

# Dispersion formulae in relation to the polarization fields in dense media.

1. ~~the~~ All theories of dispersion based on the ~~fundamental~~ <sup>fundamental</sup> postulate that the constituent ~~atoms~~ molecules of the med. are polarizable: ~~the~~ i.e. behave as ~~the~~ dipole oscillators in the oscillating electric field

of the incident light wave.  
 For an isolated oscillator ~~produced in the med.~~  
 The polarization =  $\alpha E$  where

$$\alpha = \sum_{i=1, n}^z \frac{F_i e^2}{(\Omega_i^2 - \omega^2)}$$

$F_i$  strength  $\Omega_i$  freq.,  $\omega$  of incident wave.

this does not take into account

frictional forces or damping =

2) In studying the dispersion of a dense medium  
~~an assembly~~ consists of  
 such oscillators per unit vol;

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~~the polarization~~  
~~the field acting on the oscillator~~  
~~the medium naturally gets polarized~~  
~~Point in calculating and~~  
we are naturally interested in  
~~determination~~ knowing the pol.  
per unit volume of the med.  
in terms of  $\chi$   
one requires to know the polarization  
 $\chi$  produced in unit volume  
per unit field in the medium  
& the dielectric constant  $K$   
is given by

$$K - 1 = 4\pi \chi.$$

~~the med.~~ Hence our main  
objective is to calculate  $\chi$   
in terms of  $\alpha$ .

3. Two questions arise immediately  
(a) How is the field  $F$  actually acting  
on the oscillator, and which  
produces the polarization related

to the field in the medium, ~~as~~  
in terms of which  $\chi$  is defined  
(b) ~~How~~ What relation does

~~the~~ the polariz  $\chi$  produced  
in the assembly related to  
the polariz  $\chi$  of the ~~medium~~  
isolated oscillator?

3) Obviously the answers to (a) & (b) will enable  
the purpose of this article

to review the ~~last~~ current-  
dispersion formulae  
from this point of view.

us to formulate  
the dispersion i.e.  
to formulate explicitly  
to select  
the const. or ref.  
index of a med  
dispersion  
in terms of the  
applied frequency

(in the medium is defined in the following  
manner. ---)

4. Now the field, ~~actually~~  
this will not however be the field  
of an oscillator  
and which produces the  
polariz  $\chi$ . ~~may be regarded as~~  
the latter field will consist  
of include in addition  
to the fd in the med, the  
polarization field. i.e.  $F = E + P$

5)

In the special case

when the oscillators are polarizable dipoles, and they are arranged either at the pts of a regular cubic lattice, ~~or~~ are distributed at random. Lorentz showed that

$$P = \frac{4\pi}{3} N E$$

$$E + P = E \left[ 1 + \frac{4\pi}{3} N \right] = E \times \frac{K+2}{3}$$

$\chi$  will then be  $= n\alpha$

so that one obtains the well known formula of Lorentz

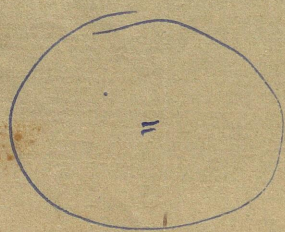
$$\frac{K-1}{K+2} = \frac{4\pi}{3} n\alpha$$

6. Now what is the significance of this polarization field  $P$ ?

# From definition  $E = F +$  field due to the surface charges:

~~Sometimes~~  $P$  is calculated by Lorentz in the

following manner: Scoop out



Surface charge on the inner face of sphere  $P$

Since the dipoles make up the sphere  
[if pt dip. & if cubic array or random]  
give zero field at centre.

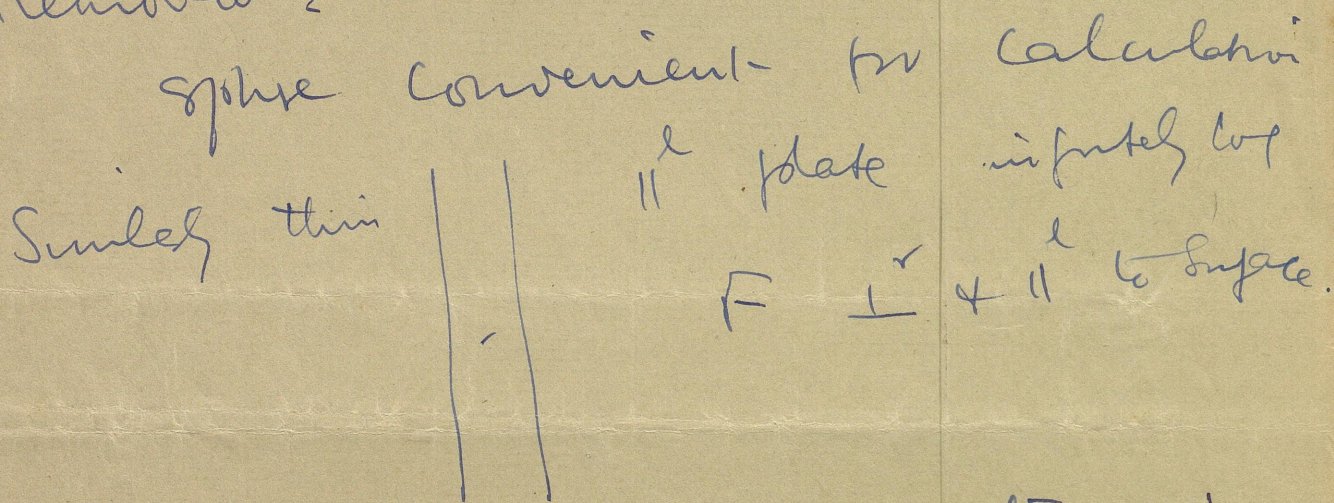
Hence ~~usually description~~  
~~of P as due to effect of dipoles~~  
~~dipoles would be this way~~

At 1st sight it might appear from this that the distant dipoles produce the polarization field & not the near ones. This would not be correct.

Actually the polariz<sup>n</sup> field is the in a sense <sup>an</sup> ~~self~~ <sup>natural</sup> ~~field~~ <sup>is -ve, +ve</sup> which is <sup>excluded</sup> from E but included in F. Difference bet<sup>n</sup> field at pt P when the doublet at P is removed (without disturbing the rest of the med.) & field at P when the doublet is there!

7) the choice of spherical shape

for cavity - may seem arbitrary.  
 It is not so. By definition the  
 pot. field is just the difference between  
 the field that would obtain at  
 pt P when the molec. at P is  
 removed, and when it is not  
 removed =



Demonstrate:  $\epsilon_{11}^3$  factor in  
 these 2 cases: Cubic distri. -  
 pt diple.

Ionic crystal: Displacement - pot.  
 $\epsilon_{11}^3$  factor verified:

But - electronic dipole not  
 pt-dipole: Distribution of surrounding  
 not cubic: or spherical =  
 with general  $\mu$   $\frac{k-1}{k+\beta} = -$

More generally

$$\mu_{ij} \quad K-1 = 4\pi X$$

$$= 4\pi n \sum \frac{e^2 F_i}{m_i (\omega^2 - \omega_{i0}^2)} \epsilon (1 + \mu_{ij} x_j)$$

can not be expressed in a simple form -  
when all  $\mu_{ij}$ 's =

Now ~~⊕~~ when the  $\mu_{ij}$ 's are  
all = 0 <sup>reduces</sup> to Drude's  
which is as it should be since Drude's  
did not contemplate the presence  
of ~~any~~ <sup>any</sup> polarization field:

Hence ~~it has~~  $D$ 's is regarded  
as a special case of  $L$ 's applicable  
to the case when all  $\mu_{ij} = 0$

this is not true

~~the effect~~ show that the general  $L$ -  
Expression above reduces to Drude's.

The obvious conclusion to draw is this:  
That effect of polariz. field can be taken  
into acc in 2 other ways.

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In L's explicitly = the field  
 primary the pol<sub>1</sub> is not field E is  
 the med. but niches pol<sub>2</sub> field also.  
 Hence K corresponds greater - the  
 for  $F_i$  and  $\Omega_i$  same as for  
 the isolated oscillator.

2)  $F_i + \Omega_i$  changed to  $f_i + \omega_i$   
 char. of the assembly  $\equiv$   $f_i + \omega_i$   
 field activity is just  $E$ .

whether  
 the = a  
 pol<sub>1</sub> field  
 or not

This greatly simplifies - however  
 complicated the pol<sub>1</sub> field may  
 be, D's will hold.  
 one needs to know

All that ~~is necessary~~ to  
 describe uniquely the dispersion  
 is  $f_i$  and  $\omega_i$  for all the 2 oscillator  
 types:  $\leftarrow$  A from p. 9

Conversely the dispersion data  
 can supply only the 2 Z constants  
 which would be needed just to describe  
 the class of the oscillators, & would  
 not give any information regarding the  
 polarization fields.

A  
To go  
into p. 8

Illustrate in case of ~~infra~~ polar  
osc. of NaCl:

$$\Omega \text{ vs } \omega:$$

Pol. field  
separately taken into acc.

Effect pol.  
included ahead -

$$E_e - \text{vs.} - E_e$$

Energy

Any formula which involves  
2 constants for each oscillator will  
fit with the same ~~the~~ data

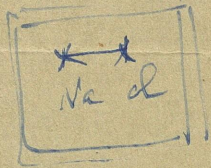
$$\frac{K-1}{K+2} \quad \text{or} \quad \frac{K-1}{K+\alpha}$$

where  $\alpha$  is given

will fit equally well. Such a  
fit does not prove anything about  
the <sup>actual</sup> magnitude of the <sup>pol.</sup> field.

If ~~if~~ independent considerations  
one knows either  $\Omega_i$ 's ~~from separate~~  
~~state~~, or the pol. field for the  
~~dependence~~ <sup>separation</sup> on ~~separation~~ one would  
be able to include find the other.

$\Omega_i$  from vapour for <sup>at all</sup> elec. = freq.  
 $\hbar \omega_i$  for essent. vibr. freq.



In benzene get  $\Omega_i$ 's for vapour  
 $\hbar \omega_i$ 's for liq.  
 whence get ph. field : take the same  
 for all  $\nu_{ij}$ 's.

or rare gases :  
<sup>their</sup>  
liquids.