$N = 1000a + 100b + 10c + d$

$S = 3d + 2c - b - 3a$

$N = a \cdot 10^5 + b \cdot 10^4 + c \cdot 10^3 + d \cdot 10^2 + e \cdot 10 + f$

$S = 10f - 4e - d + 3c + 4b + a$

$N + 9S = 91f - 26c + 91d + 1027c + 10036b + 100009a$ is $\div$ by 13.

What about divisibility by 17?

$N = a_0 + a_1 \cdot 10^1 + a_2 \cdot 10^2 + a_3 \cdot 10^3 + \cdots$

Let sequence of multipliers (q) be $\alpha_0, \alpha_1, \alpha_2, \alpha_3 \cdots$

$S = a_0\alpha_0 + a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3 + \cdots$

$N + \lambda S = a_0(1 + \lambda\alpha_0) + a_1(10 + \lambda\alpha_1) + a_2(10^2 + \lambda\alpha_2) + a_3(10^3 + \lambda\alpha_3) + \cdots$

$1 + \lambda\alpha_0 = 17K_0$
$10 + \lambda\alpha_1 = 17K_1$
$10^2 + \lambda\alpha_2 = 17K_2$
$10^3 + \lambda\alpha_3 = 17K_3$

$\alpha_1 - 10\alpha_0 = 17(K_0\alpha_1 - K_1\alpha_0)$
$10\alpha_2 - 10^2\alpha_1 = 17(K_1\alpha_2 - K_2\alpha_1)$
$10^2\alpha_3 - 10^3\alpha_2 = 17(K_2\alpha_3 - K_3\alpha_2)$

$\alpha_0(17K_1 - 10) = \alpha_1(17K_0 - 1)$
$\alpha_1(17K_2 - 10^2) = \alpha_2(17K_1 - 10)$
$\alpha_2(17K_3 - 10^3) = \alpha_3(17K_2 - 10^2)$

$\alpha_1 - 10\alpha_0$
$\alpha_2 - 10\alpha_1$
$\cdots$
have all 17 as a factor.

If
$\lambda\alpha_0 = 17K_0 - 1$
$\lambda\alpha_1 = 17K_1 - 10$
$\lambda\alpha_2 = 17K_2 - 10^2$
$\lambda\alpha_3 = 17K_3 - 10^3$
$\cdots$

$\lambda\alpha_0 = 16, 33, 50, 67, 84, 101, 118, 135, 152, 169$
$\lambda\alpha_1 = 7, 24, 41, 58, 75, 92, 109, 126, 143, 160$
(3) (3) (3)

$\lambda = 3, K_0 = 2, 5, 8$
$\alpha_0 = 11, 28, 45$
$\alpha_1 = 8, 25, 43$

$17K_1 - 10 = 24, 75, 129$, $17K_1 = 34, 85, 139$
$K_1 = 2, 5$

$3\alpha_2 = -83, -66, -49, -32, -15, 2, 19, 36, 53, 70$
$\alpha_2 = -22, -5, 12$; $K_2 = 2, 5, 8$

How to derive this sequence ⑧, 3, 2, −1, −3, −2, 1, .. in case of 7?

Here $1 + \lambda\alpha_0 = 7K_0$, $10 + \lambda\alpha_1 = 7K_1$, $10^2 + \lambda\alpha_2 = 7K_2$, ---

$\lambda\alpha_0 = 7K_0 - 1 = \cdots, -29, -22, -15, -8, -1, 6, 13, 20, 27, 34, 41, 48, 55, 62, 69$
$\lambda\alpha_1 = 7K_1 - 10 = \cdots, -80, -73, -66, -59, \cdots, -10, -3, 4, 11, 18, 25, \cdots$

$\cdots, -$

$\lambda\alpha_0 = \cdots, -71, -64, -57, -50, -43, -36, -29, -22, -15, -8, -1, 6, 13, 20, 27, 34, 41, 48, 55, 62, 69, \cdots$
$\lambda\alpha_1 = \cdots, -80, -73, -66, -59, -52, -45, -38, -31, -24, -17, -10, -3, 4, 11, 18, 25, 32, 39, 46, 53, 60, \cdots$

Writing all the eight cases together

	α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7
$\lambda=1$	-1,	7,	2,	3,	-4,	-6,	8,	-5)
$\lambda=2$	8	-5	1	-7	-2	-3	4	6
$\lambda=3$	-6	8	-5	1	-7	-2	-3	4
$\lambda=4$	4	6	-8	5	-1	7	2	3
$\lambda=5$	-7	-2	-3	4	6	-8	5	-1
$\lambda=6$	-3	4	6	-8	5	-1	7	2
$\lambda=7$	-5	1	-7	-2	-3	4	6	-8
$\lambda=8$	2	3	-4	-6	8	-5	1	-7
ordrd remainders	1	-7	-2	-3	4	6	-8	5

234,352,604 × 17

3983,994,268

3,983,994,268

1 × 8 = 8
-7 × 6 = -42
-2 × 2 = -4
-3 × 4 = -12
4 × 9 = 36
6 × 9 = 54
-8 × 3 = -24
5 × 8 = 40
1 × 9 = 9
-7 × 3 = -21

147
103
44

1000000000

Remainders when $1, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, \ldots, 10^{16}$ are divided by 17

are (1, 10, 15, 14, 4, 6, 9, 5, 16, 7, 2, 3, 13, 11, 8, 12, 1

(1, -7, -2, -3, 4, 6, -8, 5, -1, 7, 2, 3, -4, -6, 8, -5, 1, - - - - -)

1 × 8 = 8.
-7 × 6 = -42
-2 × 2 = -4
-3 × 4 = -12
4 × 9 = 36
6 × 9 = 54
-8 × 3 = -24
5 × 8 = 40.
-1 × 9 = -9
7 × 3 = 21

+159.
- 91
68 divisible by 17

30/6/63 – So the whole thing is trivial. Simply find remainders when ($n = 2m+1$)

($1, 10, 10^2, \ldots 10^m, \ldots$) are divided by n

the remainders {modulo (n) if necessary, so that each | ~~mod~~ each remainder | $\leq m$} can be written as

$$[1, r_1, r_2, \ldots r_{m-1}, -1, -r_1 - r_2, \ldots -r_{m-1}.]$$

for 7: $(1, r_1, r_2) = (1, 3, 2)$

for 13 $(1, r_1, r_2, r_3, r_4, r_5) = (1, -3, -4, -1, 3, 4)$ ie still further $(1, r_1 r_2)$ alone are sufficient

Case of 7. $\alpha_1 - 10\alpha_0$ having 7 as a factor does not determine α_0 & α_1 uniquely.

If α_0 be given, say $\alpha_0 = 3$ as is the case, $\alpha_1 - 30$ having 7 as a factor also does not determine α_1 uniquely, for α_1 may be $= 2, 9, -5$ [with $|\alpha_i| \leq 10$]

Ans, $\alpha_2 - 10\alpha_1$ having 7 as a factor means for these three cases

(i) $\alpha_2 - 20$ having 7 as a factor $\rightarrow \alpha_2 = -1, -8, 6$

(ii) $\alpha_2 - 90$ " $\rightarrow \alpha_2 = -1, -8, 6$

(iii) $\alpha_2 + 50$ " $\rightarrow \alpha_2 = -1, -8, 6$

$\alpha_1 - 10\alpha_0 = 7(K_0\alpha_1 \cdot$ $1 + \lambda\alpha_0 = 7K_0$

$10 + \lambda\alpha_1 = 7K_1$

$10^2 + \lambda\alpha_2 = 7K_2$

$10^3 + \lambda\alpha_3 = 7K_3$

$10^4 + \lambda\alpha_4 = 7K_4$

$10^5 + \lambda\alpha_5 = 7K_5$

$10^6 + \lambda\alpha_6 = 7K_6$

$\alpha_0 = 3, \lambda = 2, K_0 = 1$

$\alpha_1 = 2$, " , $K_1 = 2$

$\alpha_2 = -1$, " , $K_2 = 14$

$\alpha_3 = -3$, " , $K_3 = 142$

$\alpha_4 = -2$, " , $K_4 = 1428$

$\alpha_5 = 1$, " , $K_5 = 14286$

100 - 2 = 98

1000 - 6 = 994

10000 - 4 = 9996

100002

How to show $\alpha_6 = \alpha_0$?

$\therefore \alpha_1 - 10\alpha_0 = 7(K_0\alpha_1 - K_1\alpha_0) \rightarrow 2 - 30 = 7(2-6)$ ✓

$2 - 30 = 7(2 - 6)$

$10(\alpha_2 - 10\alpha_1) = 7(K_1\alpha_2 - K_2\alpha_1) \rightarrow 10(-1-20) = 7(-2-28)$

$= 10(-21) = 7(-30)$

$10^2(\alpha_3 - 10\alpha_2) = 7(K_2\alpha_3 - K_3\alpha_2) \rightarrow 10^2(-3+10) = 7(-42+142)$

98

24
18
42

only consider $N = a_1 a_0$, can we find α_0, α_1 ?

$1 + \lambda\alpha_0 = 7K_0$
$10 + \lambda\alpha_1 = 7K_1$ $\rightarrow \alpha_1 - 10\alpha_0 = 7(K_0\alpha_1 - K_1\alpha_0)$

$a_0\alpha_0 + a_1\alpha_1$ is divisible by 7 if $a_0 + 10a_1$ is divisible by 7 & conversely.

56

18
10
28

$a_0 + 10a_1 = 7l$

$a_0\alpha_0 + a_1\alpha_1 = 7m$

49

27
8
35

14 $a_0 = 4, a_1 = 1 \rightarrow 4\alpha_0 + \alpha_1 = 7l_1$ $\quad \lambda\alpha_0 = 7(2l_1 - l_2)$

21 $a_0 = 1, a_1 = 2 \rightarrow \alpha_0 + 2\alpha_1 = 7l_2$ $\quad \lambda\alpha_1 = 7(4l_2 - 4l_1)$

28 $a_0 = 8, a_1 = 2 \rightarrow 8\alpha_0 + 2\alpha_1 = 7l_3$

$N = 10a_1 + a_0$

$S = \alpha_1 a_1 + \alpha_0 a_0$
$= 2a_1 + 3a_0$

$N + 2S = 14a_1 + 7a_0$

with $\lambda\alpha_0 = \ldots, 6, \ldots$

$\lambda\alpha_1 = \ldots, -3, \ldots$

$\lambda \neq 3$

$1 + 3\alpha_0 = 7K_0$
$10 + 3\alpha_1 = 7K_1$

& taking $\lambda = 3$

$N + \lambda S = a_0(1 + \lambda\alpha_0) + a_1(10 + \lambda\alpha_1)$

$1 + \lambda\alpha_0 = 7K_0$ $\quad \frac{7K_0 - 1}{\alpha_0} = \frac{7K_1 - 10}{\alpha_1} = \lambda$

$10 + \lambda\alpha_1 = 7K_1$

$\lambda = 4$

$1 + 4\alpha_0 \longrightarrow \alpha_0 = 4$
$-7 + 4\alpha_1 \longrightarrow \alpha_1 = 6$
$-2 + 4\alpha_2 \longrightarrow \alpha_2 = -8, 9$
$-3 + 4\alpha_3 \longrightarrow \alpha_3 = 5$
$4 + 4\alpha_4 \longrightarrow \alpha_4 = -1$
$6 + 4\alpha_5 \longrightarrow \alpha_5 = -10, 7$
$9 + 4\alpha_6 \longrightarrow \alpha_6 = 2,$
$5 + 4\alpha_7 \longrightarrow \alpha_7 = 3,$

$(4, 6, -8, 5, -1, 7, 2, 3)$

$\lambda = 5.$

$1 + 5\alpha_0 \longrightarrow \alpha_0 = -7, 10$
$-7 + 5\alpha_1 \longrightarrow \alpha_1 = -2$
$-2 + 5\alpha_2 \longrightarrow \alpha_2 = -3$
$-3 + 5\alpha_3 \longrightarrow \alpha_3 = 4$
$4 + 5\alpha_4 \longrightarrow \alpha_4 = 6$
$6 + 5\alpha_5 \longrightarrow \alpha_5 = -8, 9$
$9 + 5\alpha_6 \longrightarrow \alpha_6 = 5$
$5 + 5\alpha_7 \longrightarrow \alpha_7 = -1$

$(-7, -2, -3, 4, 6, -8, 5, -1)$

$\lambda = 6$

$1 + 6\alpha_0 \longrightarrow \alpha_0 = -3$
$-7 + 6\alpha_1 \longrightarrow \alpha_1 = 4$
$-2 + 6\alpha_2 \longrightarrow \alpha_2 = 6$
$-3 + 6\alpha_3 \longrightarrow \alpha_3 = -8, 9$
$4 + 6\alpha_4 \longrightarrow \alpha_4 = 5$
$6 + 6\alpha_5 \longrightarrow \alpha_5 = -1$
$9 + 6\alpha_6 \longrightarrow \alpha_6 = -10, 7$
$5 + 6\alpha_7 \longrightarrow \alpha_7 = 2$

$(-3, 4, 6, -8, 5, -1, 7, 2)$

-55

$\lambda = 7$

$1 + 7\alpha_0 \longrightarrow \alpha_0 = -5,$
$-7 + 7\alpha_1 \longrightarrow \alpha_1 = 1$
$-2 + 7\alpha_2 \longrightarrow \alpha_2 = -7, 10$
$-3 + 7\alpha_3 \longrightarrow \alpha_3 = -2,$
$4 + 7\alpha_4 \longrightarrow \alpha_4 = -3$
$6 + 7\alpha_5 \longrightarrow \alpha_5 = 4$
$9 + 7\alpha_6 \longrightarrow \alpha_6 = 6$
$5 + 7\alpha_6 \longrightarrow \alpha_6 = -8, 9.$

$(-5, 1, -7, -2, -3, 4, 6, -8)$

~~-59~~ ~~-63~~

~~-64~~

-61

-65

$\lambda = 8$

$1 + 8\alpha_0 \longrightarrow \alpha_0 = 2$
$-7 + 8\alpha_1 \longrightarrow \alpha_1 = 3$
$-2 + 8\alpha_2 \longrightarrow \alpha_2 = -4$
$-3 + 8\alpha_3 \longrightarrow \alpha_3 = -6$
$4 + 8\alpha_4 \longrightarrow \alpha_4 = 1, -9, 8$
$6 + 8\alpha_5 \longrightarrow \alpha_5 = -5$
$9 + 8\alpha_6 \longrightarrow \alpha_6 = 1$
$5 + 8\alpha_7 \longrightarrow \alpha_7 = -7, 10$

$(2, 3, -4, -6, 8, -5, 1, -7).$

-76

-74

91

-75

~~4,164,239~~

7,246,239

7239×13

94107

658749×11

7246239

9
5
8

(A) with only $N = a_1 a_0$:

$\lambda = 3$ $\quad 1 + \lambda a_0 = 7k_0 \qquad 10 + \lambda a_1 = 7k_1$

$\lambda = 3, \quad 1 + 3a_0 = 7k_0 \qquad 10 + 3a_1 = 7k_1$

$a_0 = 2$ ~~$a_0 = 9$~~.

$a_1 = -1$

$N = 10a_1 + a_0$

~~$S = 20\,7a_1 + 2a_0$~~ $\quad S = -a_1 + 2a_0$

$N + 3S = 7a_1 + 7a_0 = 7(a_1 + a_0)$ ✓ $\quad (2, -1)$ is possible $\quad 84$

$8 - 8 = 0$

56 → 12 − 5 = 7
49 → 18 − 4 = 14 ✓
63 → 6 − 6 = 0
14 → 8 − 1 = 7
21 → 2 − 2 = 0.
28 → 16 − 2 = 14
35 → 10 − 3 = 7
42 → 4 − 4 = 0

$1 + 3a_0 =$ −29, −26, −23, −20, −17, −14 ✓ $a_0 = -5$, −11, −8, −5, −2, 1, 4, 7 ✓ $a_0 = 2$, 10, 13, ~~16~~, ~~19~~, ~~22~~, ~~25~~, ~~28~~ ✓ $a_0 = 9$, 31.

$10 + 3a_1 =$ −20, −17, −14 ✓ $a_1 = -8$, −11, −8, −5, −2, 1, 4, 7 ✓ $a_1 = -1$, 10, 13, 16, 19, 22, 25, 28 ✓ $a_1 = 6$, 31, 34, 37, 40

ie 9 combinations are possible for (a_0, a_1) with $\lambda = 3$. [Simplest is $(2, -1)$

with $\lambda = 2$: $\quad 1 + 2a_0 = 7k_0, \quad 10 + 2a_1 = 7k_1$

$1 + 2a_0 =$ −19, −17, −15, −13, −11, −9, −7 ✓, −5, −3, −1, 1, 3, 5, 7 ✓, 9, 11, 13, 15, 17, 19, 21 ✓

$a_0 = -4$ $\qquad a_0 = 3$ $\qquad a_0 = 10$

$10 + 2a_1 =$ ~~0~~ −10, −8, −6, −4, −2, 0 ✓, 2, 4, 6, 8, 10, 12, 14 ✓, 16, ~~18~~, 20, 22, 24, 26, 28 ✓, 30

$a_1 = -5$ $\qquad a_1 = 2$ $\qquad a_1 = 9$

ie again ~~nine~~ 9 combinations for (a_0, a_1) with $\lambda = 2$. [Simplest is 3, 2)

14 → 12 + 2 = 14
21 → 3 + 4 = 7
28 → 24 + 4 = 28
35 → 15 + 6 = 21
42 → 6 + 8 = 14
49 → 27 + 8 = 35
56 → 18 + 10 = 28
63 → 9 + 12 = 21

with $\lambda = 1$. $\quad 1 + a_0 = 7k_1$, $\quad 10 + a_1 = 7k_2$

a_0: −9, −8, −7 ✓ $a_0 = -8$, −6, −5, −4, −3, −2, −1, 0 ✓ $a_0 = -1$, 1, ⋮, 7 ✓ $a_0 = 6$, ⋮, 11

a_1: 0 ✓ $a_1 = -10$, 1, 2, 3, 4, 5, 6, 7 ✓ $a_1 = -3$, ⋮, 10, 11, ⋮, 14 ✓ $a_1 = 4$, ⋮, 20

again 9 combinations of (a_0, a_1)

[Simplest being $(-1, -3)$]

14 → −4 − 3 = −7
21 → −1 − 6 = −7
28 → −8 − 6 = −14
35 → −5 − 9 = −14
42 → −2 − 12 = −14
49 → −9 − 12 = −21
56 → −6 − 15 = −21
63 → −3 − 18 = −21

Now let us go to case of divisibility by 17 } which is not given in Davis' book.

Remainders when $1, 10^1, 10^2, \ldots 10^{15}$ are divided by 17 are given by.

(1, 10, 15, 14, 4, 6, 9, 5, 16, 7, 2, 3, 13, 11, 8, 12) repeated

or (1, -7, -2, -3, 4, 6, 9, 5, -1, 7, 2, 3, -4, -6, -9, -5)

10000

~~or (1, -7, -2, -3, 4, 6, 9, 5~~

~~or (1, -7, -2, -3, 4, 6, -4, 5, -1, -6, 2, 3, -4, -6, -4, 5)~~

1	4
-7	6
-2	9
-3	5

$1 + \lambda\alpha_0 = 17k_0$

$-7 + \lambda\alpha_1 = 17k_1$

$4 + \lambda\alpha_5 = 17k_5$

with $\lambda = -4$, consider $1 - 4\alpha_0 = 17k_0$

and $4 - 4\alpha_5 = 17k_5$

$\alpha_0 = -4$, and $\alpha_5 = 1$

~~with λ~~ = what should be λ if $\alpha_0 = 1$, $\alpha_5 = -1$.?

with $\lambda = 16$, $1 + 16\alpha_0 = 17k_0 \rightarrow \alpha_0 = 1$.

$4 + 16\alpha_5 = 17k_5$

$\lambda = -13$, $4 - 13\alpha_5 = 17k_5 \rightarrow \alpha_5 = -1$

$4\alpha_5 \equiv -1$

$4\alpha_6 \equiv -7$

$4\alpha_7 = -2$

$4\alpha_8 = 3$

We need only take $\lambda = 1, 2, \ldots 8$., the values of $(\alpha_0, \ldots)$ obtained for 9, .. 16 being merely those with change of sign.

For $\lambda = 1$,

$1 + \alpha_0 = 17k_0$ $\quad \alpha_0 = -1$

$-7 + \alpha_1 = 17k_1$ $\quad \alpha_1 = -10, 7$

$-2 + \alpha_2 = 17k_2$ $\quad \alpha_2 = 2$

$-3 + \alpha_3 = 17k_3$ $\quad \alpha_3 = 3$

$4 + \alpha_4 = 17k_4$ $\quad \alpha_4 = -4$ $\rightarrow$

$6 + \alpha_5 = 17k_5$ $\quad \alpha_5 = -6$

$9 + \alpha_6 = 17k_6$ $\quad \alpha_6 = -9, 8$

$5 + \alpha_7 = 17k_7$ $\quad \alpha_7 = -5$,

ie (-1, 7, 2, 3, -4, -6, -9, -5)

or (-1, 7, 2, 3, -4, -6, 8, -5)

$\lambda = 2$,

$1 + 2\alpha_0$

$-7 + 2\alpha_1$

$-2 + 2\alpha_2$

$-3 + 2\alpha_3$

$4 + 2\alpha_4$

$6 + 2\alpha_5$

$9 + 2\alpha_6$

$5 + 2\alpha_7$ $\rightarrow$

$\alpha_0 = -9, 8$

$\alpha_1 = -5$

$\alpha_2 = 1$

$\alpha_3 = -7, 10$

$\alpha_4 = -2$

$\alpha_5 = -3$

$\alpha_6 = 4$

$\alpha_7 = 6$

ie (8, -5, 1, -7, -2, -3, 4, 6)

$\lambda = 3$, ~~$\alpha_0 = -6, \alpha_1 =$~~

$1 + 3\alpha_0 \rightarrow \alpha_0 = -6$

$-7 + 3\alpha_1 \rightarrow \alpha_1 = -9, 8$

$-2 + 3\alpha_2 \rightarrow \alpha_2 = -5$

$-3 + 3\alpha_3 \rightarrow \alpha_3 = 1$

$4 + 3\alpha_4 \rightarrow \alpha_4 = -7, 10$

$6 + 3\alpha_5 \rightarrow \alpha_5 = -2$

$9 + 3\alpha_6 \rightarrow \alpha_6 = -3$

$5 + 3\alpha_7 \rightarrow \alpha_7 = 4$

ie (-6, 8, -5, 1, -7, -2, -3, 4).

For any λ : it is similarly seen that for each λ, there are 9 combinations (α_0, α_1) & in each case the simplest may be chosen.

So case (A) with $N = a_1 a_0$ appears trivial having no unique (α_0, α_1). *

Case (B) $N = a_2 a_1 a_0$. – Here $1 + \lambda\alpha_0 = 7k_0$, $10 + \lambda\alpha_1 = 7k_1$, $100 + \lambda\alpha_2 = 7k_2$ or $1 + \lambda\alpha_0 = 7k_0$, $3 + \lambda\alpha_1 = 7k_1$, $2 + \lambda\alpha_2 = 7k_2$ or $\lambda + \lambda\alpha_0 = 7k_0$, $3 + \lambda\alpha_1 = 7k_1$, $5 + \lambda\alpha_2 = 7k_2$

(i) $\lambda = 1$, $1 + \alpha_0 = 7k_0$, $3 + \alpha_1 = 7k_1$, $2 + \alpha_2 = 7k_2$

leading to $\alpha_0 = -8, -1, 6$; $\alpha_1 = -10, -3, 4$; $\alpha_2 = -9, -2, 5$ — ie 27 combinations [simplest being $(-1, -3, -2)$]

$N = 100a_2 + 10a_1 + a_0$

$S = -2a_2 - 3a_1 - a_0$

$N + 5S = 98a_2 + 7a_1 = 7(14a_2 + 1)$.

(ii) $\lambda = 2$; $1 + 2\alpha_0 = 7k_0$; $3 + 2\alpha_1 = 7k_1$, $2 + 2\alpha_2 = 7k_2$

leading to $\alpha_0 = -4, 3, 10$; $\alpha_1 = -5, 2, 9$; $\alpha_2 = -8, -1, 6$ — ie 27 combinations [simplest being $(3, 2, -1)$]

$N + 2S = 100a_2 + 10a_1 + a_0 + 2(-a_2 + 2a_1 + 3a_0)$

$= 98a_2 + 14a_1 + 7a_0$

$= 7(14a_2 + 2a_1 + a_0)$

(iii) $\lambda = 3$; $1 + 3\alpha_0 = 7k_0$, $3 + 3\alpha_1 = 7k_1$, $2 + 3\alpha_2 = 7k_2$

leading to $\alpha_0 = -5, 2, 9$; $\alpha_1 = -8, -1, 6$; $\alpha_2 = -10, -3, 4$ — ie 27 combinations [simplest being $(2, -1, -3)$]

$N + 3S = 91a_2 + 7a_1 + 7a_0 = 7(13a_2 + a_1 + a_0)$

* Note that for each case $\lambda = 1, 2, 3$ above, the three values of each $\alpha_0, \alpha_1, \alpha_2$ are obtained by suitably in A.P with 7 as the adding number, so that the simplest one is unique in a sense [so that the remark in * above is not true.

(iv) $\lambda = 4$, $1 + 4\alpha_0 = 7k_0$, $3 + 4\alpha_1 = 7k_1$, $2 + 4\alpha_2 = 7k_2$

$\rightarrow$ $\alpha_0 = -9, -2, 5$; $\alpha_1 = -6, 1, 8$; $\alpha_2 = -4, 3, 10$ — simplest being $(-2, 1, 3)$

$N + 4S = 112a_2 + 14a_1 - 7a_0$

$= 7(16a_2 + 2a_1 - a_0)$.

(v) $\lambda = 5$, $1 + 5\alpha_0 = 7k_0$, $3 + 5\alpha_1 = 7k_1$, $2 + 5\alpha_2 = 7k_2$

$\rightarrow$ $\alpha_0 = -10, -3, 4$; $\alpha_1 = -9, -2, 5$; $\alpha_2 = -6, 1, 8$ — simplest being $(-3, -2, 1)$

$N + 5S = 105a_2 - 14a_0 = 7(35a_2 - 2a_0)$

(vi) $\lambda = 6$, $1 + 6\alpha_0 = 7k_0$, $3 + 6\alpha_1 = 7k_1$, $2 + 6\alpha_2 = 7k_2$

$\rightarrow$ $\alpha_0 = -6, 1, 8$; $\alpha_1 = -4, 3, 10$; $\alpha_2 = -5, 2, 9$ — simplest being $(1, 3, 2)$.

$N + 6S = 112a_2 + 28a_1 + 7a_0$

$= 7(16a_2 + 4a_1 + a_0)$

Cases $\lambda = 0$ & $\lambda = 7$ need not be considered since that would imply, 1, 10 & 100 being multiples of 7

Also cases $\lambda > 7$ need not be considered since for, eg $1 + 8\alpha_0 = 7k_0$ is equivalent to $1 + \alpha_0 = 7k_0$, –

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(*a*) Strike out the instructions not wanted
(*b*) Here insert any further instructions in clear and legible writing.

Signature of Setter

(YEAR)..... (SYMBOL).....

Candidates are required to give their answers in their own words as far as practicable
The figures in the margin indicate full marks (*a*)
The questions are of equal value (*a*)
(*b*)

What about number of possible values of α_i in this case of 13?

$$10^n + \lambda\alpha_n = 13K_n, \quad n = 0, 1, \ldots 11$$

$$1 + \lambda\alpha_0 = 13K_0$$

with $\lambda = 1$, $1 + \alpha_0 = 13K_0$, $10 + \alpha_1 = 13K_1$, ..

with α_0 taking values from -10 to $+10$ i.e 21 values, there can be only one value of α_0 satisfying the above eqn $\text{I}\left(\frac{21}{13}\right) = 1$

& obviously $\alpha_0 = -1$, is the only value.

Using the eqns
$1 + \alpha_0 = 13K_0$
$10 + \alpha_1 = 13K_1$
$9 + \alpha_2 = 13K_2$
$12 + \alpha_3 = 13K_3$
$3 + \alpha_4 = 13K_4$
$4 + \alpha_5 = 13K_5$
~~$1 + \alpha_6 = 13K_6$~~

lead to
$\alpha_1 = 3$
$\alpha_2 = 4$
$\alpha_3 = 1$
$\alpha_4 = -10$
$\alpha_5 = -9$
~~$\alpha_6 = 12$~~

or $(-1, 3, 4, 1, -3, -4)$ repeated

For $\lambda = 12$, we had on p. 6, $(1, -3, -4, -1, 3, 4)$ repeated.

$\lambda = 4$, $1 + 4\alpha_0 = 13K_0$
$10 + 4\alpha_1 = 13K_1$
$9 + 4\alpha_2 = 13K_2$
$12 + 4\alpha_3 = 13K_3$
$3 + 4\alpha_4 = 13K_4$
$4 + 4\alpha_5 = 13K_5$

$\rightarrow$
$\alpha_0 = 0, -10, 3, 0$
$\alpha_1 = -9, 4$
$\alpha_2 = 1,$
$\alpha_3 = -3$
$\alpha_4 = -4, 9$
$\alpha_5 = -1,$

Simplest being $(3, 4, 1, -3, -4, -1)$.

100

-31
-28
-37,
-36

& so for $\lambda = 9$ we have sequence of Davis $(-3, -4, -1, 3, 4, 1)$
or $(10, -4, -1, 3, 4, 1)$.

$[1, 10, 10^2, \ldots$
$1, 3, 2$

the former being more appropriate

Perhaps there is not much to choose between our $(-1, 3, 4, 1, -3, -4)$ & Davis' version $(-3, -4, -1, 3, 4, 1)$

1 10 100 1000, 10000, 100000
1, -3, -4 -1, 3, 4, ..

Also in the 6 cases of λ_e the simplest values are

$(-1,-3,-2)$, $(3,2,-1)$, $(2,-1,-3)$, $(-2,1,3)$, $(-3,-2,1)$, $(1,3,2)$

Really reducing to three cases $\lambda = 1, 2, 3$ for $x = -6x \pmod 7$
$2x = -5x$ (")
$3x = -4x$ (")
} explaining that in the last three cases of $\lambda = 4, 5, 6$

the values of $\alpha_0, \alpha_1, \alpha_2$ are obtained from the cases $\lambda = 3, 2, 1$ respectively by mere change of sign.

Cases (C) - (E) ie $N = a_3 a_2 a_1 a_0$, $a_4 a_3 a_2 a_1 a_0$, $a_5 a_4 a_3 a_2 a_1 a_0$

ie to consider in addition the eqns $10^n + \lambda\alpha_n = 7K_n$ $(n = 3, 4, 5)$.

$10^3 = 6 \pmod 7 = -1 \pmod 7 = 7m - 1$ $(1, 3, 2, 6, 4, 5)$
$10^4 = 4$ (") $= -3$ (") $= 7m - 3$
$10^5 = 5$ (") $= -2$ (") $= 7m - 2$

Cases $n = 6, 7, \cdots$ etc, $\rightarrow$ $10^6 = 7m - 20 = 7m' - 6 = 7m' + 1$
$10^7 = 7m + 10 = 7m + 3$.
$10^8 = 7m + 30 = 7m + 2$.
$10^9 = 7m + 20 = 7m - 1$
$10^{10} = 7m - 10 = 7m - 3$
$10^{11} = 7m - 30 = 7m - 2$.

$10^6 - 1 \equiv 0 \pmod 7$
$10^7 - 3 \equiv 0 \pmod 7$
$10^8 - 2 \equiv 0$ (")
— —
— —
— —

ie multipliers are repeated as $(1, 3, 2, -1, -3, -2)$, (— —)

since remainders when $1, 10, 10^2, 10^3, 10^4, 10^5$ are divided by 7 are $(1, 3, 2, 6, 4, 5)$

Can we prove that the cases $\lambda = 1, 2, 3$ for case (B) ie $N = a_2 a_1 a_0$ are equivalent?

ie $(-1,-3,-2)$, $(3,2,-1)$, $(2,-1,-3)$

in the three cases $\lambda\alpha_0 = -1, 6, 6$
$\lambda\alpha_1 = -3, 4, -3$
$\lambda\alpha_2 = -2, -2, -9$
} all equivalent (mod 7).

p. 20

So in the case of divisibility by 7, the most natural sequence of multipliers to take is $(1, 3, 2, -1, -3, -2)$ and not $(3, 2, -1, -3, 2, 1)$ as done in Davis.

$1, 10, 10^2 \div 7$
$\rightarrow 1, 3, 2, -1, -3, -2$

Taking the Example on p. 87 of Davis viz $N = 3,201,828$

$8 + 6 + 16 + 1 + 0 + 8 + 3$
$= 42$

$1 \times 8 = 8$
$3 \times 2 = 6$
$2 \times 8 = 16$
$-1 \times 1 = -1$
$-3 \times 0 = 0$
$-2 \times 2 = -4$
$1 \times 3 = 3$
Total $= 28$ (ie divisible by 7).

for $(1, 3, 2)$, $(3, 2, -1)$
$\lambda\alpha_0 = 6, 6$
$\lambda\alpha_1 = 18, 4$
$\lambda\alpha_2 = 12, -2$
} equivalent (mod 7)

Having solved the case of divisibility by 7 let us consider divisibility by 13. Case 13.

Remainders when $1, 10, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}, 10^{11}, 10^{12}$

are divided by 13 are

(i) $(1, 10, 9, 12, 3, 4, 1, 10, 9, 12, 3, 4)$ repeated in view of $10^{12} - 1 \equiv 0 \pmod{13}$, etc

(ii) $(1, -3, -4, -1, 3, 4)$ repeated; say corresponding to λ

In Davis the sequence is (ii) $(10, -4, -1, 3, 4, 1)$. - Suppose these correspond to λ'

$1 + \lambda\alpha_0 = 13K_0$, $10 + \lambda\alpha_1 = 13K_1$, $9 + \lambda\alpha_2 = 13K_2$, $12 + \lambda\alpha_3 = 13K_3$, $3 + \lambda\alpha_4 = 13K_4$, $4 + \lambda\alpha_5 = 13K_5$

In case (i) $\lambda = 12$, and in case (ii) $\lambda = 9$, and $\lambda\alpha_0$ etc are for (ii) $(90, -36, -9, 27, 36, 9)$ and for (i) $(12, -36, -48, -12, 36, 48)$ which are $\equiv \pmod 7$

As in the case of 7 it can be shown from the equations

$$10^n + \lambda\alpha_n = 13K_n \quad (n = 0, 1, \ldots 11)$$

that $\alpha_i = \alpha_{13-i}$ $(i = 1, \ldots 12)$

that for λ and $13 - \lambda$, we have α_n and $-\alpha_n$ resp., for

$$10^n + (13 - \lambda)\alpha_n = 13k_n \text{ is } \equiv 10^n + \lambda(-\alpha_n) = 13K_n$$

Further in this case $\alpha_{6+i} = \alpha_i$ $(i = 0, 1, 2, 3, 4, 5)$

$\alpha_3 = -\alpha_0$
$\alpha_4 = -\alpha_1$
$\alpha_5 = -\alpha_2$

for,

$$10^i + \lambda\alpha_i = 13k_i$$
$$10^{6+i} + \lambda\alpha_{6+i} = 13K_{6+i} \text{ or } (13l_6 + 1)10^i + \lambda\alpha_{6+i} = 13K_{6+i}$$
$$10^i + \lambda\alpha_{6+i} = 13K'_{6+i} \rightarrow \alpha_i = \alpha_{6+i}$$

Again we have further $\alpha_{3+i} = -\alpha_i$ $(i = 0, 1, 2)$, for

$$10^{3+i} + \lambda\alpha_{3+i} = 13K_{3+i}$$

is equivalent to

$$10^i(13l + 12) + \lambda\alpha_{3+i} = 13K_{3+i}$$
$$10^i(13m) - 10^i + \ldots -$$
$$10^i - \lambda\alpha_{3+i} = 13K'_{3+i}$$

but $10^i + \lambda\alpha_i = 13K_i$

we have $\alpha_{3+i} = -\alpha_i$

So in case 13: we have in reference to the sequence given by Davis viz $(10, -4, -1, 3, 4, 1)$, the sequence $(1, -3, -4, -1, 3, 4)$ as found above.

Taking Ex. on p. 88 of Davis viz $N = 3,046,583,852$

2×1	$= 2$	$= 52$
5×-3	-15	-65
8×-4	-32	
3×-1	-3	$= -13$ divisible by 13.
8×3	24	
5×4	20	
6×1	6	
4×-3	-12	
0×-4	0	
3×-1	-3	

$aa' + bb' + cc' = 0$

Block H – No. 45.

Hostel, I.I.Sc, Blore – 12

Book No. 10

(Started at Inst. on 31/10/72)

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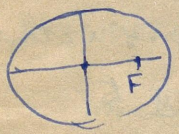
[continued from previous book]

Thus $\frac{x^2}{K^2\cosh^2 u} + \frac{y^2}{K\sinh^2 u} = 1$

or if $a = K\cosh u$, $b = K\sinh u$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Focal distance $= \sqrt{a^2 - b^2} = \sqrt{K^2(\cosh^2 u - \sinh^2 u)} = K$

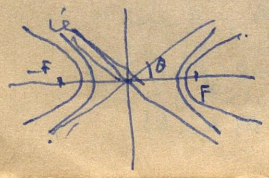
Hence curves u = const are a family of confocal ellipses all having a focal distance K

If $u = 0$, $a = K$, $b = 0$ ∴ this represents a degenerate ellipse and is a st: line from $-K \le x \le K$

Now $\lim_{u\to\infty}\left(\frac{b}{a}\right) = \lim_{u\to\infty}(\tanh u) = 1$. Hence as $u \to \infty$ the ellipses become more & more circular. Therefore all the z-plane is covered if u takes the range $0 \le u \le \infty$.

To obtain curves v = const, write $\cosh u = \frac{x}{K\cos v}$ ∴ $\sinh u = \frac{y}{K\sin v}$, we get $\frac{x^2}{K^2\cos^2 v} - \frac{y^2}{K^2\sin^2 v} = 1$

∴ if $a' = K\cos v$, $b' = K\sin v$, $\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1$ i.e. v = const



represent a family of hyperbolas. Asymptotes are given by $\frac{x}{a'} = \frac{y}{b'}$ & focal distance by

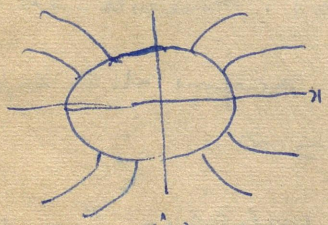
$$F = \sqrt{a'^2 + b'^2} = k$$

∠ angle θ is given by $\tan\theta = \frac{y}{x} = \frac{a'}{b'} = \tan v$

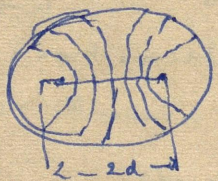
If $v = 0$, we get a straight line starting from F & extending to ∞ along x-axis. This is the case of only a degenerate hyperbola.

Thus from the geometrical configuration of curves $u = \text{const}$ & $v = \text{const}$, we see that with proper choice of u or v to represent the potential for the transformation $z = k\cosh w$ gives solns to the following problems

(1) Electric field round a charged elliptic conducting cylinder



(i)



(ii)

Here let $\phi = u$. The equipotential surfaces are now a family of confocal ellipses & the lines of force are a family of confocal hyperbolas.

Next consider a condenser formed by a flat strip and an elliptic cylinder surrounding this strip as in (ii)

of width of strip be $2d$, where $d = \sqrt{a^2 - b^2}$, the strip is given by the degenerate ellipse $u = 0$.

To find the capacitance C_0 of this cylinder, we determine C_0 by $C_0 = \dfrac{K[v]}{|u_2 - u_1|}$

[Basic eqns of electrostatics are $\nabla \times \vec{E} = 0$, $\nabla \cdot \vec{D} = \rho$, $\vec{D} = K\vec{E}$

From these we can ~~deduce~~ eqn ~~of~~ (4.43) on p. 489 of Pipes]

Here $u_1 = 0$, $\cosh u_2 = \dfrac{a}{K} = \dfrac{a}{d}$

Hence $u_2 = \cosh^{-1}\left(\dfrac{a}{d}\right)$ & $[v] = 2\pi$

Hence $C_0 = \dfrac{2\pi K}{\cosh^{-1}(a/d)}$.

~~Fr. H. Miller's~~ Reddick & Miller – Adv. Math. for Engineers

(1) Applying L.T.

Ex: A particle starts from rest at the point $(-1, 0)$ and moves thereafter in accordance with $D^2x = 2y$, $D^2y = -2x$ where $D = d/dt$. Find the parametric eqns of the path of the particle [Ex. 18 on p. 124 of the book]

(4) This is solved on p. 575 using L.T.

The initial conds are $x=-1,\ y=0,\ Dx=0,\ Dy=0$ for $t=0$ ——— (2)

Eliminating y from $D^2x=2y$ & $D^2y=-2x$ we get ——— (1)

$$D^4x \equiv 2D^2y = -4x$$

i.e. $D^4x + 4x = 0$ ——— (3)

Corresponding to $t=0$, we also get from (1) and (2)

$x=-1,\ Dx=0,\ D^2x=2y=0,\ D^3x=2Dy=0$ ——— (4)

[using $\mathcal{L}\{f^{(n)}(t)\} = p^n \mathcal{L}\{f(t)\} - f(0)p^{n-1} - f'(0)p^{n-2} - \ldots - f^{(n-1)}(0)$]

$$\mathcal{L}\{D^4(x)\} = p^4\mathcal{L}(x) - p^3$$

& hence $\mathcal{L}\{D^4(x)+x\} = p^4\mathcal{L}(x) - p^3 + 4\mathcal{L}(x) = 0$

$$\mathcal{L}(x) = \frac{p^3}{p^4+4} = \frac{1}{2}\left\{\frac{p-1}{p^2-2p+2} + \frac{p+1}{p^2+2p+2}\right\}$$

$$\therefore\ x = \frac{1}{2}(e^{-t}\cos t + e^{-t}\cos t) = \cos t \cosh t$$

whence $y = \frac{1}{2}D^2x = -\sin t \sinh t$. as parametric eqns

Add this for Sept. last eqn asking in the first qn on L.T

the inverse of $\mathcal{L}\{e^{bt}\sin at\} = \dfrac{a}{(b-b)^2+a^2}$

$$\mathcal{L}\{e^{bt}\cos at\} = \frac{b-b}{(b-b)^2+a^2}.$$

(2) Complex variables

(1) Definition of an analytic fn $w = f(z)$ is existence of $\dfrac{dw}{dz}$ independent of path of $\Delta z \to 0$. – Cauchy-Riemann eqns – u & v satisfy eq Laplace's eqn (u & v harmonic conjgts)

– for analytic fn, Jacobian $J = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix} = \left(\dfrac{\partial u}{\partial x}\right)^2 + \left(\dfrac{\partial v}{\partial x}\right)^2$

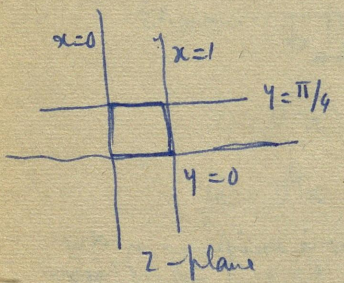
in virtue of C-R. eqns ie $= \left|\dfrac{\partial f}{\partial x}\right|^2 = |f'(z)|^2$

Hence the inverses $x = x(u,v)$, $y = y(u,v)$ exist & are single valued and continuous for any region R_1 of the z-plane in which $f'(z) \neq 0$ and consequently the region R_1 of the z-plane & the corresp. region R_2 of the w-plane [ie where $u = u(x,y)$ & $v = v(x,y)$ are single valued fns possessing continuous first partial derivatives] are mapped one-to-one on each other

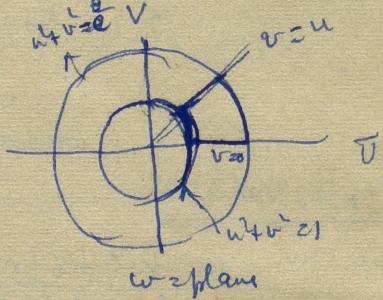
(6) p.415 · Ex. $w = e^z = e^{x+iy} = e^x(\cos y + i \sin y)$, $u = e^x \cos y$,

$v = e^x \sin y$ & by elimination

$v = u \tan y$, $u^2 + v^2 = e^{2x}$



Parallel lines



All lines $y = \text{const}$ map into the pencil of lines through the origin & all lines $x = \text{const}$ map into concentric circles. Rectangular region bounded by $x=0$, $x=1$, $y=0$, $y=\pi/4$ maps onto the sector of a ring bounded by $v=0$, $v=u$ & $u^2+v^2=1$ & $u^2+v^2=e^2$

Since $e^{z+2n\pi i} = e^x[\cos(y+2n\pi) + i\sin(y+2n\pi)]$

$= e^x(\cos y + i \sin y)$ when n is an integer,

the map w_0 of the point z_0 corresponds to all points $z_0 + 2n\pi i$

Hence if we divide the z-plane into horizontal bands by the lines $y = 2n\pi$ & let each strip include the lower but not the upper boundary, then each strip is mapped on to entire w-plane (with exception of point $w=0$ to which no finite z-point corresponds)

by the transformation $w = e^z$.

Exx: (1) For $w = z^3$, show that u & v satisfy Cauchy–Riemann & Laplace eqns & that families $u = c_1$ & $v = c_2$ are orthogonal to each other

(2) For $w = \log z$ defined as inverse of exp. for $z = e^w$ show that (a) writing z in polar form, ~~that~~ $\log z$ is infinitely many valued & (b) u & v satisfy C–R & L-eqns in any region to which the origin is not interior.

p. 458 – use of method of residues to show

$$\int_0^\infty \frac{\sin x}{x}\, dx = \pi/2.$$

5(a) ~~Exhibit on the Argand diagram geometrically~~ use the polar form of a complex number z to derive De Moivre's theorem for a positive integral index. [Churchill, p. 11)

[May]

(b) Show that ~~the result of multiplying z by i~~ when z is multiplied by i, the resulting vector iz is the one obtained by rotating z through a right angle in the counterclockwise direction without changing the length of the vector. (ibid)

(c) Find expressions for the nth roots values of $z^{1/n}$, and derive expressions for the nth roots of unity [Churchill, pp. 14-15]

5 (a) Define the modulus $|z|$, the argument $\arg z$ [Sept] of a complex number and the complex conjugate $\bar{z}$ of a complex number, and prove that

(i) $|z_1 z_2| = |z_1| \cdot |z_2|$; (ii) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

and (iii) $|z_1 + z_2| \leq |z_1| + |z_2|$ [Churchill pp. 8, 11

(b) Using the polar form of z, prove the relations

(i) $1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2\sin\frac{1}{2}\theta}$.

(ii) ~~$\sin\theta + \sin 2\theta + \cdots + \sin n\theta = \frac{1}{2}\cot\frac{1}{2}\theta - \frac{\cos(n+\frac{1}{2}\theta)}{2\sin\frac{1}{2}\theta}$~~

[(a) is obvious, To prove (b) use the geometric series formula

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z} \qquad (z \neq 1)$$

$1 + r(\cos\theta + i\sin\theta) + r^2(\cos 2\theta + i\sin 2\theta) + \cdots + r^n(\cos n\theta + i\sin n\theta)$

$$= \frac{1 - r^{n+1}\{\cos(n+1)\theta + i\sin(n+1)\theta\}}{1 - r(\cos\theta + i\sin\theta)}$$

$1 + (\cos\theta + i\sin\theta)$
$+ (\cos 2\theta + i\sin 2\theta)$
$+ \cdots + (\cos n\theta + i\sin n\theta) =$

$$1 + z + z^2 + \cdots + z^n$$
$$-z \quad -z^3 \quad -z^{n+1} = 1 - z^{n+1}$$

ie $\cancel{1 + z^2 + z^4}$ & R.H.S $= \{1 - r^{n+1}\{\cos(n+1)\theta + i\sin(n+1)\theta\}$

$$\times \{(1 - r\cos\theta)^2 + ir\sin\theta) \Big/ (1 - r\cos\theta)^2 + r^2\sin^2\theta$$

with $r = 1$, & putting R.H.S $= \dfrac{1 - \{\cos(n+1)\theta + i\sin(n+1)\theta\}}{1 - \cos\theta - i\sin\theta}$

$$= \frac{[\quad][1 - \cos\theta + i\sin\theta]}{(1 - \cos\theta)^2 + \sin^2\theta} = \frac{[\,][\,]}{2(1 - \cos\theta)}$$

$$= \frac{[\quad][\quad]}{4\sin^2\frac{1}{2}\theta} = \frac{\{1 - \cos(n+1)\theta - i\sin(n+1)\theta\}\{1 - \cos\theta + i\sin\theta)\}}{4\sin^2\frac{1}{2}\theta}$$

$$= \{1 - \cos(n+1)\theta\}(1 - \cos\theta) + \sin(n+1)\theta\sin\theta$$
$$+ i\left[\sin\theta\{1 - \cos(n+1)\theta\} - \sin(n+1)\theta\{1 - \cos\theta)\}\right] \Big/ \text{"}$$

$$= 1 - \cos\theta - \cos(n+1)\theta + \cos\theta\cos(n+1)\theta + \sin(n+1).\theta.\sin\theta$$
$$+ i\left[\sin\theta - \cos(n+1)\theta\sin\theta - \sin(n+1)\theta + \sin(n+1)\cos\theta)\right] \Big/ \text{"}$$

$$= [1 - \cos\theta - \cos(n+1)\theta + \cos n\theta] + i[\sin\theta - \sin(n+1)\theta - \sin n\theta] \Big/ \text{"}$$

Hence equating real & imaginary parts

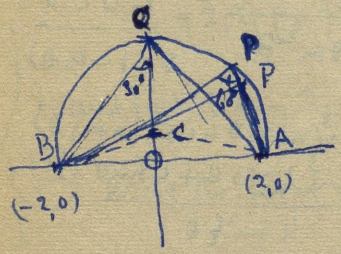
$$\cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1 - \cos\theta - \cos(n+1)\theta + \cos n\theta}{4\sin^2\frac{1}{2}\theta}.$$

(10)

$$= \frac{2\sin^2\frac{1}{2}\theta + 2\sin\frac{1}{2}\theta \cdot \sin\frac{1}{2}(2n+1)\theta}{4\sin^2\frac{1}{2}\theta}$$

$$= \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2\sin\frac{1}{2}\theta}.$$

(C) [for Sept] ~~Find the~~ Interpret geometrically or otherwise the locus, $\text{amp}\left(\frac{z-2}{z+2}\right) = \pi/3$ in the z-plane [Besant Sm. 23 (iii) p. 55]



$\text{amp}(z-2) - \text{amp}(z+2) = \pi/3$

Locus of P such that $A\hat{P}B = \pi/3$ is circle standing on segment AB and passing through the point $Q\,(0, OA\tan 60°) = (0, 2\sqrt{3})$. If $C\,(0, y_1)$ be the centre of this ⊙, $(2-0)^2 + (0-y_1)^2 = (2\sqrt{3}-y_1)^2$ i.e. $y_1^2 + 4 = 12 + y_1^2 - 4\sqrt{3}y_1$

i.e. $4\sqrt{3}\cdot y_1 = 8$ or $y_1 = 2/\sqrt{3} = \frac{2\sqrt{3}}{3}$

and radius $= 2\sqrt{3} - \frac{2\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$

Eqn of ⊙ is $(x-0)^2 + \left(y - \frac{2\sqrt{3}}{3}\right)^2 = \left(\frac{4\sqrt{3}}{3}\right)^2$

$$x^2 + y^2 - \frac{4\sqrt{3}y}{3} + \frac{12}{9} - \frac{48}{9} = 0$$

i.e. $x^2 + y^2 - \frac{4\sqrt{3}y}{3} - 4 = 0.$

Otherwise $z-2 = (x-2)+iy$

$z+2 = (x+2)+iy$

$\therefore \arg(z-2) - \arg(z+2) = \tan^{-1}\frac{y}{x-2} - \tan^{-1}\frac{y}{x+2}$

$\therefore$ locus is given by putting this equal to $\pi/3$. Hence taking tangents

$$\frac{y/x-2 - y/x+2}{1 + y^2/x^2-4} = \sqrt{3}$$

$$\text{i.e. } \frac{y(x+2) - y(x-2)}{(x^2-4) + y^2} = \sqrt{3}$$

$$\text{i.e. } (x^2-4) + y^2 = \frac{4y}{\sqrt{3}} \quad \text{i.e. } x^2+y^2 - \frac{4y}{\sqrt{3}} - 4 = 0 \quad [\text{Cartesian}]$$

Two questions on tensor analysis

4 (a) Define contravariant, covariant and mixed tensors of the second rank, and show that A^m_{npm} is a ~~covariant tensor of the second order~~ (?) the Kronecker delta δ^m_n is a mixed tensor. [May]

[Defining $\bar{A}^{mn} = \frac{\partial \bar{x}^m}{\partial x^i} \cdot \frac{\partial \bar{x}^n}{\partial x^j} A^{ij}$, & etc $\bar{A}^m_n = \frac{\partial \bar{x}^m}{\partial x^i} \cdot \frac{\partial x^j}{\partial \bar{x}^n} A^i_j$

$\bar{\delta}^m_n = \frac{\partial \bar{x}^m}{\partial x^i} \cdot \frac{\partial x^j}{\partial \bar{x}^n} \delta^i_j = \frac{\partial \bar{x}^m}{\partial x^i} \cdot \frac{\partial x^i}{\partial \bar{x}^n} = \frac{\partial \bar{x}^m}{\partial \bar{x}^n} = \delta^m_n$]

(12)

(b) If g_{ij} be a symmetric covariant tensor, explain briefly the notion of the covariant derivative of a vector A_i with respect to g_{ij} and show that this derivative is a tensor.

$$\left[\text{Cov. derivative} = A_{i,j} = \frac{\partial A_i}{\partial x^j} - \left\{ ij, h \right\} A_h \right.$$

$$\left. \text{and this can be shown to be a tensor} \right].$$

Q.4 (a) [Sept] ✓ Define symmetric and skew-symmetric tensors of the second order, and show that any such tensor can be written as the sum of a symmetric a skew-symmetric tensor

(b) If g_{ij} be a symmetric covariant tensor, show how to construct the associated contravariant tensor g^{ij} & verify that

$$g_{mn} g^{pn} = \delta^p_m \qquad g_{mn} g^{pn} = \delta^p_m$$

(c) Defining the Christoffel 3-index symbol $\left\{ ij, k \right\}$ by

$$\left\{ ij, k \right\} = \frac{1}{2} g^{kl} \left(\frac{\partial g_{il}}{\partial x^j} + \frac{\partial g_{jl}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^l} \right)$$

show that $\left\{ ij, i \right\} = \frac{1}{\sqrt{g}} \frac{\partial (\sqrt{g})}{\partial x^j}$, where g is the determinant of the components of g_{ij}

$$\left[\left\{ ij, i \right\} = \frac{1}{2} g^{il} \left(\frac{\partial g_{il}}{\partial x^j} + \frac{\partial g_{jl}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^l} \right) \right.$$

second term in brackets $\frac{1}{2} g^{li} \frac{\partial g_{ji}}{\partial x^l}$ (interchanging dummy indices i & l

and third term $= \frac{1}{2} g^{li} \frac{\partial g_{jl}}{\partial x^l}$ since $g_{ij} = g_{ji}$

ie 2nd & 3rd terms cancel, leaving

$$\{ij, i\} = \frac{1}{2} g^{il} \frac{\partial g_{il}}{\partial x^j} \quad \left[\text{Now } \frac{\partial g}{\partial g_{ij}} = G^{ij} = g \cdot g^{ij} \right.$$

$$\left. = \frac{1}{2g} \cdot \frac{\partial g}{\partial x^j} = \frac{1}{\sqrt{g}} \frac{\partial (\sqrt{g})}{\partial x^j} \right]$$

(e) [Add a bit here from Pauli or etc]

Q. 10 (c) Solve the follow system of diff. eqns using the L.T.
[Sept]

$$\dot{y} + 2\dot{z} + y + 3z = 0 \qquad y(0) = 0$$
$$\dot{y} + 2z - 4y - 8z = \sin t \qquad z(0) = -1$$

[Ans $\dot{y} + y + 2\dot{z} + 3z = e^{-t}$

$$3\dot{y} - y + 4\dot{z} + z = 0 \qquad y(0) = 1, \; z(0) = 0$$

$$\left[\text{Ans: } y = \frac{5e^{t} + 2e^{-2t} - 3e^{-t}}{4} \; ; \; z = \frac{-e^{t} - 5e^{-2t} + 6e^{-t}}{6} \right]$$

[from Wylie, Ex. 7, p. 305]

From Wylie:

p. 54 (1) In any analytic fn $w = u + iv$, (x, y) are replaced by their equivalents in terms of z & $\bar{z}$ viz

$$x = \frac{z+\bar{z}}{2}, \quad y = \frac{z-\bar{z}}{2i}$$

w will appear as a fn of z & $\bar{z}$ alone.

$$\left[\frac{\partial w}{\partial \bar{z}} = \frac{\partial (u+iv)}{\partial \bar{z}} = \frac{\partial u}{\partial \bar{z}} + i \frac{\partial v}{\partial \bar{z}} \right.$$

$$= \left[\frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial \bar{z}} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial \bar{z}} \right] + i \left[\frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial \bar{z}} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial \bar{z}} \right]$$

and $\frac{\partial x}{\partial \bar{z}} = \frac{1}{2}$ & $\frac{\partial y}{\partial \bar{z}} = -\frac{1}{2i} = \frac{i}{2}$

$$\therefore \frac{\partial w}{\partial \bar{z}} = \left[\frac{1}{2} \frac{\partial u}{\partial x} + \frac{i}{2} \frac{\partial u}{\partial y} \right] + i \left[\frac{1}{2} \frac{\partial v}{\partial x} + \frac{i}{2} \frac{\partial v}{\partial y} \right]$$

$$= \frac{1}{2} \left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right] + \frac{i}{2} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]$$

each bracket $= 0$ $\because$ of C-R. eqns

i.e. $\frac{\partial w}{\partial \bar{z}} = 0$ i.e. w is a fn of z alone

(2) p. 550 – Unworked Example.

If $f(z)$ be analytic, show that

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2.$$

(3) p. 548 – BK work

Show that the curves of the family $u(x,y)=c$ are the orthogonal trajectories of the curves of the family $v(x,y)=k$ & vice versa.

(4) p. 568

Th: If $f(z)$ is analytic within and on a circle of radius r with centre at z_0

$$|f^{(n)}(z_0)| \leqq \frac{n!\,M}{r^n} \qquad \text{[Cauchy's inequality]}$$

where M is the maximum value of $|f(z)|$ on C

[Proof: $|f^{(n)}(z_0)| = \left|\frac{n!}{2\pi i}\int_C \frac{f(z)\,dz}{(z-z_0)^{n+1}}\right|$

$\leqq \frac{n!}{2\pi}\int_C \frac{|f(z)|\,|dz|}{|z-z_0|^{n+1}} \leqq \frac{n!}{2\pi}\cdot\frac{M}{r^{n+1}}\int_C |dz|$

$= n!\,M/r^n$

For special case $n = 0$, Cauchy's inequality becomes

$$|f(z_0)| \leq M$$

ie the absolute value of a fn $f(z)$ cannot have a maximum at any point where the fn is analytic

This is the <u>maximum modulus theorem</u>

[<u>Solving Q.10(c) for Sept</u>

$$\left.\begin{aligned} \dot{y} + y + 2\dot{z} + 3z &= e^{-t} \\ 3\dot{y} - y + 4\dot{z} + z &= 0 \end{aligned}\right\} \quad y(0) = 1,\ z(0) = 0$$

Taking L.T's, $sY(s) - y(0) + 2sZ(s) - z(0) + Y(s) + 3Z(s) = \frac{1}{s+1}$

$$3sY(s) - 3y(0) - Y(s) + 4sZ(s) - 4z(0) + Z(s) = 0$$

ie $Y(s)(s+1) + Z(s)(2s+3) = \frac{1}{s+1} - 1 = \frac{-s}{s+1}$

$$Y(s)(3s-1) + Z(s)(4s+1) = 3$$

$$Y(s)\{(s+1)(4s+1) - (2s+3)(3s-1)\} = \frac{-s(4s+1)}{s+1} - 3(2s+3)$$

$$Z(s)\{(2s+3)(3s-1) - (4s+3)(s+1)\} = \frac{-s(3s-1)}{s+1} - 3(s+1)\Big]$$

Too complicated - to be replaced or Q. modified

~~F(s)~~

Q. 10(a) (May) If $\mathcal{L}\{f(t)\} = F(s)$, show that

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$$

and deduce that

$$\mathcal{L}^{-1}[F(s)] = t\,\mathcal{L}^{-1}\left[\int_s^\infty F(s)\,ds\right]$$

Hence find (i) $\mathcal{L}\left[\frac{\sin kt}{t}\right]$ and (ii) $\mathcal{L}^{-1}\left[\frac{1}{(s^2-1)^2}\right]$

[Wylie p. 315 & worked Exx. 11 & 12, p. 310.]

4(b) [May] If $\underline{g}$ is the determinant of a symmetric covariant tensor g_{ij}, show that the cofactors of g_{ij} divided by $\underline{g}$ (denoted by g) are the components of a symmetric contravariant tensor. ~~g^{ij}~~

[Eisenhart, p. 15]

~~4(b) If $a_{ij}\lambda^i\lambda^j = 0$ for an arbitrary vector λ^i, show that the tensor a_{ij} is skew-symmetric~~ [Eckt]

[If $C_{ij}A^iA^j$ is an invariant, $C'_{ij}A'^iA'^j = C_{ij}A^iA^j$

ie $C_{KL}\frac{\partial x^K}{\partial x'^i}\cdot\frac{\partial x^L}{\partial x'^j}\cdot\frac{\partial x'^i}{\partial x^m}A^m\frac{\partial x'^j}{\partial x^n}A^n = C_{ij}A^iA^j$

ie $C_{KL}\delta^K_m\delta^L_n A^m A^n = C_{ij}A^iA^j \sim C_{KL}\delta^K_n\delta^L_m A^n A^m = C_{ij}A^iA^j$

(18) ~~ie~~ $\cancel{C_{nm} A^m A^n = C_{ij} A^i A^j}$

$\cancel{A'_{ij} B'^i C'^j = A_{kl}}$

4(b) [Sept] If B^i and C^j are contravariant vectors, and A_{ij} is a covariant ~~vector~~ tensor, prove that $A_{ij} B^i C^j$ is an invariant.

$$\left[A'_{ij} B'^i C'^j = A_{kl} \frac{\partial x^k}{\partial x'^i} \cdot \frac{\partial x^l}{\partial x'^j} \cdot \frac{\partial x'^i}{\partial x^m} B^m \cdot \frac{\partial x'^j}{\partial x^n} C^n \right.$$

$$= A_{kl} \delta^k_m \delta^l_n B^m C^n = A_{kl} B^k B^l.$$ & hence result.

[Ex. on p. 9 of Barry Spain's Book on Tensors]

This will do

$$\cancel{C'_{ij} \frac{\partial x'^i}{\partial x^k} A^k \cdot \frac{\partial x'^j}{\partial x^l} A^l = C_{ij} A^i A^j}$$

$$\cancel{C'_{ji} \frac{\partial x'^j}{\partial x^k} A^k \frac{\partial x'^i}{\partial x^l} A^l = C_{ji} A^j A^i}$$

~~4(i) If A^i be a covariant~~

B.E (Paper)

4 questions on Mechanics.

2 " on Fourier series & harmonic analysis

3 " Partial diff. Eqns.

Butler's Mechanics – Problems.

(1) A uniform circular plate of radius a and a uniform square plate of side $2b$ are equimomental. Show that $a/b = \frac{2}{\sqrt{3}}$ and the ratio of their surface densities is $3/\pi$

(2) A shell of mass M moving in any direction, bursts into three equal pieces each of mass $\frac{1}{3}M$. If v_{13}, v_{23}, v_{31} are the magnitudes of the mutual relative velocities of the pieces after the explosion, prove that the increase in K.E due to the explosion is $M(v_{23}^2 + \cdots)/18$, motion being supposed two dimensional.

[There are no external impulsive forces during the explosion. Hence by principle of linear momentum

$$\frac{1}{3}Mv_1 + \cdots + \cdots - MV = 0 \qquad \begin{pmatrix} V = \text{velocity of } M \text{ before} \\ v_1, v_2, v_3 \text{ of pieces} \\ \text{afterwards} \end{pmatrix}$$

ie $V = \frac{1}{3}(v_1 + v_2 + v_3)$ —(1)

Resulting gain in K.E $= \frac{1}{2}\left(\frac{1}{3}M\right)v_1^2 + \cdots - \frac{1}{2}MV^2$

$$= \frac{M}{18}\left\{3(v_1^2 + \cdots) - (v_1 + v_2 + v_3)^2\right\}$$

$$= \frac{M}{18}\left\{(v_2 - v_3)^2 + \cdots + \cdots\right\} \quad \because v_{23} = v_2 - v_3 \text{ & } v_{23}^2 = V_{23}^2$$

(3) Worked Ex. (3) on p. 238 (2 dimensional motion)

To finish questions on complex variables nos. 6, 7, 8

6(a) If in any analytic function $w = f(z)$ $= u + iv$, the variables [May] x and y are replaced by

$$x = \frac{z+\bar{z}}{2}, \quad y = \frac{z-\bar{z}}{2i},$$

Replace this in the form of Ex (2), p. 241

use the Cauchy–Riemann equations to show that w will then appear as a function of z alone.

(b) If $f(z)$ obtain expressions for $f'(z)$ in terms of partial derivatives of u and v, and show that

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4 |f'(z)|^2$$

$$\left[|f(z)|^2 = u^2 + v^2, \quad \frac{\partial^2}{\partial x^2}(u^2+v^2) = \frac{\partial}{\partial x}\left\{ 2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x} \right\}\right.$$

$$= 2\left(\frac{\partial u}{\partial x}\right)^2 + 2u\frac{\partial^2 u}{\partial x^2} + 2\left(\frac{\partial v}{\partial x}\right)^2 + 2v\frac{\partial^2 v}{\partial x^2}$$

and $\partial^2/\partial y^2 (u^2+v^2)$

$$\text{Sum} = 2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right.$$

since $u\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = 0.$

Now $f'(z)$ is either $= \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ or $\frac{\partial u}{\partial y} - i\frac{\partial u}{\partial y}$

& hence $L.H.S = 2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right\} + 2\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\}$

$= 2\left|f'(z)\right|^2 + 2\left|f'(z)\right|^2 = 4\left|f'(z)\right|^2$.]

6 (b) Transform the Cauchy-Riemann conditions for the analytic function (except at $z=0$) $f(z) = u + iv$ into polar coordinates, and use these conditions to show that both u & v satisfy the Laplace equation in these coordinates.

[Sept]

[First part Book work, Cauchy-Riemann

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}.$$

$$\frac{\partial^2 u}{\partial r^2} = -\frac{1}{r^2}\frac{\partial v}{\partial \theta} + \frac{1}{r}\frac{\partial^2 v}{\partial r \partial \theta}$$

$$\frac{\partial^2 u}{\partial \theta^2} = -r\frac{\partial^2 v}{\partial \theta \partial r} \quad \therefore r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2}$$

$$= -\frac{\partial v}{\partial \theta} + r\frac{\partial^2 v}{\partial r \partial \theta} + \frac{\partial v}{\partial \theta} - r\frac{\partial^2 v}{\partial \theta \partial r} = 0$$

ie Laplace eqn in (r,θ) for u. Find Laplace eqn for v can be proved]

3/11/72 New Books Section

(1) Polyhedron Models by Magnus J. Wenninger – Cambridge Univ. Press, 1971 [Reference only]

At back of cover: Men have been fascinated for a long time by models of the regular and semi-regular solids. The Greeks knew the simplest of them; since their day the range of figures has extended until 75 are now known, (Cf. J. Fr. Inst. article) called more generally uniform polyhedra. Father Wenninger describes simply and carefully how models of all the known uniform polyhedra and some stellated forms may be constructed. The book is fully illustrated with drawing & photographs. This is the first book to give practical guidance in the making of all these intricate and beautiful solids.

[On cover page – a beautiful stellated solid]

[Mention this to Venkatachaliengar – Also consider the long-range idea of having a book like this Kannada!]

At end of foreword, a reference to a rhombic dodecahedron

discovered in 1960 by Bilinski which would fit snugly into a box of unit height, breadth τ and length τ^2, where τ is the "divine proportion" or 'golden section' $(\sqrt{5}+1)/2$ — foreword by H·S·M·C (obviously Coxeter).

Ref. to a statement of Bertrand Russell

" Mathematics possesses not only truth but supreme beauty, a beauty, cold and austere, like that of sculpture, sublimely pure and capable of a stern perfection such as only the greatest art can show"

Altogether instructions for making 119 types of solids — Also gives some stick models (like K·V's).

(21) At the end of the cover of the book, one finds an advertisement of a book "A book of curves" by E. H. Lockwood published by C.U.P.

(22) Applied functions of a complex variable by A. Kyrala — Wiley Interscience, 1972

Examples: (1) Show that the st. line passing through z_1 and z_2 has the equation $\begin{vmatrix} 1 & z & z^* \\ 1 & z_1 & z_1^* \\ 1 & z_2 & z_2^* \end{vmatrix} = 0$ $\{z^* = \bar{z}\}$

(2) Show that the Cauchy-Riemann equations may be replaced by

$$\frac{\partial f}{\partial \bar{z}} = 0,$$ if f is considered as fn of z and $\bar{z}$.

Ans: $$\frac{\partial f}{\partial \bar{z}} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \bar{z}} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \bar{z}} \qquad \left[\begin{array}{l} 2x = z+\bar{z} \\ 2iy = z-\bar{z} \end{array}\right.$$

$$i\,2\frac{\partial f}{\partial \bar{z}} = \left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right) = 0$$

$$\text{or } 2\frac{\partial f}{\partial \bar{z}} = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$$

[this is the same property given in Wylie, but expressed in a more intelligible form]

(3) Is e^{e^z} analytic? yes since analytic fn of an analytic fn is analytic since, if $f(z)$ and $g(z)$ are analytic

$$\frac{d\,f[g(z)]}{dz} = f'[g(z)]\,g'(z) \text{ exists \& is unique}$$

But $e^z = e^x(\cos y + i\sin y)$; $u = e^x\cos y$, $v = e^x \sin y$

and $\frac{\partial u}{\partial x} = e^x\cos y = \frac{\partial v}{\partial y}$, and $\frac{\partial u}{\partial y} = -e^x\sin y = -\frac{\partial v}{\partial x}$

ie C-R. conds are satisfied \& all derivatives are continuous

14) If $f = f(z, \bar{z})$, show that

$$\nabla^2 f = 4 \frac{\partial^2 f}{\partial z \, \partial \bar{z}}.$$

(15) Bilinear mapping $w = \dfrac{az+b}{cz+d} = \dfrac{a}{c} + \dfrac{bc-ad}{c(cz+d)}$ for which $bc-ad \neq 0$. —— (1)

These mappings are bilinear. Inverse of (1) is

$$z = \frac{b-dw}{cw-a}$$

(i) Property of invariance of cross-ratio of four points under (1)

(ii) (1) → circles & st: lines into circles & st: lines ie a circle → either a circle or a st: line, and a st: line → st: line or circle. This follows from the fact that the form of the eqn for a circle (st: line for $A=0$) is

$$A\bar{z}z + Bz + \bar{B}\bar{z} + C = 0 \quad (\text{where } A \& C \text{ are real})$$

(iii) A simple special case is conjugate inversion w.r.t a circle of radius r and centre at c. This is an ~~adopt~~ analytic mapping

$$w - c = \frac{r^2}{z-c}.$$

Since $z-c = \rho e^{i\phi}$ has an image $w-c = (r^2/\rho)e^{-i\phi}$ it is clear that the circle $|z-c| = r$ of inversion is reflected in a line through

$z = C$, parallel to the real axis, while any point outside the circle ($\rho > r$) is image inside the circle $|w - C| = r$ since for such a point $|w - C| = r^2/\rho < r$. Similarly any point inside becomes a point outside. In the terminology of mapping z-plane onto itself, we would say that the exterior & interior of the circle $|z - C| = r$ were interchanged by the mapping

Exx.

(5) Describe the mapping $w = \frac{1}{2}\left[z + (1/z)\right]$

Since $\frac{dw}{dz} = \frac{1}{2}\left[1 - (1/z^2)\right] = 0$ for $z = \pm 1$, mapping fails to be conformal at these points. Considering the circle $|z| = r$ with $z = re^{i\theta}$, and $w = u + iv$, one has

$$2u = \left(r + 1/r\right)\cos\theta$$
$$2v = \left(r - 1/r\right)\sin\theta$$

so that image of $|z| = r$ is the ellipse

$$\frac{4u^2}{\left[r + (1/r)\right]^2} + \frac{4v^2}{\left[r - (1/r)\right]^2} = 1$$

in the w-plane. This is also the image of $|z| = 1/r$. As $r \to 1$, the semi-major axis approaches 1 and semi-minor

axis approaches zero, so that the outside & inside of the unit circle in the z-plane correspond to the whole w-plane cut by a straight segment between $w=-1$ and $w=+1$. The inverse mapping is double-valued, with $z = w \pm \sqrt{w^2-1}$, the $-$ sign giving a mapping of the cut w-plane onto $|z|<1$, and the $+$ sign giving a mapping of the cut w-plane onto $|z|>1$

(6) Verify that the complex flow-function

$$f(z) = v_\infty \left(z + R^2/z\right)$$

describes a divergence-free, irrotational two-dimensional flow of an ideal fluid past a cylinder of radius R centred at $z=0$ with real velocity v_∞ parallel to the real axis at $z=\infty$

[If ϕ = vel. potential, $f(z) = \phi + i\psi$

a complex velocity $v = \overline{f'(z)} = \dfrac{\partial\phi}{\partial x} - i\dfrac{\partial\psi}{\partial x}$

$$= \frac{\partial\phi}{\partial x} + i\frac{\partial\phi}{\partial y} = v_1 + iv_2$$

by the C-R. eqns, so that divergence of v

$$\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \text{, since } \phi \text{ is harmonic}$$

The curl of $\phi = 0$, since $\dfrac{\partial v_2}{\partial x} - \dfrac{\partial v_1}{\partial y} = \dfrac{\partial^2\phi}{\partial x\partial y} - \dfrac{\partial^2\phi}{\partial y\partial x} = 0$ ~~for the flow $f(z)$~~

(28)

for the quian $f(z)$

$$v = v_\infty \left(1 - \frac{R^2}{\bar{z} z}\right)$$

with w at $z = \infty$, $v = v_\infty$ and

$$\bar{v} z = v_\infty \left(z - \frac{R^2}{z}\right)$$

also $\mathrm{Re}\left(\bar{v} z\right) = v_\infty \mathrm{Re}\left[z - R^2/z\right] = 0$ on $\bar{z} z = R^2$

so the direction of $\underline{v}$ is $\perp^l$ to the surface of the cylinder

(7) find the electrostatic potential of an infinite conducting plane, from which two strips of negligible width have been removed, if the region between the strips is maintained at unit voltage and the region outside the strips is grounded

Ans: Let the region at unit voltage lie on the real axis with $a < \mathrm{Re}\, z < b$, while the grounded region is $\mathrm{Re}\, z < a$ and $\mathrm{Re}\, z > b$. The harmonic measure of the region at unit voltage is

$$\phi = \frac{1}{\pi} \arg\left(\frac{z-b}{z-a}\right)$$

which $\to 1$ as $z \to$ portion at unit voltage and $\to 0$ as $z \to$ the grounded portion

(8) Construct the most general function $f(z)$ analytic and free of singularities in an annulus $a < |z| < b$ with real part vanishing on $|z| = a$.

Ans: For $g(z)$ analytic & free of singularities for $a^2/b \le |z| \le b$ the function

$$f(z) = g(z) - \overline{g}\left(\frac{a^2}{z}\right)$$

has the property on $\bar{z}z = a^2$ that

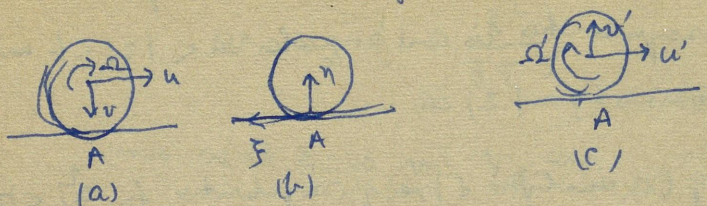
$$\mathrm{Re}\, f(z) = \mathrm{Re}\left[g(z) - \bar{g}(z)\right] = 0.$$

(9) Show that the linear distortion in a bilinear mapping is $\left|dw/dz\right| = [ad - bc] / (cz+d)^2$

Bullen: p. 243 (Mechanics)

Ex (A) A uniform spherical ball moving in a vertical plane & spinning about a diameter at rt $\angle^s$ to the plane of motion with angular velocity Ω impinges on the ground with velocity components u & v (along x-axis & $\downarrow$ y axis) The ground is horizontal and sufficiently rough to prevent slipping; and e is the coefficient of restitution at the impact

(30) Investigate the motion just after the impact



Let M = mass of ball, a its radius & components of impulse at point of contact be (ξ, η) as in (b) and (u', v') velocity components after impact of the centre & Ω' the angular vel. just after impact as in (c).

By Principle of linear momentum

$$M(u' - u) = -\xi \qquad (1)$$

$$M(v' + v) = \eta \qquad (2)$$

By principle of angular momentum

$$\frac{2}{5} Ma^2 (\Omega' - \Omega) = \xi a \qquad (3)$$

Since A is reduced to rest at the impact

$$u' = a\Omega' \qquad (4)$$

By Newton's law of impact

$$v' = ev \qquad (5)$$

From (1), (3) & (4) we have $u' = a\Omega' = (5u + 2a\Omega)/7$ (6)

$$F = 2M(u - a\Omega)/7 \qquad (7)$$

Eqns (5), (6) & (7) determine motion just after impact

Particular case

(a) If $\Omega = u/a$, $F = 0$, $u' = u$, $\Omega' = \Omega$. Thus u & Ω are unaltered after impact

(b) If $\Omega > u/a$ (case of overspin), then F is -ve, $u' > u$ and $\Omega' < \Omega$. Thus spin is reduced & forward velocity increased by the impact

(c) If $\Omega < u/a$, $F > 0$, spin is increased & forward velocity reduced by impact

(d) If $\Omega = -5u/2a$, $u' = 0$, $\Omega' = 0$ ie ball rises vertically without spin after the impact

(e) If $\Omega < -5u/2a$, u' & Ω' are both negative. Thus if before impact the ball has an 'underspin' of magnitude $> 5u/2a$ the ball will bounce backwards.

Ex(B). A uniform rod AB of length 6 ft. is smoothly hinged at A, which is fixed, and is initially at rest with B vertically

below A. find what speed must be given to the lower end B in order that B should just reach a point vertically above A [Ans. $24\sqrt{2}$ ft/sec]

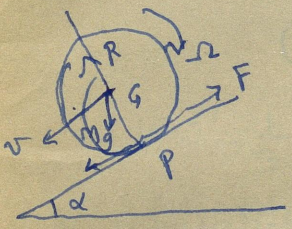
Ex(c). A uniform piece of wire ABC is bent so as to form two straight portions at rt L^s, AB of length a & BC of length b. It swings in a vertical plane freely about B which is fixed. Find the length of the simple equivalent pendulum [Ans. $\frac{2}{3}(a^2+b^2)/(a^4+b^4)^{1/2}$

Ex(d). A uniform rod AB is initially rotating about the end A which is fixed; if the end B is impulsively stopped by an impulse at rt L^s to AB, prove that the magnitude of the impulse at B is twice that at A.

Ex from Charltais Textbook of Dynamics

(α) A O^r hoop of radius a rotating in a vertical plane with spin ω and with its centre at rest, is in contact with a rough plane inclined at an angle α, the angle of friction

for the surfaces in contact also being α. Show that if the initial slip velocity is down the plane, the hoop remains stationary for a time $a\omega/(g\sin\alpha)$, and then the hoop rolls down the plane with accn $\frac{1}{2}g\sin\alpha$ [slip vel. down the plane]



Since centroid has no motion $\perp^r$ to the plane

$$R = Mg\cos\alpha$$ and so during slipping $$F = R\tan\alpha = Mg\sin\alpha \quad —(1)$$

Hence during slipping the resultant force down the plane is zero so that the hoop remains stationary while slipping occurs.

Taking moments about G

$$-F\cdot a = Ma^2\dot{\Omega} \quad —(2)$$

From (1) & (2) $-g\sin\alpha = a\dot{\Omega}$ during impact. Integrating w.r.t. t to t from $t=0$ to $t=\tau$

$$-g\tau\sin\alpha = a\int_0^\tau \dot{\Omega}\,dt = a\int_\omega^0 d\Omega = -a\omega$$

$$\therefore \tau = a\omega/(g\sin\alpha)$$

Now let Ω denote angular velocity at any time t after slipping ceases. This requires

$$v + a\Omega = 0 \qquad (3)$$

For the motion down the plane

$$mg\sin\alpha - F = M\dot{v} \qquad (4)$$

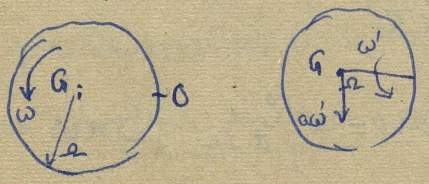
Taking moments about G leads to

$$F = -Ma\Omega \qquad (5)$$

From (3), (4) & (5), $\dot{v} = \frac{1}{2}g\sin\alpha$

Case (b) Ω to the plane

(β) A uniform $\odot^r$ disc of mass M & radius a is rotating in its plane with initial angular velocity ω, its centre being at rest. If a point on the $\odot^{ce}$ be suddenly fixed, find the new angular velocity of the disc & the velocity of the centre



Let O be the point in space coincident with the point on the rim that is to be fixed. As the centroid G is initially at rest the moment of momentum about O before fixing is $0 + \frac{1}{2}Ma^2\omega$.

Just after impact, let ω' be the angular velocity of the disc about O. Then its centroid has velocity $a\omega'$. The moment of momentum about O is now $ma^2\omega' + \frac{1}{2}ma^2\omega' = \frac{3}{2}ma^2\omega'$ since there is no force moment about O.

$$\frac{3}{2}ma^2\omega' = \frac{1}{2}ma^2\omega$$

$$\text{i.e. } \omega' = \frac{1}{3}\omega, \quad a\omega' = \frac{1}{3}a\omega$$

Hence immediately after impact, the disc rotates with angular velocity $\frac{1}{3}\omega$ about O, the centroid moving with velocity $\frac{1}{3}a\omega$.

6(a) complex variable.

Sept: Define an analytic function $w = f(z)$ of a complex variable and show that e^{e^z} is analytic. [p. 24].

7(a) [May] Use Cauchy's integral formula to prove ~~that~~ the ~~generalised~~ general result

$$f^{(n)}(z_0) = \int_C \frac{f(z)\,dz}{(z-z_0)^{n+1}}$$

and deduce that if C be a circle of radius r with centre at z_0, then

$$|f^{(n)}(z_0)| \leq \frac{n!\,M}{r^n}$$

(36) where M is the maximum value of $|f(z)|$ on C.

(b) Evaluate

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+9)(x^2+4)}$$

by means of a suitable contour integral and using the residue theorem

[Ex. 2, p. 601, Wylie modified by taking $a=3, b=2$. For method better explained vide Churchill's related Ex. p. 165]

7 (a) State and prove the Cauchy Integral formula.
[Sept]

(b) Prove that

$$\int_0^{2\pi} \frac{4\, d\theta}{5+4\sin\theta} = \frac{8\pi}{3}$$ [Churchill, Ex. p. 168]

with the aid of residues.

8 (a) For the mapping given by $z = k\cosh w$ $(w = u+iv)$ discuss
[May] the nature of the ~~mapping~~ mapping on the z-plane of curves corresponding to $u = \text{const.}$ & $v = \text{const.}$ Hence derive the nature of the electric field round a charged elliptic ~~cylinder~~ conducting cylinder.
[See pp. 1-2]

(b) For the bilinear transformation

$$w = \frac{az+b}{cz+d} \quad (ad - bc \neq 0)$$

show that (i) circles and straight lines are mapped into circles and st: lines, and (ii) the cross-ratio of four points remains invariant

8 (a) Sept. Indicate briefly the hydrodynamical application of the mapping

$$w = A(z + 1/z) \quad (A \text{ being a real constant}).$$ [Churchill] p. 213.

(b) Find the Mobius transformation ~~which maps the points~~ (i) mapping the points $z = -1, 0, 1$ into the points $w = 0, i, 3i$ resp.

[Wylie, Ex 1, p. 635]

B.E (III) Paper

1. Define the terms: Principal axes of inertia, and equi-momental systems.

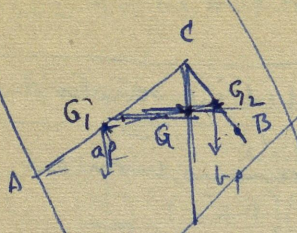
Ex. from Bullen – Problem (1) on p. 19 of this notebook [Soln on envelope on table]

2 (a) Show that in a compound pendulum, the centres of suspension and ~~the centre of~~ oscillation are interchangeable.

~~(b) Ex (C) p. 32 of this note book~~

~~If O be In eqm position G lies below C and CG is given by~~

(38)



G_1G_2 must be horizontal as its moment with vertical through C is G.

Since ACB is a $\perp^{lr}$ & CG is $\perp^{r}$ to AB

$$\frac{CG}{GG_1} = \frac{GG_2}{CG} \quad \text{or} \quad CG^2 = GG_1 \cdot GG_2$$

G divides G_1G_2 in the ratio $\frac{CG}{CG_1} = \frac{CG_2}{G_1G_2}$

$$\frac{GG_1}{GG_2} = \frac{b}{a}$$

$$G_1G_2 = \frac{1}{2}\sqrt{a^2+b^2}$$

$$CG = \frac{CG_1 \cdot CG_2}{G_1G_2} = \frac{a\rho/2 \times b\rho/2}{\frac{1}{2}(a^2+b^2)^{1/2}} = \frac{a\rho b}{2(a^2+b^2)^{\frac{1}{2}}}$$

$$a\rho + b^2\rho = CG(a+b)\rho$$

$$CG = \frac{a^2+b^2}{a+b}$$

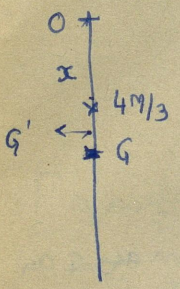
M.I about horizontal axis through C

$$= \frac{4}{3}a\rho \cdot \frac{a^2}{12} + \frac{4}{3}b\rho \cdot \frac{b^2}{12}$$

$$= \frac{1}{9}\rho(a^3+b^3)$$

(b) A uniform thin rod of mass M and length $2a$ is free to turn in a vertical plane about a fixed horizontal axis about the end O. A particle of mass $4M/3$ can be attached to the rod at any point. Prove that the length of ~~simple~~ equivalent simple

pendulum is least when the point of attachment is distant $\frac{1}{2}a$ from O.



$$OG'\left(M+\frac{4M}{3}\right) = x\cdot\frac{4M}{3} + M\cdot a$$

$$K_0'^2\left(M+\frac{4M}{3}\right) = M\cdot\frac{4a^2}{3} + \frac{4M}{3}\cdot x^2$$

$$l' = \frac{K_0'^2}{OG'} = \frac{\left(M\cdot\frac{4a^2}{3} + \frac{4M}{3}\cdot x^2\right)\Big/\left(M+\frac{4M}{3}\right)}{\left(M\cdot a + \frac{4M}{3}\cdot x\right)\Big/\left(M+\frac{4M}{3}\right)}$$

$$= \frac{4(a^2+x^2)}{3a+4x}$$

$$2x(3a+4x) - 4(a^2+x^2) = 0$$

$$6ax + 8x^2 - 4a^2 - 4x^2 = 0$$

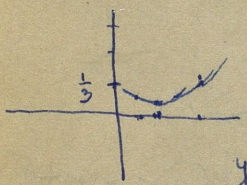
$$4x^2 + 6ax - 4a^2 = 0$$

$$\frac{dl'}{dx} = 0 \text{ if } 2x^2 + 3ax - 2a^2 = 0$$

$\left[x = \frac{1}{2}a \rightarrow \frac{a^2}{2} + \frac{3a^2}{2} - 2a^2 = 0\right]$ ✓ $\longrightarrow x = \frac{1}{2}a$.

$$x = \frac{-3a \pm \sqrt{9a^2+16a^2}}{4} = \frac{-3a\pm 5a}{4} = \frac{a}{2} \text{ or } -2a$$

$x < \frac{1}{2}a$, l'



$$y = \frac{1+x^2}{4x+3}$$

x	0	1/4	1/2	3/4	1	2
y	1/3	17/64	1/4	25/96	2/7	

$$y' = \frac{(4x+3)2x - (1+x^2)4}{(4x+3)^2} = \frac{4x^2+6x-4}{(4x+3)^2}$$

$$y'' = \frac{(4x+3)^2(8x+6) - (4x^2+6x-4)8(4x+3)}{(4x+3)^4}$$

$$= \frac{(4x+3)(8x+6) - (4x^2+6x-4)}{(4x+3)^3} = \frac{2\{(4x+3)^2 - (2x^2+3x-2)\}}{(4x+3)^3}$$

$$= \frac{14x^2+21x+11}{(4x+3)^3}$$

> 0 if $x = \frac{1}{2}a$

(40) $f'(x) = \dfrac{4x^2+6ax-4a^2}{(3a+4x)^2}$

If $x < \frac{1}{2}a$, $f'(x) < \dfrac{a^2+3a^2-4a^2}{25a^2}$ i.e $f'(x) = -ve$

$x > \frac{1}{2}a$, $f'(x) > a^2+3a^2-4a^2$ i.e $f'(x)$ $+ve$

No need to find second derivative

3 (~~a~~) Investigate the motion of a solid sphere starting from rest, down an inclined plane sufficiently rough to prevent slipping of the sphere.

(~~b~~) Verify the result by using the principle of conservation energy.

4 (a) obtain the equations of motion of a rigid body in two dimensions under impulsive forces.

(b) Ex (β) on p. 34 of this note book.

5(a) Expand ~~obtain the Fourier sine series expansion~~

$$\tfrac{1}{8}x(\pi-x) = \sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots \quad (0 \le x \le \pi)$$

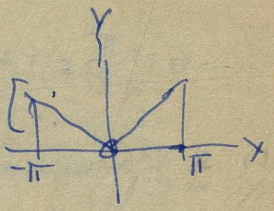
5(a) find (~~i~~) the Fourier (i) sine series, and (ii) cosine series for the function defined by $f(x) = x$ from $x = 0$ to $x = \pi/2$, and $f(x) = \pi - x$ from $x = \pi/2$ to $x = \pi$. [Gibson Ex 122, p. 196]

5(a) (Miller) p. 194

Obtain the Fourier series for the function defined by

$$f(x) = -x, \quad -\pi < x \leq 0$$

$$f(x) = x, \quad 0 < x < \pi$$



[$a_0 = \frac{1}{\pi}$ Continuous everywhere as seen from graph

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,dx$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0}(-x)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$$= -\frac{1}{\pi}\left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{x\sin x}{n} + \frac{\cos nx}{x^2}\right]_{0}^{\pi}$$

$$= -\frac{1}{\pi n^2} + \frac{\cos n\pi}{\pi n^2} + \frac{\cos n\pi}{\pi n^2} - \frac{1}{\pi n^2} = \frac{2}{\pi n^2}(\cos n\pi - 1)$$

Since $b_n = -\frac{1}{\pi}\left[-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_{0}^{\pi}$

$= \frac{1}{\pi}\int f(x)\sin nx\,dx$

$= \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} = 0$

$n = 1, 2, \ldots$

$\therefore$ we get the series

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\right]$$

(42) Putting $x = 0$ in this

$$0 = \frac{\pi}{2} - \frac{4}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right]$$

or $$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

(b) If y be expressed $= f(x)$ is given by the Fourier expansion

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

for $c < x < c + 2\pi$. Show that the root mean square value $\bar{y}$ is given by the equation

$$\bar{y}^2 = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

Reddick [Miller, p. 206–207]

6. Using the table of values given below, plot the curve $y = f(x)$ and analyse it harmonically, starting with the computation of a_5 and b_5

x	0	$\pi/9$	$2\pi/9$	$\pi/3$	$4\pi/9$	$5\pi/9$	$2\pi/3$	$7\pi/9$	$8\pi/9$
y	1.98	1.69	1.70	2.15	2.79	3.11	2.77	1.82	1.67

(i)

x	π	$10\pi/9$	$11\pi/9$	$4\pi/3$	$13\pi/9$	$14\pi/9$	$5\pi/3$	$16\pi/9$	$17\pi/9$	2π
y	−0.22	−0.61	−0.58	−0.31	0.15	0.73	1.43	1.98	2.17	1.98

(ii)

[Ibid Ex. 5, p. 2+3 unworked.

Using the harmonic analysis formulae

$$2m\,(a_m + a_{3m} + a_{5m} + \cdots) = f(0) - f\left(\frac{\pi}{m}\right) + f\left(\frac{2\pi}{m}\right) - \cdots - f\left\{\frac{2m-1}{m}\pi\right\}$$

and $2m\,(b_m - b_{3m} + b_{5m} - \cdots) = f\left(\frac{\pi}{2m}\right) - f\left(\frac{3\pi}{2m}\right) + \cdots - f\left(\frac{4m-1}{2m}\pi\right)$

and taking values up to $m = 5$, we find by computation that a_5 & $b_5 = 0$

and thus we get by going backward, the reqd harmonic analysis expn

$$1.30 + 0.92 \cos x - 0.42 \cos 2x + 0.18 \cos 3x + 1.10 \sin x - 0.68 \sin 2x - 0.21 \sin 3x \,]$$

6. Alternatives - Give Ex 1, p. 83 of Vol I of Doherty & Keller

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
y	2	$3/2$	1	$1/2$	0	$1/2$	1	$3/2$	2

the values of x & y for further intervals of $\pi/4$ repeat themselves.

~~Carry out the~~ Plot the curve $y = f(x)$ and carry out the harmonic analysis up to the third harmonic

[Ans: From the table $f(2\pi - x) = f(x)$. Hence no sine terms can appear since $\sin n(2\pi - x) \neq \sin nx$. Since

(44) $\cos n(2\pi - x) = \cos nx$ for $n = 0$ or a $+^{ve}$ integer. all Cosine terms may be present. Carrying out the analysis in tabular form with columns for y, $\cos t$, $y\cos t$, ~~$y\cos 2t$~~ $\cos 2t$, $y\cos 2t$ $\cos 3t$ & $y\cos 3t$. we find

$$a_0 = 2,\ a_1 = 0.853,\ a_2 = 0,\ a_3 = 0.147$$

Thus $y = f(x) = 1 + 0.853\cos x + 0.147\cos 3x$]

7. (a) Solve the first order partial differential equation:

$$\frac{\partial z}{\partial x} - a\frac{\partial z}{\partial y} = bz \qquad \text{(Berry, Ex. p. 348)}$$

[$(D - aD' - b)z = 0$

Put $aD' + b = m$ & use symbolic method

$(D - mz) = 0$, and

$$z = Ae^{mz} = e^{aD'+b} f(y)$$

$$= e^{bz + az D'} f(y) = e^{bx} \cdot e^{ax\frac{\partial}{\partial y}} f(y)$$

$$= e^{bx} f(y + ax) \,]$$

(b) Solve the second order partial differential equation

~~$\frac{\partial^2 z}{\partial x^2}$~~ $\partial^2 z/\partial x^2 + \partial^2 z/\partial y^2 = xy$ [Berry, Ex. 2, p. 350]

[From the form $(D^2+D'^2)z = xy$

or $(D+iD')(D-iD')z = xy$, the C.F

$$z = f(x+iy) + \varphi(x-iy)$$

for the P.I, $z = \frac{1}{D^2+D'^2}xy = \frac{1}{D^2\left(1+\frac{D'^2}{D^2}\right)}xy$

$$= \frac{1}{D^2}\left(1-\frac{D'^2}{D^2}+\cdots\right)xy = \frac{1}{D^2}xy = \frac{x^3y}{6}$$

since $D'^2xy = D'^4xy = \cdots = 0$.

General soln is $f(x+iy) + \varphi(x-iy) + \frac{1}{6}x^3y$.

8 (a) ~~Using Laplaces equation~~, find the most general solution ψ for spherical waves and ~~deduce~~ show that the wave front is always a sphere with centre at the origin

[[Berry, Ex 2(b) p. 525

[For sph. wave soln u must be a fn of t and r only. Hence we ~~express~~ $\frac{\partial^2 u}{\partial x^2} + \cdots$. Gen. wave eqn is $\frac{\partial^2 u}{\partial t^2} = a^2\left(\frac{\partial^2 u}{\partial x^2} + \cdots + \frac{\partial^2 u}{\partial z^2}\right)$

Expressing $\nabla^2 u$ in polar coords, noting that $\frac{\partial u}{\partial r}$, $\frac{\partial^2 u}{\partial \theta^2}$ & $\frac{\partial^2 u}{\partial \phi^2}$ will each be zero, the eqn becomes

50]

(4b) $$\frac{\partial^2 u}{\partial t^2} = a^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r}\right)$$

writing $v = ru$, the easily transforms to

$$\frac{\partial^2 v}{\partial t^2} = a^2 \frac{\partial^2 v}{\partial r^2}$$

with general soln $v \equiv ru = f(r-at) + \varphi(r+at)$

$$\text{ie } u = \frac{1}{r}\left[f(r-at) + \varphi(r+at)\right]$$

is most general soln & from the form the reqd result is obvious

(6) Find the ~~[illegible]~~ series soln for the transverse vibrations of an elastic string of length, given that the initial distortion is given by

$$y = \frac{2b}{l}x \quad (0 < x < l/2)$$

$$y = -\frac{2b}{l}\cdot x \quad (l/2 < x < l)$$ [Sokolnikoff §109, p. 364-5

[wave eqn is $$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad \text{——(1)}$$

& a particular soln is $y = A\cos Kat \sin Kx$ ——(2)

for arbitrary A & K

If string of length l is distorted into some curve

$y = f(x)$ & then released without receiving any initial velocity.

Soln of (1) is to chosen such that $y = f(x)$ for $t = 0$ and $\frac{\partial y}{\partial t} = 0$ when $t = 0$. Further since string is fixed at the ends, $y = 0$ when $x = 0$ & $x = l$

Consider the ∞-series

$$y = a_1 \cos\frac{\pi a t}{l} \sin\frac{\pi x}{l} + a_2 \cos\frac{2\pi a t}{l} \sin\frac{2\pi x}{l} + \ldots \quad (3)$$

each term of which is of form (2) where K is chosen so that each term reduces to 0 when $x = 0$ & $x = l$. When $t = 0$ the series becomes

$$a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{2\pi x}{l} + a_3 \sin\frac{3\pi x}{l} + \ldots$$

This can be made to represent $f(x)$ if we choose

$$a_n = \frac{2}{l}\int f(x) \sin\frac{n\pi x}{l}\, dx \qquad \text{[Fourier sine series]} \quad (4)$$

4-5

This readily implies that the derivative of (3) w.r.t t satisfies the remaining boundary cond^n $\partial y/\partial t = 0$ when $t = 0$.

Thus (3) with (4) gives formal soln

(48)

For particular initial distortion given the soln is readily found to be

$$y = \frac{8b}{\pi^2}\left[\frac{1}{1^2}\sin\frac{\pi x}{l}\cos\frac{\pi a t}{l} - \frac{1}{3^2}\frac{\sin 3\pi x}{l}\cos\frac{3\pi a t}{l} + \cdots\right]$$

9 (a) Derive the two dimensional ~~form~~ equation of heat conduction ~~in the form~~ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = a^2 \frac{\partial u}{\partial t}$, [Wylie p. 353]

~~of heat conduction~~

(b) ~~Find the steady state temperature distribution in the unit circle if the upper half of the boundary of the circle is kept at the temperature of 100° and the lower half of the boundary is kept at the temperature of 0°.~~ [Wylie, Ex. 13, p. 641 unworked]

Soln (?)

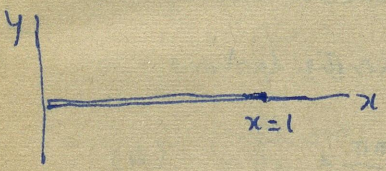
Look up some simple solved problem on 9/11/72

~~Matriculation (General Mathematics)~~

9/11/72

Q.9(b) Sokolnikoff & Sokolnikoff p. 373, §112 (Variable heat flow)

The ends of a rod of small uniform cross-section and of length l, with its surface impervious to heat, are kept at the constant temperature $u = 0$. At time $t = 0$, the distn of temp. along the rod is given by $u = f(x)$. Find the temp. at any point x of the rod at time t.



$$\left[\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, \quad \alpha^2 = k/c\rho \right.$$ (diffusivity ie rate of propagation of temp) —— (1)

Boundary conditions are

$$\left.\begin{array}{ll} u = 0 & \text{when } x = 0 \\ u = 0 & \text{" } x = l \end{array}\right\} \text{for all } t \quad \text{—— (2)}$$

$$u = f(x) \quad \text{" } t = 0$$

Let $u = X(x)\,T(t)$ & (1) gives, after simplification

$$\frac{1}{\alpha^2 T}\frac{dT}{dt} = \frac{1}{X}\frac{d^2X}{dx^2} = -\beta^2 \text{ say (a const)}$$

leading to $\dfrac{dT}{dt} + \alpha^2\beta^2 T = 0$; $\dfrac{d^2X}{dx^2} + \beta^2 X = 0$

Linearly independent solns of these are readily found to be

$$T = e^{-\alpha^2 p^2 t}$$

$$X = \cos \beta x$$

$$X = \sin \beta x$$

Since $u = TX$, possible choices are $u = e^{-\alpha^2 p^2 t} \cos \beta x$, $u = e^{-\alpha^2 p^2 t} \sin \beta x$

The first particular soln does not satisfy the first ~~one~~ condn of (2). If β be chosen as $n\pi/l$, where n is an integer

$$u = e^{-\alpha^2 \left(\frac{n\pi}{l}\right)^2 t} \sin \frac{n\pi}{l} x \qquad \text{——— (3)}$$

satisfies the first two condns of (2) but not the last one.

Consider $u = \sum_{n=1}^{\infty} a_n e^{-\alpha^2 \left(\frac{n\pi}{l}\right)^2 t} \sin \frac{n\pi}{l} x \qquad (4)$

This is a soln of (1) ~~being~~ since each term is with a_n arbitrary.

for $t = 0$, (4) reduces to $\sum_{n=1}^{\infty} a_n \sin \frac{n\pi}{l} x$, which can be made equal to $f(x)$ provided that

$$a_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} \, dx$$

Then

$$u = \sum_{n=1}^{\infty} \left[\frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} \, dx \right] e^{-\alpha^2 \left(\frac{n\pi}{l}\right)^2 t} \sin \frac{n\pi x}{l}$$

Satisfies all the condns & is therefore the required formal soln

Bennett - p. 577 (unworked) example 141

Solve the eqn $\frac{\partial V}{\partial t} = k^2 \frac{\partial V}{\partial x^2}$ given $V = 0$ when $x = l$, $\frac{\partial V}{\partial x} = 0$ when $x = 0$, and $V =$ const. c when $t = 0$. (1)

Ans on p. 772, $V = \frac{4c}{\pi} \sum_{n=0}^{\infty} (-1)^n e^{-\frac{k(2n+1)^2 \pi^2 kt}{4l^2}} \cos \frac{(2n+1)\pi x}{2l}$

[using the previous method of Sokolnikoff, Conds are

$$\left.\begin{aligned} V &= 0, \quad x = l \\ \frac{\partial V}{\partial x} &= 0, \quad x = 0 \end{aligned}\right\} \text{ for all } t \qquad \text{——(2)}$$

$$V = c, \quad t = 0.$$

Let $V = X(x)\,T(t)$. These lead to possible choices

$$V = e^{-\alpha^2 \beta^2 t} \cos \beta x, \quad V = e^{-\alpha^2 \beta^2 t} \sin \beta x.$$

The latter does not satisfy $\frac{\partial V}{\partial x} = 0$, when $x = 0$. Hence if β be chosen as

$$V = \sum_{n=1}^{\infty} a_n e^{-\alpha^2 \left(\frac{n\pi}{l}\right)^2 t} \cos \frac{n\pi}{l} x$$

$$V = \sum_{n=0}^{\infty} a_n e^{-\alpha^2 \left(\frac{2n+1}{2l}\right)^2 t} \cos \frac{(2n+1)\pi}{2l} x \qquad (3)$$

satisfies the first Cond^n $V = 0$ when $x = l$ also.

(52) (3) will be a soln provided a_n be so chosen that for $t=0$, $V=C$ i.e. for $t=0$ (3) reduces to

$$V = \sum_{n=0}^{\infty} a_n \cos \frac{(2n+1)\pi}{2l} x$$

i.e. a_n must be chosen such that

$$a_n = \frac{2}{l} \int_0^l C \cdot \cos \frac{(2n+1)\pi}{2l} \cdot x \cdot dx$$

$$= (-1)^n \cdot \frac{2}{l} \cdot \frac{2l}{(2n+1)\pi}$$

$$= (-1)^n \frac{4C}{\pi} \cdot \frac{1}{(2n+1)} \cdot$$

$$= C \left[\frac{2l}{(2n+1)\pi} \sin \frac{(2n+1)\pi}{2l} x \right]_0^l$$

$$= \frac{2l}{(2n+1)\pi} \cdot \frac{\sin (2n+1)\pi}{2} \cdot$$

$$= \frac{2l}{(-1)^n} \frac{2lC}{(2n+1)\pi}$$

Is answer in Benny correct? Put $t=0$, we get

$$V = \frac{4C}{\pi} \sum_0^{\infty} \cos \frac{(2n+1)\pi}{2l} x$$

$$= \frac{4C}{\pi} \left[\frac{\cos \pi x}{2l} + \cos \frac{3\pi x}{2l} + \cdots \right] ?$$

$$V = \frac{4C}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} e^{-k(2n+1)^2 \pi^2 t / 4l^2} \cdot \cos \frac{(2n+1)\pi x}{2l} \quad (*)$$

no, the form (*) with $\frac{1}{2n+1}$ in denominator is correct

1(a) Two prime numbers which differ by 2 are called twin primes. Find the twin primes between 55 and 75.

(b) Simplify (1) $\dfrac{8/11 - 3/7}{1 + \frac{8}{11}\times\frac{3}{7}} \div \dfrac{1}{4\frac{9}{23}}$ (2) $0.08\dot{3} + 0.\dot{4}\dot{8} + 0.\dot{3}\dot{6}$

(b) Express the sum: $0.08\dot{3} + 0.\dot{4}\dot{8} + 0.\dot{3}\dot{6}$ as a decimal.

(c) Find the square root, correct to two decimal square root of 93 correct to two decimal places (9.64)

2(a) Two persons, A and B whose salaries together amount to Rs 1,000/- spend 75% and 70% of their salaries per cent of their salaries respectively. If their savings are as 5 : 4, find their actual salaries.

2(b) Three persons whose salaries, the sum of whose salaries per month is Rs 1,440/- save $33\frac{1}{3}$%, 30% and 25% of their salaries respectively. If their expenses are as 40 : 56 : 75, find the salary of each. (Percentage & ratio & proportion)

(a) A man walks from his house to a town 10 Km. away at 6 Km./hr., and cycles back immediately at 18 km/hr. Find the average speed for the journey. (average)

3 (a) ~~The price of an article is marked at Rs 144 which~~ The marked price of an article is Rs 144/- and ~~it is~~ this price is ~~priced~~ 20% more than the cost price. If the shopkeeper allows a trade discount of 5 paise in the rupee, find (a) the net gain percent. [Redbook, p.37). [Pavate Ex. 30, p.32]

3 (b) A manufacturer makes a profit of 50%, the wholesaler 40%, and the retailer 25% on sale of an article. If its cost price to the manufacturer is Rs 16, find at what price the retailer sells it. (~~This~~ Redbook, p.38) [Profit & loss]

~~4(a)(b)(c)~~

4(a) [Red book, Ex. 3, p.61]

(b) Ex. 3, p.224 of the Madras Book.

~~5(a) Simplify by removing brackets:~~

~~5(a) If $a-b=1$, and $ab=6$, find a and b~~

5 (b) If a and b are positive, ~~and~~ and $a+b=5$, ~~and~~ $ab=6$, find a and b.

(c) ~~(b)~~ If $a+b+c=9$, and $ab+bc+bc+ca+ab=28$, find the value of $a^2+b^2+c^2$.

(a) Show that $(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)=0$

6(a) Solve the equations

$$\frac{x}{3}-\frac{y+1}{4}=\frac{3x+5y}{4}=x-2$$

[→ $8x+3y=21$, and
$x-5y=8$ giving $x=3$, $y=-1$]

(b) Ten years ~~ago~~ hence a man will be twice as old as his son, but eight years ago the man was eight times as old as his son. Find their presentages & verify the result.

[$x+10=2(y+10)$ i.e. $x-2y=10$
$x-8=8(y-8)$ $x-8y=-56$

→ $x=32$, $y=11$]

7 (a) ~~Show how to construct an angle of 60° using ruler and compasses only.~~ and 30° and 45°

(b)

7(a) Show how to construct angles of ~~90°, 45°, 60° and 30°~~ 60° and 90° 45° respectives using ruler and compasses only.

(b) A wire is to be stretched between the tops of two electric poles 11 ft. and 16 ft. high, standing vertically 12 ft. apart. Find the length of the wire required to connect their tops.

(c) ~~Give the construction for~~ Draw a st: line AB, 3.5" long, and divide it into three equal parts. [Join (A) & (C), draw $90°$

8 (a) A cow tethered to a rope $3\frac{1}{2}$ ft. long has its rope trebled in length. How much greater space is ~~it~~ it allowed now than before? [$\pi = 22/7$

$$\left[\ \pi\left\{\left(\frac{21}{2}\right)^2 - \left(\frac{7}{2}\right)^2\right\} = \pi \times 14 \times 7 = 22 \times 14 = 308\ \text{sq ft}\right.$$

(b) 8 dozen lead balls spheres each of diameter 3 cm. are melted and recast in the form of a solid cylinder 8 cm. in diameter. Find its height.

9 ~~(a)~~ Plot the following points on a graph, and find the equation of the straight line on which they lie.

x	-3	-1	1	2	4
y	-3	1	5	7	11

[$y = 2x + 3$

(b) The time of oscillation of a simple pendulum of varying length was found as follows

Length (in feet)	1	2	3	4	9	16	25
Time (in seconds)	1.1	1.6	1.9	2.2	3.3	4.4	5.6

Extra Draw ~~on the same graph~~ also a circle of radius 1.5 with centre at the origin, and find ~~the points coordinates~~ from the graph the ~~coordinates~~ points of intersection of this circle with the straight line

~~10 (a) Explain the terms: pie diagrams and bar diagrams in use in statistics.~~

~~(b)~~

~~10(a)~~ 10. Find the arithmetic mean, median and the mode for the following distribution:

x	f
0 - 9	32
10 - 19	65
20 - 29	100
30 - 39	184
40 - 49	288
50 - 59	167
60 - 69	98
70 - 79	46
80 - 89	20
Total	1,000

$$A.M = \frac{\Sigma fx}{\Sigma f} = \frac{43240}{1000} = 43.24$$

Median = 43.6

Using Mean - Mode = 3 (Mean - Median)

we get mode = 44.32

(15/1/72) – New Books Section (5-8)

(1) Introduction to Particle Physics by Roland Omnès
Translated by G. Barton (Wiley Interscience, 1970)
- Seems to be quite an elementary book and an excellent introduction to the subject.

(2) Algebraic methods in statistical mechanics and quantum field theory by Gerard Emch. (Wiley Interscience 1972)
Interscience monographs & texts in Physics & Astronomy
edited by R.E. Marshak
- All kinds of Topics brought together, quite an advanced book.
Topics :- 1. General motivation
2. Global themes
3. Canonical commutation & anticommutation relations
4. Quasilocal themes

(3) Basic math course for electronics by Henry Jacobowitz
- Tab Books – Blue Ridge Summit PA, 17214

Chapter 1 – Ohm's law arithmetic

Chapter 2 - Network algebra

Chapter 3 - From AC to complex numbers

Chap 4 - Complex numbers

Chapter 5 - Logarithms

Quite an interesting elementary book with funny illustrations for eg p. 13 on powers of 10, p. 19 on precise answer, p. 27 on percentage of permissible error, p. 37 graph of $y = 4 + 2k$ & similarly on several other pages - All with funny figures of men holding different types of sign boards explaining notions. - Specially on p. 129 relating to logarithms (Fig of a slight case of log m minus log n)

- Quite a good model for writing an elementary book ~~in Kannada~~ - might be used for the book re. Childrens Book Trust.

(4) American J. Maths Vol. 79, No. 7 - August - Sept, 1972

Quite an elementary type of journal

p. 761 - on a problem of Golamb on Powerful numbers

by Andrzej Majewski (60)

Golomb defined a powerful number as a positive integer which for every prime number p is divisible by p^2 provided it is divisible by p. He asked whether there exist positive integers $\neq$ ~~0~~ 1 and 4 which are in infinitely many ways representable as the difference of two relatively prime powerful numbers.

Gives a simple proof of 7 lines

17/11/72 — Attended Eric Lorde's Lecture from 3 to 4 P.M.

(1) Book — The number of things (Pythagoras, geometry and humming strings) by Evans G. Valens
[Methuen & Co, London, 1965.

— Elementary but interesting.

(2) Mathematical Sciences and Social sciences Edited by William Kruskal (Prentice Hall, 1970)

— Five contributors — This book brings into focus

the absurdity of the remarks made by V.K.R.V. Rao the other day at the seminar.

22/11/72 Biographical Memoirs of the Royal Society. (Vol. 13)

H. Bhabha (by Lord Penney) p. 40. – In this phase of his work he displayed knowledge of and skill in methods of modern algebra to a degree quite ~~unknown~~ unusual among theoretical physicists of the time. – – – Particles of higher spin are today part of the observed external world.

Bangalore – Reader in 1940 (Sir Dorab Tata Trust) & Professor in 1942. At Bangalore he was indeed in a class of his own. The papers which he published were more mathematical in nature than his earlier work. – While his work at Bangalore was less recognised internationally, it appeared to give him greater intellectual pleasure than to the earlier & more celebrated work he done while in Europe.

Worked 5 years at I.I.Sc. –

Vol. 17 – H. Max Born by Kemmer & Schlapp. (Born 11/12/1882)
(published 360 papers etc from 1915 to 1968

(married 1913)
Born dies 5/1/1970 & is survived by his widow
a son, and two daughters.

(2) Computers in number theory Edited by Atkin & Birch
Academic Press, 1971. (Proceedings of the Science
Research Council Atlas Symposium No. 2. held at
Oxford from 18 - 23 August, 1969.

1st article - The economics of number theoretic computations
(pp. 1-9) by D. H. Lehmer.

p. 115 - Article by Mordell on repⁿ of an integer as the sum
of four integer cubes

p. 99 - on generalized Δ^r 1 Δ numbers by A. S.
Fraenkel (Israel)

Lots of other interesting problems.

31/12/72 – At Home.

~~Queries for Guwahati 1973.~~

(a) 12/2/73 – At the Institute again after nearly 3 months thanks to Cataract operation of the right eye on 16/12/72

New Books Section

(1) ✓✓ The mathematical theory of symmetry in solids (Representation theory for point groups and space groups) by C.J. Bradley and A.P. Cracknell (Clarendon Press, Oxford, 1972)

– A huge book of 745 pages, comprehensive with a fabulous bibliography (references to almost all Indian workers

Chap 1. Symmetry & the solid state (1.2 is group theory – 1.3 is point groups & 1.4 is space groups)

Chap 2 – Symmetry-adapted functions for the point groups.

Chap 3 – Space groups

Chap 4 – The reps of a group in terms of the reps of an invariant subgroup.

Chap 5 – Single valued reps of the 230 space groups

Chap. 6 – The double valued reps of the 32 point groups and the 230 space groups.

Chap 7 – The magnetic groups & their co-representations

(2) Non-linear partial diff. eqns in Engineering by Vol IV by W. F. Ames (Academic Press)

– Very interesting material specially Chap. 4 on numerical methods (in § 4.0 Intron. – reference to non-linear Navier-Stokes eqns).

– In § 4.11, p. 269 talks of vector Laplacian with the notation $\vec{\Delta}\vec{A} = \text{grad div } \vec{A} - \text{curl curl } \vec{A}$ (*)

[In footnote remarks that except in rectangular coordinates $\vec{\Delta}\vec{A}$ and $\nabla^2\vec{A}$ are different.]

{(*) In rectangular coordinates, the usual formula is given as $\text{curl curl } \vec{A} = \text{grad div } \vec{A} - \nabla^2\vec{A}$}.

(3) The uses of Argument by S. E. Toulmin – Cambridge Univ. Press, 1969 – Very interesting for a general reading.

Green's fn:

$L(y) = p_0 y^{(n)} + \cdots$

(A) $L(y) = r(x)$

[Mathews] Soln given by $y = \int_{x_0}^{x} K(x,\xi)\, r(\xi)\, d\xi$

$K(x,\xi) = \text{Green's fn} = \dfrac{D(x,\xi)}{p_0(\xi)\, W(\xi)}$

(B) Hilbert - Courant ($n = 2$): $L(y) = py'' + p'y' - qy$. $\left(p' \neq \frac{dp}{dx}\right)$

(1) $L[K] = 0$ except at $x = \xi$, where K has a singularity $\rightarrow L(y) = -\varphi(x)$

(2) $\lim_{\epsilon \to 0} \left.\dfrac{d(K,\xi)}{dx}\right|_{x=\xi+\epsilon}^{x=\xi-\epsilon} = -\dfrac{1}{p(\xi)}$ $\qquad \left[y = \int_{x_0}^{x} K(x,\xi)\, \varphi(\xi)\, d\xi \right]$

(3) $K(x,\xi) = K(\xi, x)$

(4) Connection with integral equation theory:

consider $L(y) + \lambda \rho(x) y = \psi(x)$ -- (1) $\quad [\rho(x) > 0]$

with $\varphi(x) = \lambda \rho . y - \psi(x)$, soln is

$$y(x) = \lambda \int_{x_0}^{x} K(x,\xi)\, \rho(\xi)\, y(\xi)\, d\xi + g(x) \quad \text{-- (2)}$$

where $g(x) = -\int_{x_0}^{x} K(x,\xi)\, \psi(\xi)\, d\xi$. -- (3)

Soln of (1) is equivalent to soln of integral eqn (2). The homogeneous eqn $L(y) + \lambda \rho y = 0$ is equivalent to the integral eqn

(66) $$y(x) = \lambda \int_{x_0}^{x} K(x,\xi)\, p(\xi)\, y(\xi)\, d\xi \qquad (\text{with } q \equiv 0)$$

or putting $y(x)\sqrt{p(x)} = z(x)$ and ~~$K(x,\xi) =$~~ $\underline{K}(x,\xi) = K(x,\xi)\sqrt{p(x)\,p(\xi)}$, we have

$$z(x) = \lambda \int_{x_0}^{x} \underline{K}(x,\xi)\, z(\xi)\, d\xi \qquad \cdots (3)$$

It then follows from integral eqn theory that we can find the ∞-set of eigenvalues $\lambda_1, \lambda_2 \ldots \lambda_n$ & corresp. eigenfns $y_1 \ldots y_n$ of the diff. eqn (1) & general soln $v(x) = \sum_{n=1}^{\infty} c_n y_n(x)$, $c_n = \int_{x_0}^{x} w\rho\, y_n\, dx$ –

Also orthogonality and completeness of the system of eigenfunctions can be proved. – Also connection with resonance theory in oscillations.

(5) Construction of Green's function in the ordinary and generalised sense – with the ∞-set of above eigenfns given we can get

$$K(x,\xi) = \sum_{n=1}^{\infty} \left[y_n(x)\, y_n(\xi) / \lambda_n \right]$$

(6) Partial Diff. Eqns of 2nd order of type $\Delta v = -\Phi(x,y)$ – use of Green's fn $K(x,y;\xi,\eta) = K(\xi,\eta;x,y)$ – Soln v

$$v(x,y) = \iint_G K(x,y;\xi,\eta)\, \Phi(\xi,\eta)\, d\xi\, d\eta$$

– corresponding eigenvalue problem.

(7) Examples of Green's functions (involving transcendental fns)

(8) Green's fns of Δv for $\mathcal{D}^{le}$ of sphere. (Bessel fn)

(9) Green's fn and conformal representation – Potential eqn

$\Delta \Lambda = 0$ with $\frac{\partial \Lambda}{\partial x} = -\frac{\partial K}{\partial y}$; $\frac{\partial \Lambda}{\partial y} = \frac{\partial K}{\partial x}$ [Case of $\mathcal{D}^{le}$]

(10) Green's fn ~~of for~~ potential equation for a spherical surface.

15|2|73 Institute

(11) Springer Tracts on Modern Physics – Vol. 43

Recent developments in Lattice theory by W. Ludwig

p. 155 – The phonon Green's function – For the investigations of the dynamics of lattices we have seen the Green's function to be a powerful tool. Especially in scattering problems the Green's function formulation is the best one. – Talks of pure mechanical Green's fn & thermodynamic Green's function (p. 282).

2|3|73 At Institute – New Books Section

The method of weighted (MWR) residuals and variational Principles by Bruce A. Finlayson

(68)

Part I - The method of weighted residuals

Chap 1 - Introduction

Chap 2 - Boundary-value problems in heat and mass transfer

Chap 3 - Eigen-value & initial-value problems in heat and mass transfer.

Chap 4 - Applications to Fluid Mechanics

Chap 5 - Chemical Reacting Systems.

Chap 6 - Convective instability problems

Part II

Chap 7 - Intro to variational analysis

Chap 8 - Variational Principles in Fluid Mech.

Chap 9 - " in heat & mass transfer problems

Chap 10 - on the search for variational Problems

Chap 11 - Convergence & Error bounds.

(Quite a beautiful book)

Questions for Andhra M.Sc (Tech) – GeoPhysics 1973.

Section B

Define the spherical ~~excess~~ excess, and

9(a) Show that the area of a spherical triangle is given by $E r^2$, where E is the spherical excess & r the radius of the sphere [Benny, p. 636]

(b) ABC is a spherical triangle in which side $c = \pi/2$. Show that $\cos A = \frac{\cos a}{\sin b}$; $\cos B = \frac{\cos b}{\sin a}$; $\cos C = -\cot a \cot b$

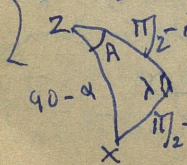
[using cosine rule, $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \longrightarrow \cos A = \frac{\cos a}{\sin b}$

putting $C = \pi/2$

Simily $\cos B = \frac{\cos b - \cos c \cos a}{\sin c \sin a} \longrightarrow \cos B = \frac{\cos b}{\sin a}$

$\cos C = \frac{\cos c - \cos a \cos b}{\sin a \sin b} \Longrightarrow -\cot a \cot b$]

10(a) Define the terms altitude and azimuth of a star, and show how the declination of ~~the~~ star ~~of Knowing right ascension~~ can be found ~~to~~ when ~~the~~ its altitude & azimuth are ~~given~~ known for a place of latitude λ

[

Using cosine rule

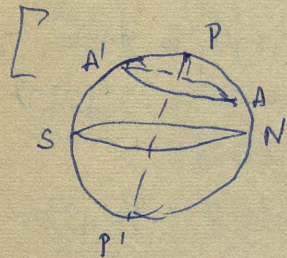
$\sin \delta = \sin \alpha \sin \lambda + \cos \alpha \cos \lambda \cos A$]

[α = altitude]

(70)

(b) Show how the latitude of a place can be found by ~~the~~ observing the meridian altitudes of ~~two~~ a circumpolar ~~stars~~ star.

[Benny, p. 639]



$$\lambda = \frac{1}{2}(\alpha_1 + \alpha_2)$$

$$\alpha_1 = NA = NP - PA$$

$$\alpha_2 = NA' = NP + PA'$$

Adding $\alpha_1 + \alpha_2 = 2NP$, since $PA = PA'$]

Part A.

Q.1(a) If $\underline{a}$, $\underline{b}$, $\underline{c}$ be the position vectors of three points A, B, C respectively referred to any origin O, ~~show that~~ find the position vector of the centroid G of the triangle ABC.

[Benny, unworked Ex. 9, p. 170, unworked]

[Ans: $3(\underline{A}+\underline{B}+\underline{C})$

(a) ~~(b)~~ Evaluate the scalar triple products

~~(b)~~ $[\underline{B}+\underline{C},\ \underline{C}+\underline{A},\ \underline{A}+\underline{B}] \longrightarrow = 2[\underline{A}\,\underline{B}\,\underline{C}]$

and $[\underline{B}-\underline{C},\ \underline{C}-\underline{A},\ \underline{A}-\underline{B}] \longrightarrow$ ed.

(b) ~~(c)~~ ~~For~~ four vectors $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$, prove that $(A\times B)\times(C\times D)$

$$= [CDA]B - [CDB]A = [ABD]C - [ABC]D$$

2 ~~(a)~~ (b) State Green's theorem, and ~~put~~ derive its cartesian form

$$\iiint (U\nabla^2 V - V\nabla^2 U)\,dx\,dy\,dz = \iint\left(U\frac{\partial V}{\partial n} - V\frac{\partial U}{\partial n}\right)dA$$

~~and deduce that~~ $\cancel{\iiint \frac{1}{r}\nabla^2 V\,dx\,dy\,dz =}$ [Benny Bk work, pp. 577–78]

2 (a) Prove that (ii) curl grad $\underline{A} = 0$, and div Curl $\underline{A} = 0$.

(i) div $[\underline{A}\ \underline{B}] = \underline{B}$ Curl $\underline{A} - \underline{A}$ curl $\underline{B}$.

[Bookwork]

~~2(c)~~ [1(b) is bookwork in Wylie, p. 472]

Section B.

7 (a) Derive ~~the perpendicular form of a plane $lx + my + nz = p$~~ of a ~~plane, and~~ find the eqn of the plane through the point $(-3, 2, 5)$ which is $\perp^r$ to the two planes ~~$2x + 3y$~~ $2x + y + 3z - 1 = 0$ and $3x - y + 2z + 4 = 0$

[Bennyworth Ex. 1, p. 140]

(b) Show that the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+2}{1}$ is $\frac{5}{6}\sqrt{6} = \frac{5}{\sqrt{6}}$.

[Unworth Ex. 16, Benny, p. 150

[In general S.D = ~~Σ~~ between $\frac{x-\alpha}{l} = \ldots$ & $\frac{x-\alpha'}{l'} = \ldots$

is $\Sigma\,(\alpha - \alpha')(mn' - m'n) \Big/ \sqrt{\Sigma\,(mn' - m'n)^2}$

Substitute for ~~α, β, γ & δ~~ the particular case]

6 (b) The tangent plane at (x, y, z) on the ellipsoid $x^2/a^2 + \ldots = 1$ cuts the coordinate axes in L, M, N. Show that the area of the triangle LMN is $a^2b^2c^2/2px, y, z$,

[Benny, Unworth Ex 13, p. 161]

(72) (c) Describe with a sketch the surface represented by

$x^2 + 2y^2 = 4z$. [Benny V D, Q.1 (iv)

(Q) Find the eqn of the right circular cone with vertex at O, axis along the line $x = y = z$ and semi-vertical angle 45°. [Benny V D, Q.6

[Ans: $x^2+y^2+z^2-4yz-4zx-4xy=0$

[Soln as in worked Ex 2, p.151]

3(a) Prove that

(i) $\lim_{x\to\infty} (x^n e^{-x}) = 0$, ~~and (ii) $\lim_{x\to 0} (x^n \log x) = 0$~~,

n being a +ve integer [Gibson Exx. 9 & 10, p.99]

(b) ~~Show that~~ Find the sum to infinity of the series

$\frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \cdots$

[$\log_e 2$ using $\log(1+x)$ & collecting 2 terms each]

[$(1 - \frac{1}{2}) + (\frac{1}{3} - \frac{1}{4}) + \cdots$

(c) Prove that the coeff of x^n in the expansion

$$1 + \frac{1+2x}{1!} + \frac{(1+2x)^2}{2!} + \frac{(1+2x)^3}{3!} + \cdots = \frac{2^n e}{n!}$$

[use Binomial & exponential series]

4(a) State the mean-value theorem, and use it to show that if $f''(x) = 0$ for every point x of (a,b), then $f(x)$ is cont. in (a,b)

(b) Evaluate

$$x \tan x - \frac{\pi}{2} \sec x, \text{ when } x = \pi/2 \qquad [\text{Gibson Ex 8, p 422}$$

$$\left[\lim_{x \to \pi/2} \frac{x \sin x - \pi/2}{\cos x} = \lim_{x \to \pi/2} \left[\frac{\sin x + x \cos x}{-\sin x}\right] = -1\right.$$

(c) use Taylor's theorem to derive the series expansion of $\sin x$.

5(a) If $u = f(x, y)$, and x, y are functions of t, find

$$\frac{d^2u}{dt^2}. \qquad [\text{Gibson, Ex. 4, p. 223}]$$

(b) Find the value of $\int x^2 dv$ taken throughout the volume v of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ [Gibson, Ex. 2, p. 339

$$\left[\text{Integral} = \int x^2 dx \int dy dz = \int_{-a}^{a} x^2 dx \left[\pi bc\left(1 - x^2/a^2\right)\right]\right.$$

$$= 4\pi a^3 bc/15\Big]$$

6(a) If $f(x) = 0$ from $x = -\pi$ to $x = 0$, and $f(x) = 1$ from $x = 0$ to $x = \pi$, ~~show that~~ derive the Fourier expansion

$$f(x) = \frac{1}{2} + \frac{2}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right)$$

[Gibson, Worked Ex. 3, p. 486]

(b) Explain the notions of complete, singular, and general integrals of a partial differential equation. ~~Illustrate~~

(74) obtain these integrals for the equation $F(P,Q)=0$,
where $p = \partial z/\partial x$ and $q = \partial z/\partial y$ [Ramanathan, pp 735 (Vol III) -36

10/3/73 - At Institute - New Books Section

(1) Functional Analysis by Michael Reed & Barry Simon (Vol I)* (Acad. Press N.Y) - 1972

~~- This is to be a three volume series as mentioned in the Preface.~~

Introduction - Contains very interesting remarks about maths & Physics etc

* In the series of modern mathematical Physics
this is the first volume -

Contents - Preliminaries - Hilbert spaces -
Banach spaces - Topological spaces - Locally convex spaces - Bounded operators - The Spectral theorem - unbounded operators (enclosing math. problems in quantum mechanics, Section 11, p. 302)

Vol II to be Analysis of Operators (5 Chapter)

Vol III to be Operator Algebras 7 Chapters.

(Both vols. deal with applns in quant. mech)

(2) 250 problems in elementary number theory by Waclaw Sierpinski — Elsevir, N.Y, London and Amsterdam — This book is No. 26 of the Series "Modern Analytic and Computational methods in Science & Technology" edited by Richard Bellmann.

— Sierpinski is from the Polish Academy of Sciences

— From Book Cover reverse re this ancillary textbook is intended for everyone interested in number theory. It will be of special value to instructors and students both as a textbook and a source of reference in mathematical study groups"

— Other books in this Bellman series include a book in preparation by T. Parthasarathy & T. E. S. Raghavan

(76) On some topics in 2-person games.

Contents of Sierpinski's book.

1 - Divisibility of numbers
2 - Relatively prime numbers
3 - Arithmetic Progression
4 - Prime and composite numbers.
5 - Diophantine equations.
6 - Miscellaneous.

Book contains solutions to the problems also

Quite a beautiful book (may be used for setting problems in Math. Newsletter)

Problems on Complex Variables for Andhra (Chem. Engg) Papers.

(1) Small book by E. G. Phillips.

(1) Show that in a conformal trn $w = f(z)$ [$f'(z) \neq 0$ in the domain in z-plane], show that the amplified magnification is $|f'(z)|^2$.

If Δ be closed domain in the w-plane corresp. to a closed domain D in the z-plane, area A of Δ is given by $A = \iint_{\Delta} du\,dv = \iint_{D} \left\{ \frac{\partial(u,v)}{\partial(x,y)} \right\} dx\,dy$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2$$

by the Cauchy-Riemann equations, but as we have seen

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 = \left| \frac{\partial w}{\partial x} + i\frac{\partial v}{\partial x} \right|^2 = \left| f'(z) \right|^2$$

$$\therefore A = \iint_{D} |f'(z)|^2\, dx\,dy \quad \text{— Proved.}$$

(2) Bilinear transformation

If we write eq^n of a circle as

$$A z\bar{z} + Bz + \bar{B}\bar{z} + C = 0 \qquad (A, C \text{ real}) \quad (1)$$

& then substitute $z = \frac{-\delta w + \beta}{\gamma w - \alpha}$ [inverse to $w = \frac{\alpha z + \beta}{\gamma z + \delta}$ (3)]

$$\therefore \bar{z} = \frac{-\bar{\delta}\bar{w} - \bar{\beta}}{\bar{\gamma}\bar{w} - \bar{\alpha}}, \text{ we get}$$

$$D w\bar{w} + Ew + \bar{E}\bar{w} + F = 0 \qquad (2)$$

where $D = A\delta\bar{\delta} - B\bar{\gamma}\delta - \bar{B}\gamma\bar{\delta} + C\gamma\bar{\gamma}$

$F = A\beta\bar{\beta} - B\bar{\alpha}\beta - \bar{B}\alpha\bar{\beta} + C\alpha\bar{\alpha}$

are both real, and also that coeffts of w & $\bar{w}$ in (2)

are conjugate complex. Hence (2) represents a circle in the w-plane since it is of the same form as (1)

(3) Geometrical inversion under B.T (3)

Theorem – The B.T. transforms two points which are inverse w.r.t circle (1) into two points which are inverse w.r.t (2)

Proof :– If z & z_1 be inverse points w.r.t circle (2) it can be shown that z & z_1 satisfy eqn (1) and also

$$A z_1 \bar{z} + B z_1 + \bar{B} \bar{z} + C = 0 \quad \text{—(4)}$$

[If K be centre of $\odot^{le}$, P, P_1 the inverse points & corresponding complex nos be K, z, z_1 condns for inverse is $KP \cdot KP_1 = r^2$ ie

$$(z_1 - K)(z - K) = r^2 \text{ \& } \arg(z_1 - K) = \arg(z - K)$$

second condn arising since K, P, P_1 are collinear. Both these eqns are satisfied if & only if

$$(z_1 - K)(\bar{z} - \bar{K}) = r^2 \quad \text{—(5)}$$

writing (1) as $\left(z - \frac{\bar{B}}{A}\right)\left(\bar{z} + \frac{B}{A}\right) = \frac{B\bar{B} - AC}{A^2}$ —(1')

ie circle with centre $\left(-\bar{B}/A\right)$ and radius $\sqrt{\frac{(B\bar{B}-AC)}{A^2}}$

Hence eqn (3) becomes

$$\left(z_1+\frac{\bar{B}}{A}\right)\left(\bar{z}+\frac{B}{A}\right)=\frac{B\bar{B}-AC}{A^2}$$

which on simplification reduces to (4).

[Borrow this book]

Benny

(1) If $\alpha'=\frac{[\beta\gamma]}{[\alpha\beta\gamma]}$, $\beta'=$, $\gamma'=$.

(α',β',γ') are said to be reciprocal to (α,β,γ).

$$(\alpha'\alpha)=\frac{([\beta\gamma]\alpha)}{[\alpha\beta\gamma]}=\frac{[\alpha\beta\gamma]}{[\alpha\beta\gamma]}=1$$

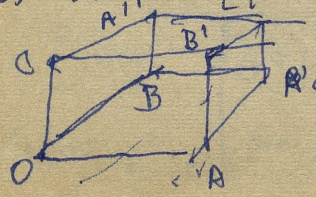
Simf $(\beta'\beta)=(\gamma'\gamma)=1$.

and $(\alpha'\beta)=(\alpha'\gamma)=(\beta'\alpha)=(\beta'\gamma)=(\gamma'\alpha)=(\gamma'\beta)=0$

$\because (\alpha'\beta)=\frac{([\beta\gamma]\beta)}{[\alpha\beta\gamma]}=0$

(2) Any $\xi=\frac{[\xi\beta\gamma]}{[\alpha\beta\gamma]}\alpha+\cdots+\cdots$

(3) Interpret $[\beta+\gamma,\ \cdots\ ]=2[\alpha\beta\gamma]$ geometrically



ie $\square^d$ with sides OA', OB', OC'

= twice $\square^d$ of sides OA, OB, OC

(80) (4) From $(\alpha \times \beta) \times (\gamma \times \delta) =$ - $=$ -, to prove that

$$([\alpha\beta])^2 + (\alpha\beta)^2 = (\alpha)^2 (\beta)^2$$

If α, β be unit vectors & $\widehat{\alpha\beta} = \theta$, this $\longrightarrow \sin^2\theta + \cos^2\theta = 1$

(5) $[\alpha+\beta+\gamma, \beta+\eta, \gamma] = [\alpha\beta\gamma]$ — worked Ex. 13 (ii) p.187

{ L-H.S $= (\alpha+\beta+\gamma) \cdot [\beta+\gamma, \gamma] = \alpha \cdot [\beta+\gamma, \gamma] + 0 + 0$
$= \alpha \cdot [\beta, \gamma] + \alpha [\gamma, \gamma] = [\alpha\beta\gamma]$. }

(5) If α', β', γ' be reciprocal to α, β, γ prove that

$$[\alpha'\beta'\gamma'][\alpha\beta\gamma] = 1$$

{Using $[\beta\times\gamma, \gamma\times\alpha, \alpha\times\beta] = [\alpha\beta\gamma]^2$, result follows.}

(6) If α', β', γ' be reciprocal to α, β, γ, show that α, β, γ are reciprocal to α', β', γ'

(7) Any vector $\underline{r} = (r\alpha)\alpha' + (r\beta)\beta' + (r\gamma)\gamma'$

(8) If $\underline{P} = a_1\alpha + a_2\beta + a_3\gamma$; $Q = b_1\alpha + -$, $R = c_1\alpha + -$

prove that $[PQR] = \begin{vmatrix} a_1 & a_2 & a_3 \\ - & - & - \\ - & - & - \end{vmatrix} [\alpha\beta\gamma]$

(9) Prove that $[PQR][\alpha\beta\gamma] = \begin{vmatrix} [P\alpha] & [P\beta] & [P\gamma] \\ - & - & - \\ - & - & - \end{vmatrix}$

{use (7) (8) & (5)}

(10) $\nabla[\nabla(\alpha\beta)]$ being a constant vector, show that

(i) Show: $[\nabla[r u]u] = (u\nabla)r - (\nabla r)u$ [r scalar]

(i) $\left(\underline{a}\,\nabla\right)\frac{1}{r} = \frac{-(\lambda r)}{r^2}$ ($\underline{a}$ being a const vector

(11) Show that $[\nabla[\nabla\alpha]] = \nabla(\nabla\alpha) - (\nabla\nabla)\alpha$

Margenau & Murphy – Maths of Physics & Chemistry

(1) For tensor g_{mn} define $g^{mn} = \frac{G^{mn}}{g}$ [g = det; G^{mn} = cofactor of g_{mn} in g; g_{mn} symm]

$g_{mn}\, b^{bn} = \delta_m^b$ [from exp^n of dets]

$g_{mn} B^n = g^{mn} B_m$

Let A^n be a vector, then $B_m = g_{mn} A^n$ is also a vector, moreover

$$g^{mn} B_m = g^{mn} g_{mb} A^b = \delta_b^n A^b = A^n$$

so that g^{mn} changes a covariant vector into a contra-variant vector & hence it must itself be a tensor

[Sepn: $\bar{A}^m = \frac{\partial \bar{x}^m}{\partial x^i} A^i$ & $\bar{A}_m = \frac{\partial x^i}{\partial \bar{x}^m} A_i$]

Vectors $A^m = g^{mn} A_n$ and $A_m = g_{mn} A^n$ are called associated

(82) Suil $A^{mn} = g^{mp} g^{nq} A_{pq}$

and Also $A_{mn} B^{mn} = A^{mn} B_{mn}$ &

$A_{mn} B^{b}_{n} = A^{n}_{m} B^{p}_{n}$

$A^{n}_{m} = g_{mp} A^{pn}$ & $A^{n}_{m} = g_{pm} A^{np}$ are not identical

unless A is a symmetrical tensor

At Institute – 22/3/73

New Books Section

(1) Concepts of space – The history of theories of Space in Physics by Max Jammer with a foreword by Albert Einstein

– Camb. Mass. 1969

The last chapter on "Space in modern science appears quite interesting.

(2) Seeing Sound by Winston E. Kock (83)
(References)
Wiley Interscience 1971
– Many examples of visual presentation of phenomena relating to sound – Mentions in preface "Seeing is believing"

(3) Contributions to the theory of weak and strong interactions of elementary particles by V B. Berestetskii (Editor) – Translated from Russian & published by U.S. Atomic Energy Commission – Many interesting articles, especially on the S-matrix

(4) Multivariate Data Analysis by William W Cooley & Paul R. Lohnes – Quite a modern & advanced book on Statistics – John Wiley, 1971

Contents:

(84)

Part I - ~~Chapt~~ Intro

Chap 1 - Multivariate analysis and computer programming

Chap 2 - The basic vectors & matrices

Part II - Studies of a single population

Chap 3 - Multiple correlation and the method of least squares.

Chap. 4 - Principal components

Chap 5 - Selected factor analysis procedures

Chap 6 - Canonical correlation

Chap 7 - Multiple partial correlation

Part III - Multiple population studies

Chap. 8 - Multivariate analysis of variance

Chap. 9 – Discriminant Analysis

Chap. 10 – Classification procedures.

Chap 11 – Multiple covariance analysis

Chap 12 – Factorial discriminant analysis

Chap 13 – Strategy considerations in multivariate research.

References & appendices

– At end of each chapter Exercises are given

Appears elementary, comprehensive & a fine text book (?)

2/4/73 New Books Section

(1) Categories & groupoids by Phillip. J. Higgins* – Von Nostrand Reinhold Mathematical Studies 32.

[* King's College, London]

– quite a nice book – mentions in the Introduction how

(86) the theory of Categories of groupoids have" provides a good model for certain aspects of homotopy theory. In it there are algebraic analogues of such notions as path, homotopy, deformation, covering & fibration - In Chap. 17 " groupoids are used to compute fundamental groups". [For Calcutta Higher Algebra topic?]

p. 1 - Defines category - "morphisms" "functors" etc.

p. 5 - A groupoid A is a category in which every edge is invertible (edge = morphism)

(2) Transmission of information by orthogonal functions

by Henning. F. Harmuth - Springer Verlag (1972) - 2nd Edition (enlarged & modified)

From Preface: Emphasis change from sine-cosine fns (sinusoidal fns) to non-sinusoidal fns - The former are projections of the exponential fn which, in turn, is the Character group of the real numbers. The topology of the real numbers is

formally accepted to be the same as that of time or of one-dimensional space, and this is the basis for a variety of claims that sinusoidal fns are unique & superior to all others.

Time may well have the same topology as the real numbers, but we will never know. We can only make a finite number of measurements & there must be a finite distance between them, either in space or time. The topology of time or space which permits experimental verification is thus that of a finite number of integers. Small as this theoretical point might appear, it provides enough material uncertainty to admit at least denumerably many systems of functions to applications that were previously believed to be reserved for sine-cosine functions. Asynchronous filters and mobile radio communications based on nonsinusoidal functions are carried in this book all the way to block circuit diagrams, primarily to help demonstrate this point.

. . . The long years between the dawn of a

(88) Theoretical conception and working experiment can only be bridged in a country that tolerates unsinusoidal actuates

—

An introductory introduction of 8 pages dealing with the topics: Historical background and motivation for the use of non-sinusoidal functions — orthogonal fns, Walsh functions and other basic mathematical concepts — filtering of Time & space signals — Direct and carrier transmission of signals — Non-sinusoidal electromagnetic waves — Statistical theory of Communication

Read through the Introduction hastily — to be read again

Walsh functions (those assuming the two values +1 & −1 only) are presently the most important example of non-sinusoidal fns in communication engineering) — — these functions may be found on ancient temples & the pattern of the checker board used for playing checkers or Chess turns out to be a two-dimensional

Walsh functions.

[p. 22 - 23. The Walsh fns wal $(0, \theta)$, sal (i, θ) and cal (i, θ) are of considerable interest in communications.* There is a close connection between sal & sine fns, as well as between cal and cosine fns. The letters s and c were chosen to indicate this connection, while the letters al are derived from the name Walsh. ~~Just~~ as $\sin\theta$ & $\cos\theta$ are to

{* The probably oldest use of Walsh fns in communications is for the transposition of conductors}.

are convenient at different times, we have a similar duality existing for Walsh fns. A single function wal (j, θ) may be used instead of the three fns ~~wal (0, θ)~~, wal $(0, \theta)$, sal (i, θ) and cal (i, θ).

wal $(2i, \theta)$ = cal (i, θ), wal $(2i-1, \theta)$ = sal (i, θ)

$(i = 1, 2, \ldots$

The fns wal (j, θ) may be defined by the following difference eqns — —— .

[p. 23 - 30 contain lots more interesting properties of Walsh fns - In particular it is shown that Walsh fns

(90) form a group w.r.t multiplication (p. 240) and which, mathematically speaking, is isomorphic to the discrete dyadic group ~~Hadam~~

pp. 30-43 contain Hadamard matrices - such a matrix of rank 12 reported by Paley as being of interest in ~~communication~~ since it is usual to multiplex twelve telephony channels into a group. This → to Paley fns hal (j,θ) - Haar fns har $(0,0,\theta)$ (ie Thiele-value fns) and har $(0,1,\theta)$, identical with wal $(0,\theta)$ and wal $(1,\theta)$ - Haar fns leading to her (j,θ) and ter (t,θ) - again blo & ~~wal~~ fns - the fns had $(4,k|K,x)$ - All sorts of fns with beautiful illustrative diagrams - good for an article in Math. Newsletter. - To be read through carefully later.

(3) Selected papers of Cecil Frank Powell by E. H. S. Burhop, W. O. Lock & M. G. K. Menon - North Holland 1972

- Tribute by Weisskopff - general articles of Powell are very interesting & would be useful for the Symposium on "Science & Society".

(4) An introduction to mathematical learning theory

by Atkinson, Bower and E. J. Crowther - John Wiley & Sons, 1965

First sentence of Preface - "In my recent developments in the social sciences, the application of mathematical methods has become an important and rapidly expanding sphere of activity — the psychology of learning is the major focus of this book." — the book is a form of mathematical psychology — It is a book on psychology and not on mathematics

(See Poor V.K.R.V. Rao)

Questions on ordinary diff. eqns

(1) Show that there exists at most one solution of the homogeneous equation $y' + a(x)y = 0$ —— (1) with the initial condition $y(x_0) = b$

[Suppose ϕ, ψ are two solns. with $\phi(x_0) = \psi(x_0) = b$.

Putting $\chi = \phi - \psi$, we have

$$\chi' = \phi' - \psi' = a(x)\{\psi(x) - \phi(x)\}$$

$$= -a(x)\chi.$$

Hence χ is also a soln of (1) ie $\chi' + a\chi = 0$.

Multiplying by $e^{-A(x)}$, $e^{-A(x)}\chi' + a e^{-A(x)}\chi$

$$= e^{-A(x)}\chi' + \frac{d}{dx}\left(e^{-A(x)}\right)\chi$$

$$= \frac{d}{dx}\left\{e^{-A(x)}\chi\right\} = 0.$$

ie $e^{-A(x)}\chi = \text{const} = c$, say, but this must be zero and since $\chi(x_0) = \phi(x_0) - \psi(x_0) = b - b = 0$

ie $e^{-A(x)}\chi(x)$ is identically zero & since $e^{-A(x)}$ is never zero χ must be $= 0$ ie $\phi \equiv \psi$.

[This is Coddington Ex. 4 (c) – the solution above

is from Linear analysis & Diff. equations by MacCamy and Mizel*, p. 108.

[From ⊛ further examples etc:

(1) If y_1 be some particular soln of $y' + ay = b$. Then every soln of this can be written in the form

$$y = y_1 + y_0$$

where y_0 is a soln of $y' + ay = 0$. (Proved in Coddington)

(2) If $y_1(x)$ & $y_2(x)$ be both solns of the eqn

$$y' = e^{x^2} y + \sin x$$

and $y_1(2) > y_2(2)$ show that $y_1(x)$ is always $> y_2(x)$

[Hint. Consider eqn satisfied by $y_1 - y_2$]

(3) If the soln of $y' + a(x) y = 0$ is zero at one point, then it must be identically zero.

(4) Method of variation of parameters for $y'' + ay' + by = f(x)$ [a, b consts]

Ex: $y'' - 4y = x$. Here $a = 0$, so $y_1(x) = e^{2x}$, $y_2(x) = e^{-2x}$. Then α and β are given by $\alpha'(x) e^{2x} + \beta'(x) e^{-2x} = 0$

and $2\alpha'(x) e^{2x} - 2\beta'(x) e^{-2x} = 1$

(94)

The determinant $D(x) \neq 0$ & soln of the system gives

$$\alpha'(t) = \tfrac{1}{4} t e^{-2t}, \quad \beta'(t) = -\tfrac{1}{4} t e^{2t}$$

By integration by parts. $\int \tfrac{1}{4} x e^{-2x} = -\tfrac{1}{8} x e^{-2x} - \tfrac{1}{16} e^{-2x} + \text{const} (= c_1 \text{ say})$

~~Sing~~ $\int$ ie $\alpha(x) = -\tfrac{1}{8} x e^{-2x} - \tfrac{1}{16} e^{-2x} + c_1$

Sing $\beta(x) = x e^{2x} + \tfrac{1}{16} e^{2x} + c_2$

Then any fn of the form

$$y(x) = \alpha(x) e^{2x} + \beta(x) e^{-2x} = -\tfrac{1}{4} x + c_1 e^{2x} + c_2 e^{-2x}$$

is a soln.

(B) Theory of ordinary diff eqns by Eastham — Mere theory:

(C) Elementary Diff Eqns by Wilcox & Curtis — Rather elementary

(D) ordinary Diff eqns by William T. Reid — Pure Theory.

24/4/73 At Institute

Refereeing shapes by Mishra & Chowdhury on the restricted problem of a spheroid and an ellipsoid

References:

(1) M.N.R.A.S. 1928, 88, p.641.

H.N. Russell – on the advance of Periastron in eclipsing binaries – It is well known that if a small satellite moves about an oblate ~~spheroid~~ planet in an orbit of small eccentricity with a period P, the periastron will advance with a period P' such that $P/P' = \frac{r^2}{a^2}(e - \frac{1}{2}\phi)$

[a = radius of orbit, r = that of planet, e the superficial ellipticity of the latter, and ϕ = ratio of centrifugal force to gravity at its equator] – .

For two oblate spheroids in orbital motion about one another, the motion of periastron will be given, to the first order of small quantities by

$$P/P' = \frac{1}{a^2}\left\{r_1^2(e_1 - \frac{1}{2}\phi_1) + r_2^2(e_2 - \frac{1}{2}\phi_2)\right\}$$

(96) or putting $e - \frac{1}{2}\phi = k\phi$, $P/P' = \frac{1}{a^2}\left(k_1 r_1^2 \phi_1 + k_2 r_2^2 \phi_2\right)$

In eclipsing binaries direct evidence regards periods of revolution and rotation of the stars coincide (at least as regards). If m_1 & m_2 are the masses,

$$\phi_1 = \frac{m_1 + m_2}{m_1} \cdot \frac{r_1^2}{a^2}$$

& $$P/P' = \frac{m_1 + m_2}{a^2}\left\{k_1 \frac{r_1^2}{m_1} + k_2 \frac{r_2^2}{m_2}\right\}$$

-- Consideration of tidal distortion in star 1 is $\frac{6m_2}{m_1 + m_2}$ times the rotational etc etc — gets numerical values table for ρ & ρ_0 $\left[P = \frac{R}{n}\rho T \frac{\rho_0}{\rho_0 - \rho}\right]$ —

Applies results to normal models like Emden's polytropes, γ Cygni, stars built on Eddington's model. - - -

(2) M.N.R.A.S. 98, 1938, p. 448.

Zdenak Kopal — on the motion of the apsidal lines in close binary systems.

Reference to earlier work of H.N. Russell & K. Walter — owing to its importance — not only from the dynamical point of view,

but also in ~~virtue of its importance~~ view of the inferences on the internal constitution of stars made in this way — Eqns of motion in cartesian coords, supposing both components to be heterogeneous ellipsoids with polytropic density distn & three unequal axes $a_i > b_i > c_i$ (ϵ_i & ϵ_i' taken to be their meridional & equatorial eccentricities) — Special cases of prolate & oblate spheroids — Derives Russell's formula for P/p' — Effects of libration and their periods & case of larger librations — Refers to comparison with observation of Y Cygni — The analysis justifies Russel's ideas & shows his results re. Y Cygni are essentially correct (Doubt raised by Walter not justified) — But it is easy to infer from the general form of our eqns that the apsidal line will advance secularly no matter what the form of the components, are provided only that the periods of revolution and of axial rotation of both components agree strictly — ie that Kepler's laws of areas holds good — Breakdown caused by large librations is a very difficult mathematical problem

<u>Summary</u> (i) If the orbital period and periods of axial rotation of both components happen strictly to agree, the

(98) longitude of periastron — apart from the periodic changes — advances secularly, the rate of motion being the higher the greater the relative dimensions of both components, the greater their distortions from the spherical form, the lower their density concentrations and the greater the orbital eccentricity. The major semi-axis and the eccentricity of the orbit remain secularly constant.

(ii) (a) librations of small amplitude

(b) " large,

General summary: The object of the present paper is to investigate the motion of the apsidal line in close binary systems the components of which are non-homogeneous ellipsoids with three unequal axes & polytropic density distributions. The mathematical analysis in the main part of the paper proceeds by direct integration of the respective eqns of motion in the Lagrangian form, and has been carried out to the second order in the ellipticities of both configurations.

(3) Ibid, p. 734 — T. G. Cowling — On the motion of the

apsidal line in close binary systems.

Refers to work of Russell & Walter – Says that Kopal's confirmation of Russell's result is too wrong. and that Walter's result is correct if his assumption as to the rigidity of the stars is adopted. This conception is however extremely dubious – A long mathematical treatment.

Summary – It is pointed out that the discussion of the motion of the apse line in binary stars given by Russell, Walter and Kopal all depend to a smaller or greater extent on an assumption that the stars can be regarded as rigid bodies during the motion. When regard is paid to the fact that the stars are not rigid, a formula for the rate of advance of perastron is obtained similar to Russell's but differing in the numerical factor attached to certain of the terms. The density considerations of the stars which are deduced from the new formula are far more in agreement with Eddington's theory than those found by Russell.

(100) 26/4/73 — Literature about Earth's magnetic field

(1) McGraw-Hill encyclopaedia — v. 341

Early theories attributed the Earth's magnetism to permanently magnetised iron in the deep interior. This is now considered untenable because of the seismic & other evidence on the fluidity of the outer core. Nevertheless, it is now well established that the earth's magnetic field does originate predominantly in the deep interior. The most widely accepted theory, the dynamo theory of Elsasser & Bullard, attributes the earth's magnetism to currents flowing inside the fluid outer core, and involves the conversion of mechanical energy into electromagnetic energy. The theory is compatible with either a molten iron outer core or with the Ramsey theory which requires the outer core, even if composed of the same ingredients as rocks, to have the electrically conducting properties of a metal. It is also compatible with the presence of a solid inner core, the boundary of which would enable convection currents of the required type to occur in the outer core. The theory requires a source of energy for the convection currents. Radioactivity has been

suggested but it is thought by many to be inadequate. (101)
This is the chief difficulty the theory has to face

References: Books by Bates (1957), Bullen (1954 2nd Ed),
Gutenberg (1959) & H. Jeffreys (1959, 4th Edn)

(2) Encyclopaedia Brittannica, p. 850

Gauss's theory of internal & external source — Early idea of origin due to being due to magnetic materials embedded inside the earth — this discarded since at high temp prevailing all substances lose their magnetisation — Two characters this viz variation of magnetic declination from place to place & secondly its gradual change in time — The first is called secular variation of earth's magnetic field. These changes show that the field does not originate in the solid mantle of the earth. Eulasser showed that the magnetic field is due to electric currents that flow in the fluid metallic core from the secular

[p. 848 — Main layers of the earth — from surface to centre — (1) the crust outermost about 40 Km ~~depth~~ deep below the continents but is extremely thin if not completely absent, its lower boundary is called the Mohorovicic discontinuity. (2) The region below this & above the core, extending to its boundary

(102) at a depth of 2900 km. Structure of mantle is not too well pronounced (3) Between 2900 km & 5000 km is the outer core which is fluid & appears quite homogeneous. (4) From 5000 km to the centre of 7000 km is the inner core of which little is known] variation and can deduce the speed of molten iron. Eulasers theory is known as the dynamo theory — What keeps the core in motion is taken by

Rock magnetism — When a lava melt is solidified ~~in~~ in a magnetic field, and the field is removed later, it is found that the solid rock formed retains a slight amount of so-called remanent magnetisation along the direction in which the original magnetic force pointed. The remanent magnetisation is due to small grains of magnetic material (iron oxide) embedded in the rock. on the basis of this, it becomes possible to infer the direction of the earth's magnetic field at the time an ancient lava flow occurred. Some sandstones have been shown to retain a small remanent magnetisation along the direction of the field in which the material was deposited. The study of rock magnetism has been developed to the level of a systematic approach, giving information about the character of the

magnetic field at earlier times in the earth's history. There seems to be little doubt that during the last 500,000,000 years or so that are accessible to detailed geological study, the earth has had a magnetic field similar to its present one

Work on rock magnetism has shown surprisingly, that apparently the magnetic field has reversed its polarity many times, perhaps hundreds of times during the past history of the earth – that the magnetic ~~North Pole~~ South Pole became the North Pole & conversely. A detailed explanation of this phenomenon is lacking; it can be said merely that its occurrence is conceivable in terms of the dynamo theory since a self-excited generator can produce electric current in one direction as well as in the other depending on its starting conditions.

Rock magnetism studies may also throw light on migration of geographic poles

(3) Physics of the Earth by Frank. D. Stacey, 1969.

Chapter 5 – Paleomagnetism – The prehistory of the

(104) earth's magnetic field, pp. 157–190.

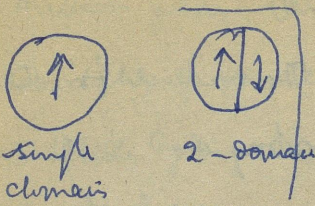
"If the facts are correctly observed there must be some means of explaining & coordinating them" Bullard, 1965

(i) Introduction – Paleomagnetism or "fossil magnetism" expands time scale by a factor of 10^6 or to the remote geologic age – is the study of the ancient magnetic field by measurement of magnetisation of rocks. – Review (1960) by Cox & Doell, Rock magnetism by Nagata (1953), Nagl & Tracey (1955 & 1963)

(2) Magnetic minerals in rocks – It is the property of remanance which is of interest in paleomagnetism – Essential difference between magnetic properties of hematite bearing sediments and magnetite-bearing igneous rocks are explicable in terms domain structure.

(3) Remanent magnetism in rocks – Thermoremanence magnetisation or TRM – blocking temperature (below the Curie point $\sim 580^\circ$ C for titanomagnetite) – Theory for single domains (multiplicity of domains depends upon

grain size (Kittel) – Basic eqn in theory of TRM



is $I_{TRM} = I_{SO} \tanh\left(\frac{\mu_B H}{K T_B}\right)$

small care integration in series. If $\mu_B H$ is small $T_{RM} = I_{SO} \frac{\mu_B H}{K T_B}$

Detrital remanent magnetisation DRM – chemical remanent magnetisation or CRM – Natural remanent magnetisation or NRM – Stability of total NRM

(4) Archeomagnetism and secular variation – Going backwards in terms we come to period from which dated archeological remains can be studied (going back from the direct record of secular variation, 1540) – pots, bricks, kiln bricks etc – intensity data show consistent record over the last 2000 years.

(5) Paleomagnetic poles and the axial dipole hypothesis –

Statistical analysis by R. A. Fisher of rock collections. The hypothesis states that the geomagnetic field has always been predominantly dipolar and when averaged over sufficient time (of order 10,000 years) the axis of the dipole coincided with the geographic pole, and the dipole field

(106) was centred at the centre of the Earth.

(6) Reversals of the geomagnetic field – The evidence for the reality of this reversal is now very strong and is of three kinds: (a) correlations between reversals observed on different continents & in different rock types, both igneous & sedimentary, (b) Sediments which were ~~then~~ baked in by contact with later igneous rocks have acquired TRM with the same polarity in almost all cases, (c) the actual process of reversal has been traced both in rapid sequences of lavas & in deep sea cores – suggesting ie. reversals occurring frequently during periods of polar wander & of continental drift. This suggests that the configuration of the mantle has a bearing on the magnetic field – difficult to explain on the convective dynamo theory but appears to follow more naturally from the precessional dynamo

(7) Polar wander & continental drift – The geomagnetic field has been predominantly dipolar for at least 300

million year (Permian period = 230 to 280 million)
Possibility of forseeable future observations of artificial satellites giving less ambiguous evidence of relative movements of continents.

(8) Paleoclimates → Paleoclimatology – discovery of coal seams in Antarctica – Connection between ancient climates & paleomagnetic latitudes of the axial dipole hypothesis is a sort of direct evidence. – Ice ages – paleomagnetic data & paleoclimate indicators

(9) Intensity of paleomagnetic field – Particular geophysical interest in extending observations back to 2×10^9 and 3×10^9 years. (according to Runcorn, this core developed since that time). If Runcorn's hypothesis be true rocks of that date should indicate no geomagnetic field. But measurements of Evans & McElhinny (1966) have extended methods back to 2.6×10^9 years & 5×10^9 years < give geomagnetic fields comparable to present fields in strength. – not sure to pre-Cambrian era. This implies that the core

 was essentially complete and producing a substantial field at an early stage of the Earth's history.

(4) Ibid – Chap. 9 on the Earth's internal heat

pp. 252–254 where Kennedy's work appears

The variation of the melting point T_m of a pure substance with pressure p is given by the Clausius–Clapeyron eqn (Vide Gross)

$$\frac{dT_m}{dp} = \frac{T_m}{L}(v_L - v_S) \qquad \text{——(1)}$$

where L is latent heat of melting & v_L & v_S are the specific volumes or reciprocal densities of the liquid & solid phases. This eqn is used to plot the variation of T_m with depth of mantle, but integration to the depth of the core with any accuracy demands a knowledge of the variations of L, v_L & v_S at very high compressions. Kraus & Kennedy (1966) examined data on the melting of materials at pressures beyond the linear compression range and concluded that the variation in melting point of a solid

even at extreme compression was proportional to the negative dilatation of the solid by virtue of the compression:

$$\frac{d T_m}{d(\Delta v_s / v_{s0})} = \text{Const} \qquad \text{—— (2)}$$

where $\Delta v_s = v_s - v_{s0}$ & v_{s0} is the specific volume of the solid at zero pressure. — Kennedy's work & probable uncertainty of 500° C. — Kennedy's value assumed in the calculations

Since outer core is fluid & according to observations of geomagnetic secular variations, it has internal motions which are very rapid by comparison with other geophysical processes, we can assume its temp. gradient to be adiabatic & thus estimate temperature of the core-mantle boundary. The variation of T with p for adiabatic law is given by Maxwells thermodynamic relations

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial S}\right)_p \quad [S = \text{entropy}$$

V = Volume of material. This gives volume expansion coeffn $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ and since entropy is defined by

(110) $dS = \frac{dQ}{T} = \frac{mc\,dT}{T}$ m = mass, c = Sp. heat

It follows that $\left(\frac{\partial T}{\partial S}\right)_p = \frac{T}{m c_p}$. Writing $m/V = \rho$ we obtain after suitable substitutions

$$\left(\frac{\partial T}{\partial \rho}\right)_S = \frac{\alpha T}{\rho c_p}$$

Also $\frac{dp}{d\rho} = -g\rho$

so that $\left(\frac{\partial T}{\partial \rho}\right)_{adiabatic} = -\left(\frac{\partial T}{\partial \rho}\right)_S \frac{dp}{dr} = \frac{\alpha T g}{c_p}$

which is applied in mantle-core calculations

—

p. 263. – Kennedy's relation also writtens

$$\frac{dT_m}{dp} \cdot K = \text{const} = \left(\frac{dT_m}{dp}\right)_0 K_0$$

Combining with (11) on p. 108

$$L = \frac{K}{K_0} \cdot \frac{(v_L - v_S)}{(dT_m/dp)_0} T_m$$

since approx. $K(v_L - v_S) = K_0 (v_L - v_S)_0$

we have $L = L_0 \left(\frac{T_m}{T_{m0}}\right)$. (III)

(5) Global Geophysics by Tuctor, Cook, Dyr, Stacey

Chap 4 – Geomagnetic field

(i) The main field

(ii) Paleomagnetism – Prehistory of the Earth's magnetic field

(iii) The magnetosphere & the radiation belts

(iv) Magnetic storms & associated effects

(6) Electromagnetism & the Earth's interior by K. Rikitake

No reference to Kennedy

(7) Physics of Geomagnetic phenomena edited by Matsushita & W. H. Campbell Vol I

II.1 – deals with Paleomagnetism (by Nagata & Ozima)

(i) Intro

(ii) Rock magnetism as a basis of paleomagnetism

(iii) Fields during last 10,000 years

(iv) Geomagnetic field in geological time

(v) Theoretical Interpretation.

— Must be interesting but no time to read.

(8) Ibid Vol II

§ 1.1 by Sydney Chapman : Perspective

Should be very interesting — talks of morphology of geomagnetic storms

VI·1 — Storms & Aurorae

Attended Kennedy's lecture on Raman Inst. on 27/4/73 & afterwards I wrote down on Board his formula

$$\frac{d\,T_m}{d(\Delta v_0 / v_{s0})} = \text{const} \times L = \lambda_0 \left(\frac{T_m}{T_0}\right)$$

and asked him if he remembered it & whether he had any other new formula. He was highly pleased & shook hands with me — otherwise it was impossible to follow what all he said, his pronunciation being unintelligible

Report on paper by Miss Anna Chowdhry

The results in the paper, some of them elegant and new, are of a purely dynamical nature. The problem dealt with is of astronomical interest viz. close binary stars & eclipsing binaries. In fact, all the references at the end of the paper deal with astronomical problems. But the authors completely ignore this aspect, and merely present the results as an exercise in the Hamilton-Jacobi theory of classical mechanics relating to rigid body motion. It would be interesting to know what new astronomical result ~~re~~ ~~motion of binary stars follow~~ generalising that of Johnson & Kane, and others, follows from the work in the present paper.

It is suggested therefore that the paper be re-written with an emphasis on the astronomical aspect, and also cutting down to a minimum the formal and cumbersome details of the dynamical calculations, and then submitted for publication

BSM

(114) 16/5/73 At Institute – New Books Section

(1) Order & Chaos (Laws of entropy & energy) by

Stanley W August } funny way
Loren G Kepler. } of having names

(~~[illegible]~~)

on the cover of book itself – discovery of the laws of thermo-dynamics & their application to technology, culture and life processes.

– Basic Book, Inc. publishers, N.Y & London (1967)

– Very interesting book – Specially the last chapter on "demons, poetry & life" – Also many funny illustrations – Very fine for a popular lecture

p. 213: (last of the ~~last~~ 9th chapter)

"Incomprehensible' murmurs theology reverently
What was that word? asks Science
'Incomprehensible, I often use it in my litanies

'Ah yes' murmurs Science, with almost equal reverence
'Incomprehensible'

'The comprehensibility of comprehension' begins Philosophy staring straight in front of him

'Poor fellow' says Theology 'he is wandering again, better lead him home'

'I haven't the least idea where he lives' says Science,

'Just below me, says Theology 'we are both above you'

The above is from Stephen Leacock's "Common Sense and the Universe" where he told the story viz. Science, Philosophy and Theology have nowadays all come together – at a funeral. The funeral is that of Dead Certainty. The ~~interfoment~~ interment is over and the three turn away together

Then follows the above lines of poetry

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p. 211. Perhaps Josh Billings was correct when he said

"I have finally come to the Conclusion that a good reliable

(116) set of bowels is worth more to man than any quantity of brains"

At the beginning of this ~~book mentions last~~ Chapter (rather Chap. 09 - Chap. 10 is "What else) mentions the folklore

Humpty Dumpty set on a wall
Humpty Dumpty had a great fall
All the kings horses
And all the kings men
Could'nt put Humpty Dumpty together again

- a nearly 1000 year old nursery rhyme - Counterpart of Humpty Dumpty in France (Boule-boule), Sweden (Thille dille) Denmark (Lille-Trille), Finland (Hillerin Lillerin) and Germany (Humpelken Rumpelken) — Second law.

p. 185 - Maxwells demon - cannot really count molecules because of (thermodynamics ?) radiation - the requires a

Hashlight (Ratio of energy emitted by this bulb divided by the temperature of the filament is called negative entropy or negentropy, word coined by Brillouin. – Concept of negentropy applies to writing of poetry (very interesting)

(Statement of Socrates " Poets ... say many things, but they understand nothing of what they say'

Chap. 10 " What else' – Story about Sommerfeld and thermodynamics; his death in an accident while he was writing a book on the subject.

¶ – All in all, many students view thermodynamics as something akin to quinine – a bitter pill alleged to have ~~metal~~ medical value.

(2) Graph theory by Watarun Mayeda. – A very comprehensive work with lots of applications to networks etc.

18/5/73 — Met Sudarshan (who bragged about his being awarded Hon. Doctorate of Delhi Univ & showed me the citation) - Later on Seshachar came & they were talking about participants in the forthcoming seminar on "Science and Society" consisting of all sorts of ~~follows~~ fools & I did not find my name in it & felt rather hurt.

Later ~~used some harsh~~ made some caustic remarks to E.C.G & B.R.S saying "That is because [E.C.G had said I was looking fresher] I have been dropped out the Symposium on Science & Society & saying "It is better to hear nonsense rather than talk nonsense" & left ~~with~~ them in a flurry

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Later saw Athreya who gave me the programme for the damned Summer School in Maths at C.C.B & found that inaugural fn had taken place on 14th - Pilani V.K. was Director of the show & Nagaraj was Dy. Director & hence concluded that the latter was the real culprit for not inviting me

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Anyway the above two things worried me the whole night & I slept very badly

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19/5/73 Again at Inst - Saw E.C.G. & made up with him; also

phoned to B.R.S & made up with him – went library upstairs & saw number of journals and articles of which were Greek to me – Did not quite decide whether to go to C.C.B or not on 26th to attend K.V's 1st lecture on Geometry & Imagination; Thought of writing to him & then find out whether it would be appropriate for me to go.

23/5/73 – At Institute again

Had some discussion with E.C.G. on Dirac's relativistic wave eqn & understood nothing what ~~I have~~ he said & I believe he understood nothing of what I said.

(1) Looked through some issues of Scripta Mathematica
(2) " " " Revue Roumaine de Math pure et appliquées

8/6/73 (1) High energy Astrophysics by Trevor. C. Weekes

(2) Lectures on -- Do – by G. R. Burbridge (T.I.F.R, 1966 Bangalore Symp)

(3) Astrophysics – Progress in Physics Series (Benjamin, 1969)

(4) Enrico Fermi Series Course 35 (1966)

New Books Section.

(1) 5th Congres international de Cybernetique [Reference]

Namur, 1967 (Assn. Int. de Cyb)

(Belgium)

All kinds of meaningless (to me) articles)

(a) p. 233 - Some recent advances in adaptively controlled teaching systems by George Maller & Gordon Pask (U.K) (System Research Ltd, Richmond, Surrey)

[Adaptive teaching machines]

p. 236 - The aim of any teaching system is defined as maximising $\frac{dP}{dt}$ where

$$\frac{dP}{dt} = \frac{1}{\eta_{max}} \left(\rho \frac{dq}{dt} + \eta \frac{d\rho}{dt} \right)$$

Two terms of R.H.S. correspond to what we term closed-loop & open-loop systems.

p. 239 - Conclusion :- By using a simple linear method

of the teaching - learning process, we have been able to derive optimal conditions for the designing a closed-loop teaching system and to ~~determine~~ derive a limiting condition which determines the point where an open-loop system becomes more effective than a closed-loop system.

(b) Applications of the algebra of relations to Cybernetics by H. Greniewski, p. 47.

(c) A maximum entropy approach to Bayes law problems etc. p. 119.

(d) Some Information-Theoretical considerations in Cosmology by V. Majernik, p. 130. [uses Planck's h]

Bibliography by same author - Some applications of information theory in quantum mechanics

(e) p. 174 - Aspect thermodynamique de la pensée humaine by D. Malic (Yugoslavia)

(172)

Section I – Principles & methods of cybernetics
pp. 1–221

Section II – Semantic machines pp. 233–369

Section III – Automation; Technical aspects
pp. 381–515

Section IV – Cybernetics & human sciences
pp. 523–658

Section V – Cybernetics & life, pp. 661–881

Lots of other interesting articles

14/6/73 – App. Math. Dept. Library

(a) Analytical inequalities by D. S. Mitrinović

Springer Verlag (1970)

– mentions my results in two places – Also T. S. N's, V. R. T's & K. S. K's but found no reference to ~~mine~~ & K, V's work (for eg. inequalities re. lattice sums).

(1-4 Feb 1972)

(h) Bombay 1972 Symposium on nuclear physics & solid state Physics – Vol. 14A – invited papers.

– Read it through in a cursory way, specially the seminar on on symmetry principles in physics, in particular:

Symmetry in particle physics by N. Mukunda p. 57

21/6/73 – Attended the last afternoon Session of the stupid seminar on Science & Society at the Faculty Hall as a Gate Crasher (as E.C.G. put it) – Also spoke something about 'Dead Certainty' & referred to C.D. Narasimhaiah's talk on Science & Poetry – Also criticised E.C.G's clubbing Phys & Maths – Anyway I wish I had not gone to that meeting where I was "unwanted" & spoke something.

22/6/73 – Went to Appl. Maths Deptt, saw E.V.K & again looked at book on Inequalities by Mitrinovic – Got from E.V.K. a typed copy of Lovelock's lectures on tensor analysis & special relativity

(174) New Books Section

(1) Man, Science & Society published by the Indian Institute of Advanced Study, Simla (Vol. 9) – General Editor – Niharanjan Ray. (Proceedings of a Seminar of 11 days) – Looking at this I thought the 4 day conference from 18th to 21st held at the Inst. was a colossal waste of money involving unnecessary duplication.

(2) Readings in Earth Sciences – Edited by Benjamin Moulton (Van Nostrand, 1972) – Number of articles by different authors divided into

Part I – Man's view of the Earth – 6 articles

" II – Dynamics of Crustal changes and Earth modifications – 7 articles

" III – Man's confrontation with the Earth – 7 articles

– Read the interesting article (the last one Part I) by Edmund M. Spieker on the Nature of Geology and its place among the natural Sciences

To learn without thinking is useless
To think without learning is dangerous
— Confucius

— Refers to the Compendious work of 1963 — Fabric of Geology (Albritton Ed)

Intn:

(1) Geology is at least as difficult as such awsomely revered subjects as theoretical physics, because to prosecute it as a science requires hard thinking largely without the powerful help of higher mathematics ... — Every student of the subject should learn early that beneath a glamorous exterior geology is a very difficult domain of intellectual endeavour.

(2) Defn, conceptual nature of Science

(3) Basic areas of Science and their Characteristics — Gives a figure illustrating the domain of natural science by making three divisions: Physical, Telluric, and Biologic

Figure

The Domain of Natural Science

Physical		Telluric	Biologic	
Physics	Chemistry	Geology	Botany	Zoology

Number of variables increase ⟶

Power of Mathematics decreases ⟶

Concentrative Distributive

(mechanics distributive elements even in Physics for eg in elementary particle theory).

(4) Philosophy of Science - Three levels of analysis (a) The perceptual (2) the epistemological (3) the speculative or metaphysical - the realm of science is almost entirely in (1)

- "Science of Science" - materials of science

(5) Generalisation

(6) Generalisation in Geology

(7) Maturity of Geology, the mathematical element - Very interesting remarks, specially the following

1. Although it is essential in much of scientific work, mathematics itself is not science. It is uniquely powerful in the derivation of conclusions from bodies of fact in areas such as physics, but it alone cannot create anything new in the matter of science proper. We must remember the age old maxim that the product of a sound mathematical sequence can never be any stronger than the factual ingredients put in at the start. Again, I must insist that in geology these facts have to come from the field before those of the laboratory can wield effective power

The fact that many men well trained in mathematics, physics & chemistry have brought the healthy influence of these basic disciplines to bear on the problems of the earth is a fine thing and a definite factor in the advancement of our science, but it alone is no signal whatever of health or maturity in geology proper. Until mathematicians have become so

competent as to deal effectively with the essentially geologic conditions, with their hosts of variables that lie at the base of our science, we shall have to depend on the more difficult devices of ordinary reasoning for the successful analyses of our complicated problems, the weighing and balancing of many factors, the thinking through of mazes of combinations; the whole requiries the utmost in rigor.

Many geologists have depricatively observed that ours is not an exact science. Exact it may not be, but exacting it most assuredly is"

(3) Particles & fields – 1971 – Proc. of the Rochester meeting of APS/DPF on 30/8 – 2/9, 1971

The article on Lepton conservation laws by S. F. Rosen on p. 226 appeared to me very interesting & understandable after reading ~~Asst~~ Asimov's book on the Neutrino. – Also an

article on solar neutrinos — number of
articles on general relativity (some of a general type)
and cosmology.

This book (C 9 B3 p 73 N – N 71) to be borrowed
✓ as soon as available.

26/6/73 — New Books Section

(1) Astrodynamics – Vol II (Samuel Herrick) – Deals
with Orbit Correction, Perturbation theory, integration
(Looks like branching celestial mechanics)
– Refers to Lagrange & Poisson brackets in three
dimensions (using spherical coordinates).

(2) Astrophysics (W. A. Benjamin, 1969).
Article on Astrophysical neutrinos by M. A. Rudermann
(Also article by Davidson & J. V. Narlikar)
Read through Rudermann's article cursorily

(B) Enrico Fermi school series Vol. 35 (Course 35) – on
(A.P, 1966)

(180)

high energy astrophysics.

pp. 367-391 - Neutrino astrophysics by W. A. Fowler

- Also looked through this cursorily.

(4) Lectures on High Energy Astrophysics by G. R. Burbridge

T. I. F. R.: 1966 Summer School on Rel. Astrophysics & Cosmology held at Bangalore, 1966.

Chapter 7 - Neutrino Astronomy. pp. 147-184

- glanced through cursorily - saw 14 references

(5) Stars, planets and life - The evolution of the cosmos by Robert Jastrow (Heinemann, London)

- Beautifully illustrated popular type of book.

(Borrowed today)

(6) Integrated electronic systems (by Aerospace Division of Westinghouse Electric Corporation)

(Prentice Hall, 1970)

Intro to systems application of integrated circuit devices & techniques

Chap 1 – Radar Systems.

Chap 2 – Communications Systems

" 3 – Television cameras

" 4 – – Functional Electronic blocks

" 5 – Passive Element design – Resistors and capacitators

" 6 – Bipolar transistors in functional electronic blocks

" 7 – Field-effect devices

" 8 – Design & final layout of an integrated circuit

[die size – device comparabilities – Resistors – P-N. jn Capacitors – Advantages of integrated circuits – Parasitics – Estimating the die area – Package choice – interconnection planning – Pin arrangement – Component arrangement

[* Advantages are (i) ~~Temp~~ Temp. compensating a transistor diode, (ii) Matching input transistors of a differential amplifier, (iii) maintaining a required dc. level from input to output in an amplifier - (iv) Fabricating temperature - compensated current sources]

Chap 9 - Materials preparation
" 10 - Epitaxial growth and related processes
" 11 - Oxidation & diffusion technology
" 12 - Microphotography and photoengraving
" 13 - Bonding & packaging
" 14 - Failure analysis and failure mechanisms
" 15 - Quality & reliability control of integrated circuits

[accelerated testing, - Scanning electron microbe - Infrared techniques - Failure as a function of time - Process control charts - Reliability verification]

Chap. 16 - Electro-optics and its application to

solid-state bonding

Chap. 17 - Semi-conductor delay lines

From introduction:

(1) Methods of introducing IC technology in systems.
(2) Introduction of digital systems.
(3) Application considerations (5 subdivisions)
(4) Available IC devices & techniques (Not 3 sections)

From Preface

Designer must also know the basic physical behavior of materials & processing methods - materials research & device developments go with increased use - & some examples justifying this point of view - Basic understanding of semiconductor formalisms - use of digital signal-processing techniques as opposed to analog processing methods.

(184) 2/7/73 – At Institute – Read through Courant-Hilbert Vol I, Chap. 4 dealing with Variational Calculus – Later attended one A. P. Rao's (from T. I. F. R.) lecture on "Unified formulation of variational problems" – found it trivial dealing only with quant. mech. problems – made some comments.

4/7/73 – At. Institute – Books on Radio-Astronomy

1) Nuclear Astrophysics by W. A. Fowler (Amer. Phil. Soc. Philadelphia, 1967).

IV. Supermassive stars, quasars, Extra-galactic radio sources and galactic explosions, pp. 89 – 106.

2×10^{62} ergs frequently quoted as a representative value of the energy requirement in the larger radio sources & is taken as the maximum value – From the relation $E = Mc^2$, this is equivalent to the

energy obtained from the complete annihilation (185)
of the mass of one hundred million suns, $10^8 M_\odot$.
Thus the problem can be taken to be the conversion of mass
into energy.— Annihilation of matter and anti-
matter may well prove the ultimate solution of the
problem.— The overall energy release in 10^6 to 10^7 years
by condensation into a single supernova of large
masses in the range 10^3 to $10^{10} M_\odot$ comes out (using
classical theory) to be of the order 10^{62} ergs.— For
nuclear generation of energy of this order requires
a mass in the upper limit $10^{10} M_\odot$, which corresponds
to the entire mass of a medium sized galaxy!—
Hoyle & Fowler thus found that nuclear energy is
probably inadequate & hence turned to another possibility
viz. gravitational energy.— After a detailed examination
using Newtonian theory & theory of general relativity, it can
be concluded that gravitational release of energy (from the
core to the outer envelope) may be somewhat more

efficient than nuclear release, but not be a
Large margin. — Reference to discovery of ~~quasars~~ radio stars by
Hoyle & Fowler (by theory) & Observational ~~confirmation~~ discovery
of Schmidt of Pasadena & Confirmation by J. B. Oke (Is
This the Schonday the 7th Lecturer at Raman Institute) (No)
& Jesse L. Greenstein & Mathews — Observations in
Australia led by Cyril Hazard ~~showed~~ showed that
radio-stars are certainly not stars in the ordinary
sense but were a new class of astronomical objects -
called quasars (with negligible gravitational redshift)
and are extra-galactic objects and that their redshifts
arise from the general cosmological expansion of the
Universe — the great quasar controversy (?) —
pp. 106 - 109 on Energy resources discusses the evolution
of radio-sources & quasars using both nuclear energy
and gravitational energy & refers to results of Feynman,
Chandrasekhar & Others [The optical luminosity of
quasars are very high $\sim 10^{46}$ erg Sec^{-1}, but there is no

convincing evidence that these objects have life-times in excess of 10^5 to 10^6 years. Thus the cumulative optical emission is of the order of 10^{59} ergs which is well within the nuclear resources of a star with $M = 10^8 M_\odot$. Only seven percent of the hydrogen of such a massive star need be converted into helium to release this amount of energy.

These considerations lead to the final question: Are the cores of supermassive stars stable or unstable during nuclear burning? For spherically symmetric non-rotating configurations the answer is "No" for the cases $M > 10^6 M_\odot$

Richard Feynman has pointed out in lectures to his class in relativity that general relativistic instabilities set in very early in the evolution of a spherically symmetric supermassive star. Icko Iben Jr. proved the existence of these instabilities in detailed numerical calculations. James Wright and I have been independently able to show that even in the

post-Newtonian approximation to hydrostatic equilibrium, general relativistic effects lead to dynamic instability for supermassive stars. Subrahmanyan Chandrasekhar has given a very elegant proof of this instability using the full panoply of the exact equations of general relativity. For supermassive stars with $M \sim 10^3 M_\odot$ to $10^6 M_\odot$ the instability sets in at approximately the stage of contraction where hydrogen burning can also take place at the centre of the star. It is possible to show that relaxation oscillations are triggered by the sudden onset of nuclear burning during contraction arising from instability. These oscillations are damped by the very great luminosity of the quasi-stellar object. The calculated periods are the order of ten years in agreement with the observations of Harran Smith and Dorrit Hoffleit

on the variability of 3C 273. Shock waves transmit the nuclear energy to the tenuous outer regions of the quasar, where optical line emission and radio emission are stimulated. The radio emission originates in the outermost regions where the shock wave becomes relativistic and electrons are accelerated to relativistic energies.

For masses greater than $10^6 M_\odot$ nuclear burning does not lead to stable oscillations but takes place explosively. This could well account for the explosion in the galaxy M82 in which Roger Lynds and Allen Sandage point out that the total mass of the expanding material could be as great as $5.6 \times 10^6 M_\odot$. There are many other galaxies which show evidence of exploding events. I recall with great delight a sentence which appeared in the Time magazine

"Astrophysicists Fred Hoyle and Williams A

Fowler, from Caltech told the American Physical Society that galaxies *often* explode with *improbable* energy" (The italics are mine)

It is important to emphasise that the general relativistic instability follows from the assumption of spherical symmetry. The instability reflects the fact that pressure support from gas and radiation alone is insufficient to guarantee stable hydrostatic equilibrium in a spherically symmetric ~~soft~~ supermassive star. It is thus necessary to give up the special symmetry inherent in spherical collapse and to look to other mechanisms which will tend to stability dynamic or static, in supermassive stars for lifetimes comparable to the estimates made for the quasi-stellar objects.. One possibility is rotation. A large initial rotation can lead to a flattened disk configuration which

can fragment into smaller stars with characteristic times of stable evolution comparable to those required. Even relatively small rotations remove the general relativistic instability in the post-Newtonian approximation. Contraction is slowed to the point where hydrogen burning can be effective in triggering and maintaining oscillations in stars with masses considerably in excess of the $10^6 M_\odot$ limit found for the non-rotating case, perhaps so great as $10^9 M_\odot$.

The wide range in masses possible for the quasars leads to a consequence of great significance. Since the absolute ~~magnitude~~ luminosity is tied to the mass, quasars will also exhibit a wide range in absolute magnitude. Thus the red shift - apparent magnitude plots for quasars should not consist of a single line, but of a

broad band with width characteristic of the absolute luminosity. The observations to date seem to ~~bear~~ bear this out.

It would take us too far afield to discuss stability and rotation in more detail in this lecture. It can be concluded that the nuclear and gravitational resources of sufficiently massive ~~stars~~ condensations are indeed able to meet reasonable estimates for the energy requirements in the strong radio sources and quasi-stellar objects. Nuclear energy is made available during the stable epoch of stellar evolution, during which rotation and general relativity play dominant roles in contrast to the situation in less massive stars. Gravitational energy, somewhat greater in magnitude than nuclear energy, becomes available in the ultimate stage of stellar evolution when the

nuclear resources are exhausted and instability leads to gravitational collapse. The quasars shine on nuclear energy. Gravitational energy release leads to the development of the extended radio sources. These conclusions are subject to the usual proviso associated with developments in a rapidly developing field of observation and theoretical study. We are almost certainly in for a number of surprises before we understand the extended radio-sources and the quasars - these celestial objects have not revealed all of their secrets by any means]

(2) Brandeis Univ. Summer Inst. of Theoretical Physics, 1968 - Astronomy & general relativity Vol I -

(a) Gravitational collapse by C. W. Misner, pp. 113-188

Preliminaries: Survey of Diff. Geometry and general

(194) relativity – Collapse studied from the exterior Schwarzschild metric – Relativistic hydrodynamics & some static solutions (including the Oppenheimer–Volkov eqns and the use of the Sch. interior metric to show necessity of gravitational collapse) – Relativistic spherical hydrodynamics – collapse and cosmology (free-fall collapse, Oppenheimer–Snyder soln; Robertson–Walker solns and inhomogeneous solutions)

(b) Extra-galactic radio-astronomy by A. T. Moffet

pp. 217–291.

Introduction – Historical review – Basic concepts – Observational characteristics of radio sources – Identifications and intrinsic properties of radio sources – Synchrotron radiation – Energetics of radio sources – Theories of the evolution of

radio-galaxies and quasi-stellar sources.

(3) Radio-astronomy & the galactic system - Symposium No. 31 of I A U (~~Int~~ Int. Astr. Union), Sept. 1966.

Part III - The galaxy as a radio source

(4) Bull. of the Pulkovo Observatory. Vol 23, Issue 3, No. 173 (Radio Astronomy) by A A. Mikhailov (1964)

- Mostly observational material

(5) Paris Symposium on Radio Astronomy (IAU and URSI) 1958 - Edited by R. N. Bracewell.

Part II - Galactic & extra-galactic sources

Part V - Discrete sources and the Universe.

(to be looked with)

6/7/73 - At Institute - Looked at various books & also issues of Astrophysical Journal for 1969 & could nowhere find any reference to radioastronomical test of general relativity.

(196) — Later found at home the A.I.P. Conference Proceedings which I had borrowed an article by Shapiro wherein Einstein's tests were verified using 4 types of (i) radar signal retardation (ii) radio source deflections by sun

(iii) retardation of radar signals used to measure advance of perihelion of Mercury. Read all this carefully on 8th & attended meeting at Raman Institute on the 9th. What the lecturer R.D. Ekers of Holland said at the meeting was exactly what I had in the article of Shapiro.

10/7/73 At Institute — Read number of Journals in the Staff Reading Room.

New Books Section — Vol. 2 of the A.I.P. Conference Proceedings (Particle Physics — Irvine Conference of 1971 published by the American Inst. of Physics — Number of topics presented — This book to be

borrowed after 15th July 73 — Read this through casually

14/7/73 At Institute

(1) Introduction to Space Science by Robert C. Haynes (John Wiley, 1971)

Cop Chap. 14 – Radio Astronomy. – cosmic rays, neutrinos, Electromagnetic radiation between 4×10^{-5} & 8×10^{-5} cm (λ) and radio channel occupied by e.m. waves having λ's between ~1 cm and perhaps 30 metres. This radio channel of information provides data for radio astronomy.

14.1 – Radio wave absorption – Big antennas

14.2 – Radio telescopes –

14.3 – Radio sources. – Galactic arms & centre,

14.4 – Emission mechanisms. ionised H-clouds, neutral H regions, supernova remnants, radio galaxies outside our own galaxy, quasars, and quasi-stellar galaxies

14.4 – Emission mechanisms – Thermal, atomic

transition mechanisms, and synchrotron emission (when magnetic fields are present.

14.5 - Thermal emission mechanism

14.6 - Thermal radiation & the H-II Regions - one of the great achievements of radio astronomy has been the understanding of the structure of our galaxy that it has brought to us.

Gives radio map of the Milky Way

14.7 - Interstellar magnetic fields - A magnetic field strength of 1 gamma would suffice to prevent the collapse of the galactic arms by gravitation.

14.8 - Emission from neutral hydrogen. - detectability of the 21-cm radiation & measure thereby of the radial component of neutral hydrogen cloud.

14.9. Structure of the galaxy

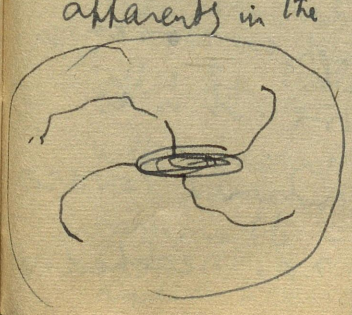
(a) Ionised hydrogen found mainly in clouds that occupy a flat disc in the galactic plane. - The clouds are regions within the much larger amount of un-ionised cold

(b) Assumption of galaxy rotating about a centre with rate of rotation decreasing with increasing distance from the centre. All gas at same distance from centre rotates with same velocity & all of galactic gas (neutral H) is at the same temp.

(c) Doppler shift used to find distance of region

(d) Most of neutral H is confined to a flat disc of diameter ≈ 80,000 light years & thickness only 800 light years & mass about 2% of whole galaxy, most of the mass of this mass being in the stars. $M_{galaxy} \sim 1 \times 10^{11} M_\odot$ ie there are 100 billion stars in the Milky Way.

(e) within the disc the gas is arranged in spiral arms. There are at least four such distinct arms, the Sagitt arms, the Cygnus, Perseus & Norma arms. ∴ the Sun is apparently in the Cygnus arm. — Spiral form, outermost part seriously distorted, the plane seems to be "twisted". Seen edge on the Milky way would have apparently an integral sign



lying down. Reason unknown

(f) At $R/2$ [$R = \frac{1}{2} \times 80,000$ l.y $= 40,000$ l.y] the neutral H density reaches a maximum. At $r = R$ density = 1/10 of max. value.

(g) At distance of earth, for eg ($r \sim 25,000$ light years) the rotational speed of galaxy is about 200 km/sec The speed is nearly constant between this distance and about $r = 9,000$ light years, where it is ~ 175 km/sec

(h) A surprising feature becomes evident at distances less than 9000 l.y. A weak wing is observed in the 21 cm emission line from these regions; a Doppler effect corresponding to about 50 km/sec is observed at $r \sim 9000$ l.y, and speeds of recession from galactic centre as high as 200 km/sec have been measured nearer the centre. It is as though some gas is moving radially outward towards us in the form of a shell that has now reached the

9000 l.y point. This raises the fascinating (201)
possibility that the centre of our galaxy has undergone
an explosion — The explosion hypothesis is supported
somewhat in that a strong non-thermal source
has been detected near the thermal centre of the
galaxy, called Sagittarius A.

14.10 - Interstellar molecules & Chemistry

14.11 - Synchrotron emission (magnetic bremstrahlung)
- Is the e.m. radiation produced by the ultra-
relativistic charged particles moving in a magnetic
field.

14.12 - The galactic ~~hal~~ halo. - At meter wavelengths
the galaxy appears to be roughly spherical rather
than disc shaped. The region outside the disc is
called the halo - It has a non-thermal spectrum
with synchrotron mechanism being perhaps
responsible — We infer that a magnetic field
and a halo of relativistic electrons pervade

and surround the galaxy of stars.

14.13 – Supernova remnants – Some astronomers believe that all non-thermal galactic radio-sources are supernova remnants. – Discussion of Crab nebula.

14.14 – Radiogalaxies –

14.15 – Quasars

14.16 – Pulsars

14.17 – Primordial fireball

14.18 – Three-Halves Power law – number of galaxies $N = \frac{4}{3} \pi \rho r^3$ (r = radius of sphere, ρ = average density)

of this number applies to all radio-galaxies that are detectable above a given flux threshold S_0. Assuming that all radio-galaxies emit approximately the same radio power, the flux S of each of them depends on distance as r^{-2} & $S = \text{const.} \cdot r^{-2}$. Hence $N = \text{const.} \times S^{-3/2}$ called the "three-halves

power law of radio-astronomy which states that the number of radio sources should increase as the 1.5 power of the threshold flux.

(2) Annual review of Astronomy & Astrophysics

Vol. 7 (1969)

Saw 2 articles:

(a) Cosmic rays in the galaxy by Peter Meyer

(b) The large scale distribution of hydrogen in the galaxy

In (b) found two references to P.B. Price (not R.M. Price), along with Rajan & Tamhane — discussion seems to be similar to reference (1) but more comprehensive

Intro: Galactic hydrogen can be studied in several different ways: the neutral gas through the 21-cm line, and the ionised component through the

(204) thermal radio continuum, the radio recombination lines, Faraday rotation, pulsars, and also through optical observations of H II regions. The radio data can, in principle, extend over the whole of the galaxy

The Dutch & Australian work of the 1950's resulted in the well-known hydrogen spiral maps, including derivation of a galactic rotation curve which was then used to estimate kinematic distances of the H-concentrations.

More recent work has yielded an enormous amount of new information, but not yet lead to a basic understanding of the problems involved. and distribution is extremely complex.

Radio methods developed for studying the ionised component of H have shown that it is possible to attempt the mapping of H II regions [clouds of ionised hydrogen] [neutral H atoms are H I & ionised oxygen is O II]

in the whole galaxy

It has become very clear that the distribution and motion problems must be be solved together & this combined problem has not yet been solved

(3) Physics of non-thermal sources edited by S.P. Maran & A.G.W. Cameron (1964) (A NASA publication June 64)

References to work of R.M. Price:

p.37 dealing with linear polarisation effects in radio sources - Cooper & Price showed that at three different points in Centaurus A, the position angle of the electric current varied as λ^2 over a considerable range of wave lengths & hence they suggested that Faraday rotation had been observed.

p.48 - Map of Centaurus A by Cooper & Price - contour map given

p. 49 = We await with interest Cooper & Price's projected study of this source (between 21 cm line & zero wave length) with the 10 Cm parametric receiver

p. 64 = Ref. to similar work with Taurus A.

pp. 144-45 = Interpretation of Faraday rotation effects (relating to Crab Neb nebula & extra galactic sources)

[Firm(1) In 1845 Faraday discovered that when a block of glass is subjected to a strong magnetic field, the glass affects the propagation of light passing through it. In particular, when plane polarised light is sent through glass in a direction ||^{l} to the applied magnetic field, the plane of the electric vector is rotated. The amount of rotation is ∴^{l} to the field strength & path length through the

medium, the constant of proportionality being a function of the medium & its density. This is the Faraday rotation effect — This is also mentioned in connection with the galactic halo.

went to Raman Institute on 16th at 4 P.M, but found that the meeting was over by 11 A.M, thanks to my not looking at the invitation letter properly — Anyway met R.M. Price himself accidentally, but could not talk anything with him since he was busy leaving — When he saw me, he automatically saluted me and introduced himself as Price.

Free neutron $n \to p^+ + e^- + \bar{\nu}_e$

$p^+ + e^- + \bar{\nu}_e \to n$

$p^+ + \bar{\nu}_e \to n + e^+$ (Detection of antineutrinos)

$n + \nu_e \to p^+ + e^-$ (Detection of neutrinos by neutrino telescope)

$e^- + e^+ = \nu_e + \bar{\nu}_e$ (Death of neutrinos → supernovae)

$\pi^- \to \mu^- + \bar{\nu}_\mu \quad \| \quad \pi^- \to e^- + \bar{\nu}_e$

$\pi^+ \to \mu^+ + \nu_\mu \quad \| \quad \pi^+ \to e^+ + \nu_e$ (rare : 1 in 7000)

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$
$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

$\nu_\mu + n \to p^+ + \mu^-$ ‖ $\nu + n \to p^+ + \mu^-$ — noticed in 2-Vexp

$\nu + n \to p^+ + e^-$ — not "

23/7/73 — At Institute.

Looked at two vols of Essays in Physics (Series Publication) and found an article on Neutrino Physics very interesting & understandable — Also another essay on SU(3). Borrowed the 2nd Vol & found that the date of return was 25/7/73 & so returned the book.

Later went to Dr B. Sanjiva Rao's house near M.E.S. M.P.L's College & Seshachar's house. Found unexpectedly that Dr B.S. had back vols. of Proc. Ind. Acad & that he wanted to get rid of them.

30/7/73 At Institute — found F.C.G. had left on 22/7/73 back to the States

— M.R. Anantharayanan told me that Dr B. Sanjiva Rao wanted time till 6/8/73 to check up the

(1) Bi-national Conference on Mathematics Research and Education - Bangalore 4/6/73 to 15/6/73

4 Vols already published by Bambah.

A lot of nonsense hastily published.

(2) Foundations of Quantum Mechanics - Proc. Fermi International School - Course 49 - Edited by d'Espagnat - (Academic Press, N.Y & Lond, 1971)

- Lots of high-brow articles specially relation between quant. mech & philosophy - opening lecture by E. Wigner 24 lectures

on p. 422 - D. Bohm mentions that Einstein when he was 15 years old asked the question "what would happen if one were to travel with the speed of light and look in a mirror" - Bohm's article continued

~~of course~~ Evidently one would see nothing

because the light from one's face would never reach the mirror . . . .

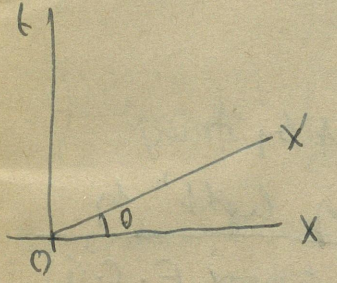
From our more modern vantage point, we can emphasise this difference yet more by considering the atomic structure of the matter out of which we are constituted. If we went faster than light, as a simple calculation shows, the electromagnetic fields that hold our ~~bodies~~ atoms together would be left behind us (as the waves produced by an aeroplane are left behind it when it goes faster than sound). As a result our atoms would disperse, and we would fall apart. So it would make no sense to suppose that we could go faster than light

It leads to absurdities to suppose that we can catch up with and overtake light.

All laws have to be invariant relationships independent of the speed of the observer. (In a space

ship, without windows, all phenomena will proceed in the same way, independently of the speed of the ship.) Such invariance is possible if relations between frames be

$$x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}} \; ; \; t' = \frac{t - Vx/c^2}{\sqrt{1 - V^2/c^2}}$$



From this it can be seen that two observers moving at different speeds will assign the property of simultaneity to a different set of events.

If OX represents a set of events that are regarded as simultaneous in the frame of the laboratory, then a non-horizontal line OX' (with slope $\tan\theta = V/c$) will give those events that are regarded as simultaneous in the frame of a space ship moving at the velocity V. — This relativity of time is one of the radically new features of Einstein's theory.

The speed of light is taken not as a possible

(212) speed of an Object, but rather as the maximum speed of propagation of a signal

(2) Relativity & Common sense by Hermann Bondi – Book to be borrowed.

(3) Space & time in special relativity by A.A.D. Mermin. – to be borrowed.

1/8/73 Span July 1973 (Brought by Arun)
Article on "Faster than speed of light" by Isaac Asimov [No reference to poor E.C.G]

3/8/73 – At Institute – Andhra Appl. Maths & Appl. Physics Paper Setting for 1973

(A) Applied Maths – II Semester – Maths II

Complex Variables – 3 Qns → 6
Solid Geometry – 3 Qns → 6
F.T & L.T – 3 Qns → 6
for the 2 papers.

Complex Variables.

(1) State and prove Jordan's lemma (§ 4 8, p. 122, Phillips) and use it to evaluate the integral

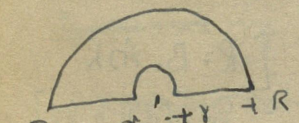
I 22

$$\int_0^\infty \frac{\sin x}{x}$$

by using an indented contour (Sansone & Gerretsen VR I p. 131

[Consider $\int_C \frac{e^{iz}}{z}\,dz$

the contour C consisting of the real axis from r to R ($r<R$) the upper half of the circle $|z| = R$, the real axis from $-R$ to $-r$ and the upper half of the circle $|z| = r$



From Jordan's lemma, the integral along the large circle tends to zero as $R \to \infty$. The integral along the small circle $\to -i\pi$, since $z e^{iz}/z \to 0$ & the circle is traversed from the left to the right. There are no singularities inside the contour. Hence

$$\int_{-\infty}^{\infty} \frac{e^{iz}}{z}\,dz = \pi i$$

(2 1 4) where the integral is a principal value at $z = 0$.

on equating the real & imaginary parts & taking account of the fact that $\sin x/x$ is an even function, we obtain

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{1}{2}\pi \Big]$$

Q(1') (a) State and prove Cauchy's Residue Theorem (Phillips p. 114) and ~~deduce~~ show that if $f(z)$ has a simple pole at $z = z_0$, ~~then~~ then

II. Q.2

$$\operatorname{Res}(f, z_0) = \lim_{z \to z_0} \left[(z - z_0) f(z)\right]$$

(b) use the Calculus of residue to show that

$$\int_0^{2\pi} \frac{\cos\theta}{5 + 4\cos\theta}\,d\theta = -\frac{\pi}{3}$$ [R. B. Ash Ex. (d), p. 90]

[Soln on p. 223:

given integral $= \displaystyle\int_0^{2\pi} \frac{e^{iz} + e^{-iz}}{2\{5 + 4(e^{iz} + e^{-iz})}\,d\theta$

$$= \int_{|z|=1} \frac{z + z^{-1}}{2\{5 + 2(z + z^{-1})} \cdot \frac{dz}{iz}$$

$= 2\pi i \sum$ residues of integrand

inside inside the unit circle

Now. $\dfrac{z+z^{-1}}{2\{5+2(z+z^{-1})} \cdot \dfrac{1}{iz} = \dfrac{z^2+1}{2iz(2z+1)(z+2)}$

Residue at $z=0$ is $\frac{1}{4i}$ & at $z=-\frac{1}{2}$ is given by

$\dfrac{5/4}{2i(-1/2)2(3/2)} = -\dfrac{5}{12i}$. Hence the integral

$= 2\pi i\left(\dfrac{-2}{12i}\right) = -\dfrac{\pi}{3}$.

(2) (a) Illustrate the ~~Explain the~~ notions of Riemann surfaces and branch points ~~for~~ associated multiple valued functions, ~~and illustrate the same~~ by considering the function $w^n = z^{1/2}$ (~~a positive integer~~)

[Phillips p. 28]

(b) Find all the bilinear conformal transformations which transform the half-plane $I(z) \geq 0$ into the unit circle $|w| \leq 1$. [Phillips, p. 51] BK work ~~worked~~

[Ans: $w = \dfrac{z-a}{z-\bar{a}} e^{i\theta}$]

(2') (a) If $f(z)$ is ~~regular~~ analytic at the point ~~P~~ z, with $f'(z) \neq 0$, show that the mapping ~~is conf~~ $w = f(z)$ is conformal at the point, and find expressions for the angle of rotation and the linear magnification of the mapping [$\arg f'(z)$ and $|f'(z)|$

L Q.3

(b) ~~Show that~~ find the Riemann surface and branch points associated with the mapping

$$w = \frac{1}{2}\left(z + \frac{1}{z}\right)$$ [M. Fuchs & Shabat]

[~~For~~ two-sheeted surface obtained by taking two superposed w-planes which cut along the segment $(-1, +1)$, the edges being ~~pasted~~ joined criss-cross fashion is the R-surface.

Branch points are $w = \pm 1$]

(3) (a) State and prove Cauchy's integral theorem.

II Q.1

(b) Obtain the Taylor's series expansion for the

analytic function $f(z)$ about the point $z = a$.

B')(a). Derive the Cauchy-Riemann conditions for an analytic function $f(z)$, and deduce that the real and imaginary parts u & v of $f(z)$ are both harmonic functions.

I-B.1

(b) If, in particular, $u(x,y) = e^x(x\cos y - y\sin y)$ find $f(z)$. [Phillips, Ex. p. 17

[Here $\phi_1 = \frac{\partial u}{\partial x} = e^x(x\cos y - y\sin y + \cos y)$

$\phi_2 = \frac{\partial u}{\partial y} = e^x(-x\sin y - \sin y - y\cos y)$

Hence $f'(z) = \phi_1(z,0) - i\phi_2(z,0) = e^z(z+1)$

$\therefore f(z) = \int e^z(z+1)\,dz + C = ze^z + C$.

(h) Find the Laurent series expansion of the function $f(z) = 1/(z-2)$ in the neighbourhood of the points $z = 0$ and $z = \infty$ [Berry & Kovari, p. 64

[$-\frac{1}{2}\sum_{n=0}^{\infty}(z/2)^n$ for $|z|<2$, and $\sum_{n=0}^{\infty}\frac{2^n}{z^{n+1}}$ for $|z|>2$]

(218)

(a) (c) Show that two of the lines $\mathbf{r} = \vec{a} + k\vec{u}$, $\mathbf{r} = b + h\vec{v}$ intersect if $[\vec{v}\ \vec{b}\ \vec{u}] = [\vec{v}\ \vec{a}\ \vec{u}]$ and the point of intersection is

$$\vec{a} + \frac{[a\,b\,v]}{[v\,a\,u]}\vec{u} \text{ or } \vec{b} + \frac{[a\,b\,u]}{[v\,b\,u]}\vec{v}$$

(b) Show that the equation of the $\perp^r$ from the point $\vec{h}$ to the line $\vec{r} = \vec{a} + k\vec{t}$ is

$$\vec{r} = \vec{b} + k\vec{t} \times \{(\vec{a} - \vec{b}) \times \vec{t}\}.$$

(c) Show that the plane through the points $\vec{r}_1, \vec{r}_2, \vec{r}_3$ has the equation

$$[r, r_2, r_3] + [r_1, r_3, r] + [r_1, r, r_2] = [r_1 r_2 r_3]$$

[unworked Examples from Rutherford's vector methods]

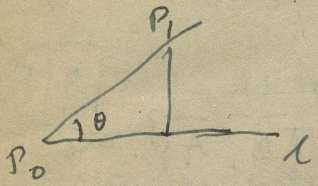
(d) Skew lines : P_0 & P_1 have $\vec{r}_0$ & $\vec{r}_1$ & unit $\vec{v}_0 = \cos\alpha_0\,\vec{i} + \ldots$, $\vec{v}_1 = \cos\alpha_1\,\vec{i} + \ldots$

$$\cos\theta = \pm \frac{\vec{v}_0 \times \vec{v}_1}{|\vec{v}_0|\,|\vec{v}_1|}$$

$$S.D = d = \frac{|\overrightarrow{P_0P_1} \cdot \vec{r}|}{|\vec{r}|} \quad \text{where } \vec{r} = \vec{v_0} \times \vec{v_1}$$

($i \cdot \vec{r}$ along SD)

(e) Distance between point & line



$d = |\overrightarrow{P_0P_1}| \sin\theta.$

unit vector $\parallel d$ to P_0P_1 is

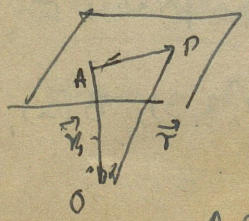
$$\frac{(x_1 - x_0)\,i}{|\overrightarrow{P_0P_1}|} \; i + \cdot \; + \cdot$$

& unit vector $\parallel d$ to l is $\cos\alpha \cdot i + - +$

Hence $\cos\theta \longrightarrow \sin\theta$.

[(d) & (e) Vectors & applns by A. J. Pelletreggio]

(f) Vector eqn of the plane through a given point and $\parallel$ to two given lines



Let $A(\vec{r_1})$ be given point & $P(\vec{r})$ any point on the req'd plane and $\vec{i_1}$, $\vec{i_2}$ the direct vectors of two given lines. Since AP is coplanar with $\vec{i_1}$ & $\vec{i_2}$ we may

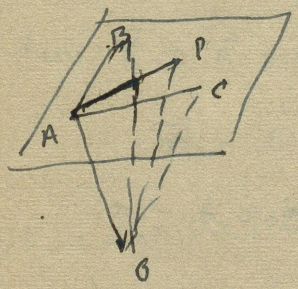
(220)

$$\overrightarrow{AP} = \lambda \vec{d}_1 + \mu \vec{d}_2$$

Hence $\vec{r} - \vec{r}_1 = \lambda \vec{i}_1 + \mu \vec{i}_2$

ie $\vec{r} = \vec{r}_1 + \lambda \vec{i}_1 + \mu \vec{i}_2$.]

(g) Same as (c) from Rutherford



Since vectors $\overrightarrow{AP}$, $\overrightarrow{AB}$, $\overrightarrow{AC}$ are coplanar, we may write

$$\overrightarrow{AP} = \lambda \overrightarrow{AB} + \mu \overrightarrow{AC}$$

ie $\vec{r} - \vec{r}_1 = \lambda(\vec{r}_2 - \vec{r}_1) + \mu(\vec{r}_3 - \vec{r}_1)$

& reqd eqn of plane is

$$\vec{r} = (1 - \lambda - \mu)\vec{r}_1 + \lambda \vec{r}_2 + \mu r_3$$

[(f) & (g) are from Intro to Vector Analysis by J. C. Tallack]

Form given by Rutherford in (c) is preferable to (g)

But in a numerical case ~~where~~ with A(1, 2, 3), B(−1, 2, 0) and C(2, −1, −1) we have

$$x = 1 - 2\lambda + \mu$$

$$y = 2 - 3\mu$$

$$z = 3 - 3\lambda - 4\mu$$

Eliminating λ & μ we get $9x + 11y - 6z = 13$.

7/8/73 — At Institute

(1) Bells coordinate Geometry.

(a) If any secant through a given point O meets a given sphere in P & Q, $OP \cdot OQ = \text{Const}$

[$\frac{x-\alpha}{l} = \ldots = r$, Putting $x = \alpha + lr$, ... in eqn of sphere

$$F(x, y, z) = a(x^2+y^2+z^2) + 2ux + 2vy + 2wz + d = 0$$

we have $ar^2 + r\left(l\frac{\partial F}{\partial \alpha} + \ldots\right) + F(\alpha, \beta, \gamma) = 0$

Hence $OP \cdot OQ = F(\alpha, \beta, \gamma)/a$ = same for all secants

d = power of O w.r.t. sphere

Radical plane of two spheres - Locus of pts of equal power

Given Putting $a = 1$ for both spheres $F_1 = F_2 = 0$.

Locus is $S_1 - S_2 = 0$.

Radical plane of three spheres taken two by two pass through one line (eqn is $S_1 = S_2 = S_3$

Radical plane of four spheres taken two by two pass through a point (given by $S_1 = S_2 = S_3 = S_4$

Coaxial system : $x^2+y^2+z^2+2\lambda x+d=0$ (taking (λ variable)
line of centres as x-axis, and radical plane as $x=0$)

Examples :

(1) A plane passes through a fixed point (a, b, c), and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=0$$

(2) If r is the radius of the circle

$$x^2+y^2+z^2+2ux+2vy+d=0, \quad lx+my+nz=0$$

show that $(r^2+d)(l^2+m^2+n^2) = (mw-nv)^2 + - + -$

(3) Prove that the eqn of the sphere which lies in the octant ~~OABC~~ OXYZ, and touches the coordinate planes is of the form

$$x^2+y^2+z^2-2\lambda(x+y+z)+2\lambda^2=0$$

(4) OA, OB, OC are mutually $\perp^r$ lines through the origin & their direction cosines are (l_i, m_i, n_i) $(i=1,2,3)$

If $OA = a$, $OB = b$, $OC = c$, prove the eqn of sphere $OABC$ is

$$(x^2+y^2+z^2) - x(al_1 + bl_2 + cl_3) - y(am_1 + \cdots) - z(an_1 + \cdots) = 0$$

---

(5) Constn that (i) a line be $\perp^r$ to a plane &

(ii) a line may lie on a plane

(i) $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$ (ii) $al+bm+cn=0$ and $a\alpha+b\beta+c\gamma+d=0$

(6) Constn that three planes

$$u_i = a_i x + b_i y + c_i z + d_i = 0$$

should intersect in a line

If any two of the four determinants $\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{vmatrix} = 0$ ← (1)

(60)

(5) Discussion of the intersection of three planes viz

$$u_i = a_i x + b_i y + c_i z + d_i = 0 \quad (i = 1, 2, 3)$$

(i) If $\Delta = |a_1 b_2 c_3| \neq 0$, planes have a common pt & only

(ii) If $\Delta = 0$, and ~~$|b_1 c_2 d_3| \neq 0$~~, planes form one of the other three determinants in (1) above $\neq 0$, like

planes form a Δ^r pencil

(iii) If any two of four determinants in (1) are not zero, and one of their common minors is not zero, the remaining two determinants are zero & the three planes have a line of intⁿ at a finite distance

Alternative proof of (ii) Any plane thro' int of $u_1=0, u_2=0$ is given by $\lambda_1 u_1 + \lambda_2 u_2 = 0$. If the planes through the three planes has through one line

$$\lambda_1 u_1 + \lambda_2 u_2 = -\lambda_3 u_3 \quad -(a)$$

$$\text{or } \lambda_1 u_1 + \lambda_2 u_2 + \lambda_3 u_3 = 0 \quad -(b)$$

converse is easily proved by going from (b) to (a).

considering coefficients of (b) we have

$$a_i \lambda_i = 0,\ b_i \lambda_i = 0,\ c_i \lambda_i = 0,\ d_i \lambda_i = 0$$

Eliminate the λ_i's we have (1)

(6) Line intersecting two given lines $u_1=0=v_1$ and $u_2=0=v_2$ has the eqns $u_1 + \lambda v_1 = 0$, $u_2 + \lambda v_2 = 0$

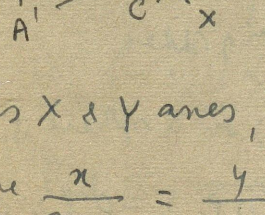
[Reqd. each plane contains the two given lines resp]

17) Line intersecting three given lines viz $u_i = 0 = v_i$ $(i=1$

If the three planes $u_i - \lambda_i v_i = 0$ ~~are coplanar~~ have a line of nits, this line is coplanar with each of the three given lines & intersects all the three given lines

The conds for existence of such a line of nits are two in number say $F_1(\lambda_1, \lambda_2, \lambda_3) = 0$, $F_2(\lambda_1, \lambda_2, \lambda_3) = 0$ If the λ_i be chosen to satisfy these two conditions, then any two of $u_i - \lambda_i v_i = 0$ represents a line which intersects the three given lines. Thus an infinite no. of lines can be drawn to intersect three given lines.

18)



(i) For two skew lines AB, A'B' let CC' be S.D. Take mdpt of CC' as origin and the bisectors of angles between ||ls to AB & A'B' through O as X & Y axes, OC' as z-axis, the eqns of AB & A'B' are

$$\frac{x}{\cos\alpha} = \frac{y}{\sin\alpha} = \frac{z-c}{0}, \quad \frac{x}{\cos\alpha} = \frac{y}{-\sin\alpha} = \frac{z+c}{0}$$

2α = angle between AB & A'B

(ii) Obliging ans - Taking ||ls to AB & A'B' through O as X & Y

(226) ans, eqns of AB & A'B' are

$y = 0,\ z = c$ and $y = 0,\ x = 0,\ z = -c$.

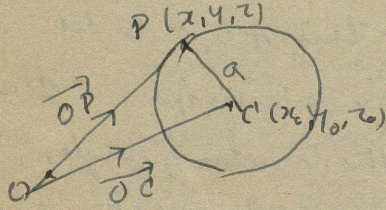
Problem. Find locus surface generated by a st. line which intersects two given lines and is ∥ to a given plane.

Solⁿ: with axes oblique & given plane as $lx + my + nz = 0$

the locus is $\frac{lx}{z+c} + \frac{my}{z-c} + n = 0$.

(9) From Pettofrezzo:

p. 81



(a) & $|\overrightarrow{OP} - \overrightarrow{OC}| = a$

$(\overrightarrow{OP} - \overrightarrow{OC}) \cdot (\overrightarrow{OP} - \overrightarrow{OC}) = a^2$

$\overrightarrow{OP} = ix +$ —

$\overrightarrow{OC} = ix_0 +$ ·

& these lead to $(x - x_0)^2 +$ — $= a^2$

eqn of sphere

(b) Eqn tangent plane at $P_1(x_1, y_1, z_1)$ on a sphere is

$(\overrightarrow{OP} - \overrightarrow{OP_1}) \cdot (\overrightarrow{OC} - \overrightarrow{OP_1}) = 0$ ($\because CP_1 \perp PP_1$)

where P is any point (x, y, z) on the tangent plane

using i, j, k | vectors vector eqn reduces to usual eqn

$$(x-x_1)(x_0-x_1)+(y-y_1)(y_0-y_1)+(z-z_1)(z_0-z_1)=0$$

(10) From ~~Tallack~~ Benny.

(a) In a tetrahedron, show that the lines joining ~~of midpts of opposite faces~~ the vertices to the centroids of the opp. faces, and the lines joining the midpts of pairs of opp. edges are all concurrent.

[Taking vertices as (x_i, y_i, z_i) $(i=1,\dots 4)$

~~m~~ x-coords of midpts of one pair (14 & 23) are $\frac{x_2+x_3}{2}$ and $\frac{x_1+x_4}{2}$, & that of midpt of line joining these is $\frac{1}{4}(x_1+x_2+x_3+x_4)$. Sim for y & z coords

C.G of BCD is $\left(\frac{x_2+x_3+x_4}{3}, \dots\right)$ say G_1

& pt. dividing AG_1 in ratio 3 : 1 is

$$\left(\frac{\frac{x_2+x_3+x_4}{3}+\frac{1}{3}x}{1+1/3}, \dots\right)$$

ie $\left(\frac{x_1+x_2+x_3+x_4}{4}, \dots\right)$. Hence result

Put this in vector form - ~~Done in Ex.~~ unworked Ex. 12, p. 170.

(b) Condn of intn of two lines viz $(\alpha-\alpha')(mn'-m'n)+\dots=0$

Questions on Fourier Series etc

(D) (11) Fundamentals of mathematical analysis by Fikhtengol'ts. Vol II (Tr. by Sneddon).

Ex: p. 444 – Find the Fourier cosine and Fourier sine transforms of the function

$$f(x) = e^{-ax} \quad (a>0,\ x \geq 0)$$

[Cosine transform is

$$F_c(x) = \sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty e^{-az} \cos zx\, dz = \sqrt{\left(\frac{2}{\pi}\right)} \frac{a}{a^2+x^2}$$

$$F_s(x) = \sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty e^{-az} \sin zx\, dz = \sqrt{\left(\frac{2}{\pi}\right)} \frac{x}{a^2+x^2}$$

~~Deduce~~ Hence use the inverse transforms to evaluate the integrals $\int_0^\infty \frac{\cos xz}{a^2+z^2}\, dz$, and $\int_0^\infty \frac{\sin xz}{a^2+z^2}\, dz$

$$= \frac{\pi}{2a} e^{-ax} \qquad \frac{\pi}{2} e^{-ax}$$

[Fourier transform in general is

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(u)\, e^{izu} \cdot du$$

Inverse $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(z)\, e^{-izx} \cdot dz$

For F_c & F_s these become

$$\left.\begin{aligned} F_c(z) &= \sqrt{\frac{2}{\pi}} \int_0^\infty f(u) \cos zu \, du \\ f(x) &= \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(z) \cos xz . dz \end{aligned}\right\}$$

$$\left.\begin{aligned} F_s(z) &= \sqrt{\frac{2}{\pi}} \int_0^\infty f(u) \sin zu \, du \\ f(x) &= \sqrt{\frac{2}{\pi}} \int_0^\infty F_s(z) \sin xz \, dz . \end{aligned}\right\}$$

Using these the results given can be obtained]

(12) Vector Analysis by Max. Stein (At home)

Ex. 7, p. 18 - If $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are coplanar show that

$$(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = 0$$

$$\left\{ (\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = [ABD]C - [ABC]D \right.$$

& each of the triple scalar products $= 0$ since A, B, D are coplanar, as also A, B, C }

Soln of Rutherfords (C) - If r_1, r_2, r_3 be the vectors of A, B, C ~~are coplanar~~, let r be any point in the plane of ABC, then $r - r_1, r_1 - r_2, r_1 - r_3$ are coplanar ie $[(r - r_1), (r_1 - r_2), (r_1 - r_3)] = 0$

ie $(r-r_1)\cdot\{(r_1-r_2)\times(r_2-r_3)\}=0$

Simplifying & putting $[r_1 r_1 r_2]=[r_1 r_3 r_1]=0$, we get

$[r r_1 r_2]-[r r_1 r_3]+[r r_2 r_3]-[r_1 r_2 r_3]=0$

ie $[r r_1 r_2]+[r r_2 r_3]+[r r_3 r_1]=[r_1 r_2 r_3]$ —(1)

Stein Ex. 3, p. 26.—2 show that $(A\times B)+(B\times C)+(C\times A)$ is $\perp^r$ to plane of A, B, C.

~~Putting $r=(r_1\times r_2)+(r_2\times r_3)+(r_3\times r_1)$~~

~~and using $(A\times B)\cdot($~~

~~(1) can be written as~~

~~$\{(A\times B)+(B\times C)+(C\times A)\}=[A B C]$~~

A. It is easy to show that the vector sum of the three cross products is $\perp^r$ to each of the sides AB, BC, BC

thus $(r_1-r_2)\cdot\{(r_1\times r_2)+(r_2\times r_3)+(r_3\times r_1)\}$

$=[r_1 r_1 r_2]+[r_1 r_2 r_3]+[r_1 r_3 r_1]$

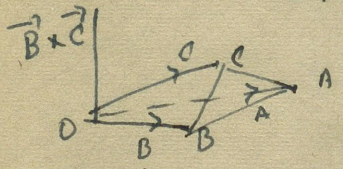
$-[r_2 r_1 r_2]-[r_2 r_2 r_3]-[r_2 r_3 r_1]=0$ since $[r_1 r_2 r_3]=[r_2 r_3 r_1]$

Since the sum of cross products is $\perp^r$ to BC & CA

& hence the result.

To verify if A, B, C are coplanar $[ABC] = 0$ &
conversely, we note that (i) if A, B, C be coplanar

~~OA, OB, OC~~ OA_1 lies in the plane OB & OC



ie $\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$

Sml if $A\cdot(B\times C) = 0$

OA is $\perp^r$ to $(B\times C)$ ie OA lies in plane of OB & OC

(Trivial)

Indian Who's Who, 1971

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Has the pen gone?
No thank God

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cos\frac{(2n+1)\pi x}{2l} = \cos\frac{\pi x}{2l} - \frac{1}{3}\cdot\cos\frac{3\pi x}{2l} + \frac{1}{5}\cos\frac{5\pi x}{2l}$$

$$\text{sens} = \cos\frac{\pi x}{2l} - \frac{\cos 3\pi x}{2l} + \frac{\cos 5\pi x}{2l} - \cdots$$

$$\cos\theta - \cos 3\theta + \cos 5\theta - \qquad = -2i\sin\theta$$

$$e^{i\theta} - e^{3i\theta} + e^{5i\theta} -$$

$$e^{i\theta}(1 - e^{2i\theta} + e^{4i\theta} - \cdots) = e^{i\theta}\left(\frac{e^{i\theta}}{1-e^{2i\theta}}\right) = \frac{1}{e^{-i\theta} - e^{i\theta}}$$

$$= -\frac{1}{e^{i\theta} - e^{-i\theta}}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta \qquad = -\frac{2i}{\sin\theta}$$

$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$1 - x^2 + x^4 -$$



$$\frac{1}{1+x} = 1 - x + x$$

$$\cos\theta + \frac{\cos 3\theta}{3} + \cdot$$

$$= \frac{1}{2}\left[(e^{i\theta} + e^{-i\theta}) + (e^{3i\theta} + e^{-3i\theta}) + \cdots\right.$$

$$= \frac{1}{2}\left[(e^{i\theta} + e^{3i\theta} + \cdots) + (e^{-i\theta} + e^{-3i\theta} + \cdots)\right]$$

$$= \frac{1}{2}\left[e^{i\theta}(1 + e^{2i\theta} + \cdots) + e^{-i\theta}(1 + e^{-2i\theta} + \cdots)\right]$$

$$= \frac{1}{2}\left[e^{i\theta}\cdot\frac{1}{1-e^{2i\theta}} + e^{-i\theta}\cdot\frac{1}{1-e^{-2i\theta}}\right.$$

$$\begin{array}{r} 280 \\ 264 \\ \hline 160 \\ 132 \\ \hline 28 \end{array}$$