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SRIKRISHNARAJENDRA ROAD DISHANGUDI

8 .8. 4

Right groups (i.e. right simple semigroups containing an idempotent) – Chap I is a comprehensive introdn to the theory – Two elements of S are $\mathcal{L}$ equivalent ($\mathcal{R}$ equivalent) if they generate the same principal left (right) ideal – Join of the equivalence relations $\mathcal{L}$ & $\mathcal{R}$ is denoted by $\mathcal{D}$ and their intn by $\mathcal{H}$ – A class $D \subseteq S$ is regular if for every $a \in D$ there is an $x \in S$ such that $a = axa$. Properties of resulting classes (in particular of regular $\mathcal{D}$-classes) in an arbitrary S are described. – Properties of 0-minimal ideals and 0-simple semigroups – Completely 0-simple semigroups – Complete detn of all possible completely 0-simple semigroups – Repn of a semigroup by means of matrices – Rees repn theorem of completely 0-simple semigroup – Faithful repn of a regular semigroup – decomposition of a semigroup into union of simple subsemigroups of various kinds, including groups – Tamura-Kimura description of any commutative semigroup as a semilattice of so-called "Archimedean" semigroups – Representations of semigroups by means of finite matrices over a field Φ. – Method of constructing all irreducible representations of S from those of its principal factors – The authors reproduce not only the <u>best results</u> known up to now, but also new and original material generalising known results and give new presentations which often able one to prove results more simply. These contributions are scattered all over the book. – 228 exercises many including 2 to 4 questions contain much material previously published in a great number of papers, results that, in the time they have been published, have given valuable contributions to the subject. Not all of the exercises can be solved by inspection for they are often meant to supplement the text. Some of them will serve as sources for future research. – Excellent & <u>clearly</u> written book.

Th. 11. S regular iff $(a)_{(1,1)} = (a)_R (a_L)$ ... (6) for every element of S.

Here $(a)_L$ denotes the principal left ideal of S generated by element a in S.

A semigroup is said to be a duo-semigroup if every one-sided (left or right) ideal of S is a two-sided ideal. Th. 9 has an interesting consequence for the case of regular duo semigroups.

Th. 12. Let S be a regular duo semigroup. Then every bi-ideal of S is a two-sided ideal of S.

It is known that a semigroup S which is a semilattice of groups is both regular and duo semigroup [Ref. paper 10. Acta. Sci. Math 30, 133-38 (1969)]. Th. 12 implies the following result.

Th. 13 — Let S be a semigroup which is a semilattice of groups. Then each bi-ideal of S is a two-sided ideal of S

Cor. For such an S as in Th. 13, every quasi-ideal Q of S is a two sided-~~semi~~ ideal of S.

It may be noted that Ths. 12 & 13 remain true with (m, n) ideal instead of bi-ideal [Ref. 11 — Elem. Math 24, 39-40 (1969)]

Th. 14. Suppose S is a regular duo semigroup, and m, n are arbitrary non-negative integers such that m+n > 0. Then every (m, n)-ideal of S is a two-sided ideal of S.

Th. 15 — Let S be a semigroup which is a semilattice of groups, then the (m, n)-ideal of S [same conditions for m & n as in Th. 14] is a two-sided ideal of S.

[This paper is a model of brevity, and it is clear what exactly are the new results obtained in this paper.]

Math. Reviews. Vol. 24 (July - Dec. 62), p. 488) — Review of Clifford & Preston of Vol I of the book "Algebraic theory of semi-groups" (Math. Surveys, No. 7 of A.M.S. (1961) [This is the book referred to by Sen] [Review by S. Schwarz] simple special semigroup called bicyclic. — Simple & 0-simple semigroups-

An important property of bi-ideals proved by the author [Ref. (6)] is

Th. 5. If A be a bi-ideal, and B a non-empty subset of S, then the products AB and BA are bi-ideals of S

If $\overline{S}$ be the multiplicative semigroup of all non-empty subsets of S, and S_1 is the set of all bi-ideals of S, then by Th. 5, the set S_1 is a semigroup under the multiplication of subsets, and the semigroup S_1 is a two sided ideal of $\overline{S}$.

Th. 6. Let A, B be bi-ideals of S; then AB and BA are also bi-ideals of S.

As a simple consequence of Th. 6, we obtain

Th. 7 Let P, Q be quasi-ideals of S. Then PQ & QP are bi-ideals of S

It is known the following characterisations of the bi-ideal [Ref. 5 & 11 of author]

Th. 8. A non-empty subset B of S is a bi-ideal of S if & only if any one of the following assertions holds:

(A) There exists a left ideal L of S such that B is a right ideal of L;

(B) There exists a right ideal R of S so that B is a left ideal of R.

(C) There exist a left ideal L & a right ideal R of S such that

$$RL \subseteq B \subseteq R \cap L. \quad \text{--- (1)}$$

In what follows we shall say that S is regular if to any element $\underline{a}$ of S, there exists an element $\underline{x}$ in S such that the condition

$$axa = a \quad \text{--- (2)}$$

holds. It is known that a semigroup is regular if & only if the relation

$$L \cap R = RL \quad \text{--- (3)}$$

holds any left ideal L & for any right ideal R of S. This criterion & Th. 8 imply the following result

Th. 9. Let S be a regular semigroup and A a non-empty subset of S. Then A is a bi-ideal if & only if it may be represented in the form

$$A = RL \quad \text{--- (4)}$$

where R, (L) is a right (left) ideal of S.

The author recently obtained the following characterisations of regular semigroups by means of bi-ideals.

Th. 10 – A semigroup S is a regular iff $(a)_{(1,1)} = aSa$ --- (5) for each element $\underline{a}$ of S.

Lajos' paper – On the Bi-ideals in semigroups, p. 710. (Paper of just 3 pages)

Let S be a semigroup & A non-empty subset of S. We shall say that A is a bi-ideal or (1-1) ideal of S if the following conditions hold:

(i) A is a subsemigroup of S.

(ii) $ASA \subseteq A$.

The notion of bi-ideal was introduced by R. A. Good & D. R. Hughes (Amer. Math. Soc 58, 624-25 (1952). It is also a special case of the (m, n)-ideals introduced by the author [Ref. No. 4 out of the 9 references to the author himself. Ref. (10) here refers is given as "On semi-groups which are semilattices of groups: Acta Sci. Math. 30, 133-35 (1969)]

In this short note we give a summary of some results concerning the bi-ideals of semi-groups, and we ~~assume~~ announce some new results. For the terminology not defined here we refer to the books by A. H. Clifford and G. B. Preston [Amer. Math. Soc. 1961, 1967] referred to I & II]. Proofs of the results will not be given. (Note)

Th. 1. Let S be an arbitrary semigroup. Then any left (right, two sided, and quasi-) ideal of S is a bi-ideal of S.

Th. 2 – Intn of bi-ideals $A_1, \ldots A_n$ viz $E\bigcap_{i=1}^{n} A_i$ is either empty or a ~~bi-ideals~~ bi-ideal of S.

Defn Proper bi-ideal A if S − A is not empty. It is easy to see that a group has not proper bi-ideals, and what is more this property characterises the class of groups among semigroups.

Th. 3. A S is a group iff it has no proper bi-ideals.

Defn: Bi-ideal of S generated by A is meant the smallest bi-ideal of S containing A. denote this by $(A)_{(1,1)}$. If A has a single element, then $(A)_{(1,1)}$ is said to be a principal bi-ideal of S

Th. 4 – If a be an arbitrary element. Then $(A)_{(1,1)} = A \cup A^2 \cup ASA$ and $(a)_{(1,1)} = a \cup a^2 \cup aSa$.

~~(1) The use of the word "iff" in definitions is unnecessary.~~

(2) The essential idea of the papers is the introduction of the notion of a "pseudo-ideal" ~~analogous~~ ~~similar~~ ~~analogous~~ in addition to the notions of bi-ideal and quasi-ideal known in the literature, and ~~use~~ the use of this notion to derive ~~theorems relating to~~ ~~groups, regular semi-groups,~~ criteria for a semi-group being regular, being a group or a semilattice of groups. ~~Although the relevant theorems are not~~ The relevant theorems serve to bring out the usefulness of the ~~role of~~ concepts and properties of ideals in semi-group theory. The paper may be accepted for publication.

The following modifications are suggested so as to make the paper more concise:

(1) The use of the word "iff" in definitions ~~is unnecessary~~ may be avoided

~~(2) Most of the examples are trivial and may be omitted with the exception~~ of Ex. 3.

~~(3) Similarly many of the propositions are obvious as can be seen by inspection~~. Thus ~~Propositions~~ 1,

(2) ~~The emphasis of most~~ In most of the recent work on semigroups the emphasis is mostly on the presentation of new results rather than on giving detailed proofs of these results. ~~except, of course, in the when the results are structure theorems of depth~~ Accordingly it is suggested that

(a) the several examples be merely mentioned without any detailed explanation; ~~and~~

(b) the proofs of Theorems 1, 2, and 3 and 6 may be omitted;

(c) Theorems 4 and 5 and 7 may be combined into a single theorem, and the proofs ~~also be~~ shortened;

(d) Theorems 8 and 9 ~~also to~~ may also be combined into a single theorem;

~~(e) so Proposition 10,~~

~~These suggestions are made with a view~~

(3) ~~The bibliography refers to only to publications in book form on semi-groups only up to 1963. So~~ The author may consult the following more recent papers ~~As regards Theorem 10, the author's attention is drawn to a~~

(i) ~~paper by~~ Lajos: " Semigroups that are semilattices of groups I – Math. Japan 16 (1971), 25–36

(ii) T. E. Hall: "On regular semigroups", J. Algebra, 24 (1973), pp. 1–24

(iii) ~~Mohan S. Putcha: Semi Articles~~ Papers on related topics in Semigroup Forum 6 (1973); ~~and these~~ may be included in the bibliography if the author finds them relevant.

BULLETIN
OF THE
CALCUTTA MATHEMATICAL SOCIETY

92, Acharyya Prafulla Chandra Road,
Calcutta-9 (India)

To

Reference : Paper No. 58/73 Date 28.6.74

Title of paper On Pseudoideals of semi-groups

Name of the author M. K. Sen

Paper No. 58/73

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8. The referee may comment if the paper is original but does not contain substantial new material, whether this should be published as a letter to the Editor or not.

Math. Reviews

(1) Sept. 73, p. 626. Bruce W. Mielke, Publ. Math. Debrecen 18 (1971), 73-75 (1972)

A note on bi-ideals and quasi-ideals in semigroups.

Let B be an equivalence relation defined on a semigroup S by aBb if $a = b$ or if there exist u and v in S such that $aua = b$ and $bvb = a$. Let B_a denote the B-class containing a in S. The author studies this equivalence relation B and bi-ideals of a semigroup S.

(2) March 73, p. 664

S. Lajos - A remark on inverse Clifford semi-groups. Mat. Vesnik, 8 (23), 1971, 25-26

A semigroup S is a semilattice of groups (or equivalently, S is an inverse Clifford semigroup) if & only if $L \cap R = LR$ for every left ideal L of S and every right ideal R of S. Replacing "left ideal" or "right ideal" or both by "bi-ideal" or "quasi-ideal" one obtains eight other statements, proved in this paper, characterising inverse Clifford semigroups.

Ibid, p. 315-16. - The paper contains seven conditions necessary and sufficient for S to be a semilattice of eight groups. For instance $R \cap I \cap B = RIB$ or $R \cap I \cap Q = RIQ$ or $R \cap B = RSB$ or $R \cap B = RB$ for every right ideal R, every two sided ideal I, every bi-ideal B, and every quasi-ideal Q of S.

(3) Feb. 73, p. 272. Note on anti-ideals in semigroups by J. Fabricius

normal Bands (idempotent semigroups satisfying the identity $zyzx = xzyx$)

p. 374 - paper by Lajos - Note on regular semigroups - Announces (without proof) some results concerning regular semigroups.

(4) August 73, p. 330. Lajos - Semigroups that are semilattices of groups I, Math. Japon. 16 (1971), 25-31.

Study of ideal-theoretical characterisation of semigroups which are semilattices of groups. - Many conditions expressed in terms of left ideals, right ideals, bi-ideals, quasi-ideals and two-sided ideals of S.

Ibid, on regular right duo semigroups, Elem. Math 27 (1972), 86-87

Mention of condition $I \cap Q = QI$ for every quasi-ideal Q and two-sided ideal I for S being a regular right duo semigroup. Suff. equivalent condition $B \cap R = RB$ (B being bi-ideal)

(5) April 73, p. 965 - Structure theorems for regular semigroups by G. Lallement

Semigroup Forum, 4 (1972), 95-123

Regular semigroups which are subdirect products of completely 0-simple semigroups. Ref. to Clifford's theory of radicals [Semi Forum I (1970), no. 2, 103-127]

(6) December 73, p. 1591

p. T. E. Hall - On regular semigroups, J. Algebra 24 (1973), 1-24.

p. 1592 S. Lajós - Notes on regular semigroups IV - The author proves that a semigroup is a semilattice of left groups if & only if $B \cap L = BL$.

p. 1593 – Two papers by R. J. Warne on structure of a class of regular semigroups in Kyungpook Math. J. 12 (1972) & 11 (1971), pp. 13-17 & 165-67 respectively

Nov. 73, p. 1280, A. S. Prosvirov (1971) some Russian Journal – Periodic semigroups.

Let e be an idempotent of a periodic semigroup S. The set K_e of elements $a \in S$ such that $a^n = e$ for some $n = n(a)$ is called a torsion class

October 73. p. 939. Janet E. Ault – Pac. J. Math. 41 (1972), 303-306. – Regular semigroups which are extensions of groups – A semigroup T is an extension of a group by a 0-categorical regular semigroup if & only if T is a regular semigroup which contains a categorical minimal left ideal that is also a minimal right ideal

Zentralblatt, Bd. 251 – 1973, 29/8/73,

p. 155 – Notes on regular semi-groups IV by S. Lajos.

Ideal theoretic characterisations are found for semigroups that are semilattices of left (or right) groups.

on exponential semigroups If $(xy)^n = x^n y^n$ for all +ve integer n

p. 156. Ideals of a semilattice by M. Petrich, Czech. Math. J 22 (97), 361-67 (1972)

Deals with retract & other classes of ideals

Uber Homomorphismen auf regularen Halbgruppen by M. M. ~~Lahorkin~~ Lesohin (Russian)

Ibid – Deals with accessible semigroups (Commutative)

Ibid – 14/11/73

p. 127 – Categorical semigroups by Monzo – Semigroup Forum 6, 59-68 (1973)

Question raised by McMorris & M. Satyanarayana [Proc. Amer. Math. Soc. 33, 29 271-77 (1972)] whether a regular semigroup with a tree of idempotents is categorical This question is answered in the affirmative.

Ibid, 18/12/73.

p. 127. M. Petrich: Regular semigroups satisfying certain conditions on idempotents & ideals (Trans. Amer. Math. Soc. 170, 245-67 (1972)

An ideal I of a semigroup (regular) S is called categorical if $abc \in I$ implies either $ab \in I$ or $bc \in I$. Regular semigroups all of whose ideals are categorical are considered ... The paper contains quite a few definitions, theorems, propositions, corollaries, lemmata which cannot & need not be reproduced here, though almost every fancy regular semigroups will find something to his own taste.

Ibid. 3/12/73. p. 130, Mohan S. Putcha – Semigroup Forum 6, 12-34 (1973)

Putcha's theorem that a semigroup S is a semilattice of archimedean semigroups

(Contd)

[archimedean if for any two elements a, b in S, $b^m \in S'aS'$ and $a^n \in S'bS'$ for some positive integers m, n] It is ~~known that S is~~ if and only if for $a, b \in S$, $a|b$ implies $a^2 | b^m$ for some positive integer m.

p. 134. G. Lallement – Structure theorems for regular semigroups.

Basic theorems in alg. theory of S concern regular S – Suschkewitch – Rees theorem – Cliffords theorem (describing the structure of a union of groups with commuting idempotents. This had great influence on the whole of semigroup theory) – 0-simple & 0-bisimple semigroups

Math. Reviews. Vol. 44. No. 4 (Oct. 72)

Ault & Patrick – Structure of semigroups J. R & A. Math 251 (1971), 110–141.

Nov. 72 (p. 981) on "Hopficity"

p. 984 – Paper by Namboodripad & Subramoniam – Semigroup Forum 2 (1971), 264–70

on some classes of regular semigroups