A Universe with a Conical Singularity

We shall begin with what we have called .

1 Einstein – Gödel Metric

$$ds^2 = 2(du + g\sin\alpha\, d\beta)\,dt - 2L(du + g\sin\alpha\, d\beta)^2 - fR(d\alpha^2 + \sin^2\alpha\, d\beta^2)$$

$2L = \epsilon \quad (\epsilon^2 = 1) \qquad f = \frac{1}{2}g_\alpha + \frac{1}{2}g\cot\alpha$, satisfies

$$\frac{1}{2}\left(\frac{f_\alpha}{f}\right)_\alpha + \frac{1}{2}\frac{f_\alpha}{f}\cot\alpha - 1 = -\frac{(1+3\epsilon)f}{R}.$$

$8\pi p = \Lambda - \frac{\epsilon}{R^2}, \qquad 8\pi\rho = -\Lambda + \frac{1+\epsilon}{R^2}$

$\epsilon = 1 \longrightarrow$ Einstein's universe

$\epsilon = -1 \longrightarrow$ ~~Cx~~ Gödel's universe

2. A mass particle in E G universe

~~Metric in the ... M~~

Non rotating particle : McVittie

Kerr-particle

Metric as above with

$2L = \epsilon + \mu, \qquad \mu = m e^{2\epsilon y/R}\sin\frac{2z}{R} \qquad z = u - \epsilon t$

$y = \int g\, d\alpha.$

$8\pi p = \Lambda - \frac{\epsilon}{R^2} + \frac{2\mu}{R^2}$

$$8\pi\rho = -\Lambda + \frac{2+\epsilon}{R^2} - \frac{6\mu}{R^2} \,. \quad \cancel{16}$$

Perfect fluid not generally possible

$$16\pi\sigma\,(1-2\mu) = (\epsilon-2\mu)^2 - (1-2\mu)^2$$

$\epsilon = 1.$ Perfect fluid possible. $\sigma = 0$

$\epsilon = -1$ $\qquad 2\pi\sigma = \dfrac{\mu}{1-2\mu}$

A particle in the rotating background will radiate!.

3. Solutions sensative to choice of Constants,

$2L = \epsilon + \mu$ But now $\mu = m\, e^{\frac{2ky}{k}} \sin \dfrac{2L(t-ky)}{k}$

Noted if we replace k by ϵ, we get para 2.

~~$8\pi p$~~ or $8\pi p = \Lambda - \dfrac{\epsilon}{R^2} + \dfrac{2\mu}{R^2}$, $8\pi\rho = -\Lambda + \dfrac{2+\epsilon}{R^2} - \dfrac{6\mu}{R^2}$

$$4\pi\sigma = (\epsilon+1-2k)\,\frac{\mu}{1-2\mu}.$$

$\epsilon = 1$ $\qquad 4\pi\sigma = \dfrac{(1-k)\,2\mu}{1-2\mu}.$

$\epsilon = -1$ $\qquad 4\pi\sigma = \dfrac{-2k\mu}{1-2\mu}$

4 Conical singularity

Consider the metric

$$ds^2 = 2(du + g\sin\alpha\, d\beta)(dt + H\sin\alpha\, d\beta) - 2L(du + g\sin\alpha\, d\beta)^2 - fR(d\alpha^2 + \sin^2\alpha\, d\beta^2)$$

We shall consider only Einstein Universe

so $f = \dfrac{R}{1+3\epsilon}$ $\quad \epsilon = 1 \quad \therefore f = \dfrac{R}{4}$

$2L = 1 + \dfrac{4h}{R}$ $\quad \cancel{2L = \epsilon - \frac{h}{Rf} = \epsilon - \frac{4h}{R}}$

$h = \frac{1}{2}H_\alpha + \frac{1}{2}H\cot\alpha$

$$8\pi p = \Lambda - \frac{1}{R^2} + \frac{4}{R^3}h$$

$$8\pi\rho = -\Lambda + \frac{3}{R^2} + \frac{20}{R^3}h$$

$$16\pi\sigma\left(1 + \frac{12h}{R}\right)R^2 = -16\left[\left(\frac{h}{R}\right)_{\alpha\alpha} + \left(\frac{h}{R}\right)_{\alpha}\cot\alpha + 8\left(\frac{h}{R}\right)^2 + 2\frac{h}{R}\right]$$

The velocity of the fluid is $\left(\frac{1}{\xi}, 0, 0, \frac{L}{\xi} + \xi\right)$

then $\xi^2 = 1 + \dfrac{12h}{R}$

Consider $\dfrac{h}{R} = -n\,\mathrm{cosec}^3\alpha$ $\qquad \xi^2 = 1 - 12n\,\mathrm{cosec}^3\alpha$

$\therefore \pi\xi^2R^3 = 4n(1-2n)\,\mathrm{cosec}^3\alpha$

$n = 0 \Rightarrow h = 0 \Rightarrow H = 0$, Einstein Universe

Since $\xi^2 > 0$ $\quad 12n < \sin^3\alpha$ $\quad$ Let $12n = \sin^3\theta$

$\sin^3\alpha > \sin^3\theta \quad \therefore \alpha > \theta$