A. M. D. G.

ST. JOSEPH'S COLLEGE
TRICHINOPOLY

NOTE BOOK

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Form B.Sc. (Hons)

Subject Properties of matter

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Compound Pendulum

Corrections to be made.

1). Viscous resistance of the air.

Its effect may be represented by introducing a term α to the velocity in the equation of motion. The equation thus becomes $\frac{d^2\theta}{dt^2} + \nu\frac{d\theta}{dt} + \mu\theta = 0$.

μ is known to be $\left(-\frac{hg}{k^2+h^2}\right)$ h being distance of the centre of gravity from the axis of rotation, k [illegible] radius of gyration about an axis through the C.G. ||l to the axis of rotation. ν is a constant depending upon the medium.

The indicial eqn for the above differential eqn is $m^2 + \nu m + \mu = 0$

$$\therefore m = -\frac{\nu}{2} \pm \sqrt{\frac{\nu^2}{4} - \mu}.$$

$$\therefore \theta = A e^{\left(-\frac{\nu}{2} - \sqrt{\frac{\nu^2}{4} - \mu}\right)t} + B e^{\left(-\frac{\nu}{2} + \sqrt{\frac{\nu^2}{4} - \mu}\right)t}$$

If $\frac{\nu^2}{4} > \mu$ the indices of e are $-$ive, & ∴ whatever be the value ~~to~~ of t, that of θ never reverses its sign, being ~~α~~ $A+B$ when $t=0$, & 0 when $t=\alpha$.

s a constant. This constant is called he logarithmic decrement. The Log. Dec s the diminution of the logarithm (Naperian) f one amplitude below that of the mplitude immediately preceding.

Period $T = \frac{2\pi}{\sqrt{\mu - \frac{v^2}{4}}} = \frac{2\pi}{\sqrt{\mu}}\left(1 + \frac{v^2}{8\mu}\right)$ nearly.

Hence period is increased by the iscosity in the ratio $\left(1+\frac{v^2}{8\mu}\right) : 1$ [or since $= \frac{4\pi^2}{T^2}$ nearly] in the ratio

$$\left(1 + \frac{v^2T^2}{32\pi^2}\right) : 1 \quad \left[\text{about } (1+6\times10^{-9}) : 1\right]$$

orrection when the <u>periods</u> about the wo knife edges are <u>not exactly equal.</u>

Let T_1, T_2 be periods about the knife-dges A & B whose distances from the entre of gravity are h_1 & h_2.

$$T_1^2 = 4\pi^2 \frac{k^2+h_1^2}{h_1 g}$$

e $\frac{g}{4\pi^2} T_1^2h_1 = k^2 + h_1^2$ Similarly $\frac{g}{4\pi^2} . T_2^2h_2 = k^2 + h_2^2$.

$$\therefore \frac{g}{4\pi^2} \frac{T_1^2h_1 - T_2^2h_2}{h_1 - h_2} = h_1 + h_2$$

Let $T_1^2h_1 - T_2^2h_2 / h_1 - h_2$ be put as ~~the~~ T^2 ~~mputed time &~~ T may be called the mputed time since it is the period of a simple pendulum of length $h_1 + h_2$.

It may be expressed in a more convenien form as follows.

Let $t^2 = \frac{T_1^2 + T_2^2}{2}$ & $\alpha^2 = \frac{T_1^2 - T_2^2}{2}$.

$$T^2 = \frac{T_1^2 h_1 - T_2^2 h_1}{h_1 - h_2}$$

$$= \frac{(t^2+\alpha^2)h_1 - (t^2-\alpha^2)h_2}{h_1 - h_2}$$

$$= t^2 + \alpha^2 \frac{h_1 + h_2}{h_1 - h_2}$$

$$= \frac{T_1^2 + T_2^2}{2} + \frac{h_1 + h_2}{h_1 - h_2} \cdot \frac{T_1^2 - T_2^2}{2}.$$

ie $\frac{8\pi^2}{g} = \frac{T_1^2 + T_2^2}{h_1 + h_2} + \frac{T_1^2 - T_2^2}{h_1 - h_2}$.

The value $(h_1 + h_2)$ is measure with great exactitude, but $h_1 - h_2$ is not determined with nearly such accuracy. But the error of observat will have only an inappreciable resul effect on the final result since it occurs in the small term $T_1^2 - T_2^2$.

3) Air effect — air drag and buoyancy

We may represent the buoyancy as an upward force acting at the centre of gravity of the displaced air nd equal to its weight mg. Let this ntre of gravity be at distance s_1 from he knife edge A.

The mass of air flowing with the pendulum ill have no effective weight to increase he restoring couple, for it is buoyed up y the surrounding air. But it adds to he mass moved and increases the oment of inertia by an amount say $m_1 d^2$.

$$\therefore T_1 = 2\pi\sqrt{\frac{(k^2+h_1^2)M + m_1 d^2}{Mgh_1 + mgs_1}}$$

&

$$\text{ie } \frac{g}{4\pi^2} T_1^2 h_1 = \frac{(k^2+h_1^2)M + m_1 d^2}{M + m\frac{s_1}{h_1}}$$

$$= \left(k^2+h_1^2+\frac{m_1}{M}d^2\right)\left(1+\frac{m}{M}\frac{s_1}{h_1}\right)$$

$$= k^2+h_1^2+\frac{m_1}{M}d^2+\frac{m}{M}\frac{s_1}{h_1}.k^2+\frac{m s_1}{M}h_1$$

neglecting the last term since it is the product of two very small quantities.

Now If if A & B are accurately oints of suspension and oscillation,

$h_1 + h_2 = \frac{k^2 + h_1^2}{h_1}$ ie $h_1 h_2 = k^2$.

In the above case k^2 is nearly equal to $h_1 h_2$.

Putting this value in the fourth term of the above expression, the error committed would be negligible, since the term is small. Thus

$$\frac{g}{4\pi^2} . T_1^2 h_1 = k^2 + h_1^2 + \frac{d^2}{r} m_1 + \frac{m}{r} k^2 (h_1 + h_2$$

On inverting the pendulum let the air drag change to $m_2 d^2$ & the ~~be~~ Centre of gravity of the displaced air be at distance s_2 from the knife-e

$$\frac{g}{4\pi^2} T_2^2 h_2 = k^2 + h_2^2 + \frac{d^2}{r} m_2 + \frac{m}{r} k^2 (h_2 + h_1)$$

∴ If T is the computed time,

$$\frac{g}{4\pi^2} T^2 = \frac{g}{4\pi^2} (T_1^2 h_1 - T_2^2 h_2)/(h_1 - h_2)$$

$$= h_1 + h_2 + \frac{d^2}{r(h_1 - h_2)}(m_1 - m_2)$$

$$+ \frac{m_2 k^2}{r(h_1 - h_2)}(h_1 + h_2)(s_1 -$$

The above expression gives the chang in the computed time due to the air effec

This source of error can be eliminated if the pendulum is made ymmetrical in form. Thus $m_1 = m_2$;

$= s_2$ & $\frac{\cancel{4}\, g}{4\pi^2} T^2 = h_1 + h_2$.

The computed time is not altered by

e air effect.

) Rounding of the knife-edge.

Let ρ_1, ρ_2 the knife-edges be supposed
 be cylindrical in form of radii ρ_1 & ρ_2.
e moment of inertia is to be calculated
bout the different axes of contact. But
ese axes are so near each other that
e may consider the moment of in. to be
e same. The restoring couple here is
t due to the rotation of the centre of
ravity in a circle (not of radius $h_{1,2}$ but)
 radius $h_1 + \rho_1$. The horizontal travel of
e centre of ~~an~~ curvature does not
lter the amount of work. Hence we have

$$T_1 = 2\pi \sqrt{\frac{K^2 + h_1^2}{(h_1 + \rho_1) g}} \quad \text{ie} \quad \frac{g}{4\pi^2} T_1^2 h_1 = (K^2 + h_1^2) \Big/ 1 + \frac{\rho_1}{h_1}$$

$$= (K^2 + h_1^2)\left(1 - \frac{\rho_1}{h_1}\right)$$

$$= K^2 + h_1^2 - \frac{\rho_1}{h_1} . K^2 \cancel{a} - \rho_1 h_1$$

$= K^2 + h_1^2 - \rho_1 (h_1 + h_2)$ putting $h_1 h_2$ as an approximation for k^2 in the 3rd term which is small.

Illrly $\frac{\cancel{4\pi} g}{4\pi^2} T_2^2 h_2 = k^2 + h_2^2 - \rho_2 (h_1 + h_2)$

∴ If T is the computed time, $\frac{g}{4\pi^2} T^2$

$$\frac{g}{4\pi^2} \cdot \frac{T_1^2 h_1 - T_2^2 h_2}{h_1 - h_2} = h_1 + h_2 - \frac{h_1 + h_2}{h_1 - h_2} \cdot (\rho_1 - \rho_2)$$

Hence the unequal rounding of the knife-edges alters the computed time.

The error can be eliminated by interchanging the knife-edges. The new computed time T' is given by

$$\frac{g}{4\pi^2} T'^2 = h_1 + h_2 - \frac{h_1 + h_2}{h_1 - h_2} (\rho_2 - \rho_1)$$

$$\therefore \frac{g}{4\pi^2} \frac{T^2 + T'^2}{2} = h_1 + h_2$$

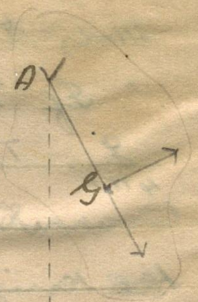
Hence the correct computed time $= \frac{T^2 + T'^2}{2}$.

5) <u>Yielding of the support.</u>

<u>Vertical yielding.</u>

At any instant t let the deflection be θ. The centrifugal force on the support due to θlar motion is mh $\frac{d\theta}{dt}$ $mh \left(\frac{d\theta}{dt}\right)^2$ where m is the mass of the

endulum & h the depth of the centre
f gravity G below the support A.
his force acts along AG.
he tangential force due to
he accel. $\frac{d^2\theta}{dt^2}$ of the body is $mh\frac{d^2\theta}{dt^2}$.



he sum of the vertical compo-
nts is ~~mg~~ $mh\left(\frac{d\theta}{dt}\right)^2\cos\theta$

$- mh\frac{d^2\theta}{dt^2}\sin\theta$

e $mh\ \cancel{\theta}\left\{\left(\frac{d\theta}{dt}\right)^2 - \theta\frac{d^2\theta}{dt^2}\right\}$ since θ is small.

$\frac{d\theta}{dt}$ is evaluated as follows.

Let α be the maximum ~~value~~ amplitude of θ.
Then K.E at instant t ie $\frac{1}{2}I\omega^2$ is the
ork done in rotating through $\alpha - \theta$

e $\frac{1}{2}I\omega^2 = mgh(\cos\theta - \cos\alpha)$

$= mgh\ \cancel{\theta}\left(\frac{\alpha^2}{\lfloor 2} - \frac{\theta^2}{\lfloor 2}\right)$

ie $\left(\frac{d\theta}{dt}\right)^2$

$= \frac{gh(\alpha^2-\theta^2)}{k^2+h^2}.$

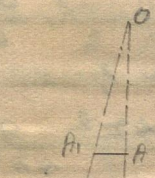
∴ vertical force
n the support $= mh\left\{\frac{gh(\alpha^2-\theta^2)}{k^2+h^2} - \frac{gh\theta^2}{k^2+h^2}\right\}$

$\alpha^2.\frac{mgh^2}{k^2+h^2} - 2\theta^2.\frac{mgh^2}{k^2+h^2}$

The first ~~point~~ term of the force

is constant, & the other term is variable. ~~depen~~ The variable term depen only on the second ~~ter~~ power of θ whic is small & ∴ we may consider the force on the support to be practically constant. In other words the support does not move vertically to any apprecia extent during the oscillations.

Horizontal yielding.



As the pendulum swings the support also moves to & fro horizontally. If A G be the equilibrium positions of the support & the C.G. & A₁ G₁ their positions at any instant, A G & A₁ G₁ produced ~~met~~ will meet at a point O which is fixed relative to the support & the pendulum. The pendu effectively ~~moo to~~ swings about the pt. O. Thus the effect of the horizontal yield s to ~~increase the bar bar raise up the point~~

~~f support~~ make the C.G. swing in an arc of radius OG

The ~~force or~~ horizontal force on the ~~pendulum~~

upport is $mh\left(\frac{d\theta}{dt}\right)^2 \cdot \theta + mh\frac{d^2\theta}{dt}\cos\theta$

$= mh_1\frac{d^2\theta}{dt^2}$ nearly

Let e be the horizontal yielding due to

nit horizontal force

$\therefore$ The yielding d_1 of the support

$= m h_1 \frac{d^2\theta}{dt^2} \cdot e.$

Now $T_1 = 2\pi\sqrt{\frac{k^2+(h_1+d_1)^2}{(h_1+d_1)g}}$

$\therefore \cancel{4\pi^2}\ \frac{g}{4\pi^2} T_1^2 h_1 = \{k^2+(h_1+d_1)^2\}\left(1-\frac{d_1}{h_1}\right)$

$= k^2 + h_1^2 \cancel{+} - \frac{k^2 d_1}{h_1} + h_1 d_1$

$= k^2 + h_1^2 + d_1(h_1 \cancel{+} h_2)$ Putting h_1h_2 for k^2

ly if d_2 is the yielding ~~about~~ when supported

the other knife edge

$\frac{g}{4\pi^2} T_1^2 h_2 = k^2 + h_2^2 \cancel{+} d_2(h_2 - h_1)$

$\therefore \frac{g}{4\pi^2} \frac{T_1^2h_1 - T_2^2h_2}{h_1-h_2} = h_1+h_2 \cancel{+} (d_1+d_2)\cancel{(h_1+h_2)}$

~~$= h_1+h_2 - (h_1+h_2)\, me\, d^2$~~

But $d_1 = me\, h_1 \frac{d^2\theta}{dt^2}/\theta = me\, h_1 \frac{h_1 g}{h_1^2+k^2}$

$= me\frac{h_1 g}{h_1+h_2}$ – putting h_1h_2 for k^2.

$$\therefore \frac{g}{4\pi^2} \cdot \frac{T_1^2 h_1 - T_2^2 h_2}{h_1 - h_2} = (h_1 + h_2) +$$

$$+ (h_1 + h_2) \frac{\lambda mg}{h_1 + h_2}$$

$$= h_1 + h_2 + \lambda mg.$$

Thus the effective increase in lengt of the simple equivalent pendulum is the yielding due to a horizontal force equal to the mass of the pendulum.

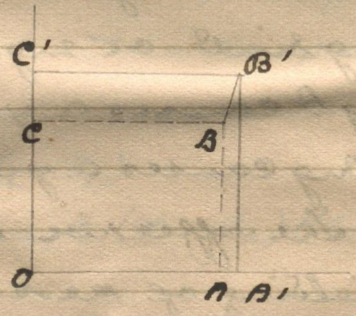
In the case of a simple pendulum the effective increase is calculated more easily.
The tension of the string is
$mg \cos\theta$ or mg
& the horizontal pull is
$mg \sin\theta$ or $mg\theta$
$mg\theta$ causes a yielding of AB
$\therefore mg$ causes a yielding $\frac{AB}{\theta}$ or AC.
$\therefore$ The effective increase is the horizonta yielding of the support due to a horizon force equal to the mass of the pendulum

It can also be calculated by observing periods on this support & a non-yielding o

Elasticity

This chapter deals with homogeneous trains only, that is strains in which ines which were parallel to each other emain ∥l after the strain, parallel lanes remain ∥l planes, & the ratios tween any two lengths, or any two areas emain unaltered.

In general it can be proved that ny homogeneous strain is a superposition f uniform dilatations and shears. The roposition is proved here in the case f a plane surface OABC.



n account of the strain let the surface take up its new osition OA'B'C'.

t e & f be the elongtions along the x & y axis.

$$AA' = OA \times e = \left\{\frac{e+f}{2} + \frac{e-f}{2}\right\} OA$$

$$BB' = OC \times f = \left\{\frac{e+f}{2} - \frac{e-f}{2}\right\} OC.$$

Thus the strain is equivalent to
1) a uniform dilatation $\frac{e+f}{2}$ along the two axes (2) ~~log~~ & an elongation $\left(\frac{e-f}{2}\right)$ along the x axis together with a contraction $\left(\frac{e-f}{2}\right)$ along the y-axis.
The former ~~stain~~ strain alters the size, but not the shape & hence is a uniform dilatation. The latter stai alters the shape but not the size & ∴ is a shear.

Longitudinal elongation without lateral contraction.

We have seen that if a unifor cube is subjected to stress P, Q, R on its ~~tw~~ three pairs of faces, all outwar & if λ, μ be resp^ly^ the ~~contion~~ ~~ca~~ elongation due to unit stress in its own dire tion & the contraction due to the same in the two ⊥r directions, then ~~extension~~ elongati along the x-axis $e = P\lambda - \mu(Q+R)$; along the y-axis $f =$ ~~R~~ $Q\lambda - \mu(P+R)$ &

long the z axis $g =$ ~~R~~ $R\lambda - \mu(P+Q)$.

Now longitudinal elongation without ateral contraction can be produced if

$f = g = 0$ ie $R\lambda = \mu(P+Q)$

$Q\lambda = \mu(P+R)$

$\therefore (R-Q)\lambda = \mu(Q-R)$ & since $\frac{\lambda}{\mu}$ $\frac{\mu}{\lambda} = \sigma$

$(R-Q) = \sigma(Q-R)$

ie $(Q-R)(1+\sigma) = 0$ & since σ is not egative, $Q-R$ $1+\sigma$ is +ive & $\therefore Q-R=0$

$Q = R$.

$R = \sigma(R+P) \therefore R = \frac{P\sigma}{1-\sigma}$

$\therefore$ Elongation along the x-axis

$= P\lambda - \mu(Q+R)$

$= P\lambda\left(1 - \frac{2\sigma^2}{1-\sigma}\right)$

If contraction had been permitted take place e would have been $P\lambda$

$\therefore$ The new condition to which the body subjected reduces the elongation in ratio $1 : \left(1 - \frac{2\sigma^2}{1-\sigma}\right)$.

Torsional Couple. It has been proved at the torsional couple per unit twist

~~is~~ (μ) in the case of a wire of ○lar section is $\frac{1}{2}\pi n a^4/l$.

$$\therefore \frac{\mu}{\text{Sectional area}} = \frac{1}{2} n a^2/l.$$

In the Case of a wire of elliptic secti the ~~area being~~ axes being major a, mino b, μ, $= \frac{1}{2}\pi n a b^3/l$.

$$\therefore \frac{\mu}{\text{Sectional area}} = \frac{1}{2} n b^2/l.$$

Hence it is seen that beating out a wire into one of elliptic Section decreases the torsional couple to a great extent without thereby diminishing its strength. In the Phosphor bronze suspenders of galvanometer Coils the less μ is the greater beco the sensibility. Hence the suspenders are thin strips, not fibres.

Bending

σ in terms of the two Curvatures.

Let the figure ① represent the sectio of a bent bar in the plane of the ben AB being neutral axis, O the Centre

f curvature. Consider
ny fibre ||l to AB, say
D at the top most layer.
rem the property of ||r
egments $\frac{CD}{AB} = \frac{C.B}{OA}$.
$\therefore \frac{CD - AB}{AB} = \frac{CA}{\rho}$ where
ρ = ~~rad~~ OA.
But $\frac{CD - AB}{AB} = \lambda$ elongation
l the fibre $\therefore \lambda = \frac{CA}{\rho}$.
||rly it can be shown that
very fibre ||l to the neutral
urface undergoes an elon-
ation which is $= \frac{1}{\rho} \times$ (the
e distance of the fibre
rom the neutral surface)

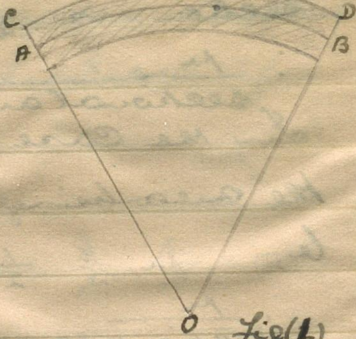


Fig (1)

Fig (2)

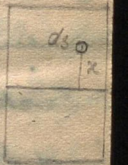
Now the elongation is
ccompanied by a contraction μ in the
r direction. Fig (2) shows a section of
e ~~rod~~ bar taken ⊥r to its length, A,B, being
e section of the neutral surface.
As before μ is $\frac{A_1'B_1 - C_1D_1}{A_1B_1} = \frac{1}{OB'} \times C_1A_1$

If ρ_1 is the radius of curvature in this case, $\mu = \frac{C_1 A_1}{\rho_1}$ i.e. $\frac{CA}{\rho_1}$

But $\sigma = \frac{\mu}{\lambda}$ $\therefore$ $\sigma = \frac{\rho}{\rho_1}$

$\therefore$ Poisson's ratio is ~~the~~ also the ratio of the two radii of curvature of the neutral surface of a bent rod, the numerator radius being taken in a plane ⊥r to the plane of bending & the denominator radius in ~~a~~ the plane of bending.

Bending moment.



Let the figure represent the section of the bar ⊥r to its length. Elongation of any area ds at dista

x from A, B, $= \frac{x}{\rho}$

$\therefore$ ~~Force per unit~~ Stress acting on it $= \frac{qx}{\rho}$

$\therefore$ Force on the area $= \frac{qx\,ds}{\rho}$.

$\therefore$ Moment of the force is $q\frac{x^2}{\rho}ds$. This gives the restoring couple called into play by the elasticity of the bar material o this element ds. $\therefore$ Total restoring ouple or bending moment $= \frac{q}{\rho} \Sigma x^2 ds.$

$\frac{qAk^2}{\rho}$ A being area, k the radius of yration about $\bar{A}B,$.

ag and deflection in a Cantilever.

t the Cantilever

at AB be

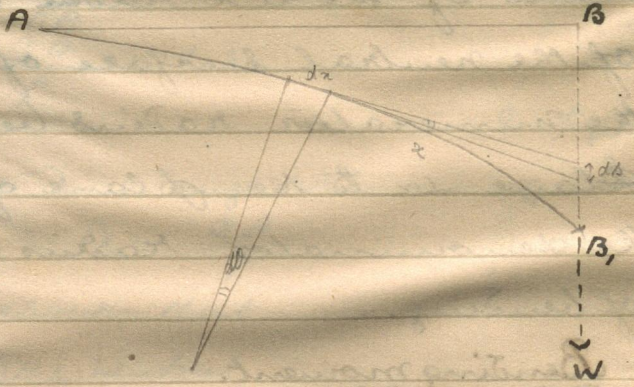
ent to the

osition AB,

e to a wt W.

ag due to an

lement dx



t distance x from B' is ds & $\frac{ds}{x} = d\theta = \frac{dx}{\rho}$

$\therefore d\theta = \frac{dx}{\rho} = \frac{Wx\,dx}{qAK^2}.$

If θ be the ∠ between the tangents rawn to the bent bar at B, & another t l distant from B,,

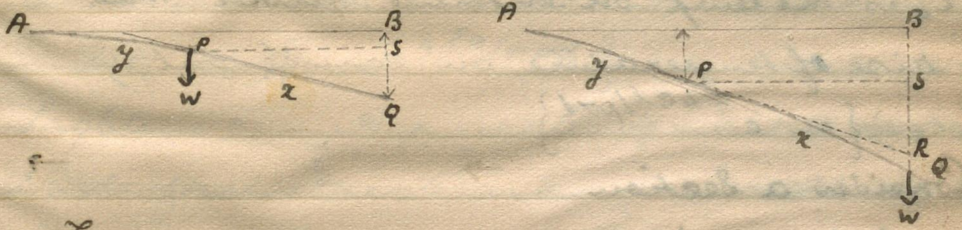
$$\theta = \int_0^l \frac{Wx\,dx}{qAK^2} = \frac{W}{qAK^2} . \frac{l^2}{2}$$
$$= C\frac{l^2}{2} \text{ where C is a constant.}$$

Sag $ds = \frac{x\,dx}{\rho} = C x^2 dx$

$\therefore$ If s be the sag due to the portion of the ar of length l measured from B,,

$$s = \int_0^l C x^2 dx = C\frac{l^3}{3}.$$

Reciprocal relation in a cantilever.



When the cantilever is loaded at ~~p~~ P the depression at Q is the same as ~~thd~~ the depression at P when it is loaded at Q.

Proof Case i Loaded at P. Let AP = y, PQ = x

$$BQ = BS + SQ = k\frac{y^3}{3} + x.k\frac{y^2}{2} = ky^2\left(\frac{y}{3}+\frac{x}{2}\right)$$

Case ii Loaded at Q.

Depression at P = BS = BQ − RQ − SR

$$= k.\frac{(x+y)^3}{3} - k\frac{x^3}{3} - x\left\{k\frac{(x+y)^2}{2} - k\frac{x^2}{2}\right\}$$

$$= k\left\{x^2y + xy^2 + \frac{y^3}{3} - x^2y \mp \frac{xy^2}{2}\right\}$$

$$= k\,y^2\left\{\frac{y}{3}+\frac{x}{2}\right\}$$

Problem I A tube of length l, diamet d through which a liquid is passing with a velocity v is surrounded by steam at temperature t_1. The heat bstracted from the steam per unit area

er unit time = $cv(t_1 - t)$ where t is

e temperature of the liquid at that point.

f t_0 is the temp on entering prove that

e rise of temperature on coming out is

$t_1 - t_0)\{1 - e^{-4cl/\rho v d}\}$.

Consider a section

f the tube of thickness dx at distance x from

e end. The temp. of the liquid that passes

hrough this can be ~~cons~~ taken to be constant

y t

Heat conducted per unit ~~area~~ time

$2\pi r\, dx \times cv(t_1 - t)$ calories.

l. of liquid that passes through it per unit

ime = $v\pi r^2$.

If dt is the rise in temperature,

$$dt \times \sigma\rho v\pi r^2 = 2\pi r\, dx\, cv(t_1 - t)$$

ie $dx = \dfrac{\sigma\rho r}{2c}\dfrac{dt}{t_1 - t} = -\dfrac{\sigma\rho r}{2c}\dfrac{d(t_1 - t)}{t_1 - t}$

Let T be the temperature on

coming out

$$\therefore \int_0^l dx = -\frac{\sigma\rho r}{2c}\int_{t_0}^{T}\frac{d(t_1 - t)}{t_1 - t}$$

ie $l = \frac{-\sigma p r}{2c} \log_e \frac{t_1 - T}{t_1}$

$\therefore \frac{t_1 - T}{t_1 - t_0} = e^{-\frac{4lc}{\sigma p d}}$

ie $t_1 - T = (t_1 - t_0)\, e^{-4lc/\sigma p d}$

ie $T - t_0$ or increase in temperature

$= (t_1 - t_0)\left\{1 - e^{-4lc/\sigma p d}\right\}$

Problem II A particle of unit mass starts from rest, moves in a straight line, under a constant accelerating force f & a retarding force av^2 where v is the velocity. Show that the velocity after passing over a distance x is given by $v^2 = \frac{f}{a}(1 - e^{-2ax})$

~~ds~~ The space travelled in time dt when vel. is v is given by $ds = v\,dt$.

$\frac{dv}{dt} = \cancel{f} f - av^2 \therefore ds = \frac{v\,dv}{f - av^2}$

ie $ds = \frac{d(f - av^2)}{-2a(f - av^2)}$

If V is the final velocity

$\int_0^x dx = \frac{1}{-2a}\int_0^V \frac{d(f - av^2)}{f - av^2}$

ie $x = \frac{1}{-2a} \log_e\left(\frac{f - aV^2}{f}\right)$

ie $1 - \frac{aV^2}{f} = e^{-2ax}$

ie $V^2 = \frac{f}{a}\left(1 - e^{-2ax}\right)$

Problem III A conical vessel radius r, eight h, base upwards, filled with ater, discharges the water through an orifice f area a at the vertex. Show that the ime taken for complete discharge $= \frac{2\pi r^2}{5a}\sqrt{\frac{2h}{\cancel{2}g}}$

When the level falls by dx e water that gets out is $\pi \frac{r^2}{h^2} x^2 dx = a v dt$. (1)

he work done by the centre of gravity owering by $\frac{3}{4} dx$ is = the work done in iving velocity v to the water

i $\frac{1}{3} \pi \frac{r^2}{h^2} x^3 \frac{3}{4} dx . g = \frac{1}{2} a v^3 dt$. (2)

Dividing (2) by one, $v^2 = \cancel{\frac{xg}{2}} . 2xg$

Substituting in (1)

$\pi \frac{r^2}{h^2} . x^2 dx = a \sqrt{\frac{xg2}{\cancel{2}}} dt$

f ~~t~~ T is the total time taken for discharge

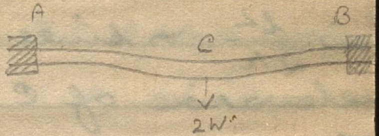
$\int_0^T dt = \frac{\pi r^2}{h^2} . \sqrt{\frac{\cancel{x}1}{2g}} . \frac{1}{a} \int_0^h x^{3/2} dx$

ie $T = \frac{\pi r^2 \cancel{\sqrt{2}} \cancel{h} h^{5/2} \times 2}{\sqrt{2} h^2 g a 5}$

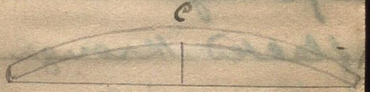
$= \frac{2\cancel{\pi}}{5} \frac{\pi r^2}{a \cancel{\sqrt{2}}} \sqrt{\frac{2h}{\cancel{2}g}}$.

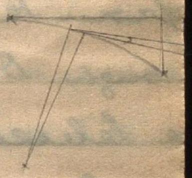
Bar clamped at both ends.

Let A B be the bar of length 2l, loaded t its midpt with a wt 2W. If the rod were resting on two knife-edges at A & B, the portion CB would have been a cantilever. So too the portion AC. The tangent at B wou have made an ∠ $\frac{3Wl^2}{2qAK^2}$ with the horizon (q being young's mod, A area of section, & K radius of gyration about the neutra surface) Due to the clamping the ∠ is z Hence the effect of the clamp is to exer a couple opposite to the one due to the loa acting in the same plane. Let us suppose it to be a uniform couple ϕ.



If ϕ alone were acting the ~~rod~~ bar would have bent upwards as the arc of a ⊙ of radius say R. The ∠ between the tangents at the end B & the vertex C is $\int_0^l \frac{dx}{R} = \frac{l}{R}$





$\therefore \frac{l}{R}$ by supposition $= \frac{Wl^2}{2qAK^2}$.

Now the elevation of C due to the couple is $\frac{l^2}{2r}$ & since $\frac{l}{r} = \frac{2Wl^2}{2gAK^2}$

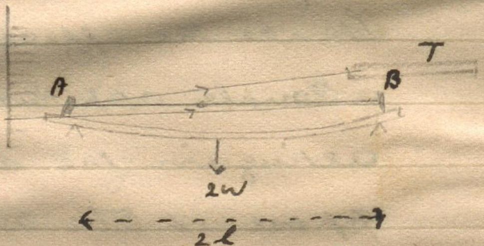
elevation of C = $\frac{Wl^3}{4gAK^2}$.

Hence the ~~to~~ couple due to the load tends to de- ~~presses~~ C by $\frac{Wl^3}{3gAK^2}$ & that due to the clamps nds to levate~~s~~ C by $\frac{Wl^3}{4gAK^2}$.

Hence the depression of C is $\frac{Wl^3}{12gAK^2}$.

oung's Modulus by bending

Accurate method for determining he sag.



A, B are two plane irrors attached to the ar just beyond the nife edges. A telescope T views a scale as eflected through both the mirrors. Suppose is the distance AB & D the distance of from the scale. When the mirrors rotate rough θ ie $\frac{Wl^2}{2gAK^2}$ there is a shift say S in e telescope reading.

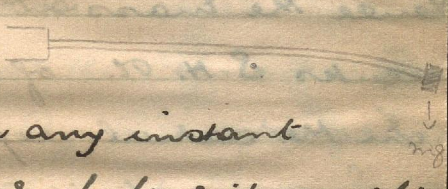
Due to the rotation of A, the shift is 2dθ if there had been a scale at B, the reading would have been $2d\theta$ below the former reading; or if ~~the~~ B also had rotated thro θ in the same direction as A, the readin in T would have been $2d\theta$ ~~be~~ lower.

Now B ~~also~~ rotates, ~~but~~ in the opposite direction. Its equivalent rotation is 2θ, & ∴ the shift due to B is $4D\theta$ ∴ $\delta = \theta(2d+$

Thus δ is known & q is calculated.

<u>Oscillations of a Cantilever.</u>

A mass m is ~~rigidly~~ hung ~~fixed at~~ from the end of a cantilever. Consider any instant when the depression of m below its equilibrium position is y



When the depression is x, the load is W such that $x = \frac{Wg\,l^3}{3qAK^2} = C\,Wg$ C being $\frac{l^3}{3qAK^2}$

∴ ~~Work done~~ $Wg = \frac{x}{C}$

Work done when force Wg works through dx is $\frac{x\,dx}{C}$ ∴ Work done on the mass to cau the sag $y = \int_0^y \frac{x\,dx}{C} = \frac{1}{C}\frac{y^2}{2}$

~~This is equal to the~~ K.E. $\frac{1}{2}m\left(\frac{dy}{dt}\right)^2$ ~~wha~~

f the system mass $\frac{1}{2}m(dy/dt)^2$ (neglecting for a first approxi-ation the wt. of the Cantilever)

$$\therefore \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}\frac{y^2}{c} = \text{Constant}$$

$$\therefore m\frac{d^2y}{dt^2} + \frac{1}{c}\cancel{dt}\, y = 0$$

$$\text{ie} \quad d^2y/dt^2 + \frac{y}{mc} = 0$$

$$\text{ie } y = Ae^{i\sqrt{1/mc}\,t} + Be^{-i\sqrt{1/mc}\,t}$$

$$= K\cos\left(\sqrt{\frac{1}{mc}}\,t - \varphi\right) \quad K \,\&\, \varphi \text{ being}$$

Constants.

This is the eqn to S.H.M of period $2\pi\sqrt{mc}$

Hence the mass at the end of the Cantilever ecutes S.H.M. of period $2\pi\sqrt{\frac{ml^3}{3AK^2}}$.

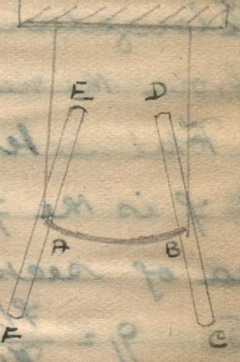
In the actual experiment l must e very great if T should be measurable.

The K.E. ~~of~~ due to the motion of the particles of the bar ~~are~~ is not negligible since it is mparable to $\frac{1}{2}m\left(\frac{dy}{dt}\right)^2$. The bar actually dds to the moving parts of the system a say m' $\therefore T = 2\pi\sqrt{(m+m_1)c}$

& since m' is ~~not~~ cannot be directly easured, the method adopted is to find T when o masses M_1 & M_2 are respectively attached to the end of the bar. $T_1^2 - T_2^2 = 4\pi^2 C (M_1 - M_2)$ Hence C & M, ∴ g are determined. Success of the expt. requires that M_1 & M_2 should be as remote from each other as possible.

Searle's Expt for σ

Two moment of inertia rods are rigidly fixed to the ends of ~~the~~ a wire of the material of which the Poisson's ratio is to be determined. The wire along with the rods form an H. By two strings attached to the centres of the rods the H is hung horizontally. The ends E D of the rods are drawn together by ~~pass~~ slippin a string loop over them. On burning the strings, the connecting wire unbends itself, bends in the opposite direction, and so on thus executing oscillation.



Consider the moment when the inclina of either rod to the equil. position is θ.

between the rods, ie ∠ subtended by e bent wire at its centre of curvature is θ. If l is the length of the wire & ρ radius f curvature at this instant, $2\theta = \frac{l}{\rho}$

e $\rho = \frac{l}{2\theta}$.

Elongation of any fibre at ~~length~~ distance from the neutral axis $= \frac{x}{\rho}$

. Total extension of the fibre $= \frac{xl}{\rho}$

If f is the force per unit area, s the rea of section of the fibre, q the Young's od $q = \frac{f/s}{x/\rho}$ ie $f = \frac{xqs}{\rho}$

$\therefore$ Work done on the fibre $= \frac{1}{2}$ final orce × elongation $= \frac{1}{2}\frac{xqs}{\rho} \times \frac{xl}{\rho}$

$= \frac{1}{2}\frac{ql}{\rho^2} x^2 s$

$\therefore$ Work done on the whole bent wire

$= \frac{1}{2}\frac{ql}{\rho^2}\{sx^2 = \frac{1}{2}\frac{ql}{\rho^2}Ak^2$ where A = rea of section, k = radius of gyration about e neutral axis

$= \frac{1}{2}\frac{ql}{\rho^2}. \pi a^2 \times \frac{a^2}{4}$ & since $\rho = \frac{l}{2\theta}$ ie $\frac{1}{\rho^2} = \frac{4\theta^2}{l^2}$

$= \frac{1}{2}\frac{\pi q a^4}{l}. \theta^2$ a being radius of the wire

For P.E Couple $= \frac{qAk^2}{\rho}$; deflection $= \frac{l}{2\rho} = \theta \therefore$ Couple $= \frac{2qAk^2\theta}{l}$

$\therefore$ Work $= \frac{1}{2}\frac{l}{\rho} \cdot \frac{qAk^2\theta}{\rho}\}$ $T = 2\pi\sqrt{\frac{I}{\text{couple per unit defl.}}}$

$= 2\pi\sqrt{\frac{Il}{2qAk^2}}$ $= 2\pi\sqrt{\frac{2Il}{\pi q a^4}}$

If ω be the ∠lar vel of the rods at this instant, I the mom. of in. of eac rod about the axis of oscl^{n}, the combine Kin. E. of the system = ~~2~~ $\frac{1}{2} \times 2I \times \omega^2$

$= I\omega^2$ neglecting the K.E. of the wire as be too small.

Since P.E. + K.E is constant

$\frac{1}{2}\frac{\pi q a^4}{l}\theta^2 + I\omega^2$ = Constant

Differentiating w.r.t time

$$\frac{1}{2}\pi q\frac{a^4}{l}\theta\frac{d\theta}{dt} + I\omega\frac{d\omega}{dt} = 0$$

$$\therefore \frac{d\omega}{dt} = -\frac{\pi q a^4}{2lI}\theta$$

If T_1 be the period of oscillation

$$T_1 = 2\pi\sqrt{\frac{2Il}{\pi q a^4}}$$

One of the rods is now clamped to a fixed stand & the other is ~~to~~ suspen from it by means of the connecting wire. If T_2 be the period $T_2 = 2\pi\sqrt{\frac{2Il}{\pi n a^4}}$ where n is the rigidity modulus.

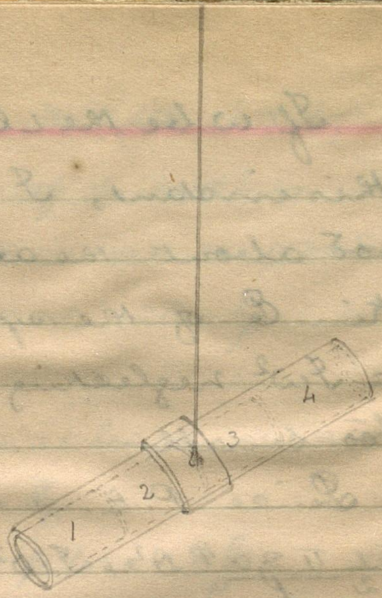
$\therefore \frac{T_2^2}{T_1^2} = \frac{q}{n}$ & since $q = 2n(\sigma + 1)$

ie $\sigma = \frac{q}{2n} - 1$

$$\sigma = \frac{T_2^2}{2T_1^2} - 1$$

axwell's needle.

In the determination
f rigidity modulus
he chief source of
rror is the possible
naccuracy in the
lculation of the mom-
ent of inter inertia
f the rod or disc. Max-



well's arrangement eliminates this quantity.
e so-called needle is a brass tube into
hich are fitted 4 perfectly similar cylinders
of brass & two of wood. Let their mom-
ts of inertias be (about an axis through
e C.G ⊥r to the length) be I_1, I_2, I_3, I_4. Let I
e the mom. of in. of the tube about its axis
ose. From the usual formula $T = 2\pi\sqrt{\frac{I}{\mu}}$
e have $I = \frac{4\pi T^2 \mu}{4\pi^2}$ μ being restoring
uple per unit twist of the wire.
et T_1 be the period where the two extreme
ods are brass, & the inner ones wood.

$$\frac{\mu}{4\pi^2} T_1^2 = I + I_1 + I_2 + I_3 + I_4 + m_1(d_1+a)^2 + m_2(d_1+b)^2$$
$$+ m_3(d_2+c)^2 + m_4(d_2+d)^2 \quad \text{--- (1}$$

Where m_1, m_2 are the masses of the wood cylinders, m_3, m_4 of the brass cyl., d_1, d_2 distances of the centres of fig of the wood & brass cylinders from the axis of osc. a, b, c, d, distances of the centres of gravity of the cylinders from their centres of figure.

Now each of the cylinders ~~are~~ is reversed in its own position. If T_2 be the new period we have

$$\frac{\mu}{4\pi^2} T_2^2 = I + I_1 + I_2 + I_3 + I_4 + m_1(d_1-a)^2 + m_2(d_1-b)$$
$$+ m_3(d_2-c)^2 + m_4(d_2-d)^2 \quad \text{--- (2)}$$

Adding (1) & (2) & dividing by 2

$$\frac{\mu}{4\pi^2} \cdot \frac{T_1^2+T_2^2}{2} = I + I_1 + I_2 + I_3 + I_4 + m_1(d_1^2+a^2)$$
$$+ m_2(d_1^2+b^2) + m_3(d_2^2+c^2) + m_4(d_2^2+d^2$$

Next interchanging the positions of the brass & wood cylinders, the expt is repeated. If T_3 & T_4 are the periods,

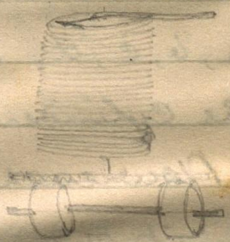
$$\frac{\mu}{4\pi^2} \cdot \frac{T_3^2+T_4^2}{2} = I + I_1 + I_2 + I_3 + I_4 + m_3(d_1^2+c^2)$$
$$+ m_4(d_1^2+d^2) + m_1(d_2^2+a^2) + m_2(d_2^2+b^2) -$$

ubtracting (4) from (3)

$$\frac{}{\pi^2}\left\{\frac{T_1^2+T_2^2}{2} - \frac{T_3^2+T_4^2}{2}\right\} = (m_3+m_4)(d_2 - d_1^2)$$
$$+ (m_1+m_2)(d_1^2 - d_2^2)$$
$$= (d_2^2 - d_1^2)(\overline{m_3+m_4} - \overline{m_1+m_2}).$$

Hence can be calculated μ v. i. n.

<u>pirals</u> We consider



ere a flat spiral ~~of~~ i.e.
ne in which the wire
s practically ⊥r to the
is of the spiral. If a
t is attached to the end of the spiral, by the
t. can be made to execute vertical as
ell as angular oscillations.

<u>ertical oscillations.</u>

It is the twisting, untwisting or twisting in the opposite direction of the wire that main-
ins the vertical oscillations. At any
nstant let the displacement of the
ci wt. from its equil. position be x.
t φ be the twist per cm of the wire
, the spiral. Hence the depression of the

~~At midpt~~ wt. due to one Cm of the spire

is $a\varphi$ a being ~~the~~ radius of the spiral.

Depression due to the whole spiral $= la\varphi$

(l = length of the wire) But $la\varphi = x$ $\therefore \varphi = \frac{x}{la}$

Couple acting $= Mga = \frac{1}{2}\pi n b^4 \varphi$

where n = rigidity modulus, & b = radius of the wire. $\therefore$ Work done $= \frac{1}{2} \times$ Couple $\times$ total twist $= \frac{1}{2} \cdot \frac{1}{2} \pi n b^4 \varphi \times l\varphi = \frac{1}{4} \pi n b^4 l \varphi^2$

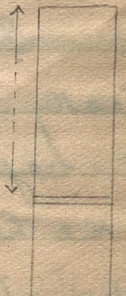
$= \frac{1}{4} \pi n b^4 l . \frac{x^2}{l^2 a^2} = \frac{\pi n b^4}{4 l a^2} x^2$

Hence P.E stored up in the wire

$= \frac{\pi n b^4}{4 l a^2} x^2.$

The kinetic energy is ~~p~~ partly in the moving ~~body~~ mass M & partly in the spiral

K.E. in the mass $= \frac{1}{2} M v^2$ v being the vel. at the instant under consideration.



Let L be the length of the Spiral, ~~this~~ m its mass. Consider a portion of the spiral, of thickness dy at epth y below the highest pt. Its vel $= \frac{y}{L}v$

s mass $=$ ~~α~~ $\frac{m}{L}dy$

$\therefore$ Its K.E $= \frac{1}{2}$ ~~$\frac{y^2}{L^2}$~~ $\frac{m\,dy}{L} \times \frac{y^2v^2}{L^2}$

$$= \frac{1}{2}\frac{mv^2}{L^3}y^2dy$$

K.E. of the whole Spiral

$$= \frac{1}{2}m\frac{v^2}{L^3}\int_0^L y^2dy = \frac{1}{2}\left(\frac{m}{3}\right)v^2$$

The sum of the P.E. & K.E. is a constant

e $\frac{\pi n b^4}{4la^2}x^2 + \frac{1}{2}\left(M+\frac{m}{3}\right)v^2 =$ Constant

Differentiating with respect to t (time)

$$\frac{\pi n b^4}{2la^2}x\frac{dx}{dt} + \left(M+\frac{m}{3}\right)v\frac{dv}{dt} = 0$$

$\therefore$ The linear accel$^{\underset{\cdot\cdot}{n}}$

$$\frac{dv}{dt} = -x\cdot\frac{\pi n b^4}{2la^2}\Big/ M+\frac{m}{3}$$

& $\therefore$ it is $\propto$ ~~&~~ the displacement directed towards the equil. position.

Hence motion is S H. & $\therefore$ period

$$T = 2\pi\sqrt{\frac{M+m/3}{\pi n b^4/2la^2}} = 2\pi\sqrt{\frac{a^2(M+\frac{m}{3})}{\mu}}$$

In practice neither M nor m are irectly determined. The period is found t before and after placing a wt m, the hanger attached to the spiral

T_1, T_2 be the periods,

$$T_2^2 - T_1^2 = 4\pi^2 . \frac{m_1}{\pi n b^4/2la^2}.$$

b & a are directly measured.

If t be the number of turns in the spiral

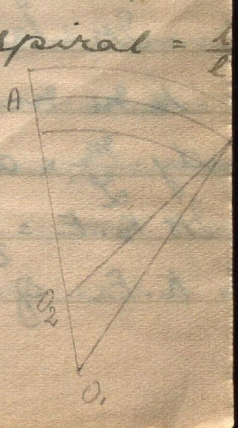
$l = t \times 2\pi a$.

n can thence be calculated.

Angular oscillations.

The rod attached to the spiral is deflected through a small ∠ and then let go. The spiral executes ∠lar oscillations the period of which can be determined as follows.

. Consider an instant when the deflect is θ. θ is likewise the increase (or decrease) ~~of~~ in the ∠ between the tangent drawn to the neutral surface at the top & bottom. Alteration in curvature per unit length of the wire in the spiral = $\frac{\theta}{l}$ where l is the length of the wire.



Let AB be unit length of the wire, O_1 ~~or~~ & O_2 the centres of curvature before and fter the strain. $\angle O,BO$ is evidently $\frac{\theta}{l}$

f it is also equal to the $\frac{e}{x}$ where e

the elongation of a fibre at unit

istance from the neutral axis ∴ $\frac{\theta}{l} = \frac{e}{x}$

e = $\frac{x\theta}{l}$. Now $q = \frac{f/s}{e}$ where f is

e force & s the area of section of the

ibre ∴ $f = qse = qs\frac{\theta}{l}x$

Bending moment $= \sum q\frac{\theta}{l} s x^2$

$= q\frac{AK^2}{l}\theta$

.E. of the ~~wir~~ spiral $= \frac{1}{2} q\frac{AK^2}{l}\theta \times \theta$

$= \frac{1}{2} qAK^2\frac{\theta^2}{l}$.

e K.E. is partly in the suspended weight

partly in the ~~move~~ spiral.

K. E. of the former $= \frac{1}{2} I\omega^2$

t dy be the section of a

piral of at depth y below the

. Its Lar vel. $= \omega \times \frac{y}{L}$

its ~~m~~ mom. of inertia

$dy . \frac{m}{L} \times a^2$

Its K.E. $= \frac{1}{2}\frac{ma^2\omega}{L^2} y\,dy$

. K.E. of the whole spiral $= \int \frac{1}{2}\frac{ma^2\omega}{L^2} y\,dy$

$= \frac{1}{2}\frac{m}{3}a^2\omega$

$\therefore \frac{1}{2} q\frac{Ak^2}{l}\theta^2 + \frac{1}{2}\omega\left(I + \frac{ma^2}{3}\right) =$ Consta

Proceeding in the usual manner, the period can be shown to be

$$2\pi\sqrt{\frac{I + ma^2/3}{qAk^2/l}}.$$

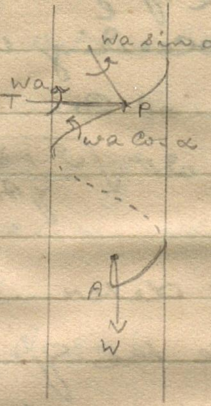
In practice I & m are not determined when it is reqd to Cal late ~~the~~ q. There are two moving masse each ~~w~~ of wt w on the horizontal bar. They are kept at distance δ_1 from the axis & the period T_1 is determined. Next they are moved to distance T_2 & period aga determined. $T_1^2 - T_2^2 = \frac{4\pi^2}{qAk^2/l}(d_1^2 - d_2^2)\,2w.$

Oblique Spiral

When the wt. acting along the axis of the cylinder on which the spiral is wound, moves vertically downwar the torsion alone of the wire is called into play if the ~~w~~ spiral be a flat o But if the spiral is oblique, i.e the wire is ~~bent~~ inclined at an ∠ α (not zero) to

orizontal, then bending as well as
orsion ~~to~~ are called into play.

Considering the equilibrium
f the portion PA of the
wire, the stresses at P
re equivalent to a couple
f moment Wa having as
xis PT the tangent (horiz-
ntal) to the cylinder at P.



his can be resolved into two :— ① one
a cos α, axis being along the wire PQ, tending
twist the spring, ② the other to Wa sin α
ving as axis PN, at right ∠s to the length
Q, tending to bend the spring.

If φ be the twist per unit length
we have $Wa\cos\alpha = \frac{1}{2}\pi n b^4 \varphi$

$$\therefore \varphi = \frac{Wa\cos\alpha}{\frac{1}{2}\pi n b^4}$$

If θ be change in the inclination be-
ween the tangents at pts P & Q ~~a unit distance a~~
ery near each other is :—

$$Wa\sin\alpha = qAk^2 \times \frac{\theta}{PQ}.$$

$$\therefore \theta = \frac{W a \sin\alpha}{g A k^2} . PQ.$$

Let us consider the effect of these changes on the radial arm~~s~~ which we imagine fixed to the spring. Due to torsion the relative motion at the end is $PQ\,\phi\,a$ & in consequence of the inclination of the spring this motion takes place at an angle α to the vertical $\therefore$ vertical component of the motion $= PQ\,\phi\,a \cos\alpha$

$$= \frac{W a^2 \cos^2\alpha}{\frac{1}{2}\pi n b^4} . PQ.$$

Due to bending the end of the radial arm moves through $\theta \times a$ & th motion is inclined at α to the horizont or $(90 - \theta)$ to the vertical $\therefore$ Component along the vertical $= \theta a \sin\alpha$

$$= \frac{W a^2 \sin^2\alpha}{g A k^2} . PQ.$$

$\therefore$ Total vertical displacement

$$= W a^2 \left(\frac{\sin^2\alpha}{g A k^2} + \frac{\cos^2\alpha}{\frac{1}{2}\pi n b^4}\right) PQ.$$

& $\therefore$ due to the whole wire in the spir vertical displacement $= W a^2 l . \Big(\frac{\sin^2\alpha}{g A k^2} + \frac{\cos^2\alpha}{\frac{1}{2}\pi n}$

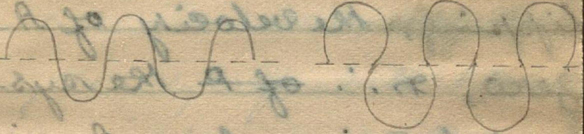
Elastic Curves or Elasticas.

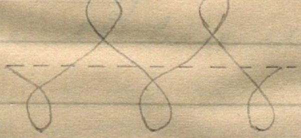
Let ACB repre-
ent a bow bent
y the tension T
long the string ANB. The portion CB is
n equilibrium under the stresses at C
the tension in the string. Hence the
tresses at C must be equivalent to a couple
× CN and a force T (CN is the ⊥r from C to
e line of action of the force in the string).
aking into consideration the couple alone,
e see that the radius of curvature ρ of
e neutral axis at C is given by

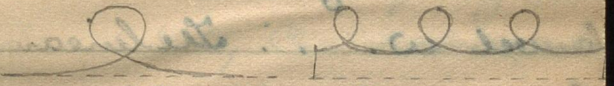
$$q\frac{Ak^2}{\rho} = T.\ CN$$ where q = Young's

odulus, Ak^2 = mom. of. in. of the cross-section
the rod about an axis through its centre at
t. ∠s to the plane of bending. Hence the
urve into which the bow bends is such
at the reciprocal of the radius of curvature
or in other words, the curvature) at any
t is ∝ to the distance of the pt. from the a

fixed straight line. Such curves are called elastic curves or elasticas.



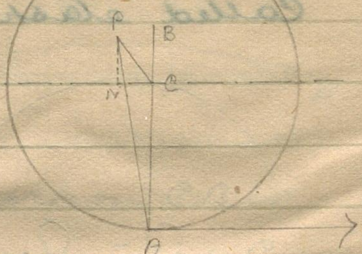




All the curves traced above belong to the group of elasticas. The last one ~~alone~~ where the pts on the curve are all very near the fixed line is of specia[l] importance. This curve can be shown to be the path traced by a pt. which is very near the centre of a circle when the ○ rolls without slipping, with a uniform ∠lar vel. (ω) a[long] a st-line.

Let C be the centre of the ○, P the pt. of instantaneous contact of the ○ with the ground, & P the corresponding position of a point very near the Centre.

ince the ⊙ ~~mo~~ rolls without lipping, the velocity of A zero, & ∴ ~~of P~~ the sys- is turning about A with larvel ω. ∴ The linear locity of P = ω × PA.



∴ The Centri~~fugal~~petal acceleration of P towards = $\frac{\omega^2 \times PA^2}{\rho}$ where ρ is the radius of cur- ature of the path of P at the pt. P.

Now the accel. of P towards A = sum of e components of the accel^{ns} of P to C & of to A resolved ||l to PA.

Accel. of P to C = $\omega^2 \times PC$

ts Component ||l to PA = $\omega^2 \times PC \times \cos \angle CPA$

ut in the limit P tends to C, ∠CPA = ∠PCB ∠CPN where PN = ⊥r from P to the line aced by the Centre C of the ⊙.

∴ Comp. of the accel. of P to C ||l to PA

$= \omega^2 . PC \times \frac{PN}{PC} = \omega^2 PN.$

ompon Acceleration of C to A = 0 since the path of C is a st. line.

$\therefore \omega^2 \times \frac{PA^2}{\rho} = \omega^2 \times x$ where $x = PN$

$\therefore \cancel{\omega^2} \frac{1}{\rho} = \frac{x}{a^2}$ where a = radius of the ⊙

& ∴ very nearly equal to PA.

Thus $\frac{1}{\rho}$ is $\propto$ to the distance of P from the St. line described by C.

The curve ABCD represents the path of the moving pt. P At positions, A, B C & D, P is at height a above the ground, & AB = BC = ... = πa. ~~B & C are~~ A, B... are pts of inflexi ie at these pts the curvature $\frac{d^2y}{dx^2}$ of the curve~~s~~ changes from +ive to −ive or vice-versa. At A', B' etc which are pts most removed from the axis of reference or path of the centre of the ⊙, the tangents drawn to the curves are ||l to the axis of refer

It is easily seen that the shap

ssumed by a loaded pillar must e that of an elastica if the load be uch as just to make the pillar end; for here $gAK^2 \times \frac{1}{\rho} = Wx$ (where is the radius of curvature at any t, W the load, & x the distance of the t. from the line of action of the load.

~~If the pillar is clamped~~

$$= \cancel{W \frac{x}{gA}} \quad \frac{W}{gAK^2} \cdot x$$

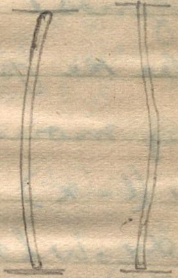
he Constant of ~~p~~Proportionality s $\frac{W}{gAK^2}$ which takes the place $\left(\frac{1}{a^2}\right)$ in the formula for the lastica.

Hence the Curve ~~trace~~ ~~traced~~ formed by the illar is that traced by a pt very near centre of a revolving ⊙, the radius a of hich is such that $\frac{1}{a^2} = \frac{W}{gAK^2}$ ie $a = \sqrt{\frac{gAK^2}{W}}$.

Let the ~~two~~ pillar be clamped at the ttom & let its length be such that it st begins to bend. The top must be a pt of inflection & the bottom a pt on the elastica ~~ma~~ that is most removed from the line of action of the force. The pill corresponds to a quarter of a complete undulation of the elastica & ∴ the length l_* of the pillar $= \frac{\pi}{2} a = \frac{\pi}{2} \div \sqrt{\frac{W}{qAk^2}}$.

ie ~~&~~ $l^2 = \frac{\pi^2}{4} \cdot \frac{qAk^2}{W}$

$$\therefore W = \frac{\pi^2 qAk^2}{4l^2}.$$



Next, if both ends are unclamped, (~~&~~ the load ~~is~~ being W_1), the pillar comprises one half of a complete undulation & $l = \pi a = \pi\sqrt{\frac{qAk^2}{W_1}}$ ie $W_1 = \frac{\pi^2 qAk^2}{l^2}$.

In the third place, if both ends are clamped, the ~~length~~ load is $\frac{4\pi^2 qAk^2}{l^2}$.

The bending of beams

The depression caused any pt on a beam can be investigated as follows.

The curvature at any pt. on a cu

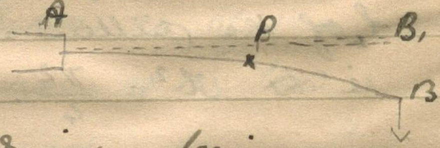
s given by $\dfrac{d^2y/dx^2}{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}$ In the Case of bending

f beams, $\frac{dy}{dx}$ is practically negligible

ompared ~~to the~~ ~~one~~ 1 & ∴ the Curvature

an be taken to be $\frac{d^2y}{dx^2}$.

Cantilever, clamped at A, loaded at B with



t W, the axes of coordinates being

e horizontal & vertical through A.

Let P be the pt (x, y)

Bending moment $= gAK^2 \frac{d^2y}{dx^2}$

$= W(l - x)$ where l = length AB.

$AK^2 \frac{dy}{dx} = Wlx - W\frac{x^2}{2} + C_1$

$gAK^2 y = Wl\frac{x^2}{2} - W\frac{x^3}{6} + C_1x + C_2$

When $x = 0$, $dy/dx = 0$ ie $C_1 = 0$

& also $y = 0$ ie $C_2 = 0$

∴ ~~g~~ The sag at any pt distant x from A

given by $\frac{W}{gAK^2}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$

∴ at the end of the bar where

$= \frac{l}{\cancel{1}}$, the sag is $\frac{W}{gAK^2} \cdot \left(\frac{l^3}{2} - \frac{l^3}{2 \times 6}\right) = \frac{Wl^3}{3gAK^2}$.

2) Beam ~~loaded~~ placed on two supports and loa
in the middle.

$$qAk^2 \frac{d^2y}{dx^2} = \frac{W}{2}x$$

$$\therefore qAk^2 \frac{dy}{dx} = \frac{Wx^2}{4} + C_1$$

$$\& \; qAk^2 \, y = \frac{Wx^3}{12} + C_1 x + C_2$$

When $x = \frac{l}{2}$, $dy/dx = 0$ ie $0 = \frac{Wl^2}{16} + C_1$

$$\therefore C_1 = -\frac{Wl^2}{16}$$

When $x = 0$, $y = 0$ $\therefore C_2 = 0$.

$$\therefore y = \frac{W}{qAk^2}\frac{x^3}{12} - \frac{Wl^2}{16qAk^2}x$$

$$= \frac{Wx^{\cancel{4}}}{4qAk^2}\left(\frac{x^2}{3} - \frac{l^2}{4}\right)$$

Hence depression at the midpt is given by $y = \frac{Wl}{8qAk^2}\left(\frac{l^2}{12} - \frac{l^2}{4}\right) = \frac{-Wl^3}{48qAk^2}$

the -ive sign shows that y is the direction opposite to that of $\left(\frac{W}{2}\right)$.

3) Beam clamped at both ends.

If the clamps are replaced by knife edges the ends would be inclined to the horizontal at a certain ∠. ∴ The effect of the clamps is to make the ends horizontal, i.e. to exert a couple opposite in direction to that caused by the load. Let this be quivalent to a uniform Couple of moment

$$\therefore\ gAk^2\frac{d^2y}{dx^2} = \frac{w}{2}x - \varphi$$
$$\therefore\ gAk^2\ \frac{dy}{dx} = \frac{w}{4}x^2 - \varphi x + C_1$$
$$gAk^2\ \ y = \frac{w}{12}x^3 - \frac{\varphi}{2}x^2 + C_1 x + C_2$$

When $x = 0,\ y = 0 \quad \therefore\ C_2 = 0$

When $x = 0,\ dy/dx = 0 \quad \therefore\ C_1 = 0.$

When $x = \frac{l}{2},\ dy/dx = 0 \quad \therefore\ \frac{w}{16}l^2 - \frac{\varphi l}{2} = 0$

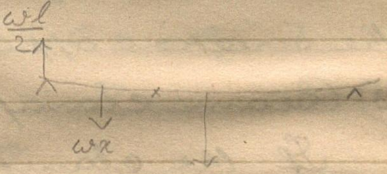
ie $\varphi = \frac{wl}{8}$

$$\therefore\ \ gAk^2 y = \frac{w}{12}x^3 - \frac{wl}{16}x^2$$

∴ Sag at the midpt is given by

$$y = \frac{w}{4gAk^2}\frac{l^2}{4}\left(\frac{l}{6} - \frac{l}{4}\right) = \frac{wl^3}{192gAk^2}.$$

Beam bending under its own wt — Supports on two Knife-edges.



~~Thrust~~ Thrust on the nife edges is $\frac{wl}{2}$ where is wt. per unit length of the beam.

$$gAk^2\frac{d^2y}{dx^2} = \frac{wl}{2}x - wx.\frac{x}{2}$$
$$= \frac{w}{2}(lx - x^2)$$
$$gAk^2\frac{dy}{dx} = \frac{w}{2}\left(l\frac{x^2}{2} - \frac{x^3}{3}\right) + C_1$$
$$gAk^2\ \ y = \frac{w}{2}\left(l\frac{x^3}{6} - \frac{x^4}{12}\right) + C_1 x + C_2.$$

When $x=0$, $y=0$ ∴ $C_2=0$

When $x=\frac{l}{2}$, $\frac{dy}{dx}=0$

$$\therefore \frac{w}{2}\cdot\left(\frac{l^3}{8}-\frac{l^3}{24}\right)+C_1=0$$

ie $$-\frac{w}{\cancel{2}}\times\frac{l^3}{8\times3}=C_1$$

$$C_1=-\frac{wl^3}{24}$$

$$\therefore qAk^2y=\frac{w}{12}\left(lx^3-\frac{x^4}{2}\right)-\frac{wl^3}{24}x$$

When $x=\frac{l}{2}$, the sagg is given by

$$y=\frac{1}{qAk^2}\frac{w}{12}\cdot\left(\frac{l^4}{8}-\frac{l^4}{32}\right)-\frac{wl^4}{48}$$

$$=\frac{-1}{qAk^2}\frac{wl^4}{12\times8}\left(1-\frac{1}{4}-\frac{1}{6}2\right)$$

$$=\frac{5wl^4}{384qAk^2}.$$

In a similar manner it can be shown that if the rod be clam at both ends, the sag is $\frac{wl^4}{384qAk^2}$.

Young's Modulus ~ adiabatic (Saha 4

It is a ~~an~~ ~~general~~ universal fact that thermal changes which take place when ~~we~~ any alteration in the Stat of a body is caused are such as to oppose the change. Hence suppose the strain of a body is increased, and the nature of the body is such that it

o stiffer at ~~&~~ a higher temperature
an at a lower one. The body ∴ absorbs
eat from outside in order to resist
change & rises in temperature ~~If the change is so quick that~~
~~ere is no time to absorb heat from outside~~
~~t takes thermal energy from itself &~~
~~cools.~~

The thermal effects due to a change
l strain can be calculated from
hermodynamical considerations.
et us suppose there are two chambers
t o maintained at absolute temperatures
& T_0 $(T_1 > T_0)$. In the cool chamber
o placed a stretched wire under stress
Let its elongation e be increased by
e. Work done on the wire = force × distance
Pa × lde (a = area of section of
e wire, l = ~~thickness~~ length)
Now transfer the wire with its
ength unaltered into the hot chamber.
e shall suppose that the thermal

Capacity of the wire to be so small tha
no work is done in heating up the wi
If the stiffness of the wire changes
with temperature the new tension P
reqd to keep the wire stretched will not
be the same as P. Let the wire
contract in the hot chamber until
its elongation diminishes by de.
Work done by the wire = P'a l de.

Now transfer the wire with length
unaltered into the cold chamber; it
will be in the same state as before.
The work done by the wire exceeds tha
done on it by (P'–P)a l de. Hence the
arrangement constitutes a heat
engine. The ~~arg~~ process is evidently
reversible.

Hence it must obey the laws of
reversible engines. These engines
work by absorbing heat dH from
the hot chamber and giving heat dh

the Cold Chamber

s. $\frac{dH}{T_1} = \frac{dh}{T_0} = \frac{(P'-P)\,a\,l\,de}{T_1 - T_0}$

Since $dH - dh$ is the net amount f work done.

Now $\frac{P'-P}{T_1-T_0}$ is the rate of change of stress with temperature when elongation Constant & ∴ can be written as

$\left(\frac{dP}{dT}\right)_e$. $\therefore dh = T_0 \left(\frac{dP}{dT}\right)_e a\,l\,de.$

If instead of giving this heat to the chamber the wire were to absorb it, its temperature would ~~rise~~ ~~Change~~ rise by say $d\theta$

where $dh = a l \rho \times s \times J \times d\theta$

(where ρ = density, s = sp. heat, J = Joule's equivalent)

Hence $d\theta = T_0 \left(\frac{dP}{dT}\right)_e de \,/\, J\rho s$ -- (1)

This expression proves the statement made at first that the change of temp is such as to resist further strain.

If $\frac{dP}{dT}$ is +ive $d\theta$ is +ive ie if the body is stiffer at a higher temperature, it rises in temp. due to increase of strain

& thus resists further strain.

If $\frac{dP}{dT}$ is −ive, the body Cools. For in this Case the body is stiffer at a lowe temperature & ∴ Cooling & not heating can offer resistance to further strain.

The expression for $d\theta$ can be thro into a more convenient form as follows. If the body is allowed to expand under Constant tension, increa in elongation due to a rise dT in temp $= \omega dT$ (ω being coeff^t of linear exp^n) Now diminish the tension keeping temp. Constant, until shor ing due to tension ~~of ωdT~~ just Compensates for lengthening due to rise in temperature. The necessary diminutio in tension is $q\omega dT$ where (q = young's mo ulus) ∴ $dP = -q\omega dT$

$$\therefore d\theta = -\frac{T_0 q \omega de}{J\rho s}$$

Now $q\,de$ is the additional tension

P req^d to cause the elongation de.

$\therefore d\theta = -\frac{T_0 w\, dP}{J\rho s}$ . . . . . (2)

Dr. Joule~~s~~ tested the correctness
f this formula experimentally. The
hanges in temperature were measured
y thermoelectric couples inserted in the ~~bars~~.
ars on which he experimented.

A qualitative expt can be performed
y suddenly stretching a band of India
ubber & holding it against the ~~ch~~ lips.
t is found warmer. $d\theta$ in this
ase is +ive because w is −ive. In
ll other elastic substances increase
stress causes a cooling effect.

If the heat dh is not allowed to
escape the elongation caused is
artly due to the increase in stress.
partly due to the change of temperature.
ue to stress ~~is~~ it is $\frac{dP}{q_i}$ where q_i is
e isothermal young's mod ~~elasticity~~. Due to temp.
is $w\,d\theta$. Hence the net strain

which is $\frac{dp}{q_a}$ (q_a being adiabatic ~~ela~~ youngs mod.) $= \frac{dp}{q_i} + \omega\, d\theta$

$$\therefore \frac{1}{q_a} = \frac{1}{q_i} + \omega \frac{d\theta}{dp}$$

$$= \frac{1}{q_i} - \frac{T_0 \omega^2}{J \rho s}$$

$\therefore \frac{1}{q_a} < \frac{1}{q_i}$ (even in the case of rubber where ω is $-$ive)

& $\therefore q_a > q_i$

∥ When q_i is known at any temp. q_a can be calculated. It is seen that in the case of metals q_a is nearl 1% greater than q_i.

The value of q_a can also be experimentally determined, for in al acoustic experiments the modulus involved is the adiabatic modulus. Such experiments however show that q_a is ~~a~~ nearly 20% greater than q_i. This ~~ex~~ discrepancy has not yet been ~~ac~~ satisfactorily accoun for.

(Notes taken from Poynting & Thoms

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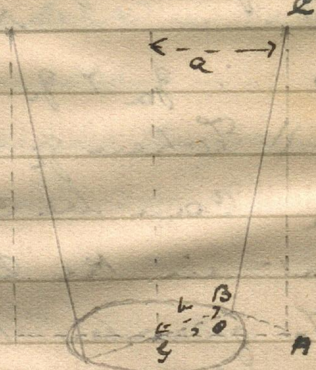
Bifilar suspension

The body is suspended by two
trings equally long, equally inclined
the vertical, such that the body is
ymmetrical about a vertical axis through
Centre of gravity.



If I is the moment
f inertia about this
ert. axis, & C the
estoring couple per unit
lar displacement, the
eriod is $T = 2\pi\sqrt{\frac{I}{C}}$.
C is evaluated as follows.
When the deflection is θ, couple = $C\theta$;
ork done for a further deflection $d\theta = C\theta d\theta$.
y is the depth of the ~~centre of gravity G~~ pts of attachment of the body to the string below
level of suspension, m the wt. of the
dy, work done = $mg\,dy$
$\therefore mg\,dy = C\theta d\theta$.
et 2a be distance between the strings at
e top & 2b at the bottom.

The figure gives a ~~ax~~ perspective view when the deflection = θ = $\angle BGA$.
CA is ⊥r from ~~the~~ C to the line joining the pts of attachment when the body is at rest.
G is the midpt of the pts of attachment.
$\angle CAB$ is evidently a rt. ∠, & CA = y
CB = l (the length of the string)

$$y^2 = l^2 - AB^2 = l^2 - (a^2 + b^2 - 2ab\cos\theta)$$

$$\therefore y = (l^2 - a^2 - b^2 + 2ab\cos\theta)^{1/2}$$

$$\therefore \frac{dy}{d\theta} = \frac{1}{2(l^2 - a^2 - b^2 + 2ab\cos\theta)^{1/2}} \times -2ab\sin\theta$$

$$= \frac{-ab\sin\theta}{l\left(1 - \frac{a^2 + b^2 - 2ab\cos\theta}{l^2}\right)^{1/2}}$$

Since usually a & b are very small compared to l, $\frac{a^2}{l^2}$ etc can be neglected
Also θ being small $\sin\theta = \theta$

$$\therefore \frac{dy}{d\theta} = \frac{-ab\theta}{l}$$

$$\therefore C = \frac{mg}{\theta}\frac{dy}{d\theta} = \frac{-mg.ab}{l}$$

(The −ive sign in the value of $\frac{dy}{d\theta}$ shows that as θ increases, y decreases)

$$\therefore T = 2\pi\sqrt{\frac{Il}{mgab}}$$

Surface Tension

Forces that govern the ~~size~~ shape of a drop

If the drop is spherical, of radius r
ts potential energy due to gravity = mgr.
due to surface tension = $4\pi r^2 T$.
The total energy = $mgr + 4\pi r^2 T$
hen the drop flattens out the
irst term diminishes, but the 2nd
ncreases. Hence an intermediate shape
taken up where the sum of the two
rms is a minimum.

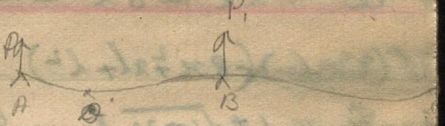
The ∠ of contact is the ∠ between the tangent to the
quid surface at a pt where three
edia solid, air & the given liquid meet
gether & the interface of liquid & solid.

Problem :- A bar of uniform cross-section
supported on ~~two~~ three knifeedges ~~two~~ on the
me horizontal level, two at the ends &
e in the middle. Show that the pts of
aximum sag are nearly $\frac{1}{5}$ of the length of the
od from each end

Let P be the thrust

at A, (x, y) ~~the~~ a

pt D between A & B



axes being the horizontal & vertical throu

A.

$$gAk^2 \frac{d^2y}{dx^2} = Px - m\frac{x^2}{2}$$

$$\therefore gAk^2 \frac{dy}{dx} = P\frac{x^2}{2} - m\frac{x^3}{6} + C_1$$

$$gAk^2 \; y = P\frac{x^3}{6} - m\frac{x^4}{24} + C_1x + C_2.$$

When $x = 0$, $y = 0$ $\therefore C_2 = 0$

When $x = l$, (2l being total length)

y & $\frac{dy}{dx}$ are zero

$$\therefore \quad P\frac{l^2}{2} - m\frac{l^3}{6} + C_1 = 0 \quad \text{---} (1)$$

$$P\frac{l^3}{6} - m\frac{l^4}{24} + C_1 l = 0$$

$$\text{ie } P\frac{l^2}{6} - m\frac{l^3}{24} + C_1 = 0 \quad \text{--} (2)$$

$$\therefore P\frac{l^2}{2} - m\frac{l^3}{6} = P\frac{l^2}{6} - m\frac{l^3}{24}$$

$$\text{ie } \quad P = \frac{3.ml}{8}$$

Substituting in (1) $l^2\left(\frac{ml}{6} - \frac{3}{16}ml\right) = C_1$

$$C_1 = -\frac{ml^3}{48}.$$

$$\therefore gAk^2\frac{dy}{dx} = \frac{3}{8}ml\frac{x^2}{2} - m\frac{x^3}{6} - \frac{ml}{48}$$

Where sag is maximum $\frac{dy}{dx} = 0$

$$\therefore \frac{3}{16}mlx^2 - \frac{1}{6}mx^3 - \frac{1}{48}ml^3 = 0$$

ie $9lx^2 - 8x^3 - l^3 = 0$

ie $(x-l)(-8x^2 + xl + l^2) = 0$ ie $8x^2 - xl - l^2 = 0$

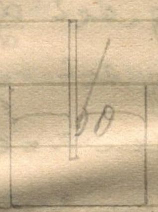
ie $x = \frac{1 \pm \sqrt{1+32}}{2 \times 8} l = \frac{6.745}{16} \times l$

$x \qquad = \frac{2}{5} l$ nearly

Hence the pts of maximum depression

re nearly $\frac{1}{5}$ from either end.

Angle of Contact



If a glass plate is dipped

to a basin of mercury, the

ercury ~~level~~ ~~goes~~ surface bends downwards

meets the glass. When the glass plate is

lowly rotated from the vertical, a position

reached when this bending ceases. If φ be

∠ of rotation, $(180° - φ)$ is the ∠ of contact.

As however it is not easy to measure

rotation, a different method is employed.

A spherical glass bulb B

ith two orifices has its

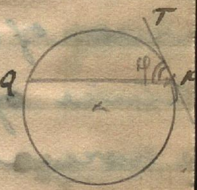
wer orifice connected

a cylindrical tube through

capillary. The mercury

is poured into the ~~C~~ wide tube, & as it ~~p~~passes slowly through the capillary into the bulb. At first the bent on the bound of the mercury surface is very well mark Slowly it lessens, & a limit is reached wh the surface is perfectly horizontal. ~~The~~ If some printed matter is held above the mercury, the disappearance of distortion i the image will show clearly when this ~~level~~ limit is reached. The flow of mercury is stopped & a trace of the bulb with the mercury level is taken on paper

A tangent PT is drawn at the pt of contact of the mercury level PQ with the bulb.



$180° - Q\hat{P}T$ gives the ∠ of contact.

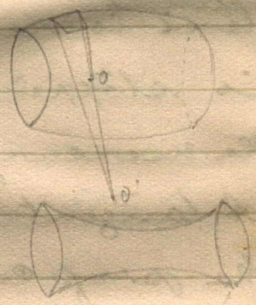
The wide tube is replaced by a capil lary & from the consequent depressio the surface tension can be calculated, assuming the value for ∠ of contact.

For the experiment tobe successful

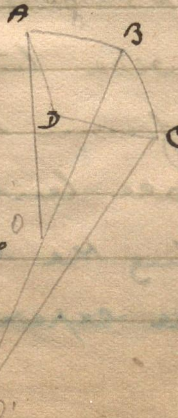
o essential that the mercury is pure.
f impure, it will have to be cleaned by
ixing it with nitric acid & heating the
ixture, or again by filtering it several times.

Mercury alone has an $\angle$ of contact
above $\frac{\pi}{2}$. Several liquids including water
et glass & so the $\angle$ of contact is zero.

Excess of pressure inside bubbles.



very small rectangle
BCD is taken on the
urface, AB, BC being
ies at elements of the
urves of principal cur-
ature of the surface.
e normals to the sur-
aces are drawn at
, B & C. Those at A & B
eet in O & those at B & C
eet in O'. BO & BO' are
alled the principal radii of
urvature of the surface.



If O & O′ are on the same side of the Surface as in a sphere or ellipsiod, the surface is called synclastic (συν-κλαω: break) If O & O′ are on opposite sides the Surface is anticlastic. The former class includes ~~the~~ spheres & ellipsoids & the latter is shaped like a saddle or dice-box. In practice anticlastic bubbles are formed by drawing apart two funnels whose mouths are originally together.

To find the excess of pressure in the surface, we employ the principle of virtual work, ie when a ~~system~~ body is in equilibrium under the action of a system of forces, if a slight displacement is given to the body, the algebraic sum of the work done by the various forces is zero.

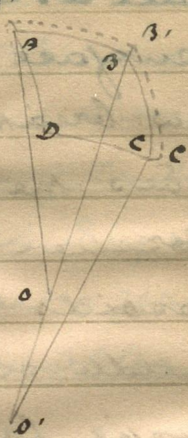
The film ABCD is displaced through a small distance x ~~which is x~~ normal to itself, e sides AB, BC now

ccupying the positions

'B', B'C'.

'B' = AB $(1 + \frac{x}{r_1})$ where r_1 = BO

B' = BC $(1 + \frac{x}{r_2})$ where r_2 = BO'



The new area of the film.

= S $(1+\frac{x}{r_1})(1+\frac{x}{r_2})$ S being ~~old a~~

itial area

S $(1 + x\,\overline{\frac{1}{r_1} + \frac{1}{r_2}})$ neglecting

e ~~of~~ product $\frac{x^2}{r_1 r_2}$ as too small

∴ Work done ~~in~~ ~~by~~ against surface

nsion = surface energy × increase in area

= $T . S . x . \overline{\frac{1}{r_1} + \frac{1}{r_2}}$ (T being surface tension)

his is ~~m~~ equal to the work done by

e excess of Pressure P, ie by ~~Pa~~ by

force PS acting through ~~an~~ a distance x

ie $P \times S \times x$

∴ $P = T . (\frac{1}{r_1} + \frac{1}{r_2})$

In the case of a bubble where there

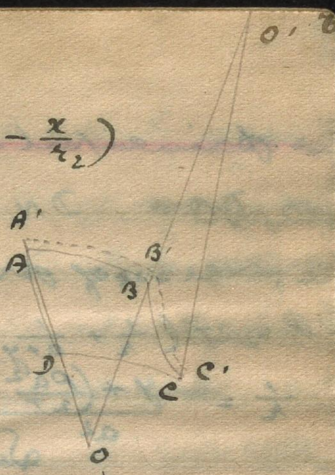
e two surfaces $P = 2T (\frac{1}{r_1} + \frac{1}{r_2})$

In an anticlastic drop the proof is

Similar. Only $B'C' = BC(1 - \frac{x}{r_2})$

& ∴ we get

$$P = T\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$



Stability of films that approximate to the Cylindrical shape.

Let the fig. represent the trace of ~~a~~ the film, CDE being one of the principal lines, CFE the position this line would have occupi if the film were perfectly cylindrical, AB the axis of the cylinder, PQ a line ||l & very close to CFE.

We shall prove that CDE is an ela ca, in other words, that the curvature at any point it is ∝ to the distance of the pt from PQ.

Consider the curvature ~~k~~ at D.

Drop DFN ⊥r to AB.

he principal radii of curvature at D are

& DN DN = $a+y$ where $a = NF$ & $y = FD$.

e excess of pres. within the bubble is

$P = 2T\left(\frac{1}{r} + \frac{1}{a+y}\right)$ ie $\frac{1}{r} = \frac{P}{2T} \mp \frac{1}{a+y} = \frac{P}{2T} - \frac{1}{a}\left(1 - \frac{y}{a}\right)$

é $\frac{1}{r} = \frac{y + \left(\frac{Pa^2}{2T} - a\right)}{a^2}$

~~Since~~ If the film had been rfectly cylindrical, excess of press. P_1 ould have been $\frac{2T}{a}$ ∴ $\frac{P_1 a^2}{2T} - a^2 = 0$ ince P_1 is very nearly equal to P, $\left(\frac{Pa^2}{2T} - a\right)$ almost zero, or is a small quantity

∴ The ~~radius~~ curvature at any pt is ery nearly ∝ to the distance of the pt from FE or more accurately is ∝ to the distance of the pt from PQ.

Hence the ~~tw~~ principal lines taken long the axis are elasticas, & since constant of proportionality is a^2, the dius of the generating ⊙ is a, & ∴ the points of inflection of the curve are distance of πa from each other.

We shall now consider four cases

of nearly cylindrical films. In each Case ~~sign~~ let P be the excess of pres. & P' what the excess would have been if the film were perfectly cylindrical.

~~1) length < πa & [illegible] synclastic~~

Let AB be the axis of the Cylinder, CD the principal line which is an elastica, EF the axis of ~~[illegible]~~ the elastica, which is at distance a from AB, & let the length EF be exactly πa. Let DN be ⊥r on AB from

1) CD < ~~λ~~ πa & the ~~bubble~~ bulging out

To calculate P we shall suppose ~~the~~ C to extend to F. F is a pt of inflection for the curve & ∴ curvature is zero.

$\therefore P = 2T\left(\frac{1}{a}+0\right) . \frac{2T}{a}$

If the ~~curve~~ bubble ~~[illegible]~~ were perfectly cylindrical, CD would have been a straight line & ∴ $P' = 2T\left(\frac{1}{DN}+0\right) = \frac{2T}{DN}$

Since DN > a, $\underline{\underline{P > P'}}$

2) CD < πa, & Concave

$P = \frac{2T}{a}$; $P' = \frac{2T}{DN}$

$DN < a$ ∴ ~~$P > P'$~~ $\underline{\underline{P < P'}}$

) $CD > \pi a$, bulging out

Here $DN < \cancel{>} a$

∴ $P < P'$

) $CD > \pi a$, Concave

Here $DN > a$

∴ $P > P'$

Hence when length is less than πa the xcess of press is greatest when ~~it~~ the bubble is ynclastic, less when cylindrical & east when anticlastic. In this case a synclastic bubble is ~~flow~~ blown & e air is allowed to escape through an ifice, it becomes cylindrical, then ticlastic & finally Collapses.

On the contrary if the length of the bble is greater than πa, the allowing the air to get out makes it bulge out ore & then Collapse. If on the other and ~~if~~ air is blown into it, it becomes

Cylindrical, then anticlastic & finally collapses. The figure shows how the foregoing results can be experimentally verified, by forming a bubble between two funnels.

Fig 1

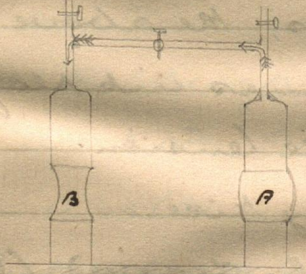


Fig 2

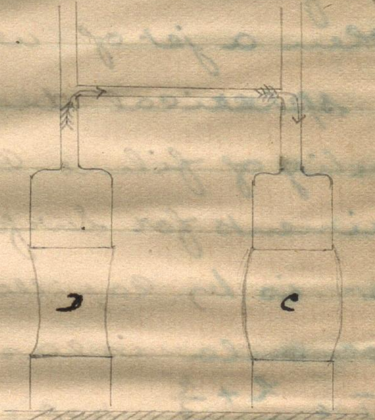


Fig 3

In fig (2) the distance between the upper & lower pairs of funnels is less th half the $\bigcirc^{ce}$. & ~~Te~~ the excess of press in the film that bulges out is greater that in the other & so when the two are put in communication air flows from A to B & both tend to become cylindrical. In fig (3) where $l > \pi a$ the excess of press in the fil

ubble that bends in is greater & ∴ when
e two are connected air flows from D to C
ereby giving further swelling to ~~D~~ C & greater
ending in to D. ~~It~~ The ~~fre~~ bubbles Collapse.
ens it is seen that ~~films~~ bubbles of length greater
an ~~half~~ the $\odot^{ce}$ is are ~~ti~~unstable [cf Newman & ~~Starke~~ Searle Page 153]

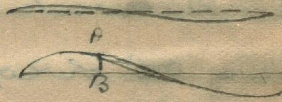
When a jet of water breaks up into
mall spherical drops, the above factor
stability of films comes into play.

ⓐ experiments for surface tension.
he best is by capillary rise.
The formula is easily derived (cf B. A.
otes) $T = \frac{h + \frac{r}{3}}{2 \cos\theta} . r g \rho$

Strictly speaking the meniscus is
araboloidal, not spherical & ∴ the
ien $\frac{r}{3}$ is only a first approximation.
is the radius of that portion where
e meniscus is formed, hence it should
measured by a ~~column~~ small column
mercury introduced at that spot, or else
breaking the capillary there.

n &
Pgl 153

Theory of ripples and waves.



Let A be any pt on the surface of the wave & AB the ⊥r dropped from A to the line joining the undisturbed pts on the surface. The wave considered is for convenience sake a plane wave, i.e. one i which the crests and troughs are ∥l st. lines, not ○les as ~~when~~ are form when a stone is dropped on the surface of water. Waves at a great distance from the pt of disturbance are practically plane waves. The excess of pressure at B is due partly to the curvature of the film, and partly to the height o the water column above B. Due to the height of the water column press = $AB \times g\rho$. Due to curvature of the film, press = $T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ Now

in the Case of the plane wave one of
he Curvatures is ~~tw~~ zero, & the other
the Curvature of the line in the
lane of propagation (say $\frac{1}{r}$)
$\therefore$ Total press at $B = AB \times \rho g + \frac{T}{r}$.
Since the wave surface is an elastica
$\frac{1}{r} = \frac{AB}{a^2}$ where a is the radius of
e circle but by the rotation of which
is curve is generated. If λ is the
avelength $2\pi a = \lambda$ & $\therefore \frac{1}{a^2} = \frac{4\pi^2}{\lambda^2}$.
Excess of press at $P = AB \times \rho g + T \times AB \times \frac{4\pi^2}{\lambda^2}$

$$= AB\rho\left(g + \frac{4T\pi^2}{\rho\lambda^2}\right)$$

Hence we see that the action of the
urface tension is effectively to increase
e gravity. Stokes has proved that when
ave disturbances are propagated
gravity alone, their velocity of
ropagation is the same as that acquired
a body falling through $\frac{\lambda}{4\pi}$ controlled
gravity. If v is the velocity, $v^2 = 2g\frac{\lambda}{4\pi}$
$\frac{g\lambda}{2\pi}$.

In this case we have instead of g the quantity $g + \frac{4\pi^2 T}{\rho\lambda^2}$.

Hence the velocity of propagatio is such that $v^2 = \frac{\lambda}{2\pi}\left(g + \frac{4\pi^2 T}{\rho\lambda^2}\right)$

$$= \frac{\lambda g}{2\pi} + \frac{2\pi T}{\rho\lambda}$$

In the case of waves where λ i very small, the second term is the controlling factor, the other one being negligible. The ten surface-tension alone determines the velocity of propagation of the wave. Such waves are called ripples.

If λ is very great, the second term is negligible, gravity alone is the controlling factor, & then the wa is called a gravity wave.

Now the product of the two ter $\frac{\lambda g}{2\pi}$ & $\frac{2\pi T}{\rho\lambda}$ is $\frac{gT}{\rho}$ which is a constant whatever be the wave length. Now the sum of two terms whose product is a constant is a minimum when e two terms are equal ∴ v^2 is a inimum when $\frac{\lambda g}{2\pi} = \frac{2\pi T}{\lambda\rho}$. ie r the minimum value of v,

$^2 = \cancel{\frac{4\pi T}{\lambda\rho}} = \cancel{\lambda g}$ Since $\frac{\lambda g}{2\pi} = \frac{2\pi T}{\lambda\rho}$

$^2 = \frac{4\pi^2}{\rho g} T$ & $T = \frac{\lambda^2 \rho g}{4\pi^2}$.

$v^2 = \frac{\lambda g}{2\pi} + \frac{2\pi T}{\lambda\rho} = \cancel{2}\frac{\lambda g}{\pi}$ ∴ $v^4 = \frac{\lambda^2 g^2}{\pi^2}$

ie $\lambda^2 = \frac{v^4 \pi^2}{g^2}$

∴ $\frac{v^4 \pi^2}{g^2} = \frac{4\pi^2 T}{g\rho}$

∴ $v^4 = \frac{4Tg}{\rho}$

~~which~~ In the case of water, the least velocity of propagation f waves is $\sqrt[4]{4 \times 72 \times 981} = \underline{23.05}$ cms/sec.

The formula $v^2 = \frac{\lambda g}{2\pi} + \frac{2\pi T}{\lambda\rho}$ an be utilized for the experimental etermination of T of a liquid like mercury. ercury is taken in a wide basin d it is waves are continuously roduced in it by a pin which is ttached to one of the prongs of an lectrically maintained tuning fork. n is the frequency of the tuning fork, $v = n\lambda$

$$\therefore n^2 \lambda^2 = \frac{\lambda g}{2\pi} + \frac{2\pi T}{\lambda \rho}$$

All the quantities are known except λ. λ is observed as follows Any pt A on the surface of the mercury is a crest turn n times a second. If an intermittent illumination is caused on the surface n times a second, due to the persistance of vision it will appear as if the mercury surface is formed into a permanent wave. At the crests bright illumination is seen due to reflection (A liquid other than mercury will not show this effect with sufficient clearness) By means of a travelling microscope the distance between successive crests can be measured The intermittent illumination may be caused in either of the foll

ays. 1) A tuning fork with ~~tw~~ a per-
forated thin lamina attached to each
rong allows a spot of light to pass
rough these perforations $2n$ times a
econd. Thus illumination is caused
ot when A is a trough & when A is a
rest. 2) A phonic wheel ~~to~~ i.e. one
aving a number of holes ~~attat~~ about
ear its rim is so adjusted as to
otate with the same frequency ~~as~~
t n/m where m is the no. of holes.
f a light is placed above the wheel,
llumination is caused n times a sec.

scillations of a drop.

We consider the case of drop that
s not controlled by gravity, for instance
drop of water placed in ~~an~~ ~~enclosu~~
ixture of oils having density one.
e factors on which t (the time of osc.) ~~density~~ depends
re only surface tension T, radius a,
density ρ. Let $t \propto T^x a^y \rho^z$

From the theory of dimensions, the dimensions on both sides must be identical in length l, time t & mass m

$$\therefore t^{1} \times l^{0} \times m^{0} = \left(\frac{m \times l}{t^{2} \times l}\right)^{x} \times l^{y} \times \left(\frac{m}{l^{3}}\right)^{z}$$

$$= t^{-2x} \times l^{y-3z} \times m^{x+z}$$

$\therefore -2x = 1 \;\therefore x = -\frac{1}{2}$; $x + z = 0 \;\therefore z = \frac{1}{2}$;

$y - 3z = 0 \;\therefore y = \frac{3}{2}$.

$$\therefore t \propto \sqrt{\frac{a^{3}\rho}{T}}$$

The constant of proportionality can be ~~term~~ determined by experimenting on a liquid of known S. T. The constan being known, the T for any other liqui can be determined.

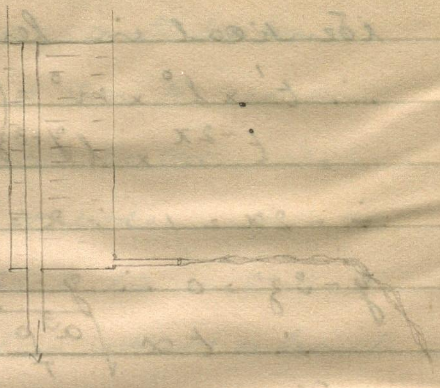
Lord Rayleigh's expts shows that the constant is $\frac{\pi}{\sqrt{2}}$

$$\therefore t = 2\pi\sqrt{\frac{a^{3}\rho}{8T}}.$$

In practice it is not necessary to eliminate the action of gravity by a liquid of the same sp. gravity.

A tube of a radius a is flattened out into an elliptical shape, and ater is made to flow out from it under

Constant pressure head.



r a certain length

e ~~of~~ ~~Clo~~ Column is

orizontal, thereby

howing that the

ction of gravity is

on-existent. In this

a length, the variations in the shape

re controlled by a, density ρ & surface

nsion T. The period of oscillation

the time taken to ~~rav~~ travel the

istance between two crests. If λ is this

istance, since velocity is ~~g~~ $\sqrt{2gh}$,

$= \frac{\lambda}{\sqrt{2gh}} = 2\pi\sqrt{\frac{a^3\rho}{8T}}$ according to the

revious formula.

urface Tension by Jaegar's Method.

f Record Book

The ~~p~~ excess of pressure within

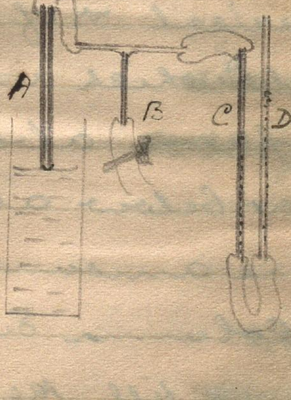
$hg\rho + \frac{2T}{r} = h'g\rho'$ where h' ~~& ρ' are~~

~~heigh~~ difference in manometer levels &

ρ' = density of the liquid in the manomet

Now ~~the~~ at the moment of detaching the pressure is slightly greater than $hg\rho + \frac{2T}{r}$ by a quantity which is necessa to cause the escape of the bubble. Hence the eqn written above is not quite correc Also it is tobe noted that the method isnot applicable for finding the variation in surface tension over a small range of say 10 or 20 degrees. The error committed in reading h & h' ~~are~~ is ordinarily greater than ~~that~~ the variation tobe determined.

Modification of Jaegar's method



A is a capillary which is connected to a capil ~~manometer~~ side tube C and to a rubber tubing B which communicates with the outside atmosphere.

The communication can be closed ith a pinch cock. C is connected to nother vertical tube by means of a ubber tubing and some liquid is poured n the two limbs so that the arrangement erves as a manometer. When B is open the capillary dips in the given liquid e level in A rises. B is now closed. vidently the pressure above the meniscus, the atmospheric press. is greater ~~less~~ than the press. at a pt in the liquid which is immediately below e meniscus, the difference being $\frac{T}{r}$ (assuming the meniscus to be sherical & of radius r) If now the imb D which is attached to D C is raised e press. above the meniscus increases; that below also increases by the ame amount. Hence the height of e column diminishes. Suppose D is aised till the miniscus is in level with the liquid in the vessel. Then the press. below the meniscus is ~~#~~ P ~~$+ \frac{2T}{r}$~~ P being atmospheric press. The press above is $P + hg\rho$ where h = difference in the levels of C & D & ρ is density of liquid in C & D The difference between press. below & above the meniscus must be $\frac{2T}{r}$ $\therefore \frac{2T}{r} = hg\rho$ $\therefore T = \frac{r}{2} hg\rho$. This method is used to find ~~sp. hea~~ T at different temps. That the meniscus is in level with the liquid can be ~~seen~~ easily seen by observing the image.

Surface Tension and Surface energy.

Sudden alteration in the area of a film is accompanied by a change of temp. which makes the film stiffer to resist the change.

Let us suppose there are two chambers of temps θ_1 & θ_2, ~~&~~ θ_1 being > than, but very near to θ_2. Let

film be taken ~~of area~~ of small
eat Capacity. 1) At temp. θ_1 let
ts area be increased by A. Work
one on the film is $2A$ ~~θ~~ T_1 where T_1
S. T. at temp. θ_1. Heat q_1 is absorbed
y the film. 2) Transfer the film ~~for~~ to
he enclosure at temp. θ_2. No work is
one on or by the film since by supposi-
ion heat Capacity is small. 3) At θ_2
et the area decrease by a, work done
y the film being $2AT_2$ (T_2 = S. T. at θ_2)
heat given out being q_2. 4) Transfer
e film to the other; no work is done.
The film is now in its original Con-
ition, & the Cycle is reversible. Hence
y the law of reversible engines

$$\frac{q_1}{\theta_1} = \frac{\text{available work}}{\text{diffrence of temp}} = \frac{2A(T_2 - T_1)}{\theta_1 - \theta_2}$$

Writing β for $\frac{T_1 - T_2}{\theta_1 - \theta_2}$

e have $q_1 = -\theta_1 \times 2A\beta$

If β is +ive, ~~the~~ ie, if surface tension
greater at a higher temperature, ie. if

the film stiffens with temperature, then q is -ve, ie heat is not absorbe but given out when the film increas in area. On the other hand is ~~sod~~ S. T is less at a higher temperature, the film absorbs heat from the outside. Now for all liquids, the surface tension becomes less with temp. ∴ all liquids absorb heat from the outside when the film increases in area & if the change is rapid, so that there is no tim to absorb heat, then the film cools

In the case of water $\beta = -\cdot 14$

∴ At 27°C ie 300° (abs) heat absorbe in order to keep the ~~area~~ temp. constant $= -2A \times \theta, \times \beta = -300 \times \cdot 14 \times$

$= 84 A$

Work done by the external force in increasing the area by $2A = 2A \times T$

$= 150 A$ (taking T as 75)

Thus the heat absorbed is nearly

~~ouble~~ half of work done by external rces.

At temp. θ, ~~energy~~ if T is the surface nsion & $\frac{dT}{d\theta}$ $\overset{\text{rate of}}{\wedge}$ increase of T with temp. then heat absorbed per unit increase f area = $\theta\frac{dT}{d\theta}$.

Work done by external forces = T

he increase in surface energy = um of work done & heat absorbed.

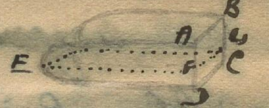
$\therefore$ Surface energy = $T + \theta\frac{dT}{d\theta}$

Since $\frac{dT}{d\theta}$ is +ive

Surface energy is always > S. T.

uincke's drop. The surface tension and of contact can be found by ~~means~~ ~~a drop~~ measuring the dimensions of drop of the liquid provided it does ot spread on the surface of glass. it ~~sep~~ spreads, say like water, dimension of a bubble of air formed low a glass plate placed on the surface e to be measured.

The drop must be such that a certa



area, however small, at its top is perfectly horizontal. Let a central slab be cut out of the drop by two vertical planes at unit distance apart, & suppose that this slab is cut in two equal parts by a vertical plane at rt. ∟s to its length. Consider the equilibrium of this portion of the slab above the horizontal section of greatest area, i.e. above E F C. The horizontal forces acting on this portion are T × A B ie T (since A B = unity) & the difference in horizontal Components of the pressures over the curved surface & over the plane surface A B G F. Since A B is a

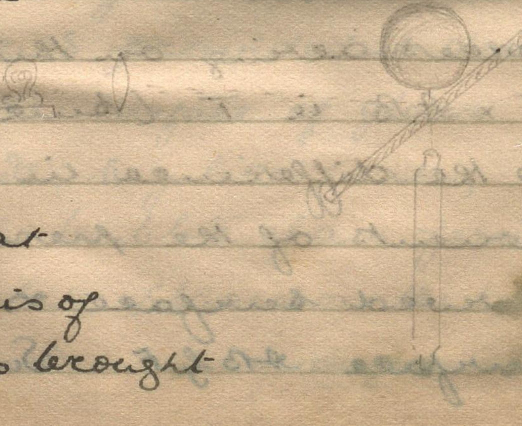
lane, the pressure below the sur-
lace is the same as that above it &
. the difference is $\frac{1}{2}h^2g\rho$ where
= AF, ρ = density of mercury.
These two forces balance each other
e . T $\frac{1}{2}h^2g\rho$.

Considering the equilibrium of the
hole ~~the~~ portion of the slab, &
quating the horizontal forces,

$$T(1 \mp \cos\theta) = \frac{1}{2}H^2g\rho$$

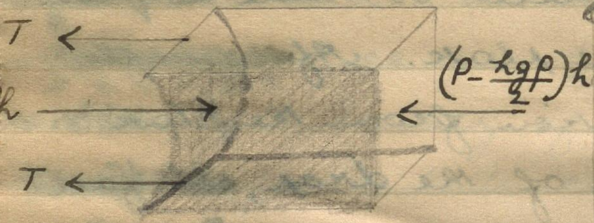
where H is the total height of the
rop.

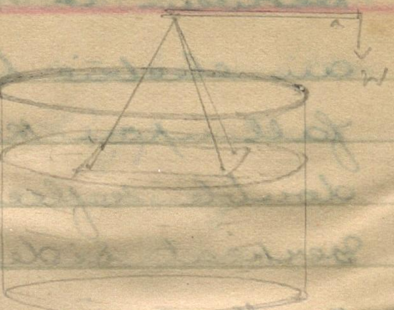
To find H a spherometer can be
sed. For h,
travelling micro-
cope is directed
wards the
rop, a glass
late inclined at
5° to the ~~ba~~ axis of
e microscope is brought



between it & the drop. & light from an electric bulb is made to pass fall upon the plate. After ~~refle~~ double reflection from the plate & the vertical side of the drop, it falls on the microscope & a streak of light is seen in the plane of the eye-piece. If the incident and reflected beams are not ~~on~~ both in a horizontal plane a double streak will be seen. They can be made to merge into one by adjusting the light or the glass plate. The microscope reading is taken. Next a spherometer is placed over the drop, & its central leg is made to touch the top of the drop. The microscope is focussed on to the tip of the leg. The difference between the two readings gives h.

__Surface Tension by detatchment of an area__





As in the case of the Quincke's drop t can be shown that $2T = \frac{1}{2}H^2 g\rho$ hence T can be calculated. But e theory supposes that the plate hile being detatched is perfectly arallel to the surface of water. f however the plate is ever so lightly disturbed it ceases to be arallel & detatchment becomes asy. Hence the expt. is only theorecally possible.

$2T = \cancel{8}\,\frac{1}{2}H^2 g\rho \therefore H = 2\sqrt{\frac{T}{g\rho}}$

he pressure above the plate just efore detatching is greater than that elow by $Hg\rho$ & $\therefore$ the force which

pulls the plate down to the surface
the liquid is $AHg\rho = wg$, where
w is the wt. that balances the ~~the~~
detatching plate.

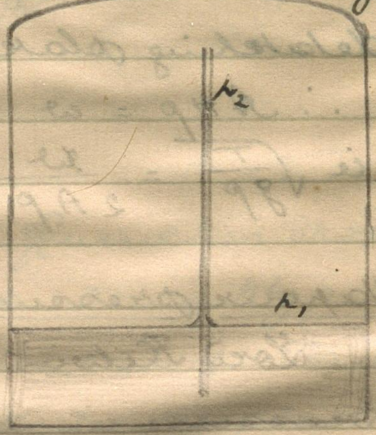
$$\therefore AH\rho = w \quad \text{ie} \quad A\rho . 2\sqrt{\frac{T}{g\rho}} = w$$

$$\text{ie} \sqrt{\frac{T}{g\rho}} = \frac{w}{2A\rho} \quad \text{ie} \quad \underline{\underline{T = \frac{w^2 g\rho}{4A^2\rho}}}$$

<u>Vapour pressure over a Curved Surf</u>

Lord Kelvin showed that vapour pressure in equilibrium with a curved surface is not the same as ~~that~~ the pressure of the vapour in equilibrium with a flat one. If a spherical drop evaporates, area of surface, & ∴ potential ene decreases & hence, the surface tension is a factor which promote increased evaporation from a curved surface, & hence we conclude that the pressure of the vapour in equilibrium with a

onvex surface must be greater.
he exact value of the difference
an be calculated from the case of
Concave surface.



Let a capillary
e dipped in a liquid
laced in a closed
essel. When the
apour in the vessel
as become saturated,
& no more evaporation takes place,
s attaining a state of equilibrium,
t p_1, p_2 be the vapour pressures above
the flat & concave surfaces of the
essel & capillary respy

$p_1 - p_2 = h \sigma g$

where σ = vapour density, assuming
to be uniform.

ow $\cancel{p_2}$ pressure above the meniscus is greater than the pressure
ust below the meniscus by $\frac{2T}{r}$.

ie $\cancel{p_2} + h(\rho - \sigma) g = \frac{2T}{r}$

$$\therefore h = \frac{2T}{r(\rho-\sigma)g}$$

$$\therefore p_1 - p_2 = \frac{\sigma}{\rho-\sigma}\cdot\left(\frac{2T}{r}\right) --$$

When h becomes very large on account of r being small, the assumption that σ is uniform becomes ~~ta~~ untenable, & then we proceed as follows.

If σ be the pressure at a layer thickness dh at height h above the layer, the difference dp between vapour pressures above & below the layer~~s~~ ~~are~~ is $-dp = dh\,\sigma g$

Integrating both sides

$\int_{p_1}^{p_2} -dp = g\int_0^h \sigma\,dh$ Now $\frac{p}{\sigma} = R\theta$

where R is the gas constant of the eqn $PV = R\theta$

$$\therefore \frac{dp}{\sigma} = \frac{dp}{p} \times R\theta$$

$$\therefore -\frac{R\theta}{g}\log\frac{p_2}{p_1} = h = \frac{2T}{r(\rho-\sigma)g}$$

$$\therefore \log\frac{p_2}{p_1} = \frac{-2T}{R\theta\, r(\rho-\sigma)}$$

This eqn can be shown to be a generalized form of I

$$og\left(\frac{p_2}{p_1}\right) = \log\left(1+\frac{p_2-p_1}{p_1}\right)$$

$= \frac{p_2-p_1}{p_1}$ when p_2-p_1 is ~~se~~ very mall compared to p_1

$$\therefore p_1 - p_2 = \frac{p_1 2T}{R\theta r(\rho-\sigma)} = \frac{\sigma}{\rho-\sigma}\cdot\frac{2T}{r}.$$

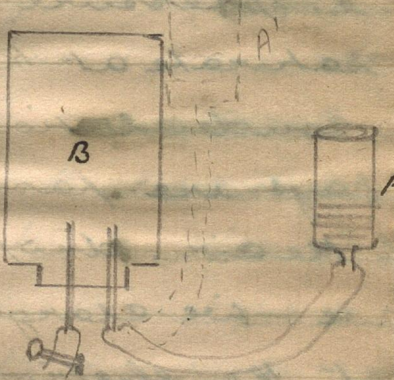
We may assume the formu-a to hold good in ~~ca~~ the case of onvex surfaces as well. Here r becomes ive & ∴ we have

$$p_2 - p_1 = \frac{\sigma}{\rho-\sigma}\cdot\frac{2T}{r}.$$

∴ the pressure of vapour contact with the surface of apour is drop of dew or a aindrop is greater that the normal ressure of the water vapour by $\frac{\ }{\sigma}\cdot\frac{2T}{r}$. This difference is very reat when r is small, & infinite hen r approximates to zero. Hence e see that ~~a~~ even a very high degree of persaturation is insufficient to ake ~~d~~ raindrops in a purely dust-ree atmosphere, for in the initial

Stage of formation requires an infinite vapour pressure if no nucleus for formation is present. Actually there are suspended dus particles or atleast ionized molecules of air which act as nuclei.

The ~~importance of~~ action of nuclei ca be demonstrated by a simple expt. devised by Aitken.

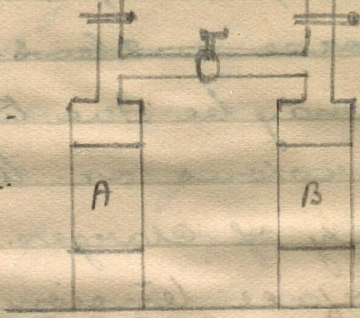


A & B are two closed vessels connected by an flexible pipe. A is partly filled with water. When A is raised up the pressure in B is high. If sudden lowered, pressure decreases, & due to cooling some water vapour condenses ~~so~~ using a part of the suspended dust as nuclei. By repeating this process, all the dust can be used up, a stage hich is indicated by cessation of e ~~co~~ forming of cloud. If more ust is introduced through the extra ipe supplied, cloud is again formed. f the air is dust-free & the temp-erature lowered ~~to four f~~ so as to ake the saturation four-fold i.e. pressure is 4 times that reqd saturate at the cold temp.) ~~the~~ o condensation is seen. If now rays are passed, causing ionization f the air, droplets are seen which ave -ive charges, thereby showing at the electrons act as nuclei. At a further stage of supersa-uration, +ive charged drops are een, which show that protons too ave become nuclei. If further aturated, uncharged drops are seen. hus the condensing power of the neutral molecule is less than that of the proton, which again is ~~a~~ <? ~~worse nucleus~~ than that of the electron. It may be due to difference in size of electrons & protons or to the attractive property of charges.

Surface tension of very thin films

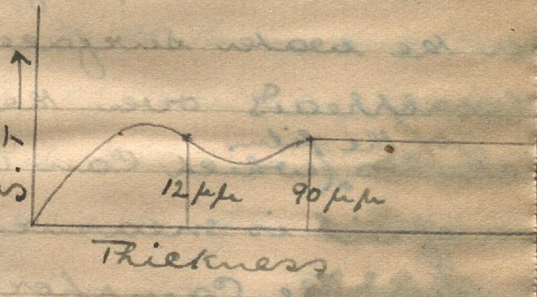
If we observe a film that is draining away slowly, certain portions of it are seen coloured, and certa other portions black. Hence we conclude that films of different thicknesses can have the same surface tension, for if the surface tension of the thinner part were, say, less, ~~it would~~ that part wo be pulled by the thicker part ~~t~~ & th the film ~~will~~ would collapse.

A quantitative investigation of the variation of S. T. with thickness was made by Rücker & Reinold in 1886. The diagrammatic representation o ir apparatus is shown in the figure.



& B are cylindrical lms of equal dimensions, put in communication with each other. is made to drain ay, its upper portions thereby getting inner; but B is maintained at constant ickness by passing an electric current om bottom to top. If A becomes of ess surface tension than B, the essure in it would be less, & nce it will shrink. But it was served that until about 12 $\mu\mu$ thickness hen the film in its upper part was lack ~~stable film~~ ~~B~~ A could be aintained. But at this stage no part A could be found of thickness between $\mu\mu$ & 90 $\mu\mu$. There was a harp discontinuity shown in the

figure. Hence we conclude that when the thickness decreases from 1350 $\mu\mu$ (the upper limit of the experiments) to 90 $\mu\mu$ the S.T. is constant. ~~Be~~ After 90 $\mu\mu$ S.T. is less, & the lower parts ~~are~~ pull away the upper par When ~~it~~ ~~t~~ thickness is about 50 μ S.T. again increases, i.e. draining increases the S.T., the film is Stable. At 12 $\mu\mu$ S.T. is the same as for thickness above 90 μ After 12 $\mu\mu$ S.T. again increas for a while & then diminishes indefinitely.



The relation between S.T. & thickness is shown on the adjoining graph.

The methods used for thickness

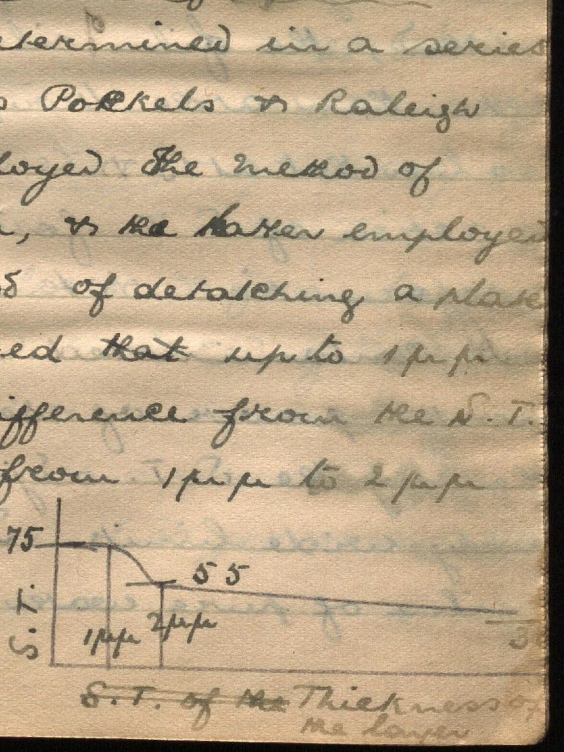
re to measure the shifting of the inter- erence bands or of to find the electric esistance of the film.

Movement of Camphor on water.

When Camphor ~~Sta~~ Shavings ~~are~~ rown on the surface of pure water, e Camphor particles move to & fro ith great velocity. [Camphor dissolves n water; the solution has a smaller T. than pure water &. ∴ the liquid n the neighbourhood of a camphor article is pulled by the adjacent ater surface; hence the motion]

a small drop of oil is thrown ver the water surface, a thin oil ilm spreads over the water. Let the ickness of the film (which can be known if the ol. of oil is measured) be less than o μμ. The Camphor moves as before. the thickness be increased (by confining e oiled surface between two strips of metal dipping below the surface) when it reaches a value about 1 $\mu\mu$ the motion appreciably decreases. When thickness is 2 $\mu\mu$ there is no motion at all. Raleigh who performed the above expt concluded that the ~~di~~ lessening of motion takes place when one molecular layer of oil completely covers the water surface & ∴ that 1 $\mu\mu$ is approximately the diameter of a molecule.

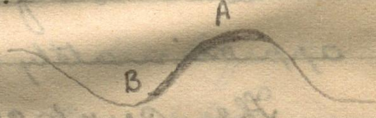
The surface tension of greased surfaces was determined in a series of expts by Miss Pokkels & Raleigh. The former employed the method of detatching an area, & the latter employed Wilhelmy's method of detatching a plate. The results showed that up to 1 $\mu\mu$ no appreciable difference from the S.T. of pure water, from 1 $\mu\mu$ to 2 $\mu\mu$ a sudden fall, & thenceforward



very gradual change until the S.T.
ssumes the ~~tea~~ value of the oil used.

Hence we see that so long as the
ickness of the ~~greased~~ oiled layer is less
an the minimum reqd to give the
T. of the oil, any decrease in thick-
ess is accompanied by an increase in
T. This explains the action of oil in
lming troubled waters.

hen a wave tends to be
rmed by the action of

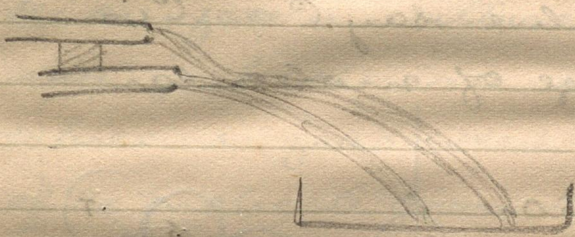


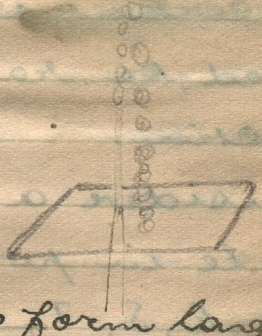
wind, the film of oil at the crest A is
icker than at the trough B. ∴ S.T. at
is less that at B & hence B pulls on A
action of S.T. forces opposes the
aveformation & tends to produce level.

A contaminated surface has a
reater power of self-adjustment
hereby the S.T. adjusts itself within
irly wide limits. Hence where in
film of pure water is unstable because

there is no force to counteract its ow weight, a film of a mixture of soa & water is stable, for there the S.T. is not exactly the same at all parts. Hence it is that bubbles are oftener seen in unclean streams than in pure water wells.

Two drops of pure water when meeting against each other ~~Co~~ rebou But the water is greased or electrifi



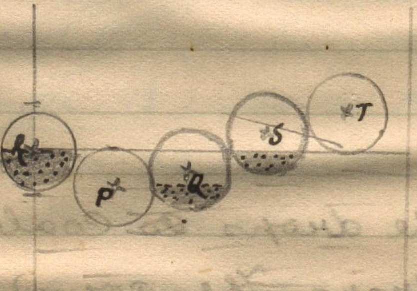


the drops ~~to~~ coalesce & form larger drops. The sound produced if the drop are made to fall on a wooden or metal plate changes appreciably. The same is seen in two jets near each other.

Laplace's Theory of Capillarity.

Laplace developed a theory by hich all surface tension & allied henomena are explained on the ssumption that molecules attract ach other. So little is known of e relation of this force with the istance z between two molecules. that at we can assume it to be a function f z & that $\varphi(z) \to 0$ when z ~~liquid~~ o a certain value say C. C is alled the range of molecular ttraction.

Consider a molecule in positions P, Q, R, S, T, ~~in~~ ith each of these ts as centre & with as radius spheres are describe(d). hen the molecule is at P there are n equal number of molecules on



every side of it, & the resultant attraction on P is zero. Part of the sphere about Q is in air or in the vapour of the liquid wherein molecules are very sparsely distribute (almost there are no molecules compared to those in water) Hence there is a resultant attraction dowwards. At R this attraction is a maximum. When the molecule is abo the surface an attraction is still acting, but it is a decreasing one & becomes zero when the distance of the molecule from the surface is g C. The total work done on a molecule to carry it from the surface to the space beyond C = that done to bring it from the interior.

A quantity of importance i the work done not on one molecule ent on the molecules comprised within nit volume. ~~Let it be K.~~ This quantity s usually denoted by K. K is ∴ efined as the amount of work done n unit vol. of a liquid to ~~bring~~ take it rom the surface of the liquid to the pace beyond the range of molecular attraction of the liquid. Since an equal mount of work is done to bring the olume to the surface of the liquid, 2K the total work done to change unit olume of a liquid into vapour at e same temperature. Expressed in ~~echanical~~ thermal units the work is $L\rho$ here L is the latent heat & ρ the density.

$$L\rho J = 2K \quad \therefore K = \frac{L\rho J}{2}$$

In the case of water at 100°C, we s get

$$K = \frac{4 \cdot 2 \times 10^7 \times 1 \times 538}{2}$$

$$= 1 \cdot 13 \times 10^{10}$$ ~~dynes~~ ergs / ~~per~~ cc

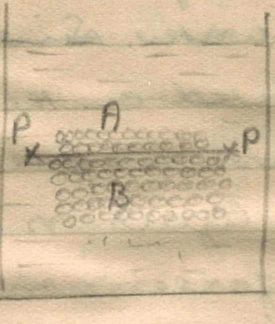
An expression for K can be erived analytically as follows.

Let z be the height of a molecule at any instant above the surface of the water. ~~Since m is its ma~~ The force exerted on the molecule is $\propto$ to the mass m of the molecule, density ρ of the liquid & a function of z say $\varphi($
$\therefore$ force $= m\rho\,\varphi(z)$ Let the molecul be carried upwards through a dista dz so small that the force during this interval can be considered uni-form. Work done in the process $= m\rho\,\varphi(z)\,dz$. Hence total amou of work done in carrying the mo-lecule from the surface to distan c (beyond the range of attraction) $= m\rho\int_0^c \varphi(z)\,dz$. Let there be n molecules in unit vol. Work done in carrying unit vol i.e. K

$$= nm\rho\int_0^c \varphi(z)\,dz \quad = \quad \rho^2\int_0^c \varphi(z)\,dz.$$

The quantity K is equal to what is known as the intrinsic pressure pull per unit area within the nterior of the liquid.

Consider a plane PP n the interior of a liquid ividing out it into wo portions A & B. e B attracts layers of



iquids so in A up to a distance of C. The total force on the layers in A ach of unit area & infinitesimal hickness is the pull per unit area n A due to B. Consider a layer in at height z above P, its thickness being dz o small that all molecules in it e at the same distance z from P. mass is $\rho\, dz$ & $\therefore$ force on it due B $= \rho\, dz \times \rho \times \varphi(z)$

$\therefore$ Total force on A due to B

$$= \int_0^\infty \rho^2 \varphi(z)\, dz = \rho^2 \int_0^c \varphi(z)\, dz$$

ince $\varphi(z) \to 0$ as $z \to c$.

Thus the pull on A due to B or the intrinsic pressure = the Laplace Const.

Vander Waal Calculated the value of K from the eqn gas-equation $(p + \frac{a}{v^2})(v - b) = RT$. When the Correction $\frac{a}{v^2}$ is applied for the value of the pressure, the eqn holds good for liquids as well as fo gases. $\frac{a}{v^2}$ is the diminution in pressure Caused per unit area due to molecular forces pulling ~~a~~ the surface layer into the interio and is ∴ equal to K. Since $\frac{1}{v^2} = \rho^2$, $a/v^2 = \rho^2 a$ & ∴ $a = \int_0^c \varphi(z)$ Since $K = \rho^2 \int_0^c \varphi(z) \, dz$.

The value of a Can be Calculat as follows:–

The density of water is 1 & b is ~~of the dimensions~~ Comparable to & ∴ a negligible quantity when Compared to the volume in the gaseou state.

ence the eqn can be written as

$$\left(p + \frac{a}{v^2}\right) v = RT.$$

At the boiling pt of water, press =

6 Cms or $1·013 \times 10^6$ dynes/Cm^2, temp is

73 abs. $\therefore\ v = \frac{22400}{18} \times \frac{373}{273} = 1672$ CC

When pressure is 14·96 at Cms of

ercury or $1·994 \times 10^5$ dynes per Cm^2,

ater boils at 60°C & the vol. of 18 gm

water vapour is $v = \frac{22400}{18} \times \frac{333}{273} \times \frac{76}{14·96}$

$= 7671$ CC

ence substituting values for p, v, T

$$1·013 \times 10^6 + \frac{a}{1672^2}\Big) 1672 = R \times 373 \quad (1)$$

$$\Big(1·994 \times 10^5 + \frac{a}{7671^2}\Big) 7671 = R \times 333 \quad (2)$$

ividing (1) by (2) R can be eliminated,

solving for a we get $a = 4·7 \times 10^{10}$

Since a gram of water occupies a vol.

04 CC at 100°C, we have

$K = \frac{a}{v^2} = 4·35 \times 10^{10}$ dynes/Cm^2.

This eqn value is of the same

der of magnitude as the value obtained

bove.

Van der Waal employed data obtained from a different source & got a value nearly 1·05 × 10^{10} dyne

Let ~~a~~ qu ~~unit~~ one gram of a liquid at temp T_1, press p_1 be taken, & let the pressure be decreased to p_2 the process being isothermal. Let the corresponding change in vol. be from v_1 to v_2

$$\left(p_1 + \frac{a}{v_1^2}\right)(v_1 - b) = R\,T_1$$

$$= \left(p_2 + \frac{a}{v_2^2}\right)(v_2 - b)$$

Again let the v_1 c.c. at p_1 press be heated to temperature ~~A~~ T_2 such that the vol. changes to v_2

$$\therefore \left(p_1 + \frac{a}{v_2^2}\right)(v_2 - b) = R\,T_2.$$

$$\therefore \cancel{T_1 / T_2}$$

$$\therefore \frac{T_1}{T_2} = \frac{\left(p_1 + \frac{a}{v_1^2}\right)(v_1 - b)}{\left(p_1 + \frac{a}{v_2^2}\right)(v_2 - b)}$$

ie $\left(p_1 + \frac{a}{v_1^2}\right)(v_1 - b) = \frac{T_1}{T_2}\left(p_1 + \frac{a}{v_2^2}\right)(v_2 -$

But the L. H. H $= \left(p_2 + \frac{a}{v_2^2}\right)(v_2 - b)$

$$\therefore \frac{T_1}{T_2}\left(p_1+\frac{a}{v_2^2}\right) = \left(p_2+\frac{a}{v_2^2}\right)$$

In this eqn all quantities xcept a can be directly measured s. ∴ a can be Calculated.

The value of k thus determined was ound to agree with that Calculated rom latent heat.

Dupré was the first to apply atent heat to Calculate ~~int~~ intrinsic ressure. He overlooked the fact t work has to be done to bring molecule from the interior to the urface of a liquid, & hence wrote own the formula as $k = L\rho J$. The alculated value for k was evidently ouble the actual value.

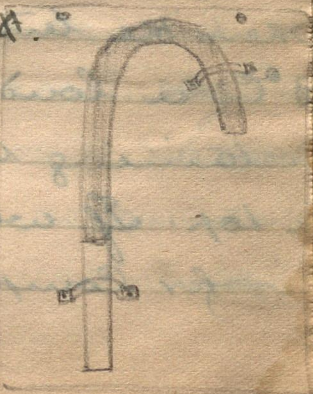
Tensile Strength of a liquid.

When a steel wire is acted by set of wts the wire extends but oes not snap. ~~The~~ Forces of attraction etween molecules keep them together.

The molecules in the portion of the wire below any horizontal plane are pulled by tho molecules above it which are within the range of attraction. Laplace's theory has shown that such a force exists in liquids also and that its magnitu per unit area is K. The tensile Stren of water is about 10000 atmosphere which is the same as that of a steel wire & half that of a pianoforte wi

The best known illustration of tensile strength in liquids is the stick of mercury at the top of a barometer tube. If the tube is brought into the vertical position slowly, the column remains entire through more than 76 Cms high.

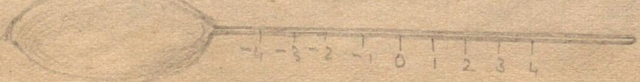
A bent tube fastened to a vertical board contains air free liquid, and its vapour.



he liquid Column does not break
rough dropped downwards from
height. The height of dropping is
creased till the Column breaks.
he force that Causes the breaking
n be approximately Calculated from
e height of falling. Hence the
nsile strength is deduced.

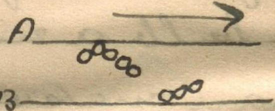
A more accurate method
what follows. A bulb
filled with liquid Completely
ir free with no vapour
the top, the temperature
eing about 90°C. The bulb is Cooled;
ut the water does not Contract; in-
ead it remains in a state of tension.
hen the temp. has fallen to about
8°C a loud report is heard & a space
ntaining water vapour is seen at
e top. If we assume the bulk modu-
s for Compression to be the same as

for dilation, knowing the strain from the vol. of space produced, the pressure to which the liquid was subject, i.e. the tensile strength can be calculated. The above expt by Berthelot was improved upon by Prof. Worthington who placed within the bulb a mercury manometer. The manometer was an ellipsoidal vessel fitted with a calibrated capillary & is containing mercury. The cooling caused an outward pressure to act on the manometer which depressed the mercury level in the capilla The sides of the manometer were thin steel plates. By exerting 1, 2, 3 . . . atmospheres the +ive divisions could be marked. ~~By~~ Equal divisions were marked below ~~th~~ 1st to read 0, 1, 2, . . . etc.



Viscosity

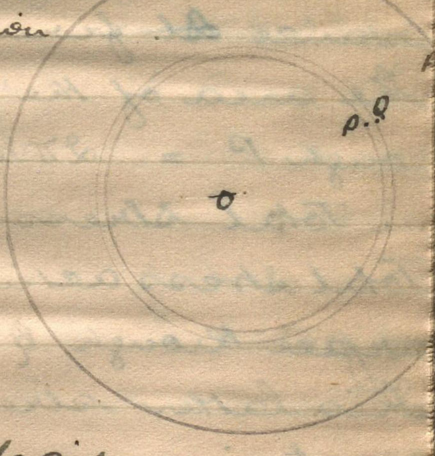
That property of fluids in virtue f which relative motion between e adjacent layers is retarded is scosity. It can easily be explained rom the assumption that molecules re carriers of momentum. Let et A, B be two l planes, A moving B at rest. When a molecule mpinges on A it acquires momenum which after successive imacts is transmitted to B and tends make B move in the same direction s A. Meanwhile since A loses art of its momentum to the moleules impinging on it A tends to top. Thus ~~that~~ relative motion tween A & B is retarded.



B exerts a tangential stress on & vice versa which is proportional to the relative velocity gradient i.e. $\frac{F}{A} \propto \frac{dv}{dx}$ or $\frac{F}{A} = \eta \frac{dv}{dx}$ The constant of proportionality η is called the coefft of viscosity, and can be defined as the force per unit area exerted by one layer of the liquid upon another ||l to it at unit distance, the relative velocity being unity.

<u>Flow of a viscous fluid through a cylindrical capillary tube.</u>

Let the cross-section of the tube be a ~~cyl~~ ⊙ of radius OA = a. Let v be the velocity at P (OP = r). It is supposed that the motion is not turbulent, &. ∴ Velocity at every pt is ||l to the axis of the tube and velocity all over the surface of



ny cylinder coaxial with the tube is e same. It is evident that since the iquid is practically incompressible, the locity at one end ~~on entrance~~ for any such surface he same as that at the other end.

At P. velocity gradient = $\eta \frac{dv}{dr}$. angential stress causing this gradient $\eta \frac{dv}{dr}$. Consider the portion of the luid bounded by two coaxial cylinders rough P & Q ($OQ = r + \Delta r$) & by two lanes ⊥r to the axis of the tube at istance ~~At~~ from Δz from each other.

The area of the surface of the cylinder rough P = $2\pi r$ ~~Δz~~ Δz

∴ Total stress on the area = $2\pi r \Delta z \times \eta \frac{dv}{dr}$.

Total stress acting on the area of the urface through Q = $2\pi\eta\left\{r\frac{dv}{dr} + \frac{d}{dr}\left(r\frac{dv}{dr}\right)\Delta r\right\}\Delta z$

This latter stress acts in the direction v to increase the relative velocity, the former acts in the opposite direction. The resultant stress tending to increase

$$v = 2\pi\eta\left\{\frac{d}{dr}\left(r\frac{dv}{dr}\right)\Delta r\right\}\Delta z$$

Let P denote the pressure gradient i.e. the increase of pressure in the direction of v. The effect of the pressu over the ends of the ring is equivalent to a force $2\pi r\,\Delta r \times P\,\Delta z$ tending to diminish v. Since the motion is stea there is no change of momentum in th ring &c. ∴ the two forces must be equal.

$$2\pi\eta\left\{\frac{d}{dr}\left(r\frac{dv}{dr}\right)\Delta r\right\}\Delta z = 2\pi r\,\Delta r \times P$$

~~Since~~ i.e. $\eta\frac{d}{dr}\left(r\frac{dv}{dr}\right) = rP$

Since the motion of the liquid is ‖l to the axis, pressure must be the same all over a cross-section ∴ P is independent of r. Again velocity is independent of length, depends only on r ∴ P is a constant.

Hence integrating

$$\eta r\frac{dv}{dr} = \frac{r^2}{2}P + C$$

$\therefore \eta \frac{dv}{dr} = \frac{r}{2} P + \frac{C}{r}$

Integrating again

$\eta v = \frac{r^2}{4} P + C \log r + C_2$

C & C_2 are constants of integration.

ince velocity is not infinite when $r = 0$,

must vanish.

$\therefore \eta v = \frac{r^2}{4} P + C_2$

~~Now~~ Careful expts have shown

at the layer in contact with the

urface of the liquid does not move,

at there is no slipping. Hence when

$= a, \; v = 0 \quad \therefore \quad C_2 = - P\frac{a^2}{4}$

$\therefore \eta v = P\left(\frac{r^2}{4} - \frac{a^2}{4}\right)$

Now P is the rate of increase of

ressure in the direction of velocity,

$P = - \frac{p_1 - p_2}{\mathcal{L}}$ where p_1 & p_2 are the

eloci pressures at the two ends where

liquid enters and leaves the tube,

l is the length of the tube

~~Again~~ Vol. of water that passes

unit time across a section

of the tube $= \int_0^a 2\pi r\,dr \times v$

$= 2\pi \frac{P}{4\eta}\int_0^a (r^3 - a^2 r)\,dr = \frac{-\pi P a^4}{8\eta}$

$= \frac{\pi(P_1 - P_2)a^4}{8\eta l}$

Viscous flow of gases through a Capil

Using the same notation as in the previous case we obtain the equatio

$$\eta \frac{d}{dr}\left(r\frac{dv}{dr}\right) = r\frac{dp}{dl}$$

Now it can easily be seen that dp/dl is not a constant ove the surface of the cylinder of radius r, for if through an area ds a volume V enters its press on coming out diminishes & ∴ V becomes $V\frac{p_1}{p_2}$ & ∴ the velocity of flow is greater, changing from v_1 to $v_1\frac{p_1}{p_2}$. Hence, v is no independent of l; but the produc of the velocity & ~~&~~ pressure is a constant independent of l

∴ multiplying both sides by p

$$\eta \frac{d}{dr}\left(r\frac{dv}{dr}\right) p = r p \frac{dp}{dl}$$

Now $\frac{d}{dr}(pv) = v \cdot \frac{dp}{dr} + p\frac{dv}{dr}$

But $\frac{dp}{dr} = 0$ ∴ $\frac{d}{dr}(pv) = p\frac{dv}{dr}$

& again $\frac{1}{2}\frac{d}{dl}(p^2) = \frac{dp}{dl} \times p$

∴ The above eqn can be written as

$$\eta\frac{d}{dr}\left\{r\frac{d(pv)}{dr}\right\} = \frac{r}{2}\frac{dp^2}{dl}$$

pv is indepent of l & $\frac{dp^2}{dl}$ is a constant

∴ Integrating

$$\eta r\frac{d(pv)}{dr} = \frac{r^2}{4}\frac{dp^2}{dl} + C_1$$

i.e

$$\eta\, d(pv) = \frac{dp^2}{dl}\frac{r}{4}dr + \frac{C_1}{r}dr$$

Integrating

$$\eta pv = \frac{dp^2}{dl}\frac{r^2}{8} + C_1 \log r + C_2$$

$C_1 = 0$

& $C_2 = -\frac{dp^2}{dl}\frac{a^2}{8}$

Vol. of gas flowing per unit time through an section

$$= \int_0^a 2\pi r\,dr \times v$$

$$= \int_0^a 2\pi r\,dr.\ \frac{dp^2}{dl}\left(\frac{r^2}{8} - \frac{a^2}{8}\right)\frac{1}{\eta p}.$$

$$= \frac{dp^2}{dl}\cdot\frac{1}{\eta p}\cdot\frac{\pi}{4}\int_0^a\left(r^3dr - a^2r\,dr\right)$$

$$= \frac{dp^2}{dl}\cdot\frac{\pi}{4\eta p}\cdot -\frac{a^4}{4}$$

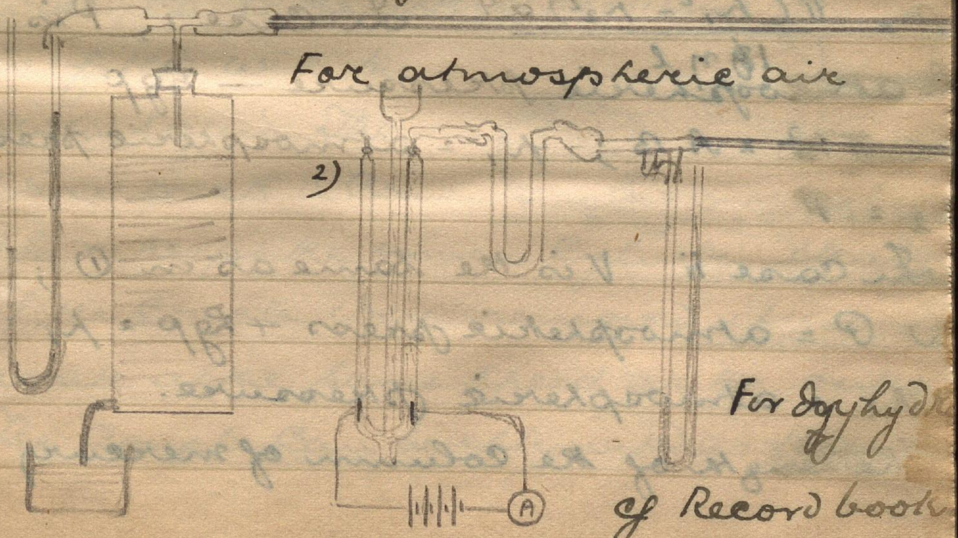
& since $\frac{dp^2}{dl} = -\frac{p_1^2}{l}$

$$= \frac{\pi(p_1^2 - p_2^2)a^4}{16\eta l p}$$

Hence vol V that pass through any cross-section where press is P is given by

$$PV = \frac{\pi(p_1^2 - p_2^2)a^4}{16\eta l}$$

Expts for viscosity of a gas.



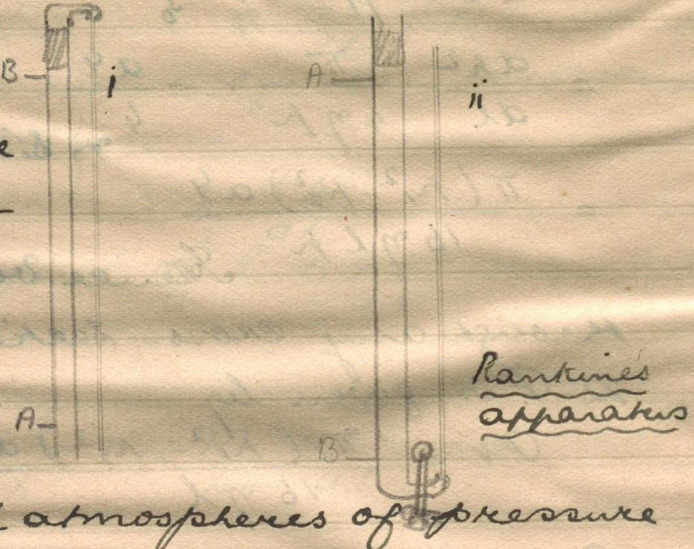
For atmospheric air

For dry hydr

cf Record book

The above expt. cannot be employed
or oxygen since oxygen easily
ttacks the rubber connections supplied.

3)



Rankine's apparatus

In expt No 3
ne should be
aken that mer-
ury does not
low into the
hin capillary.
f it does,
en several atmospheres of pressure
ill not be sufficient to drive it out.

$P = \frac{\pi(p_1^2 - p_2^2)a^4}{16\eta l}$ In case i, P is
atmospheric pressure $- hg\rho$
$= \pi r^2 \times AB$; p_1 = atmospheric press;
$_2 = P$

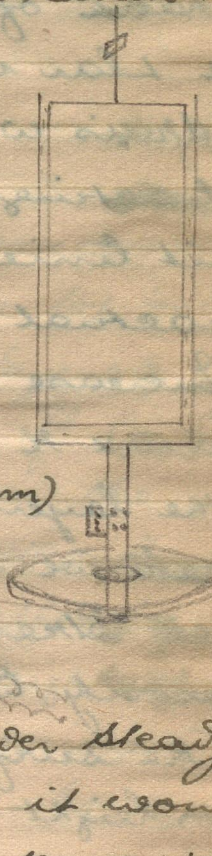
In case ii V is the same as in ①;
ut P = atmospheric press $+ hg\rho = p_1$
p_2 = atmospheric pressure.

The length of the column of mercury

Should be such as to allow a suffic
long time for descending. But it sho
not be too small lest after moving
for some time it come to a Stand
Still. The bore of the wider tube
Should be of about 1 mm radius.

Revolving Cylinder

The outer cylinder
to two Coaxial Concentric Cylinders
Can be rotated at a Constant
Speed, say 20 revol. per min.
The inner is suspended
within it by a torsion
wire, & the interspace (1 mm)
is filled with the
given liquid. The
wire exerts a Cou-
ple & keeps the inner Cylinder steady.
If the Couple were removed it woul
rotate at the Same Speed as the outer.
The Couple Can be estimated as follows.



It is supposed that the motion is not
urbulent, that the liquid everywhere
oves in coaxial layers. Let Ω be the
e $\angle$lar vel. of the outer cylinder,
ω that of any layer of radius r, a & b
e radii of the outer & inner cylinders.
he $\angle$lar vel. of a layer of radius
$+ dr$, is $\omega + d\omega$. If the couple were
ot acting its displacement in
nit time would have been $\omega(r+dr)$;
e actual displacement is $(\omega + d\omega)(r+dr)$
Increase in displacement $= d\omega(r + dr)$
$d\omega \times r$ ~~This g~~ ie relative velocity
f the layers of radii r & $r + dr = r\,d\omega$
relative velocity gradient $= r\frac{d\omega}{dr}$.
~~orce~~ Stress per unit area exerted by
iscosity to cause this relative vel $= \eta r \frac{d\omega}{dr}$.
n the surface of the layer of radius
& height unity total tangential force
$2\pi r \times \eta r \frac{d\omega}{dr}$
oment of the force about the axis

$= 2\pi\eta r^3 \frac{d\omega}{dr}$ This moment is balanced by the Couple C

$$\therefore\ C = 2\pi\eta r^3 \frac{d\omega}{dr}$$

$$\text{ie}\ \frac{C}{2\pi\eta}\int_b^a \frac{dr}{r^3} = \int_0^\Omega d\omega$$

Integrating

$$\frac{C}{2\pi\eta}\cdot\frac{1}{2}\left(\frac{1}{b^2}-\frac{1}{a^2}\right) = \Omega$$

$$\text{ie}\ \ C = 4\pi\eta\Omega\frac{a^2b^2}{a^2-b^2}.$$

In practice an end correction is necessary since at the top & bo of the inner cylinder the layers are not moving ~~At~~ to the axis. If C_1, C_2 be the Couples acting when the inner cylinders are of lengths l_1, l_2 & l is supposed to be the effective increase in length (the height of water above & below the inner cylinder being the same in both cases) we have two expression involving l, whence l can be liminated

$$C_1 = 4\pi\eta\Omega(l_1+l)\frac{a^2b^2}{a^2-b^2}$$

$$C_2 = 4\pi\eta\Omega(l_2+l)\frac{a^2b^2}{a^2-b^2}$$

$$\therefore \eta\left\{l_1-l_2\right\}4\pi\Omega\frac{a^2b^2}{a^2-b^2} = C_1 - C_2.$$

If ~~Ω~~ ω_1, ω_2 are the ∠lar velocities the two cases, the eqn reduces to

$$\eta\left\{l_1-l_2\right\}4\pi\frac{a^2b^2}{a^2-b^2} = \underline{\underline{\frac{C_1}{\omega_1} - \frac{C_2}{\omega_2}}}$$

Searle's Viscometer

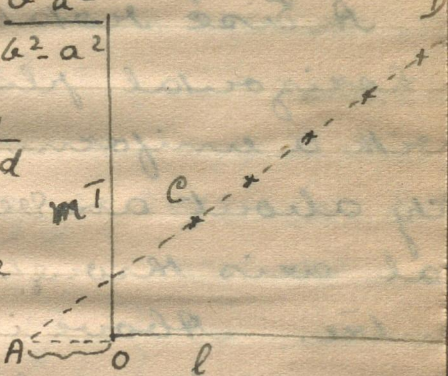
In this apparatus the inner ylinder is capable of rotation about s axis under the action of two wts ting downwards at the end of strings ould round an axle. The couple is mgd where m = mass, & d diameter f the axle. The outer is can be moved or down, but does not rotate. ~~length~~ height of the rotating film of water can thus be adjusted. The wt. m should be such that ~~the~~ it moves down with a constant velocity.

If T is the time for 1 complete revolut of the cyl. $\cancel{\frac{T}{2\pi}}$ $\frac{2\pi}{T}$ = ∠lar vel.

& $\frac{mgd}{t}$ = Couple per unit length

$$\therefore \frac{mgd}{l} = 4\pi\eta \frac{2\pi}{T} \frac{b^2a^2}{b^2-a^2}$$

$$\therefore \frac{mT}{l} = 8\pi^2\eta \frac{a^2b^2}{b^2-a^2} \frac{1}{gd}$$



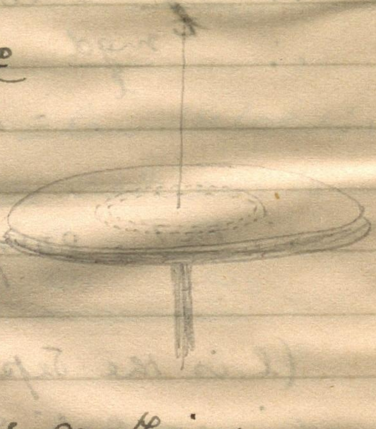
(l is the depth of the inner cylinder below water. The formula holds only if end correctio is zero)

If ~~A~~ mT is taken on the y-axis & l on the x-axis and a ~~cur~~ graph is drawn it is seen to be a st. line which meets the x-axis not at the origin but at some other pt. A.

This shows that AO is the end correction.

the ~~graph~~ eqn had $l + AO$ instead of l graph would have passed through the igin. The slope of the curve gives $\pi\eta \frac{a^2b^2}{b^2-a^2} \cdot \frac{1}{gd}$ whence η can be calculated.

iscosity by Revolving disc



A disc rotates in horizontal plane ith a uniform velo ity about an verti al axis through its ntre. Above it hangs an thin late ∥ to it from a ~~suspension~~ rsion ~~of~~ fibre which lies in the me line as the axis of the lower isc. When the lower rotates, upper remains stationary under action of the torsional couple which alances the viscous force exerted by intervening fluid. ~~If~~ The relative vel.

of the lower disc at a pt r from its centre is ~~vr~~ ωr

$\therefore$ The vel. gradient $= \frac{\omega r}{d}$ δ bein distance ~~fro~~ between the plates.

$\therefore$ Viscous force over an area $2\pi r\,dr$ (a narrow ring of radius

$= 2\pi r\,dr \times \eta \times \frac{\omega r}{d}$

The moment of the force about the centre $= \frac{2\pi\eta\omega}{d} r^3 dr$

$\therefore$ Couple acting on the upper plate $= \int_0^a 2\pi\eta \frac{\omega}{d} r^3 dr$ (a = Rad of the plate)

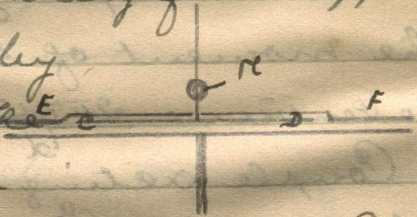
$$\left.\begin{aligned} &= \frac{1}{2}\pi\eta a^4 \frac{\omega}{d} \\ &= \frac{1}{2}\pi n b^4 \frac{\theta}{l} \end{aligned}\right\}$$

Where $\frac{1}{2}\pi n b^4/l$ = torsional Couple & θ is the twist measured by a scal & mirror.

The eq^n $\frac{1}{2}\pi\eta a^4 \frac{\omega}{d} = \frac{1}{2}\pi n b^4 \frac{\theta}{l}$ shows that Viscosity in fluids is analogous to rigidity in solid

In deriving the above eq^n it has

een tacitly assumed that the viscosity
el. gradient $\frac{\omega r}{d}$ is not affected either
y the end altered here at the our
dge of the upper plate, & that there
s no viscosity drag at the upper
urface of the plate. These condi-
ions are satisfied only if the upper
late is protected by
guard ring EF. ~~The~~
late merely fills a



lar hole in the ~~upp~~ larger plate EF &
ence the flow is practically the same
s when there is only one plate EF
The viscosity of air can be determined
ery accurately by the above method
a = 20 cms, d = 3 mm & ~~the~~ $\omega = \frac{2\pi}{3}$
This arrangement has been used to
aluate very low pressures, from
knowledge of the law connecting
essure & viscosity.

Effect of pressure & temp. on viscosity

Viscosity is practically unaffected by pressure, except when the pressure is extremely low. At low pressure viscosity falls very rapidly. Unlike in the case of liquids the ~~pressure~~ viscosity of ~~vapour~~ gases increases with increase of temperature. Many empirical laws connecting viscosity & temp. have been formulated one of which is $\eta \propto \sqrt{T}$ where T is the abs. temp. (Cf later)

Correction for K.E. in Poiseuille's flow

We have tacitly assumed whil calculating q $\left(= \frac{\pi (p_1 - p_2) a^4}{8 \eta l}\right)$ that all the pressure head h of the liquid is utilized in overcoming viscosity & that $p_1 - p_2 = hg\rho$ *
But some work is done in imparting K. E to the outcoming liquid.

ince total work done p.s. $= q h g \rho$,

s the work done against viscous resistance $= \cancel{\pi} (p_1 - p_2) q$ where $(p_1 - p_2)$ is

e effective pressure head ie $hg\rho$ minus certain quantity, we have the eqn

$$q h g \rho = K.E + (p_1 - p_2) q.$$

K. E. of the liquid that flows out p.s. n be evaluated as follows.

Liquid flowing through an annular ection of radius r p.s. is of vol. $\pi r dr \times v$ & its mass is $2\pi \rho r dr \times v$

Its K. E. $= \pi \rho r dr v^3$.

Now we have seen $v = \frac{\cancel{p}_1 - p_2}{4 l \eta} (a^2 - r^2)$

∴ ~~K. E = πρ~~ & again

$$q = \frac{\pi (p_1 - p_2) a^4}{8 \eta l}$$

$$\therefore v = \frac{2q}{\pi a^4} (a^2 - r^2)$$

$$\therefore K.E = \pi \rho \times \frac{8 q^3}{\pi^3 a^{12}} (a^2 - r^2)^3 \times r dr$$

$$= \frac{8 \rho q^3}{\pi^2 a^{12}} \times -\frac{1}{2} d(a^2 - r^2) \times (a^2 - r^2)^3$$

Total K.E. $\int_0^a -\frac{4\rho q^3}{\pi^2 a^{12}} \left(a^2 - r^2\right)^3 d\left(a^2 - r^2\right)$

$$= -\frac{4\rho q^3}{\pi^2 a^{12}} \left(\frac{\overline{a^2 - r^2}^4}{4}\right)_0^a$$

$$= -\frac{4\rho q^3}{\pi^2 a^{12}} \times -\frac{a^8}{4}$$

$$= \frac{\rho q^3}{\pi^2 a^4}.$$

$$\therefore \; q h g \rho = \frac{\rho q^3}{\pi^2 a^4} + q\left(p_1 - p_2\right)$$

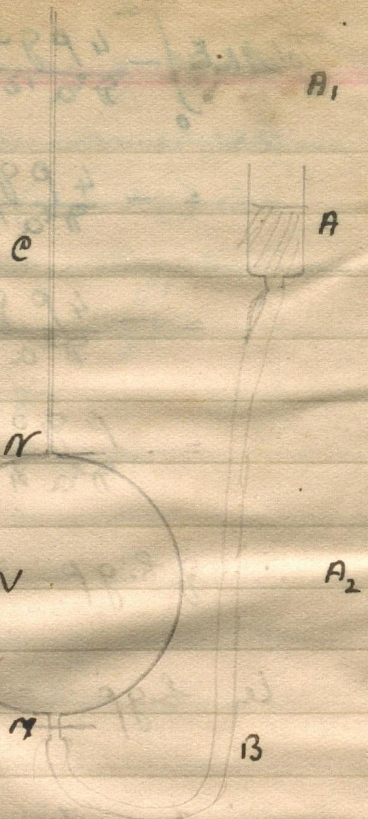
i.e. $hg\rho = \frac{\rho q^2}{\pi^2 a^4} + \left(p_1 - p_2\right)$

$$\therefore \; p_1 - p_2 = g\rho\left\{h - \frac{q^2}{\pi^2 a^4 g}\right\}$$

Thus on account of K.E. imparted to the liquid the pressure head is effectively decreased by a quantity $q^2/\pi a^4 g$

η of air under varying pressure.

A bulb ~~B~~ has two orifices, one above which is in communi



ation with the Cap-
llary C, the other below
hich is connected to
side vessel A
hrough a rubber
ubing. Water is poured
nto ~~the~~ A until it
eaches a mark M
n the stem of V.
A is lowered below
l & air allowed to
nter into V. Then
e tip of the Capillary
closed, A is raised to a position A_1,
the tip is opened. Air escapes thus
nding to ~~lower the pressure in V~~
aise the level M. Rising of the
vel is prevented by lowering A.
et A be lowered from A_1 to A_2 the
ime taken being t seconds.
When A is at A let the pressure

in V be p_1 & when it is at A_2, let press be p_2; & let p be pressure at any intermediate state. If dv is the volume of air that escapes in time dt, $\frac{dv}{dt}$ = vol. that escape p.s. & we have

$$p \cdot \frac{dv}{dt} = \frac{\pi(p^2 - P^2)a^4}{16\eta l}$$

where P is the atmosp. pre

$$\therefore p\,dv = \pi\frac{(p^2 - P^2)a^4}{16\eta l}dt.$$

Assuming Boyle law to be true, $p\,dv = -V\,dp$ where V is the vol. of air in the bulb, between the mark n & the bottom v of the capillary.

$$\therefore -V\,dp = \frac{\pi(p^2 - P^2)a^4}{16\eta l}dt$$

Integrating $\cancel{-V\int_{p_1}^{p_2} dp = \pi}$

$$-V\int_{p_1}^{p_2}\frac{dp}{p^2 - P2} = \frac{\pi a^4}{16\eta l}\int_0^t dt$$

ie $$\frac{V}{2P}\int_{p_2}^{p_1}\left(\frac{dp}{p - P} - \frac{dp}{p + P}\right) = \frac{\pi a^4}{16\eta l}t$$

$$\frac{V}{2P}\left(\log\frac{p-P}{p+P}\right)_{p_2}^{p_1} = \frac{\pi a^4}{16\eta l}t$$

$$\text{ie } \frac{V}{2P}\log\left\{\frac{p_1-P}{p_1+P}\cdot\frac{p_2+P}{p_2-P}\right\} = \frac{\pi a^4}{16\eta l}t.$$

All the quantities are known except η ∴ η can be calculated.

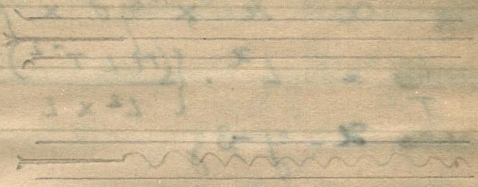
Sometimes V cannot be easily etermined. A tube ~~will~~ is to be attached t M, ~~& the~~ between the bulb & the ubber tubing of which the volume s known. If it is v_1,

~~If~~ first an expt is conducted ith the level of water at M, & then ith the level below the given tube. Thus we can find values for ~~&~~ $V\eta$ & $(V + v_1)\eta$. Subtracting we get $v_1\eta$ & hence alculate η.

Turbulent Motion

Motion of liquid in a capillary ube is turbulent when the velocity

~~are not~~ at
different pts
i.e. the cross-
section is
not ||l to the



axis of the tube. So long as the vel. is ||l, the quantity of flowing out p.s. is $\propto$ to the pressure head. When the $\propto$-ality is not kept, ~~no~~ motion is turbulent.

Turbulent motion can be easily demonstrated by introducing a tiny jet of coloured water within the capillary bore. When the mean velocity $\frac{q}{\pi a^2}$ reaches a certain value the stream of coloured water ceases to be a st. line & become sinuous.

This value of the vel. beyond which motion is turbulent is called the critical value. ~~Let~~ It depends on density & viscosity of the liquid & on e radius of the bore.

$$v_c \propto r^x \times \eta^y \times \rho^z$$

$$\frac{L}{T} = L^x . \left\{\frac{(M L T^{-2}) T \times L}{L^2 \times L}\right\}^y \times \left(\frac{M}{L^3}\right)^z$$

$$1 = x - y - 3z$$

$$-1 = -y$$

$$0 = y + z$$

$$\therefore \cancel{z} \quad y = 1; \quad z = -1; \quad x = -1$$

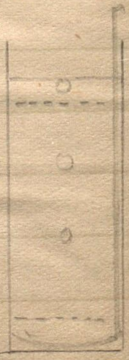
$$\therefore v_c \propto \frac{\eta}{\rho r}$$

Osbourne Reynolds to whom the bove formula is due exptly determined the value of the constant of liff for water & found it to be nearly 1000.

Poynting & Tho
Page 212

Viscous drag at high velocities.

Stokes has proved that when particle moves through an cean of a viscous medium, the riving force is proportional to e first power of the velocity if that elocity is small, & is ∝ to the w power of vel. if vel. is great. He has also shown that the force



bin the former case is $6\pi a \eta v$.

It is supposed that ~~veloc~~ the walls of the vessel containing the mediu fluid do not in any way affect the velocity, hence we have used the word "an ocean" of the fluid.

Let us now consider the case wh ~~ffel is $\propto$~~ vel. is large & $\therefore$ force $\propto v^2$.

Force is a function of viscosit radius of the moving particle & density of the fluid.

$$\text{Force} \propto a^x \times \eta^y \times \rho^z \times v^2$$

where a = radius, η = viscosity, ρ = density, x, y, z are constants.

Equating dimension ~~in~~

$$\frac{ml}{t^2} = l^x \times \left(\frac{m}{tl}\right)^y \times \left(\frac{m}{l^3}\right)^z \times \cancel{v}\left(\frac{l}{t}\right)^2$$

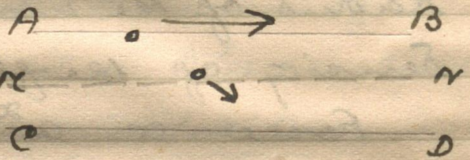
Equating the indices of t

$$2 = y + 2 \quad \text{ie } y = 0$$

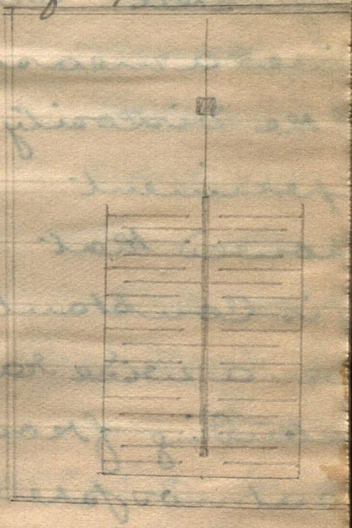
$\therefore$ The index of viscosity in the expression for force is zero, in ords at high velocities the viscosity f the fluid exerts no drag on the oving particle.

Relation between pressure & viscosity.

The ~~ote~~ phenomenon of viscosity as already been accounted for on e assumption that molecules re carriers of momentum. The oving layer has molecules mpinging upon it, ∴ imparts mom- ntum to these olecules which ∝ to its own velocity & the num- r of molecules impinging upon it p.s. onsider any intermediate layer MN. e ~~transfer of~~ momentum transferred through p.s. is the viscous drag between & D & ∴ the coeff$^{\underline{t}}$ of viscosity ∝ to the ~~transfer of~~ momentum transferred ugh unit area of MN. Now

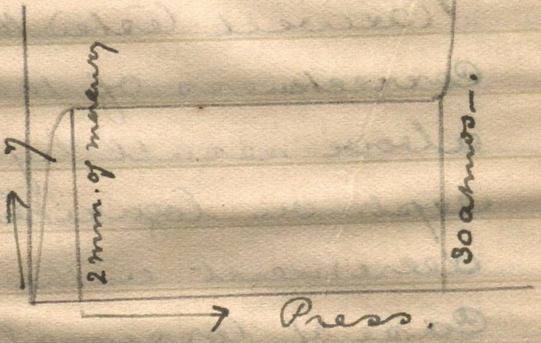


momentum transferred is $\propto$ to the no. of molecules per unit volume and also to the mean free path, the distance of through which a molecule travels on an average unhindered $\therefore \eta \propto n \times l$ When pressure is doubled n is also doubled, but l is halved, & $\therefore$ the product nl remains unchanged. Thus we see that η is ~~prop~~ independent of pressure



Maxwell tested the correctness of the above result by an expt. on logarithmic decrement in the case of torsional oscillations. Instead of one oscillating disc he used several fixed one above the other about

ommon axis and he introduced
number of vanes midway
etween each pair of discs. The
anes were fixed and attached to
e sides of a vessel. The ~~oscillations~~
iscs were suspended by a
ine torsion wire & a mirror attached
the wire. ~~to~~ The whole arrangement
t enclosed within a chamber
f which the pressure can be
djusted. The logarithmic decrement
ives a measure
f the viscosity.
periment
howed that
is Constant
ver a wide range
xtending from 2mm of Hg to ~~30 at~~ about
o atmospheres. Below 2mm ~~the~~ η
alls very rapidly ~~being~~ to zero when
essure ~~to~~ tends to zero. Above 30 atmos.



the pressure η rises slightly. These two variations are also natural consequences of the molecular theory.

Viscosity & temp.

Theoretical investigations lead to a result $\eta \propto \sqrt{T}$ where T is the absolute temp. But expt this ~~des~~ result does not hold good for small variations of temperature. Several empirica relations have been suggested one of which, due to Sutherland i

$$\eta = a\frac{\sqrt{T}}{1+C/T}$$ where a & C are constants. This approximates very nearly to exptal observatio

Compressibility of liquids.

Compr sibility is the reciprocal of bulk modulus. Its exptal determinatio is attended with serious difficul ince a liquid cannot be compressed ithout altering the volume of containing vessel. The amount this alteration has therefore to be irst investigated. We shall consider e case of cylindrical vessel & subject an internal press. p_0 & an external ressure p_1. Lamé has shown at the strain in this case is radial displacement ρ given by

$\rho = Ar + \frac{B}{r}$ where A & B are onstants & r is the distance from the axis of e pt. of which the displacement is n question. He has further hown that this is the resultant two displacements, one $\frac{d\rho}{dr}$ along the radius, and another & $\frac{\rho}{r}$ along ~~th~~ a direction at rt ∠s the radius & the axis, say the y axis.

We have seen that when an isotropic material is subjected to stresses P, Q, R the elongations e, f, g along the three ⊥r directions are given by

$$e = \lambda P - \mu Q - \mu R$$

$$f = \lambda Q - \mu P - \mu R$$

$$g = \lambda R - \mu P - \mu Q$$

& that $k = \frac{1}{\lambda - 2\mu}$ & $n = \frac{1}{2(\lambda+\mu)}$

where k & n are the bulk modulus & the rigidity modulus.

From the above eqns we can express P, Q, R in terms of e, f, g k & n

$$P = (k + \frac{4n}{3}) e + (k - \frac{2n}{3})(f+g)$$

$$Q = (k + \frac{4n}{3}) f + (k - \frac{2n}{3})(g+e)$$

& $R = (k + \frac{4n}{3}) g + (k - \frac{2n}{3})(f+e)$

By Lamé's result:

Radial elongation $= \frac{d\rho}{dr}$

$= A - B/r^2$

$\therefore e = A - \frac{B}{r^2}$

& elongation ⊥r to the radius & to the axis, ie along the y-axis s $\frac{\rho}{r}$ ie ~~A~~ $f = A + \frac{B}{r^2}$.

Consider any pt on the inner surface of the cylinder. The stress t that point is p_0. The radial longation $e = A - \frac{B}{a^2}$ where a is e internal radius of the cyl. & the long. $f = A + \frac{B}{a^2}$.

Hence we can equate

$$p_0 = \left(k + \frac{4n}{3}\right)\left(A - \frac{B}{a^2}\right) + \left(k - \frac{2n}{3}\right)\left\{A + \frac{B}{a^2} + g\right\} . (1)$$

Similarly stress at any pt on the uter surface of the cylinder is p_1 f the external radius is b,

$$p_1 = \left(k + \frac{4n}{3}\right)\left(A - \frac{B}{b^2}\right) + \left(k - \frac{2n}{3}\right)\left\{A + \frac{B}{b^2} + g\right\} \text{---} (2).$$

The pressure acting over the lat bottom along the x-axis s $p_0 \pi a^2$ on the interior, & $p_1 \pi b^2$ n the exterior. The resultant ressure $= \pi\left(p_0 a^2 - p_1 b^2\right)$ & acts n an area $\pi\left(b^2 - a^2\right)$

Hence the stress along the z-axis

is $R = \frac{\pi(p_0a^2 - p_1b^2)}{\pi(b^2 - a^2)} = \cancel{a^2}$

$$= \frac{p_0a^2 - p_1b^2}{b^2 - a^2}.$$

$$R = \left(k + \frac{4n}{3}\right)g + \left(k - \frac{2n}{3}\right)(e + f)$$

$$(e + f) = A - \frac{B}{a^2} + A + \frac{B}{a^2}$$

$$= 2A$$

Hence

$$\frac{p_0a^2 - p_1b^2}{b^2 - a^2} = \left(k + \frac{4n}{3}\right)g + \left(k - \frac{2n}{3}\right)2A. \text{---} (3$$

Thus we have three eqns containing three unknowns viz A, B & g

A, B, & g can ∴ be solved for.

Internal vol. initially before the pressure is applied $= \pi a^2 l$ where l is the length of the cylinder.

After applying the pressure a increa to $\left(a + Aa + \frac{B}{a}\right)$ & l increases to ~~$l(1+g$~~ $l(1+g)$

∴ The change in interior volume

$$\delta v_1 = \pi\left(a + Aa + \frac{B}{a}\right)^2 l(1+g) - \pi a^2 l$$

which is an expn containing

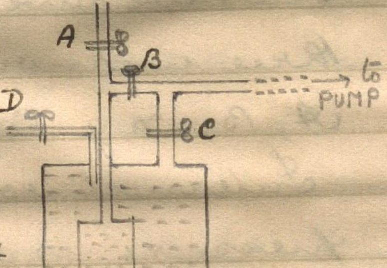
A, B, g.

Substituting values for A, B, g
simplifying we shall have

$$dv_1 = \pi a^2 l \left\{ \frac{p_0 a^2 - p_1 b^2}{b^2 - a^2} \cdot \frac{1}{k} + \frac{b^2}{b^2 - a^2} \cdot \frac{p_0 - p_1}{n} \right\}$$

Similarly if dv_2 be the change in ext. vol.

$$dv_2 = \pi b^2 l \left\{ \frac{p_0 a^2 - p_1 b^2}{b^2 - a^2} \cdot \frac{1}{k} + \frac{a^2}{b^2 - a^2} \cdot \frac{p_0 - p_1}{n} \right\}$$



Piezometer

The above results worked out by Lamé were utilized by Regnault in his expt. with the iezometer for determining the compressi-ility of liquids.

The apparatus is shown in e figure. The inner vessel which ntains the liquid is ~~tes~~ has a hemispherical bottom though a flat one is assumed in the theory. The error thus introduced is however ~~a~~ negligible. The four pinch cocks A B C D can control the pressure inside & outside the inner vessel. The stem of the inner vessel is carefully calibrated and any change in vol. can be directly measured.

(1) Suppose the pinch cocks B & C are open & A & D closed. A pressure p is exerted both inside and outside. i.e $p_0 = p_1 = p$

The increase in the interior vol.

$dv_1 = \pi a^2 l \times \frac{-p}{k} = -\pi a^2 l p \times \frac{1}{k}$

If K is the bulk modulus of the liquid the ~~increase~~ decrease in vol. of water $= \frac{\pi a^2 l p}{K}$ Both these factors depress the level of ~~wat~~ the liquid in the stem say by an amount ω

$\therefore \omega = \pi a^2 l p \left(\frac{1}{K} - \frac{1}{k} \right)$

Hence k cannot be calculated unless

e know k.

) If now B & D are open, & A & C

losed, pressure is applied in the

nterior only. $p_0 = p$ & $p_1 = 0$

Increase in volume of the interior

f the ~~ee~~ inner vessel

$$dv_1 = \pi a^2 l \left\{ \frac{p a^2}{b^2 - a^2} \cdot \frac{1}{k} + \frac{b^2}{b^2 - a^2} \cdot \frac{p}{n} \right\}$$

& decrease in vol. of the liquid

$$= \frac{\pi a^2 l p}{K}$$

$\therefore$ ~~Net Decrease~~ Apparent

ecrease in vol. of the liquid

$$\omega_1 = \pi a^2 l p \left\{ \frac{1}{K} + \frac{1}{k} \cdot \frac{a^2}{b^2 - a^2} + \frac{1}{n} \frac{b^2}{b^2 - a^2} \right\}$$

) Let B & D be closed, ~~&~~ A & C open;

$p_0 = 0$; $p_1 = p$.

Increase in internal vol. of the

essel $= \pi a^2 l \left\{ \frac{-p b^2}{b^2 - a^2} \cdot \frac{1}{k} + \frac{b^2}{b^2 - a^2} \cdot \frac{-p}{n} \right\}$

$$= \pi a^2 l p \left\{ \frac{-b^2}{b^2 - a^2} \cdot \frac{1}{k} - \frac{b^2}{b^2 - a^2} \cdot \frac{1}{n} \right\} .$$

Since the liquid undergoes no strain the above quantity is ω_2 the apparent ~~expn~~ compression of the liquid.

As is seen above ω_1 is not independent of the elastic constants of the material of the vessel.

$$\omega_1 + \omega_2 = \pi a^2 l p \left\{ \frac{1}{K} - \frac{1}{k} \right\} = \omega$$

Hence the two latter expts ~~is~~ are a check upon the value of ω. Such a check is necessary to know how far the conditions assumed in the theory are realized in the actual experiment. It may for example happen that the vessel ~~of~~ is not of perfectly isotropi material & in that case we should make sure that the defect does not affect the final value to any apprec able extent.

Regnault's expt was conducted with great care and his ~~obs~~ values for ω a even today considered standard values.

But he had no direct method for ~~alculating~~ finding k. He made use of the ormula $\frac{9nk}{3k+n} = 2n(\sigma+1)$; found out & assumed σ to be $\frac{1}{4}$ for all ubstances. Hence the final calculated alue for k showed great discrepancy ccording as different vessels were used.

Jamin proposed a modification of e expt by which he said that k could e be eliminated. D was replaced ey a calibrated tube. Pressure as applied inside. He claimed at when the vessel expands a vol. f water is displaced into the stem at ew ~~This~~ When this is subtracted ~~when~~ from e apparent diminution in vol. in e stem of the inner vessel, we get e real compression.

This result can easily be shown e incorrect.

The apparent diminution in the

Stem is $dV + dv_1$ where dV is the compression of the liquid; but the displace vol. at D is dv_2

∴ Jamin's result gives

$dV + dv_1 - dv_2$ which is not the same as dV.

$dv_1 - dv_2$ =

$$= \pi a^2 l \left\{ \frac{pa^2}{b^2-a^2} \cdot \frac{1}{k} + \frac{b^2}{b^2-a^2} \cdot \frac{p}{n} \right\}$$

$$- \pi b^2 l \left\{ \frac{pa^2}{b^2-a^2} \cdot \frac{1}{k} + \frac{a^2}{b^2-a^2} \cdot \frac{p}{n} \right\}$$

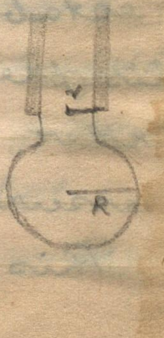
$$= \pi a^2 l p . -\frac{1}{k} .$$

$$\therefore dV + dv_1 - dv_2 = \pi a^2 l p \left\{ \frac{1}{K} - \frac{1}{k} \right\}$$

The result obtained by Jamin is not $\pi a^2 l p / k$ but. ~~a2~~ ω the ~~ol~~ apparent depression when pressure applied both inside & outside.

S.T. by detatchment of a drop

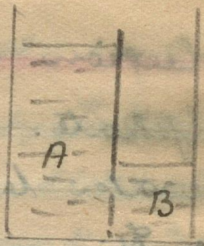
Just before detatching a cylindrical & a spherical drop are in equil. Theory shows $R = 2r$.



Osmosis and diffusion

Osmosis is the term applied to
e selective transmission of certain
articles through a semipermeable
membrane through which certain
ther particles are not transmitted.
or example a bladder filled with
lcohol & immersed in water &
ecomes distended; when a semi-
ermeable membrane of cupric
ferrocyanide separates a vessel
nto two parts one side having water
the other side a sugar solution
t is noticed that the level of liquid
n the solution side rises until a
ertain hydrostatic pressure difference
established between the two parts.
his pressure difference which prevents
rther osmosis is called osmotic
ressure.

This phenomenon can be explained



on the basis of the kinetic theory of matter. Initially when levels in A & B are equal let no. of impacts per unit area p.s. on both sides b n, of which on side A n_1 are of wat & $(n - n_1)$ of sugar. All n water moled from B are allowed to pass to A; but only n_1 ~~mol to~~ water ~~no~~ molecules pass from A to B, & the $(n - n_1)$ suga molecules are stopped. This net excess raises the level of liquid in A until the pressure is such that on side A the impacts of water alone is n.

The quantitative expts on osmosis were first performed by Pfeffer. The p semipermeable membrane he used was a film of cupric ferrocyanide deposited within the pores of a porous pot, prepared by placing the pot filled with potassium ferrocyanide

Solution in a Vessel of Copper
ulphate. The membrane is permeable
o water but not to sugar.
uts His expts showed that the
smotic pressure is $\propto$ to the concen-
ation.

Van t'Hoff later analyzed Pfeffer's
bservations & found that for dilute
olution osmotic pressure is $\propto$ the
bsolute temp.

Later it was further noticed that
e osmotic pressure of a certain
olution ~~was~~ is the same as it would be if
he same amount of solute in gaseous
tate occupied a volume equal to that
the solution. Thus $P = CRT$.

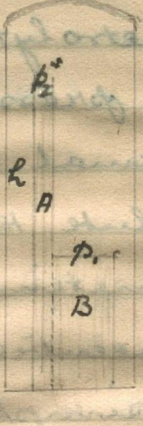
The osmotic pressure of all solutions
ntaining the same number of gm. molecules
solute per litre is the same. Thus it is
en that a substance in solution
erts an osmotic press. the same as that

if the solvent had been absent.

The above laws refer only to non-electrolytic solutions; but in electrolyt the pressure is always greater than the normal value. If the solution is very dilute the pressure is generally ~~doub~~ twice sometimes thrice, the normal value. In Concentrated solutions pressure is no a simple multiple of the normal valu This phenomenon can also be explai on the Dynamical theory by assumin that the ~~solute exists~~ substance in solution is separated into 2 or more ions; in dilute solutions all molecules are so separated; but in Concentrated solutions only a fraction are separated; each ion exerts a ~~force on the~~ pressu due to its impacts, + the net effec is that of an increase in Concentra

<u>Vapour pressure over a solution</u> Let a solution Contained in a tall cylindri vessel

e separated by a semipermeable
iaphragm from the pure
olvent contained in the vessel
. For equilibrium to exist
e free surface in A should
tand at a certain height h
bove the free surface in B.
t both the vessels be con-
ained in an isothermal
nclosure which is empty but for the
resence of the vapour of the solvent.
f p_1 denote the vapour pressure imme-
iately above the free surface of B,
e pressure at a height h above that
hould be $p_1 - hg\sigma$ where σ is the
ensity of the vapour & is supposed to
e uniform. $p_1 - hg\sigma$ should be equal
p_2 the vapour pressure of the solution;
r if it $p_1 - hg\sigma \neq p_2$ but less, then
re of the solvent will pass into the
apour at the free surface of A, thus cause



more solvent to pass from B to A. A perpetual motion will be caused without the aid of external agency, which is against the 2nd law of thermodynamics. If p_1 were less, then also motion would ensue, but in the opposite direction

$$\therefore \quad p_2 = p_1 - hg\sigma. \qquad 1)$$

Since the pressure on side A of the semipermeable diaphragm is greater than that on side B by a quantity equal to the osmotic pressure,

$$p_2 + hg\rho - p_1 = P \qquad 2)$$

where ρ is the density of the solution

$$\text{but } h = \frac{p_2 - p_1}{g\sigma} \quad \text{by (1)}$$

$$\therefore (p_2 - p_1)\left\{1 - g\rho \times \frac{p_2 - p_1}{g\sigma}\right\} = P$$

$$\text{ie} \quad p_2 - p_1 \left\{\frac{\sigma - \rho}{\sigma}\right\} = P$$

$$\text{ie} \quad p_1 - p_2 = \frac{P\sigma}{\rho - \sigma}.$$

Since $\rho > \sigma$, $p_1 - p_2$ is +ive, the vapour pressure over the solution s less than that over the pure solvent.

If σ cannot be supposed to be niform, the value of h being great, en the following modification is necessary. At any height $\cancel{x}$ the difference dp f vapour pressure between layers x distant from each other is

$$dp = -\sigma g\, dx$$ where σ is the vapour density at that point

But $p/\sigma = RT \quad \therefore \quad \sigma = \dfrac{p}{RT}$

$$\therefore -\frac{RT}{g}\frac{dp}{p} = dx \quad \text{ie} \quad \frac{RT}{g}\int_{p_1}^{p_2} = \int_{h} d\cancel{x}$$

$$\text{ie} \quad \frac{RT}{g}\log\frac{p_2}{p_1} = -h$$

$$\text{ie} \quad \Big\{ \quad \frac{RT}{g}\log\frac{p_1}{p_2} = - h$$

But $\cancel{p}$ osmotic press. $P = p_2 + hg\rho - p_1$

$$= \cancel{p}\, hg\rho - d \quad \text{where } d = (p_1 - p_2)$$

But $h = RT/g \,\log\left(1 + \dfrac{d}{p_2}\right)$

$$= \frac{RT}{g}\log\left(1 + \frac{d}{RT\sigma_2}\right)$$

$$\therefore d = hg\rho - P = g\rho \frac{RT}{g} \log\left(1 + \frac{d}{RT\sigma_2}\right) -$$

When d is small this eqn reduces to the one derived above

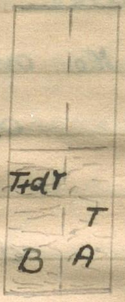
$$d = \rho RT \times \frac{d}{RT\sigma} - P$$

$$= \frac{d\rho}{\sigma} - P$$

$$\therefore d = P\frac{\sigma}{\rho - \sigma}$$ (Edser, Gen Phys

Boiling point of a solution The boiling point is that temp. at which the vapour pressure equals the external pressure. Thus if a solution & a pure solvent in two vessels are raised in temp, the vapour pressure of the sola~~tion being less,~~ pure solvent being grea ~~the~~ it will begin to boil before the solution. The B. P. of a solution is ∴ greater than that of the pure solvent.

Let a vessel separated into two parts by a semimermeable membrane contain the solvent at ~~the~~ its B. P (T) on one side A & the olution at its B. P. (T+dT) on the ther side B. The upper portions in A & B are empty except or the vapour of the solvent, Since the liquid is at its . P. the p pressures in oth the parts are ~~equal~~ the same, both eing equal to the external pressure.



ow, suppose the following cycle of oper-tions to be performed.

1) Force v c.c. of the solvent from to B against osmotic pressure. Work one is pv ergs

2) Evaporate this amount of liquid in B. Heat absorbed is $L_1 v \rho J$, L_1 being atent heat at temp. T+dT, ρ density.

3) Transfer the vapour from B to A. ince pressures on both sides are qual, no work is done in this process.

4) Condense this vapour in A; the eat given out is $L v \rho J$ ergs, L being

L = latent heat at temp. T.

The cycle is reversible & ∴ applying the II Law of thermodynammics

$$\frac{pvLJ}{T} = \frac{L_1 pvJ}{T+dT} = \frac{pv}{dT}$$

$$\therefore \quad dT = \frac{TP}{LpJ}.$$

For a normal solution i.e. one having one gm molecules of solute per litre of water,

$$P = \frac{8\cdot31 \times 10^7 \times 373}{1000}$$

$$dT = \frac{373^2 \times 8\cdot31 \times 10^7}{\cdot96 \times 537 \times 4\cdot2 \times 10^7 \times 1000}$$

$$= \cdot534.$$

The term molecular elevation is often applied to the elevation of B. P. of ~~C~~ a solution of concentration 10 times normal

The B. P. deduced in this way agrees perfectly with expt, thereby confirming the theory of osmotic pressure.

The depression of freezing point ca

e deduced in a somewhat simi-
ar manner. Let $T - dT$ be the temp. of
B.

1) Transfer v cc of liquid from A to B. ~~[illegible]~~
work pv is done. 2) Freeze this quanity.
eat L, ρ v is ~~absorbed~~ liberated 3) Transfer
e ice from B to A. No work is done.
) Melt the ice. Heat L ρ v is absorbed.

dT as before = $\frac{T p}{L \rho J}$.

Since L is only 80, the molecular
epression of F. P. is greater ~~X~~
18·4) than the mol. elevation of
. P. Hence in determinations of os-
motic pressure, the depression of
reezing point is more often utilized
han any other property of solutions.

Osmotic pressure of electrolytes.

The abnormally high value of P
electrolytic solutions is explained
 the basis of Arrhenius' theory of
ectrolytic Conduction., ~~that~~ The theory

says that electrolytic molecules are split up into two or more ions. The number of mol. so split up will be a certain fraction β of the total number of mol. present. In very dilute solution β is one; but as concentration increases β becomes less. Each one of the separat ions collides with the semipermeable membrane & replaces one molecule of water. If P is the osmotic pressure of a non-electrolytic normal solution, P_1 of an electrol normal solution of coefft of ionization β, then

$$\frac{P_1}{P} = \frac{n(1-\beta) + \alpha \times n\beta}{n}$$

$$= \frac{1-\beta+\alpha\beta}{1} = 1+(\alpha-1)\beta.$$

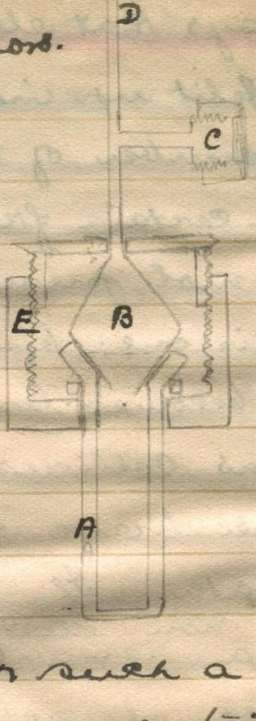
β can be determined from expts on electric conductivity. α is 2 or 3 or 4 according to the nature of the solute & hence the formula

$P_1/P = 1 + (\alpha-1)\beta$ can be verifi

Measurement of osmotic pressure.

Pfeffer's direct measurements were

ubject to many exptal errors. ater observers a utilized hange of vapour pressure, aising of B. P. & most ften lowering of F. P. But est results ~~ob~~obtained are y Morse's direct exps ~~on~~ ith a modified form of Pfeffers apparatus. He took pecial care to obtain a ot of uniform porosity; & such a ot was secured after several trials bo with different clays. Pfeffers ethod of making a chemical precipitate was found unsuitable ~~espec~~ at igh concentration, since the membrane as too weak. Hence he used a process electrical endosmose, which gave a niform strong semipermeable membrane. The pressure over the surface the solution was increased to prevent



osmosis by means of a & screw arrang
ment C. The increase in pressure wa
measured by a closed nitrogen manome
at D previously calibrated. The manometer
& the pressure screw are attached to a
Conical glass bulb B & the glass bulb
tightly fitted into the neck of the porous
pot A by a steel collar E.

(Newman & Searle 289)

Diffusion If two fluids are in contac
which can mix in all proportions, then
they do so due to the migratory movemen
which characterize fluids & form a
uniform mixture. This process is
called diffusion. This takes place
against gravitation & is not a buoya
effect as can be shown by placing
a solution of $CuSO_4$ at the bottom of
a tall vessel & squeezing water from a sponge over to a
piece of cork floated above it. The li
of separation between solution & water i
clear at first, but becomes less so in

ourse of time & finally the whole
essel contains one uniform solution.
raham's pioneer expt in this line to
howed that rate of diffusion varies
ith the kind of solute & increases with
mperature as well as with concentration.
ick's expts gave a mathematical eqn

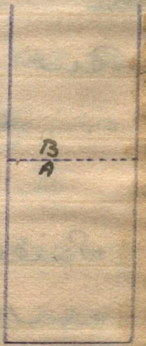
$$q = k A \frac{dc}{dx} t$$, where k is
alled the coefft of diffusion.

Expts for the coefft of diffusion
depend on the application of Fourier's
eorem which expresses in a form con-
aining k the concentration at any pt.
t a * known time subsequent to
known distribution of concentra-
ion. Hence the expt. consists in
etermining the distribution of concen-
ration at any time, & the concentra-
ion at any point after an interval t.
r this several methods have been
dopted. To take out specimens of the

liquid at various points is out of th question since that would introduc currents which completely mask the diffusion effects. Kelvin's expt was simple; he noted the positions of glass beads of varying densities. Later optical methods such as variatio of refractive indices, rotation of plan of polarization (for sugar solution), bending of a ray as it passed throug layers of varying concentrations etc Clark (in 1924) used a Raleigh refractome & obtained very consistent values over a fairly wide range of concentration & for several solutions. (~~St~~

Diffusion & Osmosis

Let C be the concentration (in gm. molecules per cc) at a layer A, & v the upward velocity of molecules in it. The no. of gm. molecules that cross upward in time



t is $dN = vaC\,dt$ a being area of

ection. But $dN = ka\dfrac{dC}{dx}\,dt$ ∴ $vC = k\dfrac{dC}{dx}$.

t A be supposed to be separated by an deal semipermeable membrane from e layer immediately above. If osmotic ressure at A is P, ~~we see~~ the relative smotic pressure is dP. ie a pressure P downward (or a force a dP) on the emipermeable membrane will prevent olecules from crossing the membrane. orce acting on each gm molecule in the ayer is ∴ $\dfrac{a\,dP}{aC\,dx} = \dfrac{1}{C}\dfrac{dP}{dx}$.

If F is the viscous drag per gram molele, i.e. F is the force reqd to give a velo-ity of one cm per sec. to a gm molecule, en $v = \dfrac{1}{CF}\dfrac{dP}{dx}$.

But $P = CRT$ ∴ $dP = RT\,dC$.

$$\therefore k\frac{dC}{dx} = vC = \frac{1}{F}RT\frac{dC}{dx}$$

$$\text{ie } k = \frac{RT}{F}.$$

Since in defining Coeff^{t} of diffusion e unit of ~~ti~~time is a day instead of a second, we have $F = \frac{24 \times 3600\, RT}{k}$.

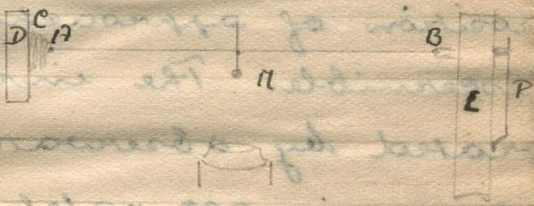
~~So~~ k for formic acid is ·472 at 0°C

$$\therefore F = \frac{24 \times 3600 \times 273 \times 8 \cdot 315 \times 10^{7}}{\cdot 472}$$

$$= 4157 \times 10^{15} \text{ dynes} = \underline{\underline{4 \cdot 249 \times 10^{12}}} \text{ gm.}$$

Gravity Balance.

Threlfall + Pollock



A ~~qu~~ fine quartz fibre AB, 30 cms long, is ~~stretched~~ horizontally between two ~~points A +~~ firm supports D + E. The fastening at end A is made by a spring C which ~~is~~ keeps the fibre in tension. At the end B the fibre can be rotated by a pointer P + the amount of rotation can be measured. A small wire n is fastened to the centre of the ~~string~~ fibre + weighted at one end so that its centre of gravity is on one side of the fibre. Hence to make the wire horizontal it is necessary to twist the fibre ~~to~~ by means of the pointe

. This ~~th~~ position of the wire is only
ust stable since a small additional
ovement of P makes the wire rotate
ompletely. A arrester prevents this excessive
otion, but its tendency to occur makes the
osition of approaching instability readily
iscernible. The instrument is cali-
rated by observations at two places
here g is accurately known. Correction
as to be made for the change in rigi-
ity of the fibre due to differences in
mperature. The instrument is portable,
ives readings quickly & has considerable
ccuracy. (Newman & Searle)

Ol. $= \frac{\pi P a^4}{8\eta l}$; $P = \frac{\pi(a_1^2 - a_2^2)}{16\eta l}$ [illegible]

Vel. $v = \frac{P}{4\eta l}(a^2 - r^2)$; $(p_1 - p_2) = g\rho\left\{h - \frac{v^2}{\pi^2 a^4 g}\right\}$ $v^2 = 2g\frac{\lambda}{4\pi}$

$t = 2\pi\sqrt{\frac{a^3\rho}{8T}}$ Bif. Susp. $mgdy = C\theta d\theta$ ∴ $t = 2\pi\sqrt{\frac{Il}{mgab}}$ Films $P = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Nearly cyl. films $\frac{1}{r} = \frac{y + \left(\frac{Pa^2}{2T} - a\right)}{a^2}$

$T = 2\pi\sqrt{\frac{\pi l^2 (W + w)}{3g\lambda\kappa^2}}$ for osc. Spiral $T = 2\pi\sqrt{\frac{\left(M + \frac{m}{3}\right)a^2}{\frac{1}{2}\pi n b^2/l}}$; $T = 2\pi\sqrt{\frac{I + \frac{ma^2}{4}}{g\lambda k^2/l}}$

Ripples vel = acquired by travelling through $\frac{\lambda}{4\pi}$