Topics

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1. **Deterministic Chaos:**
Chance out of Necessity

(1960–) In Domain of the Complex (All scales)
Infinitesimal – Finite – Infinite

2. **Quantum Indeterminacy:**
Principle of Uncertainty

(1927–) In Domain of the Infinitesimal:
very small (atoms, subatomics...)

Two major developments in our Conception of Nature —
Shifts from the paradigm of classical Determinism
(that dominated thought until turn of century 1800/1900.
In the Domain of the large:
Sensible World)

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RRI

Deterministic Chaos:
Chance out of Necessity

Nature is infinitely Diverse

Diversity: Of Being and Becoming
of Matter and Motion
of particles and Interactions
A Seeming Tyranny of the Particular

This impossible Problem of Book-Keeping
is Reduced by the Observed Regularity
of the Change ⇒ Encoded into a small
number of Finite Statements — rules of change

The Physical Laws: Themselves Changeless
Tenseless

The Many Why's & How's are Reduced
to a Manageable Number of a
few Why's and Hows

Thusly, the Universe becomes Comprehensible
— The Great Reduction.

Received Wisdom of Man & his Kind has been

Physical Laws are —

Universal
Causal
Deterministic

} Classical Paradigm (Physics)

Ex.: Newton's Laws of Motion
Newton's Law of Universal Gravitation
Maxwell's Electromagnetism
(Any Rule of the Game: for change)

The Fall of an Apple to the Earth
" " " the Moon around the Earth
" " " Earth around the Sun

caused by the same Gravitation (universal)

$$F = -\left(\frac{G M_1 M_2}{R_{12}^2}\right)$$

→ Prediction of Eclipses precise to fraction of a second centuries in advance; Determination of Particle trajectories in giant Particle accelerators, colliding beams etc.

Universal, causal Determinism ⇒
Nothing ever happens!

This Laplacean Determinism (classical Physics) turned out to be ESSENTIALLY in ERROR in TWO (distinct) WAYS →

(1) In the Domain of the Very Small: dominated by Principle of Uncertainty (1927-)

(2) In the Domain of the Very Complex dominated by Deterministic Chaos (1960-)

Both impose a Horizon of Limited Predictability which is counter-intuitive: but in their own different ways:

- Quantum Mechanics - Nature does not obey classical Deterministic Laws. Is intrinsically probabilistic.
- Chaos - obeys the letter of classical Law that be; but not the Spirit of it!

What is Deterministic CHAOS

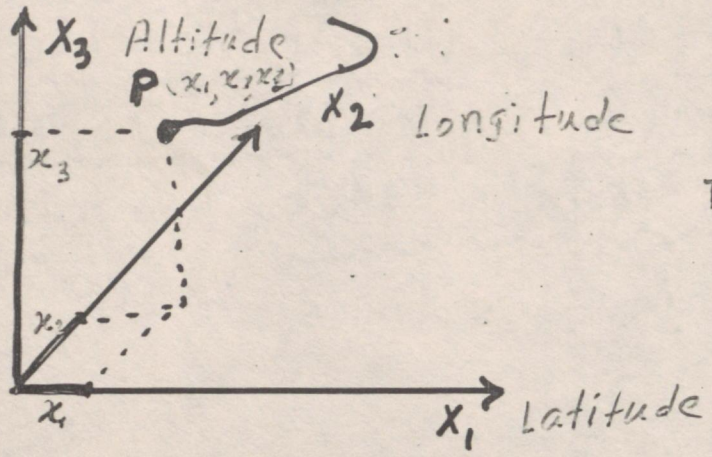
Governed by strictly Deterministic laws, but Unpredictable in the long run in a well Defined Operational Sense. Seemingly random. There is madness in order!

Questions:

1. Is there no contradiction in terms: Deterministic, yet Chaotic. NO.
2. Can small be chaotic. YES. Just two coupled pendulums will do.
3. Is Chaos Common, or a mere oddity. YES. Almost any system driven hard enough turns Chaotic. Turbulence.
4. Is every Chaotic system chaotic in its own way. YES, but there are Universal features.
5. Can chaos be measured. YES. It is an Implicate whole.
6. Can we use/control Chaos. YES. Efficient Stirring.
7. What makes for Chaos — Sensitive Dependence on Initial Conditions due to Non-Linearity

Representation of System & Evolution : State space & co-ordinates (cartesian)

Pictorial - Geometrical

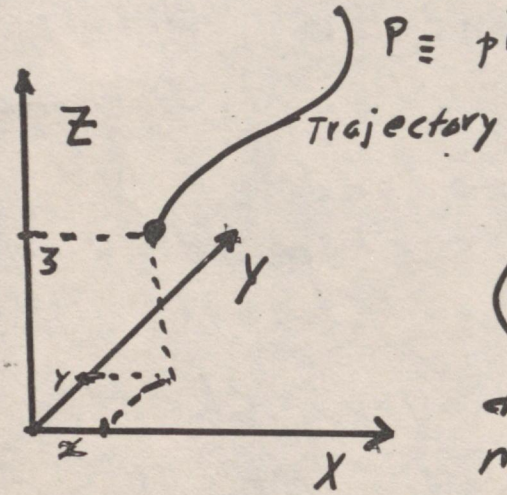


3 dimensional space

Triple (x_1, x_2, x_3) locates P in configuration space at an instant t.

For a particle: 3 position co-ordinates
+ 3 Velocity (momentum)
= 6 phase-space "

N particles ⇒ 6N dimensional phase-space



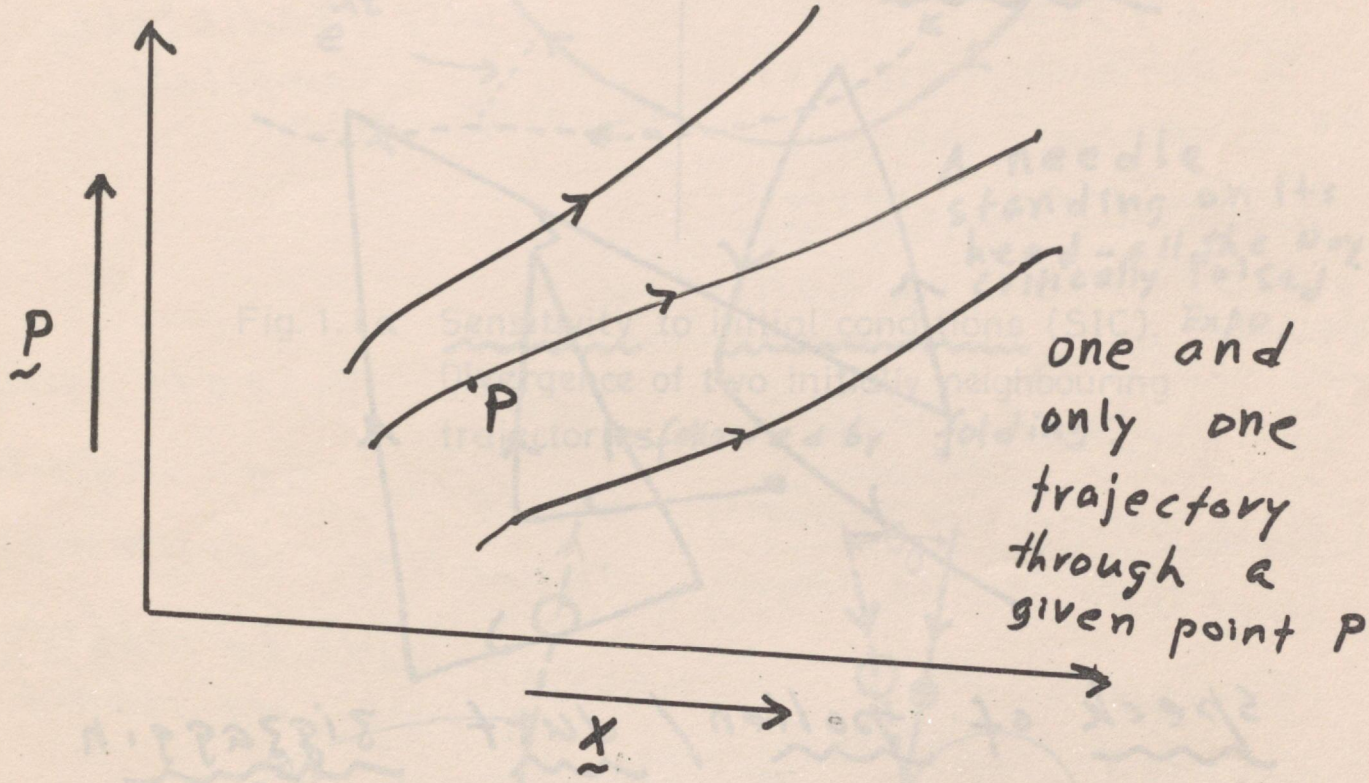
State Space
P state point
$(X, Y, Z, \ldots)$ could denote anything measurable

Examples

X = population of prey
Y = " " predator
Z = resource ...
⋮
Or X = Inflation
Y = Import
Z = Export
⋮
Or X = IQ Y = Bank Balance, Z = family size

$2H_2 + O_2 \rightleftharpoons 2H_2O$
X = con. H_2
Y = conc. O_2
Z = con. H_2O
concentrations of chemical reactants

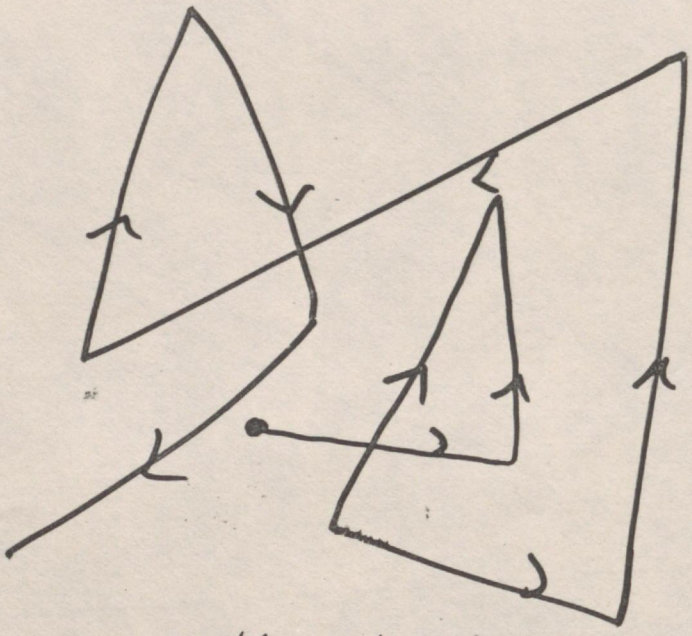
⇒ Classical Determinism : Given initial conditions (e.g. Positions and Velocities of all constituent Units) the trajectories are fully Determined as Necessary Consequence — Absolute Certainty; calculable in principle



Chance / Probability has no fundamental basis — it merely expresses our partial / incomplete knowledge of initial conditions (A practical limitation), or our lack of interest in fine details ⇒ Statistical Mechanics

Large # degrees of freedom : Will need an army of Laplacean Demons to follow these closely — Brownian motion.

Seeming Randomness (aperiodicity) of Chaos not to be confused with the stochastic (true) randomness of Brownian Motion ('Noise'):



speck of pollen / dust zigzagging in water/air : innumerable kicks by water/Air molecules → large # degrees of freedom

Chaos can occur even in two coupled pendulums; or a periodically kicked pendulum → Kinetic Toys.

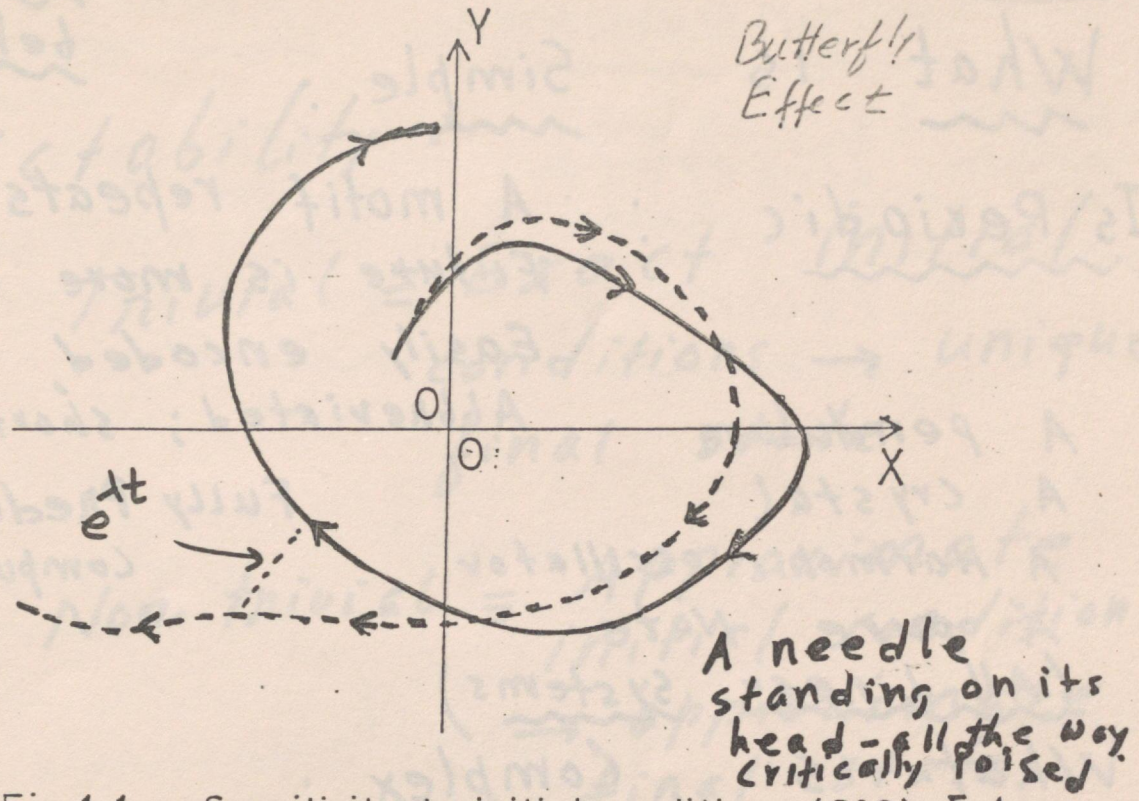


Fig. 1.1a Sensitivity to initial conditions (SIC). Expo. Divergence of two initially neighbouring trajectories followed by folding.

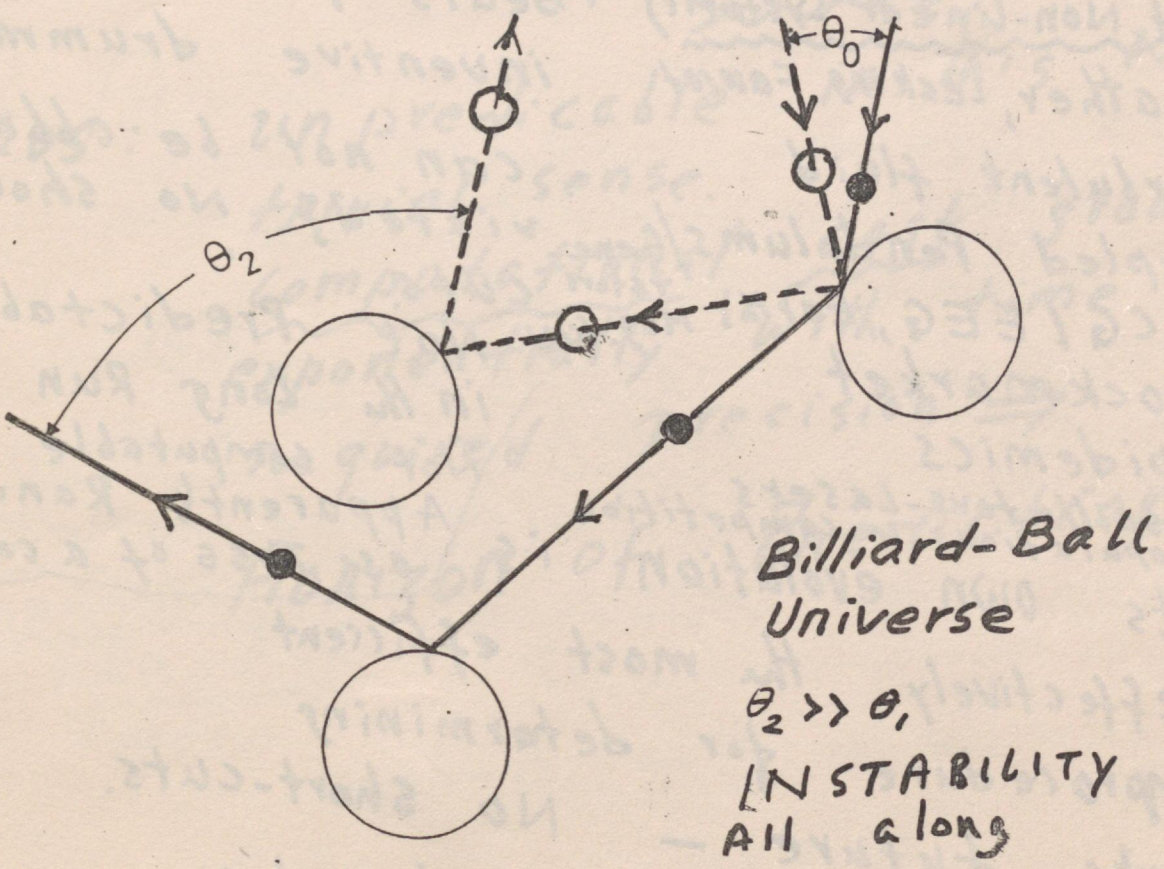


Fig. 1.1b Defocussing of trajectories by the convexity of scattering surfaces giving sensitivity to initial conditions.

CHAOS : at the Heart of Complexity of behaviour

What is Simple :

Is Periodic :

- A motif repeats itself
- Future is more of the same
- Easily encoded stored, Abbreviated; short-cuts.

A pendulum
A crystal
A Harmonic oscillator
A pure Note
(All Linear systems)

Fully Predictable
Computable

What is Complex :

Is Aperiodic :

(Most Non-linear systems)

- Never Repeats itself
- Beats of an infinitely inventive drummer
- can not be abbreviated; No short cuts.

Weather, Leaking Faucet
Turbulent fluid
Coupled Pendulums/Generators
ECG/EEG, Earth's Mag. field
Stockmarket
Epidemics
oscillators-Lasers
populations in competition

Not Predictable in the Long Run
Not computable
Apparently Random as Toss of a coin

- Its own evolution is effectively the most efficient procedure for determining its future — NO short-cuts.

It must be lived through!

Predictability:

Trivial ≡ Exact initial conditions → unique final state

Non-trivial ≡ Approximate initial conditions → approximate final state

Errors do not grow exponentially

Chaos: unpredicable in this Non-trivial sense.

'computational cost' grows exponentially with time for required precision ⇒

Horizon of Prediction.

Continuous Flow to Discrete Map: Poincare' Section

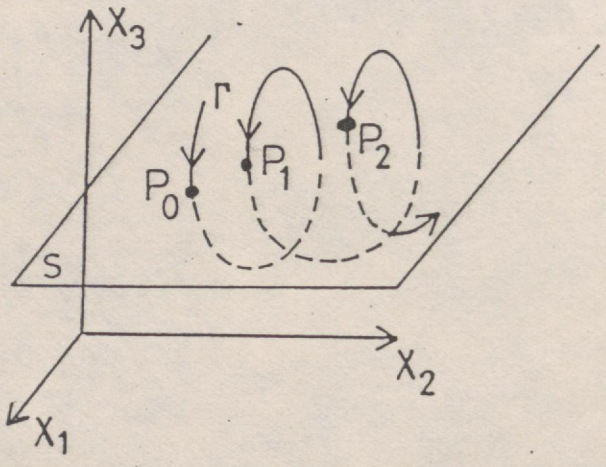


Fig. 2.4 Poincare' Section for 3-dimensional phase flow. S denotes Surface of Section.

Example: Continuous time-evolution in 3 dimensions (x_1, x_2, x_3) reduced to discrete jumps in 2 dimenensions (x_2, x_3) : Reduction

MAP: $P_0 \to P_1 \to P_2 \to \ldots$

$(x_1^0, x_2^0) \to (x_1^1, x_2^1) \to (x_1^2, x_2^2) \to \ldots$

$\ldots \to (x_1^N, x_2^N) \to \ldots$

Evolution in continuous time is conveniently viewed as repeated Iteration in Discrete time steps:

Repeated Iteration ⇒

1. Wander off to infinity (unbounded)
2. Become : simple — Periodic (bounded) Limit cycle, Fixed point
3. Become : chaotic — aperiodic (bounded) (strange Attractor. never repeating, ever wandering, confined)

No other Possibility !!

- Most Real systems obey ③, in some region of parameter space

Logistic Problem : Population X

Growth Limited by Resource and Population

$X_{n+1} = \lambda X_n$ unlimited growth/decay of pop.

annual census X_n

$\lambda < 1 \Rightarrow$ bust
$\lambda > 1 \Rightarrow$ boom

(compound interest)
stretching

$X_{n+1} = \lambda X_n (1 - X_n)$ limited by over population (folding back)

($\lambda \equiv$ boom or bust parameter,
Max. population normalized to unity)

As λ is tuned up from 0 to 4
the population growth pattern changes
from periodic to chaotic (aperiodic)
→ Period Doubling Route to Chaos.

Many competing populations show
such behaviour

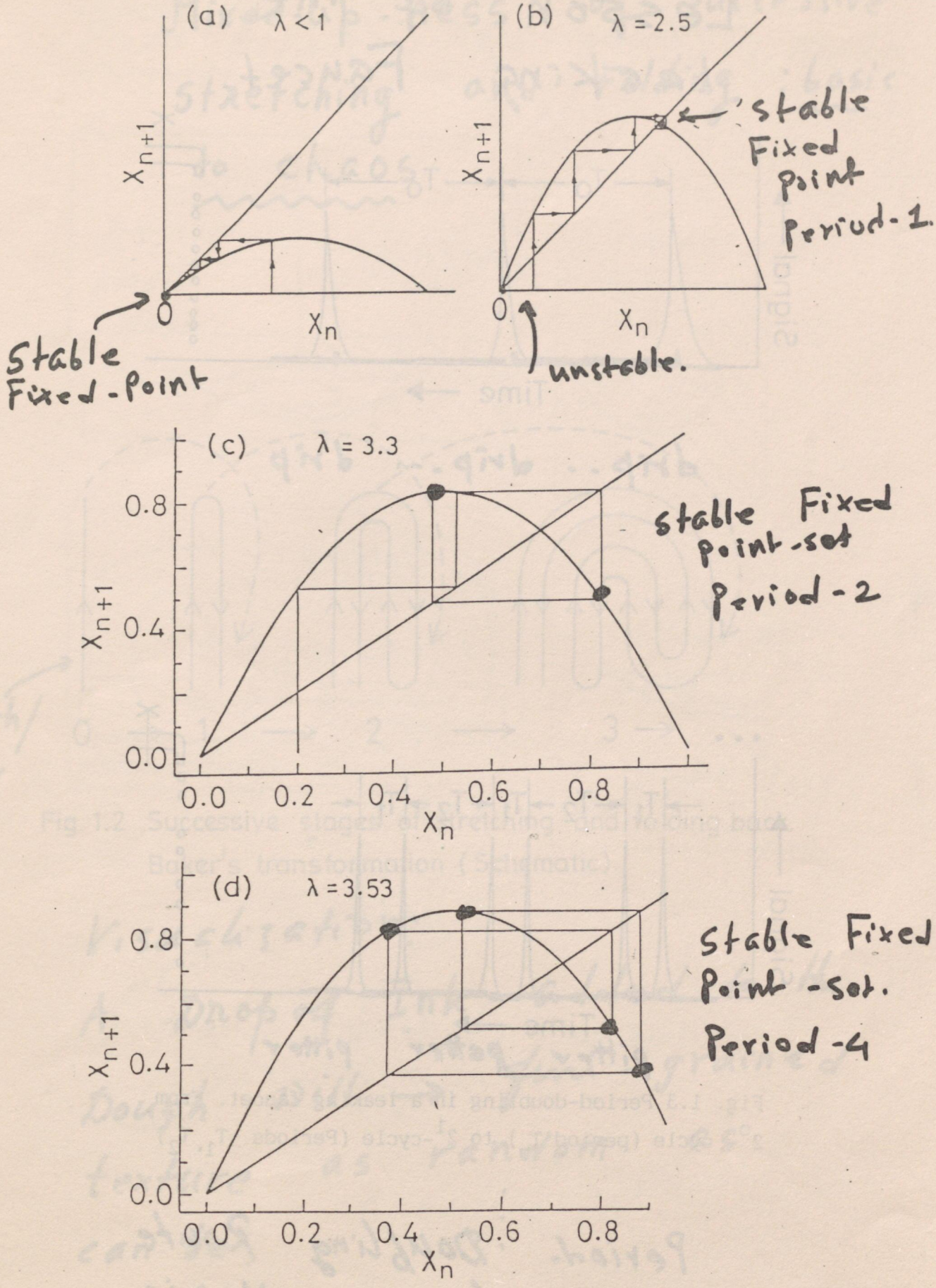
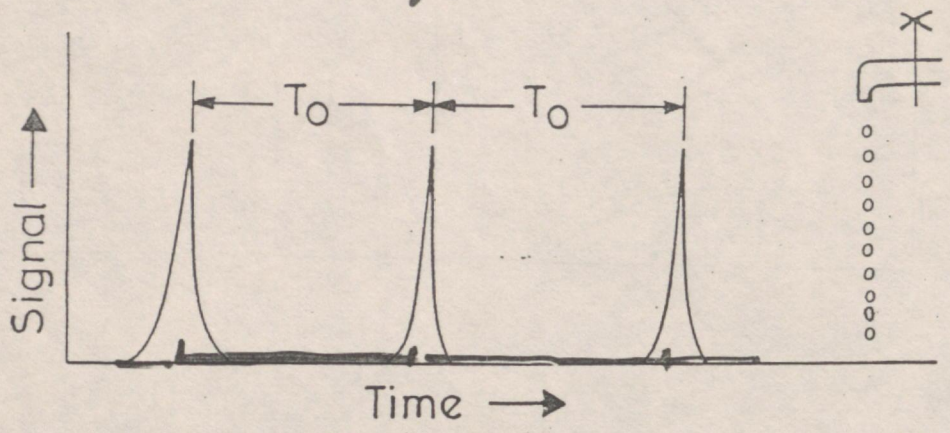


Fig. 3.1 Stable fixed point (a) at origin, (b) away from origin; (c) period-2 (d) period-4 (Schematic).

Lesson of the Leaking Faucet



drip.. drip... drip

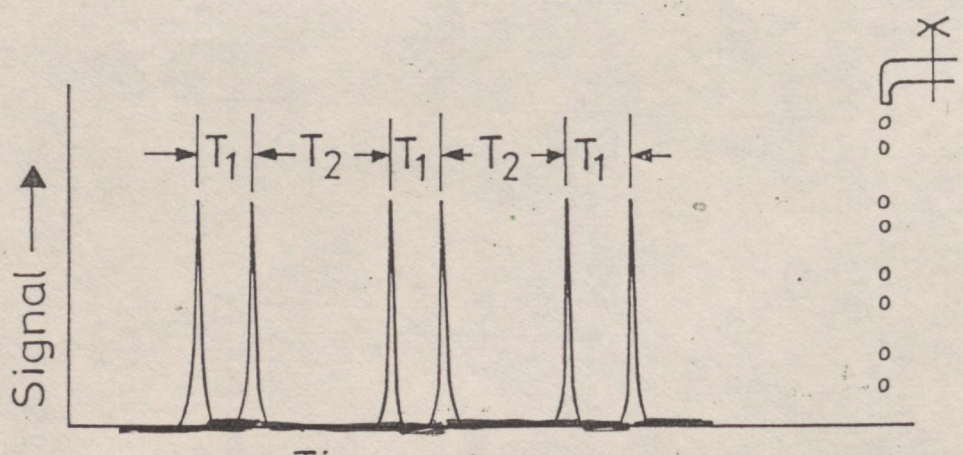


Fig. 1.3 Period-doubling in a leaking faucet. From 2^0- cycle (period T_0) to 2^1-cycle (Periods T_1, T_2)

Period-Doubling Route to Chaos as flow is turned on beyond threshold

Mixed-up-ness by successive stretching and Folding : basic to chaos

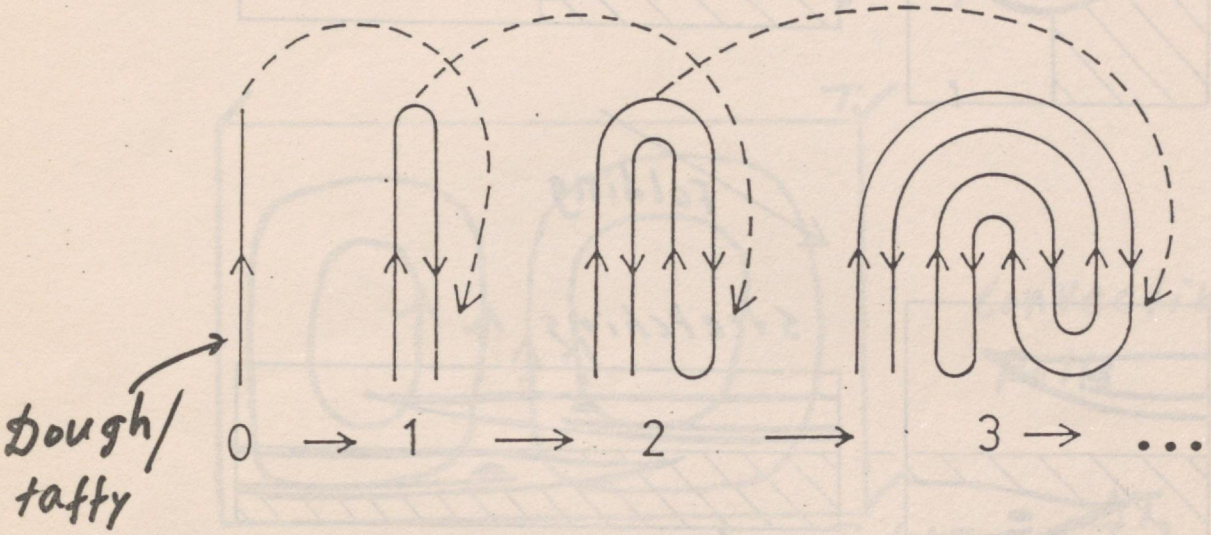


Fig. 1.2 Successive stages of stretching and folding back. Baker's transformation (Schematic).

Visualization:
A Drop of Ink added to the Dough will → fine-grained texture as random as can be.

Mixed-up Cat

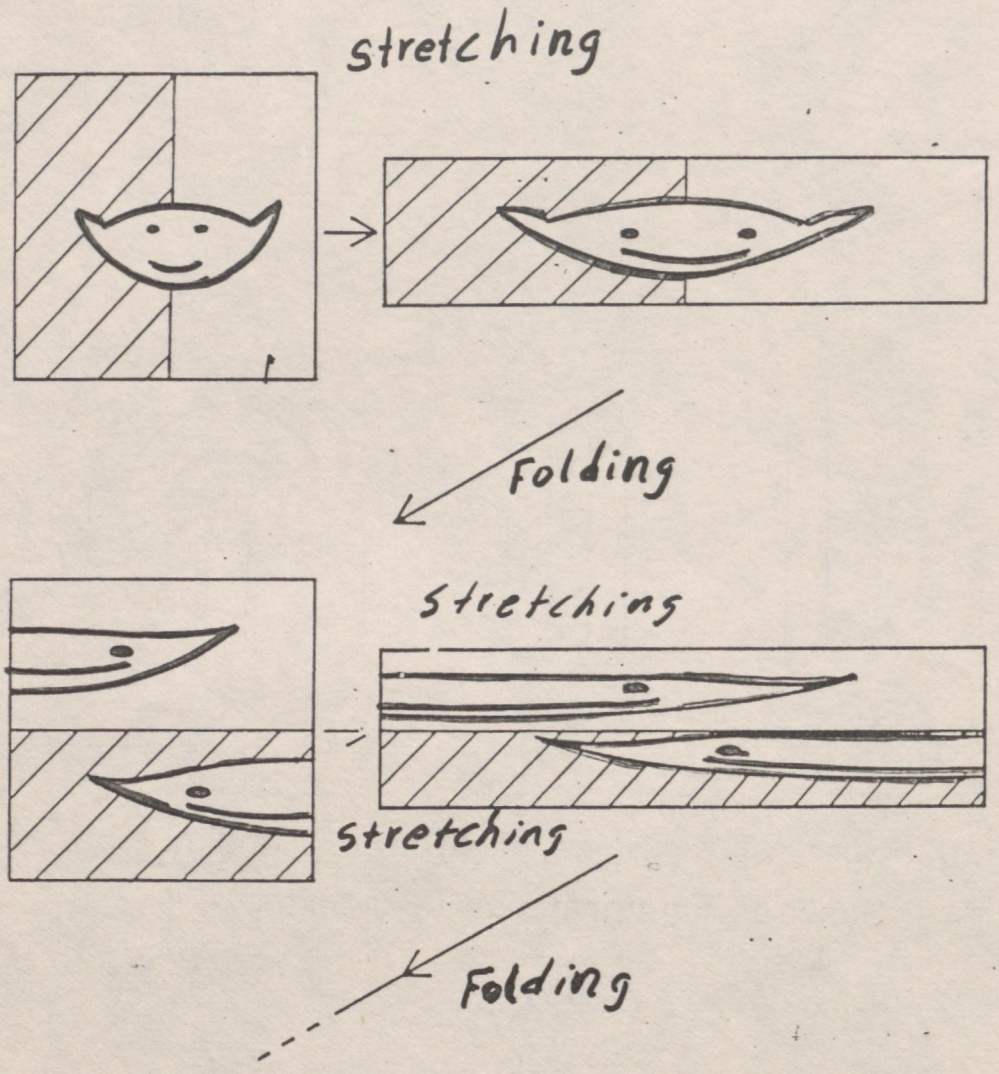
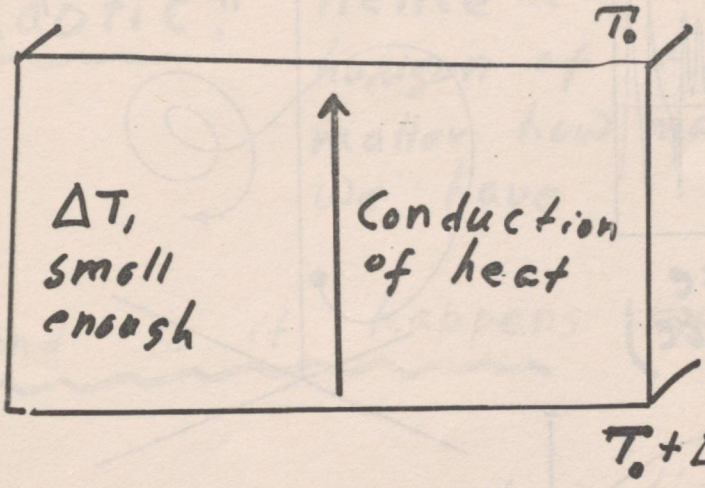
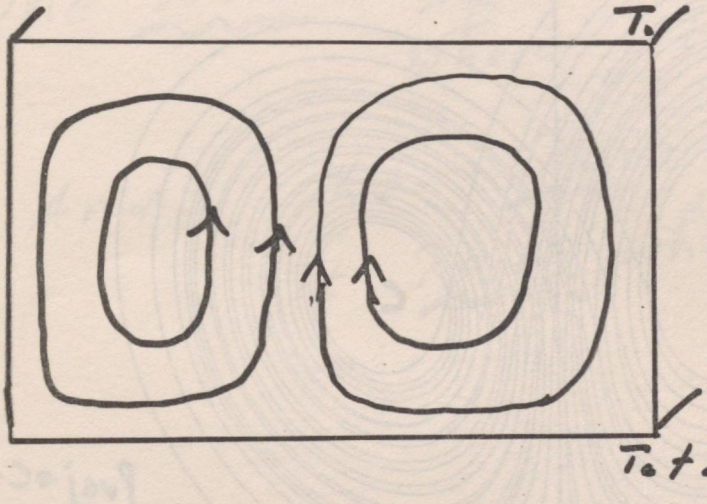


Fig. B1. Baker's Transformation. Notice the mixed-upness of "Arnold's Cat."

Convection Cell : Fluid Heated From Below: Atmosphere / Toy Weather



conduction



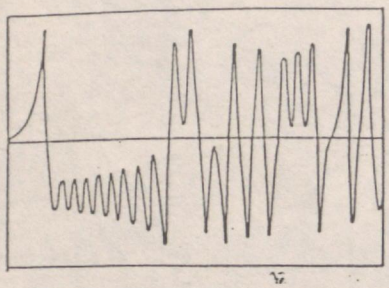
Convective Rolls

ΔT_2 large enough

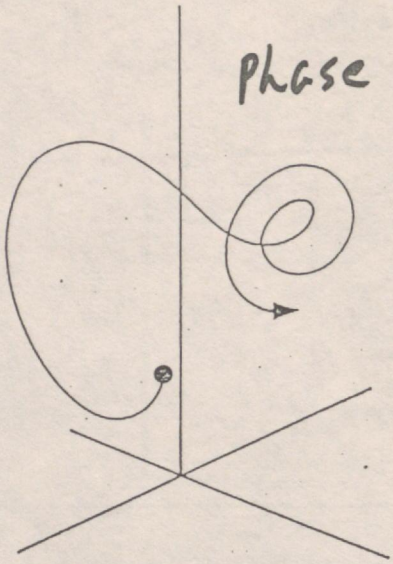
Rolls begin to Wobble

↓ Chaotic pattern

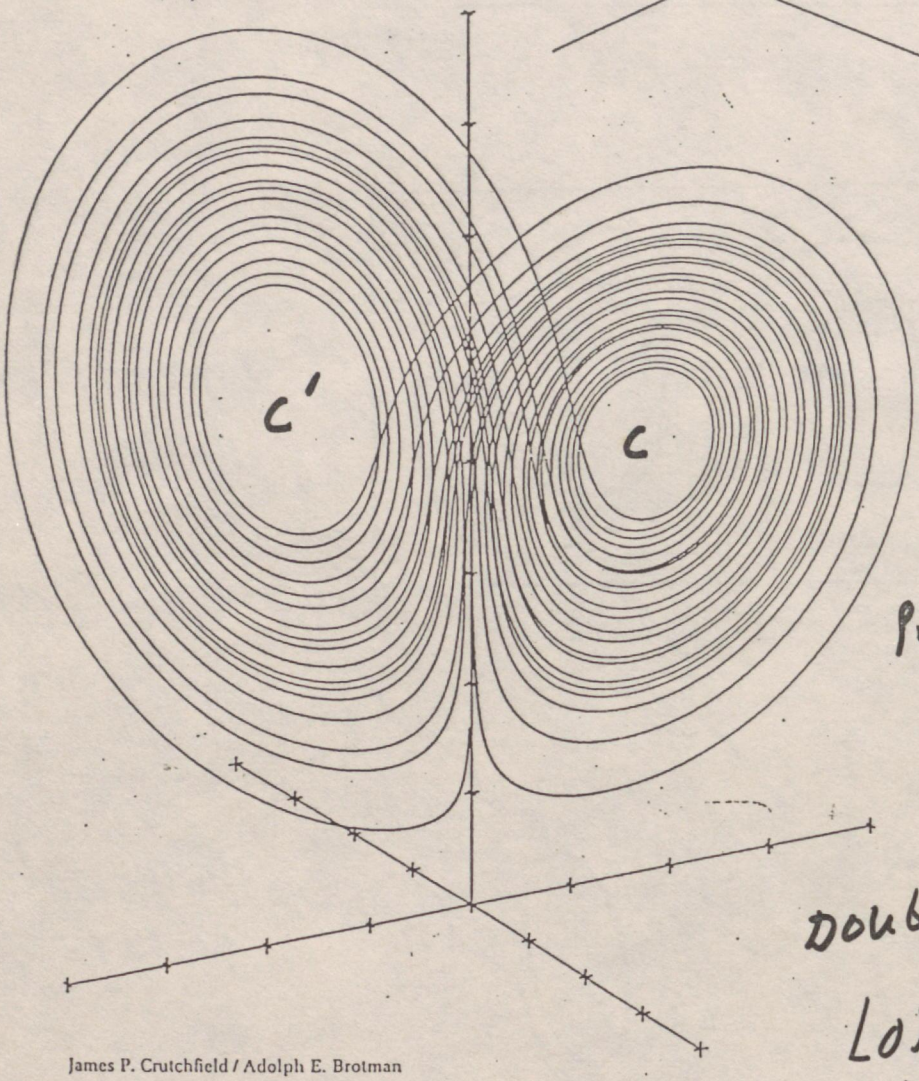
Chaotic Attractor
for the convective cell: Toy Weather



Time-trace
(Implicate)



Phase space



James P. Crutchfield / Adolph E. Brotman

Projection

Double-scroll

Lorenz
Attractor

'Random' flipping
between c and
c' ≈ Toss of a
coin!

Toy Weather
of Lorenz
(X, Y, Z): Three-
dimensional phase-
space

... so,

Weather is inherently

Chaotic: Hence the Limited Prediction horizon of a few days, no matter how many weather satellites we have

And so it happens that ...

The Wind Bloweth
where it Listeth

And the Rain
It Rains where it Wills
