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Chief Editor:

**Prof. F. J. NORONHA, M.A., (Lond.)**

Head of the Department of Mathematics  
Bangalore University.

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# THE MATHEMATICS NEWSLETTER

*Published by the Mathematical Society, Central College, Bangalore-1.*

*Chief Editor : Prof. F. J. NORONHA, M A., (Lond.)*

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## OUR NEW VICE-CHANCELLOR

We offer our hearty congratulations to Dr. H. Narasimhaiah, on his appointment as Vice-Chancellor, Bangalore University. He took charge of this responsible post on the 15th December, 1972.

Dr. Narasimaiah has been the Principal of National College for the past eleven years. A former student of the Central College where he studied for his B.Sc. (Hons) and M.Sc. degrees in Physics, he later proceeded to the Ohio University, U.S.A., where he took his Ph. D. in Nuclear Physics.

Dr. Narasimhaiah is also a keen student of Mathematics and is well aware of the importance of Mathematics for the study of Physics. We are happy to state that he is a regular contributor to the Mathematics Newsletter.

We wish him all success in his new post.

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# PROFILE

PROF. DR. C. N. SRINIVASA IYENGAR

Dr. C. N. Srinivasa Iyengar, Retired Professor of Mathematics, passed away at Bangalore on Thursday September 21, 1972 after a brief illness.

He was born on February 21, 1900, He lost his mother when he was a child and was brought up by his poor father. His educational career was outstanding. He secured the third rank for the Mysore State in his Matriculation examination, and this earned him a Scholarship of Rs. 18/- per month which helped him to pursue his studies. In 1920, he passed B Sc., Degree Examination with major Mathematics in First Class with Distinction. As there was no opportunity in Central College for M.Sc. Degree in Mathematics, he went to Calcutta University for M.Sc. The Mysore Government sanctioned Rs. 50/- per month as scholarship to enable him to continue his studies at Calcutta. He completed his M.Sc. Degree Examination in 1922 in First Class securing the First Rank of the Calcutta University with record percentage of marks.

In 1923, he was appointed as a Lecturer in mathematics in Maharaja's College at Mysore. He was promoted as an Assistant Professor in 1924 and posted to the Mathematics Department, Central College, Bangalore. He was a very popular teacher and had won the regard of his students. In 1955, he retired from the University of Mysore as Professor of Mathematics and the Principal of Central College. He accepted the post of Professor of Mathematics at Dharwar in 1953 and for a period of five years he worked very hard to build up the Postgraduate and the Research Departments of Mathematics of the Karnatak University. He retired from Karnatak University in 1963 and worked as U.G.C. Professor till 1967, the last year of his service being at the Central College.

Calcutta University awarded him D.Sc. in 1932 for his thesis entitled "Singular solutions of differential equations". He has contributed more than 15 Research Papers mostly in Geometry, to standard journals. Under his able guidance, many students have taken their doctorate degrees. He is the author of many books, a few of which are: (i) History of Ancient Indian Mathematics; (ii) Theory of Infinite Series in collaboration with Prof. P. L. Bhatnagar; and some more books in Kannada, the Regional language of Mysore State.

He was elected as the President of the Indian Mathematical Society in 1962 and his president speech at Waltair was on Geometry - Congruence.

Apart from his academic achievements which are distinct, he was responsible to build up a Ramamandir for Rama Bhajan at Malleswaram, Bangalore for promoting cultural and religious activities. He was proficient in Sanskrit and Kannada, being an ardent worshipper of Lord Shri Rama, he took up the great task of literally translating the original Valmiki Ramayana in Sanskrit to Kannada and released it for public in two volumes on Saturday February 19, 1972 in a colourful function.

## On the Art of Solving Mathematical Problems

by Dr. B. S. MADHAVA RAO

### 1. Inferential puzzles :—

Such puzzles consist of some given data from which definite conclusions have to be drawn purely by inferential methods. In very complicated puzzles of this type, use has to be made of Boolean Algebra and the notion of incidence matrices. Below is a simple example :—

Three men  $A_1$ ,  $B_1$  and  $C_1$  are married to A, B and C but not necessarily in that order. Each couple has one child, a boy  $A_2$ , a girl  $B_2$  and a boy  $C_2$ . Given that

- (i) B's child and C's child are two stars of the school football team ;
- (ii) A's son is not  $A_2$  ;
- (iii) B's wife is not  $C_1$ .

find who are the parents of each child.

*Solution* :—(a) From (ii) A's son is  $C_2$ , who from (i) must be  $B_1$ , and their child is  $C_2$ .

(b) C's child is  $A_2$  from (i) and from (iii) and (a) C's wife must be  $C_1$ , and their child is  $A_2$  ;

(c) From (a) and (b) it follows that B's wife is  $A_1$  and their child is  $B_2$ .

### 2. The following story about Gauss is due to E. T. Bell :

Gauss was a student about 10 years old in an elementary school run by a teacher one Buttner described as a virile brute. The formula he gave to the class one day was to find the sum,

$$81,297 + 81,495 + 81,693 + \dots + 100,899,$$

*i.e.* an A.P. with a common difference = 198 and number of terms = 100. Hardly had the other boys taken out their slates, Gauss wrote down the correct answer on his slate and was asked by the teacher to put it on the table.

**3. The calender trick problem :** This is discussed completely in the Issues of Scientific American, August, 1965 and May, 1967. The following extracts on the calender-calculating twins, however, worth recording here. This relates to two 24 year old twins (in 1965) named Charles and George who spent 13 years at a State Mental Institute in U.S.A., and were transferred in 1963 to a Psychiatric Institute for close observation. Their normal

I. Q. was so low that they could not add, subtract, multiply or divide single-digit numbers. Their self-taught calendar calculations, however, go far beyond the range of any hitherto reported. Although Charles is accurate only for this century, George can project the calendar calculations over a range of at least 6000 years, for example he can instantaneously identify February 15th, 2002 as a Friday or 23-8-1591 as a Wednesday. When asked in what years April 21 falls on a Sunday both will correctly answer 1968, 1957, 1953, 1946, and so on. George, when encouraged, can go back as far back as 1700. When asked in what month of the year 2002 will the first of the month fall on Friday, George gave February, March and November, all correct answers. Although both are severely myopic, there is no evidence that their common skills are genetically linked.

The fact that both twins operate in a range far beyond that of the usual 200 to 400 perpetual calendar, makes it doubtful that their feats involve only memory. Moreover they operate so rapidly that they use no formula, even if they are capable of learning one. The investigators write that they have no better explanation for the observed phenomenon than the one offered by the twins themselves, who answer the question of how they perform by saying "I know", or "It is in my head."

It is to be hoped that the above account of the two twin "idiot savants" will prevail on people from contributing any articles hereafter in the Mathematics Newsletter on the topic.

#### 4. A Problem due to Newton :-

If 12 oxen eat up 3 $\frac{1}{2}$  acres of pasture in 4 weeks and 21 oxen eat up 10 acres of like pasture in 9 weeks, to find how many oxen will eat up 24 acres in 18 weeks.

*Solution* : Let  $\alpha$  = quantity of grass per acre when pasture starts to be used ;  
 $\beta$  = quantity of grass eaten by one ox in one week ;  
 $r$  = quantity of grass that grows on one acre in one week.

$a_1, a_2, a$  the number of oxen,  $m_1, m_2, m$  the number of acres, and  $t_1, t_2, t$  the numbers of weeks in the three cases considered, then the conditions are

$$m_1 (\alpha + t_1 r) = a_1 t_1 \beta$$

$$m_2 (\alpha + t_2 r) = a_2 t_2 \beta$$

$$m (\alpha + t r) = a t \beta$$

( $\alpha, \alpha, \beta$  and  $r$  are unknown, the remaining 8 quantities are numerically given). The system for the three unknowns  $\alpha/\beta, r/\beta$  and  $a$  yields

$$a = \frac{m \{m_1 a_2 t_2 (t - t_1) - m_2 a_1 t_1 (t - t_2)\}}{m_1 m_2 t (t_2 - t_1)}$$

and with the numerical data  $a = 36$ .

### 5. Remarks about problem solving

(a) The best method of attempting to solve a problem is first to write down Q.E.D., for this act of writing down the traditional phrase which comes at the very end of proof puts the problem solver in the right mood ! This is also in accord with the proverb that " A wise man begins in the end, and a fool ends in the beginning ".

(b) Divide each problem that you want to solve into as many parts as you can, and as you need to solve them more easily — Descartes.

The above rule of Descartes is of little use as long as the art of dividing remains unexplained. By dividing the problem into unsuitable parts, the problem solver may increase his difficulty — Leibnitz.

(c) Re choice between several plans in problem solving :— Do not commit yourself too early, do not commit yourself to one course more than necessary, do one thing, but do not forget the others. A good problem solver plans like a good general ; he realises that the attack planned may fail and does not neglect the line of retreat ; his plan has some built-in flexibility, some adoptibility to unforeseen difficulties.

(d) Other important factors in problem solving :— Having a problem, mobilisation and organisation, recognising and remembering, supplementing and regrouping, isolation and combination, working from inside and working from outside, acquiring relevant knowledge ability to express oneself in mathematical language. A well stocked and well organised store of knowledge is a great asset to a problem solver ; modesty and humility when you succeed in solving a problem are quite essential in tackling the next one.

(e) Coming of the idea in solving a problem :— Ideas depend on chance. Ideas come when they want to come and not when we want them to come. Waiting for ideas is like gambling. The following witty lines about ability in solving problems being only due to chance are quite interesting :

“ All the inventions that the world contains,  
Were not by reason first found out, nor brains,  
But pass for theirs who had the luck to light  
upon them by mistake or oversight.”

### 6. Remarks about teaching and talking at mathematical meetings :—

Teaching is not a science, but an art like music, for example, saying a thing once, twice or thrice but not too many times. One has to learn teaching before actually undertaking it. The teacher's attitude and his knowledge of the ability of his pupils are essentials for good teaching. Do not teach too many things, but remember Voltaire's remark viz. " The art of being a bore is to tell everything ". Also try to follow the advice of Anatole France, who looked at all things human with a delicate irony mixed with

pity, viz. "Do not try to satisfy your vanity by teaching them great many things. Awaken their curiosity. It is enough to open their minds, do not overload them. Put there just a spark. If there is some good inflammable stuff, it will catch fire."

Regarding talking at mathematical meetings, the situation resembles a little, and differs a great deal from that of a teacher in a class room. The audience may consist of his peers and possibly his superiors and not all students. The situation is not easy and also the performance is not often successful. It is not his fault, but rather of the vast expanse of mathematics. Any one mathematician can only master only a small parcel of present day mathematics and very often knows very little about the other small parcel which the next mathematician has mastered.

In this connection, it is worth recording some remarks of Zermelo, famous for his "axiom of choice" in set theory. He liked to spend some time in coffee houses and his conversation while there was interspersed with caustic remarks about his colleagues. His remarks were very witty, very often unjust, but striking and revealing some important points. Commenting on a particular address at some meeting of the Mathematical Society of his University which had a great success, he criticised the speaker's style and dress, and remarked that the speaker might well have brought a sword with him to impress the audience still more. He eventually condensed his disapproval into *two rules* according to which, he mockingly asserted that the address must have been constructed :

- (a) You cannot overestimate the stupidity of your audience.
- (b) Insist on the obvious, and glide nimbly over the essential.

These rules, suitably interpreted, often give sound advice. Thus for eg. a speaker usually treats his audience as if everybody knew everything about each detail of his latest paper, or of his book published decades ago. Of course just the contrary is the case, and the speaker should realise it. It would be better for him to overestimate rather than underestimate the lack of knowledge of the audience about the subject. Thus, Zermelo's first rule is great profit. Again it is hardly possible to present adequately all details of a long and difficult proof at a mathematical meeting. Yet, even such proof may hinge on a central remark which in itself is intuitive and simple. The speaker should not glide nimbly over this essential point, and this is the essence of Zermelo's second rule.

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The enthusiastic algebraist finds the study of Lie Algebras amply rewarding for their own sake. But in the long run the tail cannot wag the dog.

— Irving Kaplansky

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## Fibonacci Numbers and the Golden Ratio

By

JOSEPH D'MELLO, National College

Consider a relation of the form  $x_n = Pp^n + Qq^n$ , where  $n$  is a positive integral variable and  $P, Q, p$  and  $q$  certain constants.

$$\text{Hence, } x_{n-2} = Pp^{n-2} + Qq^{n-2} \dots \dots \dots \text{(i)}$$

$$x_{n-1} = Pp^{n-1} + Qq^{n-1} \dots \dots \dots \text{(ii)}$$

$$x_n = Pp^n + Qq^n \dots \dots \dots \text{(iii)}$$

We can solve (i) and (ii) as simultaneous equations in  $P$  and  $Q$ . We find that

$$P = \frac{x_{n-1} - qx_{n-2}}{p^{n-2}(p-q)} \text{ and } Q = \frac{px_{n-2} - x_{n-1}}{q^{n-2}(p-q)}$$

Substituting these values of  $P$  and  $Q$  in (iii) and simplyfying, the following result is arrived at ;

$$x_n = x_{n-1}(p+q) - x_{n-2}(pq)$$

Hence it is plain that the sequence of terms  $x_0, x_1, x_2, \dots$  is one in which each term is (linearly) dependent on its two immediate predecessors. The relation

$$x_n = x_{n-1}(p+q) - x_{n-2}(pq)$$

is an example of recurrence relation and the equation expressing this is often called a difference equation (in this case a 2nd order difference equation) We obtain a particularly interesting sequence when we assume  $p+q=1$  and  $pq=-1$  i.e.  $p$  and  $q$  are the roots of the equation  $y^2 - y - 1 = 0$ . Then it is readily seen that  $x_n = x_{n-1} + x_{n-2}$ . We further take  $x_0 = 1$  and  $x_1 = 1$ , where  $x_0$  and  $x_1$  are respectively the first and second terms of the sequence. Note that this choice of  $x_0$  and  $x_1$  is purely a mater of discretion. When  $x_0$  and  $x_1$  have these values, we arrive at the sequence

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 \dots \dots \dots$$

This sequence is called the Fibonacci sequence, after its discoverer or inventor.

Since  $p$  and  $q$  satisfy  $y^2 - y - 1 = 0$ , we take  $p = \frac{1 + \sqrt{5}}{2}$  and  $q = \frac{1 - \sqrt{5}}{2}$

Since  $x_n = Pp^n + Qq^n$ , we write  $x_n = P \left( \frac{1 + \sqrt{5}}{2} \right)^n + Q \left( \frac{1 - \sqrt{5}}{2} \right)^n$

When  $n=0$ ,  $x_0 = 1 = P + Q$

$$\text{When } n=1, x_1 = 1 = \frac{1}{2}(P+Q) + \frac{\sqrt{5}}{2}(P-Q) \left. \vphantom{\text{When } n=1} \right\} \therefore x_n = P \left( \frac{1 + \sqrt{5}}{2} \right)^n + Q \left( \frac{1 - \sqrt{5}}{2} \right)^n$$

From these two equations we obtain

$$P = \frac{1 + \sqrt{5}}{2\sqrt{5}} \text{ and } Q = \frac{\sqrt{5} - 1}{2\sqrt{5}}$$

$$\text{Hence } x_n = \left(\frac{1 + \sqrt{5}}{2\sqrt{5}}\right) \left(\frac{1 + \sqrt{5}}{2}\right)^n + \left(\frac{1 - \sqrt{5}}{2}\right)^n \left(\frac{\sqrt{5} - 1}{2\sqrt{5}}\right)$$

$$\therefore x_n = \frac{(1 + \sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}} - \left(\frac{1 - \sqrt{5}}{2}\right)^{n+1} \frac{1}{\sqrt{5}}$$

Since  $\left|\frac{1 - \sqrt{5}}{2}\right| = 0.68 \dots < 1$  and  $\frac{1}{\sqrt{5}} = 0.4472 \dots < 1$ , it follows that the value of the term  $\left(\frac{1 - \sqrt{5}}{2}\right)^{n+1} \frac{1}{\sqrt{5}}$  is indeed small when compared to 0.5. Hence we can neglect it and say, without any loss of precision that  $x_n$  is an integer nearest to

$$\frac{(1 + \sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}}$$

It will be seen that this Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 ..... has many fascinating properties. Remember that  $x_n$  corresponds to the term in the  $(n+1)$ th place in the sequence and NOT the  $n^{\text{th}}$ , since we have assumed  $x_0$  to be the first term.

Here are some properties which can easily be verified.

$$[x_0 = 1, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 5 \dots \dots \dots]$$

1. It can be shown by induction that any two consecutive terms of the sequence are mutually prime. (except  $x_0$  and  $x_1$ )
2.  $\sum_{r=0}^n x_r = x_{n+2} - 1.$
3.  $x_m \mid x_r \iff m + 1 \mid r + 1$
4.  $\mid x_n^2 - x_{n-1} x_{n+1} \mid = 1$
5.  $\mid x_n \cdot x_{n+4} - x_{n-2} \cdot x_{n+3} \mid = 2$
6.  $\mid x_n \cdot x_{n+1} - x_{n-1} \cdot x_{n+2} \mid = 1$
7. Squaring each term of the series we have 1, 1, 4, 9, 25, 64, 169, 441, 1156..... adding in pairs we obtain, 2, 5, 13, 34, 89, 233.....  
i.e.  $x_2, x_4, x_6 \dots \dots x_r \dots \dots$  where 'r' is an even integer.
8.  $x_{2n+1} = x_{n+1}^2 - x_n^2$
9.  $(2x_{n+1} \cdot x_{n+2})^2 + (x_n \cdot x_{n+3})^2 = x_{2n+4}^2$
10. If three consecutive terms of the sequence, say  $x_n, x_{n+1}$  and  $x_{n+2}$  are taken then  $x_{n+1}^3 + x_{n+2}^3 - x_n^3$  is also a term of the sequence, In fact  $x_{n+1}^3 + x_{n+2}^3 - x_n^3 = x_{3n+5}$ .

11. Let  $R_n = \sum_{r=0}^{n-1} x_r^2 = x_0^2 + x_1^2 + x_2^2 + \dots + x_{n-1}^2$ .

Then  $R_n$  forms the sequence  $\{1, 2, 6, 15, 40, 104, 273, 714, 1870, \dots\}$

Let  $h_n$  be the highest common factor of  $R_n$  and  $R_{n+1}$

Then  $h_n$  also forms the sequence  $\{1, 2, 3, 5, 8, 13, \dots\}$  given by the recurrence relation  $h_n = h_{n-1} + h_{n-2}$ .

Also, dividing  $R_n$  by  $h_n$  we obtain the sequence,  $\{1, 1, 2, 3, 5, 8, 13, \dots\}$  or the Fibonacci sequence itself.

12. Consider the Fibonacci sequence  $\{1, 1, 2, 3, 5, 8, 13, 21, \dots, t_n, \dots\}$

and the arithmetic progression  $\{1, 2, 3, 4, 5, 6, 7, 8, \dots, s_n, \dots\}$

Then the sequence  $s_n t_n$  is  $\{1, 2, 6, 12, 25, 48, 91, 168, \dots, s_n, \dots\}$

Let  $E_n = s_n - (s_{n-1} + s_{n-2})$ . Then  $E_n$  forms the sequence  $\{3, 4, 7, 11, 18, 28, \dots, E_n, \dots\}$  also a sequence satisfying the recurrence relation  $E_n = E_{n-1} + E_{n-2}$ .

Note: This result will hold good for any two sequences  $t_n$  and  $s_n$ , one satisfying recurrence relation  $t_n = t_{n-1} + t_{n-2}$  and the other satisfying the relation  $s_n = s_{n-1} + k$  where  $k$  is a constant.

13. Finally we shall consider a very impressive result. We compute

$\frac{x_{n+1}}{x_n}$  (or  $x_{n+1} : x_n$ ) for  $n \geq 1$ . The table gives the values upto 3 decimal places for the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, .....

$x_1 : x_0 =$	1 :	1 = 1.000
$x_2 : x_1 =$	2 :	1 = 2.000
$x_3 : x_2 =$	3 :	2 = 1.500
$x_4 : x_3 =$	5 :	3 = 1.667
$x_5 : x_4 =$	8 :	5 = 1.600
$x_6 : x_5 =$	13 :	8 = 1.625
$x_7 : x_6 =$	21 :	13 = 1.615
$x_8 : x_7 =$	34 :	21 = 1.619
$x_9 : x_8 =$	55 :	34 = 1.618
$x_{10} : x_9 =$	89 :	55 = 1.618
$x_{11} : x_{10} =$	144 :	89 = 1.618
$x_{12} : x_{11} =$	233 :	144 = 1.618
$x_{13} : x_{12} =$	377 :	233 = 1.618

Notice that  $\frac{x_{n+1}}{x_n}$  converges to 1.618. Can we amount

for this? Yes, we know that  $x_{n+1} = \frac{(1+\sqrt{5})^{n+2}}{2^{n+2}\sqrt{5}}$  and

$$x_n = \frac{(1+\sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}}$$

$$\text{Hence } \frac{x_{n+1}}{x_n} = \frac{1+\sqrt{5}}{2} = \frac{1+2.236}{2} = \frac{3.236}{2} = 1.618$$

which agrees with what we have obtained. This result, in fact, is true for any sequence satisfying the recurrence relation  $x_n = x_{n-1} + x_{n-2}$ . In fact most, though not all, of the above properties hold for any such sequence.

Lastly, if  $P_n = \sum_{r=0}^{n-1} x_r = x_0 + x_1 + x_2 + \dots + x_{n-1}$ , the fact

that  $P_{n+1}/P_n$  also converges to the value  $\frac{1+\sqrt{5}}{2}$  follows directly from property 2 above i.e.

$\sum_{r=0}^{n-1} x_r = x_{n+2} - 1$ . The value  $\frac{\sqrt{5}+1}{2}$ , called phi ( $\phi$ ) and is known as the Golden Ratio.

★ - Properties 9 and 10 are due to Professor Jekuthiel Ginsburg of Yeshiva University.

The subjects to be included are: Real and Complex analysis, Analytical Geometry, Abstract Algebra, Linear Algebra and Mathematical methods including vector Analysis and Differential Equations. The first two years of the course will be spent in the first two years. In the final year students will have the option of selecting two subjects from a set of about six, including (i) Real and complex analysis (ii) Elementary Topology and Functional Analysis (iii) Abstract Algebra (iv) Mathematical methods (v) Numerical Analysis and (vi) Probability theory and statistics.

## LEADERSHIP PROJECT IN MATHEMATICS BANGALORE UNIVERSITY REPORT

The above project commenced in April 1971. This project is sponsored by the University Grants Commission who have sanctioned a sum of Rs. 3,36,000, for the purpose. The Bangalore University is one of the five Universities in India selected for a Leadership Project in Mathematics, the other four being the Universities of Punjab, Gujarat, Madurai, and Meerut.

The main aim of the Project is to raise the standard of mathematical education at the undergraduate level in all colleges of the University. The following are the areas in which work is being done:

- (i) Improvement of Curriculum and modernization of syllabus;
- (ii) Improvement of teaching methods and development of teaching aids;
- (iii) Examination reforms.
- (iv) Preparation of teacher's guides and textbooks.

(v) Improvement of the subject matter competence of teachers

(vi) Providing good textbooks and reference libraries to all colleges.

**Procedures adopted** This University has the advantage that all its Colleges are situated within a distance of atmost five miles from the University Department. It is therefore possible to involve all college teachers of mathematics in the work of the project. There are about 60 teachers in the 16 colleges in which mathematics is taught at the undergraduate level. Monthly meetings are held which are attended by all these teachers. At each meetings the progress of the work done during the previous month is reviewed and the future programme planned. Every teacher has full opportunity to express his view and also to serve on various sub-committees. Each teacher may also join one of the study group that meet regularly throughout the year.

### I Improvement of Curriculum and Modernisation of Syllabus.

This work has been entrusted to a committee consisting of a representative from each constituent college and some members of staff of the University department. After studying the syllabi of several universities in India and abroad as well as the recommendations of the committee on the undergraduate programme in Mathematics in U.S.A., the committee laid down guide lines on the content of the syllabus in Mathematics for the new B.Sc.,

course. The subjects to be included are: Real and Complex analysis, Analytical Geometry, Abstract Algebra, Linear Algebra and Mathematical methods including vector Analysis and Differential equations. There will be a common syllabus for all students in the first two years. In the final year students will have the option of selecting two subjects from a set of about six, including (i) Real and complex analysis (ii) Elementary Topology and Functional Analysis (iii) Abstract and Linear Algebra, (iv) Mathematical methods (v) Numerical Analysis and (vi) Probability theory and statistics.

The details of the curriculum in each subject were worked out by a sub-committee consisting of six to ten members. The topics in each subject have been spelt out in detail including the number of hours to be allotted to each topic. List of text books and books of reference have also been included.

The committee drew a first draft of the proposed new syllabus. This has been modified in the light of criticisms and suggestions thrown out at general body meetings. The final draft is under preparation based on the number of hours per week to be allotted for mathematics in the new B.Sc. course. It is expected that the curriculum will be duly approved by the Board of Studies in Mathematics and the Academic Council of the University. This will be the first time that the B.Sc. syllabus is drawn up by college teachers actively engaged in teaching mathematics at the undergraduate level. This is a welcome new feature made possible by the Leadership Project.

## II Improvement of teaching methods and development of teaching aids.

"The best way to learn Mathematics is to do mathematics". The Leadership Project has sent proposals to the university for the introduction of problems-working classes in mathematics in all colleges. Each such class will consist of a small batch of about 20 students working problems under the guidance of a teacher. The students will thereby have ample opportunity to solve their difficulties, to engage in discussion and obtain assistance from the teacher.

The problems working classes will correspond to practical classes in other science subjects. The teaching of mathematics will thus be brought into line with the teaching of other science subjects. This proposal is under the consideration of the university and, if implemented, is expected to bring about a significant improvement in the teaching-learning process in mathematics. In this connection it may be observed that such classes are in vogue in the U.S.A., West Germany and many other countries. In America they are sometimes called "Lab. Sessions."

## III Examination Reforms.

A new type of question paper was introduced in the pre-university and I B.Sc. examinations. Each question paper consists of two parts A and B. Part A comprises a fairly large number of questions of the short answer objective type, covering the entire syllabus.

In this part there is no choice. Hence a candidate who aims at scoring maximum marks must necessarily study all topics in the syllabus and answer correctly all questions in part A. This provides a good challenge to intelligent students. It also motivates both teachers and students to cover the syllabus completely in the classroom.

Part B consists of two or three questions of the 'traditional' essay-type, with internal choice in each question.

The Leadership Project is also aiming at compiling a set of good, objective-type, thought-provoking questions which will be supplied to teachers for use in the class room. Teachers are encouraged to give short tests at regular intervals to help students understand the subject and to stimulate their interest in mathematics.

#### IV Preparation of Teachers' Guides and Text-Books.

During the first year of the project several monographs were written on subjects where textbooks were not readily available to students. The topics included in these monographs are: Finite Differences, Errors, Difference Equations, Linear Programming, Theory of Groups and Number theory. The monographs were printed and supplied free to all teachers and student concerned.

The project is now engaged in writing text-books. Text books on Real Analysis, Abstract Algebra and Numerical Analysis are in the Press and expected to be released shortly. These will be available at cost price to students. Each text-book has been written by a small group of college teachers who are actively engaged in teaching the subject. It is hoped that these text books will make a significant contribution in improving the teaching and learning of Mathematics at the B.Sc., level.

#### V Improvement of the subject matter competence of teachers.

Refresher courses have been held during vacations. In May-June 1972 a Summer Institute was conducted for college teachers in which there were lectures on Abstract Algebra, Topology, Functional Analysis and Probability theory. Most of the college teachers have attended these refresher courses and have benefitted considerably (thereby).

Study groups have also been formed which meet regularly every week. A group of teachers take up the study of a particular subject following a chosen text-book. So far the subjects chosen have been Abstract Algebra, Topology, Functional Analysis, Real Analysis Numerical Analysis and Mathematical Methods.

To improve their subject matter competence, opportunities are afforded to college teachers to deliver lectures on topics of their choice to the students of the M.Sc. classes. At present Sri B. G. Jayaram of St. Joseph's college is delivering a course of lectures on Elementary Functional Analysis to the Sr. M.Sc. class.

**VI Strengthening the Libraries of Colleges**

Books to the value of Rs. 2000 are being added each year to the library of each constituent college. These are carefully selected and the colleges have been supplied with the best available text books on Modern Algebra, Real and Complex Analysis, Topology, Functional Analysis and Mathematical Methods.

**Mathematical Contests**

These are held to encourage undergraduate students to study modern Mathematical subjects outside the regular syllabus. The first contest held in February 1972 was a great success and nearly 100 students from various colleges participated. The contest consists of two written papers one on set topics and the other consisting of general questions involving mathematical intuition and skill.

Under the Leadership Project, the University Department of Mathematics has acquired a Film Projector and a "Mathematical" Typewriter. We also hope to get an overhead projector which will be an invaluable aid to teaching.

The Leadership project has generated considerable enthusiasm among the teachers of Mathematics in the constituent colleges. We hope to introduce a new curriculum in 1973-74 with suitable text-books and other materials. Our thanks are due to the University Grants Commission for giving us the opportunity and facilities to raise the standard of Mathematical education at the undergraduate level.

**Prof. F. J. NORONHA,**  
**Coordinator**

**V Improvement of the subject matter competence of teachers.**

Refresher courses have been held during vacations. In May-June 1972 a Summer Institute was conducted at the University of Madras. Most of the college teachers have attended these refresher courses and have benefited considerably (hereby).

Progress in Mathematics does not depend upon the maximum generality but on hitting upon the right generality.

— SAUNDERS MAC LANE

To improve their subject matter competence, opportunities are afforded to college teachers to deliver lectures on topics of their choice to the students of the M.Sc. classes. At present Sri B. G. Jayaram of St. Joseph's college is delivering a course of lectures on Elementary Functional Analysis to the St. M.Sc. class.

## On the particular Integral,

$$\frac{1}{f(D)} x^m \quad (m \text{ being a positive integer})$$

The particular integral in this case usually found by expanding  $[f(D)]^{-1}$  in powers of  $D$  or by the method of undetermined coefficients. Here is an easier method, the method of 'long division,' which proves very handy when large powers of  $x$  are involved. I illustrate the method by two examples.

(1)  $\frac{x^3}{D^2+D+1}$ . The 'division' of  $x^3$  by  $1+D+D^2$  is as shown.

$$\begin{array}{r}
 x^3 - 3x^2 + 6 \\
 \hline
 1 + D + D^2 \quad x^3 \\
 \quad x^3 + 3x^2 + 6x \quad \{ = x^3 + Dx^3 + D^2 x^3 \} \\
 \hline
 \quad \quad -3x^2 - 6x \\
 \quad \quad -3x^2 - 6x - 6 \quad \{ = -3x^2 + D(-3x^2) + D^2(-3x^2) \} \\
 \hline
 \quad \quad \quad \quad 6 \\
 \quad \quad \quad \quad 6 \quad \{ = 6 + D(6) + D^2(6) \} \\
 \hline
 \quad \quad \quad \quad \quad -
 \end{array}$$

The Particular Integral is  $x^3 - 3x^2 + 6$

(2)  $\frac{x^3 + x^2 + 2x + 1}{D^3 + D^2 - 2D + 3}$ . In this case, we avoid fractions by multiplying and dividing by 3.

$$\frac{1}{3} \frac{3x^3 + 3x^2 + 6x + 3}{D^3 + D^2 - 2D + 3}$$

The 'division' is as shown :-

$$\begin{array}{r}
 x^3 + 3x^2 + 4x - \frac{1}{3} \\
 \hline
 3 - 2D + D^2 + D^3 \quad 3x^3 + 3x^2 + 6x + 3 \\
 \quad \quad \quad 3x^3 - 6x^2 + 6x + 6 \\
 \hline
 \quad \quad \quad \quad 9x^2 \quad - 3 \\
 \quad \quad \quad \quad 9x^2 - 12x + 6 \\
 \hline
 \quad \quad \quad \quad \quad 12x - 9 \\
 \quad \quad \quad \quad \quad 12x - 8 \\
 \hline
 \quad \quad \quad \quad \quad \quad - 1 \\
 \quad \quad \quad \quad \quad \quad - 1 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad -
 \end{array}$$

Particular integral is  $\frac{1}{3} (x^3 + 3x^2 + 4x - \frac{1}{3})$

S. SUNDARARAJ,  
National College.

## Solution of Homogeneous Differential Equations by Contour Integrals

Let  $a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = 0 \dots (1)$  be an  $n^{\text{th}}$  order Linear Differential Equation ; where  $y^{(k)} = \frac{d^k y}{dx^k}$  ( $k=0, 1, 2, \dots, n$ ),  $a_0 \neq 0$  and  $y$  is  $n$  times continuously differentiable function over the Real Line I. Suppose we wish to seek a solution in the form  $y(x) = \oint_c k(x, t) f(t) dt \dots (2)$  where  $k(x, t)$  is called the 'Kernel' Suppose we take  $k(x, t) = e^{xt}$  then  $y = \oint_c e^{xt} f(t) dt \dots (3)$ . Such a solution may occur only when the co-efficients in (1) are Rational functions.

We have from (3)  $y' = \oint_c t e^{xt} f(t) dt$ ,  $y'' = \oint_c t^2 e^{xt} f(t) dt \dots y^{(n)} = \oint_c t^n e^{xt} f(t) dt$ . Substituting these values in (1) we get  $\oint_c e^{xt} \{a_0 t^n + a_1 t^{n-1} + \dots + a_n\} f(t) dt = 0$ .

This is satisfied if  $f(t) = \frac{1}{a_0 t^n + a_1 t^{n-1} + \dots + a_n}$

In which case  $y = \oint_c \frac{e^{xt} dt}{a_0 t^n + a_1 t^{n-1} + \dots + a_n} \dots (4)$

Let  $t_1, t_2, \dots, t_n$  be the simple poles of (4). Then let  $C$  be so chosen such that all these poles lie inside  $C$ .

Let  $a_0 t^n + a_1 t^{n-1} + \dots + a_n = (t-t_1)(t-t_2) \dots (t-t_n)$  (say) in which case

$$y = \oint_c \frac{e^{xt} dt}{(t-t_1)(t-t_2) \dots (t-t_n)}$$

Now Residue at  $t_1 = \lim_{t \rightarrow t_1} (t-t_1) \frac{e^{xt}}{(t-t_1) \dots (t-t_n)}$

$$= \frac{e^{x t_1}}{(t_1-t_2)(t_1-t_3) \dots (t_1-t_n)} = \lambda_1 e^{x t_1}$$

where  $\lambda_1$  is a constant

Similarly we can get Residue at  $t_2, t_3, \dots, t_n$  say  $\lambda_2 e^{x t_2}, \dots, \lambda_n e^{x t_n}$ . Now the value of the Integral (4) is equal to  $2\pi i$  times sum of Residues.

i.e.  $2\pi i (\lambda_1 e^{x t_1} + \lambda_2 e^{x t_2} + \dots + \lambda_n e^{x t_n})$

$\therefore y = A e^{x t_1} + B e^{x t_2} + \dots + P e^{x t_n}$  is the required solution of (1).

Let us illustrate this by taking some examples.

(1) To solve :  $y''' - 4y' = 0$  ;

$$\text{Let } y = \oint_c e^{xt} f(t) dt \quad \therefore y' = \oint_c t e^{xt} f(t) dt, \quad y''' = \oint_c t^3 e^{xt} f(t) dt$$

$$y''' - 4y' = \oint_c e^{xt} \{t^3 - 4t\} f(t) dt = 0.$$

$$\text{This is satisfied if } f(t) = \frac{1}{t^3 - 4t} = \frac{1}{t(t^2 - 4)}$$

$$\therefore y = \oint_c \frac{e^{xt} dt}{t(t^2 - 4)}$$

Poles are  $t=0$ ,  $t=+2$ ,  $t=-2$ . Let us calculate Residues at each of these values of  $t$ .

$$\text{Residue at } t=2 \text{ is } \lim_{t \rightarrow 2} (t-2) \frac{e^{xt}}{t(t-2)(t+2)} = \frac{e^{2x}}{8}$$

$$\text{Similarly Residue at } t=-2 \text{ is } \frac{e^{-2x}}{8} \text{ and}$$

Residue at  $0 = -\frac{1}{4}$ . Hence the value of

$$\begin{aligned} \oint_c \frac{e^{xt} dt}{t(t^2 - 4)} &= 2\pi i \left\{ -\frac{1}{4} + \frac{e^{2x}}{8} + \frac{e^{-2x}}{8} \right\} \\ &= A + B e^{2x} + C e^{-2x} \end{aligned}$$

Hence  $e^{2x}$ ,  $e^{-2x}$  and a constant ( $\lambda$ ) are 3 solutions  $y''' - 4y' = 0$ . Infact these are 3 Linearly Independent Solutions. For, the 'Wronskian' of these 3 solutions is equal to

$$\begin{vmatrix} \lambda & e^{2x} & e^{-2x} \\ 0 & 2e^{2x} - 2e^{-2x} & \\ 0 & 4e^{2x} & 4e^{-2x} \end{vmatrix} = 16, \text{ which is not zero}$$

$$(2) \quad y'' + 2y' + 2y = 0. \quad \text{Let } y = \oint_c e^{xt} f(t) dt$$

$$\therefore y'' + 2y' + 2y = \oint_c e^{xt} \{t^2 + 2t + 2\} f(t) dt = 0.$$

$$\text{This is possible if } f(t) = \frac{1}{t^2 + 2t + 2}$$

$$\therefore y = \oint_c \frac{e^{xt} dt}{t^2 + 2t + 2} \quad \text{Here } t = -1 + i, -1 - i \text{ are the two simple poles.}$$

Let C be so chosen such that both these lie inside C.

$$\therefore \text{Residue at } t = -1 + i \text{ is } \lim_{t \rightarrow -1 + i} \frac{e^{xt} \cdot (t+1-2)}{(t+1+i)(t+1-i)} = \frac{e^{x(-1+i)}}{2i}$$

$$\text{Similarly Residue at } -1 - i = \frac{e^{x(-1-i)}}{-2i}$$

$$\therefore y = \int_c^x \frac{e^{xt} dt}{t^2 + 2t + 2} = 2\pi i \left\{ \frac{e^{x(-1+i)}}{2i} + \frac{e^{x(-1-i)}}{2i} \right\}$$

$y = e^{-x} \{ A \cos x + B \sin x \}$  is the required solution.

Suppose we have a differential equation with variable coefficients. We can apply this method to solve the same.

$$\text{Ex : } xy'' + (2n+1)y' + xy = 0.$$

$y = \int_c^x e^{xt} f(t) dt$ ,  $y' = \int_c^x t e^{xt} f(t) dt$ ,  $y'' = \int_c^x t^2 e^{xt} f(t) dt$ . Then integrating by parts assuming that C is chosen so that the functional values of the initial and final points are equal (and the integrated part is zero).

$$\begin{aligned} xy &= \int_c^x x e^{xt} f(t) dt = e^{xt} f(t) \Big|_c^x - \int_c^x e^{xt} f'(t) dt \\ &= - \int_c^x e^{xt} f'(t) dt \end{aligned}$$

$$(2n+1)y' = \int_c^x (2n+1)t e^{xt} f(t) dt$$

$$xy'' = \int_c^x x t^2 e^{xt} f(t) dt = \int_c^x (x e^{xt}) t^2 f(t) dt$$

$$= e^{xt} t^2 f(t) \Big|_c^x - \int_c^x e^{xt} \{ t^2 f(t) \}' dt = - \int_c^x \{ t^2 f(t) \}' dt$$

$\therefore xy'' + (2n+1)y' + xy = 0$  will be come equal to

$$\int_c^x e^{xt} [ - \{ t^2 f(t) \}' + (2n+1)t f(t) - f'(t) ] dt = 0.$$

This is satisfied if we choose  $f(t)$  so that the integrand is zero.

$$\text{i.e. } -f'(t) + (2n+1)t f(t) - \{ t^2 f(t) \}' = 0$$

$$\text{i.e. } -f'(t) + (2n+1)t f(t) - t^2 f'(t) - 2t f(t) = 0$$

$$\text{or } (2n-1)t f(t) = (1+t^2)f'(t)$$

$$\therefore f'(t) = \frac{(2n-1)t f(t)}{(1+t^2)}$$

$$\text{or } \frac{df}{f} = \frac{(2n-1)t f(t)}{1+t^2} \quad \text{or } \frac{df}{f} = \frac{(2n-1)t}{t^2+1} dt$$

or  $\log f = \frac{2n-1}{2} \log(t^2+1) + \text{const.}$ , or  $f = A(t^2+1)^{n-1/2}$  where A is the constant of Integration.

Hence the solution of  $xy'' + (2n+1)y' + xy = 0$  is given by  $y = A \int_c^x e^{xt} (t^2+1)^{n-1/2} dt$ .

**K. S. CHANDRASHEKAR,**  
Senior M.Sc. (Mathematics)

## Linear Varieties

K. RAMAMURTHY,

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Let  $X$  be a vector space over  $K$  where  $k$  is either  $R$  or  $C$ . By a Linear variety in  $X$  we mean a subset of  $x$  of the form  $x + M$  where  $x \in X$  and  $M$  is a subspace of  $X$ .

Singleton subsets of  $X$  are linear varieties since  $\{0\}$  is a subspace of  $x$ .

Linear varieties in  $E_2$  (the 2-dimensional Euclidean space) are singleton subsets,  $E_2$  itself and lines in  $E_2$ .

The most interesting linear varieties are those known as Hyperplanes.

A Hyperplane is a linear variety  $x + M$  such that  $M \neq X$  and  $M \subset M_1 \subset X$  where  $M_1$  is a subspace of  $X$  implies  $M_1 = M$  or  $M_1 = X$ . In other words, Hyperplane is a linear variety, which results from a translation of a maximal subspace of  $X$ .

It is easy to see that  $M$  is a hyperplane through  $0$  iff it is the direct sum of  $M$  and a one-dimensional subspace iff  $M$  is a maximal subspace of  $X$ .

Hyperplanes in  $E_2$  are lines in  $E_2$  and hyperplanes in  $E_3$  are planes in  $E_3$ .

Two hyperplanes  $x_1 + M_1$  and  $x_2 + M_2$  are parallel if  $M_1 = M_2$ .

Let  $X^*$  denote the dual space  $X$ .

**Theorem:** Let  $\alpha$  be a scalar,  $f \in X^*$  and  $f \neq 0$ . Then  $M = \{x \mid f(x) = \alpha\}$  is a hyperplane. It contains zero iff  $\alpha = 0$ . To each hyperplane  $M$  containing zero there corresponds  $f$  in  $X^*$  such that  $f \neq 0$  and  $M = \{x \mid f(x) = 0\}$ .

**Proof:** Since  $f \neq 0$ , there is a  $x_1$  such that  $f(x_1) \neq 0$ . Let  $x_0 = \frac{\alpha}{f(x_1)} x_1$ .

Then  $(-x_0) + M = \{-x_0 + y \mid y \in M\} = \{-x_0 + y \mid f(y) = \alpha\} = \{y \mid f(y + x_0) = \alpha\} = \{y \mid f(y) + f(x_0) = \alpha\} = \{y \mid f(y) = 0\}$  since  $f(x_0) = f(x_1) = \alpha$ .

Since  $\{y \mid f(y) = 0\}$  is a subspace, it is clear that  $M = x_0 + \{y \mid f(y) = 0\}$  is a hyperplane.  $M$  contains zero iff  $0 = f(0) = \alpha$ .

Now let  $M$  be any hyperplane containing zero.  $M$  is a maximal subspace of  $X$ .  $M \neq X$ . Let  $x_0 \in X - M$ . Since  $M$  is a proper subspace of  $X$  and  $x_0 \in X - M$ , there is a linear functional  $f$  on  $X$  such that  $f(x_0) = 1$  and  $f(x) = 0$  if  $x \in M$ .  $M \subset \{x \mid f(x) = 0\} \subset X$ . Since  $M$  is a maximal subspace and  $\{x \mid f(x) = 0\}$  is subspace of  $X$ , it follows that  $M = \{x \mid f(x) = 0\}$  or  $\{x \mid f(x) = 0\} = X$ . Also  $f(x_0) = 1 \neq 0 \dots M = \{x \mid f(x) = 0\}$ .

$\therefore f \in X^*, f \neq 0$  and  $M = \{x \mid f(x) = 0\}$ . This completes the proof of the theorem.

The theorem gives a characterization of hyperplanes through zero :

$M$  is a hyperplane through zero iff  $M = \{x \in X \mid f(x) = 0\}$  for some  $f \in X^* - \{0\}$ .

By a topological linear space  $X$  over  $K$  we mean a linear space over  $K$  with a topology  $\tau$  on  $X$  such that if  $K$  is given the usual topology and  $K \times X, X \times X$  are given the product topologies then the functions  $f: X \times X \rightarrow X$  and  $g: K \times X \rightarrow X$  defined by  $f(x, y) = x + y$  and  $g(\alpha, x) = \alpha x$  for all  $\alpha \in K, x, y \in X$  are continuous.

**Theorem :** *The closure of a linear variety in a topological linear space is a linear variety.*

**Proof :** Let  $X$  be a topological linear space. Let  $x \in X$  and  $M$  be a subspace of  $X$ . We shall show that  $\overline{x+M} = x + \overline{M}$  and  $\overline{M}$  is a subspace of  $X$ . It can be shown that if  $h$  is a continuous map from a topological space  $Y$  into a topological space  $Z$  then  $h(\overline{S}) \subset \overline{h(S)}$  for any subset  $S$  of  $Y$ . Since  $f: X \times X \rightarrow X$  defined by  $f((x, y)) = x + y$  for all  $x, y$  in  $X$  is continuous and it follows that  $f(\overline{M \times M}) \subset \overline{f(M \times M)}$ . Also  $\overline{M \times M} = \overline{M} \times \overline{M}$  since  $M$  is a subspace.  $f(M \times M) = \{x + y \mid (x, y) \in M \times M\} \subset M$ . Hence  $f(\overline{M \times M}) \subset \overline{M}$ . Similarly, since the map  $g: K \times X \rightarrow X$  defined by  $g((\alpha, x)) = \alpha x$  for all  $\alpha$  in  $K$  and  $x$  in  $X$  is continuous, we have  $g(\overline{M \times M}) \subset \overline{M}$ . It is now clear that  $\overline{M}$  is a subspace.

We shall show that  $\overline{x+M} = x + \overline{M}$ . Let  $y \in \overline{M}$  and  $G$  be an open set containing  $x + y$ . By continuity of  $f$ , we can find open sets  $G_1$  and  $G_2$  in  $X$  containing  $x$  and  $y$  respectively such that  $f(G_1 \times G_2) \subset G$  since  $y \in \overline{M}, M \cap G_2 \neq \phi$ . If  $t \in M \cap G_2$  then  $x + t \in x + M$  and  $x + t = f((x, t)) \in f(G_1 \times G_2) \subset G$ .  $(x + M) \cap G \neq \phi$ . Since every open set containing  $x + y$  has a non-empty intersection with  $x + M$  it follows that  $x + y \in \overline{x + M}$ . Hence  $x + \overline{M} \subset \overline{x + M}$ .

Now,  $(-x) + \overline{x + M} \subset \overline{(-x) + (x + M)} = \overline{M} \therefore \overline{x + M} \subset x + \overline{M}$ . Hence  $\overline{x + M} = x + \overline{M}$ . Also  $\overline{M}$  is a subspace of  $X$ . Hence  $\overline{x + M}$  is a linear variety. The proof of the theorem is now complete.

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## AN I. Q. TEST

*Here is a test to measure your Intelligence. The duration of the test is 60 minutes. Try to solve as many problems as you can. Compare your solutions with the solutions given at the end. If you can solve*

- (i) more than 7 problems then you are highly intelligent. (IQ > 130)
  - (ii) more than 4 problems but less than 7 problems you are average. (IQ = 110)
  - (iii) more than 2 problems but less than 4, you are below average (110 > IQ > 60).
- If you cannot solve even 2 problems then you are an idiot. (IQ < 40)*

1. The smaller angle between the two hands of a clock at 3.40 A.M is [ 120°, 130°, 150°, 120° 30' ]
2. The sum of the reciprocals of 3 natural numbers is  $\frac{1}{3}$ . The numbers are \_\_\_\_\_
3. The 6th term of the sequence: 2, 4, 9, 20, 43 is \_\_\_\_\_. [ 83, 59, 90, 104 ]
4. The length of a rectangle is 5 inches and its width is less than 4 inches. The Rectangle is folded so that two diagonally opposite vertices coincide. If the length of the crease is  $\sqrt{6}$ , then the width is \_\_\_\_\_ [  $\sqrt{2}$ ,  $\sqrt{3}$ , 2,  $\sqrt{5}$ ,  $\sqrt{\frac{\pi}{2}}$  ]
5. Let  $n$  be the numbers of Integral pairs  $(x, y)$  which satisfy  $5y - 3x = 15$  and  $x^2 + y^2 \leq 16$ . Then  $n$  is \_\_\_\_\_ [ 0, 1, 2, Infinite ].
6. Let  $a$  and  $b$  be two positive numbers such that  $a < b$ . Let AM and GM be their arithmetic and geometric means respectively. Then AM - GM is always less than : \_\_\_\_\_ [  $\frac{(b+a)^2}{ab}$ ,  $\frac{(b+a)^2}{8b}$ ,  $\frac{(b-a)^2}{ab}$ ,  $\frac{(b-a)^2}{8a}$ ,  $\frac{(b-a)^2}{8b}$  ]
7. If  $n$  is a multiple of 4, the sum  $s = 1 + 2i + 3i^2 + 4i^3 + \dots + (n+i)i^n$ , where  $i = \sqrt{-1}$ , equals \_\_\_\_\_ [  $1+i$ ,  $\frac{1}{2}(n+2)$ ,  $\frac{1}{2}(n+2-ni)$ ,  $\frac{1}{2}(n^2+8-4ni)$  ]
8. The sum of all 3 digit numbers formed by 1, 2, 3, 4 taking a digit only once, in a number is \_\_\_\_\_ [ 66660, 660, 6660, Infinite ].

[ Answers: (1) 130°. (2) 6, 9, 18. (3) 90. (4)  $\sqrt{5}$ . (5) 1. (6)  $\frac{8a}{(b-a)^2}$ . (7)  $\frac{1}{2}(n+2-ni)$ . (8) 6660. ]

---

“ If we evolved a race of Issac Newtons, that wouldnot be progress. For the price Newton had to pay for being a supreme intellect was that he was incapable of friendship, love, fatherhood and many other desirable things. As a man he was a failure; as a monster he was superb,  
 — ALDOUS AUXLEY.

---

## Niels Heurik Abel



Abel, Neils Heurik, Norwegian mathematician, a pioneer in the development of modern mathematics, was born on August 5, 1802. His mathematical talent was recognised while he was a student at the cathedral school in Oslo, by a young teacher Holmboe. While still in school he believed he had found a solution at the general quintic equation, but he discovered an error before publication.

His first papers were published in 1823. At this time Abel received a small stipend from the mathematics Professor Rasmussen, which enabled him to take a trip to Copenhagen to make the acquaintance of the Danish mathematicians. Upon his return Abel found the proof for the result that the general quintic equation or higher had no solution in radical expressions; it was published in a small pamphlet at his own expense.

In 1825-26, he had the good fortune of making the acquaintance of A. L. Crelle, who became his close friend and protector. Inspired by Abel, Crelle founded the important journal *für die reine and angewandte Mathematik*, the first volume of which is filled with papers by Abel. They concern such topics as equation theory, functional equations, integration in finite form, problems from theoretical mechanics and others. The journal was the first periodical in the world devoted exclusively to mathematical research.

Abel has written a study of the binomial series, considered to be one of the classics in function theory. It contains the principles of convergence series with special applications to power series.

In 1828 Abel became ill with tuberculosis of his lungs. His poverty and ill-health had little influence, however, on scientific production. He poured forth a great number of papers, principally on equation theory and elliptic functions. Among them are found the theory of the Abelian equations with Abelian groups. The theory of elliptic functions was developed with great rapidity in competition with Jacobi. Abel died on April 6, 1829 at the age of twenty-seven. Post humously he was awarded together with Jacobi, the Grand Prix of the French academy for 1830.

It was said of him by Hermite, 'He has left mathematicians something to keep them busy for five hundred years'. To quote Crelle, 'He distinguished himself equally by the purity and nobility of his character and a rare modesty which made his person cherished to the same unusual degree as was his genius'.

—P. SHARADA

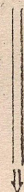
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