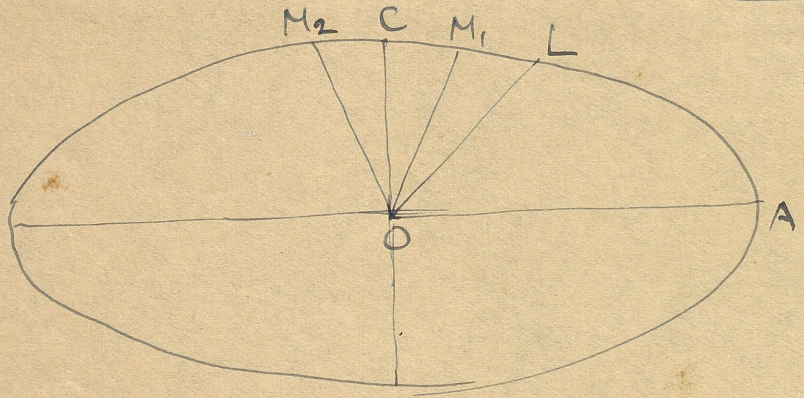
Some properties of the Magnetic Ellipsoid of a Crystal



Consider an ellipsoid whose principal semi diameters a, b, c $(a > b > c)$ are ~~respectively~~ equal to $1/\sqrt{\chi_a}$, $1/\sqrt{\chi_b}$ and $1/\sqrt{\chi_c}$ respectively, where χ_a, χ_b, χ_c are the susceptibilities of the crystal (dia- or para-magnetic) along its three principal magnetic axes OA, OB and OC resp^y. The suscep. along any direction whose direction ~~cos~~ cosines with reference to the a, b, c axes are l, m, n will be given by

$$\chi_{lmn} = 1/r^2$$

where r is the radius vector from O along that direction.

Evidently $$\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2} = \frac{1}{r^2}$$

i.e. $$\chi_{lmn} = l^2\chi_a + m^2\chi_b + n^2\chi_c \; ; \quad \text{---} (1)$$

as we should expect from direct considerations.

Let OM_1 and OM_2 be the normals to the two central circular sections of the ellipsoid and let the angles between either of these axes and OC be V. [the magnetic analogue of the optic axial angle] The magnetic anisotropy of the crystal in the planes $\perp^r$ to OM_1 or OM_2 will be zero.

Evidently

$$\frac{b^2\cos^2 V}{a^2} + \frac{b^2\sin^2 V}{C^2} = 1 \quad \text{--- (2)}$$

whence $$\tan^2 V = \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{b^2}-\frac{1}{c^2}} = \frac{\chi_a-\chi_b}{\chi_b-\chi_c}. \quad \text{---- (3)}$$

Consider a central section of the ellipsoid by a plane whose normal makes angles of θ_1 and θ_2 with OM_1 and OM_2 respectively. Let ξ and η $\cancel{(\xi > \eta)}$ be the two semi diameters of the elliptic section. If χ_ξ and χ_η are the susceptibilities along these two diameters, ~~then~~ $\chi_\xi = \frac{1}{\xi^2}$ and $\chi_\eta = \frac{1}{\eta^2}$, we shall show that the anisotropy in the plane of the section is given by

$$\chi_\xi - \chi_\eta = (\chi_a - \chi_c)\cdot\sin\theta_1\cdot\sin\theta_2.$$

Let us first consider the special case when OL lies in the ac principal plane of the ellipsoid making an angle α with OC where

$$\alpha = \frac{1}{2}\left(\theta_2 \pm \theta_1\right) \quad --- (4)$$

the upper or lower sign being taken according as ~~OL is > or <~~ $OL \gtrless OM_1$. Correspondingly

$$V = \frac{1}{2}\left(\theta_2 \mp \theta_1\right). \quad --- (5)$$

Let the diameter of the elliptic section $\perp^r$ to OL which lies in the ac plane be ξ. Then $\eta = b$. We have then the relations

$$\frac{\xi^2 \cos^2\alpha}{a^2} + \frac{\xi^2 \sin^2\alpha}{c^2} = 1 \quad --- (6)$$

and $$\frac{b^2 \cos^2 V}{a^2} + \frac{b^2 \sin^2 V}{c^2} = 1 \quad \text{same as (2)}$$

which can be rewritten in the forms

$$\frac{2}{\xi^2} = \frac{1}{a^2} + \frac{1}{c^2} + \left(\frac{1}{a^2} - \frac{1}{c^2}\right)\cos 2\alpha \quad --- (7)$$

$$\frac{2}{\eta^2} = \frac{2}{b^2} = \left(\frac{1}{a^2} + \frac{1}{c^2}\right) + \left(\frac{1}{a^2} - \frac{1}{c^2}\right)\cos 2V \quad --- (8)$$

whence $$\frac{1}{\xi^2} - \frac{1}{\eta^2} = \mp\left(\frac{1}{a^2} - \frac{1}{c^2}\right) \cdot \sin\theta_2 \cdot \sin\theta_1 \quad --- (9)$$

Since $\frac{1}{a^2} - \frac{1}{c^2}$ is negative, the upper sign will correspond to $\xi < b$, which shd. naturally be the case when $OL > OM_1$.

Now in the general case when OL may have any orientation, in view of the ~~F~~ result (proved in Salmon's Geom. of Three Dimensions, ~~Art~~ 245) that the length of an axis of a central section of a quadric completely determines the sum or difference of the angles which its plane makes with the two circular sections — whether it is the sum or the diffce depends on whether the ~~scale~~ selected axis shorter or longer than the diamr of the circular section — we should have the relations

$$\frac{2}{\xi^2} = \frac{1}{a^2} + \frac{1}{c^2} + \left(\frac{1}{a^2} - \frac{1}{c^2}\right) \cos 2\varphi$$

$$\frac{2}{\eta^2} = \frac{1}{a^2} + \frac{1}{c^2} + \left(\frac{1}{a^2} - \frac{1}{c^2}\right) \cos 2\varphi'$$

where $2\varphi = \theta_2 \pm \theta_1$

and $2\varphi' = \theta_2 \mp \theta_1$

according to the 2 alternatives ~~select~~ chosen,

whence $$\frac{1}{\xi^2} - \frac{1}{\eta^2} = \mp \left(\frac{1}{a^2} - \frac{1}{c^2}\right) \cdot \sin\theta_2 \cdot \sin\theta_1 .$$

The upper sign corresponding to ξ being less than η.