Notes on operations research.

1. P.M. Morse: Comm. Pure & App. Math. Vol. 8 (1955), 1-12.

Definition – waiting line or queueing Theory. Monte Carlo technique – linear programming problems. geometrical picture. problems of bounded optimisation – game theory. games of mixed strategy. the duel problem – optimum distribution of effort. rate of search. problems of industry and business – more than application of statistics.

Special problems treated: (a) arrival & docking of ships in harbour.

✓ (b) aircraft stacking over airports. Monte Carlo procedure

(c) optimisation of the crude-oil-cracking problem.

(d) Production planning.

✓ (e) Submarine avoiding searching aircraft – tactics used in air engagement between bomber & opposing fighter.

✓ (f) operation of search for enemy vessel, or submarine or aircraft.

(g) assignment of sales effort.

2. Methods of operations research by Morse & Kimball (1950) – Members of the O.R. Group U.S. Navy. Not appreciated during war time because information not freely available – Formation of Committee on O.R of the U.S. National Research Council, O.R. Club in London, classes in O.R. in several Universities, publication of J.O.R & O.R. Qly – Introduction: Chap I

(a) Definition: O.R is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control.

Cf. with defn in 1 viz. That O.R is research at the operational level just as Physics & Chemistry correspond to research at the basic or material level, and bridges & television sets to research at the engineering or applied level.

Implications of the definition · a scientific method · an applied science · basis for decisions means that duties of O·R· worker & the executive officer should be separate. Over & above quantitative basis there are other aspects like politics, morale etc. O·R· worker may also take these into consideration but he should not take decisions · proper presentation of results to non-scientists. O·R is not a branch of mathematics (Is this true? In later work Morse puts it under applied mathematics), nor Engineering. Requirements for scientific experience & research ability are of the highest in O·R. The creation of this new field of applied science is a result of World War II.

Value of O·R – Admiral Kings report. In five years non-military examples of applns will emerge.

(b) Simple examples to illustrate points made in the definition.

(i) Rearrangement of use of equipment – line-up of soldiers washing their mess kits after eating at a field mess station. 2 tubs for washing + 2 tubs for rinsing ⟶ 3 tubs for washing + 1 tub for rinsing (no queues later on). Soln depended on noticing that time for washing is thrice time for rinsing. No gadgets used. The O·R· worker could get change done & although soldiers may have noticed it they had no authority to get the change effected. Result of abolition of waiting line is a self-aggravating property & conversely it is an operational property of waiting lines that the longer they get, the longer they tend to get. These are non-linear effects and can be handled by known mathematical techniques based on probability theory.

(ii) Changes in setting, rearrangement of unit size – Depth setting of anti-submarine depth charges to improve U-boat sinking (3.4.8) & increasing the size of convoys & reducing average ship loss (3.2.2) are examples like (i) where no change is made in equipment but only simple changes like changing depth or size of convoys. Also in former case a knowledge of physical properties of depth charge fuse was needed unlike as in (i). In latter case no such technical knowledge reqd; in fact details are sometimes a handicap. Also, the latter case involves a choice of measure of effectiveness. Here it is percentage of ships lost, not the actual number.

(for eg. in traffic improvement, what is measure of effectiveness? No. of accidents lessened or increase in speed or ratio of the two?).

(iii) Dependence on an operational parameter – Example of finding percentage of total sales in a city made by a particular store as a good ~~example~~ one – O.R. is often an experimental science also. Use of chance fluctuations in an operation to fine parameters – Proper statistical manipulation of unpromising data gives good results – Changes in radar techniques – decreasing width of road to study traffic flow are examples of exp. method.

(iv) Problem of finding the problem – Case of putting anti aircraft guns on merchant vessels (3.4.4) lessening chance of merchant vessel being sunk – sending advertisements to satisfied customers

(v) Find the sensitive parameters – Need for a detailed understanding of the operation occurs in the problem of search of submarines by aircraft (5.2)

(vi) mathematical analysis – analytical study of flow of buses etc – maths is means to an end – ranges from simple arithmetic to most advanced maths, and sometimes use of modern computing machines (Monte Carlo technique of Morse?).

(vii) Requirements of secrecy – O.R. worker to cope with them intelligently & not emotionally – Because of this many extremely instructive examples of O.R. will perhaps be never reported in the open literature

(c) Methods of O.R.

(i) few data as in biology or economics, not like physics – must use its own methods.

(ii) Statistical methods – most impt. simple math. tool is probability & statistics – applicable where there is large amount of data for eg. bombing – Strategy & tactics will always be an art though O.R. may help it.

(iii) Field assignments. Collection of data – hazards of collection of data less in War II than in I – quick collection of data & report – interrelation between field observers & central group.

(iv) Limitations of operational data – only small changes can result – variational method & 'mutatis

(v) Limitations of 'expert opinion' — importance of actions of O.R. men & not opinions - need for unbiassed impersonal facts.

(vi) operational experiments — need for O.R. expts in peacetime — great importance of such ~~exp~~ tactical experiments (See Chap. 8)

(vii) Analytical methods — O.R. must also use theoretical methods (Chap. 4) — Von Neumann & Morgenstern on theory of games (Chap. 5) — Because theoretical studies limit themselves to simple cases, they should not be dismissed as impracticable

(viii) Summary of methods — measures of effectiveness — need to know dependence of results on all the operational parameters — executive deplts. to make a choice if there be several measures of effectiveness — Presentation of results to executives is an important part of O.R. These reports should contain only conclusions & not recommendations (for eg. Blackett's O.R. lecture on this count is faulty)

(ix) General comments — O.R. worker is not a 'gadgeteer'. Improvement in equipment to be done by scientists & not O.R. men — sense of urgency in O.R. first results are most valuable & O.R. men should not be loaded with other scientific work.

(d) Personnel and organisation.

(i) O.R. worker should have considerable experience in research whether he be a physicist, mathematician, biologist or etc.

(ii) Choice of personnel — particular O.R. mentality found in physicists & biologists & less commonly found in mathematicians, engineers & economists although there are brilliant exceptions — O.R. worker's personality. 'security-minded'.

(iii) importance of mutual understanding between scientist & administrator — partnership most important. There is always a psychological difficulty in mutual understanding — During world War II they were not foreseen & caused confusion & inefficiency — reaction of executive. feeling that there is no impt. problem for O.R. worker to tackle. suspicion & impatience. suspicion that O.R. worker wants to take command from the executive if O.R. findings

are successful & vary from that of the executive officer. He should realise that this is only a part of the findings. He should combine O.R. findings with other qualitative aspects & take final action (Chap. 8).

(iv) Possible peacetime applications — traffic studies - difficult problem of city housing and municipal facilities - O.R. in telephone operation - O.R. in house-heating (for eg. in event of next war should coal, oil or electric heating be encouraged in U.S.A.?) - O.R. fruitful only when it studies actual operations and partnership is with an administrator who has authority - otherwise O.R. is a philosophy & not a science.

Chapter II - Probability

Fundamental concepts - Probability - distribution functions - distribution functions in several variables - compound probabilities - Expected values - the simple distribution laws - Binomial distribution - Normal distribution - Poisson distribution - Sampling - the χ^2-test - Some examples.

Chapter III - Use of measures of effectiveness

Hemibel thinking (a bel = unit in log scale corresp to factor of 10 & hemibel corresponds to factor $\sqrt{10}$ ≈ 3) ie knowing the constants of the operation correct to within a factor of 3 - If actual value is within a hemibel of theoretical value, then it is unlikely that O.R. will help.

(a) Sweep rates - operation of patrol or search.

(i) calculation of constants - No. of contacts (C) with enemy units in time (T) gives the simple measure (C/T) which depends on efficiency of search and also on the number N of enemy aircraft in the area A under search.

(ii) operational sweep rate $= Q_{op} = \left(\frac{CA}{NT}\right)$ sq. mls/hr (or day)

$= (C/T) \div (N/A)$ = no. of contacts per hour (or day) / average density of target craft.

(iii) Theoretical sweep rate — $Q_{th} = 2RV$ (R = effective lateral range of detection in miles, V = average speed of search aircraft in mls/hr (or da)) — Proved in Div^{n} 6, Vol 2B (?) — Comparison of two rates (Q_{op}/Q_{th}) is a powerful means of following fluctuations in efficacy

(iv) Submarine patrol — a simple example of above of submarines sighting merchant ships

(v) Aircraft search for submarines — another example.

(vi) Distribution of flying effort — (v) above analysed more carefully

(vii) Antisubmarine flying in the Bay of Biscay — use of sweep rate for following tactical changes in case of RAF Coastal Command struggle against German U-boats in the Bay of Biscay — watching overall course of warfare by watching fluctuations of a measure of effectiveness

(b) Exchange rates — Ratio between enemy loss & own loss — If m & n be own & enemy units employed & k, l respective losses in single engagements, k & l depend on m & n & nature of dependence depends on tactics involved — we may have $k \propto n$, $l \propto m$ or (l/k) independent of m & n.

(i) Air-to-Air Combat — American & Japanese aircraft combat in Pacific in 43-44 — analysis of exchange rates for both sides

(ii) Convoys vs submarines — units of different sorts — no. of escort vessels also to be taken into consideration — conclusion is no. of merchant vessels lost per back attack $\propto$ no. of U boats in pack & $\propto$ 1/no. of escorts. $\left(k \sim \frac{n}{c}\right)$.

(iii) Resulting exchange rates — same problem as (ii) — If l = no. of U-boats sunk per attack, k = no. of merchant vessels sunk per attack, n = no. of U boats in pack, c = no. of escorts: $k \cong 5\left(\frac{n}{c}\right)$; $l \simeq \left(nc/100\right)$ $\therefore \frac{l}{k} \cong \left(c^2/500\right)$

ie case for large convoys is a persuasive one.

(c) Comparitive effectiveness – Comparison of relative effectiveness of two different weapons or tactics – choice of an equitable & usable unit of comparison – Objective choice

(i) Effectiveness of anti-ship weapons – conclusions of study of table

(ii) Bombing U-Boat pens versus Escorting Convoys – relative value of using aircraft to escort convoys to bomb submarine facilities or to hunt down submarines in the Bay of Biscay

(iii) Submarine vs Aircraft as anti-ship weapons – roughly a single submarine is equivalent as far as sinking enemy shipping goes to a squadron of long range antishipping planes.

(iv) anti-ship vs anti-city bombing – unit of measure to be man-months spent in rebuilding.

(d) Evaluation of equipment performance – How to evaluate weapons – not really for O.R. but better for O.R. men to know this.

(i) First use of new equipment – O.R. workers to go to the front.

(ii) Devising operational practice training – practice routine with means of scoring

(iii) Equipment evaluation – general questions to be considered – four types of questions raised

✓(iv) anti aircraft guns for merchant vessels – Here the measure of effectiveness should not be percentage of enemy planes shot down but whether the ship is less damaged if it has A.A. guns & uses them than if it had not & does not use them – the answer is the former the damage is less if it had A.A guns – The A.A. fire affects accuracy of plane's attack.

(v) Anti-torpedo nets – Report of findings showed that the programme of fitting with nets had not paid itself

(vi) magnetic airborne detector – to detect submerged submarines from aircraft – Low MAD detection range (100 yds for aircraft at 250 ft. altitude & submarine 250 ft depth) makes it useless for random search. Comparable to a blind man trying to draw a pencil line through a single dot on a large floor – But may be effective as a blockade across a restricted area ~~& area~~ one which does not provide warning as would surface craft. Eg. during Italian Campaign against Nazi U-boats through Gibraltar straits enroute to the Mediterranean.

(vii) ~~Aircraft antisubmarine depth bombs~~ Submarine torpedo evaluation – seven conclusions arrived at

(viii) Aircraft antisubmarine Depth bombs – ie bombs not exploding on surface of water. question of depth setting of the depth charges – In consequence of O.R. analysis it was made a doctrine to set of depth of explosion for aerial depth bombs at 25 ft. & not to drop the bombs if submarine had already submerged for more than half a minute.

(ix) Importance of maintenance ~~– Checking of CAP equipment & training~~

(xi) Radar Bomb sights – Conclusion that radar bombing equipment be used only at nights & only in non-restricted areas.

~~(x)~~

(x) Height finding radars – Checking of CAP equipment & training

(xii) Effects of supervised practice – question whether addition of new equipment or further training of personnel to use old equipment would be more effective

(xiii) Evaluating enemy's countermeasures – Case of aircraft warning radar ~~by~~ used by U.S. submarines to give early warning of enemy aircraft & hence time to dive – found really very useful

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painting the planes (Is this all the part we took in the OR war research?)

Chap. XVI – Wizards of ether & wire – The electrical eye-and-ear specialists.

p. 255 – Special need for civilian scientists to help army – Bungling re. radar sets

p. 261 Sloppy diction of average American another OFS problem – made to give speeches – work of OFS men in radar etc would take volumes

p. 274 – Tribute to radar

Chapter XVII – Fantasies, phantoms & facts – Rockets, Ram-jets & Robots.

p. 287 Jet propulsion Committee with Karman as head – guided missile – Science has created new challenges; ~~and~~ our greatest need is for statesmen to determine whether such rockets and robots are loaded to deliver penicillin or plutonium.

p. 275 – Third paragraph & first paragraph.

Chap. XVIII – Brain Trust in Hawaii – The balanced team at GHQ.

Gen. Richardson's area – Unique ORS – balanced team at ~~Oho~~ Oahu –

Hist p. 293. Richardson's tribute

Chap XIX – Island-hopping – Field office with MacArthur

Chap XX – Miniature in Manila – The pacific branch OSRD

Chap XXI – Lessons for tomorrow – The message of experience

Future of ~~OSR~~ OR

Chaps XXII to the end – on amphibious navigation.

6. Design for decision by I. D. J. Bross

Supported by ONR – Book about statistical decisions – mathematics used not beyond high school algebra – This is statistics & probability treated in an elementary way. No. OR

7. Decision processes by Thrall, Coombs & Davis

Statistical theory – also deals with theory of games.

8. Blackett's article in Defence Science Journal, Oct 55 on O.R.

Caution against theoretical investigations running riot & undue sophistication of analysis – Aim should be to help somebody take decisions in both types of work viz OR & evaluation on the basis of arguments – To keep this practical end in view – DSL to help decide what kind of armaments to buy & how to use them – Example of bombsights having difference of few yards from height of 20,000 ft, but bombing error itself was 200 yds – work of an App. Math. man on no. of bombs reqd to drop on German battle cruiser Scharanhost so as to have 50-50 chance using complicated maths to "impress the Air Marshall" (use of 1000 bombs gave probability of 0.24) – Blackett says he fought shy of mathematics in his work on O.R & kept it in the background. Often, of course, not possible to avoid it but should not be exaggerated; only get right order of magnitude rather than detailed figure (he mibel thinking?) – Same way I look on statistics; you should know it in order never to have to use it (!) – χ^2-test use is going off the rails – correlation coefficient almost never useful in O.R

Gen

Clear presentation of results – Case of a biologist lecturing before the Royal Society & the President's questions.

- Rule $n = 3 + N/10$ about Convoys in the Admiralty - Big convoys & small convoys theory - Rule re. depth of depth charges - Service people to read Service manuals carefully & look into them critically.

Profitable to check planning operations numerically - Example of Normandy landing. Damage done to big guns & small guns (analogous to tube problem?)

Blackett's operational calculus - Introduces notion of "worth whileness" - Probability is much more like a wave fn - a fn of two variables in which end state & initial state come in - (A new idea ☺ or is it a minimax problem?)

Kind of aircraft for India - What airpower can & cannot do in modern warfare - "An army with complete air superiority is like an army twice the size but without any air support" This is a guess, ~~perhaps~~ the factor is ~~ten or ten~~ not ten in any case, may be three or five - air superiority over extended frontier is hard to achieve

Re. factor two - Calculation of Clausewitz re. Napoleon ie a general who is a genius is twice as great - Story books may make us think this to be 100 or 1000

Reference to Air Commodore Dale's & Gen. Chaudhuri's talks - Suggestion to get French Potez planes & try them in India and to make indigenous aircraft

~~Next are to go and O.R.~~ Recommendation of Liberators with good radar for long-range work - Gnat for high altitude & Canberra

Studies in O.R. should be continued in collaboration with uniformed colleagues

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9. Operational Research Quarterly Volumes of 1954 & 55

Vol. 5. No. 1. March 54: (1) O.R & accounting - A friend asked "Is there one (connection?) and the lecture is to justify this

(2) Conferences in U.S - two in Jan. 54 on Production & Inventory Control and

another on O.R. At latter O.R. techniques ~~related~~ including experimental design, waiting problems, linear programming, symbolic logic, decision theory and information handling were discussed.

(3) Capacity: Its meaning, use & measurement — Capacity of Roads under Road research — Capacity in manufacturing processes — Flow of motor spirit from refinery to consumer.

(4) Reviews.

June 54. (1) Supply & demand.

(2) Reflections on O.R. by E.C. Williams (Director O.R. Admiralty) — Purely confined to military O.R — term coined by A.P. Rowe in 1937–39 to describe activities of a small section of the Air Ministry Research Station at Bawdsey (O.R in Britain in Air Force) — Activities of O.R. groups familiar to all

After war, similarly work has led to extension into other fields & this extension has created difficulties in understanding of the subject — O.R. is really old, but what is new is its importance — Rapid growth in military fields & industry make it important — Old time trial & error would now be 'trial & catastrophe' —

Four classes of work under O.R: (i) ~~effe~~ Operational effectiveness of weapons & equipment (ii) analysis of results of operation or of exercises to determine effectiveness, the influence of weapons on tactics & tactics on weapons (iii) prediction of results of future operations (iv) analysis of efficiency of organisation or of methods.

O.R. is best viewed as ~~dynamics~~ study of dynamics of situations etc. This appears best way of describing content of O.R. and as in any other branch of Science pursuit for its own sake is entirely justifiable, but application is important.

Type of man needed — four qualifications (a) — (d) — well analysed

- Lateral philosophical basis of O.R. is still being built & training O.R. formally is not yet justified

- needs enumerated to be six. ~~- no need for special training otherwise demand~~

(3) Survey of O.R. in U.S.A by Ellis Johnson (Director, ORO, John Hopkins) - Growth of military O.R - ORSA of America - Breakdown of OR members on 1/7/53 - Questionnaires sent to outstanding companies - Fields in which OR is conducted - Scope of OR - cost of OR - Education for OR, Universities & other courses - Gives facts & figures impressively

Hist (4) Abstracts & Reviews - Road Research (6 summaries)

(5) Large number research & frequency analysis — Use of probability paper - called Schwachstellenforschung (weak point research)

(6) Courses on OR - at Madrid University.

September 54 - (1) meetings of ORS, London.

(2) Inspection in relation to OR.

(3) Assessing priorities for technical effort

(4) Criminology & prediction - Borstal research project.

(5) Abstracts & Reviews

Centre Français de Recherche Opérationnelle at 115, Boulevard St., Germain, Paris 7e.

December 54 - (1) An application of the birth and death process to the provision of spare machines - Summary - Interesting conclusions.

(2) Queueing systems with phase type service

(3) Army OR in the Far East 1952-54

Gen (4) Abstracts & Reviews - Blackett's book on Recollections of OR problems studied 1940-45 - personal memoir.

Vol. 6. No. 1. March 1955 – (1) Measurement & management – warning to U.S. development in O.R. that quality may fall due to too rapid increase – In Britain fairly steady & healthy progress

(2) Opening of O.R. Centre in Milan, Italy.

(3) Accuracy & use of time study (in industry) – Discussion

✓ | (4) Use & abuses of analogy – Interesting paper (what is OR in it?)

(5) Abstracts & Reviews

June 1955 – (1) The handling of uncontrolled data.

(2) Road research summaries – 5.

✓ | (3) Mechanising ticket printing & issue

✓ | (4) OR & Railways – need for British Rep. wanting an OR Division

(5) Isoplus n – work in nomography

(6) Abstracts & Reviews – Article by Morse on "OR, past, present & future"

Sept. 1955 – (1) Meetings of ORS

(2) Isoplus n – Corrections in above article

(3) Scale factors in the management of coal mines.

✓✓ ‖ (4) Monte Carlo methods in heavy industry

(5) Courses on OR – Account of several courses in several countries.

✓ | (6) The risk taken in crossing a road

(7) OR in rubber.

Dec. 1955 – (1) ORS meetings & Library service

✓✓ ‖ (2) Generalisation of the warehousing model – uses a duality theorem

due to von Neumann : max π = min E — Big long math. paper.

(3) A mined strategy in action — tactical problem using Theory of games. The setting is that of a military escort protecting a convoy of food lorries against guerilla attack — Example from Malaya — by Beresford (Major, 13/18 Royal Hussars) & M. H. Peston (AORG).

Games

10. Journal of the O.R.S of America.

February, 55. (1) Observation, Exptl., and measurement in O.R. by O. Solandt (Canada) — History of O.R etc — very interesting article — Effective use of 3 instead of six to effectively man antitank guns — Report sent to gunners & their reaction "Burn the Report" — Holding the horses — Expert packing up the cameras — military operation in peace-time — striving for reality in our exercises — I for one hope that we shall never come to the day when a knowledge of the latest mathematical gadgets for war games, of communication theory & so on becomes the badge of office of OR worker (very prejudiced) — OR is systematic application of common sense as defn.

Gen

(2) Proposal for revising New York's subway fare structure.

(3) Approximate salvo-killing probabilities (mathematical)

(4) Statistical guide for accident research

(5) Discussion on Education for OR

(6) Letters to Editor (a) Why O.R ?

(7) Abstracts & Reports

(8) Abstracts of papers.

(9) Book reviews.

May 1955. (1) The competence and limitations of scientific method.

(2) Direct use of extremal principles – applies to electronic digital computers
Problems in structural analysis lying in the field of OR – Problem in game theory

(3) Mathematical models for Highway traffic – model for motion of cars like motion of molecules in a gas by kinetic theory.

(4) Use of storage water in a hydroelectric system – Scheduling of water of dams as an inventory problem – Result to be obtained by use of a digital computer – interesting conclusions.

Appl

(5) Poisson distn for dependent events –

(6) numerical soln of transportation problem – Simple & short paper

Lin. prog.

(7) Letters to Editor

(8) Linear programming in Capital budgeting – Elementary explanation of linear programming – Mrs E's adventures.

Simplify Lin Pr

(9) Announcements

August 1955. (1) Terminal Ballistics by J. Steinhardt – opening sentences very appropriate

(2) From Plato ~~the~~ to the linear problem by Earl of Halsbury – very interesting

(3) Stochastic properties of waiting lines by P. M. Morse – highly mathematical – uses auto-correlation function (disproof of Blackett's statement)

Gen.

(4) Discrete variable servo theory – use of servomechanisms in OR.

(5) Road safety research in Britain

(6) Production scheduling (mathematical)

(7) Letters to Editor (a) Some thought on OR on municipal operations (b) Some implications

/ | 7 OR - what is OR? It is a philosophy

November 1955 · (1) A challenge to OR

(2) Where is the new blood? by P.M. Morse - Some parts of it excellent

Gen. (3) Lanchester prize acceptance speech by L.C. Edie

February 1956 · (1) Statistics and operation research by P.M. Morse - waiting line theory - linear programming - search theory or optimum distribution of effort - mention-keeping abreast in games theory, communication theory and cybernetics also - OR is not history, it is an exptl. science with the goal of active control of future operations before it -.

Gen.

(2) OR in production control

(3) Shock waves on the highway - Theory of traffic flow.

applin

(4) Criteria for selection of water power projects - Dept. in India

applin

(5) Travelling salesman problem - uses König's theorem on matrices - High flown math. theory

(6) Machine repair as waiting line problem

(7) Polar-planimeter method for probability of hitting a target

(8) competitive bidding strategy - problem one of non-linear programming

(9) Letter on OR & design of expts - Refers to paper by Rehman & Zaher in ORQ of 1952 saying OR is not exptl. but wants only data given by expts (wrong I think)

(10) Qualities in OR personnel - Highly sophisticated

(Army)

11. Some reports of the AORG from 1944-50 (British)

(1) Lethality of the German "S" Mine - Report No. 201 - Recommendations also

(2) Estimation of the reflection coeffts of the ground for radio waves.

(3) Rel. values of diff. types of Defence Work

(4) Some characteristics of solar radio emissions

(5) Rearward vision from the driver's seats of army vehicles - Recommendations interesting.

(6) Promotion of national service men to non-commissioned rank.

(7) Physical requirements for drivers of heavy vehicles.

(8) Motion studies - 4 reports - one on study of loading drill for the recoil-less gun

(9) Application of soil mechanics to trafficability - Present Knowledge

(10) Method of Ground/Air Signals

(11) Reports on Sherman tank & Churchill tank & Chaos

(12) Theoretical investigations of the optimum capacity of rocket shell - mathematical

(13) Tracks of enemy Aircraft over London. Oct - Dec. 43 - Radar devices etc.

(14) Location of German long-Range Rocket launching sites by R.A.F. using Radar Equipment

12. Defence Science Organisation (New Delhi) – Reports.

(1) Study of weapon training at the Infantry School, Mhow – Patnaik & Seetharamaiah – interesting but more quantitative results should have been obtained.

(2) Tests of mental capacity for army personnel by M.S. Prakasha Rao – not much use

(3) Effect of throwing Crow's feet over an area for puncturing vehicles by M.N. Bhattacharya & Mahendra Singh – useless – trivial problem which solu is well known.

(4) Analysis of preliminary measurements of right foot of forty eight persons – what is the purpose served?

(5) on the accuracy of visual estimation of range – trivial collection of data work

(6) Influence of new armaments on tactics & vice versa by E. Dimir – In paras 39 to 52 makes some irresponsible remarks on OR quoting from Clausewitz.

(7) Armament of aeroplanes etc by Do – This is no report based on actual work, but merely a contribution – last sentence – "for these problems numerous OR was done & must be done in future – useless.

(8) Guide lines for the judgment of performance. by Do – Same type as (7) above – Last article talks of experimental OR – Quotes from Science in World War II & from Clausewitz – copied out from other books – mere quotations..

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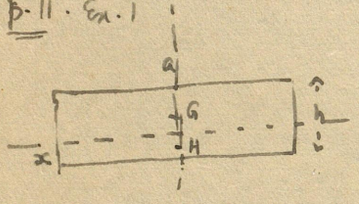
Plan for the three lectures:

What to use out of this mass? Marked red.

p. 11 · Ex. 1



$HG = \frac{1}{2}(h-x)$

$HM = AK^2/V = \dfrac{a^2 \cdot a^2/12}{a^2 x} = \dfrac{a^2}{12x}$

$a^2 x = a^2 h \cdot s$

$x = hs.$

$HM > HG, \quad \dfrac{a^2}{12x} > \frac{1}{2}(h-x)$

$a^2 > 6x(h-x). \qquad a^2 > 6hs(h-hs)$

$a^2 > 6h^2 s(1-s).$

$s(1-s) < \dfrac{a^2}{6h^2}. \qquad s(1-s) - \lambda^2 < 0, \quad s - s^2 - \lambda < 0.$ ~~$s^2 - s + \lambda > 0$~~

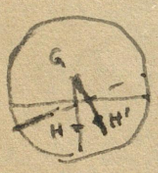
etc. ~~$(s-\frac{1}{2})^2 + (\lambda - \frac{1}{4}) > 0.$~~

~~If $\lambda > \frac{1}{4}$~~

Ex. 2: For rolling displacements $M_1 \equiv G$ ie $HM = HG$ ie eq^m is neutral

For tilting displacement $HM_2 = AK_2^2/V = \frac{A}{V} \cdot b^2/12$ $\qquad l$ = length of cylinder

rolling " $HM_1 = AK_1^2/V = \frac{A}{V} \cdot b^2/12$ $\qquad b$ = breadth of water line section.



$\therefore$ If $l > b$, $HM_2 > HM_1$ ie $HM_2 > HG$ ie stable for tilting & rolling also

Ex. 3: In (2) thrust = wt. of fluid $= \frac{1}{2}\pi \cdot 3^2 \cdot 15 \cdot w$



$\left.\begin{array}{l} x = a\sin\theta \\ y = a\cos\theta \end{array}\right\}$ (1) thrust $= \cancel{g\ 2xl\,dx\,w} = \int g\rho \cdot y \cdot 2x\,dx.$

$= 2w\int xy\,dx = 2w \cdot a^3 \int_0^{\pi/2} \sin\theta\cos^2\theta.$ $\qquad \left\{\begin{array}{l} dp = g\rho_1 dh \\ = g\rho_2 dh. \end{array}\right\}$ $= 2w \cdot 3^3 \cdot \frac{1}{3} = \underline{18w}$

Chapter 4

Strategical kinematics

A simple comparison of two measures of effectiveness will sometimes be sufficient to answer a strategical question for eg. 3 (c)(iii) — Measures of effectiveness can be statistically or analytically determined – examples below of mathematical theory basic to strategical applications of the constants measuring effectiveness.

(a) Force requirements – Case of antisubmarine war in the Atlantic – Calculation of number of antisubmarine patrol planes needed in a sea frontier to escort shipping –

L = transit time = $\frac{1}{V}(100 + 2D)$

(i) Requirements for air escort – No. of plane req^d at base so as to be able to give continuous escort = $\frac{720\,T}{N(T-L)}$ (T = max. no. of flying hours for a plane, N = av. no. of hrs a month a plane can be operational)

(ii) Expenditure of depth charges –

(b) Lanchester's equations – Differential eqns involving measures of effectiveness – A great number can be set up each corresponding to a tactical or strategic situation – Examples given below give indications for further investigation rather than as already proved ones.

(i) Description of combat – Simple diff. eqns studied by Lanchester during World War I – principle of concentration – old & new wars – in new war concentration of superior numbers gives immediate superiority – Red & blue forces

(ii) Linear law – If m = no. in Red force, n = no. in the Blue force, T = time for sequence Δ t for continuous time variable, E = exchange rate = dn/dm

$$\frac{dm}{dT} = -\frac{1}{1+E}\,;\quad \frac{dn}{dT} = -\frac{E}{1+E}\,;\quad m = m_0 - \frac{T}{1+E},\quad n = n_0 - \frac{TE}{1+E}\,;\quad n_0 - n = E(m_0 - m)$$

In terms of t, $\frac{dm}{dt} = -\frac{1}{1+E} F(m,n,t)$; $\frac{dn}{dt} = -\frac{E}{1+E} F(m,n,t)$ giving $\frac{dn}{dm} = E$.

- Solns represent average values in the sense of probability theory - If $E = \frac{n_0}{m_0}$ then one has

$(n_0 - n) m_0 = n_0 (m_0 - m)$ ie $\frac{n}{m} = \frac{n_0}{m_0}$. no advantage in concentration.

(iii) Square law - modern case - Here $\frac{dm}{dt} = -\frac{n}{1+E} G(t)$; $\frac{dn}{dt} = -\frac{mE}{1+E} G(t)$

$\frac{dm}{dn} = \frac{n}{mE}$; $n_0^2 - n^2 = E(m_0^2 - m^2)$. This is Lanchester's square law.

- Example of 1000 men on each side & tracing of the engagement.

(iv) Fighting strength — Eqns & solns have great range of approx. solns & suggest useful investigations & an important part of O.R. is to find how nearly Lanchester's laws apply — another quotation from Lanchester.

(v) Mathematical solution - Kospman's solns of square law - Let $t \to \tau$ by $\tau = \frac{\sqrt{E}}{1+E} \int G(t)\, dt$

$$\frac{dm}{d\tau} = -\frac{n}{\sqrt{E}}; \quad \frac{dn}{dt} = -m\sqrt{E}; \quad \left.\begin{aligned} m &= m_0 \cosh\tau - (n_0/\sqrt{E}) \sinh\tau \\ n &= n_0 \cosh\tau - (m_0\sqrt{E}) \sinh\tau \end{aligned}\right\}$$

- interesting property of hyp. solns is the acceleration towards the end - Eg. German collapse in Tunisia & air struggle of 1943-44.

(C) Probability analysis of Lanchester's equations — Assuming that results of each engagement are subject to laws of probability, probabilities of various outcomes can be computed.

(i) The linear law — After T contacts ($T < n_0$ or m_0), multinomial distn shows that probability of there being α Red units lost & β Blue units lost is

$$P(\alpha,\beta) = \frac{T!}{\alpha!\beta!} \frac{E^\alpha}{(1+E)^T}; \quad T = \alpha + \beta \leqq m_0, n_0$$

Also $P(\alpha, n_0)$, $P(m_0, \beta)$ can be separately calculated, $P(m_0, n_0) = 0$.

(ii) The square law - Probabilities that α Red units are hit out of the total of n Red units, & β are given by $P_r(\alpha) = \ldots$, $P_b(\beta) = \ldots$

from which we confine α_{avg} & β_{avg} - If τ be not particularly small (τ = duration of engagement)

$$\alpha_{avg} = m\left[1 - e^{-(n\tau/m\sqrt{E})}\right] ; \quad \beta_{avg} = n\left[1 - e^{-(m\tau\sqrt{E}/n)}\right]$$

– We can also write diff. eqns for the probabilities themselves. 3 eqns. ~~cal~~ initial condns. calculation tedious but straight forward.

(iii) An example – Two numerical examples to illustrate use of above eqns.

(d) Generalised Lanchester equations – Generalisation to wars from battles – notion of rates of production P and Q.

(i) Loss rates – assuming P & Q constant, generalised L-eqns are

$$\frac{dm}{dt} = P - an - cm ; \quad \frac{dn}{dt} = Q - bm - dn . \quad (a, b > c, d)$$

Such diff. eqns occur in chemical kinetics & struggle between animal species – Solns – Particular case of $a = b$, $c = d$ (opposed forces equally effective).

✓ (ii) Typical solutions – Solns in particular cases shown graphically.

✓ (iii) Destruction of production – ~~If~~ P & Q ~~be~~ not constant – If $m = m_t + m_s$, $n = n_t + n_s$ (tactical & strategic), we can take red production $= P\left(1 - \beta \cdot \frac{n_s}{m_t} n_s\right)$

Blue " $= Q\left(1 - \beta \frac{m_s}{n_t} m_s\right)$

& eqns become
$$\frac{dm}{dt} = P - P\beta \frac{n_s^2}{m_t} - \alpha(n_t + m_t)$$
$$\frac{dn}{dt} = Q - Q\beta \frac{m_s^2}{n_t} - \alpha(m_t + n_t)$$

✓ (iv) Tactical & strategic forces – To solve eqns above we must devise rules re. rel. strength of t & s forces – This has to be done such that for each side one's own loss rate is as small as possible ie Red general should strive to make the expn

$$\frac{dm}{dt} - \frac{dn}{dt} = P - Q - \beta\left[P\frac{(n-n_t)^2}{m_t} - Q\frac{(m-m_t)^2}{n_t}\right] = L(m_t, n_t)$$

as large as possible & simly Blue general strives to make this as small as possible – ie R-general tries to maximise L by adjusting m_t & B-general to minimise L by adjusting n_t ~~&~~ This is the minimax principle

requiring $\frac{\partial L}{\partial m_t} = 0$, $\frac{\partial^2 L}{\partial m_t^2} < 0$; $\frac{\partial L}{\partial n_t} = 0$, $\frac{\partial^2 L}{\partial n_t^2} > 0$ - corresp. soln is called basic soln & is best soln when opponents have equal intelligence.

(v) The minimax principle — Solution for the L in (iv) represented graphically - the values of m_t & n_t obtained here if substituted in the diff eqns of (iii) are of Lanchester generalised forms & their solns can be studied graphically and are quite striking.

(e) Reaction rate problems — Analysis of increase & attrition of forces by eqns related to chemical reactions - example from antisubmarine war in Atlantic - three distinct types of offensive action (1) hunted in Atlantic, sunk & damaged there (2) repair bases bombed (3) factories in Germany bombed.

(i) Circulation of U-boats — Study of relative effects of above three types of action — study of quantities A, B, P, S, t, L, M, CB, 1/K relating to circulation of the U-boats. $(P - S = I)$

(ii) Eqns of flow — Eqns for change of A & B (A = average no. in Atlantic, B = " " in bases)

Putting $x = \frac{CA}{M}$, $y = \frac{CB}{M}$, $b = \frac{I}{M}$, $k = \frac{K}{C}$, $u = Ct$, operations become

$\frac{dx}{du} = b - kx + (1 - e^{-y})$; $\frac{dy}{du} = kx - (1 - e^{-y})$ - using series expansions for e^{-y} & taking large y we get approx.

(iii) Typical solutions — Graphically represented & conclusions drawn.

Chapter V

Tactical Analysis

Contributions of O.R. to tactical analysis - manner to be found for new equipment & tactics by enemy — observational & analytical approaches — to obtain measures of effectiveness by observational data or studying tactical tests & then devise analytical study (See also Chap. 7).

(a) Statistical solutions — O.R. data making clear the appropriate tactics for ship

manoeuvring to dodge an incoming suicide plane. Necessary to find whether radical manoeuvring would spoil aim of incoming Kamikaze more than spoil aim of defensive AA fire.

(i) Damage due to suicide planes – Study of 477 such attacks.

(ii) Effects of manoeuvring – Destroyers & small fleet should not employ radical manoeuvres

(iii) Effect of angle of approach – A ship is safer if it presents its beam to a high diver, and turns its beam away from a low diver.

(iv) Reasons for the results – For conclusions in (iii)

(v) Suggested tactics – Based on (i) to (iv).

(vi) Submarine casualties – Proper comparisons necessary – Pacific war

(vii) comparison with Japanese submarine casualties.

(viii) operational data – Showed that Japanese submarines were likely causes of some of the U.S. submarine casualties, a probability not considered before.

(ix) Suggested manoeuvres – on basis of (viii)

(b) Analytical solution involving search theory – Great deal of tactical analysis involves principles of the theory of search – Details presented in another volume of this series (?) – The important theorem called "mean free path theorem" to be discussed here. Examples below.

(i) Covered area – equals twice the effective lateral range of vision of plane, times the speed of plane, times the length of time spent in searching the area A if search object be a patrol plane – similarly we have depths for a 16"-shell or a torpedo as search objects – The mean free path theorem gives the probability of success as a fn of the ratio between the (covered) area & total area.

(ii) Probability of hit – If α be (covered) area, $P(0,\alpha)$ = probability that target is not yet discovered (or damaged) before this instant – $\delta P(0,\alpha) = -\left(\delta\alpha/A\right) P(0,\alpha)$ of which soln is $P(0,\alpha) = e^{-\phi}$, $\phi = \left(\frac{\alpha}{A}\right)$ = coverage factor (A = total area). Probability of hit $= P(>0) = 1 - e^{-\phi}$. This is a Poisson distn – Generalising for m times hit, $P(m,\phi) = \left(\frac{\phi^m}{m!}\right) e^{-\phi}$; $P(>0) = \sum_{m=1}^{\infty} P(m,\phi) = 1 - P(0,\phi)$ $= 1 - e^{-\phi}$.

(iii) Merchant vessel sinkings — Probability of coming through unscathed in n trips is $P(0, \phi)$ with $\phi = n/10$ if vessel can make $2\frac{1}{2}$ trips across ocean before being sighted by submarine.

(iv) Area bombardment — applns to mortar getting hit by shells and to study of the effectiveness of mine fields.

(v) Anti aircraft splash power — defn of splash rate for a given battery of AA guns — probability of given plane being shot down between range r & $r+dr = s(r)\,dr$ [$s(r)$ = splash rate) — eval^{n}

(vi) Ships sighted and sunk by submarines — comparison of effectiveness of submarines used on independent patrol against those used in coordinated attack groups — saturation effect with increase of no. of submarines

(vii) Independent patrol — further detailed discussion from (vi) — property of Poisson dist^{n}

(viii) Group operation — further detailed discussion — yield per submarine in a group of 3 (these gave optimum results] ~~[3 not sugar~~ was about 50% greater than yield per ind. submarine. — analysis borne out in practice.

(ix) Disposition of CAP protection against task forces — It is advantageous to place the CAP units in such a way that the interception will take place, on the average, as far from the task force as possible (combat air patrol protecting task force from enemy planes).

(x) Analysis of tactical situation — $W_n(\rho) = \int_{\rho}^{\infty} F(R)\,\overline{E}_n(R,\rho)\,dR$ = effectiveness of CAP patrol disposition

(xi) Simple example — above calculation with simple expns assumed for E & F and conclusions — if only a single CAP unit is aloft at a given time, its most effective pos^{n} is directly over the task force (unless dir^{n} of enemy attack be known).

(xii) Several CAP units — Integration of eq^{n} in (x)

(xiii) Tactics to evade torpedoes — continuation of submarine vs submarine problem of (a) (vi)

(xiv) Geometrical details – continuation of (xiii) with geometrical illustrations – During Pacific war at least 4 U.S. submarines were saved by adopting these tactics.

(c) Measure and countermeasure – exciting opportunities for O.R. workers lie in devising countermeasures – requires good deal of technical background

(i) countermeasures to acoustic torpedoes – torpedoes steering steering towards noise.

(ii) Radar Countermeasures – Radar field the greatest arena of countermeasures

(iii) Naxos search receiver – Radar intercept receiver covering the 8–12 cm band

(iv) Allied reaction – first error was overestimating the efficiency of Naxos GSR

(v) Intermittent operation – intensive allied use of search radar.

(vi) Estimate of effectiveness of enemy's measures – Prefer estimation of the time to introduce the countermeasure.

(vii) discrete operation trials –

(viii) Mathematical Details – Calculation of probability that Tactic 1 or Tactic 2 is more successful – Ratio successes to trials for Tactic 1 against do for Tactic 2, graph –

(ix) Rules for Trials – The above conclusions (Probabilistic) reduced to approximate rules.

(x) continuous operations – (viii) & (ix) for discrete trials extended here to continual effort – Similar rules obtained.

(d) Theoretical analysis of countermeasure action – Analysis in advance of four cases viz: Enemy measure, Enemy no measure, use of Countermeasure & no use of Countermeasure – $W_{11}, W_{12}, W_{21}, W_{22}$ – Adjustment of Tactics to make W_{ij} as large as possible while enemy tries to make it as small as possible – minimax principle

(i) Definite case – $W_{21} > W_{11}$ (assumed also $W_{22} > W_{12}$ & $W_{11} > W_{12}$) – In this case best always to use countermeasure

(ii) Indefinite case – $W_{11} > W_{21}$ and $W_{22} > W_{21}$ – To be more specific take $W_{22} > W_{11} > W_{21} > W_{12}$ –

In definite case minimax = maximin & in indefinite case minimax ≠ maximin – correct soln is to alternate solns at random with some predetermined relative emphasis.

(iii) Soln for the indefinite case – Choosing

2	0
1	3

as the W-measures – illustration by figures of choice of tactics at random.

(iv) Case of three choices

4	1	1
0	3	1
0	0	2

for W-measures

– use of triangular chart for discussion – mixed soln is the safest – figures given

(v) Barrier patrol for submarines – of a plane guarding a channel from passage by a submarine – detailed discussion introducing probability fns $P(x)$, $\phi(x)$, $\psi(x)$ and G. (probability of contact) – Safe tactics for side B (submarine) shown analytically & by drawing – Safe tactics for side A (barrier) also obtained – Safe tactics for both sides – Studies of Von Neumann & Morgenstern on theory of games shows that there are solns to each problem, but technique of soln is not obtained – only three cases considered above & more investigations are necessary on this interesting and fruitful subject.

Chapter VI

Gunnery & Bombardment Problems. (mathematical)

Evaluation of weapons & analysing best methods for their use – These should be carried out by scientists in laboratories – However O. R. worker should be familiar with techniques of evaluating weapon performance just as he should know theory of probability – probability

that target is destroyed by hit & probability that it will hit target, both to be considered.

(a) Destructiveness of weapons - lethal area - lethal area or lethal volume in case of depth charge.

(i) Multiple hits - single hits sometimes not sufficient - Probability $P = D_1 P_1 + D_2 P_2 + \cdots$ where $P_1, P_2, \ldots$ are probabilities of 1, 2, .. hits and $D_1, D_2 \cdots$ are probabilities of sinking ship by 1, 2, .. hits. $D_1, D_2, \ldots$ are damage coefficients - a good approximation is $D_n = 1 - (1 - D_1)^n$ which is law of composition of independent probabilities - vital spot theory. Here lethal area = effective area $\times D_1$ - D as a fn of the spot (x, y) or (x, y, z), then lethal area $L = \iint D(x,y)\,dx\,dy$.

(ii) Random or area bombardment - $P_1 = L/A$ & $P_n = 1 - e^{-nL/A}$ - numerical example.

(iii) Aimed fire - small targets - Here $P(x,y)\,dx\,dy = \frac{1}{2\pi\sigma_c\sigma_d} e^{-\left(x^2/2\sigma_c^2\right) - \left(y^2/2\sigma_d^2\right)} dx\,dy$

& near target $P(0,0)\,dx\,dy = \frac{dx\,dy}{2\pi\sigma_c\sigma_d}$ (σ's are deviations)

Probability of destroying target $P_1 = L/2\pi\sigma_c\sigma_d$ & $P_n = 1 - \left(1 - \frac{L}{2\pi\sigma_c\sigma_d}\right)^n$

$\simeq 1 - e^{-nL/2\pi\sigma_c\sigma_d}$.

- numerical example.

(iv) Aimed fire - large targets - Here no deviations to consider & $P_1 = \iint D(x,y) f(x,y)\,dx\,dy$

$-P_n = 1 - (1-P_1)^n \simeq 1 - e^{-nP_1}$ - Chance of sinking ship with n shots - from emplacements.

(b) Pattern firing - no ballistic dispersion - firing shots simultaneously instead of aiming each shot individually & firing consecutively - salvo firing (same direction) or pattern firing (slightly displacement) - aiming error & ballistic error - Here we consider case where the second error is negligible - Basic philosophy underlying

(i) Example from Train bombing – Fundamental problem of pattern firing is to find the best pattern to use – to find best stick ~~placing~~ spacing of 2 bombs carried by a plane flying $\perp^r$ to rail road – Pattern damage function – ~~three cases~~ figure for damage function for spread of three torpedoes (corresp. to bombs in salvo) – Three cases considered & ~~not~~ the corresp. probabilities tabulated & compared.

(ii) Squid problem – effectiveness of antisubmarine device (squid) throwing three proximity ~~fuzes~~ fuzed depth charges ahead of the launching ship in a triangular pattern – Damage fn for squid pattern shown by a figure – finding of the best pattern.

(c) Pattern firing – Ballistic dispersion present – bombs, shells etc. do not hit the targets even if aim is perfect due to ballistic dispersion of the projectiles – reconsideration of problem of airplane dropping bombs on railroad – graphical representation of damage function.

(i) Approximate soln for large patterns – simple approximate solution

(ii) Example of the approximate method – Illustration of above method by design of a depth charge pattern.

(iii) Probability estimates by the approximate method – Continuation of (i).

(d) Sampling method – (c) gives rapid approx. soln but if exact solns are req'd analytical methods are impractical or too difficult to carry out – method of by a simple process of sampling – Description of sampling process by taking a sufficiently large number of trials & averaging the results for ~~estimation~~ & finding expected damage for the pattern, and repeating process for various patterns thereby

Calculating the best pattern - technique described below.

(i) Construction of sampling populations - to basis for the construction of new sampling populations of aiming errors, ballistic errors & the like, we shall use a table of random digits - methods of preparing such tables, drawing of balls from urns or rolling a die twice for each digit & translating the rolls into digits by a table or using published tables of random digits like Tippett - sampling a stochastic variable - sampling a normal distribution

(ii) a rocket problem - figure illustrating sample pattern of salvos of two rockets

(iii) Short cuts in sampling method - short cuts & tricks used

(iv) Train bombing - Rules for computing the result.

Chapter 7

Operational experiments with equipment and tactics

operational data are observational & not experimental - Supplementing operational data from operational experiments. These cannot take place of actual data obtained from battle but usually they are of great help & should be gathered during peacetime carefully - enables a country to keep abreast of new technical developments during peace rather than have to catch up at cost of lives when next war begins - Imp. & useful role for O.R. for armed forces in peacetime.

(a) Planning the operational experiments - results aimed at should be a series of numerical answers for measures of effectiveness - maintenance problems of equipment - to handle all gear entering into the operation by crews & scientists to observe & not interfere in the operation and not take part in the exptl. for eg. in AA firing

(i) Preliminary theory – Helps planning – computers – Data to be reduced as fast as expt. goes ahead.

(ii) Preliminary write up – fine items not to be written down in detail.

(iii) Plan of procedure – the fifth item mentioned above – recording of facts & not impressions – use of movie cameras – Data to be understood during the testing period – Interim reports if necessary.

(iv) Significant data – At least one person with field experience to be in the team – Sometimes not necessary to get a spread of data – data leading to tactical Conclusions.

(v) Conclusions – interpretation & conclusions.

(b) Accuracy measurements – test programmes to improve accuracy of weapons – analysing sources of errors to improve accuracy – separation of effects of aimer errors & ballistic errors.

(i) Calculation of standard deviation – use of probability paper.

(ii) An example – to show details of above calculation

(c) Evaluation of detection equipment – To find numerical value of measures of effectiveness in the tactical situations in which the equipment is to be used.

(i) Sweep width – From direct trial – difficult to make because large no. of flyings required – Hence use of range distribution methods.

(ii) detection expectancy – $P = 1 - e^{-\phi}$ – use of this to detect 'bad spots' in the performance of an experiment.

(d) Survival problems of gunnery – analysis similar to (c) (ii).

(i) AA gunnery – practice firings in which a target (a drone) is made to approach the gun position, using evasive measures – hit expectancy

(ii) Hit expectancy – $P = 1 - e^{-E}$ – use of this in testing the expt. performed.

Chapter 8

Organisational and procedural problems

O.R. has no separate existence. separate existence would be like separate existence of the frontal lobe of a brain without the rest of the brain and body – organisational problems depend upon whether the organisation is industrial, governmental or military – most examples below refer to last category – during World War II it took a great deal of missionary work to persuade security officers to release highly secret information to O.R. workers even though they did nothing more than think about the information given – Impt. requirement for O.R. worker is to become acquainted with operations first hand. He must go into the field, fly in the bomber plane, operate the radar etc. This sometimes means living away from Central group of O.R. workers.

(a) Organisation of an O.R. group for military service – O.R. groups must have broad knowledge but should have very little power & responsibility ie not to issue orders which should be left to Officers – question of transmission of technical knowledge & hierarchy of Officers – Cases of urgency.

(i) Importance of contacts with several echelons – O.R. group should have access to different levels in the hierarchy – problem of supplementing 'normal channels' is diffl. for O.R. Workers – otherwise it will be waste of technical talent.

(ii) Assignment of group – O.R. group to be assigned directly to operating command & to make reports directly to the various echelons of the command.

(iii) Subgroups in the field – relations to subgroups of O.R. workers & central group – circulation of workers between field & centre.

(iv) Reports & memoranda – to be carefully drawn – allegorically group to be considered to be separated from rest of military organisation by a semi-permeable membrane – internal stories & notes & Reports for circulation to the rest of the service – must be brief – Oral briefing

(v) Status of groups members – to be left in civilian status – in outlying theatre may be in uniform – sometimes given temporary rank. its dis advantages. un mobilisation. Cannot approach lower ranks – special insignia – no general solution possible

(vi) Implications for non-military application – industrial & Govt. problems.

(vii) Recruiting & training of O.R. worker – No university having a special course – experience in research – physicists specially useful – Training in physical Chemistry is a good background – mathematicians are particularly useful. capacity for abstract thought useful in analytical stories – biologists, psychologists & economists – need of preliminary training

(b) <u>Operations research in the field</u> – aspects of field work – ORG working with C-in-C of U.S. fleet as an example. Purposes served by field representatives <u>three</u> in number

(i) Assignments: Liaison, Staff, Operations, training, experimental (operational), experimental (equipment) – remarks on above five types – Combinations of above types – Case of non-military O.R. men

(ii) Types of field work – Analytical, statistical, Liaison, Experimental, Educational, Publication – Remarks on above six types –

different kind of subdivision into strategic & tactical work - Example of O R G men at Argentina working on routing of convoys (strategy) & O R G men in London working on problems of optimum size of convoys (tactics).

(iii) General Comments - Selection of location for a new field assignment & initial installation of a man - personality of the man. Background to include physics, engineering & matters including statistics.

(1)

3. Science at War by Crowther & Whiddington - Chapter II on O. R. (1947)

Development of O. R. a chief scientific feature of war - Science as idea for new weapons & improving old ones. This is old notion. Examples - modern form science entering in a new way - Starting point of O. R. in study of radar equipment - Short history of T. R. E. at Bawdsey - G. A. Roberts & Smith - airborne radar for night fighters - Ground control interception systems - O. R. sections started in R. A. F - Blackett as head of O. R. section at Coastal command - A A command research group under Blackett called Blackett's 'circus', this consisting of physicists, two math. physicists, an astrophysicist, an army officer & an ex-Surveyor - Blackett's connections with Army, Navy & Air force - mats of wire netting on the ground round radar receiver, grass growing under them luxuriantly. This caused by introducing geese to keep the grass down & asking for 12 A. T. S. girls to look after the geese - E. J. Williams' work on setting of depth charges - Blackett a former naval officer - Zuckerman's investigations on effects of bombs - Bernal's forecast - guide to future bombing policy - Eight section of AACRG - Sunday Soirees

(a) Applications of operational research:

(i) Depth setting for depth charge as 2 ft. by E. J. Williams

(ii) Optimum size of convoys - Haldane & Whitehead.

(iii) Making a little go a long way – Air cover to convoys against U-boats – outstanding part played by O.R. sections in this

Ex

(iv) Getting more out of aircraft – E.J. Williams as a biologist trained in statistical analysis – theory of flying as much as possible ie to reduce grounding & improving maintenance – 100% increase.

Ex

(v) Bomber losses – Concentration of raids by 1000 bombers – forecast of plans needed to attack the Tirpitz. Recommendations & their exact verification (3 hrs)

Ex

(vi) Land fighting – varieties of operations

(vii) Shooting from tanks – O.R. recommendations

(viii) Laying mines – Steps recommended to increase efficiency of mine laying

(ix) The Jap trap – warning device against Japanese creeping up at night from forest.

(x) Bird echoes – Reducing these echoes which are ~~not~~ spurious & differentiating them from echoes due to aircraft

(b) The principles of operational research:

(i) Study of weapons (ii) study of tactics (iii) study of strategy.

use of biologists in O.R. for in War as in biology we have little information about phenomena of great complexity – use of probability. sometimes against common sense (for eg. continuous prosperity of bookies) – Commanding officer & scientist. War cannot be won by gusts of emotion & hunches – Inter-relationship of O.R. workers & service men – Remark of an eminent officer re. O.R. team "Those boys won't even use a soldering iron until they know the whole of the policy of the Pacific War" – Paper on methodology by Blackett.

Gen

(c) Quantitative theory of modern warfare: a general quantitative theory of warfare, a major military calculus – problem of destruction of aircraft by gunfire – kind

of shell required – Dr. Paxman's solⁿ – attack on a whole country, a town, a division or a ship. Maximising efforts with two variables: damaging power & accuracy (dealt with in M & K). Invasion of the problem – Nine function of Commander – 'logistics' used by American to describe the technique of packing stores & calculating how much of each is wanted. Comes not from logic but from French word "maitre de logis" – most effective weapon is one which defeats the enemy without killing any one. Hence pushing human & psychological factors. For eg. study of dive-bombing on troops. one sergeant major said "I dislike dive-bombing because it is so accurate", and yet his platoon had been dive-bombed six times without suffering a single casualty – fear of wounding has a powerful effect on morale. Effects of wounding should be better known – Air warfare largely susceptible to calculation & human factors involved are courage of skill of individuals (with automation may become purely scientific). Naval warfare is also susceptible to calculation to a lesser degree. Land warfare involves additional human factors & hence judgment & capacity to command are specially important here. The commander must appear to be master of situation, at least give impression of being able to control things. the air commander has no such difficulties. He can approach problem with more objective, calculated detachment & a less emotional attitude – modern technique of warfare shows tendencies towards graded sets of developments today to reduce war to a completely automatised and mechanised activity. The final aim is the completely automatic war – using favourised radio – what about atom & H. bombs? – ~~Hitler~~ Hitler's romantic view of war & systematic scientific view of war. The reason why Hitler lost – traditional romanticism of war is the contrary of the civilian scientific spirit, and it is therefore natural that when the scientist begins to join in the conduct of war, he enters into it as a civilian. That is why directors of O.R. are generally civilians, and one reason why war in the future will tend more & more to be conducted in a civilian spirit

p.171: Groodeve's suggestion that the ship's natural magnetic field be neutralised by magnetising the ship permanently in the other direction. He arranges that ships be "wiped" by with temp. hor. coils through which immense currents of thousands of amperes could be sent. This 'wiping' technique got rid of the nuisance of permanent coils in the ship. This had a tremendous effect on morale. Crew sailed with enthusiasm in wiped ships. Aged mariners came up to the scientists and shook their hands for saving their lives. Myths arose. One Captain is reported to have said after his ship had been mined: "Why, my dear chap, you could see the torpedoes going in all directions harmlessly" a tribute to de-gaussing which could sweep magnetic mines

Hist ✓

4. Scientists against time by J.B. Baxter (1952).

Official history of the office of scientific research & development of World War II. It recites extraordinary evolution of weapons from radar, amphibious warfare, aerial combat, the proximity fuze, and the atom bomb - introduction by Vannevar Bush - only parts of the book relating to O.R. have been summarised here.

p.24: O.R. workers acted as catalysts in many developments of outstanding importance. Admiral Furer wrote to Bush "That your group of would contribute brilliant ideas and achievements to the war effort was expected, but that you would be so versatile, and that the scientists and the Navy would find themselves so adaptable to each other's way of doing business was unexpected by many"

Hist ✓

p.41-42. war against U-boats - behaviour of sound in ocean lacking - dearth of OR men in early stages. In Dec. days of 1942 no Hd. Qrs staff had at its disposal adequate no. of personnel sufficiently versed in modern mathematical techniques to derive from the statistics of past & current operations important tactical lessons - barrier patrols devised - use of Magnetic airborne detector (MAD) - depth charges -

Hist ✓

Doenitz's tribute to the part played by Allied scientists in the defeat of the U-boat.

p. 84-97. The applied mathematics panel of the NDRC developed for the army air forces a bombardier's calculator, which would so determine the spacing of bombs in a train of bombs released from a single aircraft as to maximise the chance of hitting a target (See M & K also) – on basis of calculations the panel developed a bomb-spacing calculator – Other assignments to this panel – The mathematicians calculated the bomb loads necessary to destroy given fractions of the target in area bombing of cities – Air-ground cooperation (lack of it killed Gen Mc Nair by our own bombs). Airborne rockets & effect on German morale – contributions to the effectiveness of the B-29 by Appl. Math & Appl. Psychology panels – Ground controlled night interception is most complex form of aerial combat – jamming by window & Carpet together – Gee & Loran –

p. 105 – Neuropsychiatric cases treated by psychiatrists – relation of clothing to fatigue – Care of combat soldier's feet, skin trouble etc.

p. 122 - 123 – ~~Dr Conant ascertained that teams American scientists & engineers be dis~~ American & English cooperation in scientific effort – A. V. Hill in 1940 in Washington – Tizard's arrival in U.S. with secret documents for interchange. In the early days U.K. gave more than what they received) – Cockcroft & R. H. Fowler's report of scientific liaison approved by U.S – Establishment of London office of NDRC due to Conant's efforts – Arrival of Conant in London in the middle of the German Blitz – Signing on March 11, 1941 by Roosevelt of Lend-lease Act & issuing a challenge to Germany greatly moved England – ~~Perfect~~ cooperation assured – sending of teams of American scientists & engineers to England to familiarise themselves with operation of rockets, fuzes, radar etc, and sending on Electronics Training Group sent to England by Conant – Establishment in Washington in April 41 of British Central Scientific office headed by Darwin – Pooling of reports & exchange of visits – Extension of Bush's activities to research on military medicine & his appointment as Director OSRD – Conant is Chairman NDRC.

p. 125 - 126

- Responsibilities of NDRC & OSRD precisely keeps & bounds - Reorganisation of NDRC ~~Setting up of~~ into 18 new divisions - setting up of App. Math. panel in Sept. 43 & also App. Psychology Panel - A third subdivision of OSRD viz Office of Field Service under K. T. Compton formed — National Roster of scientific men made - Choice & allocation of scientific personnel, overall plans re. this. long & difficult struggle for adequate scientific personnel & problem of choice of manpower for fighting forces or the army - Case of Mosely. Bush's letter to Secretary of War " In 1914 Britain sent to the front line trenches one of the greatest modern physicists. He had revealed for the first time some of the secrets of the nucleus of the atom, and he was known throughout the world as one of the geniuses of his day. His name was Moseley. He was soon killed in action .... This example was repeated with many others of lesser calibre by Britain in 1914. In the present war Britain has learnt a lesson & utilised its young scientists well." — Wars can be lost if gifted and well-trained scientists are in the wrong place, in uniform or at the front or wholly outside war effort.

p. 156 - Radar effort.- Oboe - Gee - Oboe enabled two men sitting in trucks on British soil to know within a few yards each way the position of a British or American plane over the Ruhr, and to know it better than did the crew of the plane.

p. 166 - Jamming carpet - airborne transmitters tuned to a slightly different frequency in each aircraft, radiate a torrent of 'noise' which obscures the position of the formation in the enemy radar, causing inaccurate gunfire (See also M & K) — Window

p. 168-69 - Study of anti-jamming techniques - program of Counter - Counter measures - Formation of anti-jamming Committee of Div. 15 — OSRD contract with Harvard University for operation of the American British laboratory of Div. 15 (ABL-15) - novel enterprise for a University but a great success - Operational use of countermeasures equipment involved some of the most extraordinary duels fought in the history of war & did a mighty job for early victory - deception in causing landings in Normandy & N. France of American troops.

p.172 - Contributions to submarine warfare - geometry of attack

p.176-78 - Introduction of pulse techniques - sonar technique - The antisubmarine ~~Warfare~~ Warfare Operations Research Group - Its scientists flew on long patrols & took active part and did a hundred & odd trips · Many of them were nomads of science. One of them Shea travelled over 300,000 miles going from activity to activity. Wherever they went they were welcomed.

- British 'hedgehog' & American 'mousetrap'

p.180-86 - use of bathythermograph - Bearing teachers & attack teachers - MAD and expendable sonobuoy - Suppl. work of Div. 6 in the field of mines & torpedoes under Houston is still secret - Operational analysis indicate that aircraft were to become more important if anything than surface craft in antisubmarine work - antisubmarine work of Div. 6 transferred to pro submarine work to be used against Japan - In Div. 6. Collaboration between scientists & navy was superb & had the two triumphs of the defeat of the U-Boat & strangulation of Japan by submarines

p.195 - The Pelican - idea of a glide bomb which would follow a radar beam directly to the target was the idea used in this - Navy's verdict of 'no operational use' against the Pelican - Development of the Bat, the first and so far the only completely automatic target-seeking missile to be used in warfare · It a glider bomb which located its target by radar, then, like a bird dog, tracked down its prey, steering itself towards the chosen target by following the reflected radar beam - Aerodynamic difficulties overcome by the Roc using a suitable radar receiver - Felix came too late - guided missiles project - Aim is to develop a sort of V-2 to fly from one hemisphere to the other - ~~by guided~~ trans-ocean This is a problem high on the applied scientists' list of unsolved problems.

p.208 - Rocket - recoilless - the V-2 fastest missile yet made by man -

p.244-47

p.222 - To find a fuze operated not by time but by proximity to the target - proximity fuze - a purely American invention - Next to atom bomb the most remarkable scientific achievement of the WAR - radio proximity fuzes with a conventional 5" Cathode ray oscilloscope & a submarine radio tube used - done with highest degree of cooperation between American science, American industry & the armed forces.

p.244-47 - DUKW & WEASEL - Special vehicles.

p.288-89 - Used smoke screens - incendiary bombs & high explosive aerial bombs - relative merits.

p.292-96 - Flame throwers - Role played by Chemists - hydraulic flames, insecticides & penicillin - explosives of unexcelled power, screening them with new & better smokes, flame throwers & incendiary bombs ✓✓

p.375 - medical research

p.377-393 - Aviation medicine - physiologist - oxygen supply - High altitude problems on human body - 'bends altitude' - lighting / wacher on airfields & aircraft - A.V. Hills work - designing goggles - Seeing at night - courses in visual physiology - 'rough air' - motion sickness - 'pullouts' - human centrifuge - heart action & blood pressure - 'anti-g-suit' - forces of acceleration.

p.395-417 - Use of psycholists to pick men - AAP - classification program - three assignments to AAP - work on effective use of machines by AAP - P.M.S. Blackett father of O.R - O.R. under Army, Navy & Airforce - Blackett said "Many war operations involve considerations with which scientists are specially trained to compute, and in which young officers are not in general trained. This is specially the case with all those aspects of operation into which probability

Hist ✓✓

considerations & theory of errors enter ... the scientist can encourage numerical thinking on operational matters, and so can help avoid running the war by gusts of emotion ...?" – Early development of O.R. in submarine warfare. Creation of ASWORG – By summer 43, Morse had recruited 44 men: 6 mathematicians, 14 actuaries, 18 physicists, 3 chemists, 2 biologists & an architect (Is this Morse's Census?) – effort at search problem – convoy barrier patrols – Best course for destroyer in trying to make sound contact with submarine forced down by aircraft – search rate – operational data punched on IBM cards & then analyzed by machine methods – Studies for radar & sonar search plans, air-sea rescue, use of MAD & radio sonobuoy – depth setting doctrine – field of countermeasures – Formation of Central ORG with Morse as head & sub-groups – Airforce interest in OR – Princeton Univ. courses of six to eight weeks on probability, matrices, mechanics & dynamics, in weapon analysis & ballistics & air photographic interpretation & and bomb damage analysis to small groups of four to six scientists at a time – Supp. training by AMP – Increase in its staff and assigning of men to as scientific consultants to about 15 Other subdivisions actuaries. – Work of AMP did not include only analysis of bombing accuracy but also analysis of rocket accuracy & multitude of problems re. gunfire (See M & K) – This worked proved extremely effective in developing new weapons & tactics – By summer 43 the no. was in hundreds – Still great need – met by transfer of men from labs to field – Scientific consultants to remain civilians so as to retain primary loyalty to the lab – formation of OFS. Its men to be sent to help several projects & missions from OFS to be sent wherever wanted – Operational analysis for Navy by ASWORG in which OFS also cooperated – Civilian in field service did not become a member of the armed services, but though he wore an army uniform with a shoulder patch inscribed "Scientific consultant". He was accorded assimilated rank which entitled him to the privileges of an officer as to messing, housing & transportation. The way in which he bore this sudden thrust imposed glory was one measure

of his competence as a combat scientist. If he took it in his stride, it was probable that he would be all right. Most men did. A few became more military than the military — Example of a 62 year old dermatologist being a great success — Other examples — a metallurgist who ruled as a major for three days in an emergency — only one civilian died was killed in the field by enemy action. only one wounded. 42 were picked out for special mention

Pacific branch of OSRD — History of OFS — wonderful expt. in the relations of civilians with officers. a successful expt.

5. Combat Scientists by L.R. Thiesmeyer & Burchard (1907)

Foreword by K.T. Compton — Combat scientists in OFS. Their ability & good judgment won the recognition and support of the overburdened officers of the area, and their sense of humour enabled them to survive the inevitable difficulties in launching an activity without military precedent. Dr. Morse (ORSG in Navy) was rarely in active theatres but he had the attributes of a Combat scientist — Appointment of OFS by Bush & Compton chief of this — The book is mainly a record of OFS activities

Chap I — Blue print for field service — Civilian specialists goes up front — Description of Prof. Compton's assignment — dressed as an officer. Card in his pocket mentioning him as consultant. It specified that if he were captured by the enemy he should be treated as a Colonel in the regular army — not inoculations he took — trip to California — military prejudice against civilians — reluctance of Pacific officers to ask for OSRD help — missionary aspect of Compton's trip — Generals receptive — relations between officers & scientists — officers welcoming of O.R. men working with them — Decision to retain civilian status — Question of rank — combat scientists to move from theatre to theatre to avoid disturbing administrative work — Gen. MacArthur's welcome to Compton

p. 23 — terms "operational analysis" & "operations research" used more or less interchangeably — In literal connotation it meant any careful, systematic & critical study of

military or naval operations – many functions of O.R. described – type of men req'd – British were pioneers. mention of Blackett – In 1942 due to Bush O.R. groups were formed in U.S – ORD under Signal Corps – OAD in Air Force did lots of work to boost OR – OR sections in all theatres demanded. More than 100 civilians – ASWORG in Navy. Civilian specialists loaned by Columbia Univ. to Navy. The work of this group greatly eliminated Nazi submarine threat – Difference between operation analysts and groups of field engineers or panels of scientific consultants for new weapons – OR services & OSRD supplying men. new subdivision of OSRD to plan loan & return of scientists from & to OSRD. This recommended formation of OFS to regularise work of OR men. The work pool of scientific manpower has been repeatedly tapped for field work but never replenished. Also OSRD men (other OR workers also) were in the London liaison office (ABL & BBRL), in ASWORG in Navy was growing & in Air Force had fan-flung OR sections. Hence a division of OSRD called OAD was formed & this became OFS. – Bush formed this on 15/10/43 with Compton as Chief, Waterman as Deputy Chief & Thiessmeyer (geologist) of ASWORG as Technical Aide – Compton meeting the three generals – Return of Compton to Boston. He had opened the way.

Chapter II – From VT-fuze to fungus – Scope of the office of field service.

types of men that the officers in S.W. Pacific wanted to have – OFS also in other theatres – once prejudice against scientific civilian help was broken, the demands for it became a crescendo that mounted steadily – Navy first to use OFS & had the rule that scientists thus made available would be under Navy regulations & under Navy supervision re. security & duties – Army used it later. out 87 OFS projects, 55 were Army sponsor & ten were under joint auspices of Army & Navy – Redtape – basic principle of OFS in supplying men – Transfer of ASWORG to OFS (Take & Morse named Deputy Chief of OFS) – Places where OFS men were to be found – Hist

Dr Morse came to occupy in this country a position comparable with that of Blackett. He was the Navy's foremost expert & was consulted by both Services. As work of ASWORG became more & more complicated it was renamed ORG. At end of war it was taken over from the OSRD & permanently attached to the Navy.

100 OFS men sent to Pacific – OFS in Central Pacific Area under Marshall & Burchard had nearly 50 men & was designated by Army as ORS – work of OR men in going into ~~[illegible]~~ territory won in German & collect ~~scientif~~ information re. German scientific war research – stupendous task of these men (attached to ALSOS) – work of ALSOS of course a secret

p. 53 Adventures of OFS men & experiences – interesting.

Chapter III – Man power headache – The search for personnel.

Difficulties of finding suitable men – In less than a year OFS ~~[illegible]~~ had many men and at close of ~~[illegible]~~ war its roster included 464 names – nature of men recruited. Engineers, mathematicians, marine biologists & ornithologists etc – versatility of OFS men. Biologists studied behaviour of submarines, convoys & patrolling aircraft. Geologists became administrative officers, studied land-mine countermeasures & helped Army officers to develop a manual on jungle warfare. One Navy commander saying on telephone "The Navy needs a solid practical, down-to-earth scientist for this job, not an 'Oh-the-wonder-of-it all' type?" – Recruitment of men to OFS. Procedure unlike Civil Service appointments. In all income categories not a single man taken on contract hesitated to take a field assignment because of financial considerations – Company contracts brought in nearly 50% OFS men

Hist

Hist

Low casualty lists of OFS men – Age of OFS men 19 to 62 – Brian O'Brien (19) expert on infra-red devices deferred acceptance of a Navy Commn to spend months as OFS man in Navy uniform on a Pacific carrier & Dr J. G. Hopkins (62) who flew to New Guinea & stayed with troops throughout Philippines campaign. – Top men like Compton, Morse etc. loaned to Govt. without compensation – Personality & temperament qualifications also impt – modest & allowing others to take credit etc etc. – The supply of scientific men as a whole was meagre enough – the supply of combat scientists distressingly scanty (If this be so in America, what about less advanced countries?)

p. 68

Chapter IV – High-pressure Education – Basic training for the field man.

Unorganised, informal indoctrination of early days – Nature of requests made by the Services – They deliberated for some weeks before crystallizing a request to OFS, but once it had been made, they usually insisted that it be filled "the day before yesterday" – OFS established a pool of scientists who were immunised & received basic indoctrination & were then available on call.

Several types of indoctrination now arranged – In Navy indoctrination was simple & could begin after he joined it. ~~[illegible]~~. Excellent work of ORG (ie Navy Divn.) (p. 77). Meetings for discussion & British Admiralty Delegations invited at these meetings. Papers rehearsed and monitored with meticulous care ~~omitting~~ and presented omitting some of the more abstruse math. considerations & concentrating on conclusions & results that would guide Navy doctrine. Prof. Blackett remarked after attending one of these ~~meetings~~ sessions that the quality of the presentations compared very well with those of the Royal Society in England.

Hist

Plan for special indoctrination by OFS – Round table conference of OFS, AMP, OAD, and ATAD (in the Navy) presided over by H. P. Robertson decided on a 6 to 8 weeks course (Vide 4) – Nature of this course – The plan of developing a high-powered indoctrination never materialised.

Chapter V – Red tape & Brass buttons – Putting the scientist into uniform. – Adventures of getting into uniform.

Chapter VI – Checks & Channels – Keeping out of trouble – Communications with OFS men at the centre allowed in special cases directly in the Navy – more liberal in the Army

Chapter VII – Conning tower to quarterdeck – Science backs up the fleet.

p. 106 – Example of searching for a submarine with sound-detection gear as per tables prepared by ORG civilian scientists of OFS at Washington – analytical wizards in the ORG. –

p. 108 – ~~Pellam's~~ Fine description of Pellam's work in intercepting submarines crossing the Mediterranean.

Hist

p. 109 – Brilliant work of J. Steinhardt (Harvard biochemist) to stop German blockade runners crossing Atlantic bringing strategic materials to Japan – Surprise of Navy.

p. 110 – Convoy problem solved by Thimann (biologist from Harvard).

p. 112 – Introduction of "Schnörchel" by Germans via the fantastic breathing mechanism making it possible for submarines to run many hours without surfacing

– ASWORG (ORG) → (Air ORG) + (Phib ORG) + (AAORG) + (Space ORG).

The remainder (ORC) dealt with theoretical & analytical problems – ORC dealt with 1600 reports a month – 15000 items – ORC maintained a machine section equipped with IBM (International Business Machine) machinery for recording nearly ½ million mech cards & turned 5000 tabulations & summaries.

"Combat team without guns" (CE ORG) was under rigid security restrictions

p. 115 Nazi acoustic torpedo – Vehlup's countermeasures more effective – found in one month. Germans dumfounded at the speed of the Americans in diagnosing their secret weapon' – The Germans had no ORG's.

p. 116 – Rinehart's brilliant work – Disclosure prevented by security

p. 117 – 126 – Many important researches described.

Chapter VIII – Square pegs & round holes – Improving service placement of its scientific men.

Chapter IX. – The fight against disease & deterioration

p. 145 – 152 – psychiatric research.

Chapter X – Scientific sleuthing – Research behind those axis boasts

Ex use of OR

Letter of Gen. Keitel to German Physicists' letter. Physicists like Hahn, Heisenberg, Bothe and Geiger.

Hist

Incident of Dr A. A. Bates (metallurgist of Westinghouse Corporation) ruling a conquered town for a few days

Features of ALSOS (intelligence mission to Europe) – Goudsmit (nuclear physicist) to work with Col. Pash – Dr W. A. Noyes – Capture of launching sites of V-1 & V-2 – Nazi murder of 16 year old workers – E. C. Kemble – German physics & Jewish Physics cause of deterioration

Nazi scientists themselves at fault. When a Professor at Kaiser Wilhelm Institute used the word 'relativity' in a lecture, he was threatened with loss of his job – Capture of radium – Japanese scientists mostly educated in U.S & hence not given assignments by the Japs – Jealousy between Japanese army & navy

Goudsmit's report on German science in war

Chapter XI – Analysts of Air Attack – Selecting bombs & fuzes.

mass of information to be channeled to those responsible for ultimate plans is prodigious. It comes from many sources, mathematicians working on probability, expt. tests, reconnaissance photographs, & analysis of actual damage – analytical quality of mind possessed by mathematician, the lawyer or chess player to work at Hd. Drs of each Air Force necessary

Ex Joint

p. 189 – 90 – Bitters team heterogeneous – strong statistical bias – joint target group

Ex Chem

p. 193 – Joint work of (Air ORG), (ORG) (AFEB) – also Chemical work

Ex Chem OR

p. 196–97 – Ratio of (HE/IB) – Effect of I.B's on p. 197–98 most dramatic – 3 million tons of bombs dropped – Superiority of Air Power.

Chap XII – Less motion wasted – The army simplifies its work.

p. 202 'Flow – process chart' by Joe Hollywood cartoon character – Report of Mogensen & Goodwin on simplification – Hansen's study.

Chap XIII – The trucks that went to sea:

p. 214 – Description of amazement of two Naval reserve officers taken in DUKW driven by a Civilian – Stephen's expt. with a 105-mm Howitzer gun being transported by a DUKW – Brig. Gen. Bell's rebuke ordering Stephens to come back. Explanation by Stephens & invitation to lunch by the Gen. – Weasels, + Water Buffaloes

p. 225 and Penguins – tribute to combat scientists who designed these trucks that went to sea

Chap. XIV – From test tube and easel, Dam site & Zoo – Artists of Sabotage, Dexterous Chemists and versatile Engineers.

Russel – Cleaves – Dietz – Coon – Showers. Conference on Chemical warfare – FETU.

Chap XV – Hunting by formula & flying unseen – Defence against fighters & flak.

p. 240 – 42 – Parsons's work on flexible gunnery – contact with APM & Aberdeen Proving Ground (versatile) – his excellent work

Gunnery

– de Galley & Groves – math & physical factors affecting performance of A A guns

p. 244 – Darlington & Gen. Marquat – Research needs "~~Send us~~ Have the Japs send us more planes to observe" – His certificate to Darlington

p. 246 – Repainting planes & making them invisible ~~by~~ due to specular reflection in the searchlight beam – Right paint – Selbert went to India & selected Bangalore and a site in Assam (China? Good God) as the best places for

His

Operational Research

Kirkee, Poona. 12/12/55

(1) Operational Research Quarterly (O.R) - Vol 5, No. 1, March 1954

(a) operational research and accounting by Stafford Beer.

(b) Conference on O.R. techniques on experimental design, waiting problems, linear programming, symbolic logic, decision theory and information handling.

(c) The capacity of roads by J. G. Wardrop of Road Research Laboratory, Dept of S. & I. R.

q = flow of traffic = no. of vehicles per hour passing a given point in both directions

Capacity = flow of traffic which moves at the minimum acceptable average speed (an arbitrary element)

A test car was driven in the traffic stream at about the speed of the traffic round it along a number of streets of various widths. one observer timed its journey with a stop watch, and another counted the traffic that was met. Over 80 runs were made along the ~~roads~~ roads being studied at different times of day, equally divided between the two directions.

The streets were grouped according to widths, and the running speed (excluding delay at intersections) was plotted against the flow. In almost all cases, the speed is unaffected by flow up to a certain point beyond which the ~~flow~~ speed falls as the flow increases. A simple empirical formula can be fitted to these results:

$$q = 3(31 - v)(w - 6) - 430$$

~~q = flow~~ v = speed m.p.h (to be less than 24), w = width in ft.

Above is an average formula. Other factors affecting capacity viz parking, flow of pedestrians, particularly on crossings & presence of large or slow-moving vehicles — Use in planning of roads of road capacity measurements. ~~etc~~ is well illustrated.

(* Operational & Operations – U.K & U.S.A – Op[illegible]ch)

(1) What is operations research? military, surgical or op. calc? Ans is yes & no – Is it a branch of P.M or App.M? Probability & Statistics? Other Sciences like Psychology? or No – indep. Science* – young & vigorous – 20 years old – History – Blackett – Theoretical Physicists – best O.R. workers – not P.M, why? App.M's are good – academic research & fishing – in Air, Navy & Army, radar, anti sub & anti aircraft reply – time fuze of a bomb to be set – war time research classic – Battle of Atlantic won by O.R – After war extension to other fields: accounting, inspection, road research, hydroelectric scheme, Factory production, sales, municipal administration, Railways, ticket, stacking of aeroplanes, criminology & prediction, spare machinery, heavy industries, highway traffic, transportation problems. one or two will be illustrated later – O.R. perhaps not new, what is new is its importance, no question of trial & error, urgency – Its status

(2) What is operations research? still unanswered – simple examples – trivial ones; coloured crayons, time & motion study, head downwards – bath tub & 3 instead of 6 – fuze setting & convoys – 7(c) – 9(d) – 9(e) – Calculating machine.

(3) We are perhaps now in a position to define O.R – Study of the dynamics of situations, of processes, of organisation or of methods – appln of scientific method to determine how best to prosecute a war, run a business or in general to conduct any operation or group of operations – O.R. is concerned with optimising the performance of a system regarded as given – O.R. is a systematic appln of common sense – is a philosophy – Case history definition – Research at the operational level – Best definition is that of Morse.

(4) Characteristics of O.R from defn – 10(l) – 2(b) – 8(a) – 8(h) – math. ability required 2(a) + ? – Probability & statistics most imp – 14 – military O.R. 2(b). – O.R & decision & secrecy – expt & analytical, no reliance on "expert" opinion – O.R & practical problems.

(5) ~~Is O.R~~

(2) O.R.Q – June 54.

(a) In U.S.A. demand for operational research workers has exceeded the supply – standards to be maintained for recruiting workers – Doctorate in the subject have been instituted at M.I.T & Case I.T. – best way of recruiting good workers is a real problem.

(b) Reflections on O.R by E.C. Williams, Director of O.R, Admiralty U.K.
– O.R. Coined by A.P. Rowe to describe activities of a group of the Air Ministry Research Station at Bawdsey in 1937-39. to distinguish this from normal research in radar – later on study of tactics & policy – development & success of war time O.R. is now classic – After war, extension to other fields has taken place – O.R. is not new even in the military as shown by Military history – also it is not new in Industry either – What is new is its importance – it is of urgent importance, no question of trial & error, otherwise trial & catastrophe – In military field four classes of work:–
(i) to find operational effectiveness of weapons & equipment
(ii) analysis of results of operations or (in peace) of exercises to determine effectiveness of tactics, influence of weapons on tactics & conversely
(iii) prediction of the results of future operations
(iv) analysis of efficiency of organisations or of methods – O.R. can be defined as the study of the dynamics of situations, of processes, of organisations or of methods – Type of individual needed for O.R
(i) ability to see the matter as a functional whole & to see what wants analysing,
(ii) expertise to do the necessary functional, dynamical or statistical analysis, (iii) ability to do things the simple way, (iv) the ability to present the answer in a straightforward & understandable manner – Expertise required varies with the problem, – mathematical ability up to stage of partial diff. eqns, analysis of variables, probability theory, statistical theory, information theory, cybernetics & so on and skill

(5) Is O.R. a mere collection of problems with no general techniques? No — four types — queueing or waiting line & Monte Carlo method (from Morse) — linear programming (McKinsey medicine) — theory of games (fingo) — optimum distn of effort (Morse) — one example of each.

(6) Prejudice against mathematics among O.R. workers wrong — fundamental research to be fostered — future prospects (School populations, atomic attack — Govt. Depts).

(7) ~~Thanks~~ Fun about lin. prog — eh. Thanks.

Queue Theory: If $A > S$, Q forms. Even if $S > A$, Q may be formed because of randomness. If both A & S be random pure with distn on Poisson law, the ~~mean length of waiting line~~ chance or probability P_n of having n units in the waiting line is given by

$$P_n = \left(\frac{A}{S}\right)^n \left(1 - \frac{A}{S}\right). \quad (S > A)$$

If $S = 2A$, $P_n = \left(\frac{1}{2}\right)^{n+1} \longrightarrow 0$ rapidly

If $A \cong S$, P_n diminishes slowly with n. The mean length of the ~~line~~ waiting line, $L^* = A/S-A \longrightarrow \infty$ as $A \to S$.

Re. ~~costing~~ cost of having n units in waiting line balanced against cost of increasing service rate — Ex of ships arrival & docking at ports, generally random. A shipping Co. decides to route all ships to one efficient port. This made $A = 0.9\,S$ & waiting line grew 3 times. Management misinterpreted this. ~~It is better to send~~ They delay was due to increase in A & not decrease in S at the port. A diversion of few ships to less efficient ports would reduce L. A small use of less efficient (ie small S) ports would produce best results overall.

In case of aeroplanes stacking, the problem is more

$$\left[* \; L = \sum_{n=0}^{\infty} n P_n = \frac{A/S}{1 - A/S} = \frac{A}{S-A} \right]$$

in the design of experiments – Pure mathematicians want to approximate to the problem ie to get exact solns, what is wanted is to get approximate solns to the problem as it is; this is really branch of applied mathematics – O.R should be as experimental as it is analytical.

(C) A survey of O.R in the U.S.A. – Called 'Operations' R in U.S.A & 'Operational' R in U.K – by E. Johnson – Director, O.R. Office, John Hopkins Univ – O.R. Soc of America recognised by National Research Council & subject is made respectable –

(i) Growth of military O.R – Began in 1941–42 & by 1945 there were 400 analysts – Now there are nearly 800 in 1954 in U.S. military in the proportion, WSEG : Navy : Army : Air Force = 1 : 2 : 10 : 20. Peacetime ceiling of 100 has been reached.

(ii) O.R.S.A in March 54 had 870 members – Perhaps there are 1500 O.R. workers in U.S.A, half in industry – two thirds of the companies in U.S.A. have no knowledge of O.R – fields where O.R. is used are: aircraft industries, Elec. machinery, Chemicals, Instruments & machine manufacturing, Petroleum. – 50 companies do their own O.R.

(iii) Scope of O.R – At least 20 more items than mentioned above are dealt with – for eg. to put 10 year forecasts of requirements on a national basis for school population, traffic investigations carried out on toll booths by N.Y. Port A– disaster plans as for methods of coping with atomic attack – also in other Govt. Depts.

(iv) Cost of O.R – costly – ultimately U.S.A. may have 10,000 analysts – no reqd may become ∴ recruits available, leading to fall in standard, already noticeable.

(v) Edn for OR – 12 Universities giving seminars – M.I.T. to give a Ph.D – Case Inst. has 5 to 10 candidates working for Doctorate – Shorter courses are very popular & given at least at 12 institutions

– great need to maintain standards

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difficult because servicing is always random thanks to the weather, although arrivals may be scheduled to be regular. In such case where each arrival is exactly $(1/A)$ time after the previous one, we find that P_n is less half of what it was when both A & S are random. Usually in this case both S & A are "partially random" because however much arrivals are scheduled & services also scheduled, a certain amount of randomness creeps in. In a case like this one uses the Monte Carlo method ie exploit the laws of chance.

To get data re. arrival & servicing & law of distribution, one uses a table of random numbers and of empirically determined probability distributions for scheduled and non-scheduled arrivals and for landings, a whole series of virtual arrivals & landings are worked out on a high speed computing machine simulating in a way actual arrivals & landings. In this manner, data can be obtained in a few hours of what actual observation would have taken years to obtain at an airport. On the basis of the data thus acquired, a theory could be worked out & this shows that there is a small reduction of queue length with improvement in scheduling arrivals & it makes it possible to find relative cost of tight scheduling as against reduction of delay time.

Linear programming.

A man has take at least u_1 units of vitamin V_1 & u_2 units of vitamin V_2. He cannot get them pure but only in medicines containing V_1 & V_2. Each oz of medicine M_1 contains ~~n_1 oz of V_1 & n_2 oz of V_2~~ n_1 oz of V_1 and n_2 oz. of V_2 & each oz of M_2 contains n_1 oz of V_1 & n_2 oz. of V_2. The problem is what amounts of M_1 & M_2 he should buy to get u_1 & u_2 at minimum cost. Suppose he buys x oz of M_1 and y oz of M_2. Let M_1 and M_2 cost p_1 & p_2 annas per oz.

The problem is to minimise $p_1x + p_2y$ given

$$xm_1 + yn_1 \geq u_1$$
$$xm_2 + yn_2 \geq u_2.$$
$$x \geq 0,\ y \geq 0$$

Take a numerical case of $v_1 = v_2 = 15$

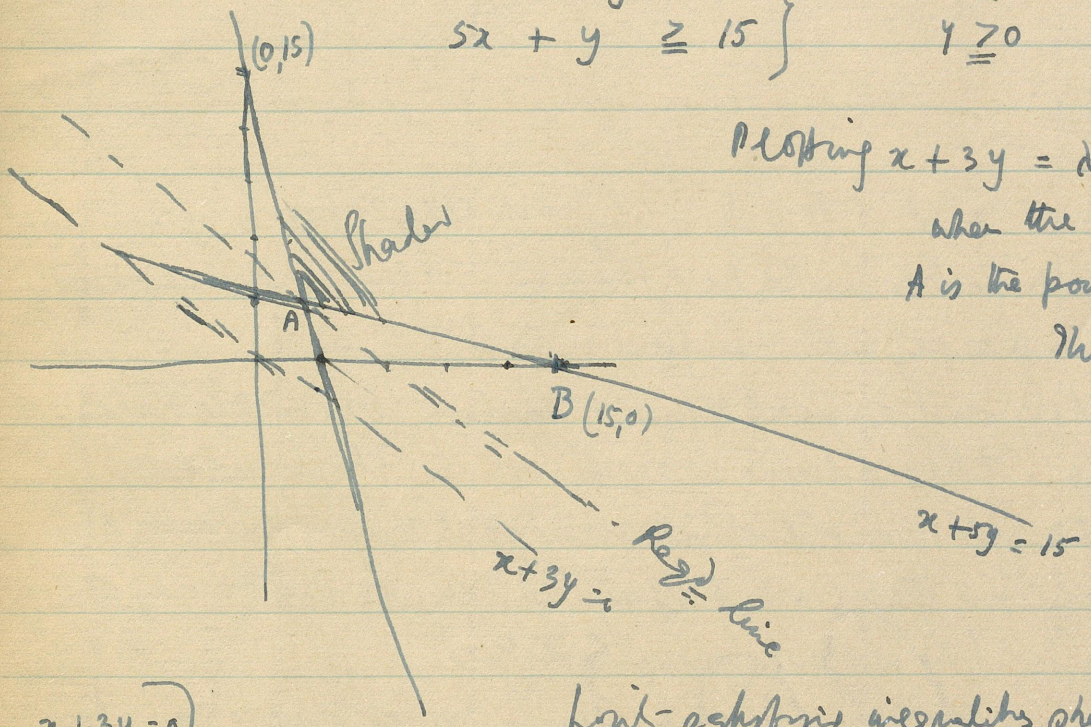
$m_1 = 1$, $n_1 = 5$

$m_2 = 5$, $n_2 = 1$.

and $p_1 = 1$, $p_2 = 3$.

ie to minimise $x + 3y$ given

$$\left.\begin{aligned} x + 5y &\geq 15 \\ 5x + y &\geq 15 \end{aligned}\right\} \qquad \begin{aligned} x &\geq 0 \\ y &\geq 0 \end{aligned}$$



Plotting $x + 3y = \lambda$, λ is least when the line passes through A

A is the point $x = y = 5/2$

This gives $\lambda = 10$

$x + 3y = 0$

$y = -\frac{1}{3}x$

Points satisfying inequalities should be in shaded region (x & y) in shaded region.

Start problem itself with number words. He has to take 15 units of each of two vitamins. Can only be obtained pure but only in two medicines. First medicine contains 1 unit per oz of (v_1) & 5 units per oz of (v_2). Second one contains 5 units per oz of (v_1) & 1 unit per oz of (v_2). First medicine costs 1 Rupee per oz & the second 3 Rs per oz. How much of each medicine should he buy to spend minimum? Let him buy x & y.

$$x + 5y \geq 15, \quad x \geq 0$$
$$5x + y \geq 15 \qquad y \geq 0.$$

$x + 3y$ to be minimum.

(d) ~~topics – Productivity studies as Human factor in~~

(d) Road research – This issue has 6 topics

(e) Grosszahl – Forschung or large number research using graphical methods like probability paper & logarithmic scales.

(f) London Conference on 4/5/1954 on linear programming at which 50 organisations were represented.

(3) O.R.Q. Sept. 54.

(a) Q. Inspection & its relation to O.R – Why inspection is necessary – Principles on which inspection should be organised – organisation needed for inspection – Techniques required – inspection as O.R.

(b) Assessing priorities for technical effort – analogy between academic research & fishing – If the academic research workers appears more brilliant than the applied or industrial scientist it may be because he merely catches any fish that comes his way, whereas the other one has to catch one particular fish within a prescribed time.

(c) Criminology & prediction – Borstal research project – people coming back to crime after release from gaol – statistical in nature – A. Wald's book on statistical decision functions was found to be of fundamental importance in this investigation

(4) O.R.Q. Dec. 54

(a) Provisioning of spare machines – Accountants want minimum stocks of spare parts & engineers want to carry large stocks since failures are 'random' – The problem of this paper is to reconcile these – highly interesting mathematical paper – A sort of queueing problem

(b) Queueing systems with phase type service – for eg. in shops where parts of the service are given at different counters – also mathematical

(c) ~~Army O~~ Army O.R. in the Far East, 1952/54 by K. Pennycuick – O.R. sections in Korea, Malaya & Singapore are described – work on psychological warfare.

(What about O.R. workers attached to military establishments?)

(d) O.R – Recollections of problems studied 1940 – 45 by P.M.S. Blackett – Review – 1940 with A.A Command, Operational use of new radar sets – three problems (i) Providing link between good sets & good guns in practice on gun sights, (ii) siting of gun batteries for A.A. defence of London, (iii) Checking of intelligence reports on the number of raiders shot down – 1941, Coastal Command – best depth setting for depth charges, right colour for painting anti-submarine aircraft, Calculation of probabilities of success of various methods of patrolling a given area – 1942, Dept of Naval O.R – actively associated with anti-submarine campaign culminating in the victories of 1943 – involved controversies about diversion of heavy bombers from the bombing of Germany to U-boat hunting & the optimum size of convoys – difficulties of making people believe in O.R.

(e) Maintenance of a car.

(5) O.R.Q – March 1955 – Vol. 6. No. 1.

(a) measurement & management.

(b) Accuracy & use of time study – a symposium

(c) The uses and abuses of analogy by Vera Daniel – Analogy has high reputation in physics in deriving theories that can be checked by expt – in social science reputation of analogy is very low – interesting paper exploiting the analogy of the 'Curie point' in magnetism to the behaviour of social groups.

If with same values for all except costs quantities, let these be Re 1/- & Rs 20/- then we have to find min of $x + 20y$. This happens when

$x + 20y = \lambda$ passes through B (Shortest fig)

ie $\lambda = 15$. $(x = 15, y = 0.)$

ie he buys only first medicine (Cheapest) & nothing of second & gets at a cost of 15 Rs, 15 units of v_1 & 75 units of v_2.

Game theory: (A) n-person - Zero sum games. - Players be $P_1 \ldots P_n$ & P_i gets p_i at end of a play & $\sum_{i=1}^{n} p_i = 0$ for every play, we have zero sum game.

Consider 2 person 0-sum games - rectangular games, P_1 chooses some integer from the first m integers and P_2 from first n. Numbers chosen are compared & one of the players pays something to the other for every choice according to rules of the game. For eg. P_1 from $\{1,2,3\}$, P_2 from $\{1,2,3,4\}$

	1	2	3	4
1	2	1	10	11
2	0	-1	1	2
3	-3	-5	-1	1

$$\begin{Vmatrix} 2 & 1 & 10 & 11 \\ 0 & -1 & 1 & 2 \\ -3 & -5 & -1 & 1 \end{Vmatrix}$$

Pay off matrix

ie What P_2 pays to P_1.

(B) Two finger game.

	(1,1)	(1,2)	(2,1)	(2,2)
(1,1)	0	2	-3	0
(1,2)	-2	0	0	3
(2,1)	3	0	0	-4
(2,2)	0	-3	4	0

(6) O.R.Q – June 1955.

(a) Handling of uncontrolled data.

(b) Road Research – (i) Braking performance of motorcycles in use on roads (ii) Vehicle speeds on straight & bottom sections, (iii) Accident rates (iv) Parked vehicles in "no-waiting" streets, (v) Time lost in parking a car in West End.

(c) Mechanising ticket printing issue – Key position of O.R. in Railway & Development plans – requires ~~first~~ immediate expansion.

(d) Isoplus n – describes a nomograph method for routine calculations – mathematical paper.

(e) O.R: Past, present & future by P.M. Morse (M.I.T) – Queue theory, Monte Carlo method, linear programming & game theory are discussed – continual checking of mathematical models with experiments is impt.

– O.R is scientific basis for decisions.

(7) O.R.Q – Sept 1955.

(a) Scale factor in the management of coal mines

(b) Monte Carlo methods in heavy industry by Jones & Lee – Monte Carlo methods means exploiting the laws of chance – hypothetical furnacetime cycles calculated by "Monte Carlo" methods agree closely with actual shop performance – interesting paper

(c) Courses on O.R by R.S. Gander – Discussion of modern developments & trends in the application of O.R. to industrial problems.

(d) Risk taken in crossing a road – study of 491 persons crossing or not crossing a road against oncoming traffic – 1189 observations recorded – 50% crossed when vehicle was about 4.5 seconds away – No one crossed when vehicle was less than 1½ secs away & everyone crossed when vehicle was more than 10.5 seconds distant – when % ages crossing at different intervals of time were plotted against logarithms of time intervals, a good fit to a normal ogive was obtained – maximum risk-taking

In (A) each element of first row > corresp. elements of the remaining two rows & so each element of second column < corresp. elements of the other two column. Hence regardless of what P_2 chooses P_1 should choose 1 rather than 2 or 3. ~~Similarly~~ i.e. optimal way for P_1 is to choose 1. Similarly to make pay off as small as possible P_2 should play 2 i.e. it is optimal for him. This is a game of pure strategy – saddlepoint (no saddle point).

(B) is a game of mixed strategy i.e. there is no such row or column. Solution in such a case – take a (2,2) pay off matrix

$$(C) \quad \begin{Vmatrix} 1 & 3 \\ 4 & 2 \end{Vmatrix}$$

Let P_1 & P_2 play 1 with frequency x & plays 2 with frequency $1-x$ (i.e. both x & $1-x$ +ve). Similarly P_2 plays 1 with frequency y & 2 " $1-y$.

Math. expectation of P_1 is

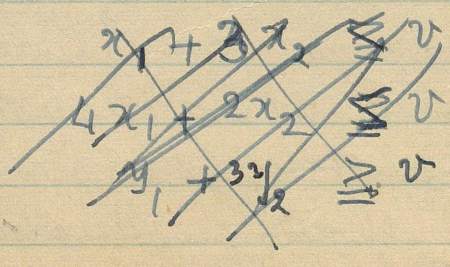
$$\begin{aligned} E(x,y) &= 1.xy + 3x(1-y) + 4(1-x)y + 2(1-x)(1-y) \\ &= -4\left(x-\tfrac{1}{2}\right)\left(y-\tfrac{1}{4}\right) + \tfrac{5}{2} \end{aligned}$$

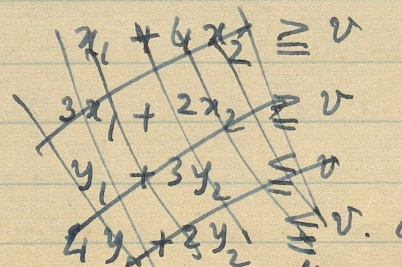
i.e. expectation is at least $5/2$ when $x = \frac{1}{2}, y = \frac{1}{4}$ - i.e. optimal ways of P_1 & P_2 are given here as $x = \frac{1}{2}, y = \frac{1}{4}$ & value of game $= 5/2$.

~~In general for~~ Alternatively we can solve (C) by choosing four numbers x_1, x_2, y_1, y_2 and v such that

$$\begin{aligned} x_1 &\geq 0 & x_1 + 4x_2 &\leq v \\ x_2 &\geq 0 & 3x_1 + 2x_2 &\leq v \\ y_1 &\geq 0 & y_1 + 3y_2 &\geq v \\ y_2 &\geq 0 & 4y_1 + 2y_2 &\geq v \end{aligned}$$

$$x_1 + x_2 = 1, \quad y_1 + y_2 = 1.$$





In this case all equalities only

$4v = 5/2$ as found above

$\frac{1}{2} + \frac{3}{2}$

$\frac{1}{4} + \frac{3}{4}$

$\frac{1}{4}$

– Connection bet. lin. programming & game theory.

levels when measured in terms of log seconds are normally distributed – Study indicates where controlled crossings are necessary.

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(8) J.O.R.S.A – Vol. 3. No. 1, February 1955.

* (a) Observation, experiment & measurement in O.R by O. Solandt – Defence Research Board of Canada.

Prof. Blackett is said to have started O.R. in all three services – first started in Air Force centering round problems of the use of radar, in the navy it centred round anti-submarine problems, in the army round anti aircraft problems & radar – spectacular successes of the early days no longer possible – German Physiologists' big paper on screw drivers, conclusion being that the best handle was a simple cylinder – Infantry problems belong to a difficult field; it has been taught that you must never fire a long burst from a machine gun because the gun vibrated so much it shook you off your aim; it was only the first two or three bullets out of a ~~burst~~ burst that hit the target. An young scientist who did not believe anything he was told took some coloured crayons & coloured a bunch of bullets. He was able to show that, consistently, the only rounds that hit the target were the last rounds of the burst & not the first – Time & motion study in the Armed Services. Looking over gun drills in Artillery it was shown that the anti-tank guns could be more effectively manned by three men instead of six. When the report was written & passed on to the gunners they said "We never use more than three men in action, but we want enough men to do the cooking, bring up the ammunition & to go scrounging around for supplies for the Officers' mess etc, and here you are showing us how we can get rid of these men. Burn the report!" We did – time & motion studies of the gun drill for the 25-pounder. Movies of the gun unit going into action were taken. Everything looked quite good except that there was one man who spent a long time just standing still doing nothing. When we looked

Optimum distⁿ of effort — operation search for an enemy vessel or submarine or aircraft. To obtain search plans so as to spend min. effort.

Time-fuze of bomb (Blackett) — Setting time-fuze for bombs from ~~aeroplanes~~ aircraft against submarines. The usual setting of exploding at depth of 150 ft was useless - then changed to 50 - A statistical analysis showed that 25 ft. depth was the best ie most likely to explode on impact.

Convoys — Convoys vs Submarines - Let m = no. of merchant vessels, c = no. of escorts, n = no. of U. Boats in pack; K = no gm. r sunk per engagement, l = no of U. B. sunk per engagement. By considering large no. of data, Blackett showed that approx

$$K \simeq 5\left(n/c\right), \quad l \simeq \left(nc/100\right)$$

$$\left(\frac{l}{K}\right) \simeq \left(c^2/500\right)$$

(i) no. of ships lost is independent of ~~convoy~~ size of convoy

(ii) exchange rate $\propto c^2$.

Hence the ~~size of~~ Admiralty on B's advice increased size of convoys so that escort also was increased & exchange rate (l/K) increased & the U-Boats left North Atlantic & went elsewhere because this was not profitable. This is the "Battle of the Atlantic.

Status (2, a, c, f) Doctorate in O.R at M.I.T & C.I.T — In U.S. more demand than supply — O.R. Soc. recognised by National Research Council — Has no. of Journals for itself — In military O.R. originally 450, now 800 in: " W.S.E.G : Navy : Army : Air = 1 : 2 : 10 : 20.

O.R.S.A. has 900 members — U.S. has 1500 O.R. Workers 50 Companies do their own O.R - pay twice as much as for Engrees

back to find what he was supposed to be doing, we found that he was holding the horses!. The horses had disappeared about 20 years earlier, but this task was not eliminated when the gun drill was rewritten.

Essential features of O.R – I always studies the organisation in Operation – I prefer term 'Operational' to 'Operations' because latter suggests military bias – process of O.R. consists of three stages (i) to discover the purpose of the operations (ii) to find some measure of its effectiveness, (iii) ~~st~~ to start on the task of trying to improve the effectiveness – mathematics alone can never be a substitute for careful observation, measurement & experiment. The three steps are observation, measurement & experiment – authors poor mathematical equipment, used only elementary statistics & probability & knew that errors are not added arithmetically but by root-squares. His denying of use of math. models partly due to this background.

Problems of bombardment – an outstanding success of war time O.R. was understanding problems of bombardment, how it should be used in war, how to make it succeed & what were its limitations. Example of 25-pounder gun problem. What is its purpose? no general agreement among military people. Majority view is that it is anti personnel. Next stage was to see if it was effective. Not at all. Many deficiencies were noticed for eg. effect of irregularities in the ground. An enthusiastic mathematician found that if he used the poles that surveyors used, he could stand on his head in the middle of an array of poles & read off the heights. So far as he was concerned, however, this method was not good because one day he was found by the police standing on his head in the middle of Barnes Common in London. He refused to say what he was doing except that it was highly secret. Naturally, he was marched off to jail & we had considerable difficulty in getting him out – Improvements were soon effected

The tank, gun weapons system – the case of the German tanks in Libya – O.R. work on this. Accurate work on tank, gun weapons system a

Design of a new organisation – Locating launching sites of V-2 rockets by sound ranging ie listening to the shock waves.

A/S-A (20) : 1/2

great scope ahead — U.S. may require 10,000 workers ultimately — danger of falling standards — 12 Univ. giving seminars — short courses at 12 institutions

Coloured crayons — told not to fire a long burst from a machine gun because vibration shook off the aim & only the first two or three out of a burst hit the target — O.R. worker took crayons & coloured the bullets a bunch of bullets & showed that only the last rounds consistently hit the target

Time & motion study — Gun drill for the 25 pounder — The man who held the horse

Head downwards — 25 pound Bombardment problem — 25 pounder gun firing what is its purpose, anti-personnel ? or what else. While studying data & trying to make improvements, the level of ground had to be taken into account. Young O.R. worker had idea of using poles used by surveyors & using them to make a contraption to stand head downwards & read the heights — Consequence.

Bath-tub — Line up of soldiers washing their mess kits after eating at a field mess station. — The night question asked by O.R. worker Theoretically this should increase flow by 50% (A/S-A from A = S = 2 to A = 1, S = 3) — But here it is zero. — Self aggravating operations — traffic jam example.

3 instead of 6 — Looking over gun drills in Artillery O.R. men found that our A-A guns could be more effectively manned by 3 men instead of 6 — results of the report.

Road crossing — Study of 491 persons — 1189 observations — 1½ secs & 10 secs — 4.5 seconds normal — % yes crossing at diff. intervals of time plotted against log (time intervals) a good fit to a normal ogive — max. risk-taking levels against log (secs) normally distributed — Help of this study re. location of controlled crossings.

Freely flowing traffic — Treated by analogy with flow of molecules in a gas.

Military O·R. in peace time - I for one hope that we shall never come to the day when a knowledge of the latest mathematical ~~gag~~ gadgets for war games, of communication theory and so on, becomes the badge of office of the O.R. worker. I think he should be known as a hard headed experimental scientist with a very broad vision, who trusts only what he can measure - O.R. is systematic applⁿ of common sense.

(b) Simulation as an aid in model building - Used for evaluating anti-aircraft guided missiles in the air defence of naval units.

(c) Proposal for revising New York's subway fare structure - long complicated paper

(d) Salvo-firing - Investigates the effects of salvo-firing of ammunition rounds at a specified target - gives approximate ~~probability~~ expressions for the probability that at least one round of the salvo kills the target - mathematical

(e) Statistical guide for accident research - applied to non-air-carrier flying

(f) Discussion of education for O·R - held in Oct 54 at Cornell Univ -

(g) Why O·R? - O.R. is simply the application of the scientific method in determining how best to prosecute a war, run a business or in general to conduct any operation or group of operations.

(h) Report of Committee on O·R:

(i) Level of math. equipment implied by successful completion of a course in diff. eqns should be a prerequisite (ii) Adequate courses in probability & in math. statistics & specific techniques like sampling theory, linear programming, machine computation & symbolic logic are recommended.

(iii) Course on O.R. should emphasise methodology rather than merely specific techniques (mathematics).

(iv) social scientists to be acquainted with physical science & vice versa

(v) should engage in team research on actual problems

(vi) Should understand relation between maths & science

(vii) should not be social scientists

Qualitative agreement with facts specially about congestion - Collision means vehicles crossing each other.

Hydroelectric system: - Big water reservoirs of a hydroelectric system collect water during high river flows for use during low flows. The problem is how to use such stored water in the face of uncertain future flow. If future river flow be known at least a year in advance in detail, the operational plans maximised expected return by supply of water for power can be worked out by method of calculus of variations. For random flows, a hydroelectric model is constructed assuming a river flow characterised by probability densities empirically assumed & solutions are worked out on a digital computer.

also * linear programming] is considered by B. Klein.

Calculation - 10 8 years to invert 20 x 20 matrix by usual method - Change-method of use of Calc.-machine reduces this to a few hours.

Qualifications (i) math ability - partial diff eqns, analysis, probability, statistics (mostly information theory, Communication theory, servomechanisms, cybernetics, symbolic logic, numerical analysis, calculus of variations, algebra & prop theory, topology.

(ii) ability to state a problem, construction of model or hypothesis, appropriate math. sophistication, assessment of effectiveness of the conclusions of the study - Prize award to L. C. Edie

(iii) maths alone no substitute for observation, measurement & experiment - evil day when (i) become basis of effici of O.R. Worker -

(iv) O.R. worker & decision - (v) Secrecy (vi) no reliance on expert opinion

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(9) J.O.R.S.A. — May 1955:

(a) Competence and limitations of scientific method by H. Margenau — philosophical & metaphysical.

(b) Influence of vehicular speed & spacing on tunnel capacity — Traffic engineering is barely 25 years old — long & complicated paper.

(c) Use of extremal principles in solving optimising problems involving inequalities — A purely mathematical paper — A problem in game theory is considered*.

(d) Mathematical models for freely-flowing highway traffic — Treated on the analogy of between flow of cars on a highway and flow of molecules in a gas as interpreted through the kinetic theory of gases. Collision means one car passing another — Models considered have more than fulfilled the author's expectations. Despite the crudeness of models we get results at least in qualitative agreement with observation for example in problems on congestion etc.

(e) Use of storage water in a hydroelectric system — scheduling of water collected during high river flows is an inventory problem. If the future river flow be known at least not a year in advance in detail, the operations plan of maximised expected return can be worked out by calculus of variation methods. Otherwise we have to use probabilistic arguments — Theory — Hydroelectric model — Solution ~~on~~ numerically on a digital computer — Good piece of work.

(f) Poisson distribution as a limit for dependent binomial events with unequal probabilities by J. E. Walsh — Purely mathematical

(g) Numerical solution of the transportation problem — The problem consists in finding a pattern of shipments between m origins with given surpluses and n destinations with given deficits such that the total cost of transportation is minimised — Done numerically by first setting a good first approximation by inspection & then iterating.

(h) Use of linear programming in Capital budgeting by P. Gunther — Linear programming has found wide applcn in determining optimum utilisation of machines, most efficient scheduling procedures, most economical routing of transportation, best system of salary scales etc. Here it is applied to Capital budgeting — Very interesting case of Mrs. E's apple pie.

(i) Review of Blackett's book - He pinpoints the revolutionary nature of military O.R ie the use of civilian scientists on tactical & strategic problems until recently almost exclusively a matter for the uniformed Service personnel.

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(10) J. O. R. S. A — August 1955.

(a) Terminal Ballistics by J. Steinhardt (O. Evaluation Group, M.I.T) — It may well be that the main contribution of O.R is conceptual. Catch phrases like pay-off function, decision processes, measures of effectiveness, suboptimization, linear or dynamic programming, game theoretical are not always fads but represent concepts — What kind of training is wanted for O.R. workers? Ability to communicate with non-scientists very essential — In O.R. formulation of the problem is half the battle.

(b) From Plato to the linear program by Earl of Halsbury — N.R.D.C, London — O.R in U.S & U.K are two great nations divided by a common language. 'Operations' & 'Operational' — O.R is concerned with optimising the performance of a system regarded as given for the purposes of the problem. — Historically the first such problem to receive the name was concerned with how to set the time fuze of a bomb to be dropped from an aircraft onto a submarine. It was solved by Blackett who showed that the fuze should be set to explode on impact, and not as the Royal Navy and Air Force believed it should be set to explode after a delay period — Paper deals with distribution of the

wealth of Society conceived as a problem in linear programming. This is not a new subject – a simple mathematical model – The Earl makes this ridiculous statement "Four hundred millions of peasants in India are rushing headlong upon a food crisis which must sooner or later accomplish their downfall in tragic circumstances because they are below the level of education at which they are accessible to propaganda on family limitation" – Concept of a saint as a non-linear discontinuity. Also a type who will work hard for a period to accumulate a minimum sum enabling him to retire early & devote the remainder of his life to water-color painting would also be a non-linearity – individual variability – the future & linear programming – modern types of digital computing machines give great opportunities for sociological work. Just as electrical engineering techniques have shed light on theory of economic stability, and Von Neumann's theory of games is shedding light on the true nature of a market, so also I believe that linear programming can shed light on distribution of wealth among competing groups in the community – Colombo Plan as an example.

(c) Stochastic properties of waiting lines by P. M. Morse — Highly mathematical

(d) Analysis of a rail road classification yard – waiting time or queue problem — analysis shows great potentialities of O. R. in the field of railroading.

(e) Applⁿ of discrete variable servo-theory to inventory control – mathematical

(f) Road safety & Traffic research in Great Britain by Glanville – R. R. Lab Road research consists not merely of building & metalling but also wider aspects of traffic & safety – trends – London traffic & intersections – pedestrian research (Zebra marking) – motor cyclists – skidding

(g) Production scheduling – highly mathematical

(h) Production & inventory control in a chemical process – This paper describes how a mathematical model was constructed for a complex chemical process, and why the use of the model was abandoned in favour of a simple operating schedule rule.

x They use the word "opsearch" for "operational research"

(i) O.R on municipal operations

(j) Social implications of O.R — What is O.R? 'It is a philosophy' — O.R. is the first attempt in the history of science to socialize scientific method on a planned & conscious basis.

(K) "Such solutions are very little solved" by Charnes & Cooper* — Readers should Klein's serious article humorously & Gunther's humorous article seriously — Indeed unless Mr. Klein has access to a computer which is vastly superior to currently available ones, he is open to Poincaré's charge "Such solns are very little solved!"

(l) Prize of the Society awarded to the paper appearing in May 54 issue of JORSA by L. C. Edie on "Traffic delays at Toll Booths" as having all the following characteristic attributes of sound O.R.

(i) statement of a problem & its importance, in a real context
(ii) Constn of a cogent quantitative hypothesis or model supported by experimental results.
(iii) an amount of mathematical sophistication appropriate to the problem.
(iv) Assessment of the effectiveness of the implemented conclusions of the study.

(m) Methods of O.R by Morse & Kimball — Revised 1950 Edition.

(n) Communications of Pure & Appl. Maths — Symposium on Appl. Math — Operations Research by P. M. Morse, 1955.

— O.R is part of applied maths. For definition refer to JORSA. But this is case-history definition & not satisfactory — O.R. is research at the operational level — Definition (Good one and comprehensive) — waiting line or queue theory. Application of Monte Carlo method to it — linear

programming problems and more general problems of bounded optimisation (non-linear) — game theory; mixed strategy; duel problem — Optimum distribution of effort problems, its naval aspect, business activities — team work wanted — O.R is more than statistics.

(12) Working in numerical analysis by J. Todd — ibid p.97. — Game theory & related developments — Applin to assignment problem inclusive transportation problem, caterer problem, problem of contract awards, travelling salesman problem, organisation theory — Monte Carlo processes.

(13) Numerical computations in Ordnance problems by A.A. Bennett, ibid, p.117.

Inversion of a matrix of order 20 by traditional methods would take 108 years of errorless operation on the fastest computing machine but by a change of method (example of OR) a 35x35 matrix could be inverted in a few hours.

(14) Methods of O.R. by Morse & Kimball — Definition — O.R is a scientific (Chap I - Introduction)

method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. This defn to be taken with all its implications & generality — O.R is an applied science, is not a branch of mathematics — O.R. worker does not and should not make the decision; Separation of O.R from executive decision — Early development because competent men were drawn in — Value of O.R — Some simple examples. (i) Rearrangement of use of equipment; involving the line-up of soldiers washing their mess kits after eating at a field mess station. Instead of two tubs for washing & two for rinsing O.R. worker suggested three for washing & one for rinsing because ~~latter~~ former took 3 times as much time. Change was made & line of waiting soldiers not only diminished & on most days there was no line.

(@) Here the right question occurred to the O.R. worker; no added requirement was needed; the executive did the thing. Theoretically the change 2+2 → 3+1 should reduce waiting by 50% but here it became zero. This is a self aggravating property. Conversely it is an operational property of waiting lines that the longer they get, the longer they tend to get for eg. slowing down in free-flowing traffic brings in a traffic jam. Valuable O.R. work has been done on such self-aggravating problems which are non linear — (ii) Problem of depth setting of the aircraft antisubmarine depth charges to improve U-boat sinkings (§3.48). This is what is mentioned in Halsbury's article also. This problem is not so simple as the wash-out problem (i) since some physics was needed. Again only a rearrangement as in (i) was made (iii) Decision to increase size of convoys & the result that of reducing average ship loss (§ 3.22). Battle of the Atlantic won by O.R. Here the measure of effectiveness is percentage of ships lost. This also shows O.R. is experimental (iv) Putting anti aircraft guns on merchant vessels (§ 3.4.4). Here again a proper measure of effectiveness was found — mathematical analysis. The standard needed varies from problem to problem — Requirements of secrecy; essential & many examples will never be reported in the literature — Methods of O.R. (i) Statistical methods; most important mathematical tool is probability & statistical theory — limitations of "expert opinion". shall be avoided, (ii) Operational experiments, (iii) Analytical methods: (a) theoretical work on the effect of various strategic distⁿ of forces (Chap. 4), (b) Theory of games of von Neumann (Chap. 5) — Personnel & organisation — Peacetime applications.

McKinsey — Introduction to Theory of Games — Ex. of medicine mixture on p. 292 as example of linear programming is very interesting — Proof of fundamental theorem on rect. games based on the Brouwer Fixed-Point Theorem of topology — Iteration for lin. prog — Sasuli 18×34 matrix solved in 29 iterations in 20 mins by machine

(Tolman – Principles of Statistical Mechanics)

2/5/57 (1)

Theory of Games.

McKinsey

Chap. I – Game a play – move a choice – number of players – payments – zero-sum games – number of moves; finite & infinite games – amount of information available.

Two-person zero-sum games – rectangular games with pay-off matrix $\|a_{ij}\|$ (i from 1 to m, j from 1 to n) – rect. games with saddle point. In general $\max_{x \in A} \min_{y \in B} f(x,y) \leq \min_{y \in B} \max_{x \in A} f(x,y)$. If these are equal and equal to $f(x_0, y_0)$, then (x_0, y_0) is a saddle point. This condition is necessary and sufficient. for rect. games, $\max_i \min_j a_{ij} = \min_j \max_i a_{ij} = a_{i_0 j_0}$ then (i_0, j_0) is saddle point

Chap. II – Rect. games without saddle points. Mixed strategy. $S_m = (x_1, \cdots x_m) \in S_n (y_1 \cdots y_n)$ with $\sum x_i = 1 \in \sum y_j = 1$. Numbers $(1, \cdots m)$ & $(1, \cdots n)$ are pure strategies – If $E(X,Y) = \sum \sum a_{ij} x_i y_j$, and if for some X^* in S_m and Y^* in S_n $E(X, Y^*) \leq E(X^*, Y^*) \leq E(X^*, Y)$ then X^* & Y^* are optimal strategies and $E(X^*, Y^*)$ is the value of the games and $\|X^*, Y^*\|$ a solution of the game or strategic saddle point – fundamental theorem for rect. games:

$\max_{X \in S_m} \min_{Y \in S_n} E(X,Y) = v_1$ and $\min_{Y \in S_n} \max_{X \in S_m} E(X,Y) = v_2$ always exist

and $v_1 = v_2 (= v)$ i.e. every rectangular game has a value; a player of a rectangular game has always an optimal strategy – Notions used in the proof of this theorem: Distance between $X^{(1)}$ & $X^{(2)}$ in Euclidean E_n, bounded subset X of E_n, limit point x of a subset A of E_n (x may not be in A), closure of a set, closed set, open set, interior of a set, boundary of a set, connected set, convex linear combination of $x^{(1)}, \cdots x^{(r)}$ with weights $a_1, \cdots a_r$, convex subset X of E_n, convex hull of a set X, hyperplane and two half spaces corresponding to a hyperplane, extremal set of X { $K(X)$ = set of points x of X which cannot be represented as $x = \frac{1}{2} x_1 + \frac{1}{2} x_2$ } – Properties of optimal strategies: (i) X^* is optimal strategy for P_1 if & only if $v \leq E(X^*, Y)$ for every Y of S_n and similarly if & only if (for P_2) $E(X, Y^*) \leq v$ for every X of S_m

(ii) If X_i means has i^{th} component 1 & all others zero and if $E(X_i, Y)$ be denoted by $E(i, Y)$, necessary & sufficient condn that v be value of game & X^*, Y^* be optimal strategies is that $$E(i, Y^*) \leq v \leq E(X^*, j)$$

(iii) $\max_{1 \leq i \leq m} E(i, Y^*) = \min_{1 \leq j \leq n} E(X^*, j)$ (E is called expectation function) — Number of examples — notions of dominance. vector $\underline{a}$ (row or column of matrix) dominates $\underline{b}$ if $a_i \geq b_i$ $(i = 1, \ldots n)$ & strictly dominates if $a_i > b_i$ — i-place extension of a mixed strategy $x = \| x_1 \ldots x_n \| \in S_n$ is the vector $\| x_1, \ldots x_{i-1}, 0, x_i \ldots x_n \|$ (where $1 \leq i \leq n+1$) — Theorems about games where dominance relations exist for the matrix of the game — graphical solutions of rectangular games having $2 \times n$ or $m \times 2$ matrices illustrated by simple examples — original proof of von Neumann on the fundamental theorem on rect. games depended on the use of Brouwer's fixed point theorem of topology. Later Neumann & Morgenstern gave a simpler proof based on theory of convex sets which is the one given in McKinsey.

Chapter III — Solutions of a rectangular game — Study of the sets $T_1(\Gamma)$ & $T_2(\Gamma)$ which are sets of optimal strategies for P_1 & P_2 for the game Γ (general rect. $m \times n$ game). This study leads to a general method for finding all solns of a rect. game better adapted to machine computation. Theorems:

(i) $T_1(\Gamma)$ & $T_2(\Gamma)$ are non-empty, bounded, convex and closed subsets of E_m, E_n — From convexity it follows that if T_1 or T_2 has more than one member, then it has infinitely many members ie a rectangular game has just one solution or infinitely many —

Definitions re. matrices — unit matrix I_n — unit vector J_n — zero vector O_n — transpose A^T of a matrix A — product of two matrices — inverse of a square matrix — adjoint of a square matrix — Sum & difference of two $m \times n$ matrices — Sub-matrix of a matrix —

(ii) If $v(\Gamma)$ be value of Γ $(\neq 0)$ & $X \in T_1(\Gamma)$, $Y \in T_2(\Gamma)$, then a necessary & sufficient condn that $X \in K[T_1(\Gamma)]$ & $Y \in K[T_2(\Gamma)]$ is that there exists a non-singular submatrix B of order r of the pay-off matrix A such that

$$v = \frac{1}{J_r B^{-1} J_r^T} \; ; \; \dot{X} = \frac{J_r B^{-1}}{J_r B^{-1} J_r^T} \; ; \; \dot{Y} = \frac{J_r (B^{-1})^T}{J_r B^{-1} J_r^T}$$

where $\dot{X}$ is the vector obtained from X by deleting the elements corresponding to the rows deleted to obtain B from A & simly $\dot{Y}$ for columns.

(iii) The previous theorem modified by replacing B^{-1} by adj B in expns for $\dot{X}$ and $\dot{Y}$, and with $v = \dfrac{|B|}{J_r (\text{adj } B)\, J_s^T}$.

(iv) main theorem: The sets $K[T_1(\Gamma)]$ & $K[T_2(\Gamma)]$ are finite sets with T_1, T_2 as their convex hulls resply. Thus T_1 & T_2 are hyperpolyhedra having $K(T_1)$ & $K(T_2)$ resply as vertices —

Remarks: (a) method given is highly systematic & quicker than methods of Chap-2 but since number of sub-matrices increases rapidly with high order of the matrices, it becomes laborious (say for 100×100 matrices for eg) even with modern electronic computing machinery

(b) out of ∞ solns of Γ, how to choose the best? Use the notion that mixed strategy X dominates mixed strategy X' if for every pure strategy j for P_2, $E(X,j) \geq E(X',j)$ & there exists at least one strategy j for P_2 such that $E(X,j) > E(X',j)$.

(c) X is called a best strategy if it is optimal and is not dominated by any other strategy (simly for Y of P_2).

(d) Every rect. game has best strategies

(e) Conds to be satisfied by two sets X & Y in order that for some Γ, one has $X = T_1(\Gamma)$ & $Y = T_2(\Gamma)$ (Solved in Ann. Math. Stud. 24).

Chapter IV: Method of approximating to the value of a game — not clearly explained — illustrated by a 3×3 game — Reference to Brown & Robinson (Annals of Math, Vol 54, 1951, p. 296-301) & Brown [Iterative solns of games by fictitious play, Conference on Research in Econ. Monograph. 13, John Wiley & Sons, 1951].

Chapter V: Games in extensive form — General games where each player has several moves instead of one and where their knowledge gained has gone before can vary from move to move, and where there are chance moves, and where $n > 2$ — Any game in extensive form where players make their choices from finite sets is an finite game

in extensive form can be shown equivalent to a rectangular game ie can be normalised, the equivalent rect. game being the normal form as opposed to the original extensive form — In this Chapter this is shown by simple examples

Ex (i) — move from finite set {1,2} & information perfect (complete) — graphical representation of the game by means of "trees" (topology) & obtaining the strategy matrix

Ex (ii) — finite {1,2} game & information imperfect — graphical representation by a tree with vertices enclosed in a broken line & obtain strategy matrix — other modifications with more than one broken line and curved broken lines in the "tree" repn

Ex (iii) — Chance moves of three types, (a) affecting the pay off (b) affecting size or nature of sets for making choices (c) determining order of making moves. In all these three cases, "trees" with curved broken lines normalise the game

Ex (iv) — $n > 2$ — Case of 4 players can also reduced to normal form by graphical repn, but method of solving these normalised games so far not explained (See Chap. 15).

Remarks: (i) Restrictions on information: In order that a game be capable of realisation, it must not happen that any play intersect the same information set more than once

(ii) normalised games have saddle points if the original extensive game information is perfect.

Chapter VI — General theory of games in extensive form — (i) A finite n-person game regarded as a system defined by trees & broken lines; definition of a strategy; definition of play pay-off functions & strategy play-off functions, $M_1 \ldots M_n$; game called zero-sum if for every n-tuple $\underline{x}$ of strategies for players $P_1 \ldots P_n$, $\sum_{i=1}^{n} M_i(x) = 0$ — (ii) Games with perfect information & Equilibrium points; For such games there are saddle points ie players have pure strategies. If player P_i has pure strategies A_i available & A = Cartesian product of $A_1, A_2 \ldots A_n$; then an element $\| x_1 \ldots x_n \|$ of A is an equilibrium point if for each i, and for y any element of A_i, we have

$$M_i(x_1 \ldots x_n) \geq M_i(x_1 \ldots x_{i-1}, y, x_{i+1} \ldots x_n)$$

Th (i) – Let $A^{(1)}$ & $A^{(2)}$ be the sets of strategies available to P_1 & P_2 in a zero-sum two-person game with perfect information; then A has an eq^{m} point – this theorem can easily be extended to n-person games. (= $A^{(1)} \cdot A^{(2)}$)

Th (ii) – The matrix of the normal form of any zero-sum two-person game with perfect information has a saddle point

(iii) Games with perfect recall & behaviour strategies – a behaviour strategy is to use a chance device at each move in order to decide which alternative to choose at that move – Th: A game with perfect recall has optimal behaviour strategies.

Chapter VII – Games with infinitely many strategies – ie infinite games ie choices made from infinite sets – specialised to games with one move for each player and with no information given to either player about the choice of the other player ie generalisation of ~~notion~~ rectangular games with pay off function $M(x,y)$ with x & y coming from infinite sets A & B. If both ~~both~~ A and B be interval $(0,1)$ game is called Continuous. Suppose

$$\max_{0\le x\le 1} \min_{0\le y\le 1} M(x,y) = v_1 \quad \& \quad \min_{0\le y\le 1} \max_{0\le x\le 1} M(x,y) = v_2$$

If $v_1 = v_2 \in M(x_0, y_0) = v$, $\underline{v}$ is value of game, there is a saddle point & optimal strategies

Possible cases (i) v_1 & v_2 need not exist (an actual exc. given)

(ii) v_1 & v_2 may exist, but $v_1 \neq v_2$

In case (i) a saddle point may still exist – example given

or although v_1 & v_2 don't exist, $\sup \inf M(x,y)$ & $\inf \sup M(x,y)$ may exist & be equal to $\underline{v}$, say. then $\underline{v}$ is the value of the game. In this case P_1 & P_2 although they have no optimal strategies can come as close as possible to an optimal method ie for every ϵ, $M(x_\epsilon, y) \ge v - \epsilon$ although for every y of P_2 there may not exist a x_0 such that $M(x_0, y) \ge v$.

For Case (ii) See Chap. X.

Chapter VIII – Distribution functions – mathematical preliminary to Chap X

Synopsis of lectures on O.R. to be delivered
at Delhi from 28/5 to 30/5/57

Game Theory.

Linear programming - General problem - Simplex technique - transportation technique - evaluation technique - Geometrical interpretation - Dual problems - ~~connection with theory of games~~ - Non-linear programming - assignment and allocation problems - Connection with theory of games - Non-linear programming (problems of bounded ~~optimization~~ (Churchill, Ackoff & Arnoff). (Morse) - Examples.

Theory of games - General theory - finite zero-sum two person games - Infinite zero-sum two person games - Games in extensive form - General n-person games. (Kuhn & Tucker)

Rectangular games - ~~continuous games~~ - Games in extensive form - Continuous games - separable games - Connection with linear programming - Zero-sum n-person games - non-zero-sum games - Games over function space - pseudo games

Measure and Countermeasure:

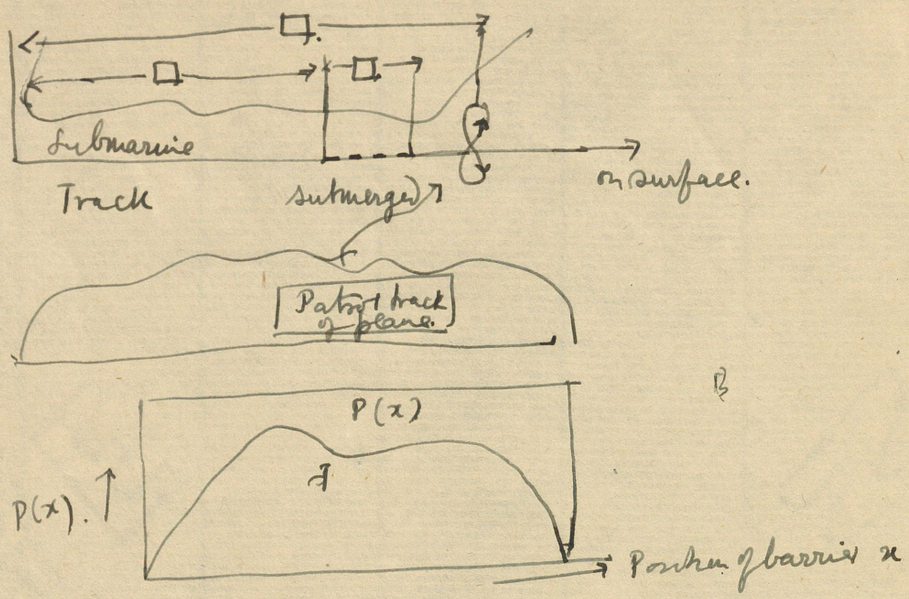
In operations research one often encounters the following sort of problem:[3] if A uses one tactic, B should use a certain defense, but if A uses another tactic, B should do something else; what are the best tactics for both A and B.? Problems of this sort are discussed by Von Neumann and Morgenstern in their "Theory of games." In some of the problems they discuss the answer is definite; there is one particular action which A must take, and likewise for B, in order that neither A nor B shall lose too much. In other words, if A took any other action than the recommended one, B could take advantage of him, and vice versa.

In other problems the answer is indefinite; any one action on the part of A will eventually lead to trouble if the 'game' is played several times; what is needed is a variation of tactics, in a random manner, so that B is never sure which action A will take next time. This sort of situation often turns up in operations research. The enemy may have a counter-measure to some weapon of ours which renders the weapon a liability in certain cases. Should we use the weapon or not?

[(3) The following examples are taken from some of the work done by the author and G.E. Kimball while they were with the operations Research Group, U.S.N.]

An example of this type of Analysis can again be taken from the theory of search. A submarine wishes to pass through a channel without being detected by a patrol aircraft, which flies back and forth across the Channel each day. The submarine can escape detection by submerging, but we assume that it can only travel a total distance $\underline{a}$ submerged and that $\underline{a}$ is shorter than the Channel length. It can surface and dive again several times, but the

total length of path submerged is no longer than a.



The channel is supposed to be wide enough so that only a few trips are possible by the plane each day. Consequently each day the plane must choose which point along the channel it is to patrol across. If it chooses a wide part the number of traverses will be fewer and the chance of detection, even if the submarine is surfaced at that point, is less than if the plane were patrolling across a narrower part. In fact, one can plot a curve giving the probability $P(x)$ that the plane will discover the submarine if it patrols across the portion a distance x from one end of the channel and if the submarine is not submerged when passing this part. Such a curve is given in Fig. 7.

It might seem, at first sight, that the aircraft should always patrol where P is largest. But if this action were taken the submarine would soon learn to dive when it goes past that point and the plane would <u>never</u> see the submarine. Obviously the plane must spread out its effort, one day patrol at one value of x, next day at another and so on, with the frequency of patrol at x governed by some predetermined function $\phi(x)$, which is called the 'density of the barrier' at x. This patrol effort should not be distributed from point to point in a regular manner, or the opponent could take advantage of this; the assignment of patrols should be in a random manner, subject to the density function $\phi(x)$.

Similarly, if the submarine is always submerged in one region the plane could patrol outside that region and the submarine would lose the advantage of its submergence. It must vary its regions of submergence in a random manner, with a relative frequency according to a density function $\psi(x)$. To recapitulate, equation-wise:

$$\text{Density of Patrol} = \phi(x); \qquad \int \phi(x)\,dx = 1,$$

(10)
$$\text{Prob. of submergence} = \psi(x); \qquad \int \psi(x)\,dx = a,$$

$$\text{Prob. of detection } G = \int P(x)\,\phi(x)\,[1-\psi(x)]\,dx.$$

First let us consider the way in which the submarine Commander must analyze the situation to decide on the submergence density $\psi(x)$. He must choose ψ so that if the opposing side learned what ψ he was using they could not gain an advantage, but if he learned the patrol plans of the plane (the value of ϕ) he might be able to gain an advantage.

If the airplane patrol-squadron commander found out that the submergence density function for the submarine was to be $\psi(x)$, he could make the probability of contact G a maximum by flying his patrol plane at the point x where $P(x)\,[1-\psi(x)]$ has its maximum value. The submarine Commander would of course, like to make the quantity $P(x)\,[1-\psi(x)]$ equal to zero, everywhere, but this is not possible, since the channel is longer than the maximum range of submergence, a. The next best thing for the submarine Commander to do is to arrange so that $P(x)\,[1-\psi(x)]$ has no single maximum, that is, is flat along the top. This is done as follows.

SAFE TACTICS FOR SUBMARINE: over as great a range of x as possible, choose $\psi(x)$ so that $P(x)\,[1-\psi(x)] = H$, a constant, that is

(11)
$$\psi(x) \begin{cases} 1 - \dfrac{H}{P(x)}, & \text{when } P(x) > H \quad (\text{over the range } B \text{ of } x) \\ 0, & \text{when } P(x) < H \quad (\text{outside range } B \text{ of } x) \end{cases}$$

The value of H is determined by the submergence limitations of the submarine,

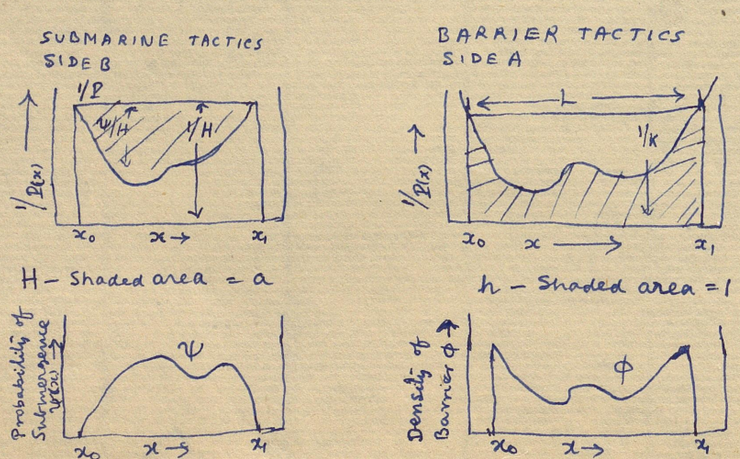
$$\int \psi \, dx = \int_B \left(1 - \frac{H}{P(x)}\right) dx = a,$$

where the integration is only over the range B of x.

If the submarine follows these submergence tactics then no matter what tactics the patrol plane follows, the probability of contact will never be greater than H, and it may well be less than H if the patrol plane is badly placed. The integral determining H ensures that H is as small as possible, consistent with the submergence capabilities of the submarine.

These are the safe tactics for the submarine. If the submarine Commander learns the plans of the aircraft patrol, he can alter ψ so that G, the contact probability, is less than H; but if he is not sure about the air patrol plans then a choice of the tactics of Equation (11) will ensure that the probability of contact is never larger than H even if the opposing side has a perfect spy system.

The details of the solution are shown on the left-hand side of Fig. 8. We plot the Reciprocal of $P(x)$ and draw a horizontal line of height $(1/H)$ so adjusted that the area between this line and the curve $(1/P)$ is equal to $\left(\frac{a}{H}\right)$. The difference between this line and the $(1/P)$ curve multiplied by H, will then be ψ which is plotted in the lower left-hand figure. If the air patrol is placed anywhere between x_0 and x_1 the probability of contact G is equal to H: if it is placed outside this range G will be less than H.



[FIG. 8] Solutions of problems shown in (FIG 7) for safe tactics for both sides.

we note that if a is small enough, or if the value of $D(x)$ fluctuates more widely, the line at height $(1/H)$ may cut the line $(1/D)$ at more than two points. when this is the case the region for submergence is divided into 2 or more parts; those regions for which $(\frac{1}{D})$ is larger than $(\frac{1}{H})$ are excluded from the schedule ($\psi = 0$ there) and for those regions where $(1/D)$ is smaller than $(1/H)$, $\psi = 1 - (H/D)$ as before. This illustrates again a property of many solutions of operational problems: if the resources available are limited they should be used only in more promising regions, the less promising regions 'should be disregarded entirely'. The same result turned up in the example, quoted earlier, of the search of two areas. It turns up in many problems discussed by Von Neumann and Morgenstern.

This property of solutions of operational problems should not be surprising to us (though it is not a usual result in the classical examples of the calculus of variations) for many of these problems are minimax problems with definite boundaries imposed on the parameters by reason of force limitations. Hence it often happens that the minimax is on one of the boundaries (where one of the parameters is zero, for instance) instead of inside the boundary.

Now let us see what the patrol squadron commander must do to ensure that the contact probability $\underline{G}$ be as large as possible. We assume that he knows that the submarine can only travel a distance $\underline{a}$, under water. If he knew, in addition, that the submarine was submerging according to the schedule ψ he could fly his patrol anywhere between x_0 & x_1 & get same value of G. But flying the patrol at ~~one~~ always at one point (or even at several points) is a dangerous thing to do, for if ~~so~~ the sub-marine commander were to learn this he could alter ψ so as to reduce G. The aircraft commander must fly his patrol at random along x, with a frequency ~~for~~ determined by a probability density factor

$\phi(x)$ (see eqn (10)) such that if the submarine commander found out the shape of ϕ he would not gain any advantage (he could not lower G) but if the aircraft commander found out the submarine's schedules, a change of patrol might enable G to be increased. This requirement, we see, is symmetric to that for the submarine. As with the submarine, the barrier density ϕ must be adjusted so that the quantity $P(x)\,\phi(x)$ which comes into the integral for G has no maxima but is flat on top. Therefore no barrier will be flown for those ranges of x where P is less than some limiting value k; and where P is $> k$, the barrier density ϕ will be $\propto 1/P$, so that $P\phi =$ const in these regions. Therefore we have for safe tactics for patrol aircraft

$$(12)\qquad \phi(x) = \begin{cases} \dfrac{h}{P(x)} & \text{when } P(x) > k \text{ (over range C of } x) \\ 0 & \text{" } P(x) < k \text{ (outside range C of } x. \end{cases}$$

where h is chosen so that $\int \phi\, dx = 1$ i.e. $1/h = \int_C (1/P)\, dx$, etc. . —

The difficulties of solving such problems are not ones of tedious detail but are often due to lack of known techniques. Much more basic research must be carried out before many problems of practical importance can be solved.