = z^2 \qquad (1)$$

has no integer solutions.

Proof. We first observe that x cannot be even. For, if x were even, then z is even and the second equation in (1) gives

$$2 \equiv 0 \pmod 4$$

which is impossible. Hence x must be odd. Then y and z are both odd. From the first equation in (1) we have then

$$5x \equiv 1 \pmod 8$$

This congruence has the unique solution

$$x \equiv 5 \pmod 8 \qquad (1a)$$

Now we write the second equation in (1) as

$$z^2 + x^2 = 2(2x+1)^2 = 2w^2, \text{ say} \qquad (2)$$

Set $z = r + s$, $x = r - s$ where r and s are integers of opposite parity.

Substituting in (2) we get the Pythagorean equation

$$r^2 + s^2 = w^2 \qquad (3)$$

Here $(r, s) = 1$. For, if $(r, s) = d > 1$, then d is odd and, $d \mid z$, $d \mid x$, which, by the second equation in (1), imply $d \mid 2$, which is impossible, since d is odd > 1. By the well known formula for all integer solutions of (3) we have (assuming r to be even, and s odd as we can do in view of symmetry in r, s)

$$r = 2RS,\ s = R^2 - S^2,\ w = R^2 + S^2$$

where R and S are integers of opposite parity, $(R, S) = 1$.

Thus, since $w = 2x + 1$, we have

$$2x + 1 = R^2 + S^2 \qquad (4)$$

Supposing R odd, S even and considering mod 8 we have from (4), in view of (1a),

$$10 + 1 \equiv 1 \text{ or } 4 \pmod 8 \qquad (5)$$

according as $S \equiv o \pmod 4$ or $S \equiv o \pmod 2$.

In either case, the congruence (5) is impossible. This proves the theorem.