

9/2/70

Saw at Institute an interesting article by M.J. Moravcsik on "High Energy Physics" in Physics Today Jan 1969, pp. 119-

The Bootstrap idea in particle Physics.

5/2/70

Thursday

(1)

(* Lipkin's funny remarks)

(1) Introduction -- (Remarks if my paper happens to be the last) -- Strong interactions -- notion of Complex A.M. -- Regge Pole Theory & S-matrix, ~~some relevant~~ - mention of field-theoretic approach - relevant parts of S-matrix & Regge Poles -- Six axioms = Complex variable theory (my special subject at one time, contain essential singularity) -- Bootstrap hypothesis -- Constitution of Hadron Gort! -- ~~Bootstrap~~ Calculations N/D eqns -- Partial exchange forces -- Examples of Bootstrap Calculations -- Bootstrap trajectories -- Ambiguity & democracy (Chew) -- further remarks re. field theory work, (Univ of Colorado series work) -- Quarks & Bootstrap & General remarks.

(Preliminary remarks)

Zemmel's two notes -- At the outset, let I wish to make it clear that what I am presenting to you is just a review of the notion of bootstrapping the bootstrap in elementary particle physics. This notion ~~mainly~~ ^{only} pertains to the strong α interactions associated with the hadrons. It appears to ~~more~~ ^{me that this is a very} elegant and natural to ~~develop~~ ^{develop} this idea via the Regge - Pole - S-matrix channel, rather than the so-called ~~field-theoretic approach~~. It is a little bit surprising ^{many of} how the people who work on the S-matrix theory consider ~~themselves~~ ^{to be} their subject so different from what other people are doing for eg. those working on field-theory and group symmetries. I have a feeling that they are all working within the same framework, but do not realise it, because they have ^{no} ~~time~~ ^{find leisure} to discuss and reflect on these things in the midst of the time they ~~devote~~ ^{almost exclusively} spend on making detailed calculations. (It was therefore a refreshing contrast to find from the ~~stimulating~~ ^{stimulating} lectures of Ramath Chaud, Ramamathan and Santhanam ~~on~~ ^{related to} strong interactions that the Matscience ^{group} ~~people~~ ^{that they} have no such prejudices, and use all the techniques as occasion demands. This remark of mine should not be taken to mean that I do not ~~have~~ ^{except} ~~appreciate~~ for the work presented before the seminar by other workers from Matscience. I am only confining myself to the ~~facts~~ ^{relative} to strong interactions). I, too have no personal predilections for or against any of these types of approach, but the ~~exotic~~ ^{nomenclature} names used in the Regge & S-matrix formalisms appeal to me very much. Thus, in these, we talk of the irreducible representations of the non-compact conspirators, super-covariant infinite number of daughters, fixed Poles as against moving Italians, ghost-killing ~~fields~~ ^{mechanisms}, all names bootstraps etc ~~ghosts~~.

A review of the notion of
the Bootstrap idea

Somewhat the ink does not flow very freely.
Somewhat the ink does not flow quite well
but it does in the other pen, and the nib
does not seem to be quite all right.

• happens to be Boon
14/4/70

all coined by crazy Americans, and all of them blindly copied by the quarrelsome Indians! Nevertheless, I hope to be able to say few words ^{at the end} about the Lagrangian field theoretic approach towards the Bootstrap idea.

In view of the ~~extra~~ commitments that I have imposed on myself, I am afraid I have ^{to} say something about the Regge Pole and S-matrix theories before I ~~go to the~~ actually go to the Bootstrap idea. A significant step towards the building up of these theories was the technique employed by Regge ⁽¹⁹⁵⁹⁺⁶⁰⁾ of introducing the notion of complex angular momentum to explain the Mandelstam representation for potential scattering. Regge showed that such an extension was not purely of academic interest, but opened up new possibilities in discussing the connection ^{the} between scattering amplitudes potentials and the scattering amplitudes. In particular he showed that for special classes of potentials $V(x)$ which are analytically continuable into a function $V(z)$, the range of validity of the dispersion relations becomes much enlarged. This kind of extension of the notion of complex A.M. was known to Watson and Sommerfeld ⁽¹⁹¹⁸⁾ but used only in problems relating to classical physics. To be more precise, what Regge ~~showed~~ originally showed was that for a wide class of potentials, the only singularities of the non-relativistic scattering amplitude in the complex A.M. l -plane were poles which move with energy i.e. poles at positions, say given by $l = \alpha(s)$. These are now called Regge poles, and $\alpha(s)$ is called a Regge trajectory. The fact that Regge trajectories correspond to physical particles (or their resonances) when $\alpha(s)$ is equal to an integer ^{or half-integer} for positive s , attracted the attention of elementary particle physicists. It was also found later that relativistic considerations could also be fitted into the theory, and this gave rise to the existence of cuts besides poles in the l -plane. Thus a new variety of mathematical techniques connected with the theory of functions of a complex variable began to come in for investigations in particle physics, besides the with frequent use of higher transcendental functions, and integral transforms. ~~It is, however, interesting to find that physicists~~ Two mathematical points specially worth spelling out are (i) notation about asymptotic behaviour of functions, and (ii) The frequent use of the word 'analytic' functions. For mathematicians, the latter means a function which is free from singularities (holomorphic) in a region. Physicists tend to use the word much more loosely to mean reference to isolated singularities such as poles and branch points, and not essential singularities and other types of singularities of a "non-smooth" behaviour. [It is suggestion worth examining whether inclusion of such latter types will enrich the Regge theory]. ~~Reference~~ In any case, the notion of "meromorphic" functions (containing only poles in a given region) is retained. This is just the barest outline of this theory. I have to include topics like the partial wave amplitudes,

their signature, singularities, unitarity and so on. In addition the notion of analytic continuation of A.M. using Carlson's theorem, properties of Regge functions, several types of Regge pole representation, and last, but not least application of group theoretic methods to Regge Theory, sense-nonsense amplitudes, and last, but not least, the notions of "conspiracy", and "evasion", and "daughter trajectories". I have purposely mentioned a huge list to frighten you and prevent you from asking any questions about these notions of reggeisation.

Regge pole theory, being of a phenomenological nature, is quite satisfactory, and shows that in particle physics we have other things besides algebra and group theory. Now coming to the S-matrix theory, we have, in addition, dynamics also. ~~But~~ As usual, dynamics is a very bothersome subject incapable of meeting all general situations comprehensively. But it can be shown that by using suitable approximations in consonance with experimental results, the S-matrix theory gives significant results. In fact, S-matrix theory and Regge Theory are intimately connected in as much as the latter ~~was~~ was introduced within the framework of the former. I shall not enter into the formalism of S-matrix theory which ~~could~~ can be found in most standard books on Elementary Particles, except for calling your attention to the well-known Mandelstam repⁿ of 2-body scattering (also called the 4-line connected part) using the s, t and u. I shall, however, examine, from a general point of view, the essential postulates of the S-matrix Theory which will be useful when I go over to ~~present~~ the bootstrap idea. I take these postulates as follows:-

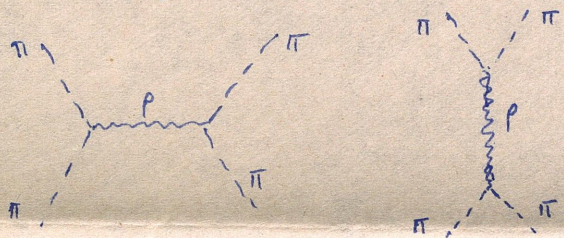
- (1) Superposition principle of quant. mech.;
- (2) Existence of an unitary S-matrix;
- (3) Lorentz invariance of the S-matrix;
- (4) Disconnectedness of S-matrix due to the finite range of the strong interaction forces;
- (5) Maximal analyticity of the 1st kind - That the only singularities of the S-matrix should be poles corresponding to stable or resonance particles;
- (6) Maximal analyticity of the 2nd kind - The S-matrix should be continuable in A.M. throughout the complex A.M. plane.

The first four axioms are not controversial and may well be accepted. There has been much controversy about (5) & (6) and their inter-relationship. But let us accept them, and proceed to examine their ~~implicit~~ implications as regards the Bootstrap idea. It is enough to consider only (5) and (6). ~~Considering for simplicity the Mandelstam repⁿ?~~

Considering for simplicity the Mandelstam representation for the four line connected part, we know that the double spectral functions P_{st} , P_{su} and P_{tu} (defined in terms of the scattering amplitude $A(s, t)$) play a fundamental role in interpreting

Connecting S-matrix theory and Regge pole theory. It can be shown by simple arguments that (5) tells us that a knowledge of the poles determines the double spectral functions, completely, while (6) tells us that a knowledge of the double spectral functions enables us to determine the poles completely. Thus we have a very strong self-consistency condition here. In spite of the above conditions, we might enunciate the Bootstrap Hypothesis i.e. the only set of particles (poles), consistent with the principles (1) to (6), is the actual set of strongly interacting particles found in nature. I can call it Article One (and the only one) of the Constitution of the Government of Hadrons! (Remarks about Govt. of India)

The word "bootstrap", of course, derives from the proverbially impossible feat of "pulling oneself up by ~~one's~~ own bootstraps", which the above self-consistency requirement (perhaps unfortunately) suggests. Originally the idea was that a single particle might be approximately self-consistent, so bootstrapping itself (for eg. the ρ in the π - π interaction) as in the figure below.



But the use of single particle bootstrapping picture is very trivial. Instead, we have to test the bootstrap hypothesis from a "global" point of view. But, unfortunately, the solutions of the unitarity equations involve the solution of infinite sets of coupled non-linear, singular integral equations & so it appears impossible to test the hypothesis as a whole. Attempts have, however, been made to demonstrate that some small subset of these equations the particles is approximately self-consistent, within some subset of the equations which is believed to be approximately decoupled from the rest. These are called bootstrap calculations, and a sufficiently large number of such calculations having been performed, and partial successes having been achieved, make the more optimistic feel that the hypothesis may be true. Some recent theoretical work on the derivation of (6) from (5) has met with partial success thereby strengthening the bootstrap hypothesis.

~~Now coming to~~ The most commonly employed integral equations ~~employed~~ for bootstrap calculations are the so called N/D equations, which consist of a pair of integral equations for $N_2(s)$ and $D_2(s)$ coupled to each other (under the condition that a certain type of poles called the CDD poles do not appear), and can be solved by substituting one into the other in view of the fact that such a substitution leads to a Fredholm integral

equation which can for each of N and D - These eqns introduced by Chew and Mandelstam are likened in importance for bootstrap calculations to the Schrödinger equation of non-relativistic calculations. These equations can be applied to all cases where amplitude $A(s, t)$ for the case of 2 particle scattering leads to Fredholm type N/D equations. It can also be shown that methods can be devised to transform the non-Fredholm type of integral equations for the elastic case to inelasticity in N/D eqns which can be solved in the usual way. Mention might also be made of Cutkosky's bootstrap eqns for 3 particles.

I shall now present a simple example to show a minor advantage of the bootstrap idea over the field theoretic approach. Let us imagine -- (p. 24) --

Thus the bootstrap hypothesis results in establishing the idea of a ~~new~~ democracy among elementary particles, while the search for more and more fundamental particles like the quarks based on considerations of symmetry. I shall present here the views of proponents one of the strong proponents of the former idea viz. G. Chew on this question -- p. 28 bottom to p. 30 up to end of (A).

~~Next I shall say a few words about the derivation attempts to derive the bootstrap hypothesis on the basis of field theory. Some of these have confined themselves only to mesons and not to hadrons in general. Also the difficulties connected with the structure of renormalised Green's function in field theory compel the making of assumptions more ~~than~~ drastic than those employed in the bootstrap theory.~~

This is perhaps too drastic ^a view, ^[P.T.O.] specially if ~~we~~ remembers that some fundamental questions ^{re} hadron-scattering have yet to be answered. No doubt, Mandelstam has been able to show that it is quite possible to have fixed poles in hadron-scattering amplitudes, in particular, they are supposed to occur through the same kind of mechanism which predicts the cuts, at a point where a trajectory passes through an integer or half-integer with the wrong signature to make a ~~particle~~ real particle. But the actual existence of a such a pole has not been demonstrated, because it has not been demonstrated yet that its coefficient is actually non-zero. This could and should be done since at some particular points of the s -variable, things can be done exactly, with no approximation. ~~Further~~

I shall now state ^{two recent} ~~some miscellaneous~~ topics connected with the bootstrap ~~notion~~. One of them is the notion of ~~recip~~ the reciprocal bootstrap in which

The final particles are identified with the particles responsible for the forces in order to fix some parameters (~~the~~ the masses and coupling constants). Although the idea of reciprocal bootstrap remains a very attractive speculation, there is by now no satisfactory evidence for it. Another type of work recently undertaken is to investigate the nature of rising trajectories (to at least value of $j = 11/2$ and $s = 46 \text{ GeV}^2$), and to adapt a new form bootstrap adapted to such trajectories. ^{in π - π scattering}

I finally wish to say a few words, ~~on~~ ^{on} ~~quite a technical~~ ^{quite a technical} about the possible relationships relations between the bootstrap idea on the one hand and the quark model on the other. If the bootstrap hypothesis ^{or the Conshho} ~~is~~ ^{is} accepted, then, of course, quarks do not exist because they have ~~never~~ ^{not} been observed ^{so far}! Then they are purely illusory, a passing phase in our description, which will go away after a while, when we will learn how to use the bootstrap method, and solve our equations without quarks. We can also ask how is it that while both the ~~mesons and nucleons and mesons~~ ^{mesons and nucleons and mesons} ~~are~~ ^{considered separately} look as if they were made ^{in different ways} of quarks, the bootstrap ~~has~~ ^{has} makes no distinction between the two types? of course, one possibility, a terrible possibility, is that quarks exist and bootstrap themselves! More likely, it might be that quarks are mathematical and the description of baryons and mesons in terms of quarks corresponds to a rough sketch of the symmetry properties, if any, of the solution of the Bootstrap equations. Another line of approach would be to find the quark model as a representation of current algebra, then apply the Bootstrap conditions somehow, and obtain a mass formula preferably agreeing with the expts. These are however ideas which I consider worth pursuing; it must be said, however, that both the ideas are of a crucial nature, ~~so~~ ^{so} either one of them is wrong, or both of them are wrong, or somehow they are compatible.

In conclusion, I might say that we have had had a very successful symposium, a special feature of which, from the point of view of the elementary particle workers, was not merely the presentation of ~~a~~ ^{the} completed pieces of work, but a survey of problems still waiting for solution. I only hope that during the next year we will find more occasions to think about these unsolved problems, before we sit down to make detailed calculations.

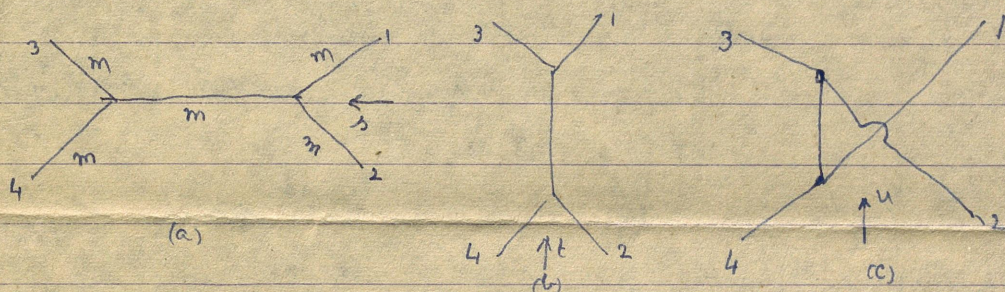
Let me thank you for the patient hearing you have given me.

Suppose the hadron properties are explained by the Bootstrap; then we are faced with a situation which is completely different from the lepton case. Here we have a small number of well-defined \mathcal{Q} elementary particles (not at all aristocratic), which act as elementary, and correctly described by quantum electrodynamics, but in the ~~hadron~~^{other} case we have a system of democratic hadrons which are all made up of one another. In either case, this is a very peculiar situation. Moreover, we should also remember

in the t -channel, corresponding to the first and higher Born approximations - considers of some anomalies (Srivastava & Nath).

(5) Some simple bootstrap calculations.

(a) As a very simple example of how one can try to test the bootstrap hypothesis, let us imagine the simplest possible world of strongly interacting particles which might satisfy the axioms, a world containing just one type of particle. This should be a meson of zero spin, since the choice of higher spin would almost certainly result in lower spin particles being generated; scalar, rather than pseudoscalar like the pion; and of zero baryon number, and other additive quantum numbers, so that a three particle vertex is allowed. In the elastic unitarity approximation of Section (2) we can regard the meson as a bound state pole of two such mesons. The force to generate



such a particle should come, in the 1st approxⁿ, from the corresponding pole terms in the t & u -channels [(b) and (c)]. Solving the N/D eqns which is quite simple in this

$$\text{case, we find } \left(\frac{N_0(s)}{D_0'(s)} \right)_{s=m^2} = \frac{g'}{16\pi} \quad (g' = \text{output pole}, g = \text{residue})$$

By numerical computation it was found $g/m^2 = 16.5$ to get $D_0(m^2) = 0$, but that then $g'/m^2 = 10.5$. Hence evidently the bootstrap does not work. The discrepancy is so wide that we can safely conclude, without undue surprise, that a universe which consisted of nothing but scalar mesons (as its only type of strongly interacting particles; there might of course be leptons etc. as well) could not satisfy our analyticity properties. In the field theory such a universe is not eliminated [since renormalisability conⁿ holds, ~~and~~ and the soln of field eqns with interaction (Lagrangian) has CDD poles at $s = m^2$ corresponding to fig (a) above]. Thus we have a simple example of a minor advantage of bootstraps over field theory in that such a hypothetical world can be eliminated.

(b) Consider the ρ in π - π scattering - Let g work done on this by several people, but the final conclusion must be that although the ρ produces a strong force, this

$$N_e(s) = \frac{1}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'-s} t_e(s') D_e(s')$$

$$D_e(s) = 1 - \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'-s} P_e(s') N_e(s') \quad \dots (A)$$

This pair of coupled integral equations can be solved by substituting one into the other. These give, with $N_e(s)$ & $D_e(s)$ for eq

$$D_e N_e(s) = B_e^L(s) + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{B_e^L(s') - B_e^L(s)}{s'-s} N_e(s') ds' \quad \dots (B)$$

is a Fredholm integral eqn which can be solved to find $N_e(s)$ & from (A)

find $D_e(s)$ $\left[B_e^L(s) = \frac{1}{\pi} \int_{-\infty}^{s_L} \frac{\text{Im} \{ B_e^{\pm}(s') \}}{s'-s} ds' \right]$ & $B_e^{\pm}(s) \xrightarrow{l \rightarrow \infty} B_e^L(s)$

s_0 is lowest threshold and s_L the beginning of the left hand cut

In all this it is assumed that elastic unitarity holds for all $s > s_0$. This sometimes results in non-Fredholm type integral eqns because of behaviour of $B_e^L(s)$ for large s . We try therefore to put in inelasticity in the eqns

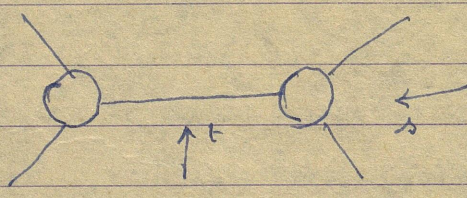
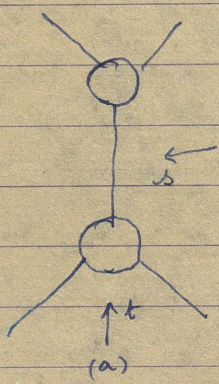
(3) Inelasticity in N/D eqns - This introduction is achieved by replacing the unitarity condition $\text{Im} [A_e^{aa}(s)] = P_e(s) |A_e^{aa}(s)|^2$ for $s < s_T$ by

$$\text{Im} \{ A_e^{ab}(s) \} = \sum_n P_e^n(s) A_e^{bn}(s_+) A_e^{na}(s_-) + (\text{many particle channels})$$

and we have $\text{Im} \{ B_e^{ab}(s) \} = \sum_n P_e^n(s) B_e^{bn}(s_+) B_e^{na}(s_-)$

An alternative method given by Froyd Warrack:

(4) Particle Exchange forces - We have noted that the left hand cut, which takes its main contribution from crossed channel singularities, corresponds to



the "force" or potential in non-relativistic scattering and that the solving the

N/D eqns is analogous in many ways to the solution

of Schrodinger's eqn for the scattering problem. Thus a pole in the t-channel corresponding to the exchange of a single particle as in (a) gives rise to a force which may generate an s-channel pole as in (b). The well-known analogy between the Yukawa potential and scalar meson exchange is being invoked here.

What are considered in this section are single & multiple particle exchanges

(9) Heidelberg International Conference on elementary particles (1968) - Ed. Fichtel

- It is hard to be original at the present rate of international conferences. In its own shortcomings it will be strictly personal - We are at a crucial point in quantum field theory. Either the determined effort of a group of "hard analysts" succeeds in constructing local quantum theories for a continuous range of masses, coupling constants and some primordial form of interaction. Apart from its bearing against a bootstrap concept of our world, this would lead to a new frontier for relativistic dynamics - need to tackle many-body problems

(p. 120) As far as the quark model is concerned, it does not seem a priori inconceivable that 4 quarks & one antiquark also form some bound state, although some of the nice simple features of the model would be lost. A new kind of nuclear chemistry for quarks would thus have to be investigated.

(p. 219) - Problem of the dips and the ghost-killing mechanism in Regge Theory

(p. 253) - Articles on quarks and symmetries by H. G. Lipkin -

In order to get a realistic picture of quark model predictions, one must look between the lines of these papers and decide what is really assumed and what is tested.

But there are many papers & life is short.

Nobody knows how to justify the quark model from its first principles. It gives sets of rules for calculating relations between exptl. quantities. These relations seem to be in agreement with expt. The choice of rules is guided by intuition based on atomic and nuclear models, but has no rigorous basis. It is therefore important to keep track of the particular rules or assumptions which are needed to obtain a particular result. One can then see whether these rules are also obtainable from other models which do not need quarks. I shall therefore ~~take~~ try to take you on a "guided tour of the quark model maze" (for pedestrians of course), pointing out the various sets of assumptions used at various stages.

Our basic ~~basic~~ approach is the following. Quarks and symmetries are very interesting, but nobody really understands why. The secret may lie deep in the mysteries of the irreducible representations of the noncompact conspirators, supercovariant daughters, fixed Poles, moving Italians and crazy Americans (Add "bloody Indians" ! ?). However, I prefer to think that the mysteries will be unravelled by experimentalists working in the laboratory. I therefore look at the quark model as a guide to interesting experiments [The finding of appropriate references is left as an exercise to the reader]

to $J = 1/2$ non-Regge poles. In the absence of experimental evidence, why resistance to notion of complete nuclear democracy? or why are quarks popular in certain circles

There seem to be three outstanding reasons. The first is the precedent of the electromagnetic theory, which controls the overwhelming majority of observable phenomena. An aristocratic structure is the essence of electromagnetism, where there exists, a priori a central component of the theory (say photon) accepted as given and not to be necessarily to be explained in terms of general principles. Certain superficial aspects of strong interaction theory resemble e.m. theory & hence people accept a central core.

The author shares the feeling that the unity of physics is its most attractive aspect. There is, nevertheless, a possibly unavoidable reason for a difference in status between e.m. & strong interactions. In contrast the parameters of strong interactions - being remote from the measurement process - stand a chance of being determined through dynamics. Perhaps, the answer to the charge that S-matrix theory destroys the unity of physics, by separating the two categories, is that, without such a separation, there would be no physics.

It is worth adding that apart from Lorentz invariance none of our S-matrix principles can tolerate zero-mass particles. It seems pointless, consequently, to speculate on the possibility that the photon might after all be plebeian & somehow emerge from the bootstrap. . . .

A second source of doubt about nuclear democracy stems from the observed nuclear multiplet structure for low baryon number, which has been related to symmetry groups. SU_2 symmetry (isospin) is exactly maintained by strong interactions and correspondingly arouses no suspicion about nuclear democracy. It is broken by the aristocratic photon, which we have exiled from nuclear society. The breaking of SU_3 , however, is due to strong interactions, and so reasoning by analogy one might expect to find as the culprit a nuclear aristocrat. The danger of arguing by analogy between e.m. & strong interactions has already been pointed out. For eg. shell symmetry is broken by strong interactions & there has never been a suggestion that a mysterious particle is to blame for this. There are also many other approximate symmetries that have long been known for particles with large B. Approximate symmetry of shell structure is regarded as a "dynamical accident" & the same may be true of SU_3 (If SU_3 has the same kind of origin as shell structure, we expect it to hold only for certain groups of ~~poles~~ poles. There will be many poles for

which SU_3 is meaningless).

The third reason for dislike by some of ~~the~~ a dynamically governed democratic structure for nuclear society, is that it makes life exceedingly difficult for physicists. Swaitly new kinds of techniques in analysis to develop S -matrix theory a great deal is to be done. Emphasis on 4-line connected part has an anti-fundamental ring (approach based on potential theory which comes from E. M. J. theorem, analogy with which is dangerous). Efforts so far have yielded little progress for enormous labour. Years may be required to develop the appropriate machinery & for many physicists it is tempting to believe that such questions are not really important. Perhaps a lucky guess about some new symmetry group - with an associated set of fundamental particles, will lead us out of the wilderness. If quarks be discovered interest in S -matrix theory will fade away. But exp't. evidence of last 30 years or more makes the existence of elementary hadrons unlikely. Physicists, whether they like it or not will have to tackle strong interaction theory compatible with nuclear democracy.

(B) Experiment & theory - In fact, every advance in theoretical understanding of the S -matrix has relevance to exp't. analysis, because what the theorist studies is exactly what the experimenter studies. In this sense, the analytic S -matrix has had a unifying influence in particle physics; the language of the theorist & experimenter becomes identical. Sometimes this closeness of theory & exp't is dismissed by the uninitiated as "sophisticated phenomenology"!

(C) Present & future - So far no systematic approach to S -matrix theory exists. Unless some fantastically clever theorist appears on the scene, exp't. must be the guiding factor. The author's belief is that the theory will never stagnate as long as it is connected with exp't, unless some of the apparently established principles are overthrown.

(11) 1966 Majorana Conference on Strong & weak interactions

p. 202 - M. Gell-mann - Relativistic quark model as representation of
 @ current algebra.

p. 688 - Reggeized bootstrap by Y. N. Srivastava - Refers to work with