Gravitational Collapse of Spherical Radiating Objects.

1. Introduction.

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Solution $f = \text{const}$ requires that singular state is reached when $\xi = 2c$ and it requires infinite proper time to reach that state

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(1)' gives $$(\ ) = \frac{z r f \left(\frac{br}{2}+1\right) - r^2 f^2 z_1}{z^2}$$

Use this in (2)

$$z\left[z r f\left(\frac{br}{2}+1\right) - r^2 f^2 z_1\right] - \cancel{2}z^2 r f\left(\frac{br}{2}+1\right) + r^2 f^2 (z-c) = 0$$

$$\therefore -z^2 r f\left(\frac{br}{2}+1\right) - r^2 f^2 z z_1 + r^2 f^2 (z-c) = 0.$$

$$\therefore -z^2\left(\frac{br}{2}+1\right) + r f (z - c - z z_1) = 0$$

$$\therefore r f = \frac{z^2\left(\frac{br}{2}+1\right)}{z - c - z z_1}.$$

Check the consistency of this result as follows

$$r f = \cancel{z^2 t}\ \frac{z\left(\frac{br}{2}+1\right)}{1 - \frac{c}{z} - z_1} = \frac{z m'\left(\frac{br}{2}+1\right)}{m'\left(1 - \frac{c}{z} - z_1\right)}$$

$$= \frac{z m'\left(\frac{br}{2}+1\right)}{z'} = r(ar^2+br+1)m' = rf.$$

$$\therefore r f = \frac{z^2\left(\frac{br}{2}+1\right)}{z - c - z z_1}$$ Use this in the eqn. at the top

$$\therefore ar^2 + br + 1 = \frac{\dfrac{z^3\left(\frac{br}{2}+1\right)^2}{z-c-z z_1} - \dfrac{z_1 z^4\left(\frac{br}{2}+1\right)^2}{(z-c-z z_1)^2}}{z^2}$$

We work out an internal solution in a form which will show the case $a = \text{const.}$ $b = \text{const.}$ immediately.

$$ds^2 = -e^{\alpha} dr^2 - r^2 e^{\beta} d\Omega^2 + e^{\gamma} dt^2$$

$$e^{\alpha} = \frac{e^{\psi(t)} \cdot e^{-\int \frac{b}{f} dm} (y_1 + c y_2)^2}{ar^2 + br + 1}, \qquad e^{\beta} = e^{\psi(t)} \cdot e^{-\int \frac{b}{f} dm} (y_1 + c y_2)^2 \qquad (I.1)$$

$$e^{\gamma} = \frac{\dot{\varphi}^2 e^{\int \frac{b}{f} dm} \cdot e^{\psi(t)}}{(y_1 + c y_2)^2 (ar^2 + br + 1)} .$$

$$m'(ar^2 + br + 1) = f \quad \text{and} \quad \dot{m} = \frac{\dot{\varphi} f e^{\int \frac{b}{f} dm}}{(ar^2 + br + 1)(y_1 + c y_2)^2}$$

$$\therefore (y_1 + c y_2)^2 = \frac{\dot{\varphi}}{\dot{m}} \frac{f e^{\int}}{ar^2 + br + 1}$$

$$\therefore e^{\alpha} = \frac{e^{\psi} e^{-\int} \dot{\varphi} f e^{\int}}{\dot{m}(ar^2 + br + 1)^2} = \frac{e^{\psi} \dot{\varphi} f}{\dot{m}(ar^2 + br + 1)^2} ; \quad e^{\beta} = \frac{e^{\psi} \dot{\varphi} f}{\dot{m}(ar^2 + br + 1)}$$

$$\text{and} \quad e^{\gamma} = \frac{\dot{\varphi}^2 e^{\int} e^{\psi} \dot{m} \cancel{f} (ar^2 + br + 1)}{(ar^2 + br + 1) \dot{\varphi} f e^{\int}} = \frac{e^{\psi} \dot{m} \dot{\varphi}}{f \cancel{\varphi}}$$

Thus the solution is

$$ds^2 = -\frac{e^{\psi}\dot{\alpha} S}{\dot{m}}\left[\frac{1}{(ar^2+br+1)^2}dr^2 - \frac{r^2}{ar^2+br+1}d\Omega^2\right] + \frac{e^{\psi}\dot{m}\dot{\alpha}}{S\cancel{\alpha}}dt^2 \qquad (\text{II}.2)$$

But we could still begin with (I.1) for the purpose we have in mind

We have

$$ds^2 = -\frac{e^{\psi(t)} \cdot e^{-S} \cdot (Y_1+\alpha Y_2)^2}{ar^2+br+1}dr^2 - r^2 e^{\psi(t)} e^{-S}(Y_1+\alpha Y_2)^2 d\Omega^2 + \frac{\dot{\alpha}^2 e^{S} e^{\psi}}{(Y_1+\alpha Y_2)^2 (ar^2+br+1)}dt^2$$

Let us first chose a new time coordinate T

such that $\dot{\alpha}^2 e^{\psi} dt^2 = dT^2 \quad \therefore -\int e^{\psi/2} d\alpha = T$ (since $d\alpha$ is -ve)

or $T = +\int e^{\psi/2}(-\dot{\alpha})dt$.

and further put $\dfrac{e^{S}}{(Y_1+\alpha Y_2)^2(ar^2+br+1)} = F(m, r)$

Then the line-element becomes

$$ds^2 = e^{g(t)}\frac{1}{F(m,r)}\left[-\frac{dr^2}{(ar^2+br+1)^2} - \frac{r^2}{ar^2+br+1}d\Omega^2\right] + F(m,r)\,dt^2$$

t is the new time coordinate and $g(t)$ is arbitrary.

The expanding universe case is now obtained with a, b constants and $F(m, r) = 1$.

Thus we write our soln. in the form

$$ds^2 = \frac{e^{g(t)}}{F(m,r)}\left[-\frac{dr^2}{(ar^2+br+1)^2} - \frac{r^2}{ar^2+br+1}\,d\omega^2\right] + F(m,r)\,dt^2$$

where a, b are arbitrary functions of m, $F(m,r)$ is a function of m and r given below and m is a conserved function satisfying

$$(ar^2+br+1)\sqrt{F}\,\frac{m' e^{-g/2}}{\cancel{\sqrt{F}}\,\cancel{(ar^2+br+1)}} + \frac{\dot m\,\cancel{\sqrt{F}}}{\sqrt{F}} = 0 \,. \quad \text{i.e.} \quad \dot m = -\frac{\cancel{m'}\,e^{-g/2}}{F(ar^2+br+1)}$$

$$= -m' F e^{-g/2}(ar^2+br+1)$$

But $m' = \dfrac{f}{ar^2+br+1}$, $\quad \dot m = -\dfrac{f(m)\,e^{-g/2}}{F(ar^2+br+1)^2}$.

F is given by
$$F = \frac{e^{s}}{(y_1+a y_2)^2(ar^2+br+1)}$$
$$= \frac{e^{-s}\left(ary_2 + f\frac{dy_2}{dm}\right)^2}{a^2(ar^2+br+1)}$$

We shall now eliminate y_2 and write the differential equation satisfied by F. The form of this differential equation ~~wit~~ must be such as to make F constants when a and b are constants. We derive that differential equation.

$$F = \frac{e^{-S}\left(axy_2 + f\frac{dy_2}{dm}\right)^2}{a^2(ax^2+bx+1)}$$

$$\therefore\ Fa^2(ax^2+bx+1)e^{S} = \left(axy_2 + f\frac{dy_2}{dm}\right)^2 \qquad (IV.3)$$

There is another equation which y_2 satisfies. viz

$$\frac{d^2y_2}{dm^2} + \frac{dy_2}{dm}\left[\frac{f_1}{f} - \frac{a_1}{a} - \frac{b}{f}\right] + \frac{y_2 a}{f^2} = 0 \qquad (\text{IV}.4)$$

In order to do the elimination let us ~~ft~~ put

$$Fa^2(ax^2+bx+1)e^{S} = G^2$$

$$\therefore\ G = axy_2 + f\frac{dy_2}{dm}$$

$$\therefore\ \frac{\partial G}{\partial x} = ay_2 \quad \text{or}\quad y_2 = \frac{1}{a}\frac{\partial G}{\partial x}$$

Next $\displaystyle\frac{\partial G}{\partial m} = a_1 x y_2 + ax\frac{\partial y_2}{\partial m} + f_1\frac{dy_2}{dm} + f\frac{d^2y_2}{dm^2}$

$$= \frac{a_1 x}{a}\frac{\partial G}{\partial x} + (ax+f_1)\frac{dy_2}{dm} + f\left[-\frac{y_2 a}{f^2} - \frac{dy_2}{dm}\left(\frac{f_1}{f} - \frac{a_1}{a} - \frac{b}{f}\right)\right]$$

$$= \frac{a_1 x}{a}\frac{\partial G}{\partial x} + \frac{dy_2}{dm}\left(ax + f_1 - f_1 + \frac{fa_1}{a} + b\right) - \frac{a}{f}\cdot\frac{1}{a}\frac{\partial G}{\partial x}$$

$$\frac{\partial G}{\partial m} = \left(\frac{a_1 r}{a} - \frac{1}{f}\right)\frac{\partial G}{\partial r} + \frac{1}{f}\left(ar + f\frac{a_1}{a} + b\right)\left(G - r\frac{\partial G}{\partial r}\right)$$

$$= \frac{\partial G}{\partial r}\left(\cancel{\frac{a_1 r}{a}} - \frac{1}{f} - \frac{ar^2}{f} - \cancel{\frac{a_1 r}{a}} - \frac{br}{f}\right) + \frac{1}{f}\left(ar + f\frac{a_1}{a} + b\right)G$$

$$\therefore \frac{\partial G}{\partial m} + \frac{ar^2+br+1}{f}\frac{\partial G}{\partial r} = \frac{1}{f}\left(ar + b + f\frac{a_1}{a}\right)G$$

$$\therefore \frac{1}{G}\frac{\partial G}{\partial m} + \frac{1}{G}\frac{\partial G}{\partial r}\cdot\frac{ar^2+br+1}{f} = \frac{1}{f}\left(ar + b + f\frac{a_1}{a}\right)$$

Now $\quad G^2 = F a^2 (ar^2+br+1)\, e^f$

$$\therefore \frac{2}{G}\frac{\partial G}{\partial r} = \frac{1}{F}\frac{\partial F}{\partial r} + \frac{2ar+b}{ar^2+br+1}.$$

~~A~~ Also $\quad \dfrac{2}{G}\dfrac{\partial G}{\partial m} = \dfrac{1}{F}\dfrac{\partial F}{\partial m} + \dfrac{2}{a}a_1 + \dfrac{(a_1 r^2 + b_1 r)}{ar^2+br+1} + \dfrac{b}{f}$

$$\therefore \frac{1}{2F}\frac{\partial F}{\partial m} + \frac{a_1}{a} + \frac{a_1 r^2 + b_1 r}{2(ar^2+br+1)} + \frac{b}{2f} + \frac{ar^2+br+1}{2f}\left[\frac{1}{F}\frac{\partial F}{\partial r} + \frac{2ar+b}{ar^2+br+1}\right]$$

$$= \frac{1}{f}\left(ar + b + f\frac{a_1}{a}\right)$$

$$\therefore \frac{1}{2F}\frac{\partial F}{\partial m} + \frac{ar^2+br+1}{2fF}\frac{\partial F}{\partial r} = \cancel{\frac{ar}{f}} + \cancel{\frac{b}{f}} + \cancel{\frac{a_1}{a}} - \cancel{\frac{a_1}{a}} - \frac{a_1 r^2 + b_1 r}{2(ar^2+br+1)} - \cancel{\frac{b}{2f}}$$

$$- \frac{2ar + \cancel{b}}{2f}$$

$$= -\frac{a_1 r^2 + b_1 r}{2(ar^2+br+1)}$$

$$\frac{1}{F}\frac{\partial F}{\partial m} + \frac{ar^2+br+1}{f}\frac{1}{F}\frac{\partial F}{\partial r} + \frac{a_1 r^2 + b_1 r}{(ar^2+br+1)} = 0$$

What a simple equation.

$$\text{or} \quad \frac{f}{ar^2+br+1}\frac{\partial F}{\partial m} + \frac{\partial F}{\partial r} = -\frac{f(a_1 r^2 + b_1 r)}{(ar^2+br+1)^2} F$$

$$\text{But } \frac{f}{ar^2+br+1} = m'$$

$$\therefore \quad \frac{\partial F}{\partial r} + \frac{\partial F}{\partial m} m' = -F f \frac{\partial}{\partial m}\frac{1}{(ar^2+br+1)}$$

$$\text{or} \quad F' = + F f \frac{\partial}{\partial m} \cdot \frac{1}{ar^2+br+1}.$$

It is clear that if a, b are constants $F' = 0$. 1

Thus we can write our solution in a more decent form as follows.

$$ds^2 = -e^{\alpha} dr^2 - r^2 e^{\beta} d\omega^2 + e^{\gamma} dt^2$$

$$e^{\alpha} = \frac{e^{g-F}}{(ar^2+br+1)^2}, \quad e^{\beta} = \frac{e^{g-F}}{ar^2+br+1}, \quad e^{\gamma} = e^{F}.$$

$$F' = -\frac{m'(a_1 r^2 + b_1 r)}{ar^2+br+1}, \quad m' = \frac{f(m)}{ar^2+br+1}, \quad \cancel{\dot m = \frac{-m' e^{-g/2} e^{-F}}{ar^2+br+1}}$$

$$\dot m = -m' e^{-g/2} e^{F} (ar^2+br+1)$$

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We now calculate the various derivatives

$$e^{\alpha} = \frac{e^{g-F}}{(ar^2+br+1)^2} \qquad \therefore \quad \alpha = g - F - 2\ln(ar^2+br+1)$$

$$\therefore \quad \alpha' = -F' - \frac{2(a_1r^2+b_1r)m'}{ar^2+br+1} - \frac{2(2ar+b)}{(ar^2+br+1)}$$

$$= F' - 2F' - \frac{2(a_1r^2+b_1r)m'}{ar^2+br+1} - \frac{2(2ar+b)}{ar^2+br+1}$$

$$\therefore \quad \boxed{\alpha' = F' - \frac{2(2ar+b)}{ar^2+br+1}} \qquad \dot\alpha = \dot g - \dot F - \frac{2(a_1r^2+b_1r)\dot m}{(ar^2+br+1)}$$

$$\therefore \quad \dot\alpha = \dot g - \dot F + \frac{2\dot m}{m'}F' = \boxed{\dot g - \dot F - 2F' e^{-g/2} e^{F}(ar^2+br+1) = \dot\alpha}$$

$$e^{\beta} = \frac{e^{g-F}}{ar^2+br+1} \qquad \therefore \quad \boxed{\beta' = -\frac{2ar+b}{ar^2+br+1}}$$

$$\boxed{\dot\beta = \dot g - \dot F - F' e^{-g/2} e^{F}(ar^2+br+1)}$$

$$\ddot\beta = \ddot g - \ddot F - \dot F' e^{-g/2} e^{F}(ar^2+br+1) + F'\frac{\dot g}{2} e^{F}(ar^2+br+1)e^{-g/2}$$
$$- \dot F F' e^{-g/2} e^{F}(ar^2+br+1) - F' e^{-g/2} e^{F}(a_1r^2+b_1r)\dot m$$
$$+ \frac{\dot m}{m'} F'(ar^2+br+1) e^{-g/2} e^{F} F'$$

$$\therefore \quad \boxed{\ddot\beta = \ddot g - \ddot F + \cancel{K}\, e^{-g/2} e^{F}(ar^2+br+1)\left[-\dot F' + F'\frac{\dot g}{2} - \dot F F'\right] - e^{-g} e^{2F}(ar^2+br+1)^2 F'^2.}$$

$$\dot\beta = \dot g - \dot F - F' e^{-g/2} e^{F}(ar^2+br+1) \quad \therefore \quad \dot\beta' = -\dot F' - F'' e^{-g/2} e^{F}(ar^2+br+1)$$
$$- F' e^{-g/2} e^{F} F'(ar^2+br+1) - F' e^{-g/2} e^{F}(2ar+b) - F' e^{-g/2} e^{F}(a_1r^2+b_1r)m'$$
$$+ F' e^{-g/2} e^{F}(ar^2+br+1)F'$$

$$\therefore \quad \boxed{\dot\beta' = -\dot F' + e^{-g/2} e^{F}(ar^2+br+1)\left[-F'' - \frac{F'(2ar+b)}{ar^2+br+1}\right].}$$

$$\beta' = -\frac{2ar+b}{ar^2+br+1} \qquad \therefore \beta'' = \frac{-(2a_1r+b_1)m'}{ar^2+br+1} - \frac{2a}{ar^2+br+1} + \cancel{(2ar+b)(a_1r^2+b_1r)}$$

$$+ \frac{(2ar+b)(a_1r^2+b_1r)m'}{(ar^2+br+1)^2} + \frac{(2ar+b)^2}{(ar^2+br+1)^2}.$$

Now $$F' = -\frac{\cancel{f(2ar+}\, m'(a_1r^2+b_1r)}{ar^2+br+1} = -\frac{f(a_1r^2+b_1r)}{(ar^2+br+1)^2}$$

$$\therefore \dot{F}' = -\frac{\partial}{\partial m}\left[\frac{f(a_1r^2+b_1r)}{(ar^2+br+1)^2}\right]\dot{m} \quad ; \quad F'' = -\frac{\partial}{\partial m}\left[\frac{f(a_1r^2+b_1r)}{(ar^2+br+1)^2}\right]m'$$

$$-\frac{f(2a_1r+b_1)}{(ar^2+br+1)^2} + \frac{2f(a_1r^2+b_1r)}{(ar^2+br+1)^3}(2ar+b).$$

$$\therefore F'' = \dot{F}'\frac{m'}{\dot{m}} - \frac{m'(2a_1r+b_1)}{ar^2+br+1} + \frac{2(2ar+b)\,\cancel{ar}\,(a_1r^2+b_1r)m'}{(ar^2+br+1)^2}$$

$$\therefore \cancel{F}\ \frac{m'(2a_1r+b_1)}{ar^2+br+1} = \dot{F}'\frac{m'}{\dot{m}} - F'' + \frac{2(2ar+b)(a_1r^2+b_1r)m'}{(ar^2+br+1)^2}$$

$$= -\frac{\dot{F}' e^{g/2} e^{-F}}{ar^2+br+1} - F'' - \frac{2(2ar+b)}{(ar^2+br+1)}F'.$$

$$\therefore \boxed{\frac{m'(2a_1r+b_1)}{ar^2+br+1} = \frac{-\dot{F}' e^{g/2} e^{-F}}{ar^2+br+1} - F'' - \frac{2(2ar+b)F'}{ar^2+br+1}}$$

$$\therefore \beta'' = \frac{\dot{F}' e^{g/2} e^{-F}}{ar^2+br+1} + F'' + \frac{2(2ar+b)F'}{(ar^2+br+1)} - \frac{2a}{ar^2+br+1} - \frac{(2ar+b)F'}{(ar^2+br+1)} + \frac{(2ar+b)^2}{(ar^2+br+1)^2}$$

$$\therefore \boxed{\beta'' = F'' + \frac{\dot{F}' e^{g/2} e^{-F}}{ar^2+br+1} + \frac{(2ar+b)F'}{(ar^2+br+1)} - \frac{2a}{ar^2+br+1} + \frac{(2ar+b)^2}{(ar^2+br+1)^2}}$$

$$8\pi T_1^1 = -e^{-\alpha}\left[\frac{\beta'^2}{4} + \frac{\beta'\gamma'}{2} + \frac{\beta'+\gamma'}{r} + \frac{1}{r^2}\right] + \frac{e^{-\beta}}{r^2} + e^{-\gamma}\left[\ddot\beta + \frac34\dot\beta^2 - \frac{\dot\beta\dot\gamma}{2}\right] - \Lambda$$

$$= -e^{F}e^{-g}(ar^2+br+1)^2\left[\frac14\frac{(2ar+b)^2}{(ar^2+br+1)^2} - \frac{F'(2ar+b)}{2(ar^2+br+1)} - \frac{2ar+b}{r(ar^2+br+1)} + \frac{F'}{r} + \frac{1}{r^2}\right]$$

$$+ \frac{e^{-g}e^{F}(ar^2+br+1)}{r^2} + e^{-F}\Big[\ddot g - \ddot F + e^{-g/2}e^{F}(ar^2+br+1)\left(-\dot F' + \frac{F'\dot g}{2} - \dot F F'\right) - e^{-g}e^{2F}(ar^2+br+1)F'^2$$

$$+ \frac34\Big\{\dot g^2 + \dot F^2 + F'^2e^{-g}e^{2F}(ar^2+br+1)^2 - 2\dot g\dot F - 2\dot g F' e^{-g/2}e^{F}(ar^2+br+1) + 2\dot F F' e^{-g/2}e^{F}(ar^2+br+1)\Big\}$$

$$-\frac{\dot F}{2}\Big\{\dot g - \dot F - F'e^{-g/2}e^{F}(ar^2+br+1)\Big\}\Big] - \Lambda$$

$$= -\frac{e^{F}e^{-g}}{r^2}\Big[\frac14 r^2(2ar+b)^2 - (2ar+b)r(ar^2+br+1) + (ar^2+br+1)^2 - (ar^2+br+1)$$

$$-\frac{F'}{2}(2ar+b)r^2(ar^2+br+1) + F'r(ar^2+br+1)^2 + r^2F'^2(ar^2+br+1)^2$$

$$-\frac34F'^2r^2(ar^2+br+1)^2\Big] + e^{-F}\Big[\ddot g - \ddot F + \frac34\dot g^2 + \frac34\dot F^2 - \frac32\dot g\dot F - \frac{\dot F\dot g}{2} + \frac{\dot F^2}{2}\Big]$$

$$+ e^{-g/2}(ar^2+br+1)\Big[-\dot F' + \frac{F'\dot g}{2} - \dot F F' - \frac32\dot g F' + \frac32\dot F F' + \frac{\dot F F'}{2}\Big] - \Lambda$$

$$= -\frac{e^{F}e^{-g}}{r^2}\Big[a^2r^4(1-2+1) + ar^3b(1-3+2) + b^2r^2\left(\frac14 - 1 + 1\right)$$

$$+ ar^2(-2+2-1) + br(-1+2-1) + 1 - 1$$

$$-\frac{F'r(ar^2+br+1)}{2}\Big\{r(2ar+b) - 2(ar^2+br+1)\Big\} + \frac14F'^2r^2(ar^2+br+1)^2\Big]$$

$$+ e^{-F}\Big[\ddot g - \ddot F + \frac34\dot g^2 + \frac54\dot F^2 - 2\dot F\dot g\Big] + e^{-g/2}(ar^2+br+1)\Big[-\dot F' - \frac{F'\dot g}{2}_{\times} \cancel{-\frac{\dot F F'}{2}} + \dot F F'\Big] - \Lambda$$

$$\therefore 8\pi T_1^1 = -\cancel{\frac{e^{F}e^{-g}}{r^2}\left(-a + \frac{b^2}{4}\right)} - \frac{e^{+F}e^{-g}}{r^2}\Big[r^2\left(\frac{b^2}{4} - a^2\right) - \frac{F'r(ar^2+br+1)}{2}(-br-2)$$

$$+ \frac14F'^2r^2(ar^2+br+1)^2\Big] + e^{-F}\Big[\ddot g - \ddot F + \frac34\dot g^2 + \frac54\dot F^2 - 2\dot F\dot g\Big]$$

$$+ e^{-g/2}(ar^2+br+1)\left(-\dot F' - \frac{F'\dot g}{2} + \dot F F'\right) - \Lambda$$

$\underline{X}$

$$8\pi T_1^4 = -e^{-\gamma}\left[\dot\beta' - \frac{\dot\beta\gamma'}{2} + (\dot\beta - \dot\alpha)\frac{\beta'}{2} + (\dot\beta-\dot\alpha)\frac{1}{r}\right]$$

$$= -e^{-F}\Big[-\dot F' + e^{-g/2}e^{F}(ar^2+br+1)\left\{-\dot F'' - \frac{\dot F'(2ar+b)}{ar^2+br+1}\right\} - \frac{F'}{2}\Big(\dot g - \dot F - F' e^{-g/2}e^{F}(ar^2+br+1)\Big)$$

$$+ \Big[\dot g - \dot F - F'e^{-g/2}e^{F}(ar^2+br+1) - \dot g + \dot F + 2F'e^{-g/2}e^{F}(ar^2+br+1)\Big]\Big(-\frac{2ar+b}{2(ar^2+br+1)}$$

$$\Big)\frac{1}{r}\Big]$$

$+ \Big[$

$$= -e^{-F}\left[-\dot F' - \frac{F'\dot g}{2} + \frac{F'\dot F}{2}\right] + e^{-g/2}(ar^2+br+1)\cancel{e^{F}}\Big[-F'' + \frac{F'(2ar+b)}{ar^2+br+1} + \frac{F'^2}{2}$$

$$+ F'\left\{-\frac{2ar+b}{2(ar^2+br+1)} + \frac{1}{r}\right\}\Big]$$

$$\cancel{=e^{-F}} \quad = e^{-F}\left[\dot F' + \frac{F'\dot g}{2} - \frac{F'\dot F}{2}\right] - e^{-g/2}(ar^2+br+1)\cancel{e^{F}}\Big[-F'' + \frac{F'^2}{2}$$

$$+ \frac{F'(-4ar^2 - 2br - 2ar^2 - br + 2ar^2 + 2br + 2)}{2(ar^2+br+1)r}\Big]$$

$$= e^{-F}\left[\dot F' + \frac{F'\dot g}{2} - \frac{F'\dot F}{2}\right] - e^{-g/2}(ar^2+br+1)\cancel{e^{F}}\left[-F'' + \frac{F'^2}{2} + \frac{F'(-4ar^2 - br + 2)}{2(ar^2+br+1)r}\right]$$

$$e^{(\gamma-\alpha)/2}T_1^4 = e^{-g/2}(ar^2+br+1)\cancel{e^{-F}}\left[\dot F' + \frac{F'\dot g}{2} - \frac{F'\dot F}{2}\right] - e^{-g+F}(ar^2+br+1)^2\left[-F'' + \frac{F'^2}{2} + \frac{F'(-4ar^2-br+2)}{2r(ar^2+br+1)}\right]$$

$$8\pi p = -8\pi t_2^2 = e^{(\gamma-\alpha)/2}\, 8\pi T_1^4 - 8\pi T_1^1$$

$$= -e^{-F}\left[\ddot g - \ddot F + \frac{3}{4}\dot g^2 + \frac{5}{4}\dot F^2 - 2\dot F\dot g\right] + e^{-g/2}(ar^2+br+1)\left[\dot F' + \frac{F'\dot g}{2} - \frac{F'\dot F}{2} + \dot F' + \frac{F'\dot g}{2}\right.$$

$$\left. - \dot F F'\right) + e^{F}\cdot e^{-g}(ar^2+br+1)^2\left[F'' - \frac{F'^2}{2} - \frac{F'(-4ar^2-br+2)}{2r(ar^2+br+1)} + \frac{\frac{b^2}{4} - a^2}{(ar^2+br+1)^2}\right.$$

$$\left. + \frac{F'a(br+2)}{2(ar^2+br+1)r} + \frac{1}{4}F'^2\right]$$

XI

$$\cancel{8\pi T_4^{\;4} = -e^{-\alpha}\left[\beta'' + \frac{3}{4}\beta'^2 - \frac{\alpha'\beta'}{2} + \frac{3\beta'}{r} - \frac{\alpha'}{r} + \frac{1}{r^2}\right] + \frac{e^{-\beta}}{r^2} + e^{-\gamma}\left[\frac{\dot\alpha\dot\beta}{2} + \frac{\dot\beta^2}{4}\right] - \Lambda}$$

$$\cancel{= -\dot g\, e^{-g+F}}$$

$$8\pi p = -e^{-F}\left[\ddot g + \frac{3}{4}\dot g^2 - \ddot F + \frac{5}{4}\dot F^2 - 2\dot F\dot g\right] + e^{-g/2}(ar^2+br+1)\left[2\dot F' + F'\dot g - \frac{3}{2}F'\dot F\right]$$

$$+ e^{F}e^{-g}(ar^2+br+1)^2\left[F'' - \frac{F'^2}{4} - \frac{a^2 - \frac{b^2}{4}}{(ar^2+br+1)^2} + \frac{F'(br+2+4ar^2+br-2)}{2r(ar^2+br+1)} + F'(2ar+b)/(ar^2+br+1)\right]$$

$$8\pi T_4^{\;4} = -e^{-\alpha}\left[\beta'' + \frac{3}{4}\beta'^2 - \frac{\alpha'\beta'}{2} + \frac{3\beta'}{r} - \frac{\alpha'}{r} + \frac{1}{r^2}\right] + \frac{e^{-\beta}}{r^2} + e^{-\gamma}\left[\frac{\dot\alpha\dot\beta}{2} + \frac{\dot\beta^2}{4}\right] - \Lambda$$

$$= e^{F-g}(ar^2+br+1)^2\left[F'' + \frac{\dot F' e^{g/2}e^{-F}}{ar^2+br+1} + \frac{(2ar+b)F'}{ar^2+br+1} - \frac{2a}{ar^2+br+1} + \frac{(2ar+b)^2}{(ar^2+br+1)^2}\right.$$

$$+ \frac{3}{4}\frac{(2ar+b)^2}{(ar^2+br+1)^2} + \frac{1}{2}\cdot\frac{2ar+b}{ar^2+br+1}\left(F' - \frac{2(2ar+b)}{ar^2+br+1}\right) - \frac{3}{r}\frac{2ar+b}{(ar^2+br+1)}$$

$$\left. - \frac{F'}{r} + \frac{2(2ar+b)}{r(ar^2+br+1)} + \frac{1}{r^2}\right] + \frac{e^{-g}e^{F}(ar^2+br+1)}{r^2}$$

$$+ e^{-F}\left[\frac{1}{2}\left(\dot g - \dot F - 2F'e^{-g/2}e^{F}(ar^2+br+1)\right)\left(\dot g - \dot F - F'e^{-g/2}e^{F}(ar^2+br+1)\right)\right.$$

$$\left. + \frac{1}{4}\left(\dot g - \dot F - F'e^{-g/2}e^{F}(ar^2+br+1)\right)^2\right] - \Lambda$$

$$= e^{F}e^{-g}(ar^2+br+1)^2\left[F'' + F'\left\{\frac{2ar+b}{ar^2+br+1}\cdot\frac{3}{2} - \frac{1}{r}\right\}\right]$$

$$+ \frac{e^{F}e^{-g}}{r^2}\left[-2a(ar^2+br+1)r^2 + (2ar+b)^2r^2 + \frac{3}{4}(2ar+b)^2r^2 - (2ar+b)^2r^2 + (ar^2+br+1)\right.$$

$$\left. - 2r(2ar+b)(ar^2+br+1) + (ar^2+br+1)^2\right]$$

$$- \frac{3}{2}F'(\dot g - \dot F) - \frac{1}{2}F'(\dot g - \dot F)\Big] + e^{-F}\left[(\dot g - \dot F)^2\left(\frac{1}{2} + \frac{1}{4}\right)\right]$$

$$+ e^{-g/2}(ar^2+br+1)\left[\dot F' - \frac{3}{2}F'(\dot g - \dot F) - \frac{1}{2}F'(\dot g - \dot F)\right]$$

$$+ e^{-g+F}(ar^2+br+1)^2\left[F'^2\left(1 + \frac{1}{4}\right)\right] - \Lambda$$

XII

$$= e^{F} e^{-g} (ar^2+br+1)^2 \left[F'' + F' \frac{(4ar^2+br-2)}{2r(ar^2+br+1)} + \frac{5}{4} F'^2 \right]$$

$$+ \frac{e^{F} e^{-g}}{r^2} \left[a^2 r^4 (-2 + \cancel{4} + 3 - 2 + 1) + ab\, r^3 (-2 + 3 - 2 - 1 + 2) \right.$$

$$+ \cancel{ab}\, r^2 (-2a + \tfrac{3}{4} b^2 - b^2 - 2a + 2a + \cancel{a}) + b^2 + a$$

$$= - e^{F-g} (ar^2+br+1)^2 \left[F'' + F' \left\{ \frac{2ar+b}{ar^2+br+1} \cdot \frac{3}{2} - \frac{1}{2} \right\} \right]$$

$$- \frac{e^{F-g}}{r^2} \left[-2ar^2 (ar^2+br+1) + r^2 (2ar+b)^2 + \frac{3}{4} r^2 (2ar+b)^2 - \cancel{(2ar+b)^2 r^2} \right.$$

$$- \cancel{2}\, r (2ar+b)(ar^2+br+1) + \cancel{2(2ar+b) r (ar^2+br+1)}$$

$$\left. + (ar^2+br+1)^2 - (ar^2+br+1) \right]$$

$$+ e^{-F} \left[(\dot g - \dot F)^2 \left(\frac{1}{2} + \frac{1}{4} \right) \right] + e^{-g/2} (ar^2+br+1) \left[-\dot F' - \frac{3}{2} F' (\dot g - \dot F) - \frac{1}{2} F' (\dot g - \dot F) \right]$$

$$+ e^{-g} e^{-F} (ar^2+br+1)^2 \left[F'^2 \left(1 + \frac{1}{4} \right) \right] - \Lambda$$

$$= - e^{F-g} (ar^2+br+1)^2 \left[F'' + F' \frac{(4ar^2+bs-2)}{2r(ar^2+br+1)} - \frac{5}{4} F'^2 \right]$$

$$- \frac{e^{F} \cdot e^{-g}}{r^2} \left[a^2 r^4 (-2 + 3 - 2 + 1) + ab\, r^3 (-2 + 3 \cancel{-3} - 3 + 2) \right.$$

$$+ r^2 [-2a + \tfrac{3}{4} b^2 - b^2 - 2a + 2a + b^2 - a)$$

$$\left. + br (-1 + 2 - 1) + 1 - 1 \right]$$

$$+ e^{-F} \frac{3}{4} (\dot g - \dot F)^2 - e^{-g/2} (ar^2+br+1) [\dot F' + 2F' (\dot g - \dot F)] - \Lambda$$

$$\therefore 8\pi T_4^4 = - e^{F-g} (ar^2+br+1)^2 \left[F'' + \frac{F' (4ar^2+br-2)}{2r(ar^2+br+1)} - \frac{5}{4} F'^2 \right] + 3 e^{F} e^{-g} \left(a - \frac{b^2}{4} \right)$$

$$+ \frac{3}{4} e^{-F} (\dot g - \dot F)^2 - e^{-g/2} (ar^2+br+1) \{ \dot F' + 2F' (\dot g - \dot F)] - \Lambda$$

XIII

$$8\pi\rho = 8\pi\left(T_4^{\,4} + e^{(g-\alpha)/2}T_1^{\,4}\right)$$

$$= -e^{-g+F}(ar^2+br+1)^2\left[F'' + \frac{F'(4ar^2+br-2)}{2r(ar^2+br+1)} - \frac{5}{4}F'^2 - F'' + \frac{F'^2}{2}\right.$$

$$\left. + \frac{F'(-4ar^2-br+2)}{2r(ar^2+br+1)}\right] + 3e^{F}e^{-g}\left(a-\frac{b^2}{4}\right) + \frac{3}{4}e^{-F}(\dot g-\dot F)^2$$

$$+ e^{-g/2}(ar^2+br+1)\left[-\dot F' + 2F'(\dot g-\dot F) + \dot F' + \frac{F'\dot g}{2} - \frac{F'\dot F}{2}\right]$$

$$= e^{F}\cdot e^{-g}\left[\frac{3}{4}F'^2(ar^2+br+1)^2 + 3\left(a-\frac{b^2}{4}\right)\right] + \frac{3}{4}e^{-F}(\dot g-\dot F)^2$$

$$- \frac{3}{2}e^{-g/2}(ar^2+br+1)F'(\dot g-\dot F)$$

$$= e^{F}e^{-g}\left[\frac{3}{4}F'^2(ar^2+br+1)^2 - \frac{3}{2}e^{g/2}e^{-F}(ar^2+br+1)F'(\dot g-\dot F) + \frac{3}{4}e^{-2F}e^{g}(\dot g-\dot F)^2\right.$$

$$\left. + 3\left(a-\frac{b^2}{4}\right)\right]$$

$$= e^{F}e^{-g}\left[\frac{3}{4}\left(F'(ar^2+br+1) - e^{-F}e^{g/2}(\dot g-\dot F)\right)^2 + 3\left(a-\frac{b^2}{4}\right)\right]$$

$$= e^{F}e^{-g}\left[\frac{3}{4}e^{-2F}e^{g}\left\{F'e^{F}e^{-g/2}(ar^2+br+1) - \dot g + \dot F\right\}^2 + 3\left(a-\frac{b^2}{4}\right)\right]$$

$$= e^{F}e^{-g}\left[\frac{3}{4}e^{-2F}e^{g}(\dot\beta^2) + 3\left(a-\frac{b^2}{4}\right)\right]$$

$$= \frac{3}{4}e^{-F}(\dot\beta)^2 + 3\left(a-\frac{b^2}{4}\right)e^{F}e^{-g}.$$

If $a-\frac{b^2}{4}$ be +ve, the density is always +ve.

It will be a pleasure to work out $T_1^{\,1}$ over again

$$8\pi T_1^{\,1} = -e^{-\alpha}\left[\frac{\beta'^2}{4} + \frac{\beta'\gamma'}{2} + \frac{\beta'+\gamma'}{r} + \frac{1}{r^2}\right] + \frac{e^{-\beta}}{r^2} + e^{-\gamma}\left[\ddot\beta + \frac{3}{4}\dot\beta^2 - \frac{\dot\beta\dot\gamma}{2}\right] - \Lambda$$

$$\cancel{e^{-a}}\,[\;=\; -e^{-g}e^{F}(ar^2+br+1)^2\Big[\frac{1}{4}\frac{(2ar+b)^2}{(ar^2+br+1)^2} - \frac{F'}{2}\frac{(2ar+b)}{(ar^2+br+1)} - \frac{2ar+b}{r(ar^2+br+1)}$$

$$+\frac{F'}{2}+\frac{1}{r^2}\Big] + \frac{e^{-g}e^{F}(ar^2+br+1)}{r^2} + e^{-F}\Big[\ddot{g}-\ddot{F} + e^{-g/2}e^{F}(ar^2+br+1)\Big(-\dot{F}'+\frac{F'\dot{g}}{2}$$

$$-F'\dot{F}\Big) - e^{-g}e^{2F}(ar^2+br+1)^2F'^2\,\cancel{\tfrac{1}{4}} + \frac{3}{4}\left(\overline{\dot{g}-\dot{F}} - F'e^{-g/2}e^{F}(ar^2+br+1)\right)^2$$

$$-\frac{\dot{F}}{2}\left(\overline{\dot{g}-\dot{F}} - F'e^{-g/2}e^{F}(ar^2+br+1)\right)\Big] - \Lambda$$

$$= -\frac{e^{-g}e^{F}}{r^2}\Big[\frac{1}{4}r^2(2ar+b)^2 - \frac{F'}{2}r^2(2ar+b)(ar^2+br+1) - r(2ar+b)(ar^2+br+1)$$

$$+ rF'(ar^2+br+1)^2 + (ar^2+br+1)^2 - (ar^2+br+1)\Big]$$

$$+ e^{-F}\Big[\ddot{g}-\ddot{F}+\frac{3}{4}(\dot{g}-\dot{F})^2 - \frac{\dot{F}}{2}(\dot{g}-\dot{F})\Big] + e^{-g/2}(ar^2+br+1)\Big[-\dot{F}'+\frac{F'(\dot{g}-\dot{F})}{2} - \frac{F'\dot{F}}{2}$$

$$-\frac{3}{2}F'(\dot{g}-\dot{F}) + \frac{F'\dot{F}}{2}\Big] + e^{-g}e^{F}(ar^2+br+1)^2\Big[-F'^2+\frac{3}{4}F'^2\Big] - \Lambda$$

$$= -\frac{e^{-g}e^{F}}{r^2}\Big[\cancel{a^2r^4(1-2+1)} + \cancel{abr^3(1-3+2)} + r^2\Big(\frac{b^2}{4}\cancel{-b^2} - 2a \cancel{+b^2}$$

$$\cancel{+2a} - a\Big) + \cancel{br(-1+2-1)} + \cancel{1-1} + F'(ar^2+br+1)\Big(\cancel{-\frac{2ar^3}{2}} - \frac{br^2}{2}$$

$$\cancel{+ar^3} + br^2 + r\Big)\Big] + e^{-F}\Big[\ddot{g}-\ddot{F}+\frac{3}{4}(\dot{g}-\dot{F})^2 - \frac{\dot{F}}{2}(\dot{g}-\dot{F})\Big]$$

$$\cancel{+ e^{-g/2}(ar^2+br+1)\Big[-\dot{F}'-F'\dot{g}+\frac{3}{2}F'\dot{F}\Big] + e^{-g}e^{F}(ar^2+br+1)^2\Big(-\frac{F'^2}{4}\Big)}$$

$$+ e^{-g/2}(ar^2+br+1)\Big[-\dot{F}'-F'(\dot{g}-\dot{F})\Big] + e^{-g}e^{F}(ar^2+br+1)^2\Big(-\frac{F'^2}{4}\Big)$$

$$8\pi T_1^1 = -e^{-g}e^{F}\Big[\frac{b^2}{4} - a + \frac{F'(ar^2+br+1)(br+2)}{2r} + \frac{F'^2}{4\cancel{r^2}}(ar^2+br+1)^2\Big]$$

$$+ e^{-F}\Big[\ddot{g}-\ddot{F}+\frac{3}{4}(\dot{g}-\dot{F})^2 - \frac{\dot{F}}{2}(\dot{g}-\dot{F})\Big] + e^{-g/2}(ar^2+br+1)\Big[-\dot{F}'-F'(\dot{g}-\dot{F})\Big]$$

XV

$$8\pi\rho = e^{(\gamma-\alpha)/2}\,8\pi\,T_1^{\,4} - 8\pi\,T_1^{\,1}$$

$$= e^{-\gamma/2}(ar^2+br+1)\left[\dot F' + \frac{F'(\dot\gamma-\dot F)}{2}\right] - e^{-\gamma+F}(ar^2+br+1)^2\left[-F'' + \frac{F'^2}{2}\right.$$

$$\left. + \frac{F'(-4ar^2-br+2)}{2r(ar^2+br+1)}\right] + e^{-\gamma+F}\left[\frac{b^2}{4} - a + \frac{F'(ar^2+br+1)(br+2)}{2r}\right.$$

$$\left. + \frac{F'^2}{4r^2}(ar^2+br+1)^2\right] - e^{-F}\left[\ddot\gamma - \ddot F + \frac{3}{4}(\dot\gamma^2-\dot F^2) - \frac{\dot F}{2}(\dot\gamma-\dot F)\right]$$

$$- e^{-\gamma/2}(ar^2+br+1)\left[-\dot F' - F'(\dot\gamma-\dot F)\right]$$

$$= e^{-\gamma/2}(ar^2+br+1)\left[2\dot F' - \frac{3}{2}F'(\dot\gamma-\dot F)\right] + e^{-\gamma+F}\left[F''(ar^2+br+1)^2\right.$$

~~Let me also work at T_1^4 again~~

$$= e^{-\gamma/2}(ar^2+br+1)\left[2\dot F' + \frac{3}{2}F'(\dot\gamma-\dot F)\right] + e^{-\gamma+F}\left[F''(ar^2+br+1)^2\right.$$

$$- \frac{F'^2}{4}(ar^2+br+1)^2 + \frac{F'}{2r}(ar^2+br+1)(4ar^2+br-2+br+2)$$

$$\left. + \frac{b^2}{4} - a\right] - e^{-F}\left[\ddot\gamma - \ddot F + \frac{3}{4}(\dot\gamma^2-\dot F^2) - \frac{\dot F}{2}(\dot\gamma-\dot F)\right]$$

We must work at T_1^1, T_2^2, T_1^4 and T_4^4 over again and check that

$$T_1^{\,1} - T_2^{\,2} = e^{(\gamma-\alpha)/2}\,T_1^{\,4}.$$

$\overline{XV}$ 1.

To work out T_2^2 we want $\ddot{\alpha}$

$$\dot{\alpha} = \dot{g} - \dot{F} - 2F' e^{-g/2} e^{F} (ar^2+br+1)$$

$$\therefore \ddot{\alpha} = \ddot{g} - \ddot{F} - \cancel{2F}\, 2e^{-g/2} e^{F} ()\left[\dot{F}' - F'\frac{\dot{g}}{2} + F'\dot{F} + F'^2 (ar^2+br+1)^2 e^{-g/2} e^{F}\right]$$

$$= \ddot{g} - \ddot{F} + 2e^{-g/2}e^{F}()\left[-\dot{F}' + F'\frac{\dot{g}}{2} - F'\dot{F}\right] - 2e^{-g}e^{2F}(ar^2+br+1)^2 F'^2$$

$$8\pi T_2^2 = -e^{-\alpha}\left[\frac{\beta''}{2} + \frac{\gamma''}{2} + \frac{\beta'^2}{4} + \frac{\gamma'^2}{4} - \frac{\alpha'\beta'}{4} - \frac{\alpha'\gamma'}{4} + \frac{\beta'\gamma'}{4} + \frac{\gamma'}{2r} - \frac{\alpha'}{2r} + \frac{\beta'}{2r}\right]$$
$$+ e^{-\gamma}\left[\frac{\ddot{\alpha}}{2} + \frac{\ddot{\beta}}{2} + \frac{\dot{\alpha}^2}{4} + \frac{\dot{\beta}^2}{4} + \frac{\dot{\alpha}\dot{\beta}}{4} - \frac{\dot{\alpha}\dot{\gamma}}{4} - \frac{\dot{\beta}\dot{\gamma}}{4}\right]$$

$$= -e^{-\alpha}\Bigg[\cancel{\tfrac{1}{2}}\Big\{\frac{\dot{F}' e^{g/2} e^{-F}}{2(ar^2+br+1)} + \frac{F''}{2} + \frac{F'(2ar+b)}{2(ar^2+br+1)} - \frac{\cancel{4}a}{ar^2+br+1} + \frac{(2ar+b)^2}{2(ar^2+br+1)^2}\cancel{\Big\}} + \frac{F''}{2}$$
$$+ \frac{1}{4}\frac{(2ar+b)^2}{(ar^2+br+1)^2} + \frac{F'^2}{4} - \left(F' - \frac{2(2ar+b)}{ar^2+br+1}\right)\left\{-\frac{2ar+b}{4(ar^2+br+1)}\right\}$$
$$- \frac{F'}{4}\left(F' - \frac{2(2ar+b)}{ar^2+br+1}\right) + \frac{F'}{4}\cdot\left(-\frac{2ar+b}{ar^2+br+1}\right) + \frac{F'}{2r} - \left(\frac{F'}{2r} - \frac{2ar+b}{r(ar^2+br+1)}\right)$$
$$+ \frac{1}{2}\left(-\frac{2ar+b}{ar^2+br+1}\right)\Bigg] + e^{-F}\Bigg[\frac{\ddot{g} - \ddot{F}}{2} + e^{-g/2}e^{F}\overset{(ar^2+\dots)}{()}\left(-\dot{F}' + F'\frac{\dot{g}}{2} - F'\dot{F}\right)$$
$$\qquad + \cancel{1}\, e^{-g/2}e^{F}(ar^2+br+1)\left(-\frac{\dot{F}'}{2} + \frac{F'\dot{g}}{4} - \frac{F'\dot{F}}{2}\right)$$
$$- e^{-g}e^{2F}(ar^2+br+1)^2 F'^2 + \frac{\ddot{g} - \ddot{F}}{2} + \frac{1}{4}\left(\dot{g} - \dot{F} - 2F' e^{-g/2} e^{F}(ar^2+br+1)\right)^2$$
$$- e^{-g}e^{2F}(ar^2+br+1)^2 \frac{F'^2}{2} + \frac{1}{4}\left(\dot{g} - \dot{F} - 2F' e^{-g/2} e^{F}()\right)\left(\dot{g} - \dot{F} - F' e^{-g/2} e^{F}()\right)$$
$$+ \frac{1}{4}\left(\dot{g} - \dot{F} - F' e^{-g/2} e^{F}(ar^2+br+1)\right)^2 + \frac{1}{4}\left(\dot{g} - \dot{F} - 2F' e^{-g/2} e^{F}()\right)\left(\dot{g} - \dot{F} - F' e^{-g/2} e^{F}()\right)$$
$$- \frac{\dot{F}}{4}\left(\dot{g} - \dot{F} - 2F' e^{-g/2} e^{F}()\right) - \frac{\dot{F}}{4}\left(\dot{g} - \dot{F} - F' e^{-g/2} e^{F}()\right)\Bigg].$$

$$= -e^{-g}e^{F}(ar^2+br+1)^2\Bigg[F'' + \frac{F'k}{r(ar^2+br+1)}\left(ar^2 + \frac{br}{2} + \frac{ar^2}{2} + \frac{br}{4} + \frac{ar^2}{2} + \frac{br}{2} - \frac{ar^2}{2} - \frac{br}{4}\right)$$
$$+ \cancel{\frac{ar^2}{2} + \frac{br}{2} + \frac{1}{2}} + \frac{1}{(ar^2+br+1)^2 r^2}\left(-ar^2(ar^2+br+1) + \cancel{4}\, r^2(2ar+b)^2\left(\frac{1}{2} + \frac{1}{4} - \frac{1}{2}\right)\right]$$
$$+ e^{-F}\left[\ddot{g} - \ddot{F} + \frac{3}{4}(\dot{g} - \dot{F})^2 - \frac{\dot{F}}{2}(\dot{g} - \dot{F})\right] + e^{-g/2}(ar^2+br+1)\left[\dot{F}'\left(-\frac{1}{2} - 1 - \frac{1}{2}\right) + \right.$$

$$+F'\dot g\left(\tfrac{1}{2}+\tfrac{1}{4}\right) \cancel{+F'\dot F\left(-1-\tfrac{1}{2}\right)} + F'(\dot g-\dot F)\left(-1-\tfrac{1}{2}-\tfrac{3}{4}\right)$$

$$+F'\dot F\left(-1-\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{4}\right)\Big] + e^{-g}e^{F}(ar^2+br+1)^2\left[F'^2\left(-1-\tfrac{1}{2}+1+\tfrac{1}{4}+\tfrac{1}{2}\right)\right].$$

$$8\pi T_2^2 = -e^{-g}e^{F}(ar^2+br+1)^2\Big[F''-\frac{F'^2}{4}+\frac{F'(2ar+b)}{ar^2+br+1}+\frac{1}{r^2(ar^2+br+1)^2}\Big(a^2r^4(-1+1)$$

$$+ar\,r^3(-1+1)-ar^2+\frac{b^2}{4}r^2\Big)\Big]$$

$$+e^{-F}\left[\ddot g-\ddot F+\frac{3}{4}(\dot g-\dot F)^2-\dot F\frac{(\dot g-\dot F)}{2}\right]$$

$$+e^{-g/2}(ar^2+br+1)\left[-2\dot F'-\frac{3}{2}F'(\dot g-\dot F)\right]$$

$$\therefore 8\pi T_2^2 = -e^{-g}e^{F}\left[F''(ar^2+br+1)^2-\frac{F'^2}{4}(ar^2+br+1)^2+F'(2ar+b)(ar^2+br+1)+\frac{b^2}{4}-a\right]$$

$$+e^{-g/2}(ar^2+br+1)\left[-2\dot F'-\frac{3}{2}F'(\dot g-\dot F)\right]$$

$$+e^{-F}\left[\ddot g-\ddot F+\frac{3}{4}(\dot g-\dot F)^2-\dot F\frac{(\dot g-\dot F)}{2}\right]$$

$$\therefore 8\pi p = -8\pi T_2^2$$

$$= e^{-g}e^{F}\left[\left(F''-\frac{F'^2}{4}\right)(ar^2+br+1)^2+F'(2ar+b)(ar^2+br+1)+\frac{b^2}{4}-a\right]$$

$$+e^{-g/2}(ar^2+br+1)\left(2\dot F'+\frac{3}{2}F'(\dot g-\dot F)\right)$$

$$-e^{-F}\left[\ddot g-\ddot F+\frac{3}{4}(\dot g-\dot F)^2-\dot F\frac{(\dot g-\dot F)}{2}\right]$$

This expression agrees with the expression found on page $\overline{\text{XV}}$. Hence we have indirect verification of T_1^1 and $e^{(g-F)/2}T_1^4$.

Just as the density could be written in terms of β and its derivatives in a form identical with Datta's form, it may be possible to write the pressure in the same way

XVI

$$8\pi p = e^{-g} e^{F} \Big[\Big(F'' - \cancel{F'^2} + \frac{\dot F' e^{g/2} e^{-F}}{ar^2+br+1} \Big)(ar^2+br+1)^2 + F'(2ar+b)(ar^2+br+1)$$

$$+ \frac{b^2}{4} - a \Big] - e^{-F} \Big[\ddot g - \ddot F + \frac{3}{4}(\dot g - \dot F)^2 - \frac{\dot F(\dot g - \dot F)}{2} + \frac{F'^2}{4} e^{2F} e^{-g} (ar^2+br+1)^2$$

$$- e^{-g/2} e^{F} (ar^2+br+1) \dot F' - e^{-g/2} e^{F} (ar^2+br+1) \frac{3}{2} F' \dot g + \frac{3}{2} e^{-g/2} e^{F} (ar^2+br+1) \dot F F' \Big]$$

Now
$$F'' + \frac{\dot F' e^{g/2} e^{-F}}{ar^2+br+1} + \frac{2F'(2ar+b)}{ar^2+br+1} = -\frac{m'(2a_1 r + b_1)}{ar^2+br+1}$$

and
$$\ddot g - \ddot F - e^{-g/2} e^{F} (ar^2+br+1) \dot F' + e^{-g/2} e^{F} (ar^2+br+1) \frac{F'\dot g}{2} - e^{-g/2} e^{F} (ar^2+br+1) \dot F F'$$

$$- e^{-g} e^{+2F} (ar^2+br+1)^2 F'^2 = \ddot \beta$$

and
$$(\dot g - \dot F)^2 + F'^2 e^{-g} e^{2F} (ar^2+br+1)^2 F'^2 - 2F'(\dot g) e^{-g/2} e^{F} (ar^2+br+1)$$

$$+ 2F'\dot F e^{-g/2} e^{F} (ar^2+br+1) = \dot\beta^2$$

$$\therefore 8\pi p = e^{-g} e^{F} \Big[\Big(F'' + \frac{\dot F' e^{g/2} e^{-F}}{ar^2+br+1} + \frac{2F'(2ar+b)}{ar^2+br+1} \Big)(ar^2+br+1)^2 - F'(2ar+b)(ar^2+br+1)$$

$$+ \frac{b^2}{4} - a \Big] - e^{-F} \Big[\ddot g - \ddot F - e^{-g/2} e^{F} (ar^2+br+1) \dot F' + e^{-g/2} e^{F} (ar^2+br+1) \frac{F'\dot g}{2}$$

$$- e^{-g/2} e^{F} (ar^2+br+1) \dot F F' - e^{-g} e^{2F} (ar^2+br+1)^2 F'^2 + \frac{3}{4}(\dot g - \dot F)^2 + \frac{3}{4} F'^2 e^{-g} e^{2F} (\;)^2$$

$$- \frac{3}{2} F' \dot g e^{-g/2} e^{F} (\;) + \frac{3}{2} F' \dot F e^{-g/2} e^{F} (\;)$$

$$- \frac{1}{2} e^{-g/2} e^{F} (ar^2+br+1) F' \dot g + e^{-g/2} e^{F} (ar^2+br+1) F' \dot F + \frac{F'^2}{2} e^{2F} e^{-g} (ar^2+br+1)^2$$

$$- \frac{\dot F(\dot g - \dot F)}{2}$$

$$8\pi p = e^{-g} e^{F}\left[\frac{-m'(2a_1 r + b_1)}{(ar^2+br+1)}(ar^2+br+1)^2 + \frac{m'(a_1 r^2 + b_1 r)(2ar+b)(ar^2+br+1)}{ar^2+br+1} \right.$$

$$\left. + \frac{b^2}{4} - a \right] - e^{-F}\left[\ddot\beta + \frac{3}{4}\dot\beta^2 + \frac{1}{2}\left[e^{F} e^{-g/2}(ar^2+br+1)F' + \dot F\right]^2 \right.$$

$$\left. - \frac{\dot g}{2}\left(e^{-g/2} e^{F}(ar^2+br+1)F' + \dot F\right)\right]$$

$$= e^{-g} e^{F}\left[m'\left\{ -(2a_1 r + b_1)(ar^2+br+1) + (a_1 r^2 + b_1 r)(2ar+b)\right\} + \frac{b^2}{4} - a\right]$$

$$- e^{-F}\left[\ddot\beta + \frac{3}{4}\dot\beta^2 + \frac{1}{2}\left\{e^{-g/2}e^{F}(ar^2+br+1)F' + \dot F\right)\left(e^{F}e^{-g/2}(ar^2+br+1)F' + \dot F - \dot g\right)\right]$$

$$= e^{-g} e^{F}\left[m'\left(\cancel{-a_1 b r^2} - 2a_1 r + b_1 a r^2 - b_1\right)\right\} + \frac{b^2}{4} - a\right]$$

$$- e^{-F}\left[\dot\beta + \frac{3}{4}\dot\beta^2 + \frac{1}{2}\cancel{(-\dot\beta)(g-\dot\beta)}(\dot g - \dot\beta)(-\dot\beta)\right]$$

$$= e^{-g} e^{F}\left[\frac{m'}{\dot m}(ar^2+br+1)^2\left\{\frac{-(2ar+b)}{ar^2+br+1}\right\}^{\cdot} + \frac{b^2}{4} - a\right]$$

$$- e^{-F}\left[\ddot\beta + \frac{3}{4}\dot\beta^2 + \frac{1}{2}(\dot g - \dot\beta)(-\dot\beta)\right]$$

$$= e^{-g}e^{F}\left[-e^{g/2}e^{-F}(ar^2+br+1)\dot\beta' + \frac{b^2}{4} - a\right] - e^{-F}\left[\ddot\beta + \frac{5\dot\beta^2}{4} - \frac{\dot g\dot\beta}{2}\right]$$

$$= -e^{-F}\left[\ddot\beta + \frac{5\dot\beta^2}{4} - \frac{\dot g\dot\beta}{2} + e^{-g/2}e^{F}(ar^2+br+1)\dot\beta'\right\} + \left(a - \frac{b^2}{4}\right)e^{-g+2F}\right]$$

Instead of changing the time variable here we can do it on the [illegible] when pro[illegible]

Let $e^{F}dt^2 = d\tau^2$ or $e^{F/2}dt = d\tau$ $\therefore \dot\beta = \frac{\partial\beta}{\partial t} = \frac{\partial\beta}{e^{-F/2}\partial\tau} = e^{F/2}\frac{\partial\beta}{\partial\tau}$

$\tau = \int e^{F/2}dt$ r regarded as constant during integration

$$\ddot\beta = \frac{\partial}{\partial t}(\dot\beta) = e^{F/2}\frac{\dot F}{2}\frac{\partial\beta}{\partial\tau} + e^{F/2}\cdot e^{F/2}\frac{\partial^2\beta}{\partial\tau^2} = e^{F/2}\frac{\dot F}{2}\frac{\partial\beta}{\partial\tau} + e^{F}\frac{\partial^2\beta}{\partial\tau^2} \quad)$$

$$\dot\beta' = \frac{\partial}{\partial r}(\dot\beta) = e^{F/2}\frac{F'}{2}\frac{\partial\beta}{\partial\tau} + e^{F/2}\frac{\partial^2\beta}{\partial r\,\partial\tau}.$$

$$\therefore 8\pi p = -e^{-F}\Big[e^{F}\frac{\partial^2\beta}{\partial\tau^2} + e^{F/2}\frac{\dot F}{2}\frac{\partial\beta}{\partial\tau} + \frac{5}{4}e^{F}\left(\frac{\partial\beta}{\partial\tau}\right)^2 - \frac{\dot g}{2}e^{F/2}\frac{\partial\beta}{\partial\tau} \cancel{+ e^{-g/2}}$$

$$+ e^{-g/2}e^{F}(ar^2+br+1)\left\{e^{F/2}\frac{F'}{2}\frac{\partial\beta}{\partial\tau} + e^{F/2}\frac{\partial^2\beta}{\partial r\,\partial\tau}\right\} + \left(a-\frac{b^2}{4}\right)e^{-g+2F}\Big]$$

$$= -e^{-F}\Big[e^{F}\frac{\partial^2\beta}{\partial\tau^2} + \frac{5}{4}e^{F}\left(\frac{\partial\beta}{\partial\tau}\right)^2 - \frac{e^{F/2}}{2}\frac{\partial\beta}{\partial\tau}\left(\dot g - \dot F - e^{-g/2}e^{F}(ar^2+br+1)F'\right)$$

$$+ e^{-g/2}e^{F}(ar^2+br+1)e^{F/2}\frac{\partial^2\beta}{\partial r\,\partial\tau} + \left(a-\frac{b^2}{4}\right)e^{-g+2F}\Big]$$

$$\cancel{= \Big[\frac{\partial^2\beta}{\partial\tau^2}} = -e^{-F}\Big[e^{F}\frac{\partial^2\beta}{\partial\tau^2} + \frac{5}{4}e^{F}\left(\frac{\partial\beta}{\partial\tau}\right)^2 - \frac{e^{F/2}}{2}\left(\frac{\partial\beta}{\partial\tau}\right)\dot\beta + e^{-g/2}e^{F}(ar^2+br+1)$$

$$e^{F/2}\frac{\partial^2\beta}{\partial r\,\partial\tau} + \left(a-\frac{b^2}{4}\right)e^{-g+2F}\Big]$$

$$= -\Big[\frac{\partial^2\beta}{\partial\tau^2} + \frac{5}{4}\left(\frac{\partial\beta}{\partial\tau}\right)^2 - \frac{1}{2}\left(\frac{\partial\beta}{\partial\tau}\right)^2 + e^{-g/2}(ar^2+br+1)e^{F/2}\frac{\partial^2\beta}{\partial r\,\partial\tau}$$

$$+ \left(a-\frac{b^2}{4}\right)e^{-g+F}\Big]$$

$$= -\Big[\frac{\partial^2\beta}{\partial\tau^2} + \frac{3}{4}\left(\frac{\partial\beta}{\partial\tau}\right)^2 + \left(a-\frac{b^2}{4}\right)e^{-g+F} + e^{-g/2}(ar^2+br+1)e^{F/2}\frac{\partial^2\beta}{\partial r\,\partial\tau}\Big]$$

~~The last term here is~~ $\cancel{e^{-\alpha/2}\frac{\partial}{\partial r}(\partial_t\beta)}$

The last term here is $e^{-\alpha/2}\frac{\partial}{\partial r}\cdot\frac{\partial\beta}{\partial\tau}$

If $D_t = e^{-\gamma/2}\frac{\partial}{\partial t}$ $D_r = e^{-\alpha/2}\frac{\partial}{\partial r}$ the last term in the bracket is $D_rD_t\beta$.

Let $e^{-\alpha/2}\frac{\partial}{\partial r} = \frac{\partial}{\partial R}$ then the last term is $\frac{\partial^2\beta}{\partial R\,\partial\tau}$

~~XV~~ XIX

Let us work out the equations of fit.

For the line-element

$$ds^2 = \left(1 - \frac{c}{\zeta} - \frac{2\zeta'}{m'}\right)\left[-m'^2 dr^2 + m'^2 dt^2\right] - \zeta^2 d\Omega^2$$

$$m'\left(1 - \frac{c}{\zeta} - \frac{\zeta'}{m'}\right) = \zeta'$$

we have found that $\cancel{T_1^4}\ 8\pi T_4^1 = \dfrac{-m' c_1}{\zeta^2\left[\zeta' - \frac{\zeta'}{m'} m'\right]}$

For the line-element

$$ds^2 = -\frac{e^{g-F}}{(ar^2+br+1)^2}dr^2 - \frac{r^2 e^{g-F}}{ar^2+br+1}d\Omega^2 + e^F dt^2$$

$$\zeta^2 = \frac{\cancel{k^2} r^2 e^{g-F}}{ar^2+br+1} = r^2 e^{\beta} \qquad \therefore \zeta' = \left(1 + r\frac{\beta'}{2}\right)e^{\beta/2}$$

$$\therefore \zeta' = e^{\beta/2}\left[1 + r\frac{-(2ar+b)}{2(ar^2+br+1)}\right] = e^{\beta/2}\frac{(br+2)}{2(ar^2+br+1)}$$

$$\boxed{\zeta' = \frac{\zeta}{r}\frac{(br+2)}{2(ar^2+br+1)}}$$

$$\dot{\zeta} = \zeta\frac{\dot{\beta}}{2} = \zeta\left[\frac{\dot{g} - \dot{F} - F' e^{-g/2}e^{F}(\ )}{2}\right]$$

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Let us now find $\zeta' - \frac{\dot\zeta}{\dot m} m'$ on the assumption that these quantities are continuous.

$$\zeta' - \frac{\dot\zeta}{\dot m} m' = \frac{\zeta}{r \cancel{f}} \frac{(br+2)}{2(\ )} + \frac{e^{g/2-F} e\zeta}{(\ )}\left(\frac{\dot g - \dot F}{2} - \frac{F'}{2} e^{-g/2} e^{F} (\ )\right)$$

$$= \frac{\zeta}{2}\left[\frac{br+2}{(ar^2+br+1)r} - F'\right] + \frac{e^{g/2} e^{-F} (\dot g - \dot F)}{2(\ )} \zeta$$

$$= \frac{\zeta}{2}\left[\frac{br+2}{r(\ )} - F'\right] - \frac{m'}{\dot m}\frac{(\dot g - \dot F)}{2} \zeta$$

There is another way of find $\zeta' - \frac{\dot\zeta}{\dot m} m'$ viz. the continuity of g_{11}.

$$\left(1 - \frac{c}{\zeta} - \frac{2\dot\zeta}{\dot m}\right) m'^2 = \frac{e^{g-F}}{(\ )^2}$$

But $\left(1 - \frac{c}{\zeta} - \frac{\dot\zeta}{\dot m}\right) = \frac{\zeta'}{m'}$

$$\therefore \left(\frac{\zeta'}{m'} - \frac{\dot\zeta}{\dot m}\right) m'^2 = \frac{e^{g-F}}{(\ )^2}$$

$$\therefore \left(\zeta' - \frac{\dot\zeta}{\dot m} m'\right) m' = \frac{e^{g-F}}{(\ )^2}$$

$$\therefore \left(\zeta' - \frac{\dot\zeta}{\dot m} m'\right) \frac{f}{(\ )} = \frac{e^{g-F}}{(\ )^2}$$

XXI

$$\therefore\ \mathfrak{z}' - \frac{\dot{\mathfrak{z}}}{\dot{m}} m' = \frac{e^{g-F}}{f(\ )}$$

Let us equate the two values: Continuity of J_{11}

implies $$\frac{e^{g-F}}{f(\ )} = \frac{\mathfrak{z}}{2}\left[\frac{br+2}{r(\ )} - F' - \frac{m'}{\dot{m}}(\dot{g}-\dot{F})\right]$$

~~But~~ $\cancel{\mathfrak{z} = r\, e^{(g-F}}$

$$\therefore\ \frac{\mathfrak{z}^2}{r^2 f} = \frac{\mathfrak{z}}{2}\left[\frac{br+2}{r(\ )} - F' - \frac{m'}{\dot{m}}(\dot{g}-\dot{F})\right]$$

$$\therefore\ \boxed{\frac{2\mathfrak{z}}{r^2 f} - \frac{br+2}{r(\ )} + F' = -\frac{m'}{\dot{m}}(\dot{g}-\dot{F})}$$

f occurs thro' the internal soln.

Let us now use the continuity of m'.

$$m' = \frac{\mathfrak{z}'}{1-\frac{c}{\mathfrak{z}}-\frac{\dot{\mathfrak{z}}}{\dot{m}}} = \frac{f}{(\ )}$$

$$\therefore\ \mathfrak{z}'(\ ) = f\left(1-\frac{c}{\mathfrak{z}}-\frac{\dot{\mathfrak{z}}}{\dot{m}}\right)$$

$$\therefore\ \frac{\mathfrak{z}}{r}\frac{(br+2)(\ )}{2(\ \ )} = f\left[1-\frac{c}{\mathfrak{z}}-\frac{\mathfrak{z}}{\dot{m}}\left\{\frac{\dot{g}-\dot{F}}{2}+\frac{F'}{2}\frac{\dot{m}}{m'}\right\}\right]$$

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$$\frac{z}{r}\frac{(br+2)}{2f} = \cancel{f}\, 1 - \frac{c}{z} - \frac{z}{2}(\dot{g}-\dot{F})\frac{m'}{\dot{m}}\,\frac{1}{m'} - \frac{z}{2}\frac{F'}{m'}$$

$$\therefore \frac{z}{r}\frac{(br+2)}{2f} = 1 - \frac{c}{z} + \frac{z}{2}\left\{\frac{2z}{r^2 f} - \frac{br+2}{r(\ )} + F'\right\}\frac{1}{m'} - \frac{z}{2}\frac{F'}{m'}$$

$$\therefore \frac{z}{r}\frac{(br+2)}{f}\left(\frac{1}{2}+\frac{1}{2}\right) = 1 - \frac{c}{z} + \frac{z^2}{r^2 f m'}$$

$$\therefore \frac{z}{r}\frac{(br+2)}{f} = 1 - \frac{c}{z} + \frac{z^2(\ )}{r^2 f^2}$$

$$\therefore \cancel{\frac{z^3(\ )}{r^2 f^2}} + \frac{z^3(\ )}{r^2 f^2} - \frac{z^2}{r}\frac{(br+2)}{f} + z - c = 0$$

$$\therefore \boxed{z^3(ar^2+br+1) - z^2 rf(br+2) + r^2 f^2(z-c) = 0}$$

We now find $\dot{z}$ on the boundary

$$\dot{z} = \frac{z}{2}\left[\dot{g}-\dot{F} + F'\frac{\dot{m}}{m'}\right]$$

$$\therefore -\frac{m'}{\dot{m}}\dot{z} = \frac{z}{2}\left[-\frac{m'}{\dot{m}}(\dot{g}-\dot{F}) - F'\right]$$

$$= \frac{z}{2}\left[\frac{2z}{r^2 f} - \frac{br+2}{r(\ )} + F' - F'\right]$$

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$$\therefore -\frac{m'}{m}\dot{z} = \frac{z}{2}\left[\frac{2z}{r^2 f} - \frac{br+2}{r(\;)}\right]$$

$$\therefore \frac{e^{g/2}e^{-F}}{(\;)}\dot{z} = \frac{z}{r}\left[\frac{z}{rf} - \frac{\left(\frac{br}{2}+1\right)}{(\;)}\right]$$

$$\therefore \dot{z} = \frac{z\,e^{-g/2}e^{F}(\;)}{r}\left[\frac{z}{rf} - \frac{\left(\frac{br}{2}+1\right)}{(\;)}\right]$$

$$= \frac{r\,e^{g/2}e^{-F/2}}{\sqrt{(\;)}\cdot r}\,e^{-g/2}e^{F}(\;)\left[\frac{z}{rf} - \frac{\left(\frac{br}{2}+1\right)}{(\;)}\right]$$

$$= e^{F/2}\left[\frac{z\sqrt{(\;)}}{rf} - \frac{\left(\frac{br}{2}+1\right)}{\sqrt{\;}}\right]$$

$$\boxed{\dot{z} = \frac{e^{F/2}}{\sqrt{ar^2+br+1}}\left[\frac{z(ar^2+br+1)}{rf} - \left(\frac{br}{2}+1\right)\right].}$$

Next

$$\dot{z}^2 = \frac{e^{F}}{ar^2+br+1}\left[\frac{z^2}{r^2f^2}(ar^2+br+1)^2 - \frac{z}{rf}(\;)(br+2) + \left(\frac{br}{2}+1\right)^2\right]$$

$$= \frac{e^{F}}{ar^2+br+1}\left[\frac{1\,(ar^2+br+1)}{r^2f^2z}\left[z^3(ar^2+br+1) - z^2 rf(br+2)\right] + \left(\frac{br}{2}+1\right)^2\right]$$

$$= \frac{e^{F}}{ar^2+br+1}\left[\frac{1\,(ar^2+br+1)}{r^2f^2z}(-r^2f^2)(z-c) + \left(\frac{br}{2}+1\right)^2\right]$$

XIV

$$= \frac{e^F}{ar^2+br+1}\left[\left(\frac{br}{2}+1\right)^2 - (ar^2+br+1)\left(\frac{z-c}{z}\right)\right]$$

$$= \frac{e^F}{ar^2+br+1}\left[\frac{c}{z}(ar^2+br+1) - r^2\left(a-\frac{b^2}{4}\right)\right]$$

$$\boxed{\dot{z}^2 = e^F\left[\frac{c}{z} - \frac{\left(a-\frac{b^2}{4}\right)r^2}{ar^2+br+1}\right].}$$

Let us now take $p=0$ on the boundary

on the boundary $p=0$ leads to [see last eqn on p. XVII

$$\ddot{\beta} + \frac{5\dot{\beta}^2}{4} - \frac{\dot{g}\dot{\beta}}{2} + e^{-g/2}e^F(ar^2+br+1)\dot{\beta}' + \left(a-\frac{b^2}{4}\right)e^{-g+2F} = 0$$

But $\dot{\beta} = \dot{g}-\dot{F}-F'e^{-g/2}e^F(\;)$ and $(\dot{g}-\dot{F})\frac{m'}{\dot{m}} + F' = \frac{br+2}{r(\;)} - \frac{2z}{r^2f}$

$$\therefore \dot{g}-\dot{F}-F'e^{-g/2}e^F(\;) = e^{-g/2}e^F(\;)\left[\frac{2z}{r^2f} - \frac{br+2}{r(\;)}\right]$$

$$\therefore \dot{\beta} = \frac{e^{-g/2}\cancel{e^{-g/2}}e^F(\;)2z}{r^2f} - \cancel{e^{-g/2}}e^F\frac{br+2}{r}e^{g/2}$$

$$\therefore \ddot{\beta} = \frac{e^{-g/2}e^F(\;)}{\cancel{r^2f}}2\dot{z} - \cancel{\frac{e^{-g/2}e^F(\;)2z}{r^2f^2}f_1\dot{m}} + \cancel{\frac{\dot{\left[e^{-g/2}e^F(\;)\right]}2z}{r^2f}}$$

$\dot{\beta}' \!=$ $\quad -e^{-g/2}e^F$

B

XXV

Before we proceed further, we find that a_1 and b_1 can be found separately as follows:

$$F' = -\frac{m'(a_1 r^2 + b_1 r)}{(\ )} = \frac{-m' r(a_1 r + b_1)}{(\ )}$$

$\therefore a_1 r + b_1 = -\frac{F'(\ )}{m' r}$. Next $\frac{m'(2a_1 r + b_1)}{(\ )} = \dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{2(2ar+b)}{(\ )}F'$

$$\therefore 2a_1 r + b_1 = \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{2(2ar+b)}{(\ )}F'\right)$$

$$\therefore a_1 r = \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{2(2ar+b)}{(\ )}F' + \frac{F'}{r}\right)$$

$$= \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{F'}{r(\ )}\left(4ar^2 + 2br - ar^2 - br - 1\right)\right)$$

$$= \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{F'}{r(\ )}\left(3ar^2 + br - 1\right)\right).$$

and $-b_1 = \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{2(2ar+b)}{(\ )}F' + \frac{2F'}{r}\right)$

$$= \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' + \frac{2F'}{r(\ )}\left(-2ar^2 - br + ar^2 + br + 1\right)\right)$$

$$= \frac{(\ )}{m'}\left(\dot{F}'\frac{m'}{\dot{m}} - F'' - \frac{2F'}{r(\ )}\left(ar^2 - 1\right)\right).$$

On the boundary

$\dot{g} - \dot{F} = e^{-g/2} e^{F}(\ )\left[F' - \frac{br+2}{r(\ )} + \frac{2g}{r^2 f}\right]$ This is a function of time.

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$$\therefore\ \ddot g - \ddot F = e^{-g/2}\, e^{F(\ )}\Big[\dot F' - \frac{b_1 r}{r(\ )}\dot m + \frac{br+2}{r(\ )^2}(a_1 r^2 + b_1 r)\dot m + \frac{2\dot z}{r^2 f}$$

$$- \frac{2z}{r^2 f^2} f_1 \dot m + \Big(F' - \frac{br+2}{r(\ )} + \frac{2z}{r^2 f}\Big)\Big(-\frac{\dot g}{2} + \dot F + \frac{a_1 r^2 + b_1 r}{ar^2+br+1}\dot m\Big)\Big]$$

$$= \Big(-\frac{\dot m}{m'}\Big)\Big[\dot F' + \frac{\dot m}{(\ )}\frac{(\ )}{m'}\Big(F'\frac{m'}{\dot m} - F'' - \frac{2F'}{r(\ )}(ar^2-1)\Big)$$

$$- \frac{(br+2)}{r(\ )}F'\frac{\dot m}{m'} + \frac{2}{r^2 f}\frac{z}{2}\Big(\dot g - \dot F + F'\frac{\dot m}{m'}\Big) - \frac{2z}{r^2 f^2} f_1 \dot m\Big]$$

$$+ (\dot g - \dot F)\Big(-\frac{\dot g - \dot F}{2} + \frac{\dot F}{2} - \frac{\dot m}{m'}F'\Big)$$

$$\therefore\ \ddot g - \ddot F + \frac{(\dot g - \dot F)^2}{2} - \frac{\dot F(\dot g - \dot F)}{2} - \cancel{\Big(\frac{m'}{\dot m}\Big)}\Big(-\frac{\dot m}{m'}\Big)\Big(2\dot F' + (\dot g - \dot F)F'\Big) - \frac{\dot m^2}{m'^2}(F'')$$

$$\cancel{-\Big(-\frac{\dot m}{m'}\Big)^2\Big[} \quad = -\Big(\frac{\dot m}{m'}\Big)^2\Big[\frac{F'}{r(\ )}\big(-2ar^2 + 2 \mp br \mp 2\big)\Big]$$

$$-\frac{\dot m}{m'}\Big[\frac{z}{r^2 f}\Big(-\frac{\dot m}{m'}\Big)\Big(\frac{2z}{r^2 f} - \frac{br+2}{r(\ )}\Big)\Big] + \frac{\dot m^2}{m'^2}\frac{2z}{r^2 f^2} f_1 m'$$

$$= \frac{\dot m^2}{m'^2}\Big[\frac{2ar^2 + br - 4}{r(ar^2+br+1)}F' + \frac{2z^2}{r^4 f^2} - \frac{z(br+2)}{r^3 f(\ )} + \frac{2z}{r^2 f^2} f_1 m'\Big]$$

Now $p=0$ implies $\ddot g - \ddot F + \frac{3}{4}(\dot g - \dot F)^2 - \frac{\dot F(\dot g - \dot F)}{2} + \frac{\dot m}{m'}\Big(2\dot F' + \frac{3}{2}F'(\dot g - \dot F)\Big)$

$$- \frac{\dot m^2}{m'^2}\Big(F'' - \frac{F'^2}{4} + \frac{F'(2ar+b)}{(\ )} + \frac{b^2/4 - a}{(\ )^2}\Big) = 0$$

i.e. $\ddot g - \ddot F + \frac{(\dot g - \dot F)^2}{2} - \frac{\dot F(\dot g - \dot F)}{2} + \frac{\dot m}{m'}\big(2\dot F' + (\dot g - \dot F)F'\big) - \frac{\dot m^2}{m'^2}F''$

$$+ \frac{(\dot g - \dot F)^2}{4} + \frac{\dot m}{m'}\frac{(\dot g - \dot F)}{2}F' + \frac{\dot m^2}{m'^2}\Big[\frac{F'^2}{4} - \frac{F'(2ar+b)}{(\ )} - \frac{(\frac{b^2}{4} - a)}{(\ )^2}\Big] = 0$$

i.e. $\frac{\dot m^2}{m'^2}\Big[\frac{2ar^2 + br - 4}{r(\ )}F' + \frac{2z^2}{r^4 f^2} - \frac{z(br+2)}{r^3 f(\ )} + \frac{2z}{r^2 f^2} f_1 m' + \frac{F'^2}{4} - \frac{F'(2ar+b)}{(\ )} - \frac{\frac{b^2}{4} - a}{(\ )^2}\Big]$

$$+\frac{(\dot{g}-\dot{F})^2}{4}+\frac{\dot{m}}{m'}\frac{(\dot{g}-\dot{F})}{2}F' = 0$$

i.e. $$\frac{\dot{m}^2}{m'^2}\left[\frac{F'}{r(\ )}(2ar^2+br-4-2ar^2-br)+\frac{2\zeta^2}{r^4f^2}-\frac{\zeta(br+2)}{r^3f(\ )}+\frac{2\zeta}{r^2f^2}f_1 m' - \frac{\frac{b^2}{4}-a}{(\ )^2}\right]$$

$$+\frac{(\dot{g}-\dot{F})^2}{4}+\frac{\dot{m}}{m'}\frac{(\dot{g}-\dot{F})}{2}F'+\frac{\dot{m}^2}{m'^2}\frac{F'^2}{4} = 0$$

i.e. $$\frac{\dot{m}^2}{m'^2}\left[-\frac{2F'(br+2)}{r(\ )}+\frac{2\zeta^2}{r^4f^2}-\frac{\zeta(br+2)}{r^3f(\ )}+\frac{2\zeta}{r^2f^2}f_1 m' - \frac{\frac{b^2}{4}-a}{(\ )^2}\right]$$

$$+\frac{1}{4}\left(\dot{g}-\dot{F}+\frac{\dot{m}}{m'}F'\right)^2 = 0$$

i.e. $$\frac{\dot{m}^2}{m'^2}\left[-\frac{2F'(br+2)}{r(\ )}+\frac{2\zeta^2}{r^4f^2}-\frac{\zeta(br+2)}{r^3f(\ )}+\frac{2\zeta}{r^2f^2}f_1 m' - \frac{\frac{b^2}{4}-a}{(\ )^2}\right.$$

$$\left.+\frac{\cancel{X}\zeta^2}{r^4f^2}-\frac{\cancel{X}\zeta(br+2)}{r^3f(\ )}+\frac{(br+2)^2}{4r^2(\ )^2}\right] = 0$$

i.e. $$\frac{3\zeta^2}{r^4f^2}-\frac{2\zeta(br+2)}{r^3f(\ )}+\frac{2\zeta f_1}{r^2f(\ )}+\frac{\frac{(br+2)^2}{4}+(a-\frac{b^2}{4})r^2}{r^2(\ )^2} = 0$$

i.e. $$3\zeta^2(ar^2+br+1)-2\zeta rf(br+2)+2\zeta r^2 f f_1 + r^2f^2 = 0$$

i.e. $$\boxed{3\zeta^2(ar^2+br+1)-2\zeta rf(br+2-rf_1)+r^2f^2=0}$$

Now let us pick up the continuity of T_4^1.

On the external, $$8\pi T_4^1 = \frac{-\dot{m}c_1}{\zeta^2[\zeta'-\frac{\dot{\zeta}}{\dot{m}}m']}$$

on the internal $$8\pi T_4^1 = -e^{\gamma-\alpha}(8\pi T_1^4)$$

Equate the two values on the boundary

XXVIII

$$\frac{-\dot m c_1 f(\,)^2}{r^2 e^{2g-2F}}$$

$$\therefore \frac{-\dot m c_1 f(\,)}{g^2 e^{g-F}} = -e^{2F-g}(\,)^2\left[e^{-F}\left(\dot F' + F'\frac{\dot g}{2} - F'\frac{\dot F}{2}\right)\right.$$

$$\left. - e^{-g/2}(\,)\left(-F'' + \frac{F'^2}{2} + \frac{F'}{2}\frac{(-4ar^2-br+2)}{r(\,)}\right)\right]$$

$$\therefore \frac{\dot m c_1 f}{r^2} e^{-g+F} = \dot F' + F'\frac{(\dot g-\dot F)}{2} - e^{-g/2}e^{F}(\,)\left[-F''+\frac{F'^2}{2}+\frac{F'}{2}\frac{(-4ar^2-br+2)}{r(\,)}\right]$$

Now $-\frac{b_1 m'}{(\,)} = \dot F'\frac{m'}{\dot m} - F'' - \frac{2F'}{r(\,)}(ar^2-1)$

$$\therefore -\frac{b_1\dot m}{(\,)} = \dot F' + e^{-g/2}e^F(\,)\left(F''+\frac{2F'}{r(\,)}(ar^2-1)\right).$$

$$\therefore -\frac{b_1\dot m}{(\,)} - e^{-g/2}e^F(\,)\cdot\frac{2F'}{r(\,)}(ar^2-1) = \dot F' + e^{-g/2}e^F(\,).$$

Also $\dot g - \dot F = -\frac{\dot m}{m'}\left(\frac{2g}{r^2f} - \frac{br+2}{r(\,)} + F'\right)$

$$\therefore \frac{\dot g-\dot F}{2} - e^{-g/2}e^F(\,)\frac{F'}{2} = e^{-g/2}e^F(\,)\left(\frac{g}{r^2f} - \frac{br+2}{2r(\,)}\right).$$

$$\therefore \frac{\dot m c_1 f e^{-g+F}}{r^2} = -\frac{b_1\dot m}{(\,)} - e^{-g/2}e^F(\,)\left[\frac{2F'}{r(\,)}(ar^2-1) - F'\left(\frac{g}{r^2f}-\frac{br+2}{2r(\,)}\right)\right.$$

$$\left. + \frac{F'}{2}\frac{(-4ar^2-br+2)}{r(\,)}\right]$$

$$= \frac{b_1 m'}{(\,)}e^{-g/2}e^F(\,) - e^{-g/2}e^F(\,)\frac{F'}{2r(\,)}\left(\begin{array}{c}4ar^2-4+br+2\\ -4ar^2-br+2\\ \cancel{4ar^2-4+2br+4}\\ \cancel{-4ar^2-br+2}\end{array}\right)$$

$$+ F'e^{-g/2}e^F(\,)\,g/r^2f$$

~~XXVIX~~ XXIX

$$\therefore \frac{c_1 \dot{m} f e^{-g+F}}{r^2} = -\frac{b_1 \dot{m}}{(\ )} \qquad \therefore c_1 = -\frac{b_1 r^2 e^{g-F}}{f\, \cancel{e^{g-F}}\, (\ )}$$

This is surprising!

$$c_1 = -\frac{b_1 z^2}{f}.$$

$$\therefore \frac{\dot{m} c_1 f e^{-g} e^{F}}{r^2} = \frac{b_1 m' e^{-g/2} e^{F} (\ )}{(\ )} + \frac{F' e^{-g/2} e^{F} (\ )\, z}{r^2 f}$$

$$\therefore \frac{\dot{m} c_1 f e^{-g} e^{F}}{r^2} = \frac{b_1 \dot{m}}{(\ )}\left(-\frac{\dot{m}}{m'}\right) + \frac{F' z}{r^2 f}\left(-\frac{\dot{m}}{m'}\right)$$

$$\therefore \frac{c_1 f e^{-g} e^{F}}{r^2} = -\frac{b_1}{(\ )} + -\frac{m'(a_1 r^2 + b_1 r)}{(\ )} \frac{z}{r^2 f} \cdot \left(-\frac{1}{m'}\right)$$

$$\therefore \frac{c_1 f (\ )}{(\ ) e^{g-F} r^2} = -\frac{b_1}{(\ )} + \frac{z}{r^2 f (\ )}(a_1 r^2 + b_1 r)$$

$$\therefore \frac{c_1 f}{(\ ) z^2} = -\frac{b_1}{(\ )} + \frac{z(a_1 r + b_1)}{r f (\ )}$$

$$\cancel{\frac{z^3}{z} r c_1 f^2 = -b_1 (\ )}$$

$$\therefore c_1 r f^2 = -b_1 z^2 r f + z^3 (a_1 r + b_1) = 0$$

$$\boxed{\therefore z^3 (a_1 r + b_1) - b_1 z^2 r f - c_1 r f^2 = 0}$$

XXX

We now collect the 4 equations holding good on the boundary r = constant.

$$\dot z = \frac{e^{F/2}}{\sqrt{ar^2+br+1}}\left[\frac{z(ar^2+br+1)}{rf} - \left(\frac{br}{2}+1\right)\right] \qquad (1)$$

$$z^3(ar^2+br+1) - z^2 rf(br+2) + r^2f^2(z-c) = 0 \qquad (2)$$

$$3z^2(ar^2+br+1) - 2zrf(br+2) + 2zr^2ff_1 + r^2f^2 = 0 \qquad (3)$$

$$z^3(a_1r+b_1) - b_1z^2rf - c_1rf^2 = 0 \qquad (4)$$

Take (2) and differentiate it w.r.t. t

$$3z^2\dot z(ar^2+br+1) + z^3(a_1r^2+b_1r)\dot m - 2z\dot z rf(br+2)$$

$$- z^2rf_1\dot m(br+2) - z^2rfb_1r\dot m + 2r^2ff_1(z-c)\dot m + r^2f^2\dot z \;\cancel{=0}$$

$$-4r^2f^2c_1\dot m$$

$$\therefore \dot z\left[3z^2(ar^2+br+1) - 2zrf(br+2) + r^2f^2\right] + r\dot m\left[z^3(a_1r+b_1)\right.$$

$$-rf^2c_1$$

$$\left.- b_1z^2rf\right] - z^2rf_1\dot m(br+2) - 2r^2ff_1(z-c)\dot m = 0$$

$$\therefore \dot z(-2zr^2ff_1) + r\dot m\left[\cancel{c_1rf^2}\right] - z^2rf_1\dot m(br+2) + 2r^2ff_1(z-c)\dot m = 0$$

XXXI

$$\therefore \quad \dot{z}\,2z\,r^2 f f_1 = r\dot{m}\left[c_1 r f^2 - z^2 r f_1 (br+2) - 2 r f f_1 (z-c)\right]$$

$$= r\dot{m}\left[(c r f^2)_1\right.$$

$$\therefore \quad f_1\left[-2z\dot{z}\,r^2 f - z^2 r\dot{m}(br+2) + 2r^2 f(z-c)\dot{m}\right] = 0$$

$\therefore$ Either $f_1 = 0$ or

$$2z\dot{z}\,r^2 f \cancel{+\frac{z^2}{z}} = -r\dot{m}\left[z^2(br+2) \mp 2rf(z-c)\right]$$

$$\therefore \quad 2z\dot{z}\,rf = f\,e^{-g/2}e^{F}\left[z^2(br+2) \mp 2rf(z-c)\right]$$

$$= f\,\frac{e^{-F/2}\cdot r}{\sqrt{}\,z}\,e^{F/2}\left[z^2(br+2) \mp 2rf(z-c)\right] \qquad z = \frac{e^{g/2}e^{-F/2}}{\sqrt{}}\,r \qquad e^{-g/2} = \frac{e^{-F/2}\,r}{\sqrt{}\;z}$$

$$\therefore \quad 2zrf\,\frac{e^{F/2}}{\sqrt{}}\left[\frac{z(ar^2+br+1)}{rf} - \left(\frac{br}{2}+1\right)\right] = \frac{f\,e^{F/2}\,r}{\sqrt{}\cdot z}\left[z^2(br+2) \mp 2rf(z-c)\right]$$

$$2z^3\,\frac{(ar^2+br+1)}{\cancel{rf}} - z^2 rf(br+2) - r z^2(br+2)f + 2r^2 f^2(z-c) = 0$$

which is satisfied in view of (2).

This means that the 4 relations are essentially 3. They contain the functions $\zeta, a, b, c, \cancel{f}$. We choose $c = 2m$. Thus we are left with ζ, a, b, f. One can choose $f =$ const or f not constant. If we choose $f =$ constant we are left with 3 function ζ, a, b to be determined by these ~~4~~ three equations. This does not correspond to Datta's solⁿ. It could be regarded as a ~~to~~ & capable of being fitted with Datta's solution at some epoch t_0.

If f is not constant, b is arbitrary and we can as well put it equal to zero and have Datta's contracting sphere allowing some radiation to escape.

Let us first take the case $f =$ constant.

XXXIII

Take (2) and (3) ~~wif~~ with f = constant and solve them for θ (ar^2+br+1) and $[br+2]$.

$z^3(\,) - z^2 rf[\,] + r^2f^2 z - r^2f^2c = 0.$

$3z^2(\,) - 2zrf[\,] + r^2f^2 = 0.$ This second equation gives

$$(\,) = \frac{2zrf[\,] - r^2f^2}{3z^2}.$$ Put this in the first.

$$\frac{z^3\{2zrf[\,] - r^2f^2\}}{\cancel{3}z^2} - 3z^2 rf[\,] + 3r^2f^2 z - 3r^2f^2c = 0$$

$\therefore \cancel{2}z^2 rf[\,](2-3) + 2z r^2f^2 - 3r^2f^2c = 0$

$\therefore\; -z^2rf[\,] + 2zr^2f^2 - 3r^2f^2c = 0$

$\therefore\; \cancel{x}\; 2zrf - 3rfc = z^2[\,]$

$$\therefore\; [\,] = \frac{2zrf - 3rfc}{z^2} = \cancel{rf(2z - c} \frac{rf(2z-3c)}{z^2}$$

$$\therefore\; br+2 = \frac{rf(2z-3c)}{z^2}.$$

Next again take ~~(2)~~ the second equ.

$$3z^2(\,) + r^2f^2 = 2zrf[\,] \quad \therefore [\,] = \frac{3z^2(\,) + r^2f^2}{2zrf}$$

Use this value in (1)

$$2 z^3 (\ ) - z^2 rf \cdot \frac{(3z^2(\ ) + r^2f^2)}{\cancel{2} z rf} + 2r^2f^2 z - 2r^2f^2c = 0$$

$$\therefore z^3(\ )\,[2-3] + z r^2f^2 - 2r^2f^2c = 0$$

$$\therefore -z^3(\ ) + z r^2f^2 - 2r^2f^2c = 0 \quad \therefore z r^2f^2 - 2r^2f^2c = z^3(\ )$$

$$\therefore (\ ) = \cancel{z}\,\frac{r^2f^2(z-2c)}{\cancel{(\ )}\, z^3}$$

$$\therefore ar^2 + br + 1 = \frac{r^2f^2(z-2c)}{z^3}$$

Thus on the boundary

$$\boxed{\begin{aligned} br+2 &= \frac{rf(2z-3c)}{z^2} \\ ar^2+br+1 &= \frac{r^2f^2(z-2c)}{z^3} \end{aligned}}$$

We now find $\dot{z}$. But the form of ar^2+br+1 suggests that Schwarzschild's singularity develops at $z=2c$. Let us now find $\dot{z}$

$$\dot{z} = \frac{e^{F/2}}{\sqrt{(\ )}}\left[\frac{z(\ )}{rf} - \frac{1}{2}(br+2)\right]$$

$$= \frac{e^{F/2} z^{3/2}}{rf\sqrt{z-2c}}\left[\frac{z r^2f^2(z-2c)}{z^3 rf} - \frac{1}{2}\frac{rf(2z-3c)}{z^2}\right]$$

XXXV

$$\dot{z} = \frac{e^{F/2} z^{3/2}}{2f\sqrt{z-2c}} \frac{2f}{2z^2}\left[2z-4c-2z+3c\right]$$

$$= -\frac{e^{F/2}\, c}{2z^{1/2}\sqrt{z-2c}}$$. This $\dot{z}$ cannot vanish for any finite z and

$e^{-F/2}\dot{z}$ becomes infinite at $z=2c$. Schwarzschild's singularity is not reached and the field equations break down much earlier.

~~The~~ Datta's case cannot be obtained as a particular case of the above. But in Datta's case f is not constant. a and b are constants. Put $b=0$ or leave it arbitrary.

Now eqn. (4) is absent and (1), (2) and (3) are really ~~+t~~ two equations only to find z and f. Well $\dot{z}^2 = e^F\left[\frac{c}{z} - \frac{(a-\frac{b^2}{4})r^2}{ar^2+br+1}\right]$

determines $z = z(t)$ a, b any constants and then (2) will give $f = f(t)$.

So that is that!!!

XXXVI a

When f is not constant, b is ~~not~~ left arbitrary. We can choose b in such a way that $rb \to \infty$. ~~i.e. take $b = 0$.~~ We know that

if $R^2 = \frac{r^2}{ar^2+br+1}$ ~~then~~ and if $\frac{1}{R_0^2} = a - \frac{b^2}{4}$.

for $b = 0$, $r \to \infty$ as $R \to R_0$ and for

$b > 0$, $r \to \infty$ as $R \to kR_0$ $\cancel{k^2\left(a - \frac{b^2}{4}\right) = \frac{1}{a}}$

$$\cancel{k^2 = \frac{1}{a\left(a-\frac{b^2}{4}\right)} = \frac{R_0^2}{\frac{1}{R_0^2}+\frac{b^2}{4}}} \Rightarrow \frac{k^2}{a-\frac{b^2}{4}} = \frac{1}{a}$$

$$\text{i.e. } k^2 = \frac{a-\frac{b^2}{4}}{a} \quad \text{or} \quad \frac{1}{k^2} = \frac{a}{a-\frac{b^2}{4}} \cancel{= \frac{a-\frac{b^2}{4}+\frac{b^2}{4}}{a-\frac{b^2}{4}}}$$

$$1 + \frac{b^2}{a-\frac{b^2}{4}} = 1 + \frac{b^2/4R_0^2}{} \qquad \therefore k^2 = \frac{1}{1+\frac{b^2R_0^2}{4}}$$

Thus given the initial density R_0^2, one can choose the constant b ~~and the initial density~~ such that kR_0 becomes the initial boundary of the collapsing object.

Let us make such a choice of B and R_0.

XXXVI

We can arrive at an equation satisfied by ζ in the general case. We begin with

(1) $$\dot{\zeta} = \frac{e^{F/2}}{\sqrt{\ }}\left[\frac{\zeta(ar^2+br+1)}{rf} - \left(\frac{br}{2}+1\right)\right]$$ ~~But $\zeta_1 = -\frac{\dot{\zeta}}{\dot{m}}$~~

But $\cancel{\zeta_1 = +\frac{\dot{\zeta}}{\dot{m}} = -\frac{\dot{\zeta}}{\dot{m}\, e^{g/2} e^{F}(\ )} = -\frac{\dot{\zeta}\, e^{g/2} e^{-F/2} e^{-F/2}}{f}}$

$\therefore$ $\cancel{\dot{\zeta} = -\zeta_1 f\, e^{-g/2} e^{F/2}}$

But $\cancel{\zeta_1 = \zeta}$ $\dot{\zeta} = \zeta_1 \dot{m} = \zeta_1(-\dot{m})\, e^{g/2} e^{F}(\ )$

$$\therefore -\zeta_1 f\, e^{g/2} e^{F} = \frac{e^{F/2}}{\sqrt{\ }}\left[\qquad\right]$$

$$\therefore \zeta_1 = -\frac{e^{g/2} e^{-F/2}}{f r \sqrt{\ }}\left[\quad\right] = -\frac{\zeta}{rf}\left[\quad\right]$$

$$\therefore \zeta_1 = -\frac{\zeta}{r f^2}\left[\zeta(ar^2+br+1) - rf\left(\frac{br}{2}+1\right)\right]$$

$$\therefore -r f^2 \frac{\zeta_1}{\zeta} = \zeta(ar^2+br+1) - \frac{rf}{2}(br+2). \qquad (1)'$$

We write (2) as

$$\zeta^2\left[\zeta(ar^2+br+1) - \frac{rf}{2}(br+2)\right] - \zeta^2\frac{rf}{2}(br+2) + r^2 f^2(\zeta - c) = 0$$

i.e. $$-\zeta^2 r^2 f^2 \frac{\zeta_1}{\zeta} - \zeta^2 \frac{rf}{2}(br+2) + r^2 f^2(\zeta - c) = 0$$

i.e. $$rf\,\zeta\zeta_1 + \frac{1}{2}\zeta^2(br+2) - rf(\zeta - c) = 0 \qquad (2)'$$

We rewrite (3) as

$$3z\left[z(ar^2+br+1)-\frac{rf}{2}(br+2)\right]-\frac{zrf}{2}(br+2)+2zr^2ff_1+r^2f^2=0$$

$$\therefore 3z\left(-\frac{r^2f^2z_1}{z}\right)-\frac{zrf}{2}(br+2)+2zr^2ff_1+r^2f^2=0$$

$$\therefore 3rfz_1+\frac{z}{2}(br+2)-2zrf_1-rf=0. \qquad (3)'$$

Let us fix our attention on (2)' and (3)'.

We write them to-gether

$$rfzz_1+\frac{1}{2}z^2(br+2)-rf(z-c)=0 \qquad (2)'$$

$$3rfz_1+\frac{1}{2}z(br+2)-2zrf_1-rf=0 \qquad (3)'$$

~~Multiply (2)' by 3 (3)' by $-z$ and add~~

~~Subtract (2)' from (3)'~~ Multiply (2)' by -1, (3)' by z and add.

$$\cancel{2z}$$

$$-2z^2rf_1-rf\cancel{z}=0$$

$$\therefore 2rfzz_1+\cancel{rfz}-rfc \qquad \cancel{\boxed{\therefore \frac{f_1}{f}=\left(\frac{z_1}{z}+\frac{c}{2z^2}\right)}}$$

$$\therefore 2zz_1-c-2z^2\frac{f_1}{f}=0$$

$$f_1=f\left[\frac{z_1}{z}-\frac{c}{2z^2}\right]$$

This is then a very general result.

It gives $\frac{f_1}{f}-\frac{c}{2z^2}=\frac{z_1}{z}$ $\therefore$ ~~$z_1=\left[\frac{zf_1}{f}-\frac{c}{2z}\right]$~~

$$z_1=\left[\frac{zf_1}{f}+\frac{c}{2z}\right]$$

Now we can take the two cases $f=$ const. $f \neq$ constant. The case $f=$ const. has been dealt with. When f is not constant, b is left arbitrary. We can put $b=$ constant. The following simplification holds when b is constant. Write (2)$'$ as

$$rfz_1 + \frac{1}{2}z(br+2) - rf\left(1-\frac{c}{z}\right) = 0 \qquad (2)'$$

Differentiate w.r.t. m

$$\therefore rfz_{11} + rf_1 z_1 + \frac{1}{2}z_1(br+2) - rf_1\left(1-\frac{c}{z}\right) - \frac{rfc}{z^2}z_1 + \frac{rfc_1}{z} = 0$$

Take this with (2)$'$. Multiply (2)$'$ by $-z_1$, this equation by z and add them

$$\therefore rfzz_{11} + rf_1 zz_1 - rf_1 z\left(1-\frac{c}{z}\right) - \frac{rfcz_1}{z} + rfc_1$$

$$- rfz_1^2 + rfz_1\left(1-\frac{c}{z}\right) = 0$$

Devide by rf

$$\therefore zz_{11} - \frac{cz_1}{z} + c_1 - z_1^2 + z_1 - \frac{cz_1}{z} + \frac{f_1}{f}\left[zz_1 - z + c\right] = 0$$

$$\therefore zz_{11} - \frac{cz_1}{z} + c_1 - zz_1 + z_1 - \frac{cz_1}{z} + \left(\frac{z_1}{z} - \frac{c}{2z^2}\right)\left(zz_1 - z + c\right) = 0$$

$$\therefore z z_{11} - \frac{2c z_1}{z} + c_1 - z_1^2 + z_1 + z_1^2 - z_1 + \frac{c z_1}{z} + \frac{c}{2}\frac{z_1}{z} + \frac{c}{2z} - \frac{c^2}{2z^2} = 0$$

$$\cancel{\therefore z z_{11} + z_1^2 - z z_1 - \frac{c z_1}{2z} - \frac{c}{2z} + \frac{c^2}{2z^2} + c_1 = 0}$$

$$\therefore z z_{11} - \frac{3c}{2}\frac{z_1}{z} + c_1 + \frac{c}{2z} - \frac{c^2}{2z^2} = 0 .$$

It may be verified that $1 - \frac{c}{z} - z_1 = 0$ satisfies the above equation. Also if we put $1 - \frac{c}{z} - z_1 = y$ we get $\frac{y_1}{y} = \frac{c}{2z^2}$ again leads to

$\frac{y_1}{y} + \frac{f_1}{f} = \frac{z_1}{z}$ or $yf = kz$ and we can check that this is (2)' with $k = \frac{br+2}{2}$.

This means that our derivation of this diff-eqn is correct. Now when c is constant this equation must be equivalent to the equation $p = 0$.

Let us wind up with one new case that of $f =$ constant.

We add a point

On the boundary $8\pi p = 0$ $\quad 8\pi\rho = \frac{3c}{z^3}$

and $$8\pi T_4^1 = \frac{-\dot m c_1}{z^2\left[z^1 - \frac{z}{m}\dot m\right]}$$

$$= \cancel{-\dot m c_1} \frac{-\dot c\, f(ar^2 + br + 1)}{z^2 e^{g} e^{-F}}$$

$$= -\frac{\dot c f r^2}{z^2 z^2}$$

$$= -\frac{\dot c f r^2}{z^4}$$

If we assume that $z = r$ on the boundary

then $$8\pi T_4^1 = -\frac{\dot c f}{r^2}$$

The case ρ = constant

Note 2 We can regard ζ and m as finite and consider

$$\dot{\zeta} = + \frac{c}{2\zeta} m$$ and consider

$$e^F = m^2\left(1 - \frac{2c}{\zeta}\right)$$ being zero on the

boundary. Which means Schwarzschild's singularity developing. Now if

$\frac{dm}{dt}$ is finite $dm/d\tau$ be.

$$\frac{dm}{d\tau} = \frac{dm}{e^{F/2}\,dt} \to \infty. \quad \text{also } \rho \to \infty.$$

~~A~~ This means we reach ~~Schwarzschild's~~ a singularity similar to Schwarzschild's singularity and it will take ~~infin~~ infinite ~~prop~~ time to reach the singular state. Time taken to reach the stage when $\zeta = \frac{9}{4} c$ can be found

Note 3: Even in the general case we can regard f arbitrary and equation 1 2 3 giving z a and b; ~~Then we have~~ f being left to be determined by the physical condition of energy generation. Then we have again z and m finite and

$$e^F = m^2 \left(1 - \frac{c}{3} - 2z_1\right) \qquad \text{But } z_1 = \frac{c}{2z} + \frac{zf_1}{f}$$

$$\therefore e^F = m^2\left(1 - \frac{c}{z} - \frac{c}{3} - \frac{2zf_1}{f}\right)$$

$$= m^2\left(1 - \frac{2c}{3} - \frac{2zf_1}{f}\right) = \left(1 - \frac{c}{3} - 2z_1\right)$$

Now we know that $br + 2$ is always true. But

(3)' requires

$$3rfz_1 + \frac{3}{2}(br+2) - 2zrf_1 - rf = 0$$

$$\frac{3}{2}(br+2) = 2zrf_1 + rf - 3rfz_1$$

This should be looked into

Note 4: Case i $\frac{f_1}{f} = 0$, "i" $t = t_0$ Dalla's sphere

start contracting and at $t = t_1$ radiation reaches the boundary. Contraction continues and in finite proper time you reach singularity at $r = 4m$

Case ii $\frac{f_1}{f} > 0$, only possible case is $\frac{f_1}{f} > \frac{1}{32m}$ because otherwise $\dot{r}$ ~~be is~~ is always $-$ve. If $\frac{f_1}{f} > \frac{1}{32m}$, $1 - \frac{2m}{R} - \frac{2\dot{R}}{m}$

then again contracts to a finite radius $R = \frac{1}{4l^2} - \sqrt{\frac{1}{4l^4} - \frac{8m}{l^2}}$ $l^2 = \frac{f_1}{f}$ $= \text{constant}$

Case iii $\frac{f_1}{f} < 0$, One can begin with $\dot{R} = 0$ and then take $\dot{R}$ $-$ve and reach the singularity. or one can begin with non-zero but ~~ve~~ $-$ve value of $\dot{R}$ and reach singularity at

$R = -\frac{1}{4k^2} + \sqrt{\frac{1}{4k^4} + \frac{8m}{k^2}}$ $-k^2 = \frac{f_1}{f}$ $= \text{const}$

=