P. U. Press, Poona]

UNIVERSITY OF POONA

EXAMINATION, 195

Subject

Paper

Section

Examiners' Signatures

CANDIDATE'S No.	QUESTIONS 1	2	3	4	5	6	7	8	9	10	11	12	Paper Total	Paper Total	Practical	GRAND TOTAL	CANDIDATE'S No.
121																	121
122																	122
123																	123
124																	124
125																	125
126																	126
127																	127
128																	128
129																	129
130																	130
131																	131
132																	
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136																	
137																	
138																	
139																	
140																	

Dice-Cyphering x = no. of brothers

y = " sisters.

$\cancel{x+1}\ (2x+1)5+y$

$= 10x+5+y$

Subtracting 5 $\rightarrow 10x+y = xy$.

(very trivial)

Three in sequence: $x, x+1, x+2$ (all <50)

Let multiple of 3 be $3y$.

Key number is $(y+1)$.

Adding nos. in sequence $\rightarrow 3x+3$.

Adding $3y \rightarrow 3x+3y+3$.

Multiply be 67, $67 \times 3 = 201$.

$\rightarrow 201(x+y+1)$.

$x+y+1 < 80$

< 64

$3y < 40$

49

~~39~~

~~$\times 64$.~~

150

~~64×201~~

64

00

128

~~1286~~

~~50×201~~

50

00

100

~~10050~~

~~$10x$,~~

~~$10x+y,\ 10x+y+1,\ 10x+y+2$~~

~~$30x+3y+3$~~

~~$3(10x+y+1)$.~~

~~Add $3z \rightarrow 3(10x+y+z+1)$~~

~~$= 3(10x+10u+v)$~~

~~$= 3\{10(x+u)+v\}$~~

~~$(x+u)v$~~

~~$= 3(10x+z)+3(y+1)$~~

$y+z+1 = 10u+v$

$a-b=5$

$b-c=7$

$a-c=12$

$\rightarrow$ If $x+y+1$ be a two digit number multiplication by $2 \times 3 \times 67 = 201$ gives the 2 digit number preceded by its double (i.e. a 4 digit no) & the last two digits are $y+1$. Subtracting this gives x.

60
70
80
210

47
48
49
144

99
98
97
294

$a-b=10$

$a-b=4$

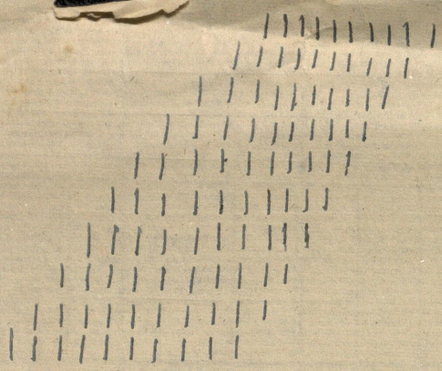
$b-c=$

675

574

671

111111 × 1111111111 (with ten 1's)



12345679009876543 21

$76923 = 9 \times 8547$

$= 9 \times 11 \times 777$ 111 × 7

$= 27 \times 11 \times 7 \times 37$

Symmetry test:

The last digit: abcd a+b+c+d

(i) crossing a → bcd, $bcd - (a+b+c+d) = 100b + 10c + d - a - b - c - d$

$= 99b + 9c - a$.

↳ This is the number given by the friend.

7128

$(1000.1 + 20 + 8) - (7+1+2+8)$

$99 + 18 - 7$.

$117 - 7 = 110$

whatever b and c, the sum of digits of $99b + 9c - a$ will be of the form $9x - a$ & so $9x - (9x - a) = a$

obvious the same is true when any other digit is crossed.

Trivial but interesting

This is obviously true of a number of any digits say $a_1, a_2 \cdots a_n$. Striking out a_r the result is $a_1 a_2 \cdots a_{r-1} a_{r+1} \cdots a_n$

$= a_1 \cdot 10^{n-2} + a_2 \cdot 10^{n-3} + \cdots + a_{r-1} \cdot 10^{n-r+1} + a_{r+1} \cdot 10^{n-r-1} + \cdots + 10 a_{n-1} + a_n$

On this subtract $(a_1 + a_2 + \cdots + a_n)$ we have

$a_1 (10^{n-2} - 1) + \cdots + 9 a_{n-1} - a_r$

The first n−1 terms constitute a multiple of 9 & hence same argument above holds. Only 4 digits are considered by Heath for sake of simplicity

: abcd.

$ab = 10a + b$.

next digit $= 10a + b + 1$

Adding → $20a + 2b + 1$.

Multiplying by 5 → $100a + 10b + 5 = ab5$

Place 0 at end → $ab50$.

Any no. between 10 & 99 say xy & subtract from ab50 } i.e. $1000a + 100b + 50 - 10x - y$.

Add to this cd → $1000a + 100b + 50 - 10x - y + 10c + d$

only subtract from this $50 + 10x + y$ → $1000a + 100b + 10c + d$

= abcd:

Conductor (50)

M. N. Quaresti — MYF. No. 674. — No. 2 - Mehkri Circle

Problem 4 - Twenty nine players take part in a knockout tennis tournament. How many matches are played in all

Soln. Each match knocks out one player, so that in order to end with a single winner 29-1 players must be knocked out. Hence there will be 29-1 = 28 matches in all

Problem 5 - A man leaves his estate to be divided between his three sons in the proportions 1/2, 1/3, 1/9

After debts & taxes his estate amounted to $17,000. How much did each son receive

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{9} = \frac{9}{18}, \frac{6}{18}, \frac{2}{18} \quad \& \text{ sum of these} = \frac{17}{18}$$

$$\left[\frac{1}{2} : \frac{1}{3} : \frac{1}{9} = 9 : 6 : 2\right]$$

ie dividing 17000 in the ∴ 9, 6, 2 gives 9000, 6000, 2000

Problem 6 - A prime no. is the sum of two other prime numbers. The sum of the two largest primes is 1,200. What are the three prime numbers?

Soln - There is only one even prime 2. Hence if the sum of two prime nos. is a third prime, one of the two smaller nos. must be 2. If the other x,

$$x + (x+2) = 1200 \quad \text{ie } 2x = 1200 - 2 \text{ or } x = 599$$

The primes are 599 and 601.

Problem 7 - What are the square roots of i

Soln: $\pm \frac{1}{\sqrt{2}}(1+i)$.

Problem 8: ~~A fam~~ A famous formula $e^{i\pi} + 1 = 0$ (including e, π, i)

Problem 9: Which of the following are unaffected by a change in the number base from decimal to binary? (i) Primes (ii) Perfect nos. (iii) an[illegible] numbers.

Soln - All three (?)

by 2 and record the remainders. Thus

2) 26	
2) 13	0
2) 6	1
2) 3	0
2) 1	1
0	1

Reading the remainders upwards the number in binary notation (for 26) is 11010 [$= 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0 = 16+8+0+2+0 = 26$]

Problem 1 – The striking clock – It takes five seconds for my clock to strike six. How long does it take to strike twelve

Ans Soln – Not ten seconds. but 11 seconds (corresponding to 11 intervals) (just as striking 6 → 5 intervals)

Problem 2 :– Joan keeps her books very tidily on her bookshelf. Her mother says that if she keeps acquiring more books at the present rate she will soon have over fifty.

Exactly 20% of Joan's books are school workbooks and exactly one-seventh were presents she received at last Christmas.

How many books has Joan on her shelf

Sol: Since 20% of five is one, the no. of schoolbooks must be an exact number, the total number of books Joan has must be a multiple of 5.

Since one-seventh were presents last Christmas, the total number of books must also be a multiple of 7.

The first two nos. which are multiples of both 5 and 7 are 35 and 70. She has not yet acquired 50 books, so she has 35 books.

Problem 3 – Given that a nine digit number beginning with 4 and ending with 4 has an exact fifth root, what is the number

Soln: $100^5 = 10^{10}$ is a ten-digit number, hence the fifth root of 9 digit number must be under 100.

Now, $50^5 = 312,500,000$

$60^5 = 777,600,000$

The fifth root is between 50 and 60 (?). Since the number 4, the fifth root ends in 4 and the reqd no is $54^5 = 459,165,024$.

[P. T.O

(A) Transfer of digits

$(100a + 10b + c)p = (100b + 10c + a)q$

$a(100p - q) + b(10p - 100q) + c(p - 10q) = 0.$

$a(100p - q) + (10b + c)(p - 10q) = 0$

$a(100p - q) = (10b + c)(10q - p)$

$\frac{10b + c}{a} = \frac{100p - q}{10q - p}.$

$\frac{100p - q}{10q - p} = \lambda$

$100p - q = \lambda(10q - p)$

$p(100 + \lambda) = q(10\lambda + 1)$

$\frac{p}{q} = \frac{10\lambda + 1}{100 + \lambda}.$

$\frac{10b + c}{a} = \frac{296}{37} = 8$

$\lambda = 8$ $\quad \frac{81}{108} = \frac{9}{12} = \frac{3}{4}$

$p = 3, q = 4.$

$10b + c = 8a.$

$b = 2, c = 4, a = 3$

$a = 1.$

$a = 4, 10b + c = 32$

$b = 3, c = 2.$

$432 \to 324$

324×3 1296

$4)972(243$

$b = 1, 130 + 13c = 19a$

$\frac{10b + c}{a} = \frac{19}{13}.$

$130b + 13c = 19a$

$324 \to 243$ (one soln)

transferring of digits.

$\frac{496}{40-5} = \frac{496}{35}$ 2

$\frac{300-2}{20-3} = \frac{298}{17}$ $\frac{595}{44}$

$\frac{100p - q}{10q - p} = \frac{296}{37} = 8$

$\frac{10b + c}{a} = \frac{19}{13}$

$130b + 13c = 19a$

$\frac{10b + c}{a} = \frac{19}{13}$

$\frac{10b + c}{a} =$

$\frac{7}{38}$

λ	$(10\lambda+1)/(100+\lambda)$				
1	—	7	71/107 X	14	141/114 X
2	21/102 X $\frac{7}{34}$	8	81/108 $= \frac{3}{4}$ ✓	15	151/115
3	31/103 X	9	91/109 X	16	161/116
4	41/104	11	111/111 no significance	17	171/117 X
5	51/105	12	121/112 X	18	181/118 X
6	61/106	13	131/113 X	19	191/ X
				20	201/120 X

$\because p < 10$ & $q < 10$

$\frac{10b + c}{a} =$

$x=2, z=5, w=\frac{250y}{y+18}$, $y=2 \to w=25 \to \frac{225}{225}$ trivial ✗

$y=7 \to w=70 \to \frac{275}{770}=\frac{25}{70}$ ✓

(D)

Addl. List Contd.

$x=2, z=6, w=\frac{260y}{y+18}$, $y=2 \to$ trivial case ✗

$y=6 \to w=65 \to \frac{266}{665}=\frac{26}{65}=\frac{2}{5}$ ✓

$\frac{266}{665}=\frac{133\times 2}{133\times 5}$ Analogously $\frac{266}{665}\ \frac{26}{65}$.

In fact same is true of $\frac{266\cdots 6}{666\cdots 5}$ ✓

$=\frac{(133\cdots 3)\times 2}{(133\cdots 3)\times 5}=\frac{2}{5}$.

Also allied to (C) on p. 5'

we also have adding 2 & 5 in numerator and denominator of (D) above we have

$\frac{2662}{6655}=\frac{22}{55}=\frac{2}{5}$.

& in general also $\frac{266\cdots 62}{666\cdots 55}$

$=\frac{(133\cdots 31)2}{(133\cdots 31)5}=\frac{2}{5}$ (E)

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17. Dr. D.M. Rao
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18. Dr. Paranjpe
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16

19. Dr. Bannerjee
Principal
Jadavpur Engineering College
JADAVPUR
W. Bengal

20. Dr. H.V. Gopal
Indian Institute of Science
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Is the same true $\frac{166}{664}$ i.e. $\frac{1661}{6644}=\frac{1}{4}$. yes, in fact $\frac{166\cdots 61}{666\cdots 44}=\frac{1}{4}$. ✓ (F)

& of $\frac{19\cdots 9}{99\cdots 5}$ i.e. $\frac{19\cdots 91}{99\cdots 55}$ yes in fact $\frac{199\cdots 91}{999\cdots 55}=\frac{1}{5}$. ✓ (G)

$x=2, z=6, y=8 \to w=80 \to \frac{286}{880}=\frac{26}{80}$ ✓

$x=2, z=7, w=\frac{270y}{y+18}$, $y=2 \to$ trivial ✗

$y=9 \to w=90 \to \frac{297}{990}=\frac{27}{90}$ ✓

λ	$(10\lambda+1)/(\lambda+100)$

other than $324 \to 243$?

$p=3,\ q=4.$; $10b+c=8a$

$b=1,\ c=6 \to a=2$
$b=2,\ c=4 \to a=3$
$b=3,\ c=2 \to a=4$
$b=4,\ c=0 \to a=5$
$b=4,\ c=8 \to a=6$
$b=5,\ c=6 \to a=7$
$b=6,\ c=4 \to a=8$
$b=7,\ c=2 \to a=9$
$b=8,\ c=0 \to a=1$
$b=9,\ c=6 \to a=12$

$216 \to 162$
~~261 → 612~~
$324 \to 243$
$432 \to 324$
$540 \to 405$
$648 \to 486.$
$756 \to 567$
$864 \to 648$
$972 \to 729$
$880 \to 808.$

13/12/73

A' B

All these relate to multiplying by 3 and dividing by 4

freq $\frac{972\times3}{2916}$ & $\frac{2916}{4}=729$

or multiplying by 4 & dividing by 3.

freq: $\frac{261\times4}{3}$: 87×4 =

$\frac{880\times3}{4} \neq 808$

About 1 & 3rd digits being interchanged

$$(100a+10b+c)p = q(100c+10b+a).$$

$$a(100p-q)+10b(p-q)+c(p-100q)=0$$

$\frac{880}{808} = \frac{110}{101}$

220×3

$p-100(p+1)$

397 298 ~~447~~ 149

Suppose $q=p+1$.

$$a(99p-1)+10b+c(-99p-100)$$

$$\frac{10b+a(99p-1)}{10b} = c(99p+100).$$

$\frac{100p-q}{p-q}$ and $\frac{p-100q}{p-q}$ to be simple integers

10,000

$\frac{100p-q}{p-q}=\lambda$, $\frac{p-100q}{p-q}=\mu$, so

$100p-q=\lambda(p-q)$
$p(100-\lambda)=q(1-\lambda)$
$\frac{p}{q}=\frac{1-\lambda}{100-\lambda}.$

$p-100q=\mu(p-q)$
$p(1-\mu)=q(100-\mu)$
$\frac{p}{q}=\frac{100-\mu}{1-\mu}.$

$(1-\lambda)(1-\mu)=(100-\lambda)(100-\mu)$
$1-\lambda-\mu=10^4-100\lambda-100\mu.$
$99\lambda+99\mu=9999.$
$\lambda+\mu=101.$

(5')

$\frac{\cancel{210}^{10} \times 3}{\cancel{21}} = 30$

$\boxed{w = \frac{(100x + 10z)y}{\cancel{y+9}\ 9x + y}}$

$\frac{\cancel{210 \times 6}}{\cancel{244}}$ $\frac{\cancel{210}^{70} \times \cancel{9}}{\cancel{27}_3}$ $\frac{\cancel{420}^{21}}{\cancel{20}}$

THE INDIAN ACADEMY OF SCIENCES

$x = 2,\ z = 1,\ w = \frac{210y}{y+18};\ y = 2 \rightarrow \frac{221}{221}$ trivial ✗

$y = 3 \rightarrow w = 30 \rightarrow \frac{231}{330} = \frac{21}{30}$ ✓

Hebbal P.O.,
BANGALORE-560006.
9th November 1973.

21

Election of Office-bearers and Members of Council for the triennium 1974-1976.

$y = 9 \rightarrow w = 70 \rightarrow \frac{291}{970} = \frac{21}{70}$ ✓ $\therefore \frac{291}{970} = \frac{97 \times 3}{97 \times 10} = \frac{3}{10}$

Dear Sir,

291

$\frac{230 \times 2}{\cancel{20}\ 10}$

In accordance with clause 4, Section V of the rules of the Academy, a circular was issued on the 1st August 1973 to the Fellows of the Academy informing them that the Council of the Academy was due to be reconstituted for the triennium 1974-76 and inviting them to make nominations in writing so as to reach this office not later than 31st October 1973.

$\frac{210 \times 2}{20}$

A list of the nominations received in response to the above circular is appended for the information of the Fellows of the Academy.

$\frac{\cancel{220}^{55} \times \cancel{8}}{\cancel{24}\ 4}$

Yours faithfully,

B. S. Venkatachar,
Secretary.

$x = 2,\ z = 2,\ w = \frac{220y}{y+18}$ ~~y = 2, 3, 9 only three cases~~ ~~$y = 2 \rightarrow w = 21 \rightarrow \frac{222}{221}$~~

$y = 2 \rightarrow$ trivial

$y = 4 \rightarrow w = 40 \rightarrow \frac{242}{440} = \frac{22}{40}$ ✓

$y = 6 \rightarrow w = 55 \rightarrow \frac{262}{655} = \frac{22}{55}$ ✓ $= \frac{131 \times 2}{131 \times 5}$ → allied to $\frac{26}{65}$

ie $\frac{26}{65} = \frac{2}{5}$

$\frac{262}{655} = \frac{2}{5}$ ✓

(5')

$x = 2,\ z = 3,\ w = \frac{230y}{y+18};\ y = 2 \rightarrow w = 23 \rightarrow \frac{223}{223}$ ✗

$y = 5 \rightarrow w = 50 \rightarrow \frac{253}{550} = \frac{23}{50}$

only 2 cases of y

131

$\frac{\cancel{23}}{\cancel{50}}$

$\frac{240 \times 6}{24}$

Colossal Rubbish

Bsm

23/11/73

$x = 2,\ z = 4,\ w = \frac{240y}{y+18};\ y = 2 \rightarrow w = 24 \rightarrow \frac{224}{224}$

$y = 6 \rightarrow w = 60 \rightarrow \frac{264}{660} = \frac{24}{60}$ ✓

$y = 9 \rightarrow w = 80 \rightarrow \frac{294}{980} = 24/80$ ✓ $\left(L.H.S = \frac{98 \times 3}{98 \times 10} = 3/10\right)$

$\frac{3}{10}$

λ	$(1-\lambda)/(100-\lambda) = \frac{\lambda-1}{\lambda-100}$
-296	$-297/-396 = 3/4$

If $\mu = 397$, $\frac{100-\mu}{1-\mu} = \frac{-297}{-396} = \frac{3}{4}$

$3\lambda - 300 = 4\lambda - 4$

$-296 = \lambda$

$\frac{-297}{-396}$

$\frac{100+296}{297} = \frac{4}{3}$.

If $\lambda = -296$

$\mu = 397$.

$\frac{100-397}{1-397} = \frac{297}{396} = \frac{3}{4}$.

$p = 3$ & $q = 4$ are allowable.

$300 - 4$

$296a + 106b - 397c = 0$ for integral values $a, b, c \leq 99$.

$148(a-c) + 5b = 0$. or $148a + 5b = 148c$.

$b = \frac{148(q-a)}{5}$ — no possible integral value of $q - a$ can be 5

divisible by 148

If $p = 4$

Hence $p = 3$, $q = 4$ looks inadmissible

Using $\lambda + \mu = 101$, If $\lambda = 56$, $\mu = 55$

$100p - q = 56p - 56q$

$44p + 57q = 0$.

11

37×8

37×16

592

788

794

397

195

$296a = 397c + 106$

$a = \frac{397c + 106}{296}$

If $p = 4, q = 3$

$397 = \lambda$

$-296 = \mu$

$c = \frac{397a + 106}{296}$

$3 \quad 397a + 106b - 296c = 0$

Suppose $p = 8$, $q = 6$.

$\frac{796}{2} = \lambda$, $\frac{-592}{2} = \mu$

$\lambda = 398$

$\mu = -297$

-296

$800 - 6$

$8 - 600$

$794a + 206b = 592c \rightarrow 397a + 106b = 296c$

$p = 2, q = 1$

$199a + 106b = 98c$

$c = \frac{199a + 106}{98}$

$p = 3, q = 2$

$298a + 106b = 197c$

$c = \frac{298a + 106}{197}$

$p = 3, q = 1$

$299a + 206b = 97c$

$c = \frac{299a + 206}{97}$

$p = 4, q = 2$, $398a + 206b = 196c$

$c = \frac{398a + 206}{196}$

$\frac{199a + 106}{98}$

686

196

296

686

140

5

398

588

398

190

1

10

388

598

598

794

784

390

1184

1776

1184

784

794

(4) $x=1,\ z=4,\ w=\frac{140y}{y+9}$; $y=5 \rightarrow w=50 \rightarrow \frac{154}{550}=\frac{14}{50}$ (4')

Additional List

(5) $x=1,\ z=5,\ w=\frac{150y}{y+9}$, $y=6 \rightarrow w=60 \rightarrow \frac{165}{660}=\frac{15}{60}$

$y=9 \rightarrow w=75 \rightarrow \frac{195}{975}=\frac{15}{75}$

1. Prof. Kesavan
Visiting Professor
Computer Group
Indian Institute of Science
BANGALORE-560012.

$\frac{75 \times 9}{18\ 2}$

2. Dr. B.S. Ramakrishna
Electrical & Communications Engg. Dept.
Indian Institute of Science
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interesting one

~~but~~ Funny one the last 5 cannot be cancelled

3. Dr. M.G. Krishna
Director, IIP
Dehra Dun

$y=\frac{9xw}{100x}$

(6) $x=1,\ z=6,\ w=\frac{160y}{y+9}$, $y=6 \rightarrow w=64$.

4. Shri Raj Mahendra
General Manager
Helicopter Division
Hindustan Aeronautics Limited
Bangalore Division
BANGALORE

$w=\frac{(100x+10z)y}{9y+9}$

ie $\frac{166}{664}=\frac{16}{64}=\frac{1}{4}$ (beautiful)

Correct result after two cancellations

5. Air Cdr. S.N. Roy Chaudhury
Director
Gas Turbine Research Establishment
BANGALORE

$\frac{32 \times 6\ 2}{15\ 5}$

In fact $\frac{166\ldots6}{666\ldots4}=\frac{1}{4}$ in general

64

6. Dr. P.K. Das
Deputy Director-General
IMD, DELHI

$\frac{166}{664}$

(7) $x=1,\ z=7,\ w=\frac{170y}{y+9}$; $y=1, w=17 \rightarrow \frac{117}{117}$ trivial

$y=8, w=80 \rightarrow \frac{187}{880}=\frac{17}{80}$

664

7. Dr. S.S.R. Murty
Aeronautics Department
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(8) $x=1,\ z=8,\ w=\frac{180y}{y+9}$

$y=3, w=45 \rightarrow \frac{138}{345}=\frac{18}{45}$

good one

$=\frac{69\times2}{69\times5}=\frac{2}{5}$

$\frac{45 \times 3}{12\ 4}$

8. Dr. Yajnik
Aeronautics Department
~~Indian Institute of Science~~ N.A.L.
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9. Dr. L.S. Srinath
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$z=9,\ w=\frac{190y}{y+9}$, $y=9$

$y=6 \rightarrow w=76 \rightarrow \frac{169}{676}=\frac{19}{76}$

$y=9 \rightarrow w=95 \rightarrow \frac{199}{995}=\frac{19}{95}=\frac{1}{5}$

69

$\frac{18}{45}$ $\frac{2}{5}$

10. Dr. G. Srikantaiya
Indian Institute of Technology
KANPUR-208016

$\frac{199}{\ }$ $\frac{38\ 190\times6\ 2}{15\ 5}$ 76

11. Dr. Badri Narayan
Indian Institute of Science
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Correct after two cancellations as in (A)

$\frac{199}{995}$

$\frac{138}{345}=\frac{69\times2}{69\times5}$

12. Dr. A.K. Kamal
Electrical Engineering Dept.
ROORKEE

More general $\frac{19\ldots9}{99\ldots5}=\frac{1}{5}$

$\frac{23\times6}{23\times15}$

13. Dr. Shankar Lal
Mechanical Engineering Dept.
ROORKEE

This finishes all possible cases for all $x=1,\ z=1\ldots9,\ \&\ y=1\ldots9$

115 115

14. Dr. S.M. Deshpande
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15. Dr. P.N. Shanta
National Aeronautical Laboratory
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Consider $x=2$, etc

$a(100p-q)+10b(p-q)+c(p-100q)=0$.

$p=4, q=1 \rightarrow 399a+30b=96c \rightarrow 133a+10b=32c$

$c=\dfrac{133a+10b}{32}$

Putting $a=2, b=1$

$c=266+10$

$p=5, q=4$ $\quad 496a+10b=395c$ $\quad c=\dfrac{496a+10b}{395}$

$p=6, q=5,$ $\quad 595a+10b=494c$ $\quad c=\dfrac{595a+10b}{494}$

$p=3, q=1,$ $\quad 299a+20b=97c$

$c=\dfrac{299a+20b}{97}$

$p=5, q=1,$ $\quad 499a+40b=95c$

$p=5, q=2,$ $\quad 498a+30b=195c$

$166a+10b=65c$

$p=6, q=3,$ $\quad 597a+30b=294c$

$199a+10b=98c$ same as 4 & 2

~~$p=5, q=3,$ $497a+20b=295c$~~

$p=4, q=3,$ $\quad 397a+10b=296c$

$p=6, q=5.$ $\quad 595a+10b=494c$ $\quad c=\dfrac{595a+10b}{494}$

If $a=4$ & $b=9$, $c=\dfrac{2470}{494}=5$

Q Found this while asleep tonight! (24/11/73)

ie the number is 495 $\quad \dfrac{495\times 6}{5}=99\times 6=594$

Alternatively the better thing is to take 594, multiply by 5 and divide by 6 giving the reversed 495

(A) About transfer of digits in pp. 1–2 are $p=3, q=4$ the only possible case.?

$$\frac{10b+c}{a}=\frac{100p-q}{10q-p} \quad \text{—(i)}$$

In (i) $p=3, q=4 \rightarrow \dfrac{10b+c}{a}=\dfrac{296}{37}=8$ ie $10b+c=8a$ —(ii)

as on p. 2.

yes. It looks as if this is the only solution.

(3')

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.....

Illegal cancellation with 3 digits in numerator & denominator

(1) $x=1, z=2, w = \frac{120y}{y+9}$, $y=1 \rightarrow$ invalid
$y=3 \rightarrow w=30$ i.e. $\rightarrow \frac{132}{330} = \frac{12}{30}$ ✓ new one! ✓
$y=6 \rightarrow w=48 \rightarrow \frac{162}{648} = \frac{12}{48}$ ✓
$y=9 \rightarrow w=60 \rightarrow \frac{192}{960} = \frac{12}{60}$ ✓

(2) $x=1, z=1, y = \frac{9w}{110-w}$ or $\frac{110y}{y+9}$, $y=2 \rightarrow w=20 \rightarrow \frac{172}{220} = \frac{11}{20}$ ✓

(3) $x=1, z=3, w = \frac{130y}{y+9}$. only one is $y=4 \rightarrow w=40 \rightarrow \frac{143}{440} = \frac{13}{40}$ ✓ new one

(C) Let us come to 4 digit transfer problems.

abcd.

(i) abcd → bcda.

(B) Are 594 & 495 with (6,5) the only solutions

Since 5+4 = 4+5 = 9 middle digit, let us try other such numbers, say

198 & 891 etc [always with b=9]

(ii) $a(100p-q) + 10b(p-q) + c(p-100q) = 0$ —(1)

(i) Try $p=9, q=2$. $898a + 70b = 191c$

$c = \frac{898a+70b}{191}$ If $a=1, b=8$ $898 + 630 = 1528$

Then $c = 8$ $= 191 \times 8$

Thus 198 works : Dividing by 2 & multiplying by 9 22

Similarly 891 : : " 9 & " 2

(ii) 297 and 792 with p & q = 8 & 3 resp (ie one added to the last digits)

Putting $p=8, q=3$ in (i) $797a + 50b = 292c$

Putting $a=2, b=9 \rightarrow c = \frac{1594+450}{292} = \frac{2044}{292}$

$= 7$ ✓

(iii) The only other one to verify are 396 and 693 2

Putting $p=7, q=4$ (i) gives $696a + 30b = 393c$

$a=3, b=9$ gives $c = \frac{2088+270}{393}$

$= \frac{2358}{393} = 6$

More generally can we try

abc → cba with a+c = b.

Then (i) → $a(100p-q) + 10b(p-q) + c(p-100q) = 0$

$a(100p-q) + 10a(p-q) + 10c(p-q) + c(p-100q) = 0$

ie $a(110p-11q) + c(11p-110q) = 0.$

$a\,a(10p-q) + c(p-10q) = 0.$

$a(10p-q) = c(10q-p).$ —(ii)

$x=7,\ y=\frac{63z}{70-z}$, $z=7$ trivial

$x=8,\ y=\frac{72z}{80-z}$ only $z=2$ trivial

$x=9,\ y=\frac{81z}{90-z}$ only $z=9$

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-o-o-o-o-o-o-o-

$\frac{10x+y}{10y+z}=\frac{x}{z}$ $\quad \frac{9x+y}{10y}=\frac{x}{z}$ $\quad 10xy=9xz+yz$

$y(10x-z)=9xz$

$y=\frac{9xz}{10x-z}$. $\quad x=1,\ y=\frac{9z}{10-z}$

$z=1,\ y=1$ trivial

$z=2,\ x$

$z=3,\ x,\ z=4,\ y=6$ ✓ 16/64 ✓

$z=5,\ y=9$, 19/95 ✓

$z=6,\ x,\ z=7$ X

$x=2,\ y=\frac{18z}{20-z}$

$z=1,\ x$

$z=2,\ y=2$ trivial

$z=3,\ x,\ z=4$ X

$z=5,\ y=6$ → 26/65 ✓

$z=6,\ x,\ z=7$ X, $z=8$ X, $z=9$ X.

$x=3,\ y=\frac{27z}{30-z}$

$z=3,\ y=3$ trivial

$z=4,5,6,7,8,9$ X

$x=4,\ y=\frac{36z}{40-z}$.

$z=4,\ y=4$ trivial

$z=8,\ y=9$ → 49/98 ✓

$x=5,\ y=\frac{45z}{50-z}$, $z=5$ → trivial
other z X

$x=6,\ y=\frac{54z}{60-z}$, $z=6$ trivial
no other z

Let us try (i) freq with $b = 8$, $a = 6$, $b = 2$

$6(10p - q) = 2(10q - p)$ ie $62p = 26q$ or $31p = 13q$

$a = 2$ & $b = 6 \rightarrow 26p = 62q$ ie $p \leftrightarrow q$ $\quad \frac{p}{q} = \frac{13}{31}$ no soln for p & q

~~(ii) $b = 7$, $a = 5$, $b = 2$~~

~~$5(10p - q) = 2(10q - p)$ ie $52p = 25q$~~

$b = 8$, $a = 5$, $b = 3 \longrightarrow 53p = 35q$ ×

$a = 4$, $b = 4 \rightarrow 44p = 44q$ ie $p = q$

(no significance since $464 \times \frac{p}{p} = 464$!)

(ii) $b = 7$, $a = 6$, $b = 1 \rightarrow 61p = 16q$ ×

" $a = 5$, $b = 2 \rightarrow 52p = 25q$ ×

$a = 4$, $b = 3 \rightarrow 43p = 34q$ ×

(iii) $b = 6$, $a = 5$, $b = 1$, $51p = 15q$ or $17p = 5q$ ×

$a = 4$, $b = 2$, $42p = 24q$ $\quad 7p = 4q$ ie $p = 4$ & $q = 7$ admissible

ie ~~246 and 642 admissible~~ 264 & 462 admissible —— (α)

~~246~~ $\quad \cancel{\frac{642 \times 4}{7}} \quad \frac{462 \times 4}{7} = 66 \times 4 = 264$ ✓

$a = b = 3$ meaningless

(iv) $b = 5$, $a = 4$, $b = 1$, $41p = 14q$ ×

$a = 3$, $b = 2$ $\quad 32p = 23q$ ×

(v) $b = 4$, $a = 3$, $b = 1$ ×

$a = 2$, $b = 2$ ×

(vi) $b = 3$, $a = 2$, $b = 1$, $21p = 12q$ ie $7p = 4q$ ie $p = 4$, $q = 7$

132 and 231 are admissible —— (β)

Note that β's are $\frac{1}{2}$ of α's. ie nothing new.

~~Then why not~~ Surely 3 β's give 396 and 693 already obtained.

In fact from (iii) of (A') we can get first (β) and then (α).

(B') ~~(a) Let us come to 4-digit problems.~~ In 3-digit problems there are $3! = 5$ cases

~~(a) abcd → bcda.~~ viz abc, acb, bac, bca, cab, cba

we have considered so far only abc → bca & abc → cba

The three other cases are (i) abc → cab ie last digit becomes first

(ii) abc → bac first two digits interchange

(iii) abc → acb last two digits

(i) Eqn is $(100a + 10b + c)p = (100c + 10a + b)q$

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$a(100p-10q)+b(10p-q)+c(p-100q)$

$(10p-q)(10a+b)=c(100q-p)$

Consider $\frac{100q-p}{10p-q}=\lambda$ ie $100q-p=10p\lambda-\lambda q$ $\qquad \frac{p}{q}=\frac{100+\lambda}{10\lambda+1}$

ie $p(10\lambda+1)=q(100+\lambda)$

λ	$(100+\lambda)/(10\lambda+1)$
1	101/11 ×
2	102/21 ×
5	105/51 ×
8	$108/81=\frac{12}{9}=\frac{4}{3}$ ✓

ie $p=4, q=3$.

$\frac{10a+b}{c}=\frac{100q-p}{10p-q}=\frac{296}{37}=\frac{79}{9}=8$

$10a+b=8c \qquad a=4, b=8, c=6.$

$\frac{486\times4}{3}=162\times4=\underline{648}$ ✓

Taking $10a+b=8c$ as an ind. eqn of 1st degree in a & b, we have

$10.1-1.9=1$ or eqn can be written as

~~$10a+b=8c(9a-b)(10-9)$~~

~~$a(10-8c)=b($~~

$a=8c-t$
$b=10t-72c$

$10a+b$
$=80c-10t$
$+10t-72c$

$a=8c-t$
$b=10t-72c$

Does $a=8c-t$
$b=10t-72c$

work? yes, it appears to

c	a	b	
1	$8-t$	$10t-72$	× $\because t=8\to a=0$ ×
2	$16-t$	$10t-144$	If $t=15, a=1, b=6 \to 162$
3	$24-t$	$10t-216$	If $t=22, a=2, b=4. \to 243$
4	$32-t$	$10t-288$	If $t=29, a=3, b=2 \to 324$
5	$40-t$	$10t-360$	If $t=36, a=4, b=0 \to 405$
6	$48-t$	$10t-432$	If $t=44, a=4, b=8 \to 486$
7	$56-t$	$10t-504$	If $t=51, a=5, b=6 \to 567$
8	$64-t$	$10t-576$	If $t=58, a=6, b=4 \to 648$
9	$72-t$	$10t-648$	If $t=65, a=7, b=2 \to 729$

Altogether 8 possible ~~cases~~ values for a, b & c.

~~Next to find the~~ Verify with $p=4, q=3$

54
216

(2) $\frac{162\times4}{3}=216$ ✓ (3) $\frac{243\times4}{3}=324$ ✓, (4) $\frac{324\times4}{3}=432$ ✓ (5) $\frac{405\times4}{3}=540$ ✓

162

(6) $\frac{486\times4}{3}=648$ ✓ (7) $\frac{567\times4}{3}=756$, (8) $\frac{648\times4}{3}=864$, (9) $\frac{729\times4}{3}=972$

189×4
243×4
972

In fact these are exactly the reverses ← of digit transfers on p.(2)

So far we have taken only $p=4, q=3$

Surely for abc → bca also we have taken $p=3, q=4$

& guessed this is the only proper choice. Are there others? Stop it at this

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and also stop the cases of the other permutations of abc.

6

29 × 83	23 × 89	21 × 21	23 · 79	29 · 73	59 × 23	53 × 29		634
87	207	441	237	87	177	477	68	4
232	184	31 × 31	158	203	118	106		
2407	2047	961	1717	2117	1257	1537		

Flood's proof that 64 is the only square with $(6+4-1) = 3^2$ and $(6\times4+1) = 5^2$

Is this proof only for 2 digits?

$w+u=5$
$w-u=2$

$$\left.\begin{aligned} x+y-1 &= \square^{u^2} \\ xy+1 &= \square v^2 \\ 10x+y &= \square_{w^2} \end{aligned}\right\} \quad 9x+1 = w^2-u^2 = (w-u)(w+u)$$

x	1	2	3	4	5	6	7	8	9
$9x+1$	10	19 ×	28	37 ×	46	55	64	79 ×	82

$x=1,\ 10 = 2\cdot5 \to w+u=5,\ w-u=2$ ×

$x=3\ \ 28 = 4\cdot7 \to w+u=7,\ w-u=4$ ×

" $= 2\cdot14 \to w+u=14,\ w-u=2 \to w=8, u=6. \to y = u^2-x+1 = 34,\ xy+1 = 102+1 \neq \square$

$46 = 2\cdot23$ ×

$x=6,\ 55 = 5\cdot11 \to w+u=11,\ w-u=5 \to w=8,\ u=3 \to y = 9-6+1 = 4,\ xy+1 = 25 = v^2 \to v=5$

$x=7,\ 64 = 2\cdot32 \to w=17,\ u=15 \to y = 225-7+1 = 219,\ xy+1 = 1534 \neq \square.$

$= 4\cdot16 \to w=10,\ u=6 \to y = 36-7+1 = 30,\ xy+1 = 211 \neq \square$

$= 8\cdot8 \to w=8,\ u=0. \to y = 0-7+1 \neq$

$x=9,\ 82 = 2\cdot41$ × The only possible case is $x=6,\ y=4$. ie 64 as shown by Flood.

What about 3 digit cases?

$$\left.\begin{aligned} x+y+z-1 &= u^2 \\ xyz+1 &= v^2 \\ 100x+10y+z &= w^2 \end{aligned}\right\}$$

$$99x+9y+1 = (w^2-u^2) = (w-u)(w+u)$$

6) 4341 16
36
12) 741 ×

100 + 40 + 221
361
$= 19^2$.

1089
4
1085

42 × 42
84
168
1764
8

(1) $x=1$; $9y+100 = (\ )(\ )$

(i) $y=1 \to 109$ ×

$y=2 \to 118 = 2\times59$ ×

$y=3 \to 127$ ×

$y=4 \to 136 = 2\times68 \to w=35, u=33$ or $4\times34 \to w=19, u=15$ or 8×17 ×

$y=5 \to 145 = 5\times29 \to w=17, u=12$

$y=6 \to 154 = 2\times77$ ×

$y=7 \to 163$ ×

$y=8 \to 172 = 2\times86 \to w=44, u=42$ or 4×43 ×

$y=9 \to 181$ ×

ie 4 cases. (i) $z = 33^2-1-4+1 = 1085$

$xyz+1 = 4\times1085+1 = 4341 \neq \square$

(ii) $z = 15^2-1-4+1 = 221.$

$xyz+1 = 885 \neq \square$

(iii) $z = 14.4-1+5+1 = 149$

$xyz+1 = 745+1 = 746 \neq \square$

(iv) $z = 42^2-1-8+1 = 1756$

$xyz+1 = 14049 \neq \square$

∴ $x=1$ does not work.

Rather tedious to go from $x=2$ to 9 & $y=1$ to 9

ie 72 cases & each with 7 cases

$x=2,\ 9y+199 = (\ )(\ )$

$y=1 \to 208 \to 2\times104 \to (53, 51)$ or $4\times52 \to (28, 24)$ or $8\times26 \to (17, 9)$

$y=2 \to 217 \to 7\times31 \to (19, 12)$

$y=3 \to 226 \to 2\times113$ ×

$y=4 \to 235 \to 5\times47 \to (26, 21)$

$y=5 \to 244 \to 2\times122 \to (62, 60)$ or 4×61 ×

$y=6 \to 253 \to 11\times23 \to (17, 6)$

$y=7 \to 262 \to 2\times131$ ×

$y=8 \to 271$ ×

$y=9 \to 280 \to 2\times140 \to (71, 69)$ or $4\times70 \to (37, 33)$ or 8×35 × or $14\times20 \to (17, 3)$

or $28\times10 \to (19, 9)$

69 × 69	33 × 33
621	99
374	99
4361	1089

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ie 11 cases. (1) $x=2, y=1, z=51^2-2-\cancel{1}+\cancel{1}=2601-2=2599$ & $xyz+1=5199=5200 \neq \square$

(2) " $z=24^2-2=576-2=574$ & $xyz+1=1149 \neq \square$

(3) " $z=9^2-2=79$ & $xyz+1=159 \neq \square$

(5) $x=2, y=2, z=12^2-3=141$ & $xyz+1=565 \neq \square$

(6) $x=2, y=4, z=21^2-5=436$ & $xyz+1=3489 \neq \square$

(7) $x=2, y=6, z=6^2-7=29$ & $xyz+1=\cancel{373 \neq \square}\ 349 \neq \square$

(8) $x=2, y=9, z=69^2-10=4351$ & $xyz+1=78319 \neq \square$

(9) $x=2, y=9, z=33^2-10=1079$ & $xyz+1=19423 \neq \square$

(10) $x=2, y=9, z=3^2-10=-1$ & $xyz+1=-17$ ✗

(11) $x=2, y=9, z=9^2-10=71$ & $xyz+1=1279 \neq \square$

78319
2) 78319 (27
4
47 | 383
329
54 | 5419

1) 19423 (13
1
23 | 94
69
26 | 2523

3) 1279 (37
9
6 | 379

(10/12/73). Instead of this wild goose chase after Flood's numbers, let us simply look a table of squares containing 3, 4, and 5 digits & verify.

42×48	81×35	4	192×6	164×7
336	63×54	168×9	1152	1134
168	252	1512		81×14
2016	315	3302		1134

3 digits

(1) 144 — Sum of digits = 9 & product = 16.
$= 12^2$.

(2) $441 = 21^2$ — "

(3) [No. 3 digit squares having Flood's property] no others with the property in (1) & (2).

Four digit squares.

(1) $41^2 = 1681$ — Sum = square, product +1 = square (Semi-Flood)

(2) $76^2 = 5776$ and $77^2 = 5929$ — Sum of digits in both cases = 25 a perfect square

(3) $85^2 = 7225$ (Sum of digits = 16 = □) & $86^2 = 7396$ — Sum of digits = 25 = □

144 × 9
$12^2 \times 3^2$
36^2
6^4

No. 4 digit squares having Flood's property.

Let us look at Clark's table regarding cubes

(1) $8^3 = 512$ — Sum of digits +1 = 9 a □ & Product of digits −1 = 9 a □ (Anti-Flood property for squares

✓(2) $14^3 = 2744$ — Sum of digits −1 = 16 & Product of digits +1 = 225 = 15^2 (Flood's property for squares)

✓(3) $27^3 = 19683$ — Sum of digits = 27 (cube 3^3) & product of digits = $36^2 = 6^4$.

✓(4) $57^3 = 185193$ — Sum of digits +9 = 27+9 = 36 = 6^2 & product of digits +9 = 1089 = 33^2

(5) $61^3 = 226981$ — Sum of digits −1 = 27 = 3^3 & product of digits = 1728 = 12^3 (Semi-Flood with for cubes)

No more cases of enough interest

Enough of Flood

Continuation from p. (24') (S-G)

Generalisation of Sebban's Goormaghtigh's problem re (5, 6) → (25, 36) and (~~2025~~ = (45, 56) → (2025, 3136) on squaring.

We have shown that instead of having difference of 1, we might consider difference of 3 viz (4, 7) → (16, 49) and (34, 67) → (1156, 4489), and (334, 667) → (111556, 444889). This is better than the S-G case of 3 digits where (445, 556) → (198025, 209136) & here we have to add 1 successively to the digits of 193025 from the right to get 209136. Similar remarks hold good for (4445, 5556), (44445, 55556) etc. Let us now try the (3334, 6667) case of our generalisation

$1111111 = (2000x + 200y + 20z + 2u + 1111)\,1111$

$10001 - 1111 = 8890 = 2000x + \quad + 4445$

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$(1000x + 100y + 10z + u)^2 = 10^7 \cdot a + \ldots + 10b + c$

$(1000x + 100y + 10z + u + 1111)^2 = 10^7(a+1) + \ldots + 10(b+1) + (c+1)$

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$(2000x + 200y + 20z + 2u + 1111)\,1111 = 11111111$

$(2000x + 200y + 20z + 2u + 1111 = 10001$

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$2000x + 200y + 20z + 2u = 8890$

$1000x + 100y + 10z + u = 4445$ ✓

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$5556 = 4445 + 1111$

$(5556)^2 = (4445)^2 + 2222(4445) + (1111)^2$

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$(5556)^2 - (4445)^2 = 1111\{8890 + 1111\} = 1111\{10001\} = 11111111$

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... 16/-

$$\begin{array}{r} 3334 \times 3334 \\ \hline 13336 \\ 10002 \\ 10002 \\ 10002 \\ \hline 11115556 \end{array} \qquad \begin{array}{r} 6667 \times 6667 \\ \hline 46669 \\ 40002 \\ 40002 \\ 40002 \\ \hline 44448889 \end{array}$$

ie again correct without any trouble with regards to the 9 being replaced by 0 (after adding 1) as in the S-G. case.

obviously the thing holds good in general. viz.

for $(\overbrace{3\cdots3}^{n}4)$ and $(\overbrace{6\cdots6}^{n}7)$ the squares are given by

$(\overbrace{1\cdots1}^{n}\overbrace{5\cdots5}^{n-1}6)$ and $(\overbrace{4\cdots4}^{n}\overbrace{8\cdots8}^{n-1},9)$.

(i) Is this possible with difference 2? $(5,7) \cancel{\to} (25,49)$; $(6,8) \cancel{\to} (36,64)$, $(7,9) \to (49,81)$ — NO

(ii) with difference of 3 — yes as we shown above

(iii) with difference of 4 — $(4,8) \cancel{\to} (16,64)$, $(5,9) \cancel{\to} (25,81)$ — NO

(iv) with difference of 5 $(1,6)$ X, $(2,7)$ X, $(3,8)$ X, $(4,9) \cancel{\to} (16,81)$ — NO

Thus only cases of $(1,3)$ with $(5,6)$ and $(4,7)$ reply

$334{,}667 \,\|\, 445{,}556$

$\cancel{(334,556)}$ X

$\cancel{34,56}$

$\cancel{34^2 = 1156}$

$\cancel{56^2 = 3136}$

12/12/73 Let us come back to transferring digits case & only confine ourselves to sending the first digit last. Try with 4 digit case (

(B)

$abcd \to bcda$

$(1000a + 100b + 10c + d)p = q(1000b + 100c + 10d + a)$

What about 2 digit case, ie reversal?

$(10a + b)p = q(10b + a)$

$a(10p - q) = b(10q - p)$

$a(1000p - q) + 100b(p - 10q) + 10c(p - 10q) + d(p - 10q) = 0$

$a(1000p - q) + (p - 10q)(100b + 10c + d) = 0$ Let $100b + 10c + d = x$ (3 digit no)

(Since $1 \le p \le 9$ & $1 \le q \le 9$, $p - 10q < 0$) & we can write

$a(1000p - q) = x(10q - p)$ or $\dfrac{x}{a} = \dfrac{1000p - q}{10q - p} = \dfrac{1000\lambda - 1}{10 - \lambda}$ $(\lambda = p/q)$

$\dfrac{1994}{8}$ $\cancel{+49}$

$\cancel{(\text{with } \frac{1}{9} \le \lambda \le 9)}$ $\dfrac{988}{19}$

$\dfrac{500 - 1}{10 - \frac{1}{2}}$

Try

(1) $\lambda = 1 \to \dfrac{x}{a} = 111$ ie trivial cases $a = 1, x = 111$; $a = 2, x = 222$

(2) $\lambda = 2 \to \dfrac{x}{a} = \dfrac{1999}{8}$ $1999a = 8x$ (no choice of a possible)

$\lambda = 1/2 \to \dfrac{x}{a} = \dfrac{998}{19} =$ ~~$98a = 19x$~~ $= ($ " $)$

$19x = 998a$ (")

(3) $\lambda = 3 \to \dfrac{x}{a} = \dfrac{2999}{7}$ $7x = 2999a$ (")

$\lambda = 1/3 \to x/a = \dfrac{997}{29} \to 29x = 997a$ (")

$\lambda = 2/3 \to x/a = \dfrac{1997}{28} \to 28x = 1997a$ (")

$37)\,2996\,(8$
296
36

(4) $\lambda = 4 \to x/a = \dfrac{3999}{6} = \dfrac{1333}{2} \to 2x = 1333a$ (")

$\lambda = 1/4 \to x/a = \dfrac{996}{39} = \dfrac{332}{13} \to 13x = 332a$ (")

$\lambda = 3/4 \to x/a = \dfrac{2996}{37} \to 37x = 2996a$ (")

$\dfrac{1000 - 3/4}{10 - 3/4}$

Instead of this systematic search, let us make a random search and find one ~~value at the~~ possible value at least.

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$\frac{x}{a} = \frac{1000\lambda - 1}{10 - \lambda}$; $\frac{2000/9 - 1}{10 - 2/9}$

$\lambda = 4/3 \rightarrow \frac{x}{a} = \frac{3997}{37} = 2996$

$\lambda = \frac{5}{6} \rightarrow \frac{x}{a} = \frac{4994}{55} = \frac{454}{5} \rightarrow 5x = 454a$

$x = 454$ & $a = 5$ is possible

ie $\frac{5454 \times 5}{6} = 909 \times 5 = 4545$. (which is correct but not quite wonderful) ?

$5454 \rightarrow 4545$ Surely $\frac{4545 \times 6}{5} = 5454$ ✓ trivial

$\lambda = \frac{4}{5} \rightarrow \frac{3995}{46}$ — X

$\lambda = \frac{2}{5} \rightarrow \frac{1995}{48} = \frac{665}{16} \rightarrow 16x = 665a = 5 \times 133\, a$ X

a is at most 8 (not possible).

$\lambda = \frac{3}{5} \rightarrow \frac{2995}{47}$ X

$\lambda = 5/3 \rightarrow \frac{4997}{25}$ X

$\lambda = 2/9 \rightarrow \frac{1991}{88} = \frac{181}{8}$ ie $\frac{x}{a} = \frac{181}{8}$ $x = 181a$ ie $a = 1, 2, 3$ or 4 or 5

ie $8x = 181 \cdot a$ ie $a = 8$

ie 1181

909

leading to $8181 \times \frac{2}{9} = 1818$

$1818 \rightarrow 8181$ & $1818 \times \frac{9}{2} = 8181$ again rather trivial

$\lambda = \frac{3}{8} \rightarrow \frac{2992}{77} = \frac{272}{7}$ ie $7x = 272 \cdot a$, $a = 7 \rightarrow x = 272$

ie $7272 \leftrightarrow 2727$ but again trivial

$\frac{909}{7272} \times 3 = 2727$

$\lambda = \frac{5}{8} \Rightarrow \frac{4992}{75} = \frac{1664}{25}$ ie $25x = 1664 \cdot a$ X

$\lambda = \frac{7}{8} \rightarrow \frac{6992}{73}$ X $\quad \lambda = 4/9 \rightarrow \frac{3991}{86}$ X

$\lambda = 1/8 \rightarrow \frac{992}{79}$ X $\quad \lambda = 5/9 \rightarrow \frac{4991}{85}$ X

$\lambda = 7/9 \rightarrow \frac{6991}{83}$ X $\quad \lambda = 8/9 \rightarrow \frac{7991}{82}$ X

$\lambda = 1/7 \rightarrow \frac{993}{69} = \frac{331}{23}$ X, $\lambda = 2/7 \rightarrow \frac{1993}{68}$ X, $\lambda = \frac{3}{7} \rightarrow \frac{2993}{67}$ X

$\lambda = 4/7 \rightarrow \frac{3993}{66} = \frac{363}{6} = \frac{121}{2}$ $2x = 121 \cdot a$, $a = 3 \rightarrow x = 121$

$a = 6 \rightarrow x = 242$

$a = 9 \rightarrow x = 363$

$a = 2 \rightarrow x = 121$

$a = 4 \rightarrow x = 242$

$a = 6 \rightarrow x = 363$

$a = 8 \rightarrow x = 484$

ie $\frac{121 \times 4}{7} = 44$

Again the four $2121 \rightarrow 1212$

$4242 \rightarrow 2424$

$6363 \rightarrow 3636$

$8484 \rightarrow 4848$

Again of the trivial type.

$\lambda = \frac{5}{7} \rightarrow \frac{4993}{65}$ X; $\lambda = 6/7 = \frac{5993}{64}$ X; $\lambda = 1/6 \rightarrow \frac{994}{59}$ X; $\lambda = 1/5 \rightarrow \frac{995}{49}$ X

$\lambda = 1/9 \rightarrow \frac{991}{89}$ X In fact, we have actually made a thorough search with only four

Case (r). $x = 9, y = 9, z = 10$ (~~2, 3, 4, 5, 6, 7, 8, 9~~), with ~~x = 9~~, Since $z = 10$ i.e. $N + E - T = 0$ (21')

~~$z = 10 \rightarrow 2+8\ (\lambda_1),\ 3+7\ (\lambda_2),\ (4+6)\ (\lambda_3)$~~ there is a carry over of 1 from 3rd column to 2nd & since $y = 9$, there is again a carry over of 1 from 2nd to 1st column. Hence $U = 0$.

13.

Medicine (1-10)

~~with (λ_1) – 3, 4, 5, 6, 7, 9 possibility is 3+6=9, 4+5=9 for x & y.~~

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~~with (λ_2). 2, 4, 5, 6, 8, 9~~
only 4+5 is available
×

& left over nos for consideration are (2, 3, 4, 5, 6, 7, 8, 9)

2. Dr. Duraiswamy
Director, Health Services
Government of India.

~~with (λ_3) 2, 3, 5, 7, 8, 9~~
only 2+7 available
×

$z = 10$ $(N+E-T=10)$ → $3+9-2=0$; $4+8-2=0$; $5+7-2=0$

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$5+8-3=0$; $5+9-4=0$; $6+7-3=0$; $6+8-4=0$; $6+9-5=0$

So ~~only case is~~

~~3 4 2~~
~~6 5 8~~

56×56 3
336
280
3136

4. Professor Ramalingaswamy
Director, All India Institute of Medical Sciences
NEW DELHI.

instead consider $x = 9$. ~~Since~~

$7+8-5=0$; $7+9-6=0$

Since $U = 0$, possibilities are $2+7,\ 3+6,\ 4+5$ (λ) (μ) (ν)

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$8+9-7=0$
11 cases.

(λ) 3, 4, 5, 6, 8, 9 should lead to $A+H-1=9$, $N+E-T=10$

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$4+9-3=10$, 5, 6, 8 ×
$5+8-3=10$, 4, 6, 9 ×
$5+9-4=10$, 3, 6, 8 ×
$6+8-4=10$, 3, 5, 9 ×
$6+9-5=10$, 3, 4, 8 ×

$\frac{10^6-1}{10^2-1}$; $\frac{10^2-1}{10-1}$

7. Dr. K. Sundaram, M.D., Gold Medalist
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$\frac{10^6-1}{10-1} = \frac{1000000-1}{9} = \frac{999999}{9} = 111111$

(μ) 2, 4, 5, 7, 8, 9

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$\frac{10^4-1}{10-1}$ $\frac{9999}{9}$

$4+8-2=0$, 5, 7, 9 ×
$5+7-2=0$, 4, 8, 9 ×
$5+9-4=0$, 2, 7, 8 ×
$7+8-5=0$, 2, 4, 9 ×
$8+9-7=0$, 2, 4, 8 ×

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$\frac{10^3-1}{10-1} = \frac{999}{9} = 111$

(ν) 2, 3, 6, 7, 8, 9

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$3+9-2=10$, 2, 6, 7 ×
$6+7-3=10$, 2, 8, 9 ×
$7+9-6=10$, 2, 3, 8 ✓ $8+3-2=9$
$8+9-7=10$, 2, 3, 6 ×

125×125
625
250
125
15625

99×99
891
891
9801

$(10x+y)^2$

999×999
891
891
891
9801

Zoology (1-8)

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So only case viz

437
589
1026

As given by Madachy

$xy = z^2$
$(x+1)(y+1) = u^2$
$10x + y = z^2 = 25$
~~$10(x+1)+(y+1) = u^2$~~ $x = 2, y = 5$
~~$z^2 + 11 = u^2$~~ ~~$u^2 - z^2 = 11$~~
$(u+z) = 11$
$u - z = 1$
$2u = 12$
$2z = 10$
$u = 6$
$z = 5$

891
891
891
8991
8991
8991
9801

possibilities viz 5/6 (6/5), (2/3)
2/9 (9/2
3/8 (8/3) leading to 7 cases viz
4 cases ← 4/7 (7/4)

(4/7) → 1212 ⟷ 2121.
(2/9) → 1818 ⟷ 8181
(4/7) → 2424 ⟷ 4242
(3/8) → 2727 ⟷ 7272
(4/7) → 3636 ⟷ 6363.
(5/6) → 4545 ⟷ 5454
(4/7) → 4848 ⟷ 8484

Nothing wonderful.

29) 9997 (34
87
129
116
139

37) 29996 (81
296
39
37
2

47) 29995 (635
282
179
141
385

(C) Let us now go to the 5 digit case:

$$(10^4\cdot a + 10^3\cdot b + 10^2\cdot c + 10\cdot d + e)\,p = (10^4\cdot b + 10^3 c + 10^2\cdot d + 10e + a)\,q$$

$$a(p\cdot 10^4 - q) + b(p\cdot 10^3 - q\cdot 10^4) + c(p\cdot 10^2 - q\cdot 10^3) + d(10p - q\cdot 10^2) + e(p - 10^3 q) = 0$$

$$a(p\cdot 10^4 - q) + (p - 10q)\{10^3\cdot b + 10^2 c + 10\cdot d + e\} = 0$$

calling $\{\ \} = x$ (a four digit number)

$$a(p\cdot 10^4 - q) = (10q - p)x.$$

$$\frac{x}{a} = \frac{p\cdot 10^4 - q}{10q - p} = \frac{10^4\cdot\lambda - 1}{10 - \lambda} = \frac{10000\lambda - 1}{10 - \lambda}$$

399993

23) 39995 (1735
23
169
161
89
69
205

59) 9994 (16
59
409
354
554

67) 29993 (44
268
319
268
513

It appears better to make a systematic search.

(1) $\lambda = 1$, $\frac{x}{a} = \frac{9999}{9} = 1111$ → trivial case

(2) $\lambda = 2$, $\frac{x}{a} = \frac{19999}{8}$ ✗; $\lambda = 1/2 \to \frac{x}{a} = \frac{9998}{19}$ ✗

(3) $\lambda = 3$, $\frac{x}{a} = \frac{29999}{7}$ ✗; $\lambda = 1/3 \to \frac{9997}{29}$ ✗; $\lambda = 2/3 \to \frac{19997}{28}$ ✗

(4) $\lambda = 4 \to \frac{39999}{6} = \frac{13333}{2}$ ✗; $1/4 \to \frac{9996}{39} = \frac{3332}{13}$ ✗; $\lambda = \frac{3}{4} \to \frac{29996}{37}$ ✗

(5) $\lambda = 5 \to \frac{49999}{45}$ ✗; $\lambda = \frac{1}{5} \to \frac{9995}{49}$ ✗ ~~1999 i.e. $\frac{x}{a} = 1999$ i.e. $x = 1999a$~~ ~~$a = 1, 2, 3, 4, 5$~~

~~$\lambda = 1/5$ leads to 5 cases 1999 → 9991~~

$\lambda = 2/5 \to \frac{19995}{48} = \frac{6665}{16}$ ✗; $\lambda = 3/5 \to \frac{29995}{47}$ ✗ $\lambda = 4/5 \to \frac{39995}{46}$ ✗

(6) $\lambda = 6 \to \frac{59994}{4} = \frac{29997}{2}$, $\frac{x}{a} = \frac{29997}{2}$ ✗ ~~$x = 4997\cdot a$, $a = 2 \to x = 4997$~~ ~~$a = 4 \to x = 9994$~~

$\lambda = 5/6 \to \frac{49994}{455}$ ✗, $\lambda = 1/6 \to \frac{9994}{59}$ ✗

(7) $\lambda = 7 \to \frac{69999}{3}$ ✗, $\lambda = 1/7 \to \frac{9993}{69} = \frac{3331}{23}$ ✗; $\lambda = 2/7 \to \frac{19993}{68}$ ✗, $\lambda = 3/7 \to \frac{29993}{67}$

$\lambda = 4/7 \to \frac{39993}{66}$ ✗ $\lambda = 5/7 \to \frac{49993}{65}$ ✗, $\lambda = 6/7 = \frac{59993}{64}$ ✗

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(A)
```
  CAN
  SHE
 ----
  QUIT
```
All ten letters differes
0, 1, .. 9 to be used ...(20')

(1) Q = 1.

100 C + 10 A + N
+ 100 S + 10 H + E
= 1000 Q + 100 U + 10 I + T

Putting Q = 1

100 (C + S − U) + 10 (A + H − I) + (N + E − T) = 1000.
x y z

or say, 100x + 10y + z = 1000

Two cases; (i) z = 0, (ii) z = 10

(i) → 100x + 10y = 1000 → 10x + y = 100 — (a)

(ii) → 100x + 10y + 10 = 1000
10x + y = 99 — (b)

(a) leads to x = 10, y = 0 — (α)
or x = 9, y = 10 — (β)

(b) leads to x = 9, y = 9 — (γ)

Case (α): x = 10, y = 0, z = 0

y = 0 & z = 0 imply that there is no carry over in the 2nd & third columns of (A) & hence x = 10 implies that U = 0. Hence we have to ascribe 2, 3, .. 9 to the remaining (1-8) 8 letters C, S, A, H, I & N, E, T such that

C + S = 9 — (λ)
A + H − I = 10 — (μ)
N + E = T — (ν)

Combinations for λ would be
2+8, 3+7, 4+6 ($\lambda_1, \lambda_2, \lambda_3$)

With (λ_1), 3+4 = 7 — (μ_1)
5, 6, 9 are left over
& no possible (ν).

Try (μ_2) as 5+4 = 9
3, 6, 7 are left over
& no possible (ν).

Try (μ_3) as 3+6 = 9
4, 5, 7 left over
& no (ν) possible

With (λ_2) left overs are
2, 4, 5, 6, 8, 9
only two possibilities
2 + 6 = 8
4 + 5 = 9
for (μ) & (ν)

With (λ_3) left overs are
2, 3, 5, 7, 8, 9
only two possibilities
2 + 7 = 9
3 + 5 = 8

Case (α) →

3 2 4	7 6 5	7 2 5	3 6 4	4 2 3
7 6 5	or 3 2 4	or 3 6 4	etc etc. or	6 7 5
1 0 8 9	1 0 8 9	1 0 8 9		1 0 9 8

Case (β): x = 9, y = 10, z = 0, this means no carry over from 1st column, a carry over from 1 from second column & with x = 9, this means that U = 0. ~~such that C + S = 9~~ so that we have

C + S = 9 — (λ)
A + H − I = 10 — (μ)
N + E = T — (ν)

to be satisfied with
2, 3, 4, 5, 6, 7, 8, 9

(μ) → 3+9−2, 5+8−3

(λ) → 2+7, 3+6, 4+5 ($\lambda_1, \lambda_2, \lambda_3$)

With (λ_1) left overs are 3, 4, 5, 6, 8, 9
4 + 9 − 3 = 10 × with (ν)
5 + 8 − 3 = 10, 4, 6, 9 ×
6 + 8 − 4 = 10, 3, 5, 9.
6 + 9 − 5 = 10, 3, 4, 8
no possible chance

With (λ_2) left overs are 2, 4, 5, 7, 8, 9
4 + 8 − 2 = 10, 5, 7, 9 ×
5 + 7 − 2 = 10, 4, 8, 9 ×
5 + 9 − 4 = 10, 2, 8, 8 ×
7 + 8 − 5 = 10, 2, 4, 9 ×
8 + 9 − 7 = 10, 2, 4, 5 ×
no possible choice

With (λ_3): Left overs are 2, 3, 6, 7, 8, 9
3 + 9 − 2 = 10; 6, 7, 8 ×
6 + 7 − 3 = 10, 2, 8, 9 ×
7 + 9 − 6 = 10, 2, 3, 8 ×
8 + 9 − 7 = 10, 2, 3, 6 ×
no possible cases

...13/-

ie Case (β) is ruled out

(8) $\lambda = 8 \to \frac{79992}{2}$ ~~x 4996 ie x~~ ; $\lambda = 1/8 \to \frac{9992}{79}$

$\lambda = \frac{3}{8} \to \frac{29992}{77}$ x, $\lambda = 5/8 \to \frac{49992}{75} = \frac{16664}{25}$ x

$\lambda = 7/8 = \frac{69992}{73}$ x

(9) $\lambda = 9 \to \frac{89999}{1}$ x, $\lambda = 1/9 = \frac{9999}{89}$ x

$\lambda = 2/9 \to \frac{19991}{88}$ x $\lambda = 4/9 \to \frac{39991}{86}$ x

$\lambda = 5/9 \to \frac{49991}{85}$; $\lambda = 7/9 \to \frac{69991}{83}$ x

$\lambda = 8/9 \to \frac{79991}{82}$; $\left[\lambda = \frac{9}{5} \to \frac{89995}{41} = 2195\right.$

$a=1$, gives $x = 2195$, $a=2 \to 4390 \leftarrow \frac{x}{a} = 2195$ $x = 2195 \cdot a$

$a = 3 \to 6585$, $a = 4 \to 8780$

A most surprising result that there are no p & q for which the transfer of digit to the last place is possible in the case of a five digit number (except the four cases. 12195, 24390, 36585 & 48780

(added on 13/12/73) See p. 15)

(D) 6-digit case. $\frac{x}{a} = \frac{10^5 \cdot \lambda - 1}{10 - \lambda}$ (x being a five digit number)

~~The~~ First let us test the known case in Geoffrey's ~~Smith~~ Smith's book, due to Dudeney viz $571428 \leftrightarrow 714285$ corresponding to $4/5$

$\lambda = 4/5 \to \frac{399995}{46}$ x

$\lambda = 5/4 \to \frac{499996}{35} = \frac{71428}{5}$. $\frac{x}{a} = \frac{71428}{5}$

$5x = 71428 \cdot a$

with $a = 5$, $x = 71428$

ie number required is 571428.

Surprising works with $5/4$ but not $4/5$!

79) 9992 (12
79
209
158
512 x

73) 69992 (95
657
429
365
642 x

~~989~~

43) 39991 (93
387
129
129

17) 49991 (294
34
159
153
69
68
11

83) 69991 (84
664
359
332
271
x

41) 79991 (187
41
359
328
319
287
321
x

23) 399995 (1739
23
169
161
8
89
69
209
287
~~255~~
25
x

(13/12/13) Perhaps not so surprising for putting $1/\lambda = \mu$, we have $\frac{x}{a} = \frac{10^5 - \mu}{10\mu - 1}$

Putting $\mu = 4/5 \to \frac{x}{a} = \frac{499996}{35} \to \frac{x}{a} = \frac{71428}{5}$ as above.

So this means that we have try all the cases dealt with above for 3, 4, 5 & 6 digits by taking $p/q > 1$ also.

(A) 3 digit case: $\frac{x}{a} = \frac{10^2 \lambda - 1}{10 - \lambda}$ (x 2 digits) or $\frac{x}{a} = \frac{10^2 - \mu}{10\mu - 1}$

(1) $\lambda = 1$ x (2) $\lambda = 2$, $\frac{x}{a} = \frac{199}{8}$ x

$\lambda = 1/2$, $x/a = \frac{98}{19}$ x x

$\mu = \frac{1}{2}$, $\frac{x}{a} = \frac{199}{8}$ x

$\mu = 2$, $\frac{x}{a} = \frac{98}{19}$ x

Work on Alphametics by Madachy

(19')

... 11.

(BE)(BE) = MOB

$\rightarrow (10x+y)^2 = 100z + 10u + x$

$10x + y = v$ [$v < 33$

$10z + u = w$

$\cancel{100x^2 + 20xy + y^2}$ $v^2 = 10w + x$

$(v+1)(v-1) = 10w$

$x = 1,\ v^2 = 10w + 1$

$w = 12,\ v = 11$ ✗

$\rightarrow x = y = 1$ ✗

$w = 36,\ v = 19$ ✓

$w =$

$x = 1, 2$ or 3:

$x = 2,\ v^2 = 10w + 2$

v even, v^2 ends with 4 or 6 ✗

$x = 3,\ v^2 = 10w + 3$.

v is odd & v^2 ends with 1, 5, 9

Hence only case $x = 1, v = 19$

$x = 1, y = 9$.

Internal Combustion

1, 4, 5, 6, 9.

1, 4, 9, 1, 6, 9, 5, 5

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$v + 11, 19, 21,\ v^2 - x = 10w$.

21, 29

31, 39 ✗

41, 49 ✗

Chemical Engineering

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91, 99

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$1000Q = 100(C+S-U) + 10(A+H-I) + (N+E-T)$

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$Q = 1 \rightarrow$

$100x + 10y + z = 1000$

$z = 0$ or 10.

$z = 0 \rightarrow 10x + y = 100 \rightarrow y = 0, x = 10$

CAN
SHE
QUIT

CAN + SHE = QUIT.

$100C + 10A + N + 100S + 10H + E$

$= 1000Q + 100U + 10I + T$.

$1000Q = 100(C+S-U) + 10(A+H-I) + (N+E-T) = 0$.

$\cancel{Q = 1 \rightarrow 1000 + 100x + 10y + z = 0}$.

$C + S - U = 9$

$A + H - I = 10$

$N + E - T = 0$

$x = 10, y = 0, z = 0$

$x = 9, y = 10, z = 0$

$x = 9, y = 9, z = 10$

Electronic Engineering

(1-3) or $y = 10, x = 9$.

$z = 10 \rightarrow 10x + y = 99$

Given in Madachy

$+x = 3 + 7 - 0 = 10$

$+y = 2 + 6 - 8 = 0$

$+z = 4 + 5 - 9 = 0$

$\rightarrow x = 9, y = 9$ (a)

or

$+x = 4 + 5 - 0 = 9$

$+y = 3 + 8 - 2 = 9$

$+z = 7 + 9 - 6 = 10$

$+900 - 90 - 10 =$

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3. Dr. Amarjit Singh, Ph.D. (Harvard)
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   Central Electronics Engineering Research Inst.
   PILANI

$\cancel{100x + 10y + z = 1000}$

$\cancel{10 + 1 + 100x \leq 1000}$

$100x \leq 891$

$x < 9$

x, y, z not necessarily +ve & not necessarily < 10

$x = 9,\ 10y + z = 100$ ~~(No soln~~

Electrical Engineering

(1-2)

0.

1000
100
900

$x: C + S - U = +10$

$y: A + H - I = 0$

$z: N + E - T = 0$

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   Indian Institute of Science
   BANGALORE-560012.

$1000x + 100x + 10y + z = 0 \rightarrow z = 0$ or 10

$z = 0 \rightarrow 100 + 10x + y = 0 \rightarrow y = 0$ ~~or 10~~ $x = +10$

$z = 10 \rightarrow 100 + 10x + y + 10 = 0$ or $99 + 10x + y = 0$

$x = 9, y = 9$:

Aeronautical Engineering

Last two → (without 1)

$2 + 1 -$

$2 + 3 - 5 = 0$

$2 + 4 - 6 = 0$

$2 + 5 - 3 = 0$

$2 + 7 - 9 = 0$

$3 + 4 - 7 = 0$

$3 + 5 - 2 = 0$

$3 + 6 - 9 = 0$

$4 + 5 - 9 = 0$

$2 + 1 - 3 = 0$

$2 + 3 - 5 = 0$

$2 + 4 - 6 = 0$

$2 + 5 - 7 = 0$

$2 + 6 - 8 = 0$

$2 + 7 - 9 = 0$

$3 + 1 - 4 = 0$

$3 + 4 - 7 = 0$

$3 + 5 - 8 = 0$

$3 + 6 - 9 = 0$

$3 + 7 - 0 = 0$

$4 + 5 - 9 = 0$

$4 + 6 - 0 = 0$

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   MADRAS.

... 12/-

$\frac{10^2\cdot\lambda-1}{10-\lambda}$; $\frac{10^2-\mu}{10\mu-1}$

(3) $\lambda=3$, $\frac{x}{a}=\frac{299}{7}x$; $\mu=\frac{1}{3}$, $\frac{x}{a}=\frac{899}{7}$

$\lambda=\frac{1}{3}$, $\frac{x}{a}=\frac{97}{29}x$ $\mu=3$, $\frac{x}{a}=\frac{97}{29}x$, $\mu=\frac{1}{3}\to\frac{299}{7}x$

$\lambda=\frac{2}{3}\to\frac{197}{28}x$, $\lambda=\frac{3}{2}\to\frac{298}{17}x$; $\mu=\frac{2}{3}\to\frac{298}{17}$; $\mu=\frac{3}{2}\to\frac{197}{28}$

(these appears no need for repetition. use the μ formula preferably by taking cases $\mu\geq 1$ or >1)

(4) $\mu=4\to\frac{96}{39}x$, $\mu=\frac{1}{4}\to\frac{399}{6}x$, $\mu=\frac{3}{4}\to\frac{397}{26}x$; $\mu=\frac{4}{3}\to\frac{296}{37}=8$. $\frac{x}{a}=8a$

leaving to cases $a=1$, $a=2$, $a=3$, $a=4$, $a=5$, $a=6$, $a=7$, $a=8$, $a=9$.

These give the cases on p.2, ~~except 880 → 808~~ where $a=1\to x=8$ and since $x=10b+c$, we can take $b=8$ & $c=0$ as mentioned on ~~p.8~~ p.2 [No. $\frac{880\times 3}{4}\neq 808$. i.e. $a=1$ not allowed].

(5) $\mu=5\to\frac{95}{49}x$, $\mu=\frac{1}{5}\to\frac{499}{5}x$; $\mu=\frac{2}{5}\to\frac{195}{48}=\frac{65}{16}$ $\frac{498}{15}=\frac{166}{5}x$, $\mu=\frac{5}{2}\to\frac{195}{48}=\frac{65}{16}x$

$\mu=\frac{3}{5}\to\frac{497}{35}x$ ~~$\to 8x=71a$ & if $a=5, 71$~~, $\mu=\frac{5}{3}\to\frac{295}{47}x$

$\mu=\frac{4}{5}\to\frac{496}{35}x$; $\mu=\frac{5}{4}\to\frac{395}{46}x$

(6) $\mu=6\to\frac{94}{59}x$, $\mu=\frac{1}{6}\to\frac{594}{4}x$; $\mu=\frac{5}{6}\to\frac{595}{44}x$, $\mu=\frac{6}{5}\to\frac{494}{55}x$

(7) $\mu=7\to\frac{93}{69}=\frac{31}{23}x$, $\mu=\frac{1}{7}\to\frac{699}{3}x$; $\mu=\frac{2}{7}\to\frac{698}{13}x$, $\mu=\frac{3}{7}\to\frac{697}{23}x$; $\mu=\frac{4}{7}\to\frac{696}{33}=\frac{232}{11}x$

$\mu=\frac{5}{7}\to\frac{695}{43}x$, $\mu=\frac{6}{7}\to\frac{694}{53}x$, $\mu=\frac{7}{2}\to\frac{193}{68}x$, $\mu=\frac{7}{3}\to\frac{293}{67}$, $\mu=\frac{7}{4}\to\frac{393}{66}x$, $\mu=\frac{7}{5}\to\frac{493}{65}$

$\mu=\frac{7}{6}\to 593/64\,x$

(8) $\mu=8\to\frac{92}{79}x$, $\mu=\frac{1}{8}\to\frac{799}{2}x$; $\mu=\frac{3}{8}\to\frac{797}{22}x$; $\mu=\frac{5}{8}\to\frac{795}{42}=\frac{265}{14}x$, $\mu=\frac{7}{8}\to\frac{793}{62}x$

$\mu=\frac{8}{3}\to\frac{292}{77}x$, $\mu=\frac{8}{5}\to\frac{492}{75}x$, $\mu=\frac{8}{7}=\frac{692}{73}x$

(9) $\mu=9\to\frac{91}{89}x$, $\mu=\frac{1}{9}=\frac{899}{1}x$; $\mu=\frac{2}{9}\to\frac{898}{11}x$, $\mu=\frac{4}{9}\to\frac{896}{31}x$; $\mu=\frac{5}{9}\to\frac{895}{41}x$

$\mu=\frac{7}{9}\to\frac{893}{61}x$, $\mu=\frac{8}{9}\to\frac{892}{71}x$

Hence the only cases are the <u>eight cases</u> on p.2.

(B) <u>Four digit cases</u> . $\frac{x}{a}=\frac{10^3-\mu}{10\mu-1}$.

Using this formula with μ, it is enough if we consider cases where $p/q<1$, since such cases have already been considered with $\lambda=p/q<1$.

(1) $\mu=1\to$ trivial, (2) $\mu=1/2\to\frac{1999}{8}x$ (3) $\mu=1/3\to\frac{2999}{7}x$; $\mu=2/3\to\frac{2998}{17}x$

(4) $\mu=1/4\to\frac{3999}{6}=\frac{1333}{2}$, $2x=1333\,a$ & $a=2\to x=1333$; $a=4\to x=2666$; $a=6\to x=3999$; $a=8\to x=5332$ → x is not a 3 digit number

$\mu=3/4\to\frac{3997}{26}x$.

(5) $\mu=1/5\to\frac{4999}{5}x$; $\mu=2/5\to\frac{4998}{15}x$; $\mu=3/5\to\frac{4997}{25}x$; $\mu=4/5\to\frac{4996}{35}x$

(6) $\mu=1/6\to\frac{5999}{4}x$; $\mu=5/6\to\frac{5995}{44}=\frac{545}{4}\to 4x=545\,a$ & $a=4\to x=545$ i.e. 4545

(7) $\mu=1/7\to\frac{6999}{3}x$, $\mu=2/7\to\frac{6998}{13}x$, $\mu=3/7\to\frac{6997}{23}x$ $\mu=4/7\to\frac{6996}{33}=\frac{636}{3}=212$ i.e.

(9) $x = 9,\ v = \frac{900y}{81+y}$. only case $y = 9 \rightarrow$ rather trivial case

(18')

...10.

• ie Barring ~~ext~~ trivial extensions of 16/64, 19/95, 26/65 & 49/98 & other trivial ones like 11/110, 22/220, etc we have found only three new cases.

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This exhausts discussion of all possible (2/3) & (3/3) digit cases single cancellation in 1st case, and single & double cancellation in the 2nd case.

Cosmic Rays, Geomagnetism, etc.

(1-2)

3/12/73 what about $\frac{xy}{zx} = \frac{y}{z}$? (a) and $\frac{xy}{xz} = \frac{y}{z}$? (b) and $\frac{xy}{zy} = \frac{x}{z}$? (c)

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(b) $xy/xz = \frac{y}{z}$ ie $\frac{10x+y}{10x+z} = \frac{y}{z}$

$\frac{10x}{10x} = \frac{y}{z}$

$y = z$.

→ trivial cases.

Metallurgy

(1-7)

$\frac{1}{2}n\{2A + D(n^2-1)\}$ $\frac{22}{22}$ $\frac{32}{32}$

$\frac{3}{2}\{2A + 8D\} = 3(A + 4D)$

956T
1085
1065R

(a) $\frac{10x+y}{10z+x} = \frac{y}{z}$

$\frac{10x}{9z+x} = \frac{y}{z}$

$10xz = 9yz + xy$

$x(10z - y) = 9yz$

$x = \frac{9yz}{10z-y}$

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$x(100-k) + 10y(1-k) + z(1-100k) = 0$.

2. Dr. Irani
Tata Iron and Steel Company

$x(100-k) + 10y(1-k) = z(100k-1)$.

If $k = 1$, $x = z$ ie $xyx = 1 \cdot xyx$

3. Dr. S.G. Paramanik
Hinustan Steel Limited

~~99x = 99z~~ $k = 2$, 1065

$98x - 10y = 199z$.

(c) $\frac{10x+y}{10z+y} = \frac{x}{z}$ ie $\frac{9x+y}{9z+y} = \frac{x}{z}$

$9zx + xy = 9zx + yz$

$xy = yz$ ie $x = z$

→ trivial cases

ie $\frac{xy}{zx} = \frac{y}{z}$ → reciprocals of the four cases of $\frac{xy}{yz} = \frac{x}{z}$.

Anyway interesting

(1) $z = 1,\ x = \frac{9y}{10-y}$.

(i) $y = 1 \rightarrow x = 1 \rightarrow$ trivial

(ii) $y = 4 \rightarrow x = 6 \rightarrow \frac{64}{16}$ (reciprocal of 16/64

(iii) $y = 5 \rightarrow x = 9 \rightarrow \frac{95}{19}$ (rec)

4. Dr. Rama Rao $98x = 10y + 199z$
Department of Metallurgy
Banaras Hindu University
VARANASI-221005.

$98x = 10y + 199$ X
$98x = 10y + 398$

1785
2643
4328

....
1...
1

(2) $z = 2,\ x = \frac{18y}{20-y}$

(i) $y = 2 \rightarrow$ trivial

(ii) $y = 5 \rightarrow x = 6 \rightarrow \frac{65}{26}$ (rec).

5. Dr. S. Ramachandran, B.Sc., D.Sc.(Met) MIT USA, MBA
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RANCHI-2.

~~424 = 1×424~~

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(3) $z = 3,\ x = \frac{27y}{30-y}$

(i) $y = 3 \rightarrow$ trivial case

7. Professor E.C. Subba Rao
Indian Institute of Technology,
KANPUR-208016 (UP)

$\frac{4251935345}{91819355185} = \frac{425345}{9185185}$

$xyz = k(zyx)$

$\frac{100x + 10y + z}{100z + 10y + x} = k$

R.H.S = $\frac{85069}{1837037}$ ~~a, b, c, d~~ xyz uvw

Liquid Crystals

(1-2)

(4) $z = 4,\ x = \frac{36y}{40-y}$

(i) $y = 4 \rightarrow$ trivial case

(ii) $y = 8 \rightarrow x = 9$

$\rightarrow \frac{98}{49}$ (reverse)

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~~1,3,5~~ 9, 11, 13, 15 9, 13, 17, 21

2. Dr. C.L. Khetrapal, M.Sc., Ph.D.
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0552

85069) 4251935345 (4998
~~340276~~
~~848~~
849175
~~5345~~
765621
835543
765621

(5) $z = 5,\ x = \frac{45y}{50-y}$.

(i) $y = 5 \rightarrow$ trivial case

Surely $z = 6, 7, 8, 9 \rightarrow$ trivial case

699224
680552
186725
170138

...11/-

ie $x = 2120a \rightarrow a = 1, 2, 3, 4$ ie the 4 cases 1212, 2424, 3636, 4848

$\mu = \frac{5}{7} \rightarrow \frac{6995}{43} x$; $\mu = \frac{6}{7} = \frac{6994}{53} x$

(8) $\mu = \frac{1}{8} \rightarrow \frac{7999}{2} x$; $\mu = \frac{3}{8} = \frac{7997}{22} = \frac{727}{2}$ ie $2x = 727a$, i.e. $a = 2 \rightarrow x = 727$ ie 2727

$\mu = \frac{5}{8} = \frac{7995}{42} x$; $\mu = \frac{7}{8} \rightarrow \frac{7993}{62} x$.

(9) $\mu = \frac{1}{9} \rightarrow \frac{8991}{1} x$; $\mu = \frac{2}{9} = \frac{8998}{11} = 818 \rightarrow 1818$; $\mu = \frac{4}{9} \rightarrow \frac{8996}{31} x$; $\mu = \frac{5}{9} \rightarrow \frac{8995}{41} x$

$\mu = \frac{7}{9} \rightarrow \frac{8993}{61} x$; $\mu = \frac{8}{9} \rightarrow \frac{8992}{71} x$

3331

ie exactly the seven cases (rather trivial) found on p.7

(C) 5 digit cases. $\frac{x}{a} = \frac{10^4 - \mu}{10\mu - 1} = \frac{10000 - \mu}{10\mu - 1}$.

14|12|73

(1) $\mu = 1$ ✗ (2) $\mu = 2 \rightarrow \frac{9998}{19} x$; $\mu = \frac{1}{2} \rightarrow \frac{19999}{8} x$

(3) $\mu = 3 \rightarrow \frac{9997}{29} x$; $\mu = \frac{1}{3} \rightarrow \frac{29999}{7} x$; $\mu = \frac{2}{3} \rightarrow \frac{29998}{17} x$

(4) $\mu = 4 \rightarrow \frac{9996}{39} x$, $\mu = \frac{1}{4} \rightarrow \frac{39999}{6} x$, $\mu = \frac{3}{4} \rightarrow \frac{39997}{26} x$

(5) $\mu = 5 \rightarrow \frac{9995}{49} x$, $\mu = \frac{1}{5} \rightarrow \frac{49999}{5} x$, $\mu = \frac{2}{5} \rightarrow \frac{49998}{15} x$, $\mu = \frac{3}{5} \rightarrow \frac{49997}{25} x$; $\mu = \frac{4}{5} \rightarrow \frac{49996}{35} x$

(6) $\mu = 6 \rightarrow \frac{9994}{59} x$; $\mu = \frac{1}{6} = \frac{59999}{4} x$; $\mu = \frac{5}{6} \rightarrow \frac{59995}{44} x$.

(7) $\mu = 7 \rightarrow \frac{9993}{69} x$, $\mu = \frac{1}{7} \rightarrow \frac{69999}{3} x$, $\mu = \frac{2}{7} \rightarrow \frac{69998}{13} x$; $\mu = \frac{3}{7} \rightarrow \frac{69997}{23} x$; $\mu = \frac{4}{7} \rightarrow \frac{69996}{33} x$

$\mu = \frac{5}{7} \rightarrow \frac{69995}{43} x$; $\mu = \frac{6}{7} \rightarrow \frac{69994}{53} x$.

(8) $\mu = 8 \rightarrow \frac{9992}{79} x$, $\mu = \frac{1}{8} \rightarrow \frac{79999}{2} x$; $\mu = \frac{3}{8} \rightarrow \frac{79997}{22} x$; $\mu = \frac{5}{8} \rightarrow \frac{79995}{42} x$; $\mu = \frac{7}{8} \rightarrow \frac{79993}{62} x$

(9) $\mu = 9 \rightarrow \frac{9991}{89} x$, $\mu = \frac{2}{9} = \frac{89998}{11} x$; $\mu = \frac{4}{9} \rightarrow \frac{89996}{31} x$; $\mu = \frac{5}{9} \rightarrow \frac{89995}{41} = 2195$

Hence $\mu = 5/9$ works giving $x = 2195a$

$41)89995(2195$
82
79
41
389
369
205
205

Verification.

$\frac{12195 \times 9}{5} = 2439 \times 9 = 21951$ ✓

$\frac{24390 \times 9}{5} = 43902$ ✓

$\frac{36585 \times 9}{5} = 7317 \times 9 = 65853$ ✓

$\frac{48780 \times 9}{5} = 87804$ ✓

$a = 1 \rightarrow$	$x = 2195 \rightarrow$	12195
$a = 2 \rightarrow$	$x = 4390 \rightarrow$	24390
$a = 3 \rightarrow$	$x = 6585 \rightarrow$	36585
$a = 4 \rightarrow$	$x = 8780 \rightarrow$	48780.

✓

5420
26100
189
142
479
426
53

$\mu = \frac{7}{9} \rightarrow \frac{89993}{61} x$; $\mu = \frac{8}{9} \rightarrow \frac{89992}{71} x$

So there are just four cases all corresponding to the single value $\lambda = 9/5$ or $\mu = 5/9$. Anyway it is quite surprising enough, and it is quite rewarding that there should be only one success in 35 cases tried altogether

For the 4 digit case we have not considered the case of single cancellation at all let one like Complete double cancellation. Then there are still the 5 digit, 6 digit & 7 digit cases (an example 7 digits in denominator & 6 in numerator is in the book on "Excursions in number theory"

Before going further let us come back to the simple case of 2 digits in numerator and 3 digits in the denominator with 1 cancellation. Perhaps it is easy to complete this investigation

$$\frac{xy}{yzu} = \frac{x}{zu}.$$

$$\frac{10x+y}{100y+10z+u} = \frac{x}{10z+u}$$

$$\frac{9x+y}{100y} = \frac{x}{10z+u} = \frac{x}{v}$$

$$100xy = v(9x+y)$$

$$x(100y - 9v) = vy$$

$$v = \frac{100xy}{9x+y}.$$

(1) $x=1, v = \frac{100y}{9+y}$.

(i) $y=1 \rightarrow v=10 \rightarrow \frac{11}{110} = \frac{1}{10}$ rather trivial

(ii) $y=6 \rightarrow v=40 \rightarrow \frac{16}{640} = \frac{1}{40}$ trivial extension $\frac{16}{64}$

(iii) $y=9 \rightarrow v=50 \rightarrow \frac{19}{950} = \frac{1}{50}$ Similar extn of $\frac{19}{95}$

(2) $x=2, v = \frac{200y}{18+y}$

(i) $y=2 \rightarrow v=20 \rightarrow \frac{22}{220} = \frac{2}{20}$ rather trivial

(ii) $y=6 \rightarrow v=50 \rightarrow \frac{26}{650} = \frac{2}{50}$ trivial extn of $\frac{26}{65}$

(iii) $y=7 \rightarrow v=56 \rightarrow \frac{27}{756} = \frac{2}{56}$ new

(3) $x=3, v = \frac{300y}{27+y}$

(i) $y=3 \rightarrow v=30 \rightarrow \frac{33}{330} = \frac{3}{30}$

(ii) $y=9 \rightarrow v=75 \rightarrow \frac{39}{975} = \frac{3}{75}$ new

(4) $x=4, v = \frac{400y}{36+y}$

(i) $y=4 \rightarrow$ rather trivial case

(ii) $y=9 \rightarrow v=80 \rightarrow \frac{49}{980} = \frac{4}{80}$

travelling case $\frac{49}{98} = \frac{4}{8}$

(5) $x=5, v = \frac{500y}{45+y}$.

(i) $y=5 \rightarrow$ rather trivial case only case.

(6) $x=6, v = \frac{600y}{54+y}$

(i) $y=6 \rightarrow$ rather trivial only case.

(7) $x=7, v = 700y/(63+y)$

(i) $y=7 \rightarrow$ rather trivial only case

(8) $x=8, v = 800y/(72+y)$

(i) $y=3 \rightarrow v=32 \rightarrow \frac{88}{332} = \frac{8}{32}$ new

(9) $x=3, v = \frac{300y}{27+y}$

(i) $y=3 \rightarrow v=30 \rightarrow \frac{33}{330} = \frac{3}{30}$

(ii) $y=9 \rightarrow v=75 \rightarrow \frac{39}{975} = \frac{3}{75}$ new

(4) $x=4, v = \frac{400y}{36+y}$

(i) $y=4 \rightarrow$ rather trivial case

(ii) $y=9 \rightarrow v=80 \rightarrow \frac{49}{980} = \frac{4}{80}$

travelling case $\frac{49}{98} = \frac{4}{8}$

(5) $x=5, v = \frac{500y}{45+y}$.

(i) $y=5 \rightarrow$ rather trivial case only case.

(6) $x=6, v = \frac{600y}{54+y}$

(i) $y=6 \rightarrow$ rather trivial only case.

(7) $x=7, v = 700y/(63+y)$

(i) $y=7 \rightarrow$ rather trivial only case

(8) $x=8, v = 800y/(72+y)$

(i) $y=3 \rightarrow v=32 \rightarrow \frac{88}{332} = \frac{8}{32}$ new

...10/-

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(D) Six digit case. $\frac{x}{a} = \frac{10^5-\mu}{10-\mu} = \frac{10^5-\mu}{10\mu-1}$, taking all values $\mu > 1$ and < 1.

$= \frac{100000-\mu}{10\mu-1}$.

(1) $\mu = 1$ (trivial), (2) $\mu = 2 \rightarrow \frac{999998}{19} x$; $\mu = \frac{1}{2} \rightarrow \frac{799999}{8} x$

(3) $\mu = 3 \rightarrow \frac{99997}{29} x$, $\mu = 1/3 = \frac{299999}{7} = 42857$ ie $x = 42857a$

$a = 1 \rightarrow x = 42857 \rightarrow 142857$ (Multiply by 3)

$a = 2 \rightarrow x = 85714 \rightarrow 285714$ "

Verification $142857 \times 3 = 428571$

$285714 \times 3 = 857142$

$\mu = 2/3 = \frac{299998}{17} x$; $\lambda = \frac{3}{2} \rightarrow \frac{199997}{28} = \frac{28571}{4}$ ie $4x = 28571a$

$a = 4 \rightarrow x = 28571 \rightarrow 428571$ (Multiply by 2 and divide 3).

Verification $\frac{428571 \times 2}{3} = 285714$

(4) $\mu = 4 \rightarrow \frac{99996}{39} = \frac{33332}{13} = 2564$ leads to $x = 2564a$ ie $a = 4 \rightarrow x = 10256 \rightarrow 410256$

(Divide by 4)

Verification: $\frac{410256}{4} = 102564$

$\mu = \frac{1}{4} \rightarrow \frac{399999}{6} x$; $\mu = 3/4 \rightarrow \frac{399997}{26} = \frac{30769}{2}$ ie $2x = 30769a$

Verification: $\frac{230769 \times 4}{3} = 307692$

$\frac{461538 \times 4}{3} = 615384$

$\frac{692307 \times 4}{3} = 230769 \times 4 = 923076$

$a = 2 \rightarrow x = 30769 \rightarrow 230769$

$a = 4 \rightarrow x = 61538 \rightarrow 461538$

$a = 6 \rightarrow x = 92307 \rightarrow 692307$

(Multiply by 4 and divide by 3)

$\mu = \frac{4}{5} \rightarrow \frac{299996}{37} = 8108 \rightarrow x = 8108a$, giving cases from $a = 2$ up to $a = 9$ (ie 8 cases).

Verification: (i) $\frac{54054 \times 3}{4} = 162162$

(ii) $\frac{216216 \times 3}{4} = 243243$

(iii) $\frac{324324 \times 3}{4} = 324324$ [2×ii]

(iv) $\frac{432432 \times 3}{4} = 405405$

(v) $\frac{540540 \times 3}{4} = 486486$ [3×ii]

(vi) $\frac{648648 \times 3}{4} = 567567$

(vii) = 4×(i), (viii) = 3×(ii).

$a = 2 \rightarrow x = 16216 \rightarrow 216216$

$a = 3 \rightarrow x = 24324 \rightarrow 324324$

$a = 4 \rightarrow x = 32432 \rightarrow 432432$

$a = 5 \rightarrow x = 40540 \rightarrow 540540$

$a = 6 \rightarrow x = 48648 \rightarrow 648648$

$a = 7 \rightarrow x = 56756 \rightarrow 756756$

$a = 8 \rightarrow x = 64864 \rightarrow 864864$

$a = 9 \rightarrow x = 72972 \rightarrow 972972$

(Multiply by 3 & divide by 4.)

(5) $\mu = 5 \rightarrow \frac{99995}{49} = \frac{14285}{7}$ ie $7x = 14285a$, $a = 7$, $x = 14285 \rightarrow 714285$

(Divide by 5)

Vern: $\frac{714285}{5} = 142857$

$\mu = 1/5 \rightarrow \frac{499999}{5} x$, $\mu = \frac{2}{5} \rightarrow \frac{499998}{15}$ ie $5x = 166666 \cdot ax$

$\mu = 3/5 \rightarrow \frac{499997}{25} x$; $\mu = \frac{4}{5} \rightarrow \frac{499996}{35} = \frac{71428}{5} = 14285$ ie $x = 14285a$

This again gives 9 cases from $a = 1$ to 9 viz 110204, 220408, 330612, 410816, 551020, 661224, 771428, 881632, 991836

Multiply by 5 & divide by 4.

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Electrochemistry & Chemical Kinetics

(1-5)

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$\frac{100000-\mu}{10\mu-1}$ ie $5x = 71428a$ ie $a=5 \to x = 71428 \to 571428$ ✓
(multiply by 5 & divide by 4)

This is the case due to Dudeney.

(6) $\mu = 6 \to \frac{999994}{59}$ ×; $\mu = \frac{1}{6} = \frac{599999}{4}$; $\mu = \frac{5}{6} \to \frac{599995}{44} = \frac{54545}{4}$ ie $4x = 54545a$

$a = 4 \to x = 54545$
$\to 454545$ ✓
Multiply by 6 & divide by 5

(7) $\mu = 7 \to \frac{99993}{69} = \frac{33331}{23}$ × $\mu = \frac{1}{7} \to \frac{699999}{3}$ ×; $\mu = \frac{2}{7} \to \frac{699998}{13} = 53846$ ie $a=1, x = 53846$

$\to 153846$ ✓ (Multiply by 7 & Divide by 2)

$\mu = \frac{3}{7} \to \frac{699997}{23}$ ×, $\mu = \frac{4}{7} \to \frac{699996}{33} = 21212$, $x = 21212a$

$\mu = \frac{5}{7} \to \frac{699995}{43}$ × $\mu = \frac{6}{7} \to \frac{699994}{53}$ ×

$a=1 \to 121212$
$a=2 \to 242424$
$a=3 \to 363636$
$a=4 \to 484848$
✓ Multiply by 7 & Divide by 4

(8) $\mu = 8 \to \frac{99992}{79}$ ×; $\mu = \frac{1}{8} \to \frac{799991}{2}$ ×; $\mu = \frac{3}{8} \to \frac{799997}{22} = \frac{72727}{2}$ ie $2x = 72727a$

$a=1 \to x = \frac{a}{2}72727 \to 272727$ ✓ Multiply by 8 and divide by 3

$\mu = \frac{5}{8} \to \frac{799995}{42} = \frac{114285}{6} = \frac{38095}{2}$ ie $2x = 38095 \cdot a$

$a=2 \to x = 38095 \to 238095$ ✓
$a=4 \to x = 76190 \to 476190$
Multiply by 8 and divide by 5.

$\mu = \frac{7}{8} \to \frac{799993}{62}$ ×

(9) $\mu = 9 \to \frac{99991}{89}$ ×; $\mu = \frac{1}{9} \to \frac{899999}{1}$ ×; $\mu = \frac{2}{9} \to \frac{899998}{11} = 81818$ ie $x = 81818 \cdot a$

$a=1 \to 181818$ ✓ Multiply by 9 and divide by 2

$\mu = \frac{4}{9} \to \frac{899996}{31}$ ×; $\mu = \frac{5}{9} \to \frac{899995}{41}$ ×, $\mu = \frac{7}{9} \to \frac{899993}{61}$ ×

$\mu = \frac{8}{9} \to \frac{899992}{71}$ ×

Thus in the case of 6 digits we have in all 25 possible cases including the case of Dudeney, and two case where there are 2 cases of mere multiplication by 3, one case of division by 4, and another case of division by 5.

We have still to consider the cases $\mu > 1$.

(10) (Ⓟ) $\mu = \frac{3}{2} \to \frac{199997}{28} \to 428571$ ie Same as on p. 16.

(11) $\mu = \frac{4}{3} \to \frac{299996}{37}$ already considered on p. 16.

(12) $\mu = \frac{5}{2} \to \frac{199995}{48}$ × (13) $\mu = \frac{5}{3} \to \frac{299995}{47}$ ×; (14) $\mu = \frac{5}{4} \to \frac{399995}{46}$ ×

(15) $\mu = \frac{6}{5} \to \frac{499994}{55} = \frac{45454}{5} \to 5x = 45454a \to 545454$ nothing new only related to 154545 of $\mu = 5/6$

(16) $\mu = \frac{7}{2} \to \frac{199993}{68}$ (does not correspond to $\mu = \frac{2}{7}$); (17) $\mu = \frac{7}{3} = \frac{299993}{67}$ ×

(18) $\mu = \frac{7}{4} \to \frac{399993}{66} = \frac{36363}{6} = \frac{12121}{2}$ ie $2x = 12121a$ (4 cases corresponding to $a = 2, 4, 6, 8$

$\to 212121$ etc $\to 121212$ on multiplying by 4 & Dividing by 7

nothing new but only variations of the 4 cases of $\mu = 4/7$

(iii) $q = 12 \to p = 11 \to \frac{1112}{1112} \to$ trivial case

(iv) $q = 32 \to p = 36 \to \frac{1862}{3632} = \frac{12}{32}$

L.H.S $= \frac{454 \times 3}{454 \times 8}$

(ii) $q = 41 \to p = 41 \to \frac{1414}{4141} = \frac{14}{41}$ (15')

trivial

just like (3)(ii) with only 1&4.

...7.

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(v) $q = 39 \to p =$ $q = 48 \to p = 66$

$\to \frac{1862}{6648} = \frac{12}{48}$

L.H.S $= \frac{1}{4}$

(iii) $q = 50 \to p = 55$

$\to \frac{1554}{5550} = \frac{14}{50}$.

like 3(iii)

(iv) $q = 63 \to p = 99 \to \frac{1764}{6399} = \frac{14}{91}$

$\to \frac{1814}{8163} = \frac{14}{63}$

L.H.S $= \frac{907 \times 2}{907 \times 4}$

(v) $q = 70, p = 70 \to \frac{1764}{7070} = \frac{14}{70}$

I think I had better postpone this Colossal longwinded game

Inorganic Chemistry

(1-4)

(vi) $q = 60 \to p = 99$

$\to \frac{1662}{6660} = \frac{12}{60}$

L.H.S $\to \frac{1992}{9960} = \frac{12}{60}$

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(vii) $q = 66, p = q \neq 60$

since then $q > 120 - q$ & p will have 3 digits

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q going through $x = 2, 3, \ldots 9$ & corresponding to each with $u = 1, 2 \ldots 9$ & then finding q & p each

(3) $x = 1, u = 3$, $p = \frac{99q}{(130 - q)}$ $(q < 65)$

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Physical Chemistry

(1-8)

(i) $q = 13 \to p = 11$

$\to \frac{1113}{1113} \to$ trivial

(ii) $q = 31 \to p = 31$

$\to \frac{1313}{3131} = \frac{13}{31}$

L.H.S $= \frac{101 \times 13}{101 \times 31}$

trivial

Very interesting using only 1&3

Just choose a random two examples & come back again later

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Say (x) $x = 6, u = 8$
(y) $x = 7, u = 5$

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(x) $p = \frac{99 \times 6q}{680 - q}$ $(q < 340)$.

(y) $= \frac{5949}{680 - q}$.

(iii) $q = 40, \to p = 44 \to$

$\frac{1443}{4440} = \frac{13}{40}$

L.H.S $= \frac{111 \times 13}{111 \times 40}$

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(i) $q = 86 \to p = 86$

$\to \frac{6868}{8686} = \frac{68}{86}$

just like 3 (ii) with only 6&8

(iv) $q = 64 \to p = 96$

$\to \frac{1963}{9664} = \frac{13}{64}$

L.H.S $= \frac{151 \times 13}{151 \times 64}$

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(ii) $\frac{3434}{4343} = \frac{34}{43}$ also works.

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Also true of any $\frac{ab}{kab} = \ldots \frac{a}{k}/-\frac{b}{a}$

(4) $x = 1, u = 4, p = \frac{99q}{140 - q}$ $(q < 70)$

(i) $q = 30 \to p = 27$.

$\to \frac{1274}{2730} = \frac{14}{30}$ $\left(\text{L.H.S} = \frac{182 \times 7}{182 \times 15}\right)$

(19) $\mu = \frac{7}{5} \Rightarrow \frac{499993}{65} = \frac{38461}{5}$ ie $5x = 38461a$, $a = 5 \to x = 38461$

leading to 538461 ✓ multiply by 5 & divide by 7 } Apparent new case but allied to $\mu = 2/7$.

(20) $\mu = \frac{7}{6} \to \frac{599993}{64}$ × (21) $\mu = \frac{8}{3} \to \frac{299992}{77} = \frac{27272}{7}$ ie $7x = 27272a$

ie $a = 7 \to$ 727272 nothing new – allied to $\mu = 3/8$

(22) $\mu = \frac{8}{5} \to \frac{499992}{75}$ × (Surprise); (23) $\mu = 8/7 \to \frac{699992}{73}$ ×

(24) $\mu = \frac{9}{2} \to \frac{199991}{88} = \frac{18181}{8}$; $8x = 18181a \to a = 8$ & $x = 18181$.

$\to$ 818181 — allied to $\mu = 2/9$

(25) $\mu = \frac{9}{4} \to \frac{399991}{86}$ ×, (26) $\mu = \frac{9}{5} \to \frac{499995}{85} = \frac{99999}{17}$ × (27) $\mu = 9/7 \to \frac{699991}{63}$ ×

(28) $\mu = \frac{9}{8} \to \frac{799991}{82}$ ×

ie the only case is that for $\mu = 7/5 \to$ 538461 (allied to $\mu = 2/7$)

$\frac{538461 \times 5}{7} = \frac{76923 \times 5}{384615}$

Let us now just try to find one case of 12 digits.

$\frac{x}{a} = \frac{10^{11} - \mu}{10\mu - 1}$ (x = a 11 digit number)

$\mu = \frac{3}{4} \to \frac{399999999997}{26}$

$= \frac{30769230769}{2}$

leading to 230769230769 ✓

Repetition of case for $\mu = 3/4$ for digits

ie multiplying by 4 & dividing by 3

13) 399999999997 (3076923
39 0769
099 119 39
91 117 39
89 29 099 117
78 26 91
89
78

dividing by 3 → 76923076923

Multiplying by 4 → 307692307692

Just try $\mu = 4/5$ (Dudeney's case for 6 digits) 71428571428

$\to \frac{499999999996}{35} = \frac{71428571428}{5}$

leads to $714285714 28a = 5x$ & with $a = 5$

leads to 571428571428 ✓✓ multiply by 5 & divide by 4

÷4 → 142857142857

×4 → 714285714285 ✓

Double Dudeney ✓✓

In fact ✓ comparing the 3 digit cases (8 in number corresponding to $\mu = \frac{4}{3}$) on p. (2), with the 6 digit cases (also 8 in number corresponding $\mu = 4/3$) on p. 16 we find repetition of digits (double)

...

Using the same parallel argument we should have for the 9 digit case a triple repetition of the 3 digit case on p. 2. Is this so? Let us try one case viz.

$10p(10x+u) = q(999x + p)$ $\quad 10p(100x+10u-q) = 999xq$

$p\{10(10x+u) - q\} = 999xq$ $\quad p = 999xq/(100x+10u-q)$ .6.

$p = 999xq/(100x+10u-q)$ $\quad p = 999xq/10(100x+10u-q)$ B(1).

(1) $x = 1,\ u = 1$

$p = \dfrac{999q}{(110-q)}$

(i) $q = 11,\ p =$ ×

(2) $x = 1,\ u = 2$

$p = \dfrac{999q}{120-q}$

(3) $x = 1,\ u = 2$

$p = \dfrac{999q}{130-q}$

$q = 19,\ p = 171$ ×

Something wrong with this whole approach.

Put $p = 10y + z$

$q = 10v + w$.

$999xq + 10qp$

$= 100p(10x+u)$

$p\{100(10x+u) - 10q\}$

$= 999xq$

$10p\{100x+10u-q\}$

$= 999xq$

$p = \dfrac{999xq}{10(100x+10u-q)}$

999 is a mistake should be 990.

$\dfrac{990x + 100y + 10z}{1000y + 100z} = \dfrac{10x+u}{10v+w}$

$\dfrac{99x + 10y + z}{100y + 10z} = \dfrac{10x+u}{10v+w}$

Let $10y + z = p$, $10v + w = q$

$\dfrac{99x+p}{10p} = \dfrac{10x+u}{q}$

$(99x+p)q = 10p(10x+u)$

$p(100x+10u-q) = 99xq$

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$\boxed{p > 10,\ q > 10}$

$999 \times 11/10 \times 999$ $\quad 111/10$ $\quad 110$ $\quad 37$ $\quad 73$

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$\dfrac{23 \times 73}{69}$

$(999 \times 20)/10 \times 90$ × $\quad 999 \times 73/10 \times 37$ $\quad 161$ $\quad 679$

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$999 \times 9/111 = 99$ $\quad \dfrac{999 \times 40}{90}$

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$171 = 100x + 10y$ $\quad 444$

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$\dfrac{xyzu}{yzvw}$

$p = \dfrac{99xq}{100x+10u-q}$ $\quad \begin{pmatrix} p > 10 \\ q > 10 \end{pmatrix}$

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(1) $x = 1, u = 1 \rightarrow p = \dfrac{99q}{110-q}$ $\quad \begin{pmatrix} p < 100 \\ q < 100 \end{pmatrix}$

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(i) $q = 11 \rightarrow p = 11$

$\dfrac{110}{11}$ $\quad$ ie $\dfrac{1111}{1111} \rightarrow$ trivial case

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99.

(ii) $q = 44 \rightarrow p = \dfrac{99 \times 44}{66} = 66$

$\rightarrow \dfrac{1661}{6644} = \dfrac{11}{44}$ Another variation 16/64

$\dfrac{99 \times 22}{88}$ $\quad 110$ $\quad 22$

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(iii) $q = 55,\ p = 99$

$\rightarrow \dfrac{1991}{9955} = \dfrac{11}{55}$ Another variation 19/95

$\dfrac{44}{66} \times 99$ $\quad 66$

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Indian Institute of Technology
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only two cases.

(2) $x = 1,\ u = 2 \rightarrow p = \dfrac{99q}{120-q}$

(i) $q = 21 \rightarrow p = 21$

$\rightarrow \dfrac{1212}{2121} = \dfrac{12}{21}$

L.H.S $= \dfrac{303 \times 4}{303 \times 7}$ new

(ii) $q = 54 \rightarrow p = 81 \rightarrow \dfrac{1812}{8154} = \dfrac{12}{54}$

L.H.S $= \dfrac{906 \times 2}{906 \times 9}$ new

$\dfrac{99 \times 22}{88}$, $\dfrac{99 \times 54}{66}$ $\quad 27 = 81$ $\quad 33$ $\quad \dfrac{906 \times 9}{54}$

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$\times 89!$ $\quad 9\ \dfrac{99}{44}$ $\quad 99$

$120 - 43 = 77$ $\quad 99 \times 21$

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F.N.A.Sc. (India)
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University of Victoria, VICTORIA
British Columbia, CANADA

$4/7$ $\quad 303 \times 4$ $\quad 303 \times 7$ $\quad 21$

216 216 216 to be multiplied by 3 & divided by 4

$$\frac{216216216 \times 3}{4} = 540540540 \times 3 = 162162162$$

So the guess is correct. obviously holds for all 8 cases on p. 2

Similarly for the 12 digit case 216216216216 must be admissible

$$\frac{216216216216 \times 3}{4} = 540540540540 \times 3$$

$$= 162162162162$$

yes it holds.

Similarly all the 6 digit cases (not the only 2 on p. 18) should be extendible to the 12 digit case say for eg:

153846153846 (corresponding to $\mu = \frac{2}{7}$).

$$\frac{153846153846 \times 7}{2} = 769230769230 \times 7$$

$$= 538461538461$$

yes it is true.

Considering cases of 3 & 6 digits are all 6 digit cases obtained by repetition of 3 digit cases, although the converse appears to be true?

Looking at the 6 digit cases, this does not appear to be the case.

For the 3 digit case, we have only the 8 cases given by $\mu = 4/3$.

Are there are other cases or the only above, just as we had as in the 5 digit case the only possible $\mu = 5/9$

Let us try again the 3-digit case

(A) 3 digits: $\frac{x}{a} = \frac{10^2 - \mu}{10\mu - 1} = \frac{100 - \mu}{10\mu - 1}$ (x = 2 digits)

(1) $\mu = 1$ ✗ (trivial) (2) $\mu = 2 \to 98/89 = \frac{49}{9}$; ~~$9x = 49 \cdot a$, $a = 9 \to x = 49 \to 949$~~

$\mu = 1/2 \to \frac{19}{8}$ ✗ (3) $\mu = 3 \to \frac{297}{29}$ ✗, $\mu = 1/3 = \frac{299}{7}$ ✗, $\mu = 2/3 = \frac{298}{27}$ ✗

(4) $\mu = 4 \to \frac{96}{39}$ ✗; $\mu = \frac{1}{4} \to \frac{399}{6} = \frac{133}{2}$ ✗, $\mu = \frac{3}{4} \to \frac{397}{26}$ ✗; $\mu = \frac{4}{3} \to \frac{296}{37}$

leading to $x = 8a$ ie the 8 cases on p. 2 ($a = 1$ not admissible)

(5) $\mu = 5 \to \frac{95}{49}$ ✗, $\mu = \frac{1}{5} \to \frac{499}{5}$ ✗; $\mu = \frac{2}{5} \to \frac{498}{15} = \frac{166}{5}$ ✗; $\mu = \frac{5}{2} \to \frac{195}{48}$ ✗

$\mu = \frac{3}{5} \to \frac{497}{25}$ ✗, $\mu = \frac{5}{3} \to \frac{295}{47}$ ✗; $\mu = 4/5 \to \frac{496}{35}$ ✗, $\mu = \frac{5}{4} = \frac{395}{46}$ ✗

(6) $\mu = 6 \to \frac{94}{59}$ ✗, $\mu = 1/6 = \frac{599}{4}$ ✗, $\mu = 5/6 \to \frac{595}{44}$ ✗; $\mu = 6/5 \to \frac{494}{55}$ ✗

16r

Shall we now undertake the four digit business? It must be colossal. (13')
Anyway let us try & see without going into it into great detail like the 3 digit stuff

... 5.

(A) $\frac{xyzu}{yzvw} \quad \frac{xyzu}{yzuv}$ (Try these cancellations first. $= x/v$

$\frac{999xv}{10x-v} = w$

$(10x-v)w = 999xv$

$v(999x+w) = 10xw$

$w = \frac{999xv}{10x-v}$.

(1) $x=1 \rightarrow w = \frac{999v}{10-v}$

(i) $v=1 \rightarrow w = 111$

$\rightarrow \frac{1111}{11111}$ trivial

(ii) $v=4 \rightarrow w = 6661$

$\rightarrow \frac{1666}{6664}$ Earlier Case

(iii) $v=5 \rightarrow w = 999$

$\rightarrow \frac{1999}{9995}$ Earlier Case

only 3 Cases

(2) $x=2 \rightarrow w = \frac{999x2v}{20-v}$

(i) $v=2, w=222$ Trivial

(ii) $v=5, w=666$

$\rightarrow \frac{2666}{6665}$ Earlier Case

(3) $x=3 \rightarrow w = \frac{999x3v}{30-v}$

(i) $v=3 \rightarrow$ Trivial

(4) $x=4, w = \frac{999x4v}{40-v}$

$v=4 \rightarrow$ Trivial

(5) $x=5, w = \frac{999x5v}{50-v}$

$v=5 \rightarrow$ Trivial case

(6) $x=6, w = \frac{999x6v}{60-v}$

$v=6 \rightarrow$ Trivial case

(7) $x=7, w = \frac{999x7v}{70-v}$

$v=7 \rightarrow$ Trivial

(8) $x=8, w = \frac{999x8v}{80-v}$

$v=8 \rightarrow$ Trivial case.

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$999 \times 42 = 666$ (333)

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MATHEMATICS AND STATISTICS (1-25)

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$\frac{4999}{9998} \times 2$

$\frac{999 \times 28}{33} \quad \frac{1000-10}{} = 990,729$

$\frac{999}{1000} \quad \frac{99.0}{}$

$\frac{xpu}{pq}$

(B) $\frac{xyzu}{yzvw} = \frac{xu}{vw}$,

$\frac{1000x+100y+10z+u}{1000y+100z+10v+w} = \frac{10x+u}{10v+w}$

ie $\frac{990x+100y+10z}{1000y+100z} = \frac{10x+u}{10v+w}$.

$999x(10v+w) + (10v+w)(100y+10z) = (10x+u)(1000y+1000z)$

Let $10v+w = q$

$100y+10z = p$!

$999xq + qp = 10xp + up$

$\frac{10xp}{q} = \frac{999x+up}{10x+u}$, ... 6/-

So it looks as if this three cancellation technique leads to nothing new, so that we have to go down to two cancellation results ie

$\frac{1000x+100y+10z+u}{1000y+100z+10v+w} = \frac{10x+u}{10v+w}$

$\frac{1000x+100y+10z+u}{1000y+100z+10u+v} = \frac{x}{v}$

$999xv + v(100y+10z+u) = 10x(100y+10z+u)$

$999xv = (10x-v)(100y+10z+u)$

$= (10x-v)w$

(9) $x=9, w = \frac{999xqv}{90-v}$

$v=9 \rightarrow$ Trivial case

N.B. - In case $x=4$, we have omitted

$v=8 \rightarrow w = 999$

$\rightarrow \frac{4999}{9998}$ Earlier Case

(7) $\mu=7 \to \frac{93}{69}x$, $\mu=\frac{1}{7} \to \frac{699}{3}x$, $\mu=\frac{2}{7} \to \frac{698}{13}$; $\mu=\frac{7}{2} \to \frac{193}{68}x$

$\mu=\frac{3}{7} \to \frac{697}{23}x$, $\mu=\frac{7}{3} \to \frac{293}{67}x$, $\mu=\frac{4}{7} \to \frac{696}{33}=\frac{232}{11}x$; $\mu=\frac{5}{7} \to \frac{695}{43}x$

$\mu=\frac{7}{5} \to \frac{493}{65}x\frac{29}{5}$ ~~$\to 5x=29a$ & with $a=5 \to x=29$ giving 529~~

$\mu=\frac{6}{7} \to \frac{694}{53}x$, $\mu=7/6 \to \frac{593}{64}x$ ~~529 x 5~~

(8) $\mu=8 \to \frac{92}{79}x$ $\mu=1/8 \to \frac{799}{2}x$; $\mu=3/8 \to \frac{797}{22}x$; $\mu=\frac{8}{3} \to \frac{292}{77}x$

$\mu=5/8=\frac{795}{42}x$, $\mu=\frac{8}{5} \to \frac{492}{75}x$; $\mu=7/8 \to \frac{793}{62}x$, $\mu=\frac{8}{7} \to \frac{692}{73}x$

(9) $\mu=9 \to \frac{91}{89}x$, $\mu=\frac{1}{9} \to \frac{899}{1}x$; $\mu=2/9 \to \frac{898}{11}x$; $\mu=\frac{9}{2} \to \frac{191}{88}x$

$\mu=4/9 \to \frac{896}{31}x$, $\mu=\frac{9}{4} \to \frac{391}{86}x$; $\mu=5/9 \to \frac{895}{41}x$; $\mu=7/9 \to \frac{893}{61}x$; $\mu=\frac{9}{7} \to \frac{691}{83}x$

$\mu=8/9 \to \frac{892}{71}x$; $\mu=\frac{9}{8}=\frac{791}{82}x$

So as guessed on p.4 the only soln for the 3-digit case is given by $\mu=4/3$ giving 8 cases analogous to the 5-digit case giving for $\mu=5/9$.

So all the 6-digit cases are not repetitions of the 3-digit cases, nor need all the 9 digit cases be mere triple repetitions of the 3-digit case. Similarly, perhaps, not all 12-digit cases are merely double repetitions cases of the 6 digit cases — Similarly 4 digit & 8 digit cases — 4 digit & 12 digit cases & so on.

All the above considerations give us plenty of unnecessary work for investigation. Let us stop here for the present.

Let us first consider the cases of enchanges of 1st & last digit in the 6-digit case

15/12/73. $(10^5a+10^4b+10^3c+10^2d+10e+f)p=q(10^5f+\dots+a)$

$a(10^5p-q)+(10^4b+\dots+10e)(p-q)+f(p-10^5q)=0$

$a(10^5p-q)+10(p-q)x+f(p-10^5q)=0$ $\quad x=10^3b+10^2c+10d+e$ (4 digit no)

$a(10^5p-q)+10(p-q)x=(10^5q-p)f$ $\quad a(p\cdot10^5-q)+10x(p-q)=f(10^5q-p)$

$a(10^5-\mu)+10x(1-\mu)=(10^5\mu-1)f$, where $\mu=q/p$ $\quad a(\lambda\cdot10^5-1)+10x(\lambda-1)=f(10^5-\lambda)$

$\lambda=p/q$

$10x(1-\mu)=(\mu\cdot10^5-1)f-a(10^5-\mu)$

$10x=\frac{f(\mu\cdot10^5-1)}{1-\mu}-\frac{a(10^5-\mu)}{1-\mu}=\mu(10^5f+a)-(10^5a+f)$

$1-\frac{4}{3}$

~~$\mu=2$, $10x=-199999f+999998a=999998a-199999f$~~

~~$\mu=5/6$~~ $\mu=3/4 \to 10x=299996f-399997a$ ×

$\mu=4/3 \to 10x=-399997f+299996a$

9 K	1	2	3	4	5	6	7	8	9
$99k/(10-k) = 10y+z$	11	x	x	66	99	x	x	x	x

ie exactly the two values given by the earlier investigation

... 4.

viz. $y = z = 1 \rightarrow$ trivial case
$y = z = 6 \rightarrow 166/664$
$y = x = 9 \rightarrow 199/995$

General Relativity, etc.

(2) $x = 2$
$\rightarrow u = \dfrac{20(10y+z)}{198+10y+z}$

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ie $20(10y+z) = k(198+10y+z)$.

Solid State Physics

(1-15)

K	1	2	3	4	5	6	7	8	9
$\dfrac{198K}{20-K}$	x	22	x	x	66	x	x	x	x

again exactly same values as before
$y = z = 2 \rightarrow$ trivial.
$y = z = 6$

$(10y+z)(20-K) = 198K$

$10y+z = \dfrac{198K}{20-K}$.

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$198 \times 2/18 \; 9 \mid 198 \times 8/12 \; 3 \times 198 \times 9/11 \times$

(3) $x = 3, \rightarrow$
$10y+z = \dfrac{99 \times 3K}{30-K}$

2. Dr. B. Dayal, Watumull Prize, 1962
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$10x - k = 11\lambda$
$x = 3, k = 8 \rightarrow \lambda = 2, \; 10y+z = \dfrac{9 \times 3 \times 8}{2} > 100$ X
$x = 4, k = 7 \rightarrow \lambda = 3, \; 10y+z = \dfrac{9 \times 7 \times 4}{3} = 84.$
$y = 8, z = 4.$

same as before except that K plays the role of u
ie $k = u$.

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$\rightarrow \dfrac{484}{847}$

ie same case on p(11') with $x = 4$.

Instead of taking specific values of x separately suppose we put.

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& here y & z are different

$10x(10y+z) = k(99x+10y+z)$

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So the statement on p(11') that putting $10y+z = v$ makes $10y+z$ always a multiple of 11 is not correct

$(10y+z)(10x-K) = 99Kx$

$10y+z = \dfrac{99Kx}{10x-K}$.

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$99Kx = \lambda(10x-K)$.
$K(99x+\lambda) = 10x\lambda$.
$K = \dfrac{10x\lambda}{99x+\lambda}$
$10x\lambda = \mu(99x+\lambda)$
$\lambda(10x-\mu) = 99x\mu$
$\lambda = \dfrac{99x\mu}{10x-\mu}$

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Thus there is no need for this investigation & the previous one putting $10y+z = v$ appears quite correct

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Let $10x - K = 11\lambda$
Then $10y+z = 9Kx/\lambda$
& need not be a multiple of 11

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LUCKNOW

[P.T.O

$10x - K = 11\lambda$
$x = 2, K = 9 \rightarrow \lambda = 1.$
Then $10y+z = 18 \times 9 = 162$
not possible since $y < 10$

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Let us ~~take~~ choose $a = 2, f = 3$ arbitrarily

$$\cancel{10x = \frac{3\mu\cdot 10^5 - 2(10^5 - \mu)}{(1-\mu)} = \frac{\mu(3\cdot 10^5 + 2) - 2\cdot 10^5}{1-\mu}}$$

$$\cancel{= \frac{\mu(300002}{} \frac{300002\mu - 200000}{1-\mu}}$$

$$\cancel{5x = \frac{150001\mu - 100000}{1-\mu}} \qquad \cancel{\mu = 5}$$

$$\cancel{\mu = 5/6 \rightarrow 5x = \frac{750005 - 600000}{} = 150005} \quad \text{ie } x = 30001 \text{ (5 digit no } \times)$$

$$\cancel{\mu = 5/7 \rightarrow 5x = \frac{750005 - 700000}{2} = \frac{50005}{2}} \times$$

$$\cancel{\mu = 2/3 \rightarrow 5x = 300002 - 300000 = 2} \times$$

$\cancel{100000}$ ~~5000~~

$\mu = 3/2$

$$10x = \frac{\mu(10^5 f + a) - (10^5 a + f)}{(1-\mu)} = \frac{q(10^5 f + a) - p(10^5 a + f)}{p - q}$$

$\cancel{a = 2, f = 3}$

$2\mu - 3 = 1$

$\frac{2q - 3p}{p - q}$ $\quad \frac{p}{q} = \frac{1}{4}$

$2q - 3p$ must contain 10 as factor.

$\cancel{2q - 3p = 10}$

$\cancel{2p - 3q = 4}$

$\cancel{3p - 3q = p + 10}$

$$\cancel{\mu = \frac{5}{7} \rightarrow 10x = \frac{\mu(3\cdot 10^5 + 2) - (2\cdot 10^5 + 3)}{1-\mu}}$$

$$\cancel{\mu = 1/4, \; 10x = 3\cdot 10^5 + 2 - 4(2\cdot 10^5 + 3)}$$

Choose $a > f$: say $3, f = 2$, $10x = \frac{\mu(2\cdot 10^5 + 3) - (3\cdot 10^5 + 2)}{1-\mu}$ ×

$a = f = 2$ say $\quad 10x = \frac{\mu(2\cdot 10^5 + 2) - (2\cdot 10^5 + 2)}{1-\mu} = \frac{2\cdot 10^5(\mu - 1) + 2(\mu - 1)}{1-\mu}$

$f\mu = a$

$\mu = a/f$

$= -(2\cdot 10^5 + 2)$ ~~absurd~~ ×

If $\mu = a/f$; $10x = \frac{a^2 - f^2}{f - a} = -$ve ×

& x cannot be 4 digit no ×

14-24

$\mu > 2/3$ $\quad$ 7/8

$\mu = 3/4$

$\mu = 4/5$

$\mu = 5/6$

$a = 2, f = 3$

$$10x = \frac{\mu(3\cdot 10^5 + 2) - (2\cdot 10^5 + 3)}{1-\mu} = \frac{10^5(3\mu - 2) + (2\mu - 3)}{1-\mu}$$

$\mu = 7/8 \rightarrow \frac{10^5(19) - 10}{}$ $\quad$ x not a four digit no

Appears rather tough. Let us try 4 & 5 digits first. $10x = \frac{\mu(1000d + a) - (1000a + d)}{1-\mu}$ $\quad$ ~~8004~~

4 digits. $\quad 10x = \frac{\mu(1000d + a) - (1000a + d)}{1-\mu}$ $\quad$ (x = 2 digit

$a = 1, d = 2 \rightarrow 10x = \frac{\mu(2000 + 1) - (1002)}{1-\mu} = \frac{2001\mu - 1002}{1-\mu}$ $\quad \mu = \frac{3}{4}$

4002
3006
996

6003 4/5
4002
~~16008~~
~~4018~~ ×
~~6990~~ 4/5

~~$\mu = 8/9 \rightarrow \mu = 1/6 \rightarrow$~~ 2001

$a = 2, d = 1$; $10x = \frac{1002\mu - 2001}{1-\mu}$ × $\quad a = 2; d = 3 \rightarrow \frac{3002\mu - 2003}{1-\mu}$

~~21007~~

~~14021~~

$a = 3, d = 2 \rightarrow \frac{2003\mu - 3002}{1-\mu}$

$3004\mu - 4003$

~~$\mu = 5/6 \rightarrow 10x =$~~

$a = 3, d = 4 \rightarrow \frac{4003\mu - 3004}{1-\mu}$

~~9012~~

$4005\mu - 5004$

$5004\mu - 4005$

$a = 1, d = 1 \rightarrow \frac{1001\mu - 1001}{1-\mu}$ × $\quad$ with $d = 5, a = 4$

$10x = \frac{5004\mu - 4005}{1-\mu}$

~~30024~~ 25020
~~24030~~ 24030
~~5994~~ 990

25020 − 24030

& with $\mu = 5/6 \rightarrow 10x =$

$10x = 990$ or $x = 99$

ie reqd number is 4995

Divide by 5 & multiply by 6 $\rightarrow \frac{999 \times 6}{5994}$

6005

$4005\mu - 5004$ also holds

$\mu = 6/5$

(9) $x = 9,\ z = 9 \rightarrow w = \frac{99y}{y+81}$

$y = 9 \rightarrow$ trivial

This finishes investigations of fractions like $\frac{xyz}{yuv}$ ... 3/-

(11')

Present try fractions of double cancellation with 3 digits each in numerator & denominator

ie $\frac{xyz}{yzu} = \frac{x}{u}$

$\frac{100x+10y+z}{100y+10z+u} = \frac{x}{u}$

$\frac{x}{u} = \frac{99x+10y+z}{100y+10z}$

$u = \frac{10x(10y+z)}{99x+10y+z}$

Let $10y + z = v$

$u = \frac{10xv}{99x+v}$

$u(99x+v) = 10xv$

$v(10x-u) = 99ux$

$v = \frac{99xu}{10x-u}$

(1) $x = 1 \rightarrow v = \frac{99u}{10-u}$

(i) $u = 1,\ v = 11 \rightarrow \frac{111}{111}$ X

(ii) $u = 4,\ v = 66$

$\rightarrow \frac{166}{664} = \frac{1}{4}$ ✓ the old case already found

(iii) $u = 5,\ v = 99$

$\rightarrow \frac{199}{995} = \frac{1}{5}$ ✓ the old case

no other cases Fine

(2) $x = 2$

$\rightarrow v = \frac{198u}{20-u}$

(i) $u = 2,\ v = 22$

$\rightarrow \frac{222}{222}$ trivial

(ii) $u = 5,\ v = 66$

$\rightarrow \frac{266}{665} = \frac{2}{5}$ old case Fine

no other cases

(3) $x = 3 \rightarrow v = \frac{99 \times 3u}{30-u}$

(i) $u = 3 \rightarrow$ trivial

(4) $x = 4 \rightarrow v = \frac{99 \times 4u}{40-u}$

(i) $u = 4 \rightarrow$ trivial

(ii) $u = 7 \rightarrow v = 84 \rightarrow \frac{484}{847} = \frac{4}{7}$ ✓ a new one

4. Prof. Rudra Pal Singh, M.Sc., Ph.D.
Senior Professor of Physics and Dean of Research
Department of Physics, Indian Institute of Technology, Powai, BOMBAY-400076.

(5) $x = 5 \rightarrow v = \frac{99 \times 5u}{50-u}$

(i) $u = 5 \rightarrow$ trivial case ie all

(ii) $\frac{99 \times 15}{44 \times 22}$

(6) $x = 6 \rightarrow v = \frac{99 \times 6u}{60-u}$

(i) $u = 5 \rightarrow v = 54 \rightarrow \frac{654}{545} = \frac{6}{5}$ ✓ new one

(ii) $u = 6$ trivial

(7) $x = 7 \rightarrow v = \frac{99 \times 7u}{70-u}$; (i) $u = 7 \rightarrow$ trivial

(ii) $u = 4 \rightarrow v = 42 \rightarrow \frac{742}{424} = \frac{7}{4}$ ✓ new one & fraction 71

(8) $x = 9 \rightarrow v = \frac{99 \times 8u}{80-u}$

(8) $x = 8 \rightarrow v = \frac{99 \times 8u}{80-u}$; (i) $u = 8 \rightarrow$ trivial case

(9) $x = 9 \rightarrow v = \frac{99 \times 9u}{90-u}$ (i) $u = 9$, trivial

ie 3 extra cases

In view of $v = \frac{99xu}{10x-u}$

v is always a multiple of 11 which makes $10y + z = 11 \cdot k$. ie $y = z$.

It should be possible to find cases where y & z are different.

For this consider

$u = \frac{10x(10y+z)}{99x+10y+z}$

(1) $x = 1 \rightarrow u = \frac{10(10y+z)}{99+10y+z}$ For this to be an integer we should have $10(10y+z)$ a multiple of $99+10y+z$

Put $10(10y+z) = k(99+10y+z)$

$(100y - 10ky) + (10-k)z = 99k$

$10y(10-k) + z(10-k) = 99k$

$10y + z = \frac{99k}{10-k}$

Crystal Physics

(1-4)

1. Prof. K.V. Krishna Rao, B.Sc., (Hons), M.Sc., Ph.D. (Osmania), Ph.D. (Lond)
Professor of Physics, Osmania University
HYDERABAD-500007, (A.P.)

$\frac{99 \times 18}{88}$

$\frac{99 \times 24}{56}$ $\frac{99 \times 30}{55}$ 11 (54)

2. Dr. P.S. Narayanan, M.A., M.Sc., Ph.D.
Associate Professor, Head of the Department of Physics, Indian Institute of Science
BANGALORE-560012.

3. Dr. A.R. Patel

$\frac{99 \times 42}{63} = 66$ ✓

4. Dr. A.R. Verma, M.Sc., Ph.D., F.N.A.
Director, National Physical Laboratory
NEW DELHI-110012.

Crystallography

(1-2)

1. Dr. R. Chidambaram, M.A., M.Sc., Ph.D.
Scientific Officer (E), Bhabha Atomic Research Centre, Trombay, BOMBAY-400085.

2. Dr. L.M. Pant
Scientist, National Chemical Laboratory
POONA.

$\frac{198 \times 2}{189} = 22$ $\frac{198 \times 9}{11}$

Field Theory, Particle Physics, etc.

(1-3)

$\frac{198 \times 82}{123}$ 66×2 $\frac{99 \times 24}{22} = 108$

1. Prof. Samarendra Nath Biswas, Ph.D. (Adelaide, 1958) D. Phil (Calcutta, 1959), F.N.A.
Senior Professor, Physics Department, Visva-Bharati University, Santiniketan, W.BENGAL.

$\frac{99 \times 16}{369} = 14$

2. Dr. Lalit K. Pandit, Ph.D. (Zurich) 1957
Associate Professor, Tata Institute of Fundamental Research, Homi Bhabha Road
BOMBAY-400005.

$\frac{99 \times 28}{33}$ 84 $\frac{121}{121}$

3. Prof. Trilochan Pradhan, Ph.D. (Chicago)
Professor, Saha Institute of Nuclear Physics
92, Acharya Prafulla Chandra Road
CALCUTTA-700009.

... 4/-

ie four digits we have $4995 \longleftrightarrow 5994$ with $\mu = \frac{6}{5}$ or $\frac{5}{6}$.

~~36032~~

$\frac{6005\mu - 5006}{1-\mu} \rightarrow$ with $\mu = \frac{4}{5}$ to ~~24020~~ ~~25030~~ −ve

$7006\mu - 6007$ ×

$8007\mu - 7008$ ×

$9008\mu - 8009$ ×

$10x = \frac{\mu(d\cdot 10^3 + a) - (a\cdot 10^3 + d)}{1-\mu}$

$= \frac{\mu(1000d + a) - (1000a + d)}{1-\mu}$

~~6993 a 3996 4662~~

~~2331~~

$a = 1,\ d = 3 \rightarrow 10x = \frac{3001\mu - 1003}{1-\mu}$ × $\quad 3001 - 2006 = 995$

$a = 2,\ d = 4 \rightarrow 10x = \frac{4002\mu - 2004}{1-\mu}$ ×

6986
7996
1010

~~7984~~
~~11994~~

Instead of choosing a & d, let us try cases for several values of μ.

(1) $\mu = 1$ ×

(2) $\mu = 2 \rightarrow \frac{2000d + 2a - 1000a - d}{-1} = \cancel{1000a}\ \cancel{1000}\ 998a - 1999d = 10x$

5988
3998
1990

$a = 2 \rightarrow 1996 - 1999d$ ×

$a = 3 \rightarrow 2994 - 1999d$ ×

$a = 4 \rightarrow 3992 - 1999d$ ×

$a = 5 \rightarrow 4990 - 1999d$ ×

$a = 6 \rightarrow 5988 - 1999d;\ d = 2 \rightarrow x = 199$ ×

$a = 7 \rightarrow 6986 - 1999d;\ d = 4 \rightarrow$ ×

$a = 8 \rightarrow 7984 - 1999d,\ d = 6 \rightarrow$ ×

$a = 9 \rightarrow 8982 - 1999d$ ×

$\mu = 1/2 \rightarrow 10x = \cancel{1000d +}\ 1000d + a - 2000a - 2d$

$= \cancel{999}\ 998d - 1999a,\quad d = 9,\ a = 8 \rightarrow$ 8982 × 10992

$a = 2 \rightarrow 998d \cancel{+ 1999}\ 3998 = \cancel{3998 - 998d}$ × $998d - 1999a$ ✓

$a = 3 \rightarrow 998d - 5997$ ×

$a = 4 \rightarrow 998d - 7996;\ d = 7 \rightarrow 6986 - 7996$ ×

$a = 5 \rightarrow 998d - 9995$ ×

$a = 6 \rightarrow 998d - 11994$ ×

$a = 7 \rightarrow 998d - \ldots 3$ ×

$a = 8 \rightarrow 998d - 15992$ ×

~~8773~~

(3) $\mu = 3 \rightarrow 10x = \frac{3(1000d + a) - (1000a + d)}{-2} = \frac{997a - 2999d}{2}$

~~2991~~
~~8973~~

$a = 4 \rightarrow \frac{1}{2}(3988 - 2999d)$ × $\quad a = 8 \rightarrow \frac{1}{2}(7976 - 11996)$ ×

$a = 5 \rightarrow \frac{1}{2}(4985 - 2999d)$ × ×

2991

$\mu = 1/3 \rightarrow \frac{(1000d + a) - 3(1000a + d)}{2} = \frac{997d - 2999a}{2}$

$997d > 2999a$

$d > \frac{2999}{997}a$

$d \gtrsim 3a$

1−3

$a = 1 \rightarrow \frac{1}{2}(997d - 2999)$ ×

$a = 2 \rightarrow \frac{1}{2}(997d - 5998)$ ×

~~$a = 4 \rightarrow \frac{1}{2}(997d - 11996)$ ×~~ enough to consider up to $a = 3$.

$a = 3 = \frac{1}{2}(997d - 8997) = 24,\ 10x = 24,\ x = \frac{12}{5}$ ×

6

8997
8973
24

6

$\mu = 2/3 \rightarrow \cancel{2000d}\ 2000d + 2a - 3000a - 3d$

$1997d > 2998a$

$d > \frac{2998}{1997}a$

$d \gtrsim \frac{3a}{2}$

$10x = 1997d - 2998a \quad a = 1,\ d = 4$

7988
2998
4990 ×
11982
11992

4

$a = 1,\ 10x = 1997d - 2998a$ ×

$a = 2,\ \text{"} = 1997d - 5996$

$a = 4,\ \text{"} = 1997d - 11992$ ×

$a = 7,\ \text{"} = 1997d - 20986 \quad (a < 6$

2998
5988
2990

1997) 2998 (1.5
1997
10010
9985
25

4

(1) $x=7, z=1, w = \frac{710y}{y+63}$.

(i) $y=7 \rightarrow$ trivial

(ii) $y=8 \rightarrow w=80 \rightarrow \frac{781}{880} = \frac{71}{80}$ ✓

Ionospheric Physics

(1-6)

(2) $x=7, z=2, w = \frac{720y}{y+63}$

(i) $y=7 \rightarrow$ true

(ii) $y=9 \rightarrow w=90 \rightarrow \frac{792}{990} = \frac{72}{90}$ ✓

(3) $x=7, z=3, w = \frac{730y}{y+63}$.

(i) only $y=7 \rightarrow$ true

(4) $x=7, z=4, w = \frac{740y}{y+63}$

only $y=7$ ✗

(5) $x=7, z=5, w = \frac{750y}{y+63}$

(i) $y=7$ ✗

(6) $x=7, z=6, w = \frac{760y}{y+63}$

(i) $y=7$ ✗

(ii) $y=9 \rightarrow w=95 \rightarrow \frac{796}{995} = \frac{76}{95}$ ✓

L.H.S $= \frac{199 \times 4}{199 \times 5} = \frac{4}{5}$

(7) $x=7, z=7, w = \frac{770y}{y+63}$.

(i) $y=3 \rightarrow w=35 \rightarrow \frac{737}{935} = \frac{77}{35}$ ✓

L.H.S $= \frac{67 \times 11}{67 \times 5}$

also fraction >1.

(ii) $y=7$ trivial

(8) $x=7, z=8 \rightarrow w = \frac{780y}{y+63}$

(i) $y=2 \rightarrow w=24 \rightarrow \frac{728}{724} = \frac{78}{24}$ ✓

L.H.S $= \frac{13 \times 56}{4 \times 56}$ Also fraction >1.

(ii) $y=7$ ✗

(9) $x=7, z=9 \rightarrow w = \frac{790y}{y+63}$

$y=7$ ✗

Now consider $x=8$ ...2.

(i) $x=8, z=1, w = \frac{810y}{y+72}$

(i) $y=8$ trivial

(ii) $y=9 \rightarrow w=90 \rightarrow \frac{891}{990} = \frac{81}{90}$ ✓

(2) $x=8, z=2, w = 8204/(y+72)$

(i) $y=8 \rightarrow$ trivial

(3) $x=8, z=2 \rightarrow w = 8304/(y+72)$

$y=8 \rightarrow$ trivial

(4) $x=8, z=4 \rightarrow w = 8404/(y+72)$

$y=8 \rightarrow$ trivial

(5) $x=8, z=5, w = 8504/(y+72)$

(i) $y=3 \rightarrow w=34 \rightarrow \frac{835}{834} = \frac{85}{34}$ ✓

L.H.S $= \frac{167 \times 5}{167 \times 2}$ fraction >1

(ii) $y=8$ ✗

(6) $x=8, z=6 \rightarrow w = \frac{860y}{y+72} \rightarrow$ (i) $y=8$ ✗

(7) $x=8, z=7 \rightarrow w = 8704/(y+72) \rightarrow y=8$ ✗

(8) $x=8, z=8 \rightarrow w = 8804/(y+72)$

(9) $x=8, z=9 \rightarrow w = \frac{890y}{y+72} \rightarrow y=8$ true ✗

Next & last by $x=9$.

(1) $x=9, z=1 \rightarrow w = 910y/(y+81) \hookrightarrow y=9$ ✗

(2) $x=9, z=2 \rightarrow w = 920y/(y+81) \rightarrow y=9$ ✗

(3) $x=9, z=3 \rightarrow w = 930y/(y+81) \rightarrow$ "

(4) $x=9, z=4 \rightarrow w = 940y/(y+81) \rightarrow$ "

(5) $x=9, z=5 \rightarrow w = 950y/(y+81) \rightarrow$ "

(6) $x=9, z=6 \rightarrow w = 960y/(y+81) \rightarrow$ "

(7) $x=9, z=7 \rightarrow w = 970y/(y+81) \rightarrow$ "

(8) $x=9, z=8 \rightarrow w = 980y/(y+81)$

(i) $y=3 \rightarrow w=35 \rightarrow \frac{938}{335} = \frac{98}{35}$ ✓

L.H.S $= \frac{67 \times 14}{67 \times 5}$ ... fraction >1

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1. Prof. Satyendra N. Ghosh, D.Sc., F.N.A., F.N.A.Sc.
Professor and Head of the Department of Applied Physics,
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2. Dr. Pradyumna K. KAW, Ph.D.
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4. Prof. B. Ramachandra Rao, M.Sc., Ph.D., F.N.A.
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Astronomy and Astrophysics

(1-3)

1. Prof. Krishna Damodar ABHYANKAR, Ph.D. (Univ. of California, Berkeley)
Director, Centre for Advanced Study in Astronomy
Osmania University, HYDERABAD-500007.

2. Dr. J.C. Bhattacharya
Indian Institute of Astrophysics, KODAIKANAL.

3. Prof. Jayant V. Narlikar, Ph.D. (Cantab)
Professor, Tata Institute of Fundamental Research
Homi Bhabha Road, BOMBAY-400005.

Radio Astronomy

1. Prof. T.K. Menon, Ph.D.
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Homi Bhabha Road, BOMBAY-400005.

Theoretical Physics

(1-6)

1. Dr. K.M. Khanna, M.Sc., (Hons), Ph.D.
Professor of Applied Physics, Birla Institute of
Technology, P.O. Mesra-835215, RANCHI.

2. Prof. P.M. Mathews
Professor and Head of the Department of Theoretical
Physics, University of Madras, Guindy
MADRAS-600025.

3. Prof. Sudhir P. Pandya, Ph.D. (Rochester, USA)
Professor and Deputy Director
Physical Research Laboratory, Navrangpura
AHMEDABAD-380009.

$\mu = \frac{3}{2} \to 10x = \frac{3000d + 3a - 2000a - 2d}{-1}$

$= 1997a - 2999d \to x = 199$ ✗

with $a = 4$ & $d = 2$

7988
5998
1990

$a > \frac{3}{2}d$. $d < \frac{2}{3}a$.

$2a > 3d$

$3d < 2a$ $\frac{10}{3} \cdot \frac{16}{3}$

39

$d < \frac{18}{3}$

$a > 4d$

~~17973~~
~~20993~~

$a = 8 \to$ " $= 15976 - 2999d$, $d = 4$ ✗

$a = 9 \to$ " $= 17973 - 2999d$, $d = 7$

(4) $\mu = 4$, $10x = \frac{4000d + 4a - 1000a - d}{-3} = \frac{996a - 3999d}{3} = 332a - 1333d$ $(d < \frac{1}{4}a)$

$\frac{6}{4}$

~~2656~~
~~2666~~ 1338

$d >$

$a = 5 \to 10x = 1660 - 1333d$ ✗

no other case ∵ of $d < \frac{1}{4}a$.

$\mu = \frac{1}{4} \to \frac{1000d + a - 4000a - 4d}{3} = \frac{996d - 3999a}{3} = 332d - 1333a$ $(d > 4a)$

Cases of $a = 1$ or 2; $a = 1$ ✗, $a = 2 \to 332d - 2666$, $d = 8$ ✗

~~$d = 8 \to 2656$~~ $d = 9$ ✗

$\mu = 3/4 \to 3000d + 3a - 4000a - 4d = 2996d - 3997a$

$d > \frac{3997}{2996}a$

$a = 2 \to 2996d - 7994$, $d = 4 \to$

$\gtrsim \frac{3}{2}a$.

1332
1332

$d = 4 \to 11984 - 7994 = 3990$ ✗

$d < 6$.

$\mu = 4/3 \to \frac{4000d + 4a - 3000a - 3d}{-3} = \frac{3997d - 2996a}{-3}$ ✗

(5) $\mu = 5$, $\frac{5000d + 5a - 1000a - d}{-4} = \frac{4999d - 995a}{-4}$ ✗ =

$\mu = 1/5 \to \frac{1000d + a - 5000a - 5d}{4} = \frac{995d - 4999a}{4}$ ✗

$d > 5a$

5885

$a = 1$, $d = 3$

$\mu = 2/5 \to \frac{2000d + 2a - 5000a - 5d}{3} = \frac{1995d - 4998a}{3}$

~~$d > 3a$~~

5885
4998
887/3 ✗

$a = 2$, $d = 5$ $9975 - 9996$ ✗

" $d = 6$ 11970
9996
1974/3 = 6 ✗

$\mu = 5/3 \to \frac{4997d - 2995a}{-2} = \frac{1}{2}(2995a - 4997d)$

~~20965~~ ✗
~~24985~~

$a = 7$,

$\mu = 3/5 \to \frac{3000d + 3a - 5000a - 5d}{2} = \frac{2995d - 4997a}{2}$, $d = 7$, $a = 5 \to$ ~~20965~~ ~~24985~~

$a = 1$, $d = 2 \to (5990 - 4997)/2$ ✗

$a = 1$, $d = 3 \to (8985 - 4997)/2 = 3988/2$ ✗

$\mu = \frac{5}{4} \to$ ~~4996~~ $\frac{4996d - 3995a}{-1}$

$= 3995a - 4996d$

$\mu = 4/5 \to 4000d + 4a - 5000a - 5d = 3995d - 4996a$

23970
24980 ✗

$a = 5$, $d = 6 \to$ 23970
−24980 ✗

(6) $\mu = 6 \to \frac{6000d + 6a - 1000a - d}{-5} = \frac{994a - 5999d}{5}$

$a > 6d$.

$\mu = \frac{6}{5} \to \frac{5995d - 4994a}{-1}$

$a = 6$, $d = 1$ $5964 - 5999 = -35/5$ ✗

$\mu = 1/6 \to \frac{1000d + a - 6000a - 6d}{5} = \frac{994d - 5999a}{5}$ ✗

$d > 5a$

$= 4994a - 5995d$

4970
5964

$\mu = 5/6 \to 5000d + 5a - 6000a - 6d = 4994d - 5995a$

$a = 4$, $d = 5 \to$ 24970
23980
990

ie $10x = 990$ ie $x = 99$

the case already found

(7) $\mu = 7 \to \frac{7000d + 7a - 1000a - d}{-6} = \frac{993a - 6999d}{6} = \frac{331a - 2333d}{2}$

$a = 7$, $d = 1 \to 3$ ✗

THE INDIAN ACADEMY OF SCIENCES

BANGALORE 560006

-ooo-

CONFIDENTIAL

Consolidated list of nominations received for the election of Fellows during the year 1973:

(9) $x=6, z=9, w=\frac{690y}{y+54}$

(i) $y=6 \rightarrow$ trivial

Spectroscopy

(1-8)

(1) $x=6, z=1, w=\frac{610y}{y+54}$

1. Dr. S.L.N.G. Krishnamachari, M.Sc., D.Sc.,
Scientific Officer, Spectroscopy Division
Bhabha Atomic Research Centre, BOMBAY-400085.

only 3 cases for $x=6$

(i) $y=6$ — trivial

(ii) $y=7 \rightarrow w=70 \rightarrow \frac{671}{770}=\frac{61}{70}$ ✓

2. Dr. P.V. Kulkarni, Ph.D. (Poona)
Associate Professor, Physical Research Lab.
Navrangpura, Ahmedabad-380009.

3. Prof. Chintamani Mande, M.Sc. (B.H.U.),
D.Phil.(Allahabad), D.Sc. (Paris)
Professor and Head of the Department of
Physics, Nagpur University, NAGPUR

(2) $x=6, z=2, w=\frac{620y}{y+54}$

(i) $y=6 \rightarrow$ trivial

(3) $x=6, z=3, w=\frac{630y}{y+54}$

4. Dr. M.V. Radha Krishna Murty, D.M.I.T. (Madras)
Ph.D. (Rochester)
Scientific Officer (SF), Spectroscopy Division
Bhabha Atomic Research Centre, BOMBAY-400085.

(i) $y=6 \rightarrow$ trivial

(ii) $y=9 \rightarrow w=90 \rightarrow \frac{693}{990}=\frac{63}{90}$ ✓

5. Dr. D. Ramachandra Rao
Associate Professor of Physics, Indian Institute
of Technology, KANPUR-208016 (UP)

(4) $x=6, z=4, w=\frac{640y}{y+54}$

6. Dr. D. Sharma
Professor of Physics, Presently Acting Vice-
Chancellor, Gorakhpur University.

(i) $y=$ ~~$6 \rightarrow w$~~ trivial

(5) $x=6, z=5, w=\frac{650y}{y+54}$

7. Dr. K. Venkata Ramaiah, M.Sc., Ph.D.
Professor and Head of the Department of Physics
Osmania University, HYDERABAD-7 (AP)

(i) $y=6 \rightarrow$ trivial

(6) $x=6, z=6, w=\frac{660y}{y+54}$

8. Prof. V.S. Venkatasubramanian
Professor of Physics, Indian Institute of Science
BANGALORE-560012.

$\frac{660}{55}$

Nuclear Physics

(1-4)

$\frac{11}{12}$

(i) $y=1, w=12 \rightarrow \frac{616}{112}=\frac{66}{12}$ ✓✓

1. Dr. S.S. Kapoor, M.Sc., Ph.D.
Scientific Officer 'E', Nuclear Physics Division
Bhabha Atomic Research Centre, BOMBAY-400085.

L.H.S $=\frac{56\times11}{56\times2}$

Again fraction >1

2. Dr. Evani Komdaiah, M.Sc., Ph.D.
Reader, Tata Institute of Fundamental Research
Colaba, BOMBAY-400005.

(ii) $y=6 \rightarrow$ trivial

~~(iii) $y=9 \rightarrow$~~

3. Dr. B.D. Nag Chaudhuri
Scientific Adviser to the Minister for Defence,
Government of India, South Block, NEW DELHI

(7) $x=6, z=7, w=\frac{670y}{y+54}$

4. Dr. M.K. Pal
Saha Institute of Nuclear Physics
92, Acharya Prafulla Chandra Road, CALCUTTA-9.

(i) $y=6 \rightarrow$ trivial

(8) $x=6, z=8, w=\frac{680y}{y+54}$.

(i) $y=6 \rightarrow$ trivial

... 2/-

$\mu = 1/7 \rightarrow \dfrac{1000d + a - 7000a - 7d}{6} = \dfrac{993d - 6999a}{6} = \dfrac{331d - 2333a}{6}$ $a = 1, d = 7 \rightarrow 2$ ✗

$\mu = 2/7 \rightarrow \dfrac{1993d - 6998a}{5}$, $a = 2, d = 7 \rightarrow$ 13951 − 13996 ✗ $a = 1, d = 6 \rightarrow$ 11958 − 6998 = 4960 $\rightarrow x = 496$ ✗

$\mu = 3/7 \rightarrow \dfrac{2993d - 6997a}{4}$ $a = 1, d = 3$ 8979 − 6997 = 1982 ($\frac{1982}{2} = 991$) ✗

~~$\mu = 4/7$~~ $a = 3, d = 7 \rightarrow$ 20957 − 20991 ✗

$\mu = 4/7 \rightarrow \dfrac{3993d - 6996a}{3} = 1331d - 2332a$ $a = 1, d = 2 \rightarrow$ 2662 − 2332 = 330 $\frac{9324}{4}$ 2331

$10x = 330$

$x = 33$ $\rightarrow$ 1332 ✓ Multiply by 7 & divide by 4.

Case of $\mu = 7/4 \rightarrow$ a & d interchange.

$\mu = 5/7 \rightarrow \dfrac{4993d - 6995a}{2}$ $a = 3, d = 5$ 24965 − 20985 = 3980 ($\frac{3980}{2}$) ✗ ~~2040~~ $x = 1990$ $x = 199$ ✗

$\mu = 6/7 \rightarrow \dfrac{5993d - 6994a}{1}$ $d = 4, a = 3 \rightarrow$ 23972 − 20982 = 2990 $\rightarrow x = 299$ ✗

(8) $\mu = 8 \rightarrow \dfrac{8000d + 8a - 1000a - d}{-7} = \dfrac{7999d - 992a}{-7}$ ✗

$\mu = 1/8$ ✗ ; $\mu = 3/8 \rightarrow \dfrac{2992d - 7997a}{5}$ $a = 2, d = 7 \rightarrow$ 20944 − 15994 = 4950 ($\frac{4950}{5} = 990$)

leading to $x = 99$

ie 2997 ✓ Multiply by 8 and divide by 3

999×8 = 7992 7 7

4, 9

$\mu = 5/8 \rightarrow \dfrac{4992d - 7995a}{3}$, $d = 5, a = 2 \rightarrow$ 24960 − 15990 = 8970 ($\frac{8970}{3} = 2990$) $x = 299$ ✗ 35919 35984

$\mu = 7/8 \rightarrow \dfrac{6992d - 7993a}{}$ $a = 2, d = 3$ 20996 − 15986 = 5010 $x = 501$ ✗

15928

(9) $\mu = 2/9 \rightarrow \dfrac{1991d - 8998a}{7}$ $a = 8, a = 1 \rightarrow$ 15928 − 8998 = 6930 ($\frac{6930}{7} = 990$) ie $x = 99$. 999 8991

$\mu = 4/9 \rightarrow \dfrac{3991d - 8996a}{5}$ ✗ $\rightarrow$ 1998 ✓ multiply by 9 & divide by 2

$\mu = 5/9 \rightarrow \dfrac{4991d - 8995a}{4}$ $d = 9, a = 5$ ✗

$\mu = 7/9 \rightarrow \dfrac{6991d - 8993a}{2}$ 44919 − 44975 ✗ 62919 − 62951 ✗ $d = 9, a = 7 \rightarrow$ ✗ 8, 9

$\mu = 8/9 \rightarrow 7991d - 8992a$ $d = 2, a = 1 \rightarrow 15992 - 8992 = 7000 \rightarrow x = 700$ ✗

So we have found just four cases viz

(1) $4995 \times \frac{6}{5} = 5994$

(2) $1332 \times \frac{7}{4} = 2331$

(3) $2997 \times \frac{8}{3} = 7992$

(4) $1998 \times \frac{9}{2} = 8991$

These cases are also true for the five digit case

$\frac{49995 \times 6}{5} = 9999 \times 6 = 59994$

$\frac{13332 \times 7}{4} = 3333 \times 7 = 23331$

$\frac{29997 \times 8}{3} = 9999 \times 8 = 79992$

$\frac{1998 \times 9}{2} = 999 \times 9 = 89991$

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(1) Now $x=4, z=1, w=\frac{410y}{y+36}$

(i) $y=4 \rightarrow$ trivial

(ii) $y=5 \rightarrow w=50 \rightarrow \frac{451}{550}=\frac{41}{50}$ ✓

(iii) $y=9 \rightarrow w=82 \rightarrow \frac{491}{982}=\frac{41}{82}$ ✓

only 3 cases

(2) $x=4, z=2, w=\frac{420y}{y+36}$

(i) $y=4 \rightarrow$ trivial

(ii) $y=6 \rightarrow w=60 \rightarrow \frac{462}{660}=\frac{42}{60}$ ✓

(iii) $y=9 \rightarrow w=84 \rightarrow \frac{492}{984}=\frac{42}{84}$ ✓

(3) $x=4, z=3, w=\frac{430y}{y+36}$

(i) $y=4 \rightarrow$ trivial

(ii) $y=7 \rightarrow w=70 \rightarrow \frac{473}{770}=\frac{43}{70}$ ✓

(iii) $y=9 \rightarrow 86 \rightarrow \frac{493}{986}=\frac{43}{86}$ ✓

(4) $x=4, z=4, w=440y/(y+36)$

(i) $y=4 \rightarrow$ trivial

(ii) $y=8 \rightarrow w=80 \rightarrow \frac{484}{880}=\frac{44}{80}$ ✓

(iii) $y=9 \rightarrow w=88 \rightarrow \frac{494}{988}=\frac{44}{88}$ ✓

(5) $x=4, z=5, w=450y/(y+36)$

(i) $y=4 \rightarrow$ trivial

(ii) $y=9 \rightarrow w=90 \rightarrow \frac{495}{990}=\frac{45}{90}$ ✓

(6) $x=4, z=6, w=460y/(y+36)$

(i) $y=4 \rightarrow$ trivial

(ii) $y=9 \rightarrow w=92 \rightarrow \frac{496}{992}=\frac{46}{92}$ ✓

(7) $x=4, z=7, w=470y/(y+36)$

(i) $y=4 \rightarrow$ trivial

(ii) $y=9 \rightarrow w=94 \rightarrow \frac{497}{994}=\frac{47}{94}$ ✓

(8) $x=4, z=8, w=480y/(y+36)$

(i) $y=4 \rightarrow$ trivial

(ii) $y=9 \rightarrow 96 \rightarrow \frac{498}{996}=\frac{48}{96}$ ✓

(9) $x=4, z=9, w=490y/(y+36)$

(i) $y=4 \rightarrow$ trivial

(ii) $y=6 \rightarrow w=70 \rightarrow \frac{469}{670}=\frac{49}{70}$ ✓

$\left(\text{L.H.S}=\frac{67\times 7}{67\times 10}\right)$

(iii) $y=9 \rightarrow w=98 \rightarrow \frac{498}{998}=\frac{49}{98}$ ✓ Entry 49/98

$=4/8$

In general we have $(499-99)/(999-98)=\frac{49}{98}$

(8') Now $x=5$.

(i) $y=9 \rightarrow w=85 \rightarrow \frac{591}{985}=\frac{51}{85}$

$\left(\text{L.H.S}=\frac{197\times 3}{197\times 5}\right)$

(1) $x=5, z=1, w=\frac{510y}{y+45}$

(i) $y=6 \rightarrow$ trivial $w=60 \rightarrow \frac{561}{660}=\frac{51}{60}$ ✓

(ii) $y=5 \rightarrow$ trivial

(2) $x=5, z=2, w=\frac{520y}{y+45}$

(i) $y=5 \rightarrow$ trivial

(ii) $y=7 \rightarrow w=70 \rightarrow \frac{572}{770}=\frac{52}{70}$ ✓

(3) $x=5, z=3, w=\frac{530y}{y+45}$

(i) $y=5 \rightarrow$ trivial

(ii) $y=8 \rightarrow w=80 \rightarrow \frac{583}{880}=\frac{53}{80}$ ✓

(4) $x=5, z=4, w=\frac{540y}{y+45}$

(i) $y=5 \rightarrow$ trivial

(ii) $y=6 \rightarrow w=60 \rightarrow \frac{564}{660}=\frac{54}{60}$ ✓

$\left(\text{L.H.S}=\frac{66\times 9}{66\times 10}\right)$

(iii) $y=9 \rightarrow w=90 \rightarrow \frac{594}{990}=\frac{54}{90}$ ✓

(5) $x=5, z=5, w=\frac{550y}{y+45}$

(i) $y=5 \times$ only one case

(6) $x=5, z=6, w=\frac{560y}{y+45}$

(i) $y=3, w=35 \rightarrow \frac{536}{335}=\frac{56}{35}$ ✓

$\left(\text{L.H.S}=\frac{67\times 8}{67\times 5}\right)$

1st time that fraction >1

(ii) $y=5 \rightarrow$ trivial

(7) $x=5, z=7, w=\frac{570y}{y+45}$

(i) $y=5 \rightarrow$ trivial

(ii) $y=9 \rightarrow w=95 \rightarrow \frac{597}{995}=\frac{57}{95}$ ✓

$\left(\text{L.H.S}=\frac{199\times 3}{199\times 5}\right)$

(8) $x=5, z=8, w=\frac{580y}{y+45}$

(i) $y=5 \rightarrow$ trivial

(9) $x=5, z=9, w=\frac{590y}{y+45}$

(i) $y=5 \rightarrow$ trivial

-o-o-o-

[P.T.O

Similarly the same cases can be extended to any number of digits also for eg:

$\frac{499995\times6}{5} = 99999\times6 = 599994$, etc. — Anyway rather trivial extn.

The repeated digits in the middle for the 4 digit case are 33 & 99. Why not other digits?

$a\,11d \times \frac{p}{q} = q(d\,11a)$; $10x(1-\mu) = \mu(1000d+a)-(1000a+d)$

(1) If $x=11$, $110(1-\mu) =$ " ie $110 = \mu(1000d+a+110)-(1000a+d)$ ✗

666

$\frac{4662}{2664} = \frac{518}{296} = \frac{259}{148} = \frac{7\times37}{4\times37}$ $\frac{5661}{1665} = \frac{629}{185}$ ✗; ~~3663~~ $\frac{4113}{3114} = \frac{457}{349}$ ✗ $\frac{4773}{3774} = \frac{1591}{1258}$ ✗

4 $\frac{4665}{5664}$ ✗; $\frac{2887}{7882}$ = ✗ Why trouble about four digit cases, which have all been exhaustively treated & only 4 have been found? ($4662 = \frac{2664\times7}{4}$; $\frac{4662}{2664} = \frac{7}{4}$

518 no See p. 26

16/12/73: Let us recapitulate results on pp. 6 onwards re. reversal of digits for the 3 digit case viz

$\frac{264\times7}{4} = 462$; $\frac{132\times7}{4} = 231$; $\frac{396\times7}{4} = 693$.

$\frac{495\times6}{5} = 594$;

As in the digit transfer case, is the repetition from the 3 to the 6 digit case possible?

for eg. ~~$\frac{462462\times7}{4}$~~, $\frac{264264\times7}{4} = 66066\times7 = 462462$

It is not ~~used~~ a reversal of 1st & last digits only but a complete reversal of the ~~digit~~ number itself. Quite a new number curio, rather unexpected

$\frac{132132\times7}{4} = 33033\times7 = 231231$

& similarly $\frac{396396\times7}{4} = 99099\times7 = 693693$

& $\frac{495495\times6}{5} = 99099\times6 = 594594$

Complete reversal of digits.

For the 3-digit case, we have found on p. 7. eight other cases leading to a circular permutation for eg: 162, 243, 324, 405, 486, 567, 648, 729 all of which give $\times\frac{4}{3}$ → 216, 324, 432, 540, 648, 756, 864, 972

Suppose we write 162162 & take $\frac{162162\times4}{3} = 54054\times4 = 216216$

This amounts to transferring the last digit to the first place (a converse of Dudeney's transfer) — Another new number curio, obviously holding for all the above 8 cases

Suppose we take $\frac{162216\times4}{3} = \cancel{43072}\ 54072$ ✗

" $\frac{162243\times4}{3} = 54081\times4 = 216324$ ✗

" $\frac{132264\times7}{4} = 33066\times7 = 231462$

280×8 / 2

280×6

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$x = 2, z = 8, \quad w = \frac{280y}{y+18}$

(i) $y = 2 \longrightarrow$ trivial case ✗

(ii) $y = 3 \longrightarrow w = 40 \longrightarrow \frac{238}{340} = \frac{28}{40}$ ✓

(iii) $y = 6 \longrightarrow w = 70 \longrightarrow \frac{268}{670} = \frac{28}{70}$ ✓

$\left(L.H.S = \frac{67\times4}{67\times10}\right)$

(iv) only 3 cases

$x = 2, z = 9, \quad w = \frac{290y}{y+18}$

(i) $y = 2 \longrightarrow$ trivial case ✗

(ii) no other cases

This finishes $x = 2$

$x = 3, z = 1, \quad w = \frac{310y}{y+27}$

(i) $y = 3 \longrightarrow w = 31 \longrightarrow \frac{331}{331}$ trivial ✗

(ii) $y = 4 \longrightarrow w = 40 \longrightarrow \frac{341}{340} = \frac{31}{40}$ ✓

(iii) only 2 cases.

$x = 3, z = 2, \quad w = \frac{320y}{y+27}$

(i) $y = 3 \longrightarrow w = 32 \longrightarrow \frac{332}{332}$ ✗

(ii) $y = 5 \longrightarrow w = 50 \longrightarrow \frac{352}{550} = \frac{32}{50}$ ✓

(iii) $y = 9 \longrightarrow w = 80 \longrightarrow \frac{392}{980} = \frac{32}{80}$ ✓

$\left(L.H.S = \frac{196\times2}{196\times5}\right)$

only 3 cases!

$x = 3, z = 3, \quad w = \frac{330y}{y+27}$

(i) $y = 3, w = 33 \longrightarrow \frac{333}{333}$ ✗

(ii) $y = 6, w = 60 \longrightarrow \frac{363}{660} = \frac{33}{60}$ ✓

(iii) $y = 9, w =$ only 2 cases.

$x = 3, z = 4, \quad w = \frac{340y}{y+27}$

(i) $y = 3 \longrightarrow$ trivial case

(ii) $y = 7 \longrightarrow w = 70 \longrightarrow \frac{374}{770} = \frac{34}{70}$ ✓

$x = 3, z = 4,$

(iii) $y = 9 \longrightarrow w = 85 \longrightarrow \frac{394}{985} = \frac{34}{85}$ ✓

$\left(L.H.S = \frac{197\times2}{197\times5}\right)$

only 3 cases!

$x = 3, z = 5, \quad w = \frac{350y}{y+27}$

(i) $y = 3 \longrightarrow$ trivial case ✗

(ii) $y = 8 \longrightarrow w = 80 \longrightarrow \frac{385}{880} = \frac{35}{80}$ ✓

only 2 cases.

$x = 3, z = 6, \quad w = \frac{360y}{y+27}$

(i) $y = 3 \longrightarrow$ trivial ✗

(ii) $y = 9 \longrightarrow w = 90 \longrightarrow \frac{396}{990} = \frac{36}{90}$ ✓

only 2 cases

$x = 3, z = 7, \quad w = \frac{370y}{y+27}$

(i) $y = 3 \longrightarrow$ trivial

no other cases.

$x = 3, z = 8, \quad w = \frac{380y}{y+27}$

(i) $y = 3 \longrightarrow$ trivial

(ii) $y = 9 \longrightarrow w = 95 \longrightarrow \frac{398}{995} = \frac{38}{95}$ ✓

$\left(L.H.S = \frac{199\times2}{199\times5}\right)$

only 2 cases.

$x = 3, z = 9, \quad w = \frac{390y}{y+27}$

(i) $y = 3 \longrightarrow$ trivial

(ii) no other cases

... 2.

P.T.O

On p. 25. we have found an extension of the 4-digit transfer case an extension to 5 and 6 & general case. Suppose we take for eg:

$$\frac{49954995 \times 6}{5} = 9990999 \times 6 = \cancel{59945994}$$
$$= 59945994 \quad \text{(8 digit case)}$$

ie not a transfer of ~~1st digit to the last~~ ~~transfer~~ ~~reversal~~ of 1st & last digits, but a complete reversal, analogous to the extn from 3 to 6 digits viz 132132 → 231231.

In fact for the cases of 3 digit & 4 digit cases we have found, the interchange of 1st & last digits are only complete reversals.

Just as we have extended 4995 to 49995, etc on top of p. 25, can we extend from the 3 digit case to 4, 5, 6, .. cases?

These 3 digit cases are (also 4 in number)

$$\frac{132 \times 7}{4}, \frac{264 \times 7}{4}, \frac{396 \times 7}{4}, \frac{495 \times 6}{5} \rightarrow 231, 462, 693, 594$$

Now $\frac{1332 \times 7}{4} = 333 \times 7 = 2331$ ✓ $\frac{2664 \times 7}{4} = 666 \times 7 = 4662$ ✓

1332 is the case already noticed & the above 2664 (not noticed on p. 24) is merely $1332 \times 2 = 2664$ (α). Since we can have $1332 \times 3 = 3996$ (β) (also not noticed on p. 24).

Again 264 → 2664 is the above case (α)
396 → 3996 " (β)
495 → 4995 is noticed on p. 24.

But have ~~we have~~ no analogues of 2997 and 1998 in the 3-digit case? yes

Is $\frac{297 \times 8}{3} = 792$; yes $99 \times 8 = 792$; $\frac{198 \times 9}{2} = 99 \times 9 = 891$.

These also have been noticed on p. 5 itself.

Thus the really distinct 3-digit cases are 132, (including 264 & 396), 198, 297 and 495 and the four cases for the 4-digit case found on p. 24 (after colossal labour) are merely simple extensions of these.

So before we undertake the 5 & 6-digit cases for ~~transfer~~ interchange of 1st & last digits; we must be prepared to meet the possibility of mere extensions of the

above 4 cases.

We can summarise our results obtained so far :-

(1) Case of transfer of first digit to last place

(a) 3 digit case :- The only possible cases are (as on p. 2) eight apparently

162, 243, 324, 405, 486, 567, 648 and 729
↕ ↕ ↕ ↕ ↕ ↕ ↕ ↕
216 432 243 540 648 756 864 972

but really they amount to (162, 243, 405, 567) only four, since

$324 = 2 \cdot 162$; $486 = 3 \cdot 162$; $648 = 4 \cdot 162$, $729 = 3 \cdot 243$

and all these correspond to multiplication by 4 and division by 3, or the multiplication by 3 & division by 4 in the reverse case.

(b) 4 digit case : The only possible cases are

(4/7) or (7/4)
1212 ⟷ 2121
2424 ⟷ 4242
3636 ⟷ 6363
4848 ⟷ 8484
(2/9) — 1818 ⟷ 8181
(3/8) — 2727 ⟷ 7272
(5/6) — 4545 ⟷ 5454

or again four cases really

Because of symmetry, the transfer to 1st digit to last place actually amounts to a complete reversal of the digits.

(c) 5 digit case : Here again there are four cases.

12195, 24390, 36585, 48780
↕ ↕ ↕ ↕
21951 43902 65853 87804.

& surprisingly enough all these correspond to only one case (9/5) or (5/9)

(d) 6 digit case : Here we have found a large number of cases which we can however classify under several heads.

(i) Case of 2 digits repeated thrice

(7/4) $\begin{pmatrix} 121212 \\ 212121 \end{pmatrix}$; $\begin{pmatrix} 181818 \\ 818181 \end{pmatrix}$ (9/2); (7/4) $\begin{pmatrix} 242424 \\ 424242 \end{pmatrix}$; (7/4) $\begin{pmatrix} 363636 \\ 636363 \end{pmatrix}$

(8/3) $\begin{pmatrix} 272727 \\ 727272 \end{pmatrix}$; (6/5) $\begin{pmatrix} 454545 \\ 545454 \end{pmatrix}$; (7/4) $\begin{pmatrix} 484848 \\ 848484 \end{pmatrix}$

This again boils down to only four cases corresponding to (7/4), (6/5), (8/3) & (9/2).

(ii) ~~(iii)~~ Cases of 3 digits repeated twice:

These correspond to repetitions of the 8 cases in (a) twice ie

$(3/4)$ $\left(\begin{matrix}216216, 324324, 432432, 540540, 648648, 756756, 864864\\ \text{and } 972972\end{matrix}\right)$ [except $405405 \times 4/3$; Vide Below]

As remarked in (a) these amount to really four independent cases.

(iii) ~~(iv)~~ All digits different.

(α) Digits being 1, 2, 4, 5, 7, 8:

$(3/1) \times \begin{pmatrix}142857\\428571\end{pmatrix}$; ~~$(5/1)$~~ ~~$\begin{pmatrix}142857\\714285\end{pmatrix}$~~ $\begin{pmatrix}285714\\857142\end{pmatrix} * (3/1)$

$(2/3)\begin{pmatrix}428571\\285714\end{pmatrix}$; $(5/4)\begin{pmatrix}571428\\714285\end{pmatrix}$ (Dudeney's case).

$(1/5)\begin{pmatrix}714285\\142857\end{pmatrix}$ – ie 5 distinct cases (or 10 if reciprocals be taken)

(β) Digits being 0, 1, 2, 4, 5, 6.

$(1/4)\begin{pmatrix}410256\\102564\end{pmatrix}$ – only one case.

17/12/23 (γ) Digits being 1, 3, 4, 5, 6, 8:

$(7/2)\begin{pmatrix}153846\\538461\end{pmatrix}$; $\begin{pmatrix}461538\\615384\end{pmatrix}(4/3)$; $(5/7)\begin{pmatrix}538461\\384615\end{pmatrix}$

– ie 3 cases.

(δ) Digits being 0, 2, 3, 6, 7, 9

~~(4)~~ $(4/3)\begin{pmatrix}230769\\307692\end{pmatrix}$; $(4/3)\begin{pmatrix}692307\\923076\end{pmatrix}$

– ie 2 cases. [The 6-digit case does not consist of merely repetitions as in (i) & (ii), but also other cases as in (iii)]

~~(ε) (ζ) 7 digit cases – we have so far not obtained any result regarding this case. Perhaps we might find only cases of (5/4) as in the 5-digit case.~~

~~(η) 8 digit case –~~

(ε) 7 digit case – No result obtained

(f) 8 digit case – We can take the two-digit cases & repeat them four times foreg. which really amounts to the double repetitions of the 4-digit cases viz. [Vide 1(a)]

12121212, 18181818, ~~2727, 4545~~ 27272727 & 45454545

There appears nothing extraordinary about these.

(g) 9 digit cases – we have obtained repetitions of the 3-digit cases thrice viz

[vide (a) above]

162162162, 243243243, 405405405 & 567567567

Again nothing extraordinary

(h) 10 digit case – (i) Double repetition of the four 5 digit cases in (c) above.

(ii) Five-fold repetition of the 2-digit cases

1212121212, 1818181818, 2727272727, 4545454545, 4848484848

Again rather trivial

(i) 11 digit case – ~~not~~ no results obtained

(j) 12 digits – Cases of 6 × 2, 4 × 3, 3 × 4, and 2 × 6

Corresponding to the last case we have the double Dudeney number

viz 571428571428 which leads after (5/4) to
714285714285

Similarly 230769230769 → 307692307092 (4/3)

and 153846153846 → 538461538461. (7/2)

(2) Case of interchange of first and last digit. –

(a) 3 digit case. – The cases obtained on pp. 4 – 6 are

$\frac{27}{72}$

$\frac{891}{198} = \frac{81}{18}$

132, 264, 396, 495, 198, 297

i.e. really 132, 198, 297 and 495 (four cases)
↓ ↓ ↓ ↓
231 891 792 594

Corresponding to $(\frac{7}{4})$, $(9/2)$ $(8/3)$ & $(6/5)$.

(b) 4 digit cases.

we have obtained only the four cases (p. 24) viz

4995 = 5 × 5 × 333
1332 = 4 × 333
2997 = 9 × 333
1998 = 6 × 333

All factors of 111

$$\left\{\begin{array}{l} (4995) \times \frac{6}{5} = 5994 \\ (1332) \times 7/4 = 2331 \\ (2997) \times 8/3 = 7992 \\ (1998) \times 9/2 = 8991 \end{array}\right\}$$

Note the analogy with the four cases (a) above

It is also to be noted that all these four cases are actually cases of complete reversal not merely exchange of 1st & last digits. This is because of repeated middle digits. But we find in Trohlichstein two such cases ~~of this sort~~ also of complete reversal but not with repeated digits in the middle ~~(i) 2178 × 4 =~~

viz (i) $2178 \times 4 = 8712$ & (ii) $1089 \times 9 = 9801$. —— (F)

It is appears as if in this case of 4 digits interchange of 1st & last digits is equivalent to complete reversal. This has to be investigated specially in view of (F) above.

(c) 5 digit cases. – just as we have obtained by analogy (b) from (a), we find here the four cases 49995, 13332, 29997, and 19998. (leading to complete reversal)

(d) 6 digit cases

(i) The extensions 133332, 199998, 299997 and 499995 (– do –)

(ii) Repetition of 3 digit cases in (a) viz:

132132, 198198, 297297, 495495 —— (– do –)

(e) 7 digit cases i.e. 1333332, 1999992, 2999997 & 4999995.

—— (– do –)

No other cases

(f) 8 digit cases (i) The above 13 ... 2, etc

(ii) The repetition of 4 digit cases viz

i.e. 13321332, 19981998, etc. } —— (do.)

—— (do)

(g) Similar results for 9, 10, & 12 digit cases.

(3) Transfer of last digit to first place (converse of Dudeney's problem)

(i) On p. 7 we have found for the 3 digit case, we have found (cf. see p. 25) four essentially distinct cases viz: 162, 243, 405, ~~405~~ & 567 which after multiplication by 4 & division by 3 give 216, 324, 540 & 756 in where last digit is transferred to the first place.

(ii) In Trohlichstein's book we have an example of this type of a 28 digit number 1,034,482,758,620,689,655,172,413,793 which on multiplication by 3 takes the last digit 3 to the 1st place giving

3,103,448,275,862,068,965,517,241,379.

on p. 132 of NBBook II, this has been discussed & found that this is equivalent to finding x, d & λ (d = last digit) in

$$\frac{x}{d} = \frac{10^{27}-\lambda}{10\lambda - 1} \qquad (\lambda = p/q) \quad \text{—— (}\alpha\text{)}$$

analogous to $\frac{x}{a} = \frac{10^{27}-\mu}{10\mu - 1}$ $(\mu = q/p)$ —— (β)

for the 1st digit transfer to the last.

Putting $\lambda = 3$ in (α) & driving the numerator $10^{27}-3$ by $30-1 = 29$ we find a 26-digit number for x/d & with $d = 3$ we get a 27-digit number & finally get $x3$ = the no. in Frohlichstein's book.

(iii) We have not made any systematic search in the case (α) as has been done for (β) for the case of any number of digits 3, or 4 or 5 & so on.

20/12/73 — The above finishes summary of what has been done so far. Many more things remain to be done for eg: $\bullet(\alpha_1)$ a more satisfactory search of illegal cancellations for higher digits & obtaining general results; (α_2) More satisfactory & general results for the cases (1) and (3) of above summary, and (α_3) similar investigation of interchange of 1st & Last digits & reversal of digits dealt with in case (2) of the Summary above.

(α_3) Frohlichstein's examples: $2178 \times 4 = 8712$ & $1089 \times 9 = 9801$

ie $abcd \longrightarrow dcba$. by operation (p/q) $\qquad (1 \leqq p, q \leqq 9)$

$$(1000a + 100b + 10c + d)p = (1000d + 100c + 10b + a)q$$

$$a(1000p - q) + b(100p - 10q) + c(10p - 100q) + d(p - 1000q) = 0$$

$$a(1000p - q) + b(100p - 10q) = c(100q - 10p) + d(1000q - p)$$

$$a(1000p - q) + 10b(10p - q) = 10c(10q - p) + d(1000q - p)$$

$$a(1000\lambda - 1) + 10b(10\lambda - 1) = 10c(10 - \lambda) + d(1000 - \lambda) \qquad (\lambda = p/q)$$

$$\frac{p}{q} = \frac{1000d + 100c + 10b + a}{1000a + 100b + 10c + d}$$

$$\frac{p-q}{p+q} = \frac{999d + 90c - 90b - 999a}{1001d + 110c + 110b + 1001a} = \frac{9(111d + 10c - 10b - 111a)}{11(91d + 10c + 10b + 91a)}$$

$$\frac{11}{9} \cdot \frac{p-q}{p+q} = \frac{111(d-a) + 10(c-b)}{91(d+a) + 10(c+b)}$$

$$\frac{10p - q}{10q - p} = \frac{222d + 20c - 111d + 10c}{222a + 20b - 111a + 10b} = \frac{10\lambda - 1}{10 - \lambda} = \frac{202d - 20a + 20c}{202a - 20d + 20b}$$

$\frac{11p - 11q}{9p + 9q}$.

$\frac{20p - 2q}{20q - 2p}$

$abcd$
$dcba$

$\frac{111d+10c}{111a+10b} = \frac{10\lambda-1}{10-\lambda}$. $\lambda=4 \rightarrow$ R.H.S $= \frac{39}{6} = \frac{13}{2}$

$222d+20c = 1443a+130b$ $\quad a=2, d=8 \rightarrow 1776+20c = 2886+130b$

$130b-20c = -1110$ $\quad$ i.e. $13b-2c = 111$

$2c-13b = 111$ (Something wrong somewhere)

$\frac{10p-q}{10q-p} = \frac{101d-10a+10c}{101a-10d+10b} = \frac{10\lambda-1}{10-\lambda}$ ——— (1)

202
80
122

with $\lambda=4, a=2, d=8 \rightarrow \frac{808-20+10c}{202-80+10b} = \frac{13}{2} = \frac{788+10c}{122+10b}$

$1576+20c = 1586+130b$ $\quad 20c-130b = 10$

satisfied by $c=7, b=1$.

$\lambda=9, a=1, d=9 \rightarrow \frac{909-10+10c}{101-90+10b} = \frac{89}{T}$. $\frac{899+10c}{11+10b} = 89$

$899+10c = 979+890b$ $\quad 10c-890b = 80$

$c-89b = 8$ satisfied by $b=0, c=8$

This is mere verification.

$(101d-10a+10c)(10-\lambda) = (10\lambda-1)(101a-10d+10b)$

$\lambda\{10(101a-10d+10b)+(101d-10a+10c)\}$

$= \{10(101d-10a+10c)+\{101a-10d+10b)\}$

This will take me back to the original eqn

$= \lambda\{10a$ Stupid idea!

Suppose we give arbitrary values to a, ~~and~~ b and c., then (1) gives relⁿ between d & λ.

(i) $a=1, b=2, c=3$, (1) $\rightarrow \frac{101d-10+30}{101-10d+20} = \frac{10\lambda-1}{10-\lambda}$

i.e. $\frac{101d+20}{121-10d} = \frac{10\lambda-1}{10-\lambda}$. Put d as a fn of λ.

$(10-\lambda)(101d+20) = (10\lambda-1)(121-10d)$

$101d(10-\lambda)+10d(10\lambda-1) = 121(10\lambda-1)-20(10-\lambda)$

$d\{101(10-\lambda)+10(10\lambda-1)\} =$

1010
1210
20

$d\{1010-101\lambda+100\lambda-10\} = 1210\lambda-121-200+20\lambda$

$d\{1000-\lambda\} = 1230\lambda-321.$

$d = \frac{1230\lambda-321}{1000-\lambda}$

1230
321
909

$$d = \frac{1230\lambda - 321}{1000 - \lambda} \cdot * \ell$$

2460
321
2139
998

$= \frac{3 \times 713}{2 \times 499}$ ×

3690
321
3369
997 ×

4920
321
4599
996

(i) $\lambda = 1$, $d = \frac{909}{999}$ × (ii) $\lambda = 2$ ×, (iii) $\lambda = 3$ ×, (iv) $\lambda = 4$ ×

$\lambda = 5$ ×, $\lambda = 6$ ×, $\lambda = 7$ ×, $\lambda = 8$ ×, $\lambda = 9$ ×

8610
321
8289
993

1533
332 ×

2763 ×
331

11070
321
10749
991) 10749 (1
991
839

6150
321
5829
995 ×

$\lambda = 1/2 \to$ ×, $\lambda =$ ~~fraction~~

123

1000
200
30

615 · 0
321 1999
294

$123d \to d321.$

$(1230 + d) \to 1000d + 321.$

1230
321
909

$909 + d \to 1000d$

Take $a = 2$, $b = 1$, $c = 7$. Eqn (1) ie $\frac{101d - 10a + 10c}{101a - 10d + 10b} = \frac{10\lambda - 1}{10 - \lambda}$ becomes

$$\frac{101d - 20 + 70}{202 - 10d + 10} = \frac{101d + 50}{212 - 10d} = \frac{10\lambda - 1}{10 - \lambda}$$

$$(10 - \lambda)(101d + 50) = (10\lambda - 1)(212 - 10d)$$

$$d\{101(10 - \lambda) + 10(10\lambda - 1)\} = 212(10\lambda - 1) - 50(10 - \lambda)$$

$$d\{1010 - 101\lambda + 100\lambda - 10\} = \{2120\lambda - 212 - 500 + 50\lambda\}$$

2120
50
2170

10 abc
100a + 10b + c
1000a + 100b + c.

$$d\{1000 - \lambda\} = (2170\lambda - 712)$$

$$d = \frac{2170\lambda - 712}{1000 - \lambda} \quad (**)$$

Cf (*) & (**) we find on the numerator on the R.H.S. $100abc\lambda - cba$

& in the denominator $1000 - \lambda$

In (**) $\lambda = 4 \to$ ~~$d = \frac{19530 - 712}{991} = \frac{18818}{991}$ 991)~~

$$d = \frac{8680 - 712}{996} = \frac{7968}{996} = \frac{1992}{249} = \frac{664}{83} = \underline{\underline{8}}:$$

Suppose we take $abc = 128$, $d = \frac{1280\lambda - 821}{1000 - \lambda}$ ×

$abc = 208$, $d = \frac{2080\lambda - 802}{1000 - \lambda}$ ×

Suppose we express λ in terms of $\underline{d}$. (Then we can take $d = 1, \ldots 9$)

11520
821
10699
991 > 1 λ

9160
802
3358
998

8320
802
7518
996

7968

14560
802
13758
9

6240
802
5438
997

$$\frac{10\lambda-1}{10-\lambda} = \frac{101d-10a+10c}{101a-10d+10b}.$$

$$(10-\lambda)(101d-10a+10c) = (10\lambda-1)(101a-10d+10b)$$

$$d\{101(10-\lambda)+10(10\lambda-1)\} = (10-\lambda)(10a-10c)+(10\lambda-1)(101a+10b)$$

$$d\{1000-\lambda) = \lambda\{10(101a+10b)-(10a-10c)\} - \{101a+10b)+10(10a-10c)$$

$$= \lambda\{1000a+100b+10c\} - 100c - 10b - a$$

$$d = \frac{\lambda(10abc)-(cba)}{1000-\lambda}.$$ quite an interesting formula

Suppose we express λ in terms of d.

$$1000d - \lambda d = \lambda(10abc) - (cba)$$

$$\lambda(10abc+d) = 1000d + (cba)$$

$$\lambda = \frac{1000d+(cba)}{10(abc)+d} \quad (***) \quad = \frac{(dcba)}{(abcd)}$$ (merely a restatement of the original problem)

~~abcd = 2178~~
~~dcba = 8712~~ (i) $abc = 123$, $\lambda = \dfrac{1000d+321}{1230+d}$

$d=1 \longrightarrow \lambda = \dfrac{1321}{1231}$ × since $\lambda = p/q$ $(1 \le p, q \le 9)$

$d=2 \longrightarrow \lambda = \dfrac{2321}{1232} = \dfrac{211}{112}$ ×

$d=3 \longrightarrow \lambda = \dfrac{3321}{1233} = \dfrac{369}{137}$ ×

{ In fact (***) is unnecessary, we simply write 1231, 1232, 1233, 1234, —
& write reverse in denominator

ie no need for all the nonsense used on pp. 28′ — 29′.

$\dfrac{1234}{4321}$ ×, $\dfrac{1235}{5321}$ ×, $\dfrac{1236}{6321} = \dfrac{412}{2107}$ ×, $\dfrac{1237}{7321}$ ×, $\dfrac{1238}{8321}$ × $\dfrac{1239}{9321} = \dfrac{413}{3107}$ ×

$abc = 123$ ×, 124. $\dfrac{1421}{1241}$, $\dfrac{2421}{1242} = \dfrac{269}{138}$ ×, $\dfrac{3421}{1243} = \dfrac{311}{113}$ ×, $\dfrac{5421}{1245}$ × $\dfrac{6421}{1246}$; $\dfrac{7421}{1247}$; $\dfrac{8421}{1248}$, $\dfrac{9421}{1249}$ ×

~~with a=1 seems difficult~~

(125) → $\dfrac{1521}{1251}$, $\dfrac{2521}{1252}$, $\dfrac{3521}{1253}$, $\dfrac{4521}{1254}$, $\dfrac{6521}{1256}$, $\dfrac{7521}{1257}$, $\dfrac{8521}{1258}$, $\dfrac{9521}{1259}$, $\dfrac{5521}{1255}$ ×

(126) → $\dfrac{1621}{1261}$, $\dfrac{2421}{1262}$, $\dfrac{3621}{1263}$, $\dfrac{4621}{1264}$, $\dfrac{5621}{1265}$, $\dfrac{7621}{1267}$, $\dfrac{8621}{1268}$, $\dfrac{9621}{1269}$ ×

(127) → $\dfrac{1721}{1271}$, $\dfrac{2721}{1272}$, $\dfrac{3721}{1273}$, $\dfrac{4721}{1274}$, $\dfrac{5721}{1275}$, $\dfrac{6721}{1276}$, $\dfrac{8721}{1278}$, $\dfrac{9721}{1279}$, $\dfrac{7721}{1277}$ ×

907/423 255/51 ~~214~~ 969/142

$\dfrac{611}{116}$

128 → $\frac{1821}{1281}$, $\frac{2821}{1282}$, $\frac{3821}{1283}$, $\frac{4821}{1284}$, $\frac{5821}{1285}$, $\frac{6821}{1286}$, $\frac{7821}{1287}$, $\frac{8821}{1288}$, $\frac{9821}{1289}$ | $\frac{711}{117} = \frac{79}{13}$ ✓

129 → $\frac{1921}{1291}$, $\frac{2921}{1292}$, $\frac{3921}{1293}$, $\frac{4921}{1294}$, $\frac{5921}{1295}$, $\frac{6921}{1296}$, $\frac{7921}{1297}$, $\frac{8921}{1298}$, $\frac{9921}{1299}$ | $\frac{769}{144}$ × | $\frac{811}{118}$

(9×9×9×9 cases to be tried $= \frac{81 \times 81}{81}$, $\frac{648}{6561}$ } ie 6561 cases – Is it worth it?

131 → $\frac{1131}{1311}$, $\frac{2131}{1312}$, $\frac{3131}{1313}$, $\frac{4131}{1314}$, $\frac{5131}{1315}$, $\frac{6131}{1316}$, $\frac{7131}{1317}$, $\frac{8131}{1318}$, $\frac{9131}{1319}$ | $\frac{459}{145}$ × | $\frac{37 \times 7}{259}$ 4 2 148

132 → $\frac{1231}{1321}$, $\frac{2231}{1322}$, $\frac{3231}{1323}$, $\frac{4231}{1324}$, $\frac{5231}{1325}$, $\frac{6231}{1326}$, $\frac{7231}{1327}$, $\frac{8231}{1328}$, $\frac{9231}{1329}$ | $\frac{359}{147}$ × $\frac{3077}{443}$ ×

133 → $\frac{1331}{1331}$, $\frac{2331}{1332}$, $\frac{3331}{1333}$, $\frac{4331}{1334}$, $\frac{5331}{1335}$, $\frac{6331}{1336}$, $\frac{7331}{1337}$, $\frac{8331}{1338}$, $\frac{9331}{1339}$ | $\frac{259}{148} = \frac{7}{4}$.

$2331 = \frac{7}{4}(1332)$ This case with middle digits equal is what we have already noticed – Vide p. 27'

134 → $\frac{1431}{1341}$, $\frac{2431}{1342}$, $\frac{3431}{1343}$, $\frac{4431}{1344}$, $\frac{5431}{1345}$, $\frac{6431}{1346}$, $\frac{7431}{1347}$, $\frac{8431}{1348}$, $\frac{9431}{1349}$ | $\frac{159}{149}$ × | $\frac{221}{122}$

135 → $\frac{1531}{1351}$, $\frac{2531}{1352}$, $\frac{3531}{1353}$, $\frac{4531}{1354}$, $\frac{5531}{1355}$, $\frac{6531}{1356}$, $\frac{7531}{1357}$, $\frac{8531}{1358}$, $\frac{9531}{1359}$ 1059 | $\frac{321}{123} = \frac{107}{41}$ | ✓

136 → $\frac{1631}{1361}$, $\frac{2631}{1362}$, $\frac{3631}{1363}$, $\frac{4631}{1364}$, $\frac{5631}{1365}$, $\frac{6631}{1366}$, $\frac{7631}{1367}$, $\frac{8631}{1368}$, $\frac{9631}{1369}$ | $\frac{421}{124}$ ×, $\frac{959}{152}$

137 → $\frac{1731}{1371}$, $\frac{2731}{1372}$, $\frac{3731}{1373}$, $\frac{4731}{1374}$, $\frac{5731}{1375}$, $\frac{6731}{1376}$, $\frac{7731}{1377}$, $\frac{8731}{1378}$, $\frac{9731}{1379}$ | $\frac{521}{125}$; $\frac{859}{153}$

138 → $\frac{1831}{1381}$, $\frac{2831}{1382}$, $\frac{3831}{1383}$, $\frac{4831}{1384}$, $\frac{5831}{1385}$, $\frac{6831}{1386}$, $\frac{7831}{1387}$, $\frac{8831}{1388}$, $\frac{9831}{1389}$ | $\frac{621}{126} = \frac{69}{14}$ ×

139 → $\frac{1931}{1391}$, $\frac{2931}{1392}$, $\frac{3931}{1393}$, $\frac{4931}{1394}$, $\frac{5931}{1395}$, $\frac{6931}{1396}$, $\frac{7931}{1397}$, $\frac{8931}{1398}$, $\frac{9931}{1399}$ | $\frac{659}{155}$ × $\frac{721}{127}$ ×

Before going further come back to 101, 111 etc.

101 → $\frac{2101}{1012}$, $\frac{3101}{1013}$, $\frac{4101}{1014}$, $\frac{5101}{1015}$, $\frac{6101}{1016}$, $\frac{7101}{1017}$, $\frac{8101}{1018}$, $\frac{9101}{1019}$ | $\frac{789}{113}$ ×; $\frac{191}{92}$ ✓

$\frac{1101}{1011}$

100 → $\frac{1001}{1001}$, $\frac{2001}{1002}$, $\frac{3001}{1003}$, $\frac{4001}{1004}$, $\frac{5001}{1005}$, $\frac{6001}{1006}$, $\frac{7001}{1007}$, $\frac{8001}{1008}$, $\frac{9001}{1009}$ | $\frac{889}{112} = \frac{127}{16}$ ×

102 → $\frac{1201}{1021}$, $\frac{2201}{1022}$, $\frac{3201}{1023}$, $\frac{4201}{1024}$, $\frac{5201}{1025}$, $\frac{6201}{1026}$, $\frac{7201}{1027}$, $\frac{8201}{1028}$, $\frac{9201}{1029}$ | $\frac{291}{93}$ ×, $\frac{689}{114}$ × 2 129

103 → $\frac{1301}{1031}$, $\frac{2301}{1032}$, $\frac{3301}{1033}$, $\frac{4301}{1034}$, $\frac{5301}{1035}$, $\frac{6301}{1036}$, $\frac{7301}{1037}$, $\frac{8301}{1038}$, $\frac{9301}{1039}$ | $\frac{391}{94}$ × $\frac{589}{115}$ ×

$104 \to \frac{1401}{1041}\ \frac{2401}{1042}\ \frac{3401}{1043}\ \frac{4401}{1044}\ \frac{5401}{1045}\ \frac{6401}{1046}\ \frac{7401}{1047}\ \frac{8401}{1048}\ \frac{9401}{1049} \Big| \frac{491}{95} \times \frac{489}{116} \times$

$105 \to \frac{1501}{1051}\ \frac{2501}{1052}\ \frac{3501}{1053}\ \frac{4501}{1054}\ \frac{5501}{1055}\ \frac{6501}{1056}\ \frac{7501}{1057}\ \frac{8501}{1058}\ \frac{9501}{1059} \Big| \frac{389}{117} \times \frac{591}{96} \times$

$106 \to \frac{1601}{1061}\ \frac{2601}{1062}\ \frac{3601}{1063}\ \frac{4601}{1064}\ \frac{5601}{1065}\ \frac{6601}{1066}\ \frac{7601}{1067}\ \frac{8601}{1068}\ \frac{9601}{1069} \Big| \frac{289}{118} \times \frac{691}{97} \times$

$107 \to \frac{1701}{1071}\ \frac{2701}{1072}\ \frac{3701}{1073}\ \frac{4701}{1074}\ \frac{5701}{1075}\ \frac{6701}{1076}\ \frac{7701}{1077}\ \frac{8701}{1078}\ \frac{9701}{1079} \Big| \frac{189}{119} \times \frac{791}{98} = \frac{113}{14} \times$

$108 \to \frac{1801}{1081}\ \frac{2801}{1082}\ \frac{3801}{1083}\ \frac{4801}{1084}\ \frac{5801}{1085}\ \frac{6801}{1086}\ \frac{7801}{1087}\ \frac{8801}{1088}\ \frac{9801}{1089} \Big| \frac{891}{99} = \frac{81}{9} = 9 \checkmark$

ie the Trowbridgeins case $1089 \times 9 = 9801$

$109 \to \frac{1901}{1091}\ \frac{2901}{1092}\ \frac{3901}{1093}\ \frac{4901}{1094}\ \frac{5901}{1095}\ \frac{6901}{1096}\ \frac{7901}{1097}\ \frac{8901}{1098}\ \frac{9901}{1099} \Big| \frac{989}{129} \times$

$110 \to \frac{1011}{1101}\ \frac{2011}{1102}\ \frac{3011}{1103}\ \frac{4011}{1104}\ \frac{5011}{1105}\ \frac{6011}{1106}\ \frac{7011}{1107}\ \frac{8011}{1108}\ \frac{9011}{1109} \Big| \frac{779}{123}$

$111 \to \frac{1111}{1111}\ \frac{2111}{1112}\ \frac{3111}{1113}\ \frac{4111}{1114}\ \frac{5111}{1115}\ \frac{6111}{1116}\ \frac{7111}{1117}\ \frac{8111}{1118}\ \frac{9111}{1119} \Big| \frac{679}{124} \times$

$112 \to \frac{1211}{1121}\ \frac{2211}{1122}\ \frac{3211}{1123}\ \frac{4211}{1124}\ \frac{5211}{1125}\ \frac{6211}{1126}\ \frac{7211}{1127}\ \frac{8211}{1128}\ \frac{9211}{1129} \Big| \frac{201}{102} = \frac{67}{34} \times \frac{579}{125}$

Before continuing this wild goose chase let us look at the two results 2178 and 1089. $1089 = 99 \times 11$, $2178 = 2 \times 1089 = 99 \times 22$

Since consider $1089 \times 3 = 3267$, $\frac{7623}{3267} = \frac{693}{297} = \frac{77}{33} = \frac{7}{3}$

(i) ie $3267 \times \frac{7}{3} = 7623$ ✓ A new case

Again $1089 \times 4 = 4356$ ∴ $\frac{6534}{4356} = \frac{99 \times 66}{99 \times 44} = \frac{3}{2}$

(ii) ie $4356 \times \frac{3}{2} = 6534$ ✓ A new case.

$1089 \times 5 = 5445$ ∴ here the factor is 1.

$1089 \times 6 = 6534$ & factor here is $\frac{2}{3}$ } Same cases as above reversed.

$1089 \times 7 = 7623$ ∴ " $\frac{2}{7}$ }

$1089 \times 8 = 8712$ ∴ $\frac{8712}{2178} = 4$ (Same factor as the 2178 case reversed)

$1089 \times 9 = 9801$ — same as 1089 case reversed

ie ~~we~~ in addition to the two cases in Frohlichstein, we get really other new (?) cases viz (i) & (ii) on previous page. But we have not obtained these independently. All these appear to be mere multiples of 99. viz:

$$1089 = 11 \times 99 = 9 \times 11^2$$
$$2178 = 22 \times 99 = 2 \times 9 \times 11^2$$
$$3267 = 3 \times 9 \times 11^2$$
$$4356 = 4 \times 9 \times 11^2$$
$$5445 = 5 \times 9 \times 11^2$$
$$6534 = 6 \times 9 \times 11^2$$
$$7623 = 7 \times 9 \times 11^2$$
$$8712 = 8 \times 9 \times 11^2$$
$$9801 = 9^2 \times 11^2.$$

I wonder if 111 and 999 could achieve similar results

$$\begin{array}{r} 999 \times 111 \\ \hline 999 \\ 999 \\ 999 \\ \hline 110889 \end{array} \qquad \begin{array}{r} 110889 \times 9 \\ \hline 998001 \end{array}$$

Does not work

$$\begin{array}{r} 111 \times 99 \\ \hline 999 \\ 999 \\ \hline 10989 \end{array} \qquad \begin{array}{r} 10989 \times 9 \\ \hline 98901 \end{array}$$

~~Also does not work~~

22222

works all right – Excellent

on p. (27') 111

So we get

$$10989 \times 9 = 98901.$$
$$21978 \times 4 = 87912$$
$$32967 \times (7/3) = 76923$$
$$43956 \times (3/2) = 65934$$

no repetition of digits

– 4 Reverse cases

54945

$$10989 \times 2 = 21978$$
$$10989 \times 3 = 32967 \qquad 87912$$
$$10989 \times 4 = 43956$$
$$10989 \times 5 = 54945$$
$$10989 \times 6 = 65934$$
$$10989 \times 7 = 76923$$
$$10989 \times 8 = 87912$$
$$10989 \times 9 = 98901$$

~~90~~ ~~8~~ 4 five digit reversals in addition to the 4 on p. 28 (c)

~~where middle~~

Try

$$\begin{array}{r} 1111 \times 99 \\ \hline 9999 \\ 9999 \\ \hline 109989 \end{array}$$

$109989 \times 9 = 989901$ ✓ Again it works for 6 digit case – Excellent

~~9999~~
~~9999~~
~~1110889~~

So we can get another 4, 6-digit reversals

4 Reversals of 6-digits in addition to those the 8 in d(i) & (ii) on p. 28.

ie
$109989 \times 9 = 989901$
$219978 \times 4 = 879912$
$329967 \times (7/3) = 769923$
$439956 \times (3/2) = 659934.$
— — — — —

$109989 \times 2 = 219978$
$109989 \times 3 = 329967$
$109989 \times 4 = 439956$
$109989 \times 5 = 549945$
$109989 \times 6 = 659934$
$109989 \times 7 = 769923$
$109989 \times 8 = 879912$
$109989 \times 9 = 989901$

Obviously this can be extended to any repunit number multiplied by 99

Suppose in particular we want to get reversals of 12-digit numbers. Consider

$\overbrace{11\cdots1}^{10\text{ times}} \times 99$

$= 9999999999$
9999999999
109999999989

109999999989×9
989999999901

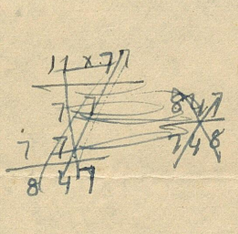
4 of 12 digits besides those of 2(g) on p. 28

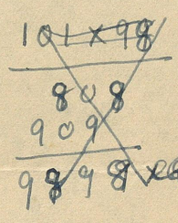
So that ~~an example of~~ we again get 4 examples of 12 digit reversals which need not be written out length

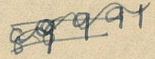
~~11×77~~
~~77~~ ~~847~~
~~77~~ ~~748~~
~~847~~

~~101×98~~
~~808~~
~~909~~
~~9898×9~~

~~89991~~







What about 10891089×9?

This equals 98019801 This is again reversed. (analogous to results on p. 28)

8 digits

21782178×4
87128712 — do — (")

What about 10892178? — No since multipliers are different.

~~Suppose we multiply by 4 & again by 9 → 43568712 → 382118308~~

$43568712 \div 2$
$= 21784356$ not reversed

$8 \times 9 \times 11^2$
$9 \times 9 \times 11^2$
$9 \times 11^2 \times 2 \times 9 \times 11^2$

$\frac{87129801}{10892178} = \frac{9681089}{1210242} = \cancel{880099\ 88099}\ \frac{880099}{11022}$

$= \frac{8009}{102} \times$

8800990
880099
9681089

~~968089~~
~~880990~~
~~88099~~
969089

So we can repeat to 12-digits for eg 108910891089. & so on.

~~9 abc~~
~~8 defg~~

965 964
874 875
91

9642
87531

A problem from L. A. Graham's book

(32')

If you multiply $\overbrace{a_1 a_2 \cdots a_n}^{z}$ by xy, the result should be

$y a_1 a_2 \cdots a_n x$.

345×83

$30008 + 3450$
3450
33458

$(10^{n+1} a_1 + \cdots + 10 a_{n-1} + a_n)(10x+y)$

$= 10^{n+3} y + 10^{n+2} a_1 + \cdots + 10^2 a_{n-1} + 10 a_n + x$

$x(10^{n+2} a_1 + \cdots + 10 a_n) + y(10^{n+1} a_1 + \cdots + a_n) = $ "

$x(10^{n+2} a_1 + \cdots + 10 a_n - 1) = y(10^{n+3} - 10^{n+1} a_1 \cdots - a_n) + 10 a^{n+2} a_1 + \cdots + 10 a_n$

$x(10z - 1) = y(10^{n+3} - z) + 10z$.

$z(10x + y) = y \cdot 10^{n+3} + x + 10z$

$z(10x + y - 10) = y0 \cdots 0x$. $\quad z = y000 \cdots 0x$

$z = \dfrac{10^{n+3} y + x}{10x + y - 10}$. $\quad z(10x+y) - 10z = 10^{n+3} y + x$

$\dfrac{3 \cdot 10^{n+3} + 8}{73} = z$.

$\dfrac{3 \cdot 10^{n+3} + 8}{73} = z$ $\quad n = 30008$

$73)30008(41$
292
80
73
78

41096×83
123288
328768
3410968

n $\quad \cancel{\dfrac{73z - 8}{3} = 3 \cdot 10^{n+3}}$

$73)308(4$
292
8

$73)3008(41$
292
88
73
15

$73)30008(411$
292
80
73
78
5

$n = 5$ $73)3000008(41096$
292
80
73
700
657
438

$3 \cdot 10^{13} + 8 = 73n$?

$3 \cdot 10^5 + 8 = 73m$ $\quad 3(10^{13} - 10^5)$ $\quad 10^5(10^8 - 1)$

$18, 112$

162 $\quad 9$

$9 \times 10^5 (10^7 + 10^6 + \cdots + 1)$

$2 \cdot 37$

$72 \cdots 3$

2
2

657
438

100000

438×73

$2 \cdots 13$
$6 \cdots 39$

10000000
1000000
100000
10000
1000
100
10
1

1111111

$73)1010101(13837$
73
280
219
611
584
270
219
511
511

$9(10a + b) = 10^n b + a$

$9(10a + 8)$
$90a + 72$
$= 8 \cdot 10^n + a$

$z = 73)300000 \cdots 8(4109589041096$
292
80
73
700
700
657
430
365
650
584
660
657
300
292
80
73
700
657
43

$\dfrac{x}{d} = \dfrac{10^n - 2}{18}$

$\dfrac{x}{d} = \dfrac{10^n - 3}{29}$.

$18)999 \cdots 98(55$
90
99
90
9
88

$29)999 \cdots 91(34$
87
129
116
139

$a_1 \cdots a_{n-1} a_n \to a_n a_1 \cdots a_{n-1}$ $\quad (10x + a_n)9$

$a_n a_1 \cdots a_{n-1} = 9(a_1 \cdots a_{n-1} a_n) = 10^n a_n + a$

27/12/73

The transfer of digits from the last place to the first :-

The two problems in Frohlichstein & Graham correspond to

$$\frac{x}{d} = \frac{10^n - \lambda}{10\lambda - 1} \quad \text{with } \lambda = 3 \text{ \& } \lambda = 9 \text{ respectively}$$

$\lambda = 3 \rightarrow \frac{x}{d} = \frac{10^n - 3}{29}$ & $10^n - 3$ is of the form 999 . . . 7

& actual division by 29 so as to end without remainder determines n

29) 999 . . . 7 (34,482,758,620,689,655,172,413,7931.

87	179	149	109	
129	174	145	87	
116	59	49	229	
139	58	29	203	
116	199	209	269	
239	174	203	261	
232	259	69	89	29
79	232	58	87	29
58	279	119		29
219	261	116		
203	189	39		
169	174	29		
145	159	10		
249	145			
232				
17				

ie the number x/d is 999,999,999,999,999,999,999,999,997

& adding 3 we get 1 followed by 27 zeros ie 10^{27} giving $n = 27$.

How does it follow that $d = 3$? ~~$29x = (10^{27} - 3)d$~~ from the actual value of x/d ?

~~1 . . . 3 × 3~~
~~3 . . . 9~~
~~1 . . . 1 × 3~~
a_1 . . . . . d
2 9 6
. 3
1 0 . . 6 . . 8
~~1 4~~

Graham states that "d is obviously 9" & also "since the first and last digits of the required number must be 1 and 9 respectively, . . ."

The first digit multiplied by 9 must give the last digit
ie first digit must be 1 & the last digit 9.

Since the 1st digit × 3 = last digit

Possibilities $1 \times 3 = 3$ = last digit 3
$2 \times 3 = 6$ = " 6
$3 \times 3 = 9$ = " 9 } How to exclude these ?

Transfer of last digit to first place :- Case of 3 digits. (multiplying by p say) ~~&~~

$abc \rightarrow cab.$

$(10^2 a + 10b + c)p = 10^2 c + 10a + b.$

If $x = 10a + b$]

$(10x + c)p = 10^2 c + x.$

$x(10p - 1) = c(10^2 - p)$

$\frac{x}{c} = \frac{10^2 - p}{10p - 1}$

$p = 1$. R.H.S $\rightarrow \frac{99}{9} = 11.$ ~~since~~

& c may have any value from 1 to 9, say $c = 3$

$x = 33$ ie $abc = 333$ hm

$p = 2$, $\frac{98}{19}$. &

$p = 3$, etc $\rightarrow \frac{97}{29}, \frac{96}{39}, \frac{95}{49}, \frac{94}{59}, \frac{93}{69}, \frac{92}{79}, \frac{91}{89}$ ✗

$\frac{x}{d} = \frac{10^3 - b}{10b-1}$ → ~~$\frac{999}{9}$~~, $\frac{999}{9}, \frac{998}{19}, \frac{997}{29}, \frac{996}{39}, \frac{995}{49}, \frac{994}{59}, \frac{993}{69}, \frac{992}{79}, \frac{991}{89}$ ×

$\frac{10^4 - b}{10b-1}$ → $\frac{9999}{9}, \frac{9998}{19}, \frac{9997}{29}, \frac{9996}{39}, \frac{9995}{49}, \frac{9994}{59}, \frac{9993}{69}, \frac{9992}{79}, \frac{9991}{89}$

409
354
554
209
158
512

Simply start with 99 ... 8 & divide by 19 $b=1$

99 ... 7 " 29 — This is Frobenius's Case

99 ... 6 " 39 $b=4$

19) 99 -- 8 (5263157894736842

95
49
38
119
114
59
57
29
19
109
95
149
133
169
152
179
171
89
76
139
133
69
57
129
114
159
152
79
76
38
19
0

By Graham's rule
$d = 2$ and
$b = 2$ ✗
$b = 2$ gives
$x = 10,5,2,6,3,1,5,7,8,9,4,7,3,6,8,4$
cyclic, given no of 18 digits wrong
$= 105,263,157,894,736,842$ ✓

(1) $\lambda = 2$

18 digits
Need carry sufficient
from 7 × 6 =
42 + 23 digits

13) 333 -- 2 (2564 Verify) 2105263157894736842

26
73
65
83
78
52
52
4

i.e. $\frac{x}{d} = 2564$. for case multiplicator is by 4.

So from Graham's argument $d = 4$

$\therefore x = 2564 \times 4 = 10,256$

Solve number is 102,564

multiply by 4, we get

102564 × 4
410256 ✓

39) 999 -- 6 (2564
78
219
195
249
234
156
156

(2) $\lambda = 4$

Another with
6 digit Case ✓

49) 999 --- 5 (20408163265306122448979 = 24/2

98
199
196
399
392
7
49
309
294
15
14
1
29
28
19
14
59
56
3
34
33
1
29
28
1
14
11
8
7
1
39
35
4
44
39
5
49
44
5

245
1
312

24/2 & with $d = 5$, the number is 102040819183673469387755

$2040816318367346938775 \times 5$
rep no is = $10204081591836734693877 55$
Verify $1,020,408,159,183,673,469,387,755 \times 5$
$= 5918367346938775$

~~49) 99995 (2045~~
98
199
196
35

7) 99995 (14285
7
29
28
19
14
59
56
35
204

49) 99 . . . . 5 (2040816326530612244897967346938775 5

$\lambda = 5$

98
199
196
399
392
79
49
309
294
159
147

129
98
319
294
259
245
149
147
299
294

59
49
109
98
119
98
219
196
239
196

439
392
479
441
389
343
469
441
289
254

359
343
169
147
229
196
339
294
459
441

189
147
429
392
379
343
369
343
269
245
245.

Elephantine number of Correct

multiply by 5.

20408163265306122448979673469387755 $\times 5 \times \frac{10}{2}$

~~1673469387755~~

1020408163265306122448983673469387755

Number is 1020408163265306122 4918 3673469387755

Verification on multiplying by 5 ie 10/2

51020408163265306122 4918 3673469387755

Number is 1,020,408,163,265,306,122,449,183,673,469,387,755

ie a number of 37 digits much bigger than Frohlichstein's 27 and smaller than Graham's 43.

4918
4898
20

8983
4836

236

$\lambda = 6$. $\frac{x}{d} = \frac{999 \ldots 4}{59}$

59) 999 . . . . 4 (1,694,915,254,237,288,135,593,220,338,983,050,847,457,627,118,644,067,966 1 ?

59
409
354
559
531
289
236
539
531
89
59
309
295
149
118
31

319
295
249
236
139
118
219
177
429
413
169
118
519
472
479
472

79
59
209
177
329
295
349
295
549
531
189
177
129
118

119
118
199
177
229
177
529
472
579
531
489
472
17

179
177
299
295
499
472
279
236
439
413
26

269
236
339
295
449
413
369
354
15

469
413
569
531
389
354
354
354
359
354
59 finishes

159
118
419
413
69
59

109
59
509
472
37

379
354
259
236
238
236

399
354
459
413

~~all wrong~~ should stop at 354

55 digits

finishes — Colossal division — Hope it is correct ~~ie x/d~~ is 49 digits

(28/12/73) – Stop this search for the present & come back again.

From Kraitchik's book: ~~for~~ 4-digit nos of form aabb = □

p. 159 of Note Book (5) ~~100~~ $1100a + 11b = \square$

$11(100a + b) = \square$ ie $100a + b = 11\square$

Just make a search: $b = 1$, $100a + 1 = 11\square$

b need only be 1, 4, 6, 9 only 1 None of 101, 201, ... contain factor 11

$b = 4$, $100a + 4$, $a = 7 \to 704 = 11 \times 64$ ✓

$b = 6$, $100a + 6$, $a = 5 \to 506 = 11 \times 46$ ×

$b = 9$, $100a + 9$, $a = 2 \to 209 = 11 \times 19$ ×

ie only case is $a = 7$ & 4 so that $7744 = 88^2$ is reqd no

(6) $abcd = \square$, $ab = \square$ & $cd = \square$

$1000a + 100b + 10c + d = \square$; $10a + b = \square$, $10c + d = \square$

ie $100(10a + b) + (10c + d) = \square$; $10a + b = \square$; $b = \cancel{1, 4, 6, 9}$ 1, 4, 5, 6, 9

$10a + b = 4^2$ or 6^2 or 7^2 or 8^2 or 9^2; $10a + 1$ ~~10a+4~~, $10a + 4 = 8^2$ $(a = 6)$

Sing $10c + d =$ " $= 9^2$ $(a = 8)$ $10a + 6 = 6^2$ $(a = 3)$, or 4^2 $(a = 1)$

$= 1^2$ $(a = 0)$ × $10a + 9 = 7^2$ $(a = 4)$

$10a + 5 = 5^2$ $(a = 2)$

$100(10a + b) = 40^2$ or 60^2 or 70^2 or 80^2 or 90^2

$= 1600$ or 3600 or 4900 or 6400 or 8100

1616, 1636, 1649, 1664, 1681

3616, 3636, 3649, 3664, 3681.

4916, 4936, 4949, 4964, 4981

6416, 6436, 6449, 6464, 6481

8116, 8136, 8149, 8164, 8181

also 2516, 2525, 2536, 2549, 2564, 258

for perfect squares using tables for squares

From the table one finds only $1681 = 41^2$

Also ~~1600~~ trivial cases of 1600, 2500, 3600, 4900, 6400, 8100.

(7) $abcd = \square = \cancel{(ab + cd)^2} = (ab + cd)^2$

$1000a + 100b + 10c + d = (10a + b + 10c + d)^2$

$100(10a + b) + (10c + d) = (10a + b)^2 + (10c + d)^2 + 2(10a + b)(10c + d)$

$100x + y = \cancel{x^2 + y}\ (x + y)^2$. $100x + y = \square$ with x & y 2 digit nos.

$y = 16, 25, 36, 49, 64, 81$

~~100x + 16~~ & x to be tested from $x = 10, \ldots 99$

~~1000~~ From tables Squares of 4 digits ending with 16, 25, — are

1225 (35^2), 1681 (41^2), 1764 (42^2), 1849 (43^2), 1936 (44^2), 2025 (45^2), 2116 (46^2)

2916 (54^2), 3025 (55^2), 3136 (56^2), 3249 (57^2), 3364 (58^2), 3481 (59^2)

4225 (65^2), 5625 (75^2), 7225 (85^2), 8281 (91^2), 8464 (92^2), 8649 (93^2),

8836 (94^2), 9025 (95^2), 9216 (96^2), 9801 (99^2), 2401 (49^2), 2601 (59^2)

1, 694, 915, 254, 237, 288

$\cancel{(10a+b)^2 = 100a^2 + b^2 + 20ab}$.

(9) $\cancel{(10a+b)^3 =}$

Sum of digits in the cube of a number = number itself

Searching tables $8^3 = 512$ & $5+1+2 = 8$; $9^3 = 729$ & $7+2+9 = 2\cdot 9$

$3^3 = 27$ & $7+2 = 3\cdot 3$.

$\cancel{m} = \cancel{n = 7m+6}$.

$(2m+1)^2$

$4m^2+4m+1$

$4m+1$: $8m+1$

$7m+6$

$= 7m^2-1$

$7n-1 \neq \square$?

(12) $x^2 + x - 9m = 0$

$x = \dfrac{-1 \pm \sqrt{\cancel{1+81m^2}\ 1+36m}}{2}$. $\cancel{81m^2+1 = \square}$.

$\cancel{m=1 \to 82 \times}$

$\cancel{m=2 \to 10}$

$\cancel{2\square}$

$36m+1 = \square$.

$m = 10 \to 19^2$

$m = 140 \to 71^2$

$m = 220 \to 89^2$

$m = 230 \to 91^2$

$x = (-1+19)/2 = 9$ & $x^2 = 81$ ×

$x = (-1+71)/2 = 35$ & x^2 has 5 ×

$x = 44, x = 45$ & x^2 " 5 ×

$\cancel{36m = 129}$

$\cancel{36m = 960}$

$\cancel{3m = 80}$

$36m = 1680$

$\cancel{3m = 140}$

$\cancel{36m = 2400}$

$\cancel{3m = 800}$

$36m = 5040$

$3m = 420$

$8m = 140$

$\cancel{140 \times 36}$

5040

35×35

$36m = 7920$

$3m = 660$

$m = 220$

$36m = 8280$

$3m = 690$

Probably we have to go higher values of m giving x in 3 digits.

$x = 567$, $x = \dfrac{-1+y}{2}$ $2x = y-1$

$1134 = y-1$

$y = 1135$.

$36m + 1 = 1135^2 - 1$

$36m = 1136 \times 1134$

$4m = 1136 \times 126$.

$m = 284 \times 126$ //

Collossal values of m.

Rather too difficult to find a method.

(13) $10^n - 1 = 9a \to a = 10^{n-1} + 10^{n-2} + \cdot + 10 + 1$

$= 111 \cdots 1$.

$1000 - 0$

$100 - 6$

$10 - \cdot$

$\cdot - 1$

$b = 9a+1 = 999 \cdots 9 + 1 = 1000 \cdots 0$.

$c = 8a+1 = 888 - 8 + 1 = 888 \cdots 89$.

$6a+1 = 66 \cdots 67$

$12a+1 = (133 \cdots 32)+1 = 133 \cdots 33$

$15a+2 = 166 \cdots 67$.

$\cancel{m1}$

1332

1665

Can we obtain any other squares?

what about $\cancel{(\lambda a + \mu b}$

$\cancel{(\lambda a + b)c}$

$(\lambda a + \mu)b + c = \square$.

$(\lambda a + \mu)(9a+1) + 8a+1$

$9\lambda a^2 + a(9\mu + 8 + \lambda) + (\mu+1)$.

$(\lambda + 9\mu + 8)^2 = 36\lambda(\mu+1)$

ie $\{(\lambda+9\mu)+8\}^2 = 36\lambda(\mu+1)$

$(\lambda+9\mu)^2 + 64 + 16(\lambda+9\mu)$

$= 36\lambda(\mu+1)$

$\lambda^2 + 81\mu^2 + 18\lambda\mu + 16 + 16\lambda + 144\mu + 64$

$= 36\lambda\mu + 36\lambda$

$(\lambda - 9\mu)^2 - 20\lambda + 144\mu + 64 = 0$.

$(\lambda - 9\mu)^2 - 4(5\lambda - 36\mu) + 64 = 0$

(i) $\mu = 0$, $\lambda = 4$, $\lambda^2 - 20\lambda + 64 = 0$

$(\lambda - 16)(\lambda - 4) = 0$

$\lambda = 16$, $\lambda = 4$.

$4ab + c$ — given by Kraitchik

$16ab + c$ is also a soln — also given

(ii) $\cancel{\lambda = 1}$, $\mu = -1 \to (\lambda+9)^2 - 4(5\lambda+36) + 64 = 0$

$(\lambda+9)^2 - 20\lambda - 144 + 64 =$

$\lambda^2 - 2\lambda + 1 = 0$ ie $\lambda = 1$ — Case (2)

P. U. Press, Poona]

UNIVERSITY OF POONA

EXAMINATION, 195 .

Subject

Paper

Section

Examiners' Signatures.

CANDIDATE'S No.	QUESTIONS 1	2	3	4	5	6	7	8	9	10	11	12	Paper Total	Paper Total	Practical	GRAND TOTAL	CANDIDATE'S No.
181	19)	99	- -	9(	526	315	789	473	684	21	= x						181
		95															
182		49	.	39													182
				38													
				19													
183				N = 105,263,157,894,736,842													183
184																	184
185																	185
186																	186
187																	187
188																	188
189																	189
190																	190
191																	191
192																	192
193																	193
194																	194
195																	195
196																	196
197																	197
198																	198
199																	199
200																	200

$(\lambda - 9\mu)^2 - 4(5\lambda - 36\mu) + 64 = 0$

$(\lambda - 9\mu)^2 - 4(4\lambda - 36\mu) - 64\lambda + 64 = 0.$

$(\lambda - 9\mu)^2 - 16(\lambda - 9\mu) - 4(\lambda - 16) = 0$

$(\lambda - 9\mu - 8)^2 - 64 - 4\lambda + 64 = 0$

ie $(\lambda - 9\mu - 8)^2 = 4\lambda.$

Kraitchiks case are (i) $\mu = 0$ leading as on p. 35 to $(\lambda - 8)^2 = 4\lambda$ ie $\lambda^2 - 20\lambda + 64 = 0$

$\lambda = 16$ or 4

ie leading to $4ab + c$

and $16ab + c$.

(ii) $\mu = -1 \rightarrow (\lambda + 1)^2 = 4\lambda \rightarrow (\lambda - 1)^2 = 0$ ie $\lambda = 1$

leading to $(a-1)b + c$. as in Kraitchik

(iii) $\mu = 3 \rightarrow (\lambda - 35)^2 = 4\lambda \rightarrow \lambda^2 - 70\lambda + 35^2 = 4\lambda$

$\lambda^2 - 74\lambda + 35^2 = 0$

$\lambda^2 - 74\lambda + 1225 = 0$

$(\lambda - 25)(\lambda - 49) = 0.$

$\lambda = 25, \lambda = 49.$

35×
49×25
4900
4
1225
41×

$(25a+3)b + c$

$= (25a+3)(9a+1) + 8a + 1$

$= 225a^2 + 52a + 3 + 8a + 1$

$= 225a^2 + 60a + 4$

$= (15a + 2)^2$

$\lambda = 25$ gives Kraitchik's case

what about $\lambda = 49$? $(49a+3)b + c$

$= (49a+3)(9a+1) + 8a + 1$

$= (21a)^2 + 4(21a) + 4$

$= (21a + 2)^2.$

49+27+8
84

~~233333×233333~~
23333×23333

11111
233331...3

233 × 233
699
699
466
54289

6999
6999
4666
543589

69999
69999
69999
46666
54435889

699999
699999
699999
699999
466666
5444358889

What about $(\lambda a + \mu)c + b$?

This $= (\lambda a + \mu)(8a + 1) + 9a + 1$

$= 8\lambda a^2 + a(8\mu + \lambda + 9) + (\mu + 1)$

λ is a □ if $(\lambda + 8\mu + 9)^2 = 32\lambda(\mu + 1)$

$(\lambda + 8\mu)^2 + 18(\lambda + 8\mu) - 32\lambda\mu - 32\lambda + 81 = 0$

$(\lambda - 8\mu)^2 - 14(\lambda - 8\mu) + 32\mu + 81 = 0$

$(\lambda - 8\mu - 7)^2 - 49 + 32\mu + 81 = 0$

$(\lambda - 8\mu - 7)^2 + 32(\mu + 1) = 0.$

(i) $\mu = 0 \rightarrow (\lambda - 7)^2 + 32 = 0$

$\lambda^2 - 14\lambda + 81 = 0$ ✗

$\mu = 1 \rightarrow (\lambda - 15)^2 + 64 = 0$

$\lambda^2 - 30\lambda + 289 = 0$ ✗

225
64
17×17

$\mu = -1 \rightarrow (\lambda + 1)^2 = 0$, ~~$\lambda = 9$ holds leading to~~ $\lambda = -1$.

~~leading to $(a-1)c + b$~~

~~$= (a-1)(8a+1) + 9a + 1$~~

~~$= 8a^2 + 2a.$~~

$b - ac$

~~$ac + b = a(8a+1) + 9a + 1$~~

~~$= 8a^2 + 10a + 1.$~~

~~$= 9a + 1 - a(8a+1)$~~

~~$= -8a^2 + 8a + 1.$~~ ✗

$9a + 1 - (8a^2 + 9a + 1)$

$= -8a^2$ //

$= (a+1)c + b = b - c(a+1) = 9a + 1 - (a+1)(8a+1)$

$(\lambda - 8\mu - 7)^2 + 32(\mu+1) = 0$

$\mu = -1$ gives $(\lambda+1)^2 = 0$ ie $\lambda = -1$ ie $b + c(-a-1) = b - c(a+1)$

$= (9a+1) - (8a+1)(a+1) = 9a+1 - (8a^2+9a+1) = -8a^2$ (−ve value) ×

$\mu = -2 \rightarrow (\lambda+9)^2 - 32 = 0$

$\lambda^2 + 18\lambda + 49 = 0$ ×

81
32

$\mu = -3 \rightarrow (\lambda+17)^2 - 64 = 0$

$\lambda^2 + 34\lambda + 225 = 0$.

289
64
225

$\mu = 2 \rightarrow (\lambda-23)^2 + 96 = 0$

$\lambda^2 - 46\lambda + 625 = 0$ ×

529
96
625

$\{\lambda - (8\mu+7)\}^2 + 32(\mu+1) = 0$

112
32

$8\mu+9$

$\lambda^2 - 2\lambda(8\mu+7) + (8\mu+7)^2 + 32(\mu+1) = 0$

$\lambda^2 - 2\lambda(8\mu+7) + (64\mu^2 + 144\mu + 81) = 0$

$\lambda^2 - 2\lambda(8\mu+7) + (8\mu+9)^2$

$\lambda = \dfrac{2(8\mu+7) \pm \sqrt{4(8\mu+7)^2 - 4(8\mu+9)^2}}{4}$

ie λ is imaginary.

So there is no choice like $(\lambda a + \mu)c + b$ giving a perfect square.

(14) $8T + 1 = S$ (for triangular T & square S figurate no.

$8 \cdot \frac{1}{2} n(n+1) + 1 = 4n(n+1) + 1$

$= (2n+1)^2$

$n = 6$: $\frac{1}{2} 6(6+1)$

$3 \times 7 = 21$

$8 \cdot 21 + 1 = 169 = 13^2$

In the figure the square $13^2 = (2 \cdot 6 + 1)^2$

& triangular no. corresponds to $\frac{1}{2} \cdot 6(6+1) = 21$

& $8 \cdot 21 + 1 = 169 = 13^2$.

(15) This is the digit transfer problem from last to first place. Kraitchik talks

$\dfrac{N-2}{10} + 2 \cdot 10^{n-1} = 2N$ (α)

132

$N = a_1 \cdots a_{n-1} 2 = 10^{n-1} a_1 + \cdots + 10 a_{n-1} + 2$

$2N = 2a_1 \cdots a_{n-1}$

$= 10^{n-1} 2 + 10^{n-2} a_1 + \cdots + a_{n-1}$

ie $\dfrac{N-2}{10} = 10^{n-2} a_1 + \cdots + a_{n-1}$

$= 2N - 2 \cdot 10^{n-1}$ (β)

Hence (α) follows.

$19N = 2(10^n - 1)$

$N = \dfrac{2(10^n - 1)}{19}$

How is Kraitchik's formula (α) ~~work in the case of $\lambda = 2$~~ related to our formula

$\dfrac{x}{d} = \dfrac{10^{n-1} - \lambda}{10\lambda - 1}$, ie $\dfrac{N-d}{10d} = \dfrac{10^{n-1} - \lambda}{10\lambda - 1}$

$N = xd$

& $x = 10^{n-2} a_1 + \cdots + a_{n-1}$

$N = 10x + d$ or $x = \dfrac{N-d}{10}$

If the number has n digits

x should have $n-1$ digits

So our formula should be on R.H.S $= \dfrac{10^{n-1} - \lambda}{10\lambda - 1}$

In Graham's formula

$a = x$, $b = d$, the formula is

$N = 10x + d$

~~$9(10x+d)$~~ =

$8x = \dfrac{(10^n - \lambda)d}{10\lambda - 1}$

$\dfrac{N-d}{10} = \dfrac{(10^n - \lambda)d}{10\lambda - 1}$

$N = \dfrac{(10^{n+1} - 10\lambda)d}{10\lambda - 1} + d$

$N = \dfrac{(10^{n+1} - 1)d}{10\lambda - 1}$

with $d = 2$, this $\rightarrow \dfrac{N-2}{10d} = \dfrac{10^{n-1} - \lambda}{10\lambda - 1} = \dfrac{10^{n-1} - 2}{19}$

$\dfrac{N-2}{2 \cdot 10} = \dfrac{10^{n-1} - 2}{9}$

$N - 2 = \dfrac{2 \cdot 10^n - 2^2 \cdot 10}{19}$

$N = \quad '' \quad + 2$

$= \dfrac{2 \cdot 10^n - 40 + 38}{19} = \dfrac{2(10^n - 1)}{19}$

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102																	102
103																	103
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120																	120

$\frac{(10^{n+1}-1)d}{89}$

$N = \frac{(10^{n+1}-1)d}{10d-1}$

$8\cancel{9}(\cancel{89}x = \frac{(10^{n-1}-9)d}{89}$

$\frac{N-d}{10} = \frac{(10^{n-1}-9)d}{89}$

$N = \frac{(10^{n}-90)d}{89} + d = \frac{(10^{n}-1)d}{89}$

$N = 10x + d$

$$\frac{x}{d} = \frac{10^{n-1}-\lambda}{10\lambda-1}$$ for a n digit number n.

$N = 10x + d$] $\quad \frac{N-d}{10d} = \frac{10^{n-1}-\lambda}{10\lambda-1}$.

$$N - d = \frac{10d\,(10^{n-1}-\lambda)}{10\lambda-1}.$$

or $N =$ " $+d$

$$= \frac{d\cdot 10^n - 10d\lambda + d(10\lambda-1)}{10\lambda-1} = \frac{d(10^n-1)}{10\lambda-1}.$$

Kraitchik takes $\lambda = 2$ & hence $N = \frac{2(10^n-1)}{19}$.

& this is what we got on p (33').

The formula $N = \frac{d(10^n-1)}{10\lambda-1}$ (1) ~~is prefer~~ appears preferable to

$$\frac{x}{d} = \frac{10^{n-1}-\lambda}{10\lambda-1}.$$

$89a = (10^{n-1}-9)$

Graham takes $d = \lambda$. How does this follow?

Can we take $\lambda = 3$. Then Kraitchik's original eqn

$$\frac{N-2}{10} + 2\cdot 10^{n-1} = 3N. \qquad N - 2 + 2\cdot 10^n = 30N.$$

$$29N = 2(10^n-1).$$

$$N = \frac{2(10^n-1)}{29}$$ also from (1) above.

For transfer of 1st → last.

$$\frac{x}{a} = \frac{10^{n-1}-\mu}{10\mu-1} \qquad \mu = q/p.$$

(a = 1st digit)

$a_1 \cdots a_n$

$N' = N/3$

~~$N = 10x +$~~

~~$N = 10^{n+1}a_1 + \cdots + 10a_{n-1} + a_n$~~

~~$= 10^{n+1}a_1 + \underbrace{10^n a_2 + \cdots + a_n}_{x}$~~

~~$N = a \cdots d$~~

$N = 10^{n-1}a_1 + \cdots + 10a_{n-1} + a_n$

$N - 10^{n-1}a_1 = 10^{n-2}a_2 + \cdots + 10a_{n-1} + a_n$

$= x$

~~$\frac{N-x}{10^{n-1}a}$~~ $\quad \frac{x}{a} = \frac{N-10^{n-1}a}{a} = \frac{10^{n-1}-\mu}{10\mu-1}$

$(N - 10^{n-1}a)(10\mu-1)$

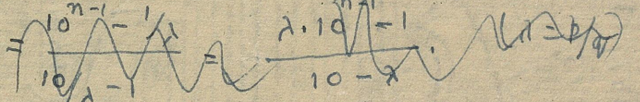
$= (10^{n-1}-\mu)a$

$N(10\mu-1) - 10^n a\mu + \cancel{10^{n-1}a}$

$= \cancel{10^{n-1}a} - a\mu.$

$N(10\mu-1) = a\mu(10^n-1)$

$$N = \frac{a\mu(10^n-1)}{10\mu-1}. \quad (i)$$



$$N = \frac{a\mu(10^n-1)}{10\mu-1}$$ as in (1) below

In (14) of Kraitchik (p. 164 of note book); $a = 3$ & $\mu = 3$.

ie $N = \frac{9(10^n-1)}{29}$.

~~This is the~~ In this case $\frac{10^n-1}{29}$ (as done for Frohlichstein's case of last → first)

$= 34,482,758,620,689,655,172,413,7931$

$= 344,827,586,206,896,551,724,137,931$

(27 digits)

$\therefore$ ~~N~~ $= 3,103,448,275,862,068,965,517,241,379$

$N = 3,103,448,275,862,068,965,517,241,379$

ie as given in Kraitchik [Ex. (16), p. 164 note book]

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(38)

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$\lambda = 4$ (3 ✗)

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200																	200

Case $\lambda = 3$, $d = 2$. Here $N = \frac{d(10^n - 1)}{9}$ [See numerical p. 38 for $(10^n - 1)/29$.

we have $N = 344,827,586,206,896,551,724,137,931 \times 2$

$N = 689,655,172,413,793,103,448,275,862$ ✗ (for verification, multiply by 3

$2068,965,517,241,379,310,344,827,586$ we get a 28-digit number which N has only 27 digits

Thus $d = 2$ fails for $\lambda = 3$ //

$d = 3$ gives $N = 758,620,689,655,172,413,793$

$d = 3$ gives $N = 1,034,482,758,620,689,655,172,413,793$

i.e. with 28 digits

and when we multiply this $\lambda = 3$ for remainder we again get a 28 digit number, multiplying the extreme.

We can in this way find $d = 3, 4, \ldots 9$. for eg: with $d = 9$

N gives a number of 28 digits only (taking two eg, even with 345,000, ---, 000 two last digits

$d = 9$ gives 3,105,000, ---, 000 i.e 28 digits)

And when we verify each with $\lambda = 3$, quotient for a 28-digit no. $\frac{4}{9}$ as the last digits.

Actually with $d = 9$, $N = 3,103,448,275,862,068,965,517,241,379$ — Incidentally this is multiplying by 3 → $9,310,344,827,586,206,896,551,724,137$ which is correct.

Refer to Ex (16) of Kreyszig p. 164 of Note Book

I think there is no need to verify for $\lambda = 4, 5, 6, 7, 8$.

when first — this is better

$\lambda = 4$ (3 ✗)

$\frac{345,c}{3,105,0--}$

$\times 9$

1

We shall examine Graham's assumption of $d = \lambda$ by taking the simple case of $\lambda = 4$ [on p. (33)']. [modification ex. (17), p. 164 of Note Book]:

In this example Kraitchik takes d also $= 4$. Suppose we take $d = 7$

$$N = \frac{d(10^n - 1)}{10\lambda - 1} = \frac{7(10^n - 1)}{39}$$

$$= 25641 \times 7$$

$$N = 179487$$

102564
230769 × 4
1923076

39) 99 . . . 9 (

or 13) 33 . . . 3 (25641

26
73
65
83
78

53
52
13

with $n = 7$

Multiplying by 4, $\frac{179487 \times 4}{717948}$ Correct ✓

So Graham's argument does not hold good.

10000000 − 1
999999
13) 333333 (25641
26
73
65
83
78

53
52
13

Let us consider Graham's example itself (Note Book, p. 152), where only $\lambda = 9$ is specified & d is said to be obviously equal to 9. Suppose we take $d = 7$.

$$N = \frac{7(10^n - 1)}{89}.$$

In fact this laborious division has not been done earlier. Let us do it now!

89) 999 . . . 9 (1,123,595,505,617,977,528,089,887,640,449,438,202,247,191

a number of 43 digits

89
109
89
209
178
319
267
529
445
849
801
489
445
449
445
499
445
549
534
159
89
709
623
869
801
689
623
66

669
623
469
445
249
178
719
712
799
712
879
801
789
712
779
712
679
623
569
534
359
356
399
356
439
356
83

839
801
389
356
339
267
729
712
179
178
199
178
219
178
419
356
639
623
169
89
809
801
89
89
0

We have now to

(i) verify Graham's result by multiplying this by 9 & getting his N.

(ii) multiply this by 7 and verify that the resulting number satisfies the criterion.

(a number of 43 digits of 99 − 9) i.e. $n = 43$

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122																	122
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128																	128
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136																	136
137																	137
138																	138
139																	139
140																	140

1,123,595,505,617,977,528,089,887,640,449,438,202,247,191 x9

10,112,359,550,561,797,752,808,988,764,044,943,820,224,719 x9 ✓ Correct as given by Graham. ✓ (for misprint)

91,011,235,955,056,179,775,280,898,876,404,494,382,022,471 ✓

Multiplication by 7.

1,123,595,505,617,977,528,089,887,640,449,438,202,247,191 x7 (9)

N = 7,865,168,539,325,842,696,629,213,483,146,067,415,730,337 x9

70,786,516,853,932,584,269,662,921,348,314,606,741,573,033 wrong in one right (0)

Explanation written & 3, Nos (A) learns

3,370,786,516,853,932,584,269,662,921,348,314,606,741,573 x9

30,337,078,651,685,393,258,426,966,292,134,831,460,674,157 wrong in method (0)

Subtraction left out = 2

2,247,191,910

Dr Lodha as it is in Graham's one & should be = 9 only

20,224,719

224,719,775

20,222,775 561,797,75

504

8,988,764,0

As seen on p.(39') for the case $\lambda = 9$, $d = 9$. by actual multiplication by $d = 7, 3, 2, 5$

Let us test the case $\lambda = 2$.

Here $N = \frac{d(10^n - 1)}{19}$.

$(10^{n-1})(10-1)$

$= 10^n - 10^{n-1} + 1$

(1 digit)

$9 \cdot 10^{n-1} - 9$

$9 \cdot 10^{n-1} - 1)$

$\frac{10^n - 1}{19} = 1,052,631,578,947,368,421$

with ~~$d = 2$~~, ~~$N =$~~

$N = 2,115,263,157,894,736,842$

$\frac{10^n - 1}{19} = 52,631,578,947,368,421.$

526315789473684212

$105,263,157,894,736,842$

with $d = 2$, $N =$ 104

$N = 10,536,317,894,736,842$

Suppose I take $d = 3$, then $\frac{d(10^n - 1)}{19} \rightarrow$ $526,315,789,473,684,213$

$N =$ $157,894,736,842,105,263 \times 2$

$3,157,894,736,842,105,26.$ ✓ correct

Suppose I take $d = 9$. $52,631,578,947,368,421 \times 9$

$473684210526315789 \times 2$

$947,368,421,052,631,578$ ✓, Again correct.

The anamoly arising in the case $\lambda = 9$ appears due to the fact that if d is taken any digit other than 9, the product of $\frac{10^n - 1}{89}$ (which of 43 digits) with d gives the number N also with 43 digits, while multiplication by 9 if $d = 9$ gives N with 44 Dyds. So that in the other cases of $d \neq 9$ when N is multiplied by 9 we get a number with 44 digits so that the transference of last digit to the first place is not possible when N has only 43 digits.

In the case of $\lambda = 4$, $\frac{10^n - 1}{39}$ leads to 25641 and multiplying this by any value of d from 4 to 9 (See reverse) gives a 6-digit number N [using $N = \frac{d(10^n - 1)}{39}$], and in all these cases when N is multiplied by $\lambda = 4$. a 6-digit number results with the last digit transferred to the first place. So that Graham's $d = \lambda$ ($= 4$) is not necessary (but sufficient) in this case.

In the case of $\lambda = 3$, $\frac{10^n - 1}{29}$ leads to $344,827,586,206,896,551,724,137,931$

having 27 digits. with $d = 2$ we get again an N with 27 digits, and on multiplying by 3 there result a number with 28 digits so that $d = 2$ is untenable. For $d = 3, 4, \ldots 9$, N has 28 digits and on multiplication by $\lambda = 3$, we again get a number of 28 digits satisfying the criterion [Vide (38')]

This suggests that in general for any given λ, corresponding d should be $\geq \lambda$, & hence for $\lambda = 9$, the only possible value of $d = 9$, thus justifying Graham's assumption (of course intuitively made ~~by~~ by a fool!). We have to verify the $\lambda =$ [illegible] to take [illegible] (Problem). This on 31/12/173

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Case $\lambda = 4$, $\frac{10^n - 1}{39} = 25641$

with $d = 3$, $N = 76923$ · Multiplying by 4 → 307692 Does not hold

(6 fixed digit)

For $d = 2$, $N = 51282$ & multiplying 4 → 205128 Does not hold

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Dated 195

(A) 21978 → (9,7) → (2,1,8) ×
23958 → (9,5) → (2,3,8) ×
25938 → (8,2) → (3,5,9) → (5,3) ✓ → 25938 + 32985 = 58923
27918 → (9,1) → (2,7,8) ×

(1) one from (A)

4 cases in the 5-digit case possible

digit case repeated

(G) 8172 → ×
8712 → ×
8352 → ×
8532 → (8,2) + (5,3) ×

nil from (G)

8532
83
5

7263 → ×
7623 → ×

nil from (F)

6174 → (6,4) + (7,1) ✓ → 6174 + 1467 = 7641
6714 → ×
6354 → ×
6534 → ×

1 from (E) ✓

4176 → (6,4) → (1,7) ×
4716 → ×
4356 → ×
4536 → ×

nil from (D)

3267 → (6,3) → (2,7) ×
3627 → ×

nil from (C)

2178 → ×
2718 → ×
2358 →
2538 → (8,2) + (5,3) ✓ → 2538 + 3285 = 5823

(1) one from (B) ✓

1269 → (9,6) + (2,1) ✓ → 1269 + 1692 = 2961
1629 → (9,2) + (6,1) ×
1359 → (9,5) → (1,3) ×
1539 → (9,3) → (1,5) ×

1 from (A) ✓

~~4 cases~~
3 cases in the case of 4 digits

3 digits case

1 case for 3 digits viz 954 − 459 = 495

(A) 123579 → (5,1), (7,3), (9,5) ‖ (5,1) → (2,3,7,9) ⇒ (9,7) + (3,2) ✓

(7,3) → (1,2,5,9) × (9,5) → (1,2,3,7) × —— 1 case

125379 → (7,1), (9,3) ‖ (7,1) → (2,5,3,9) × ‖ (9,3) → (1,2,5,7) → (7,5) + (2,1) ✓

— 1 case

142659 → (5,2), (4,1), (9,6) ‖ (5,2) → (1,4,6,9) ×

(4,1) → (2,6,5,9) × ‖ (9,6) → (1,4,2,5) × —— nil

146259 → (9,2) → (1,4,6,5) × —— nil

152649 → (4,1), (5,2), (9,6) ‖ (4,1) → (5,2,6,9) × ‖ (5,2) → (1,6,4,9) × ‖ (9,6) → (1,5,2,4) ×

—— nil

156249 → (9,2) → (1,5,6,4) → (6,1) + (5,4) ✓ —— 1 case

173529 → (5,1), (7,3), (9,5) ‖ (5,1) → (7,3,2,9) × ‖ (7,3) → (1,5,2,9) × —— nil

175329 → (7,1), (9,3) ‖ (7,1) → (5,3,2,9) × —— nil

3 cases from (A)

Altogether 3 for 6 digits

~~81972~~ 72963
1 9 23769
9 1

5 digit cases repeated

(F) 81972 → (9,7) → (8,1,2) → × ‖ 83952 → (9,5) → (8,3,2) × ‖ 85932 → (9,3) → (8,5,2) ×

87912 → (9,1) ×. nil from (F)

(E) 72963 → (9,6) → (7,2,3) × ‖ (6,3) → (7,2,9) → (9,2) ✓ →

72963
~~976~~ 23769
96732

76923 → (9,2) → (7,6,3) × —— (1) one from (E)

(D) 61974 → (9,7) → (6,1,4) × ‖ (6,4) → (1,9,7) → (7,1) ✓ →

(6,4)

61974
14967
76941

63954 → (9,5) → (6,3,4) × ‖ 65934 → (9,3) + (6,5,4) ×

67914 → (9,1) × —— (1) one from (D)

(C) 41976 → (6,4) → (1,9,7) × ‖ (9,7) → (4,1,6) ×

43956 → (9,5) → (4,3,6) ×
45936 → (9,3) → (4,5,6) ×
47916 → (9,1) → (4,7,6) ×

nil from (C)

(B) 32967 → (6,3) → (2,9,7) ‖ → (9,6) → (3,2,7) ×

36927 → (9,2) → (3,6,7) → (6,3) ✓ ⟶

36927
32796
69723

(1) one from (B)

(6 digit case repeated)

(53)

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~~A~~ G — nil since there ~~are~~ is no 9 to go with last digit 2.
nil from (G)

F → Here only cases 1, 2, 7, 8 need be considered since there is 8 to go with 3.

712683 → (6,3) → (7,1,2,8) × — nil

716283 → (8,1) × — nil

782613 → (6,3) → (7,8,2,1) × — nil

786213 — × nil nil from F

E → nil since last digit with 4 → 4+8 = 12 & (4,7) → (1,1) and digits 1 & 2 do not appear where 8 & 7 appear — nil from E

D 413586 → (5,1), ~~(6,2)~~, ~~(7,3)~~, (8,4) ‖ (5,1) → (4,3,8,6) × ‖ (6,2) → (1,3,5,6) × — nil

415386 → × nil ‖ 423576 → (6,2), (7,3) ‖ (6,2) → (4,3,5,7) → ~~(5,4)+(7,3)~~ × nil

425376 → × nil — The remaining four cases give nil since there are no 8's or 9's.

~~1 case from (D) a~~ ~~423576~~ ~~342657~~ 33

nil from (D)

C 312687 → (6,3) → (1,2,6,8) × — nil

316287 → (8,1) → (3,6,2,7) × — nil

342657 → (5,2), (6,3), (7,4) ‖ (5,2) → (3,4,6,7) × — ~~nil~~

(6,3) → (4,2,5,7) × ‖ (7,4) → (3,2,6,5) × — nil

346257 → × → nil ; 352647 → (5,2), (6,3), (7,4) ‖ (5,2) → (3,6,4,7) ×

(6,3) → (5,2,4,7) × ; (7,4) → (3,5,2,6) × — nil

356247 → × — nil — other two cases → nil nil from (C)

B 231768 → (3,1), (8,6) ‖ (3,1) → (2,7,6,8) × ‖ (8,6) → (2,3,1,7) × — nil

237168 → × — nil ; 241758 → (4,2), (7,5) ‖ (4,2) → (1,7,5,8) × ‖ (7,5) → (2,4,1,8) × → nil

All those with 71 in the middle give nil. The others are 251748 → (4,2), (7,5)

(4,2) → (5,1,7,8) × ‖ (7,5) → (2,1,4,8) × — nil & 261738 → (3,1), (8,6)

(3,1) → (2,6,7,8) × ‖ (8,6) → (2,1,7,3) × — nil

nil from B

7469253 → (9,2) ✗ ——— nil

7529643 → (5,2), (6,3), (7,4), (9,6) ‖ (5,2) → (7,9,6,4,3) ✗ ‖ (6,3) → (7,5,2,9,4) ✗

(7,4) → (5,2,9,6,3) ✗ ‖ (9,6) ✗ ——— nil

7569243 → (9,2) ✗ ——— nil

7829613 → (6,3), (9,6) ‖ (6,3) → (7,8,2,9,1) ✗ ——— nil

7869213 → (8,1), (9,2) ——— nil

2 + 2 + 0 + 0 + 0 + 0 + 0 + 0 = 4 cases for (E)

(F) 8319762 → (3,1), (9,7) ‖ (3,1) → (8,9,7,6,2) ✗ ‖ (9,7) ✗ ——— nil

8379162 → (9,1) ✗ ——— nil

8419752 → (4,2), (7,5), (9,7) ‖ (4,2) → (8,1,9,7,5) → ~~(9,5)~~ + ✗

(7,5) → (8,4,1,9,2) → (8,4) + (9,1) ✓ ——— 1 case

8479152 → (9,1) ✗ ——— nil ‖

8519742 → (3,1), (4,2), (7,5), (9,7) ‖ (4,2) → (8,5,1,9,7) ✗

(7,5) → (8,1,9,4,2) ✗ ——— nil

8579142 → (9,1) ✗ ——— nil

8619732 → (3,1), (8,6), (9,7) ‖ (8,6) → (1,9,7,3,2) ✗ ——— nil

8679132 → (9,1) ✗ ——— nil

0 + 0 + 1 + 0 + 0 + 0 + 0 + 0 = 1 case from (F

For 7-digit case

Number of cases = 5 from (A)
" = 7 " (B)
" = 5 " (C)
" = 2 " (D)
" = 4 " (E)
" = 1 " (F)

24 cases.

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4739526 → (6,2), (7,3), (9,5) || (6,2) → (4,7,3,9,5) X
(7,3) X (9,5) X — no 8 to go with 3 X — nil

4759326 — X (no 8) — nil

4839516 → (5,1), (8,4), (9,5) || (8,4) → (3,9,5,1,6) X — nil
X X

4859316 → (9,3) — nil
X

0+1+3+1+0+0+0+0 = 5 cases for (C)

(D) 6139584 → (5,1), (8,4), (9,5) || (5,1) → (6,3,9,8,4) X || (8,4) → (6,1,3,9,5) → (6,5) + (9,3) ✓

6159384 → (9,3) → (6,1,5,8,4) X — nil

1 Case

6,2,39574 → (6,2), (7,3), (9,5) || (6,2) → (3,9,5,7,4) → (7,5) + (9,3) ✓

(7,3) → (6,2,9,5,4) → X || (9,5) → (6,2,3,7,4) X

1 Case

(2,59374) → (9,3) → (6,2,5,7,4) X — nil

6739524 → (6,2), (7,3), (9,5) || (6,2) → (7,3,9,5,4) → X
(7,3) → (6,9,5,3,4) X || (9,5) X — nil

6759324 → (9,3) X — nil

6839514 → (5,1), (9,3) — nil
X X

6859314 → (9,3) X — nil

1+0+1+0+0+0+0+0 = 2 cases for (D)

(E) 7129683 → (6,3), (9,6) || (6,3) → (7,1,2,9,8) → (9,7) + (8,2)
(8,7) + (9,2) ✓

(9,6) → (7,1,2,8,3) → (3,2) + (8,1) ✓

2 cases

71692 83 → (8,1), (9,2) || (8,1) → (7,6,9,2,3) → (7,6) + (9,2) ✓

(9,2) → (7,1,6,8,3) → (7,6) + (8,1) ✓

2 cases

7H29653 → (5,2), (6,3), (7,4), (9,6) || (5,2) → (7,4,9,6,3) X || (6,3) → (7,4,2,9,5) → X

(7,4) → (2,9,6,5,3) → X || (9,6) X

nil

(B) 3129687 → (6,3), (9,6) || (6,3) → (2,1,2,9,8,7) ✗ || (9,6) → (3,1,2,8,7) ✗ — nil

3169287 → ~~(6,3) (9,6) || (6,3) → (1,2,9,8,7)~~ (8,1), (9,2)

(8,1) → (3,6,9,2,7) ✗ || (9,2) → (3,1,6,8,7) → (8,7) + (6,3) ✓
→ (3,2) + (9,6) ✓

2 cases

3429657 → (5,2), (6,3) (7,4), (9,6) || (5,2) → (3,4,9,6,7) → (7,3) + (9,6) ✓

(6,3) → (4,2,9,5,7) → (9,5) + (7,4) ✓ || (7,4) → (3,2,9,6,5) → (9,5) + (6,3) ✓

(9,6) → (3,4,2,5,7 → (7,3) + (5,2) ✓ 4 cases in:

3429657	3429657	3429657	3429657
6324579	4532697	3542796	2364975
9754236	7962354	6972453	5794632

3469257 → (9,2) → (3,4,6,5,7 ✗ || nil

3529647 → (5,2) (6,3), (7,4), (9,6) || (5,2) → (3,9,6,4,7) → (9,4) + (6,3) ✓

(6,3) → (3,5,2,9,7) ✗ || (7,4) → (3,5,2,9,6) ✗ || (9,6) → (3,5,2,4,7) ✗

1 case

3569247 → ~~(6,3)~~, (9,2) || (9,2) → (3,5,6,4,7) ✗ — nil

3829617 → (6,3), (9,6) || (6,3) → (2,8,9,1,7) ✗ — nil
✗

3869217 → (8,1), (9,2) || ~~(8,1)~~ — nil
✗ ✗

7 cases from (B)

(C) 4139586 → (5,1), (8,4), (9,5) || (5,1) → (4,3,9,8,6) ✗

(8,4) → (1,3,9,5,6) → ✗ || (9,5) → (4,1,3,8,6) → Y — nil

4159386 → ~~(7,3)~~, (9,3) || (9,3) → (~~4~~,9,4,1,5,8,6) → (6,5) + (8,4) ✓ — 1 case

4239576 → (6,2), (7,3), (9,5) || (6,2) → (4,3,9,5,7) → ✗

(7,3) → (4,2,9,5,6) → (4,2) + (9,5) ✓ || (9,5) → (4,2,3,7,6) → (4,2) + (7,3) ✓
& (6,4) + (7,3) ✓

3 cases

~~4239576~~ 4259376 → (9,5) → (4,2,5,7,6) → (7,5) + (6,2) ✓

1 case

nil for 4th col of (E)

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i.e. 1 + 0 + 1 + 0 = 4 cases for (E)

nil for (F)

No of cases for 8 digits = ~~10~~ 8 from (A)
= 10 " (B)
= ~~4~~ " (C)
= ~~0~~ " (D)
= ~~2~~ " (E)
= 0 " (F)

24 ~~30~~ cases

Again scrutinised on 16/9/74
& found 24 & not 32

(25/8/74)

Let us repeat this systematic method for cases of 7, 6, 5 & 4 digits also:

7 digits: from (A) 2319768 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)

(3,1) → (2,6,7,8,9) → (9,6) → (2,7,8) × || (8,6) → (1,2,3,7,9) → × || (9,7) → (1,2,3,6,8) → (6,3) ×

2379168 → (9,1) → (2,3,6,7,8) → (6,3) ×

2419758 → ~~(4,2), (8,6), (9,7) || (3,1)~~ → (4,2), (7,5), (9,7)

(4,2) → (1,5,7,8,9) → (9,5) → (1,7,8) × || (7,5) → (1,2,4,8,9) → (8,4) → (1,2,9) ×

(9,7) → (1,2,4,5,8) → (5,1) + (4,2) ✓ (5,1) → (7,8,9) → (9,7) ✓

2 cases

2419758 / 7128459 / 9548217 || 2419758 / 2178954 / 4598712

2479158 → (9,1) → (2,4,5,7,8) → (8,4) + (7,5) ✓

1 case

5197148 → (4,2), (7,5), (9,7) || (4,2) → (5,1,9,7,8) → ×

(7,5) → (2,1,9,4,8) × || (9,7) → (2,5,1,4,8) × ⟶ nil

5791148 → (9,1) → (2,5,7,4,8) × ⟶ nil

2619738 → (3,1), (8,6), (9,7) || (3,1) → (2,6,9,7,8) → (8,2) + (9,7) ✓ → 2619738 / 1263819 / 7

(8,6) → (2,1,9,7,3) × || (9,7) → (2,6,1,3,8) → (8,2) + (3,1) ✓

2 cases.

2679138 → (9,1) → (2,6,7,3,8) × ⟶ nil

5 cases for (A)

1st col. of (E) · 71426583 → (4,1), (5,2), (6,3), (7,4), (8,5)
× ×

(4,1) → (2,3,5,6,7,8) → (6,2) → (3,5,7,8) ✗ || (5,2) → (1,3,4,6,7,8) → (7,3) → (1,4,6,8) → ✗
(7,3) → (2,5,6,8) ✗ (8,4) ✗
(6,3) → (1,2,4,5,7,8) → (5,1) ✗ || (7,4) → (1,2,3,5,6,8) → (5,1) ✗
(8,4) ✗ (6,2) → (1,3,5,8) ✗

71462583 ⟶ (8,1) ✗ —— nil
nil

71526483 → (5,2), (6,3), (7,4) || (5,2) → (1,3,4,6,7,8) → (6,1) ✗ ||
(8,3) ✗ ||
(6,3) → (1,2,4,5,7,8) → (7,2) → (1,4,5,8) → (5,4) + (8,1) ✓
(7,4) → (1,2,3,5,6,8) → (6,1) ✗
(8,3) ✗

1 case

71562483 → (8,1) ✗ —— nil

1 case for 1st col. of (E)

2nd col. of (E) · 74126853 ⟶ (5,2), (6,3), (7,4) || (5,2) → (1,3,4,6,7,8) → (4,3)
(7,6)
(8,7) ✗
(6,3) → (1,2,4,5,7,8) → (2,1) ✗
(5,4) → (1,2,7,8) ✗
(8,7) ✗
(6,3) → (1,6,7,8)
↓
(7,6) + (8,1) ✗
(7,6) → (1,3,4,8) → (4,3) + (8,1) ✗
(7,4) → (1,2,3,5,6,8) → (2,1) ✗
(3,2) → (1,5,6,8) → (6,5) + (8,1) ✗
(6,5) → (1,2,3,8) → (3,2) + (8,1) ✗

4 cases. nil

74162853 ⟶ (8,1) ✗ —— nil
nil

74826153 ——→ nil
74862153 ——→ nil

0 + 0 + 0 + 0 = 0 nil 4 cases for 2nd col. of (E)

3rd col. of (E) · 75126843 ⟶ (5,2), (6,3), (7,4) || (5,2) → (1,3,4,6,7,8) → (4,3) → (1,6,7,8) ✗
(7,6) → (1,3,4,8) ✗
(8,7) ✗
(6,3) → (1,2,4,5,7,8) → (2,1) ✗
(5,4) → (1,2,7,8) → (7,2) + (8,1) ✓
(8,7) ✗
(7,4) → (1,2,3,5,6,8) → (3,2) → (1,5,6,8) ✗
(6,5) → (1,2,3,8) ✗

1 case

75162843 —— nil ; 75826143 ⟶ nil ; 75862143 ⟶ nil

1 + 0 + 0 + 0 = 1 case for 3rd col. of (E)

$61735284 \to (5,1), (6,2), (7,3), (8,4)$ ‖ $(6,2) \to (1,3,4,5,7,8) \to (8,1) \to (3,4,5,7) \to (5,4) + (7,3)$ (47)
$(7,3) \to (1,2,4,5,6,8) \to (8,1) \to (2,4,5,6) \to (5,4) + (6,2)$

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2 cases. nil

$61753284 \to (7,1), (8,2)$ — nil

nil

~~2 cases~~ for 1st Col. of (D)

2nd Col. of (D). $62135874 \to (5,1), (6,2), (7,3), (8,4)$

$(5,1) \to (2,3,4,6,7,8) \to (3,2) \to (4,6,7,8)$ X ‖ $(6,2) \to (1,3,4,5,7,8) \to (4,3) \to (1,5,7,8)$ X
$(4,3) \to (2,6,7,8)$ X $(5,4) \to (1,3,7,8)$ X
$(7,6) \to (2,3,4,8)$ X $(8,7) \to (1,3,4,5)$ X
$(8,7) \to (2,3,4,6)$ X

$(7,3) \to (1,2,4,5,6,8) \to (2,1) \to (4,5,6,8)$ X ‖ $(8,4) \to (1,2,3,5,6,7) \to (2,1) \to (3,5,6,7)$ X
$(5,4) \to (1,2,6,8)$ X $(3,2) \to (1,5,6,7)$ X
$(6,5) \to (1,2,4,8)$ X $(6,5) \to (1,2,3,7)$ X
$(7,6) \to (1,2,3,5)$ X

nil

2 (~~cases~~) $62153874 \to (7,1), (8,2)$ ‖ $(7,1) \to (2,3,4,5,6,8) \to (8,2) \to (4,5,6,8)$ X
$(4,3) \to (2,5,6,8)$ X
$(5,4) \to (2,3,6,8)$ X
$(6,5) \to (2,3,4,8)$ X

$(8,2) \to (1,3,4,5,6,7) \to (4,3) \to (1,5,6,7)$ X
$(5,4) \to (1,3,6,7)$ X
$(6,5) \to (1,3,4,7)$ X
$(7,6) \to (1,3,4,5)$ X

nil

62835174 —— nil

62853174 —— nil

nil for 2nd Col. of (D).

3rd Col. of (D). $67135824 \to (5,1), (6,2), (7,3), (8,4)$

$(6,2) \to (1,3,4,5,7,8) \to (3,2) \to (4,6,7,8)$ ‖ $(7,3) \to (1,2,4,5,6,8) \to (2,1) \to (4,5,6,8)$ X
$(4,3) \to (1,5,7,8)$ X $(5,4) \to (1,2,6,8)$ X
$(5,4) \to (1,3,7,8)$ X $(6,5) \to (1,2,4,8)$ X
$(8,7) \to (1,3,4,5)$ X

nil

$67153824 \to (7,1), (8,2)$ — nil

67835124 ⟶ nil

67853124 ⟶ nil

nil for 3rd Col. of (D)

nil for 4th Col. of (D)

0 + 0 + 0 + 0 = 0 cases for (D) nil for (D)

$(8,4) \to (1,2,3,5,6,7) \to (2,1) \to (3,5,6,7) \to \times$
$(3,2) \to (1,5,6,7) \times$
$(6,5) \to (1,2,3,7) \times$
$(7,6) \to (1,2,3,5) \times$ ‖ nil

42153876 → (7,1), (8,2) ‖ $(7,1) \to (2,3,4,5,6,8) \to (3,2) \to (4,5,6,8) \times$
$(4,3) \to (2,5,6,8) \times$
$(5,4) \to (2,3,6,8) \times$
$(6,5) \to (2,3,4,8) \times$

$(8,2) \to (1,3,4,5,6,7) \to (4,3) \to (1,5,6,7) \times$
$(5,4) \to (1,3,6,7) \times$
$(6,5) \to (1,3,4,7) \times$
$(7,6) \to (1,3,4,5) \times$ ‖ nil

42835176 — nil

42853176 — nil

0 + 0 + 0 + 0 = ~~0~~ ~~cases for~~ nil for 2nd Colg (C).

3rd Colg (C). 47135826 → (5,1), (6,2), (7,3), (8,4)
× ×

$(5,1) \to (2,3,4,6,7,8) \to (3,2)$
$(4,3)$
$(7,6)$
$(8,7)$ | $(7,3) \to (1,2,4,5,6,8)$ → $(2,1) \to (4,5,6,8) \times$
$(5,4) \to (1,2,6,8) \to (8,1) + (6,2) \checkmark$
$(6,5) \to (1,2,4,8) \times$

$(6,2) \to (1,3,4,5,7,8) \to (4,3) \to (1,5,7,8) \times$
$(5,4) \to (1,3,7,8) \to (8,1) + (7,3) \checkmark$
$(8,7) \to (1,3,4,8) \to \times$

2 cases.

47153826 → (7,1), (8,2) — nil
× ×

47835126 — nil

47853126 — nil

2 + 0 + 0 + 0 = 2 cases for 3rd Colg (C) ✓

nil for 4th col. of (C)

~~2~~ cases 2 + 0 + 2 + 0 = 4 cases for (C)

1st col. of (D). 61235784 → (5,1), (6,2), (7,3), (8,4)

$(5,1) \to (2,3,4,6,7,8) \to (4,2) \to (3,6,7,8) \times$
$(6,4) \to (2,3,7,8) \times$
$(8,6) \to (2,3,4,7) \times$ ‖ $(6,2) \to (1,3,4,5,7,8) \to (3,1) \to (4,5,7,8) \times$
$(5,3) \to (1,4,7,8) \times$
$(7,5) \to (1,3,7,8) \times$

$(7,3) \to (1,2,4,5,6,8) \to (4,2) \to (1,5,6,8) \times$
$(6,4) \to (1,2,5,8) \times$
$(8,6) \to (1,2,4,5) \times$ ‖ $(8,4) \to (1,2,3,5,6,7) \to (3,1) \to (2,5,6,7) \times$
$(5,3) \to (1,2,6,7) \times$
$(7,5) \to (1,2,3,6) \times$

nil

61253784 → (7,1), (8,2) ‖ $(7,1) \to (2,3,4,5,6,8) \to (4,2) \to (3,5,6,8) \times$
$(5,3) \to (2,4,6,8) \times$
$(8,6) \to (2,3,4,5) \times$

$(8,2) \to (1,3,4,5,6,7) \to (3,1) \to (4,5,6,7) \times$
$(5,3) \to (1,4,6,7) \times$
$(7,5) \to (1,3,4,6) \times$ ‖ nil

35162847 → (8,1) → (2,3,4,5,6,7) → (3,2) → (4,5,6,7) ×
(4,3) → (2,5,6,7) ×
(5,4) → (2,3,6,7) → (7,2) + (6,3) ✓
(6,5) → (2,3,4,7) → ×
(7,6) → (2,3,4,5) ×

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1 case

35826147 ——— nil
35862147 ——— nil

4 + 1 + 0 + 0 = 5 cases for 3rd col. of (B)

4th col. of (B). ——— nil since 1 appears as the last but one digit

5 + 0 + 5 + 0 = 10 cases for (B)

1st col. of (C). 41235786 → (5,1), (6,2), (7,3), (8,4) || (5,1) → (2,3,4,6,7,8) → (4,2) → (3,6,7,8) ×
(6,4) → (2,3,7,8) ×
(8,6) → (2,3,4,7) ×

(6,2) → (1,3,4,5,7,8) → (3,1) → (4,5,7,8) ×
(5,3) → (1,4,7,8) ×
(7,5) → (1,3,4,8) × || (7,3) → (1,2,4,5,6,8) → (4,2) → (1,5,6,8) ×
(6,4) → (1,2,5,8) ×
(8,6) → (1,2,4,5) ×

(8,4) → (1,2,3,5,6,7) → (3,1) → (2,5,6,7) ×
(5,3) → (1,2,6,7) ×
(7,5) → (1,2,3,6) × || nil

41253786 → (7,1), (8,2) || (7,1) → (2,3,4,5,6,8) → (4,2) → (3,5,6,8) ×
(5,3) → (2,4,6,8) ×
(6,4) → (2,3,5,8) ×
(8,6) → (2,3,4,5) → (4,3) + (5,1) ✓ ×

(8,2) → (1,3,4,5,6,7) → (3,1) → (4,5,6,7) ×
(5,3) → ~~case~~ → (1,4,6,7) ×
(6,4) → (1,3,5,7) ×
(7,5) → (1,3,4,6) ×

+ case nil

41735286 → (5,1), (6,2), (7,3), (8,4) || (5,1) → (2,3,4,6,7,8) → ~~(8,1)~~ → ×
(6,2) → (1,3,4,5,7,8) → (8,1) → (3,4,5,7) → (5,4) + (7,3) ✓
(7,3) → (1,2,4,5,6,8) → (8,1) → (2,4,5,6) → (5,4) + (6,2) ✓

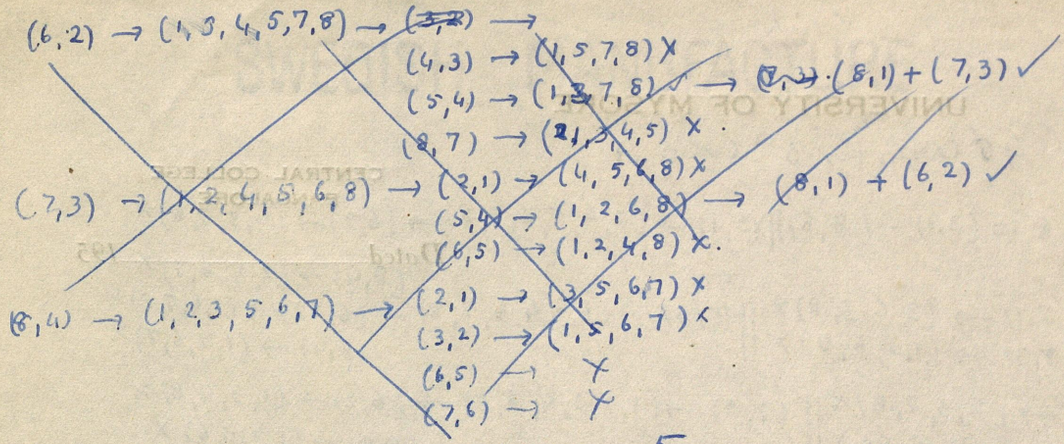
2 cases

41753286 → (7,1), (8,2) || (7,1) × (8,2) × nil

0 + 0 + 2 + 0 = 2 cases for 1st col of (C)

2nd col. of (C). 42135876 → (5,1), (6,2), (7,3), (8,4) || (5,1) → (2,3,4,6,7,8) → (3,2) → (4,6,7,8) ×
(4,3) → (2,6,7,8) ×
(7,6) → (2,3,4,8) ×
(8,7) → (2,3,4,6) ×

(6,2) → (1,3,4,5,7,8) → (4,3) → (1,5,7,8) × ~~(4,7) + (5,1)~~ ||
(5,4) → (1,3,7,8) ×
(8,7) → (1,3,4,5) × (7,3) → (1,2,4,5,6,8) → (2,1) → (4,5,6,8) ×
(5,4) → (1,2,6,8) ×
(6,5) → (1,2,4,8) ×



0 + 0 + 4 + 1 = 5 cases for 1st col. of (B)

2nd col. of B. 34126857 → (4,1) -- (8,5) ‖ (4,1) → (2,3,5,6,7,8) → (2,1) → (5,6,7,8) X
(6,5) → (2,3,7,8) X
(7,6) → (2,3,5,8) X
(8,7) → (2,3,5,6) X

(5,2) → (1,3,4,6,7,8) → (4,3) → (1,6,7,8) X
(7,6) → (1,3,4,8) X
(8,7) → (1,3,4,6) X
‖ (6,3) → (1,2,4,5,7,8)
→ (2,1) → (4,5,7,8) → X
(5,4) → (1,2,7,8) → X
(8,7) → (1,2,4,5) X

(7,4) → (1,2,3,5,6,8) → (2,1) → (3,5,6,8) X
(3,2) → (1,5,6,8) X
(6,5) → (1,2,3,8) X
‖ (8,5) → (1,2,3,4,6,7) → (2,1) → (3,4,6,7) X
(3,2) → (1,4,6,7) X
(4,3) → (1,2,6,7) X
(7,6) → (1,2,3,4) X

nil

34162857 → (8,1) → (2,3,4,5,6,7) → (3,2) → (4,5,6,7) X
(4,3) → (2,5,6,7) X
(5,4) → (2,3,6,7) X
‖ (6,5) → (2,3,4,7) X
(7,6) → (2,3,4,5) X

nil

34826157
34862157
— nil (since 1 is one of the last four digits

0 + 0 + 0 + 0 = nil for 2nd col. of (B)

3rd col. of (B). 35126847 → (4,1) -- (8,5) ‖ (4,1) → (2,3,5,6,7,8) → (3,2) → (5,6,7,8) X
(6,5) → (2,3,7,8) X
(7,6) → (2,3,5,8) → (8,3) + (5,2) ✓
(8,7) → (2,3,5,6) X

(5,2) → (1,3,4,6,7,8) → (4,3) → (1,6,7,8) X
(7,6) → (1,3,4,8) → (8,3) + (4,1) ✓
(8,7) → (1,3,4,6) X

(6,3) → (1,2,4,5,7,8) → ~~(4,3) → (1,5,7,8) X~~
~~(5,4) → (1,3,7,8) X~~
~~(8,7) → (1,3,4,5) X~~
→ (2,1) → (4,5,7,8) X
(5,4) → (1,2,7,8) X
(8,7) → (1,2,4,5) X

(7,4) → (1,2,3,5,6,8) → (2,1) → (3,5,6,8) X
(3,2) → (1,5,6,8) → (6,1) + (8,5) ✓
(6,5) → (1,2,3,8) X
‖ (8,5) → (1,2,3,4,6,7) → (2,1) → (3,4,6,7) X
(3,2) → (1,4,6,7) → (6,1) + (7,4) ✓
(4,3) → (1,2,6,7) X
(7,6) → (1,2,3,4) X

4 cases.

nil

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4 cases for 3rd col of A

4th col of (A). 26417538 → (3,1) .. (8,6) || (3,1) → (2,4,5,6,7,8) → (6,2) → (4,5,7,8) ×
(8,4) → (2,5,6,7) ×

(4,2) → (1,3,5,6,7,8) → (5,1) → (3,6,7,8) × || (5,3) → (1,2,4,6,7,8) → (6,2) → (1,4,7,8) ×
(7,3) → (1,5,6,8) × (8,4) → (1,2,6,7) ×

(6,4) → (1,2,3,5,7,8) → (5,1) → (2,3,7,8) × || (7,5) → (1,2,3,4,6,8) → (6,2) → (1,3,4,8) ×
(7,3) → (1,2,5,8) × (8,4) → (1,2,3,6) ×

(8,6) → (1,2,3,4,5,7) → (5,1) → (2,3,4,7) ×
(7,3) → (1,2,4,5) ×

nil

26517438 → (3,1) .. (8,6) || (3,1) → (2,4,5,6,7,8) → (7,2) → (4,5,6,8) ×

(4,2) → (1,3,5,6,7,8) → (6,1) → (3,5,7,8) × || (5,3) → (1,2,4,6,7,8) → (6,1) → (2,4,7,8) ×
(8,3) → (1,5,6,7) × (7,2) → (1,4,6,8) ×

(6,4) → (1,2,3,5,7,8) → (7,2) → (1,3,5,8) × || (7,5) → (1,2,3,4,6,8) → (6,1) → (2,3,4,8) ×
(8,3) → (1,2,5,7) × (8,3) → (1,2,4,6) ×

(8,6) → (1,2,3,4,5,7) → (7,2) → (1,3,4,5) ×

nil

nil from 4th col of (A)

4 + 0 + 4 + 0 = 8 from (A)

1st col of B. 31426587 → (4,1), (5,2), (6,3), (7,4), (8,5), (8,6)

(4,1) → (2,3,5,6,7,8) → (6,2) → (3,5,7,8) → × || (5,2) → (1,3,4,6,7,8) → (7,3) → (1,4,6,8) ×
(7,3) → (2,5,6,8) → × (8,4) → (1,3,6,7) ×

(6,3) → (1,2,4,5,7,8) → (5,1) → (2,4,7,8) → × || (7,4) → (1,3,5,6,8) → (6,2) → (1,3,5,8) ×
(8,4) → (1,3,5,7) → × (5,1) → (2,3,6,8) ×

(8,5) → (1,2,3,4,6,7) → (6,2) → (1,3,4,7) × || nil
(7,4) → (1,2,3,6) ×

31462587 → (8,1) → (2,3,4,5,6,7) → (6,2) → (3,4,5,7) × nil
(7,3) → (2,4,5,6) ×

31526487 → (4,1) .. (8,6) || (4,1) → (2,3,5,6,7,8) → (7,2) → (3,5,6,8) ×
(8,3) → (2,5,6,7) → (7,6) + (5,2) ✓

(5,2) → (1,3,4,6,7,8) → (6,1) → (3,4,7,8) ×
(8,3) → (1,4,6,7) → (7,6) + (4,1) ✓ || (6,3) → (1,2,4,5,7,8) → (7,2) → (1,4,5,8) ×

(7,4) → (1,2,3,5,6,8) → (6,1) → (2,3,5,8) → (3,2) + (8,5) ✓ || (8,6) → (1,2,3,4,6,7) → (7,2) → (1,3,4,6) ×
(8,3) → (1,2,5,6) × (6,1) → (2,3,4,7) →
→ (3,2) + (7,4) ✓

4 cases

31562487 → (8,1) → (2,3,4,6,6,7) → (7,2) → (3,4,5,6) → (5,4) + (6,3) ✓

1 case

(5,3) → (1,2,4,6,7,8) → (6,2) → (1,4,7,8) ✗ ‖ (6,4) → (1,2,3,5,7,8) → (5,1) → (2,3,7,8) ✗
(8,4) → (1,2,6,7) ✗ (7,3) → (1,2,5,8) ✗

(7,5) → (1,2,3,4,6,8) → (6,2) → (1,3,4,8) ✗ ‖ (8,6) → (1,2,3,4,5,7) → (5,1) → (0,2,3,4,7) ✗
(8,4) → (1,2,3,6) ✗ (7,3) → (1,2,4,5) ✗

23517468 → (3,1), (4,2), (5,3) ‖ nil (3,1) → (2,4,5,6,7,8) → (7,2) → (0,4,5,6,8)
(6,4), (7,5), (8,6) → (8,5) + (6,4) ✓

(4,2) → (1,3,5,6,7,8) → (6,1) → (3,5,7,8) ✗ ‖ (5,3) → (1,2,4,6,7,8) → (6,1) → (2,4,7,8) ✗
(8,3) → (1,5,6,7) ✗ (7,2) → (1,4,6,8) → (4,1) + (8,6) ✓

(6,4) → (1,2,3,5,7,8) → (7,2) → (1,3,5,8) → (8,5) + (3,1) ✓ ‖ (7,5) → (1,2,3,4,6,8) → (6,1) → (2,3,4,8) ✗
(8,3) → (1,2,5,7) ✗ (8,3) → (1,2,4,6) ✗

(8,6) → (1,2,3,4,5,7) → (7,2) → (1,3,4,5) → (4,1) + (5,3) ✓

4 cases for 1st col of (A).

2nd col of (A). 24317658 → (3,1), - - (8,6) ‖ (3,1) → (2,4,5,6,7,8) → (5,2) → (4,6,7,8) ✗
(7,4) → (2,5,6,8) ✗
(8,5) → (2,5,6,7) → (6,2) + (7,5) ✓

(4,2) → (1,3,5,6,7,8) → (6,3) → (1,5,7,8) ✗ ‖ (5,3) → (1,2,4,6,7,8) → (4,7) → (2,6,7,8) ✗
(8,5) → (1,3,6,7) ✗ (8,4) → (1,2,6,8) ✗

(6,4) → (1,2,3,5,7,8) → (5,2) → (1,3,7,8) ✗ ‖ (7,5) → (1,2,3,4,6,8) → (4,2) → (2,3,6,8) ✗
(8,3) → (1,2,5,8) ✗ (6,3) → (1,2,4,8) ✗

(8,6) → (1,2,3,4,5,7) → (4,1) → (2,3,5,7) ✗
(5,2) → (1,3,4,7) ✗
(7,4) → (1,2,3,5) ✗.

1 case [this is wrong with (7,4) as ✓ above]

24617358 → (3,1) - - (8,6) ‖ (3,1) → (2,4,5,6,7,8) → (7,1) → (2,4,5,8) ✗
(8,2) → (4,5,6,7) ✗

(4,2) → (1,3,5,6,7,8) → (7,1) → (3,5,6,8) ✗ ‖ (5,3) → (1,2,4,6,7,8) → (7,1) → (2,4,6,8) ✗
(8,2) → (1,4,6,7) ✗

(6,4) → (1,2,3,5,7,8) → (7,1) → (2,3,5,8) ✗ ‖ (7,5) → (1,2,3,4,6,8) → (8,2) → (1,3,4,6) ✗
(8,2) → (1,3,5,7) ✗

(8,6) → (1,2,3,4,5,7) → (7,1) → (2,3,4,5) ✗ nil

nil 1 case for 2nd col of (A)

3rd col of A. 25317648 → (3,1) - - (8,6) ‖ (3,1) → (2,4,5,6,7,8) → (5,2) → (4,6,7,8) → ✗
(7,4) → (2,5,6,8) → (5,2) + (8,6) ✓
(8,5) → (2,4,6,7) → (7,2) + (6,4) ✓

(4,2) → (1,3,5,6,7,8) → (6,3) → (1,5,7,8) ✗ ‖ (5,3) → (1,2,4,6,7,8) → (4,1) → (7,4) ✗ → (2,6,7,8) → (7,2) + (8,6) ✓
(8,5) → (1,3,6,7) ✗

(6,4) → (1,2,3,5,7,8) → (5,2) → (1,3,7,8) ✗
(8,5) → (1,2,3,7) → (7,2) + (3,1) ✓ ‖ (7,5) → (1,2,3,4,6,8) → (4,1) ✗
→ (6,3) → (1,2,4,6) ✗
→ (6,1) + (4,2) ✓

(8,6) → (1,2,3,4,5,7) → (4,1) → (2,3,5,7) → (7,2) + (5,3) ✓
(5,2) → (1,3,5,7) ✗
(7,4) → (1,2,3,5) ✗

5 cases 4 cases

25617348 → (3,1) - - (8,6) ‖ (3,1) → (2,4,5,6,7,8) → (8,2) → (4,5,6,7) ✗

(4,2) → (1,3,5,6,7,8) → (7,1) → (3,5,6,8) ✗ ‖ (5,3) → (1,2,4,6,7,8) → (7,1) → (2,4,6,8) ✗
(8,2) → (1,4,6,7) ✗

(6,4) → (1,2,3,5,7,8) → (7,1) → (2,3,5,8) → ✗
(8,2) → (1,3,5,7) → ✗ ‖ (7,5) → (1,2,3,4,6,8) → (8,2) → (1,3,4,6) ✗
(8,6) → (1,2,3,4,5,7) → (7,1) → (2,3,4,5) ✗

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7+6+2+1 = 16 cases for (F)

From (A) → 98
" (B) → 89
" (C) → 74
" (D) → 31
" (E) → ~~35~~ 34
" (F) → 16

Total for 9 digits } = 342 Cases.

The 96 numbers give 343 cases so that there must be different ways for many of the numbers.

Later found a mistake. Should be 342 BSM 17/9/74

Final Summary for this Curio! —

(1) for 3 digit numbers — 1 case.
— ~~3~~ ~~cases~~ 3 cases
(2) " 4 " — 4 cases.
(3) " 5 " — 3 cases.
(4) " 6 " — ~~19 cases~~ 24 cases
(5) " 7 " — ~~31 cases~~ 24 cases.
(6) " 8 " —
(7) " 9 " — 343

I find that for the case of 8 digits I have not attempted yet the numbers in the same systematic manner as for the case of 9 digits. This is done below! —

1st Case (A) only the 8 cases where middle digits are 17 need be considered, since 71 requires digit 9 for 2+9 ≠ 9 is not available:

23417568 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6)

(3,1) → (2,4,5,6,7,8) → (6,2) → (4,5,7,8) ✗
(8,4) → (2,5,6,7) ✗

(4,2) → (1,3,5,6,7,8) → (5,1) → (3,6,7,8) ✗
(7,3) → (1,5,6,8) ✗

(40) 3rd col.q (F). 853197642 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)
x x

(4,2) → (1,3,5,6,7,8,9) → (6,3) → (1,5,7,8,9) x || (5,3) → (1,2,4,6,7,8,9) → (7,4)
→ (8,5) → (1,5,6,7,9) x || → (1,4,6,8,9) x

(6,4) → (1,2,3,5,7,8,9) → (5,2) → (1,3,7,8,9) → (8,3) + (9,1) ✓
(8,5) → (1,2,3,7,9) → (7,2) + (9,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (6,3) → (1,2,4,8,9) → x

(8,6) → (1,2,3,4,5,7,9) → (5,2) → (1,3,4,7,9) → x
(7,4) → (1,2,3,5,9) → x

2 cases

853791642 → (9,1) — nil

856197342 → (4,2), (5,3), (6,4), (7,5), (8,6).

(4,2) → (1,3,5,6,7,8,9) → x || (5,3) → (1,2,4,6,7,8,9) → (8,2) → (1,4,6,7,9) → x
(6,4) → (1,2,3,5,7,8,9) → (8,2) → (1,3,5,7,9) x || (7,5) → (1,2,3,4,6,8,9) → (8,2) → (1,3,4,6,9) x
(8,6) → (1,2,3,4,5,7,9) → x nil

856791342 → (9,1) → nil

2 cases for 3rd col.q (F).

4th col.q (F). 864197532 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)
x x

(4,2) → (1,3,5,6,7,8,9) → (5,1) x (7,3) → (1,5,6,8,9) x
(5,3) → (1,2,4,6,7,8,9) → ~~(6,2)~~ ~~(1,3,5)~~ (6,2) → (1,4,7,8,9) x
(7,3) (8,4) → (1,2,6,7,9) x
(6,4) → (1,2,3,5,7,8,9) → ~~(8,2)~~ (7,3) → (1,2,5,8,9) → (8,2) + (9,1) ✓
(7,5) → (1,2,3,4,6,8,9) → ~~(2,5) x~~ (6,2) → (1,3,4,8,9) → x
(8,4) → (1,2,3,6,9) → x
(8,6) → (1,2,3,4,5,7,9) → (7,3) → (1,2,4,5,9) → x.

1 case

864791532 → nil

865197432 → (4,2), (5,3), (6,4), (7,5), (8,6)

(4,2) → (1,3,5,6,7,8,9) → (8,3) → (1,5,6,7,9) x
(5,3) → (1,2,4,6,7,8,9) → (7,2) → (1,4,6,8,9) x
(6,4) → (1,2,3,5,7,8,9) → (7,2) → (1,3,5,8,9) x
(8,3) → (1,2,4,5,7,9) x
(7,5) → (1,2,3,4,6,8,9) → (8,3) → (1,2,4,6,9) x
(8,6) → (1,2,3,4,5,7,9) → (7,2) → (1,3,4,5,9) x nil

864791432 — nil

1 case for 4th col.q.(F)

20 + 8 + 7 + 0 = (35) cases for (E) (34)

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(Column (F) appears most fertile) really not so

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1st column of (F). 834197562 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)

(3,1) ×, (4,2) → (1,3,5,6,7,8,9) → (5,1) × (7,3) → (1,5,6,8,9) → (8,5) + (9,1) ✓ (5,3) → (1,6,7,8,9) → (6,2) → (1,3,4,6,9) × (8,4) → (1,2,6,7,9) ×

(6,4) → (1,2,3,5,7,8,9) → (7,3) → (1,2,5,8,9) → (8,5) + (9,1) ✓ (5,2) + (9,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (6,2) → (1,3,4,8,9) × (8,4) → (1,2,3,6,9) → (6,3) + (9,1) ✓ || (8,6) → (1,2,3,4,5,7,9) → (7,3) → (1,2,4,5,9) → (5,2) + (9,1) ✓

(9,7) ×

5 cases

834791562 → (9,1) → (2,3,) × nil

835197462 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7) × || (4,2) → (1,3,5,6,7,8,9) → (8,3) → (1,5,6,7,9) ×

(5,3) → (1,2,4,6,7,8,9) → (7,2) → (1,4,6,8,9) × || (6,4) → (1,2,3,5,7,8,9) → (7,2) → (1,3,5,8,9) → (8,5) + (9,1) ✓ (8,3) → (1,2,5,7,9) → (5,2) + (9,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (8,3) → (1,2,4,6,9) × || (8,6) → (1,2,3,4,5,7,9) → (7,2) → (1,3,4,5,9) → ×

2 cases

835791462 → (9,1) × nil

7 cases for 1st col. of (F)

2nd col. of (F). 843197652 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7) ×

(4,2) → (1,3,5,6,7,8,9) → (6,3) → (1,5,7,8,9) → × (8,5) → (1,3,6,7,9) → (7,3) + (9,1) ✓

(5,3) → (1,2,4,6,7,8,9) → (7,4) → (1,2,6,8,9) → (6,2) + (9,1) ✓ || (6,4) → (1,2,3,5,7,8,9) → (8,5) (5,2) (8,5) → (1,2,3,7,9) → (7,3) + (9,1) ✓ (5,2) → (1,3,7,8,9) → (7,3) + (9,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (6,3) → (1,2,4,8,9) → (8,4) + (9,1) ✓ || (8,6) → (1,2,3,4,5,7,9) → (7,4) → (1,2,3,5,9) ×

5 cases

843791652 → (9,1) × nil

846197352 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7) × || (4,2) → (1,3,5,6,7,8,9) → × (5,3) → (1,2,4,6,7,8,9) → (8,2) → (1,4,6,7,9) →

(6,4) → (1,2,3,5,7,8,9) → (8,2) → (1,3,5,7,9) → (7,3) + (9,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (8,2) → (1,3,4,6,9) → × || (8,6) → (1,2,3,4,5,7,9) → ×

1 case

846971352 → (9,1) × nil

6 cases for 2nd col. of (F)

(38) (9,2) → (1,3,4,5,6,7,8) → (4,3) → (1,5,6,7,8) → x
(5,4) → (1,3,6,7,8) → x
(6,5) → (1,3,4,7,8) → x
(7,6) → (1,3,4,5,8) → x
(8,7) → (1,3,4,5,6) → x

nil

758296143 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → (2,3,5,6,7,8,9) → x ~~(9,1)~~ → (5,2) → ~~(3,6,7,8,9)~~ → (1,2,4,6,7,8,9) → (9,1) → (2,4,6,7,8) x

(6,3) → (1,2,4,5,7,8,9) → (9,1) → (2,4,5,7,8) → x; (7,4) → (1,2,3,5,6,8,9) → (9,1) → (2,3,5,6,8) x

(8,5) → (1,2,3,4,6,7,9) → (9,1) → (2,3,4,6,7) x; (9,6) → (1,2,3,4,5,7,8) → x

nil

758692143. → (8,1), (9,2); (8,1) → (2,3,4, --) → x
(9,2) → (1,3, 8) → x.

nil

7 cases for 3rd col of (E).

4th column of (E). 784296513 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

(4,1) → (2,3,5,6,7,8,9) → (6,2) → (3,5,7,8,9) x
(7,3) → (2,5,6,8,9) x
(9,5) → (2,3,6,7,8) x

(5,2) → (1,3,4,6,7,8,9) → (7,3) → (1,4,6,8,9) → x
(8,4) → (1,3,6,7,9) → x

(6,3) → (1,2,4,5,7,8,9) → (8,4) → (1,2,5,7,9) → x
(9,5) → (1,2,4,7,8) → x

(7,4) → (1,2,3,5,6,8,9) → (5,1) → (2,3,6,8,9) → x
(6,2) → (1,3,5,8,9) → x
(9,5) → x

nil

(8,5) → (1,2,3,4,6,7,9) → (6,2) → (1,3,4,7,9) x
(7,3) → (1,2,4,6,9) x

(9,6) → x

784692513 → (8,1), (9,2)
& both (8,1) & (9,2) → x

nil

785296413 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → x, (5,2) → (1,3,4,6,7,8,9) → (8,3) → x
(9,4) → x

(7,4) → (1,2,3,5,6,8,9) → (6,1) → x
(8,3) → (1,2,5,6,9) x

(9,6) → x

(6,3) → (1,2,4,5,7,8,9) → (7,2) → (1,4,5,8,9) x
(9,4) → x

(8,5) → (1,2,3,4,6,7,9) → (6,1) → x
(7,2) → (1,3,4,6,9) → x
(9,4) → x

nil

785692413 → (8,1), (9,2) & both give no cases.

nil

nil for 4th column of (E)

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748296153 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

(4,1) → (2,3,5,6,7,8,9) → ✗ ; (5,2) → (1,3,4,6,7,8,9) → (9,1) → (3,4,6,7,8) ✗

(6,3) → (1,2,4,5,7,8,9) → (9,1) → (2,4,5,7,8) → ✗

(7,4) → (1,2,3,5,6,8,9) → (9,1) → ✗ ; (8,5) → (1, -9) → (9,1) → ✗ ; (9,6) → ✗

nil

748692153 → (8,1), (9,2) ; (8,1) → (2,3,4,5,6,7,9) → ✗ ; (9,2) → (1,3,4,5,6,7,8) → ✗

nil

8 cases for 2nd column of (E)

3rd column of (E).

751296843 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

(4,1) → (2,3,5,6,7,8,9) → (3,2) → (5,6,7,8,9) → ✗
(6,5) → (2,3,7,8,9) → (8,3) + (9,2) ✓
(7,6) → (2,3,5,8,9) → (8,3) + (9,2) ✓
(8,7) → (2,3,5,6,9) → ✗
(9,8) → ✗

(5,2) → (1,3,4,6,7,8,9) → (4,3) → (1,6,7,8,9) → ✗
(7,6) → (1,3,4,8,9) → (9,4) + (8,1) ✓
(8,7) → (1,3,4,6,9) → ✗
(9,8) → ✗

(6,3) → (1,2,4,5,7,8,9) → (2,1) → (4,5,7,8,9) → ✗
(5,4) → (1,2,7,8,9) → (7,2) + (8,1) ✓
(8,7) → (1,2,4,5,9) → ✗
(9,8) → (1,2,4,5,7) → ✗

(7,4) → (1,2,3,5,6,8,9) → (2,1) → (3,5,6,8,9) → ✗
(3,2) → (1,5,6,8,9) → ✗
(6,5) → (1,2,3,8,9) → ~~✗~~ (8,3)
(9,8) → (1,2,3,5,6) → ✗

(8,5) → (1,2,3,4,6,7,9) → (2,1) → (3,4,6,7,9) → ✗
(3,2) → (1,4,6,7,9) → ✗
(4,3) → (1,2,6,7,9) → (6,1) + (9,2) ✓
(7,6) → (1,2,3,4,9) → ✗

(9,6) → (1,2,3,4,5,7,8) → (2,1) → (3,4,5,7,8) → ✗
(3,2) → (1,4,5,7,8) → ✗
(4,3) → (1,2,5,7,8) → (7,2) + (8,1) ✓
(5,4) → (1,2,3,7,8) → (7,2) + (8,1) ✓
(8,7) → (1,2,3,4,5) → ✗

7 cases

751692843 → (8,1), (9,2).

(8,1) → (2,3,4,5,6,7,9) → (3,2) → (4,5,6,7,9) → ✗
(4,3) → (2,5,6,7,9) → ✗
(6,4) → (2,3,6,7,9) → ✗
(6,5) → (2,3,4,7,9) → ✗
(7,6) → (2,3,4,5,9) → ✗

(36) (9,2) → (1,3,4,5,6,7,8) → (6,1) → (3,4,5,7,8) → ×

1 case

19 20 cases for 1st col. of (E)

2nd col. of (E): 741296853 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → (2,3,5,6,7,8,9) → (3,2) → (5,6,7,8,9) ×
(6,5) → (2,3,7,8,9) → (7,3) + (9,2) ✓
(7,6) → (2,3,5,8,9) → ×
(8,7) → (2,3,5,6,9) → ×
(9,8) → (2,3,5,6,7) ×

(5,2) → (1,3,4,6,7,8,9) → (4,3) → (1,6,7,8,9) → ×
(7,6) → (1,3,4,8,9) → ×
(8,7) → (1,3,4,6,9) → ×
(9,8) → ×

(6,3) → (1,2,4,5,7,8,9) → (2,1) → ×
(5,4) → (1,2,7,8,9) → ×
(8,7) → (1,2,4,5,9) → (5,1) + (9,2) ✓
(9,8) → (1,2,4,5,7) → ×

(7,4) → (1,2,3,5,6,8,9) → (2,1) → (3,5,6,8,9) → ×
(3,2) → (1,5,6,8,9) → (9,5) + (8,1) ✓
(6,5) → (1,2,3,8,9) → ×
(9,8) → ×

(8,5) → (1,2,3,4,6,7,9) → (2,1) → ×
(3,2) → (1,4,6,7,9) ×
(4,3) → (1,2,6,7,9) ×
(7,6) → (1,2,3,4,9) ×

(9,6) → (1,2,3,4,5,7,8) → (2,1) → (3,4,5,7,8) ×
(3,2) → (1,4,5,7,8) ×
(4,3) → (1,2,5,7,8) ×
(5,4) → (1,2,3,7,8) → (7,3) + (8,1) ✓
(8,7) → (1,2,3,4,5) ×

4 Cases

741692853 → (8,1), (9,2)

(8,1) → (2,3,4,5,6,7,9) → (3,2) → (4,5,6,7,9) → ×
(4,3) → (2,5,6,7,9) → ×
(5,4) → (2,3,6,7,9) → (7,3) + (9,2) ✓
(6,5) → (2,3,4,7,9) → (7,3) + (9,2) ✓
(7,6) → (2,3,4,5,9) → ×

(9,2) → (1,3,4,5,6,7,8) → (4,3) → (1,5,6,7,8) → ×
(5,4) → (1,3,6,7,8) → (7,3) + (8,1) ✓
(6,5) → (1,3,4,7,8) → (7,3) + (8,1) ✓
(7,6) → (1,3,4,5,8) → ×
(8,7) → (1,3,4,5,6) → ×

4 cases

(6,3) → (1,2,4,5,7,8,9) → (5,1) → (2,4,7,8,9) → (8,7) + (9,2) ✓ (35)

(8,4) → (1,2,5,7,9) ✗

(9,5) → (1,2,4,7,8) → ✗

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(7,4) → (1,2,3,5,6,8,9) → (5,1) → (2,3,6,8,9) → ✗

(6,2) → (1,3,5,8,9) → ✗

(9,5) → (1,2,3,6,8) → (3,2) + (8,1) ✓

(8,5) → (1,2,3,4,6,7,9) → (6,2) → (1,3,4,7,9) ✗

(7,3) → (1,2,4,6,9) ✗

(9,6) → (1,2,3,4,5,7,8) → (5,1) → (2,3,4,7,8) ✗

(7,3) → (1,2,4,5,8) → (5,4) + (8,1) ✓

(8,4) → (1,3,3,5,7) ✗

4 cases only

714692583 → (8,1), (9,2).

(8,1) → (2,3,4,5,6,7,9) → (6,2) → (3,4,5,7,9) ✗

(7,3) → (2,4,5,6,9) → (5,4) + (9,2) ✓ & (6,5) + (9,2) ✓

(9,5) → (2,3,4,6,7) ✗

(9,2) → (1,3,4,5,6,7,8) → (5,1) → (3,4,6,7,8) ✗

(7,3) → (1,4,5,6,8) → (5,4) + (8,1) ✓ & (6,5) + (8,1) ✓

(8,4) → (1,3,5,6,7) ✗ & (7,6) + (8,1) ✗

5 cases.

715296483 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → (2,3,5,6,7,8,9) → (7,2) → (3,5,6,8,9) ✗

(8,3) → (2,5,6,7,9) → (6,5) + (9,2) & (7,6) + (9,2) ✓

(5,2) → (1,3,4,6,7,8,9) → (6,1) → (3,4,7,8,9) ✗

(8,3) → (1,4,6,7,9) ✗

(9,4) → (1,3,6,7,8) → (7,6) + (8,1) ✓ & (7,6) + (8,1) ✓

(6,3) → (1,2,4,5,7,8,9) → (7,2) → (1,4,5,8,9) → (5,4) + (8,1) ✓

(9,4) → (1,2,5,7,8) → ✗

(7,4) → (1,2,3,5,6,8,9) → (6,1) → (2,3,5,8,9) → ✗

(8,3) → (1,2,5,6,9) → (6,5) + (9,2) ✓

(8,5) → (1,2,3,4,6,7,9) → (6,1) → (2,3,4,7,9) → (4,3) + (9,2) ✓

(7,2) → (1,3,4,6,9) → ✗

(9,4) → (1,2,3,6,7) → ✗

(9,6) → (1,2,3,4,5,7,8) → (6,1) → (2,3,4,5,8) ✗

(7,2) → (1,3,4,5,8) → (4,3) + (8,1) ✓ & (4,3) + (8,1) ✓

(8,3) → (1,2,5,7,8) ✗ & (5,4) + (8,1) ✓

10 cases

715692483 → (8,1), (9,2)

(8,1) → (2,3,4,5,6,7,9) → (7,2) → (3,4,5,6,9) → ✗

(4,1) → (2,3,5,6,7) → (6,5) + (9,2) ✗

4) 671593824 → (7,1), (8,2), (9,3). (7,1) → (2,3,4,5,6,8,9) → (3,2), (4,3), (5,4), (9,8)
× × × ×

(8,2) × & (9,3) ×.

nil

678395124 → (5,1), (6,2), (7,3), (8,4), (9,5).

(5,1) ×, (6,2) ×, (7,3) → (1,2,4,5,6,8,9) → (9,1) → ×, (8,4) → (1,2,3,5,6,7,9) ×, (9,5) ×

nil

78593124 → (7,1), (8,2), (9,3)
× × ×

nil

671395824
125438697
796834521

1 case only for 3rd col. of D i.e.

4th col. of (D). 682395714 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1), (6,2) → (1,3,4,5,7,8,9) → (3,1), ×, (5,3), (9,7) × → (1,4,7,8,9) ×
×
(7,5) → (1,3,4,8,9) ×

(7,3) → (1,2,4,5,6,8,9) → (4,2) → (1,5,6,8,9) ×
(6,4) → (1,2,5,8,9) → (9,1) + (8,2) ✓
(8,6) → (1,2,4,5,9) → ×

(1,2,3,5,6,7,9) → (5,3) → (1,2,6,7,9) ×
(7,5) → (1,2,5,6,9) ×
~~(9,7)~~

1 only

682395714
214357698
896753412

593714 → (7,1), (8,2), (9,3). (8,2) → (1,3,4,5,6,7,9) → (3,1) ×, (9,7) ×
× ×
(6,4)
(7,5)
(5,3)

(6,4) → (1,3,5,7,9) ×; (7,5) → (1,3,4,6,9) ×; (5,3) → (1,4,6,7,9) ×

nil

395214 → (5,1), (6,2), (7,3), (8,4), (9,5); (6,2) → (1,3,4,5,7,8,9) → (8,1) ×
× ×

7,3) → (1,2,4,5,6,8,9) → (8,1) × & (9,2) × || (8,4) → (1,2,3,5,6,7,9) → (9,2) ×

nil

87593214 → (7,1), (8,2), (9,3) || (8,2) → (1,3,4,5,6,7,9) → ×
× ×

nil

1 + 0 + 0 + 0 = 1 case only for 4th col of (D).

12 + 17 + 1 + 1 = 31 cases for (D).

1st col. of (E). 714296583 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

4,1) → (2,3,5,6,7,8,9) → (6,2) → (3,5,7,8,9) → ×
(7,3) → (2,5,6,8,9) → (6,5) + (9,2) ✓
(9,5) → ×

→ (1,3,4,6,7,8,9) → (7,3) → (1,4,6,8,9) ×
(8,4) → (1,3,6,7,9) ×

612593874 → (7,1), (8,2), (9,3)

(7,1) → (2,3,4,5,6,8,9) → (3,2) → (4,5,6,8,9) → ×

(4,3) → (2,5,6,8,9) → ×

(5,4) → (2,3,6,8,9) → (8,6) + (9,3) ✓

(6,5) → (2,3,4,8,9) → (4,2) + (9,3) ✓

(9,8) → (2,3,4,5,6) → ×

(8,2) → (1,3,4,5,6,7,9) → (4,3) → (1,5,6,7,9) → ×

(5,4) → (1,3,6,7,9) → ×

(6,5) → (1,3,4,7,9) → ×

(7,6) → (1,3,4,5,9) → ×

(9,3) → (1,2,4,5,6,7,8) → (2,1) → (4,5,6,7,8) → ×

(5,4) → (1,2,6,7,8) → (8,6) + (7,1) ✓

(6,5) → (1,2,4,7,8) → (4,2) + (7,1) ✓

(7,6) → (1,2,4,5,8) → ×

(8,7) → (1,2,4,5,6) → ×

4 cases.

628395174 → (5,1), (6,2), (7,3), (8,4), (9,5).

(5,1) → (2,3,4,6,7,8,9) → ~~(9,1)~~ → (6,2) → (1,3,4,5,7,8,9) → (9,1) → (3,4,5,7,8) → ×

(7,3) → (1,2,4,5,6,8,9) → (9,1) → (2,4,5,6,8) → (6,4) + (8,2) ✓

(8,4) → (1,2,3,5,6,7,9) → (9,1) → (2,3,5,6,7) → ~~(5,3)~~ × (9,8) → (1,2,3,4,5,6,7) → ×

1 case.

628593174 → (7,1), (8,2), (9,3).

(7,1) → (2,3,4,5,6,8,9) ×; (8,2) → (1,3,4,5,6,7,9) → (9,1) → (3,4,5,6,7) → ~~(6,4)~~ ×, (9,3) →

nil

12 + 4 + 1 + 0 = 17 cases for col. 2 of (D)

8th col of (D). 671395824 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (3,2) → (4,6,7,8,9) → ×

(4,3) → (2,6,7,8,9) → ×

(7,6) → (2,3,4,8,9) → ×

(8,7) → (2,3,4,6,8,9) → ×

(9,8) → (2,3,4,6,7) → ×

(6,2) → (1,3,4,5,7,8,9) → (4,3) → ~~(4,7)~~ (2,3) → (1,2,4,5,6,8,9) → ~~(3,1)~~

(6,4)
(8,7)
(9,8) } ×

(2,1) → ×

(5,4) → (1,2,6,8,9) → ×

(6,5) → (1,2,4,8,9) → ×

(9,8) → ×

(8,4) → (1,2,3,5,6,7,9)

→ (2,1) ×

(3,2) ×

(6,5) → (1,2,3,7,9) → (9,2) + (7,1) ✓

(7,6) → (1,3,5,9) ×

1 case only

(32) (9,3) → (1,2,4,5,6,7,8) → (4,2) → (1,5,6,7,8) → ~~(6,5)~~ + (7,1) ✓
(6,4) → (1,2,5,7,8) → ×
(7,5) → (1,2,4,6,8) → ×
(8,6) → (1,2,4,5,7) → (5,4) + (7,1) ✓

4 cases

617395284 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (9,2) → (3,4,6,7,8) → (7,6) + (8,3) ×
(6,2) → (1,3,4,5,7,8,9) → (8,1) → (3,4,5,7,9) → ~~(5,4)~~ + (9,3) ✓
(7,3) → (1,2,4,5,6,8,9) → (8,1) → (2,4,5,6,9) → (6,5) + (9,4) ×
(9,2) → (1,4,5,6,8) → ×
(8,4) → (1,2,3,5,6,7,9) → (9,2) → (1,3,5,6,7) → ~~(6,5)~~ + (7,1) ✓
(9,5) → (1,2,3,4,6,7,8) → (8,1) → (2,3,4,6,7) → (6,2) + (7,2) ×

2 cases.

6175932 84 → (7,1), (8,2), (9,3)

(7,1) → (2,3,4,5,6,8,9) → (9,2) → (3,4,5,6,8) → (6,5) + (8,3) × ◁ (5,4) + (8,3) ×
(8,2) → (1,3,4,5,6,7,9) ×, (9,3) → (1,2,4,5,6,7,8) → (8,1) → (2,4,5,6,7)
→ (6,5) + (7,2) ×
◁ (5,4) + (7,2) ×

No 0 cases.

6 + 4 + 2 + 0 = No of ~~cases~~ = 12 cases for 1st col n (D)

2nd col n (D). 621395874 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (3,2) → (4,6,7,8,9) → ×
(4,3) → (2,6,7,8,9) → (8,7) + (8,2) ◁ (7,6) + (8,2) ~~◁ (9,8)~~
(7,6) → (2,3,4,8,9) → (4,2) + (9,3) ✓
(8,7) → (2,3,4,6,9) → (6,4) + (9,3) ◁ (4,2) + (9,3) ✓
(9,8) → (2,3,4,6,7) → ×

(6,2) → (1,3,4,5,7,8,9) → (4,3) → (1,5,7,8,9) ×
(5,4) → (1,3,7,8,9) → ×
(8,7) → (1,3,4,5,9) → ×
(9,8) → (1,3,4,5,7) → (5,3) + (7,1) ✓

(7,3) → (1,2,4,5,6,8,9) → (2,1) → (4,5,6,8,9) → ×
(5,4) → (1,2,6,8,9) → ×
(6,5) → (1,2,4,8,9) → ×
(9,8) → (1,2,4,5,6) → ×

(8,4) → (1,2,3,5,6,7,9) → (2,1) → (3,5,6,7,9) → (7,5) + (9,3) ✓
(3,2) → (1,5,6,7,9) → (6 ×
(6,5) → (1,2,3,7,9) → ×
(7,6) → (1,2,3,5,9) → ×

(9,5) → (1,2,3,4,6,7,8) → (2,1) → (3,4,6,7,8) → ×
(3,2) → (1,4,6,7,8) → (6,4) + (7,1) ✓ ◁ (8,6) + (7,1) ✓
(4,3) → (1,2,4,7,8) → (4,2) + (7,1) ✓
(7,6) → (1,2,3,4,8) → (3,1) + (8,2) ✓
(8,7) → (1,2,3,4,7) → (4,2) + (7,1) ✓

12 cases

487593216 → (7,1), (8,2), (9,3)

(7,1) → (2,3,4,5,6,8,9) → (9,2) ×, (8,2) ×, (9,3) × nil

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Nod cases 4 + 1 + 0 + 0 = 5 cases for col. 4 (C)

Total number of cases = 36 + 27 + 6 + 5 = 74 for (C).

1st col of (D) 612395784 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (4,2) → (3,6,7,8,9) → (7,6) + (9,3) ✓
(6,4) → (2,3,7,8,9) → (9,8) + (7,2) ×
(8,6) → (2,3,4,7,9) → (4,3) + (7,2) × & (3,2) + (9,4) ×

(6,2) → (1,3,4,5,7,8,9) → (3,1) → (4,5,7,8,9) → (8,7) + (9,4) ×
(5,3) → (1,4,7,8,9) → (9,8) + (9,4) (7,1) ✓
(7,5) → (1,3,4,8,9) → ×
(9,7) → (1,3,4,5,8) → (5,4) + (8,3) ×

(7,3) → (1,2,4,5,6,8,9) → (4,2) → (1,5,6,8,9) → (9,8) + (6,4) ×
(6,4) → (1,2,5,8,9) → (9,8) + (6,4) ×
(8,6) → (1,2,4,5,9) → (2,1) + (9,4) ×

(8,6) → (1,2,3,4,5,7,9) → (3,1) → (2,4,5,7,9) → (5,4) + (7,2) ×
(4,2) → (1,3,5,7,9) → ×
(5,3) → (1,2,4,7,9) → (2,1) + (9,4) ×
(7,5) → (1,2,3,4,9) → (2,1) + (9,3) ✓ & (3,2) + (9,4) ×
(9,7) → (1,2,3,4,5) → ×

(9,5) → (1,2,3,4,6,7,8) → (3,1) → (2,4,6,7,8) → (7,6) + (8,2) ✓
(4,2) → (1,3,6,7,8) → (8,7) + (6,4) ×
(6,4) → (1,2,3,7,8) → (3,2) + (8,3) + (7,1) ✓
(8,6) → (1,2,3,4,7) → (4,3) + (7,2) (3,2) + (7,1) ✓

60 cases.

612593784 → (7,1), (8,2), (9,3).

(7,1) → (2,3,4,5,6,8,9) → (4,2) → (3,5,6,8,9) → (6,5) + (9,3) ✓
(5,3) → (2,4,6,8,9) → ×
(6,4) → (2,3,5,8,9) → ×
(8,6) → (2,3,4,5,9) → (3,2) + (9,4) (5,4) + (9,3) ✓

(8,2) → (1,3,4,5,6,7,9) → (3,1) → (4,5,6,7,9) → (4,5) + (9,4) × & (7,6) + (9,4) ×
(5,3) → (1,4,6,7,9) → (7,6) + (9,4) ×
(6,4) → (1,3,5,7,9) → ×
(9,7) → (1,3,4,5,6) → (4,3) + (6,5) ×

(30) (6,2) → (1,3,4,5,7,8,9) ✗ → (4,3) → (1,5,7,8,9) → ✗

(7,3) → (1,2,4,5,6,8,9) → (5,4) → (1,2,4,8,9) → (9,2) + (8,4) ✓

(6,5) → (1,2,4,8,9) → (9,2) + (8,4) ✓

(8,4) → (1,2,3,5,6,7,9) → (2,1) — ✗
(3,2) — ✗
(9,8) → ✗
(2,7) → ✗.
(6,5) → (1,2,3,7,9) → (9,2) + (7,3) ✓
(7,6) → (1,2,3,5,9) → (9,2) + (5,1) ✓

(9,3) → ✗.

(9,5) → ✗.

5 Cases.

471593826 → (7,1), (8,2), (9,3); (7,1) → (2,3,4,5,6,8,9) → (4,3) → (2,5,6,8,9) → ✗
(5,4) → (2,3,6,8,9) → ✗
(6,5) → (2,3,4,8,9) → (9,2) + (8,4) ✓

(8,2) ✗ → (1,3,4,5,6,7,9) → (4,3)
(6,5)
(4,5)
(7,6)

(9,3) ✗

1 only

(5,1) → (2,3,4,6,7,8,9) → ✗
(7,3), (8,4) → (9,1) → ✗

478395126 → (5,1), (6,2) ✗, (7,3), (8,4), (9,5) ✗

nil

478593126 → (7,1), (8,2) ✗, (9,3) ✗ & (7,1) → (9,1) ✗.

nil

No of cases 6 only for 3rd Col of (C).

4th col. of (C). 482395716 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (9,2) ✗
(3,6) ✗
(6,2) → (1,3,4,5,7,8,9) → (8,3) → (1,4,4,8,9) → (9,1) + (8,4) ✓
(9,7) ✗
(7,5) → (1,3,4,8,9) → (9,1) + (8,4) ✓

(7,3) → (1,2,4,5,6,8,9) → (4,2) → (1,5,6,8,9) → ✗
(6,4) → (1,2,5,8,9) — ✗
(8,6) → (1,2,4,5,9) → ✗

(8,4) → (1,2,3,5,6,7,9) → (5,3) → (1,2,6,7,9) → (9,1) + (6,2) ✓
(7,5) → (1,2,3,6,9) → (9,1) + (6,2) ✓

(9,6) ✗

4 cases.

482593716 → (7,1) ✗, (8,2), (9,3) ✗; (8,2) → (1,3,4,5,6,7,9) → (5,3) → (1,4,6,7,9) ✗ → (9,1)
(6,4)

(6,4) → (1,3,5,7,9) → (9,1) + (7,3) ✓

(7,5) → (1,3,4,6,9) ✗

1 case

487395216 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (9,2) → ✗; (6,2) → (1,3,4,5,7,8,9) → (9,2) ✗ ✗
→ (8,1) → (2,4,5,7,9)
→ ✗

(7,3) → (1,2,4,5,6,8,9) → (9,2) → (1,4,5,6,8) ✗, (8,4) → (1,2,3,5,6,7,9) ✗

(9,5) ✗

nil

$(8,4) \to (1,2,3,5,6,7,9) \to (2,1) \to (3,5,6,7,9) \to \times$
$(3,2) \to (1,5,6,7,9) \to (9,7) + (5,1)$ ✓

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$(6,5) \to (1,2,3,7,9) \to \times$
$(7,6) \to (1,2,3,5,9) \to (3,1) + (9,5)$ ✓

$(9,5) \to (1,2,3,4,6,7,8) \to (2,1) \to (3,4,6,7,8) \to (6,4) + (7,3)$ ✓ & $(8,6) + (7,3)$ ✓
$(3,2) \to (1,4,6,7,8) \to \times$
$(4,3) \to (1,2,6,7,8) \to (8,7) + (6,2)$ ✓
$(7,6) \to (1,2,3,4,8) \to (3,1) + (8,4)$ ✓
$(8,7) \to (1,2,3,4,6) \to (3,1) + (6,2)$ ✓

15 cases.

421593876 $\to (7,1), (8,2), (9,3)$

$(7,1) \to (2,3,4,5,6,8,9) \to (3,2) \to (4,5,6,8,9) \to (8,6) + (9,5)$ ✓
$(4,3) \to (2,5,6,8,9) \to (8,6) + (9,5)$ ✓
$(5,4) \to (2,3,6,8,9) \to \times$
$(6,5) \to (2,3,4,8,9) \to \times$
$(9,8) \to (2,3,4,5,6) \to (5,3) + (6,2)$ ✓

$(8,2) \to (1,3,4,5,6,7,9) \to (4,3) \to (1,5,6,7,9) \to (9,7) + (5,1)$ ✓
$(5,4) \to (1,3,6,7,9) \to \times$
$(6,5) \to (1,3,4,7,9) \to \times$
$(7,6) \to (1,3,4,5,9) \to (3,1) + (9,5)$ ✓

$(9,3) \to (1,2,4,5,6,7,8) \to (2,1) \to (4,5,6,7,8) \to (7,5) + (8,4)$ ✓
$(5,4) \to (1,2,6,7,8) \to \times$
$(6,5) \to (1,2,4,7,8) \to \times$
$(7,6) \to (1,2,4,5,8) \to (4,2) + (5,1)$ ✓
$(8,7) \to (1,2,4,5,7) \to (4,2) + (5,1)$ ✓

8 cases.

428395176 $\to (5,1), (6,2), (7,3), (8,4), (9,5)$

$(5,1) \to (2,3,4,6,7,8,9) \to \times$; $(6,2) \to (1,3,4,5,7,8,9) \to (9,1) \to (2,3,4,5,7,8) \to (7,5) + (8,4)$ ✓

$(7,3) \to (1,2,4,5,6,8,9) \to (9,1) \to (2,4,5,6,8) \to \times$;

$(8,4) \to (1,2,3,5,6,7,9) \to (9,1) \to (2,3,5,6,7) \to (5,3) + (6,2)$ & $(7,5) + (6,2)$

$(9,5) \to \times$.

3 cases

428593176 $\to (7,1), (8,2), (9,3)$. $(7,1) \to (2,3,4,5,6,8,9) \to \times$

$(8,2) \to (1,3,4,5,6,7,9) \to (9,1) \to (3,4,5,6,7) \to (6,4) + (7,3)$ ✓ ; $(9,3) \to \times$

1 case

15 + 8 + 3 + 1 = 27 cases for 2nd colg (C)

3rd colg (C) : 471395826 $\to (5,1), (6,2), (7,3), (8,4), (9,5)$

$(5,1) \to (2,3,4,6,7,8,9) \to (3,2) \to (4,6,7,8,9) \to \times$
$(4,3) \to (2,6,7,8,9) \to \times$
$(7,6) \to (2,3,4,8,9) \to (9,2) + (8,4)$ ✓
$(8,7) \to (2,3,4,6,9) \to \times$

$(9,8) \to (2,3,4,6,7) \to \times$

(28) (8,2) → (1,3,4,5,6,7,9) → (3,1) → (4,5,6,7,9) → (7,6) + (9,5) ✓

(5,3) → (1,4,6,7,9) → ×

(6,4) → (1,3,5,7,9) → ×

(7,5) → (1,3,4,6,9) → ×

(9,7) → (1,3,4,5,6) → (4,3) + (5,1) ✓

(9,3) → (1,2,4,5,6,7,8) → (4,2) → (1,5,6,7,8) → (7,6) + (5,1) ✓ & (8,7) + (5,1) ✓

(6,4) → (1,2,5,7,8) → (8,7) + (5,1) ✓

(7,5) → (1,2,4,6,8) → (2,1) + (8,4) ✓

(8,6) → (1,2,4,5,7,9) → ×

9 cases

417395286 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (9,2) → (3,4,6,7,8) → (7,6) + (8,4) ✓

(6,2) → (1,3,4,5,7,8,9) → (8,1) → (3,4,5,7,9) → (4,3) + (9,5) ✓ & (5,4) + (7,3) ✓

(7,3) → (1,2,4,5,6,8,9) → (8,1) → (2,4,5,6,9) → (5,4) + (6,2) ✓

(9,2) → (1,4,5,6,8) → (6,5) + (8,4) ✓

(8,4) → (1,2,3,5,6,7,9) → (9,2) → (1,3,5,6,7) → (6,5) + (7,3) ✓ & (7,6) + (5,1) ✓

(9,5) → (1,2,3,4,6,7,8) → (8,1) → (2,3,4,6,7) → (4,3) + (6,2) ✓

8 cases.

417593286 → (7,1), (8,2), (9,3)

(7,1) → (2,3,4,5,6,8,9) → (9,2) → (3,4,5,6,8) → (6,5) + (8,4) ✓

(8,2) → (1,3,4,5,6,7,9) → ×

(9,3) → (1,2,4,5,6,7,8) → (8,1) → (2,4,5,6,7) → (5,4) + (6,2) ✓

2 cases.

17 + 9 + 8 + 2 = 36 cases from 1st col 1.o.(C)

2nd Col. o.(C). 421395876 → (5,1), (6,2), (7,3), (8,4), (9,5)

(5,1) → (2,3,4,6,7,8,9) → (3,2) → (4,6,7,8,9) → [illegible] (9,7) + (8,4) ✓

(4,3) → (2,6,7,8,9) → ×

(7,6) → (2,3,4,8,9) → ×

(8,7) → (2,3,4,6,9) → ×

(9,8) → (2,3,4,6,7) → (6,4) + (7,3) ✓

(6,2) → (1,3,4,5,7,8,9) → (4,3) → (1,5,7,8,9) → (9,7) + (5,1) ✓

(5,4) → (1,3,7,8,9) → ×

(8,7) → (1,3,4,5,9) → (3,1) + (9,5) ✓

(9,8) → (1,3,4,5,7) → ×

(7,3) → (1,2,4,5,6,8,9) → (2,1) → (4,5,6,8,9) → (6,4) + (9,5) & (8,6) + (9,5) ✓

(5,4) → (1,2,6,8,9) → ×

(6,5) → (1,2,4,8,9) → ×

(9,8) → (1,2,4,5,6) → (4,2) + (5,1) & (6,4) + (5,1) ✓

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$(5,2) \to (1,3,4,6,7,8,9) \to (6,1) \longrightarrow \times$
$(8,3) \to (1,4,6,7,9) \to (9,1)+(7,4)$ ✓
$(9,4) \longrightarrow \times$

$(6,3) \to (1,2,4,5,7,8,9) \longrightarrow (7,2) \to (1,4,5,8,9) \to (9,1)+(8,5)$ ✓
$(9,4) \longrightarrow \times$

$(7,4) \to (1,2,3,5,6,8,9) \to (6,1) \longrightarrow \times$
$(8,3) \to (1,2,5,6,9) \longrightarrow (9,1)+(5,2)$ ✓

$(8,6) \to (1,2,3,4,5,7,9) \to (7,2) \to (1,3,4,5,9) \to \times$
$(9,4) \to \times$

3 cases

$385692417 \to (8,1),(9,2) \longrightarrow \times$

nil

$4+0+3+0 = 7$ cases for 4th colg (B)

So for col B, we have total number $= 39+19+24+7 = 89$ for (B)

1st colg (C). $412395786 \to (5,1),(6,2),(7,3),(8,4),(9,5)$

$(5,1) \to (2,3,4,6,7,8,9) \to (4,2) \to (3,6,7,8,9) \to (9,8)+(7,3)$ ✓
$(6,4) \to (2,3,7,8,9) \to (9,8)+(7,3)$ ✓
$(8,6) \to (2,3,4,7,9) \to (3,2)+(7,4) \times$
$(9,7) \to (2,3,4,6,8) \to (3,2)+(8,4) \& (4,3)+(6,2)$ ✓ ✓

$(6,2) \to (1,3,4,5,7,8,9) \to (3,1) \to (4,5,7,8,9) \to (8,7)+(9,5)$ ✓
$(5,3) \to (1,4,7,8,9) \to \times$
$(7,5) \to (1,3,4,8,9) \to \times$
$(9,7) \to (1,3,4,5,8) \to (4,3)+(5,1)$ ✓

$(7,3) \to (1,2,4,5,6,8,9) \to (4,2) \to (1,5,6,8,9) \to (9,8)+(5,1)$ ✓
$(6,4) \to (1,2,5,8,9) \to (9,8)+(5,1)$ ✓ & $(2,1)+(9,5)$ ✓
$(8,6) \to (1,2,4,5,9) \to (2,1)+(9,5)$ ✓

$(8,4) \to (1,2,3,5,6,7,9) \to (3,1) \to (2,5,6,7,9) \to (7,5)+(6,2) \& (9,7)+(6,2)$ ✓
$(5,3) \to (1,2,6,7,9) \to \times$
$(7,5) \to (1,2,3,6,9) \to \times$
$(9,7) \to (1,2,3,5,6) \to (3,2)+(5,1)$ ✓

$(9,5) \to (1,2,3,4,6,7,8) \to (3,1) \to (2,4,6,7,8) \to (7,6)+(8,4)$ ✓ & $(8,7)+(6,2)$ ✓
$(4,2) \to (1,3,6,7,8) \to \times$
$(6,4) \to (1,2,3,7,8) \to (2,1)+(7,3)$ ✓
$(8,6) \to (1,2,3,4,7) \to (2,1)+(7,3)$ ✓

17 Cases

$412593786 \to (7,1),(8,2),(9,3)$

$(7,1) \to (2,3,4,5,6,8,9) \to (4,2) \to (3,5,6,8,9) \to \times$
$(5,3) \to (2,4,6,8,9) \to (9,8)+(6,2)$ ✓
$(6,4) \to (2,3,5,8,9) \to \times$
$(8,6) \to (2,3,4,5,9) \to (3,2)+(9,5) \& (4,3)+(9,5)$ ✓ ✓

(26) (9,2) → (1,3,4,5,6,7,8) → (4,3) → (1,5,6,7,8) → (6,1)+(8,5) ✓
(5,4) → (1,3,6,7,8) → ×
(6,5) → (1,3,4,7,8) → (8,3)+(7,4) ✓
(7,6) → (1,3,4,5,8) → (8,3)+(4,1) ✓
(8,7) → (1,3,4,5,6) → ×

6 cases

358296147 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

(4,1) → (2,3,5,6,7,8,9) → ~~(2,3)~~ ×
(5,2) → (1,3,4,6,7,8,9) → (9,1) → (3,4,6,7,8) → (8,3)+(7,4) ✓
(6,3) → (1,2,4,5,7,8,9) → (9,1) → (2,4,5,7,8) → (7,2)+(8,5) ✓
(7,4) → (1,2,3,5,6,8,9) → (9,1) → (2,3,5,6,8) → (8,3)+(5,2) ✓
(8,6) → (1,2,3,4,6,7,9) → (9,1) → (2,3,4,6,7) → (7,2)+(6,3) ✓

(9,6) → ×

4 cases

358692147 → (8,1), (9,2)

(8,1) → (2,3,4,5,6,7,9) → ×
(9,2) → (1,3,4,5,6,7,8) → ×.

nil

14 + 6 + 4 + 0 = 24 cases for 3rd col. of (B)

4th col of (B). 384296517 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → (2,3,5,6,7,8,9) → (6,2) → (1,3,5,7,8,9) → ×
(7,3) → ×
(9,5) → ×

(5,2) → (1,3,4,6,7,8,9) → (7,3) → (1,4,6,8,9) → ×
(8,4) → (1,4,6,7,9) → (9,1) + (7,4) ✓

(6,3) → (1,2,4,5,7,8,9) → (5,1) → ×
(8,4) → (1,2,5,7,9) → (9,1) + (5,2) ✓
(9,5) → ×

(7,4) → (1,2,3,5,6,8,9) → (5,1) → ×
(6,2) → (1,3,5,8,9) → (9,1) + (8,5) ✓
(9,5) → ×

(8,6) → (1,2,3,4,5,7,9) → (5,1) → ×
(7,3) → (1,2,4,5,9) → (9,1) + (5,2) ✓
(9,5) → ×

(9,7) → ×

4 cases

384692517 → (8,1), (9,2) → ×

nil

385296417 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

(4,1) → × and (9,6) → ×.

$(8,5) \to (1,2,3,4,6,7,9) \to (9,1) \to (2,3,4,6,7) \to (6,2)+(7,4)$ ✓

$(9,6) \to (1,2,3,4,5,7,8) \to \times$

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4 cases

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348692157 $\to (8,1), (9,2)$

$(8,1) \to (2,3,4,5,6,7,9) \to \times$, $(9,2) \to (1,3,4,5,6,7,8) \to \times$

nil

$12+3+4+0 =$ 19 cases for 2nd colg B.

3rd colg (B) · 351296847 $\to (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)$

$(4,1) \to (2,3,5,6,7,8,9) \to (3,2) \to (5,6,7,8,9) \to \times$
$(6,5) \to (2,3,6,7,8,9) \to \times$
$(7,6) \to (2,3,5,8,9) \to (8,3)+(5,2)$ ✓
$(8,7) \to (2,3,5,6,9) \to \times$
$(9,8) \to (2,3,5,6,7) \to (7,2)+(6,3)$ ✓

$(5,2) \to (1,3,4,6,7,8,9) \to (4,3) \to (1,6,7,8,9) \to \times$
$(7,6) \to (1,3,4,8,9) \to (8,3)+(4,1)$ ✓
$(8,7) \to (1,3,4,6,9) \to (9,4)+(6,3)$ ✓ alt
$(9,8) \to (1,3,4,6,7) \to (6,1)+(7,4)$ ✓

$(6,3) \to (1,2,4,5,7,8,9) \to (2,1) \to (4,5,7,8,9) \to (9,4)+(8,5)$ ✓
$(5,4) \to (1,2,7,8,9) \to \times$
$(8,7) \to (1,2,4,5,9) \to (9,4)+(5,2)$ ✓
$(9,8) \to (1,2,4,5,7) \to (7,2)+(4,1)$ ✓

$(7,4) \to (1,2,3,5,6,8,9) \to (2,1) \to (3,5,6,8,9) \to (8,3)+(9,6)$ ✓
$(3,2) \to (1,5,6,8,9) \to (6,1)+(8,5)$ alt ✓
$(6,5) \to (1,2,3,8,9) \to \times$
$(9,8) \to (1,2,3,5,6) \to (6,1)+(5,2)$ ✓

$(8,6) \to (1,2,3,4,5,7,9) \to (1,2) \to (3,4,5,7,9) \to \times$
$(3,2) \to (1,4,5,7,9) \to \times$
$(4,3) \to (1,2,5,7,9) \to \times$
$(5,4) \to (1,2,3,7,9) \to \times$

$(9,8) \to (1,2,3,4,5,6,7) \to (2,1) \to (3,4,5,6,7) \to \times$
$(3,2) \to (1,4,5,6,7) \to (6,1)+(7,4)$ ✓
$(4,3) \to (1,2,5,6,7) \to (6,1)+(5,2)$ ✓
$(5,4) \to (1,2,3,6,7) \to (7,2)+(6,3)$ ✓
$(6,5) \to (1,2,3,4,7) \to (7,2)+(4,1)$
$(7,6) \to (1,2,3,4,5) \to \times$

14 cases

351692847 $\to (8,1), (9,2)$.

$(8,1) \to (2,3,4,5,6,7,9) \to (3,2) \to (4,5,6,7,9) \to \times$
$(4,3) \to (2,5,6,7,9) \to (7,2)+(9,6)$ ✓
$(5,4) \to (2,3,6,7,9) \to (7,2)+(6,3)$ ✓
$(6,5) \to (2,3,4,7,9) \to \times$
$(7,6) \to (2,3,4,5,9) \to (9,4)+(5,2)$ ✓

(24) 2nd col of (B). 341296857. → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → (2,3,5,6,7,8,9) → (3,2) → (5,6,7,8,9) → (9,8) ×
(6,5) → (2,3,7,8,9) → ×
(7,6) → (2,3,5,8,9) → ×
(8,7) → (2,3,5,7,9) → (7,3) + (5,2) ✓
(9,8) → (2,3,5,6,7) → (7,3) + (5,2) ✓

(5,2) → (1,3,4,6,7,8,9) → (4,3) → (1,6,7,8,9) → ×
(7,6) → (1,3,4,8,9) → ×
(8,7) → (1,3,4,6,9) → ×
(9,8) → (1,3,4,6,7) → (7,3) + (4,1) ✓

(6,3) → (1,2,4,5,7,8,9) → (5,4) → (1,2,7,8,9) → ×
(8,7) → (1,2,4,5,9) → (9,5) + (4,1) ✓
(2,1) → (4,5,7,8,9) → (9,5) + (7,4) ✓
(9,8) → (1,2,4,5,7) → (5,1) + (7,4) ✓

(7,4) → (1,2,3,5,6,8,9) → (2,1) → (3,5,6,8,9) → (9,5) + (6,3) ✓
(3,2) → (1,5,6,8,9) → (5,1) + (9,6) ✓
(6,5) → (1,2,3,8,9) → ×
(9,8) → (1,2,3,5,6) → (5,1) + (6,3) ✓

(8,5) → (1,2,3,4,6,7,9) → (2,1) → (3,4,6,7,9) → (7,3) + (9,6) ✓ ×
(3,2) → (1,4,6,7,9) → [illegible]
(4,3) → (1,2,6,7,9) → ×
(7,6) → (1,2,3,4,9) → ×

(9,6) → (1,2,3,4,5,7,8) → (2,1) → (3,4,5,7,8) → (7,3) + (8,5) ✓
(3,2) → (1,4,5,7,8) → (5,1) + (7,4) ✓
(4,3) → (1,2,5,7,8) → ×
(5,4) → (1,2,3,7,8) → ×
(8,7) → (1,2,3,4,5) → ×

12 cases.

341692857 → (8,1), (9,2)

(8,1) → (2,3,4,5,6,7,9) → (3,2) → (4,5,6,7,9) → ×
(4,3) → (2,5,6,7,9) → ×
(5,4) → (2,3,6,7,9) → (7,3) + (9,6) ✓
(6,5) → (2,3,4,7,9) → ×
(7,6) → (2,3,4,5,9) → ×

(9,2) → (1,3,4,5,6,7,8) → (4,3) → (1,5,6,7,8) → ×
(5,4) → (1,3,6,7,8) → ×
(6,5) → (1,3,4,7,8) → (7,3) + (4,1) ✓
(7,6) → (1,3,4,5,8) → ×
(8,7) → (1,3,4,5,6) → (5,1) + (6,3) ✓

3 cases.

348296157 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6)

(4,1) → (2,3,5,6,7,8,9) → [illegible] ×
(5,2) → (1,3,4,6,7,8,9) → (9,1) → (3,4,6,7,8) → (8,4) + (6,3) ✓
(6,3) → (1,2,4,5,7,8,9) → (9,1) → (2,4,5,7,8) → (8,4) + (5,2) ✓
(7,4) → (1,2,3,5,6,8,9) → (9,1) → (2,3,5,6,8) → (6,2) + (8,5) ✓

$(7,4) \to (1,2,3,5,6,8,9) \to (6,2) \to (1,3,5,8,9) \to \times$
$(5,1) \to (2,3,6,8,9) \to (3,2) + (9,6) \checkmark$ ◁ $(9,8) + (6,3) \checkmark$
$(9,5) \to (1,2,3,6,8) \to (2,1) + (6,3) \checkmark$

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$(8,5) \to (1,2,3,4,6,7,9) \to (6,2) \to (1,3,4,7,9) \to \times$
$(7,3) \to (1,2,4,6,9) \to (2,1) + (9,6) \checkmark$

$(9,6) \to (1,2,3,4,5,7,8) \to (5,1) \to (2,3,4,7,8) \to (3,2) + (7,4) \checkmark$
$(7,3) \to (1,2,4,5,8) \to (2,1) + (8,5) \checkmark$
$(8,4) \to (1,2,3,5,7) \to \times$

12 cases

314692587 $\to (8,1), (9,2)$

$(8,1) \to (2,3,4,5,6,7,9) \to (6,2) \to (3,4,5,7,9) \to \times$
$(7,3) \to (2,4,5,6,9) \to (5,4) + (9,6) \checkmark$
$(9,5) \to (2,3,4,6,7) \to (3,2) + (7,4) \checkmark$

$(9,2) \to (1,3,4,5,6,7,8) \to (5,1) \to (3,4,6,7,8) \to (8,7) + (6,3) \checkmark$
$(7,3) \to (1,4,5,6,8) \to (6,5) + (4,1) \checkmark$
$(8,4) \to (1,3,5,6,7) \to \times$

4 cases

315296487 $\to (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).$

$(4,1) \to (2,3,5,6,7,8,9) \to (7,2) \to (3,5,6,8,9) \to (9,8) + (6,3) \checkmark$
$(8,3) \to (2,5,6,7,9) \to (7,6) + (5,2) \checkmark$

$(5,2) \to (1,3,4,6,7,8,9) \to (6,1) \to (3,4,7,8,9) \to (9,8) + (7,4) \checkmark$
$(8,3) \to (1,4,6,7,9) \to (7,6) + (4,1) \checkmark$
$(9,4) \to (1,3,5,7,8) \to \times$

$(6,3) \to (1,2,4,5,7,8,9) \to (7,2) \to (1,4,5,8,9) \to (9,8) + (4,1) \checkmark$
$(9,4) \to (1,2,5,7,8) \to (2,1) + (8,5)$ ◁ $(8,7) + (5,2) \checkmark$

$(7,4) \to (1,2,3,5,6,8,9) \to (6,1) \to (2,3,5,8,9) \to (3,2) + (8,5)$ ◁ $(9,8) + (5,2)$
$(8,3) \to (1,2,5,6,9) \to (2,1) + (9,6) \checkmark$

$(8,5) \to (1,2,3,4,6,7,9) \to (6,1) \to (2,3,4,7,9) \to (3,2) + (7,4) \checkmark$
$(7,2) \to (1,3,4,6,9) \to (4,3) + (9,6) \checkmark$
$(9,4) \to (1,2,3,6,7) \to (2,1) + (6,3) \checkmark$

$(9,6) \to (1,2,3,4,5,7,8) \to (7,2) \to (2,3,4,5,8) \to (4,3) + (5,2)$ ◁ $(4,3) + (8,5)$ ◁ $(3,2) + (8,5)$
$(8,3) \to (1,2,4,5,7) \to (2,1) + (7,4) \checkmark$

17 cases

315692487 $\to$ $(8,1)$ ± $(9,2)$ $\to (2,3,4,5,6,7,9) \to (7,2) \to (3,4,5,6,9) \to (4,3) + (9,6)$ ◁ $(5,4) + (6,3)$
$(9,4) \to (2,3,5,6,7) \to (7,6) + (5,2) \checkmark$

$(9,2) \to (1,3,4,5,6,7,8) \to (6,1) \to (3,4,5,7,8) \to (4,3) + (8,5) \checkmark$
$(8,3) \to (1,4,5,6,7) \to (6,5) + (4,1)$ ◁ $(7,6) + (4,1)$

6 cases

12 + 4 + 17 + 6 = 39 cases from Col 1 of (B)

265197438 → (3,1), — (9,7)

(3,1) → (2,4,5,6,7,8,9) → (7,2) → (4,5,6,8,9) → ×
(9,4) → (2,5,6,7,8) → (8,2) + (7,5) ✓

(4,2) → (1,3,5,6,7,8,9) → (6,1) → (3,5,7,8,9) → (9,3) + (7,5) ✓
(8,3) → (1,5,6,7,9 → ×

(5,3) → (1,2,4,6,7,8,9) → (6,1) → (2,4,7,8,9) → (8,2) + (9,7) ✓
(7,2) → (1,4,6,8,9) → ×
(9,4) → (1,2,6,7,8) → (7,1) + (8,6) ✓

(6,4) → (1,2,3,5,7,8,9) → (7,2) → (1,3,5,8,9) → ~~(9,3)~~ ×
(8,3) → (1,2,5,7,9) → ~~(7,1)~~ ×

(7,5) → (1,2,3,4,6,8,9) → (6,1) → (2,3,4,8,9) → (9,3) + (4,2) ✓
(8,3) → (1,2,4,6,9) → ×
(9,4) → (1,2,3,6,8) → ×

(8,6) → (1,2,3,4,5,7,9) → (7,2) → (1,3,4,5,9) → ×
(9,4) → (1,2,3,5,7) → (7,1) + (5,3) ✓

(9,7) → (1,2,3,4,5,6,8) → (6,1) → (2,3,4,5,8) → (8,2) + (5,3) ✓
(8,3) → (1,2,4,5,6) → ×

7 cases

265791438 → (9,1) → (2,3,4,5,6,7,8) → (7,2) → (3,4,5,6,8) → ×
(8,3) → (2,4,5,6,7) ×

nil

15 cases from 4th col

1st col → 32
2nd col — 29
3rd col — 22
4th col — 15
98

Total number for (A) = 98

So does it mean nearly 100 for each of (A) to (F)

ie nearly 600? Sat as see

—— () ——

1st col of (B). 314296587 → (4,1), (5,2), (6,3), (7,4), (8,5), (9,6).

(4,1) → (2,3,5,6,7,8,9) → (6,2) → (3,5,7,8,9) → ×
(7,3) → (2,5,6,8,9) → ~~(6,5)~~ + ~~(2,4)~~ (9,8) + (5,2) ✓
(9,5) → (2,3,6,7,8) → (8,7) + (6,3) ✓

(5,2) → (1,3,4,6,7,8,9) → (7,3) → (1,4,6,8,9) → (9,8) + (4,1) ✓
(8,4) → (1,3,6,7,9) → ×

(6,3) → (1,2,4,5,7,8,9) → (8,4) → (1,2,5,7,9) → ×
(5,1) → (2,4,7,8,9) → (9,8) + (7,4) ✓
(9,5) → (1,2,4,7,8) → (2,1) + (7,4) ✓ & (8,7) + (4,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (8,2) → (1,3,4,6,9) → (9,4) + (3,1) ✓
(9,3) → (1,2,4,6,8) → (6,1) + (4,2) ✓

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(8,6) → (1,2,3,4,5,7,9) → (7,1) → (2,3,4,8,9) → (8,3) + (4,2) ✓
(9,3) → (1,2,4,5,7) → ×

(9,7) → (1,2,3,4,5,6,8) → (8,2) → (1,3,4,5,6) → (6,1) + (5,3) ✓

8 cases

256791348 → (9,1) → (2,3,4,5,6,7,8) → (8,2) → (3,4,5,6,7) ×

nil.

22 cases for 3rd col^n (A)

4th col^n (A). 264197538 → (2,1), - - (9,7)

(3,1) → (2,4,5,6,7,8,9) → (6,2) → (4,5,7,8,9) ×
(8,4) → (2,5,6,7,9) ×
(9,5) → (2,4,6,7,8) → (8,2) → (6,4) ✓

(4,2) → (1,3,5,6,7,8,9) → (5,1) → (3,6,7,8,9) → (9,3) + (8,6) ✓
(7,3) → (1,5,6,8,9) → ×
(9,5) → (1,3,6,7,8) → ×

(5,3) → (1,2,4,6,7,8,9) → (6,2) → (1,4,7,8,9) → ×
(8,4) → (1,2,6,7,9) → ×

(6,4) → (1,2,3,5,7,8,9) → (5,1) → (2,3,7,8,9) → (8,2) + (9,7) ✓
(7,3) → (1,2,5,8,9) → ×
(9,5) → (1,2,3,7,8) → (8,2) + (3,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (6,2) → (1,3,4,8,9) → ×
(8,4) → (1,2,3,6,9) → ×

(8,6) → (1,2,3,4,5,7,9) → (5,1) → (2,3,4,7,9) → (9,3) + (4,2) ✓
(7,3) → (1,2,4,5,9) → ×
(9,5) → (1,2,3,4,7) → (7,1) + (4,2) ✓

(9,7) → (1,2,3,4,5,6,8) → (5,1) → (2,3,4,6,8) → (8,2) + (6,4) ✓
(6,2) → (1,3,4,5,8) → ×
(8,4) → (1,2,3,5,6) → ×

7 cases

264791538 → (9,1) → (2,3,4,5,6,7,8) → (6,2) → (3,4,5,7,8) ×
(7,3) → (2,4,5,6,8) → (8,2) + (6,4) ✓
(8,4) → (2,3,5,6,7) → ×

1 case

(20)

3rd col 1(A). 253197648 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)

(31) → (2,4,5,6,7,8,9) → (5,2) → (4,6,7,8,9) → (9,4) + (8,6) ✓
(7,4) → (2,5,6,8,9) → ×
(8,5) → (2,4,6,7,9) → (7,2) + (6,4) ✓
(9,6) → (2,4,5,7,8) → ×

(4,2) → (1,3,5,6,7,8,9) → (6,3) → (1,5,7,8,9) → ×
(8,5) → (1,3,6,7,9) → (6,1) + (9,7) ✓
(9,6) → (1,3,5,7,8) → (8,3) + (7,5) ✓

(5,3) → (1,2,4,6,7,8,9) → (4,1) → (2,6,7,8,9) → (7,2) + (8,6) ✓
(7,4) → (1,2,6,8,9) → ×
(9,6) → (1,2,4,7,8) → ×

(6,4) → (1,2,3,5,7,8,9) → (5,2) → (1,3,7,8,9) → (8,3) + (9,7) ✓
(8,5) → (1,2,3,7,9) → (7,2) + (3,1) ✓

(7,5) → (1,2,3,4,6,8,9) → (4,1) → (2,3,6,8,9) → ×
(6,3) → (1,2,4,8,9) → ×
(9,6) → (1,2,3,4,8) → (8,3) + (4,2) ✓

(8,6) → (1,2,3,4,5,7,9) → (4,1) → (2,3,5,7,9) → (7,2) + (5,3) ✓
(5,2) → (1,3,4,7,9) → (9,4) + (3,1) ✓
(7,4) → (1,2,3,5,9) → ×

(9,7) → (1,2,3,4,5,6,8) → (4,1) → (2,3,5,6,8) → ×
(5,2) → (1,3,4,6,8) → (8,3) + (6,4) ✓
(6,3) ⟹ (1,2,4,5,8) → ×
(8,5) → (1,2,3,4,6) → (6,1) + (4,2) ✓

12 cases

253791648 → (9,1) → (2,3,4,5,6,7,8) → (5,2) → (3,4,6,7,8) → (8,3) + (6,4) ✓
(6,3) → (2,4,5,7,8) → ×
(7,4) → (2,3,5,6,8) → ×
(8,5) → (2,3,4,6,7) → (7,2) + (6,4) ✓

2 cases

256197348 → (3,1), - - (9,7)

(3,1) → (2,4,5,6,7,8,9) → (8,2) → ~~(4,6,7,8,9)~~ → (4,5,6,7,9) → ~~(9,5)~~ (9,4) + (7,5) ✓
(7,4)
(8,5)
(9,6)

(4,2) → (1,3,5,6,7,8,9) → (7,1) → (3,5,6,8,9) → ×
(9,3) → (1,5,6,7,8) → (6,1) + (7,5) ✓

(5,3) → (1,2,4,6,7,8,9) → (7,1) → (2,4,6,8,9) → (9,4) + (8,6) ✓
(8,2) → (1,4,6,7,9) → (6,1) + (9,7) ✓

(6,4) → (1,2,3,5,7,8,9) → (7,1) → (2,3,5,8,9) → ×
(8,2) → (1,3,5,7,9) → ×
(9,3) → (1,2,5,7,8) → ×

(6,4) → (1,2,3,5,7,8,9) → (5,2) → (1,3,7,8,9) → ×
(8,5) → (1,2,3,7,9) → ×

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(7,5) → (1,2,3,4,6,8,9) → (4,1) → (2,3,6,8,9) → ×
(6,3) → (1,2,4,8,9) → ×
(9,6) → (1,2,3,4,8) → (8,4)+(3,1) ✓

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(8,6) → (1,2,3,4,5,7,9) → (4,1) → (2,3,5,7,9) → ×
(5,2) → (1,3,4,7,9) → ×
(7,4) → (1,2,3,5,9) → (9,5) + (3,1) ✓

(9,7) → (1,2,3,4,5,6,8) → (4,1) → (2,3,5,6,8) → (6,2) + (5,3) ✓
(5,2) → (1,3,4,6,8) → (8,4) + (3,1) ✓
(6,3) → (1,2,4,5,8) → (5,1) + (4,2) ✓
(8,5) → (1,2,3,4,6) → (6,2) + (3,1) ✓

14 cases for 5th number

243791658 → (9,1) → (2,3,4,5,6,7,8) → (5,2) → (3,4,6,7,8) → (7,3) + (6,4) ◁ (7,3) + (8,6) ✓
(6,3) → (2,4,5,7,8) → (8,4) + (7,5) ✓
(7,4) → (2,3,5,6,8) → (6,2) + (5,3) ✓
(8,5) → (2,3,4,6,7) → (7,3) + (4,2) ◁ (7,3) + (6,4) ✓

6 cases for 6th number

246197358 → (3,1), (4,2), . . . (9,7)
(3,1) → (2,4,5,6,7,8,9) → (8,2) → (4,5,6,7,9) → (9,5) + (6,4)
(4,2) → (1,3,5,6,7,8,9) → (7,1) → (3,5,6,8,9) → (9,5) + (8,6) ✓
(9,3) → (1,5,6,7,8) → (5,1) + (8,6) ✓

2 cases for 7th number

246791358 → (9,1) → (2,3,4,5,6,7,8) → (8,2) → (3,4,5,6,7) → (7,3) + (6,4) ✓

1 case for 8th number in 2 cases (A)

(A) 253197648 → (5,3) → (1,2,4,6,7,8,9) → (7,1) → (2,4,6,8,9) → (6,2) + ×
(8,2) → (1,4,6,7,9) → ×

(6,4) → (1,2,3,5,7,8,9) → (7,1) → (2,3,5,8,9) → ×
(8,2) → (1,3,5,7,9) → (5,1) + (9,7) ◁ (9,5) + (3,1) ✓
(9,3) → (1,2,5,7,8) → ×

(7,5) → (1,2,3,4,6,8,9) → (8,2) → (1,3,4,6,9) → ×
(9,3) → (1,2,4,6,8) → ×

(8,6) → (1,2,3,4,5,7,9) → (7,1) → (2,3,4,5,9) → (9,5) + (4,2) ✓
(9,3) → (1,2,4,5,7) → (5,1) + (4,2) ✓

(9,7) → (1,2,3,4,5,6,8) → (8,2) → (1,3,4,5,6) → (5,1) + (6,4) ✓

8 cases for 7th number in 29 cases for 2nd Col

234791568 → (9,1) → (2,3,4,5,6,7,8) → (6,2) → (3,4,5,7,8) → (7,4) + (5,3)
(7,3) → (2,4,5,6,8) → (8,5) + (6,4)
(5,2) + (8,6) ✓
(8,5) + (4,2) ✓
(5,2) + (6,4) ✓

ie 6 cases for 2nd number

(8,4) →
(8,4) → (2,3,5,6,7) → (6,3) + (7,5) ✓

235197468 → (3,1), (4,2), (5,3), (6,4) → (7,5) → (8,6) → (9,7)

(3,1) → (2,4,5,6,7,8,9) → (7,2) → (4,5,6,8,9) → (8,5) + (6,4) ✓
(9,4) → (2,5,6,7,8) → (5,2) + (8,6) ✓

(4,2) → (1,3,5,6,7,8,9) → (6,1) → (3,5,7,8,9) → (8,5) + (9,7) ✓
(8,3) → (1,5,6,7,9) → (9,6) + (7,5) ✓

(5,3) → (1,2,4,6,7,8,9) → (6,1) → (2,4,7,8,9) → X
(7,2) → (1,4,6,8,9) → (4,1) + (8,6) ✓
(9,4) → (1,2,6,7,8) → X

(6,4) → (1,2,3,5,7,8,9) → (7,2) → (1,3,5,8,9) → (8,5) + (3,1) ✓
(8,3) → (1,2,5,7,9) → (5,2) + (9,7) ✓

(7,5) → (1,2,3,4,6,8,9) → (6,1) → (2,3,4,8,9) → X
(8,3) → (1,2,4,6,9) → (9,6) + (4,2) ✓
(9,4) → (1,2,3,6,8) → X

(8,6) → (1,2,3,4,5,7,9) → (7,2) → (1,3,4,5,9) → (4,1) + (5,3) ✓
(9,4) → (1,2,3,5,7) → (5,2) + (3,1) ✓

(9,7) → (1,2,3,4,5,6,8) → (6,1) → (2,3,4,5,8) → (8,5) + (4,2) ✓
(8,3) → (1,2,4,5,6) → (5,2) + (6,4) ✓

12 cases for 3rd number

235791468 → (9,1) → (2,3,5,7,4,6,8) → (7,2) → (3,5,4,6,8) → (8,5) + (6,4) ✓
(8,3) → (2,5,7,4,6) → (5,2) + (6,4) ✓

2 cases for 4th number ie 32 cases for 1st col. of (A)

2nd Col of (A).

243197658 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)

(3,1) → (2,4,5,6,7,8,9) →
(5,2) → (4,6,7,8,9) → (8,4) + (9,7) ✓
(7,4) → (2,5,6,8,9) → (9,5) + (8,6) ✓
(8,5) → (2,4,6,7,9) → (6,2) + (9,7) ✓
(9,6) → (4,5,7,8,8) → (8,4) + (7,5) ✓

(4,2) → (1,3,5,6,7,8,9) →
(6,3) → (1,5,7,8,9) → (5,1) + (9,7) ✓
(7,8) → (1,3,5,6,9) → (9,5) + (3,1) ✓
(7,9) → (1,3,5,6,8) → (5,1) + (8,6) ✓

(5,3) → (1,2,4,6,7,8,9) →
(4,1) → (2,6,7,8,9) → (6,2) + (9,7) ✓
(7,4) → (1,2,6,8,9) → X
(9,6) → (1,2,4,7,8) → X

19758 → (4,2), (7,5), (9,7) → (1,9,7,5,8), (2,4,1,9,8), (2,4,1,5,8)
8954
9712 ✓ already got.

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2419758
7128459
9548217

new one
17/8

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2379168 → (9,1)
→ (2,3,7,6,8)

2319768 → (3,1), (8,6), (9,7) → (2,9,7,6,8), (2,3,1,9,7), (2,3,1,6,8)

Again 2 new ones from (A)

All together 5 + 5 + 4 + 2 + 2 + 1 = 19 ~~from~~ for 7 digits

~~Before we go to 9 verify 5 again~~ Now only the case of 9 digits left (to be done using centrifugal method)

We have already obtained here 64 cases & remarked many many more out of the 96 numbers available on pp. 6–7 from (A) to (F). Shall I again scrutinise all the 96 nos & get exact possible ones? 17/8 yes

From (A). 234197568 → (3,1), (4,2), (5,3), (6,4), (7,5), (8,6), (9,7)

(3,1) → (2,4,9,7,5,6,8) → (6,2) + (8,5) + (9,7) ✓ or ~~(6,2) + (8,5) + (6,2) + (7,4) + (3,1)~~ ~~(6,2) + (7,4) +~~
(8,4) + (5,2) + (9,7) ✓ or ~~(8,4) + (5,2) + (3,1)~~ (8,4) + (9,6) + (7,5) ✓
(9,5) + (7,4) + (8,6) ✓ or ~~(9,3) + (7,4) + (8,6)~~

~~Better start from (F) together and~~

(4,2) → (1,3,1,9,7,5,6,8) → (5,1) + (6,3) + (9,7) ✓
(7,3) + ~~(8,5)~~ → × → (1,9,5,6,8) ×
(9,5) + ~~(5,2)~~ → × (3,1,7,6,8) ×

(5,3) → (1,2,4,6,7,8,9) → (6,2) + ~~(8,4)~~ → × (1,4,7,8,9) → (4,1) + (9,7) ✓
(8,4) + ~~(9,6)~~ → × (1,2,6,7,9) → ×

(6,4) → (1,2,3,5,7,8,9) → (7,3) → (1,2,5,8,9) × 1,2,3,6,9)
(9,5) → (1,2,3,7,8) ×

(7,5) → (1,2,3,4,6,8,9) → ~~(5,1)~~ (6,2) → × (1,3,4,8,9) ×
~~(6,2)~~ (8,4) + (9,6) + (3,1) ✓

(8,6) → (1,2,3,4,5,7,9) → (5,1) → (2,3,4,7,9) ×
(7,3) → (1,2,4,5,9) ×
(9,5) → (1,2,3,4,7) → (7,4) + (3,1) ✓

(9,7) → (1,2,3,4,5,6,8) → (5,1) → (2,3,4,6,8) → (6,3) + (4,2) ✓
(6,2) → (1,3,4,5,8) → (4,1) + (5,3) ✓ or (8,5) + (3,1) ✓
(8,4) → (1,2,3,5,6) → (5,2) + (3,1) ✓

(12) ie ~~11~~ cases for the first number itself of (A) itself!

(6,1,5,8,4) ×

(8,4), (9,5) → (3,9,6,8,4), (6,1,3,9,5), (6,1,

6139584
3541869
9681453

Two from (D) are already got

From (C): 4859316 → (9,3) → (4,8,5,1,6) ×

4839516 → (8,4), (9,5) → (3,9,5,1,6), (4,8,3,1,6)

4759326 → (9,3) → (4,7,5,26)

4739526 → (6,2), (7,3), (9,5) → (4,7,3,9,5), (4,9,5,2,6), ( )

4259376 → (9,3) → (4,2,5,7,6)
2534976
6794352 — Had made a mistake originally now corrected

4239576 → (7,3), (6,2), (9,5) → (4,2,9,5,6), (4,3,9,5,7), (4,2,3,7,6)

4239576
5432769
9672345 (already got)

4239576
3452967
7692543

new one 17/8

4159386 → (9,3) → (4,1,5,8,6)
4531968
8691354 ✓ already got.

4139586, → (8,4), (9,5) → (1,3,9,5,6), (4,1,3,8,6)

From (B) 3869217 → (8,1), (9,2) → (3,6,9,2,7), ( )

3829617 → (6,3), (9,6) → (8,2,9,1,7), ( )

one new one from (C)

3569247 → (9,2) → (3,5,6,4,7)

3529647 → (6,3), (7,4), (5,2), (9,6) → (5,2,9,4,7), (3,5,2,9,6), (3,9,6,4,7), (3,5,2,4,7)

3529647
2437695
5967342 (already got)

3529647
3427596
6957243

new one

3169287 → (8,1), (9,2) → (3,6,9,2,7), (3,1,6,8,7)

3169287
6217839
9387126

new one

3169287
3721986
6891273 → already got

3129687 → (6,3), (9,6) → (1,2,9,8,7), (3,1,2,8,7)

So two new ones from (B)

From (A): 2679138 → (9,1) → (2,6,7,3,8)

2619738 → (3,1), (8,6), (9,7) → (2,6,9,7,8), (2,1,9,7,3), (2,6,1,3,8)
7216389
9836127 ✓ already got

2619738
1276983
3896721

new one → (2,5,7,4,8)

2579148 → (9,1)

2519748 → (4,2), (9,7) → (5,1,9,7,8), (2,5,1,4,8)

2479158 → (9,1) → (2,4,7,5,8) →
2479158
5412987
7892145 ✓ already got

idijit
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3 & 5 dgts middle one being 9.
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have obtained 14 cases. Let us c[illegible] upagain using centrifugal method.

go backwards from (F): 8679132 → (9,1) → (8,6,7,3,2) ×

8619732 → (3,1), (8,6), (9,7) → (8,6,9,7,2), (1,9,7,3,2) ×

8579142 → (9,1) → (5,7,8,4,2) × ‖ 8519742 → (4,2), (7,5), (9,7) (8,6,1,3,2) ×

(8,5,1,9,7) × (8,1,9,4,2) × (8,5,1,4,2) ×

8479152 ×

8419752 → (4,2), (7,5), (9,7).
1452789
9872541 already obtained. ~~8397~~ 8379162 ×

8319762 → (3,1), (8,6), (9,7) ×
(8,9,7,6,2) (3,1,9,7,2) ×

only one from (F) as obtained earlier

From (E)

~~7869213~~ 7869213 → (8,1), (9,2) → (7,6,9,2,3), (7,8,6,1,3) ×

7829613 → (6,3), (9,6) × → (7,8,2,9,1) ×

7569243 → (9,2) ×; ~~756~~ 7529643 → (5,2), (6,3), (7,4), (9,6)

→ (7,9,6,4,3), (7,5,2,9,4), (5,2,9,6,3), (7,5,2,4,3) ×

7469243 ×, 7429653 → (6,3), (5,2), (9,6) ×, (7,4)

→ (7,4,2,9,5) ×, (7,4,9,6,3) ×, (2,9,6,5,3) ×

7169283 → (8,1), (9,2) → (7,6,9,2,3), (7,1,6,8,3) ×
2613879
9783162 already obtained.

7129683 → (6,3), (9,6) × → 7,1,2,9,8 ×
1267938
8397621 ✓ (7,1,2,8,3) ✓ already ga

only ~~two~~ two from (E) as already obtained

From (D). 6859314 → (9,3) ×; 6839514 → (5,1), (8,4), (9,5) ×

6759324 × 6739524 × → (7,3), (6,2) ×, (9,5) × ‖ (6,8,3,9,4) × (6,3,9,5,1) ×

6,9,5,2,4

6259374 → (9,3) → (6,2,5,7,4) ×

6239574 → (6,2), (9,5), (7,3) → (3,9,5,7,4), (6,2,3,7,4), (6,2,9,5,4) ×
3524679
9764253 ✓ already obtained

(11)

41235786
126 837
538 623

41735286
34126857
75862143

41235786
346 6357
758 143

41235786
3 26 7
7 62 3

2,4,5,3,8,6

4,5,3,8

2,5,3,6

31426857

41253786
34 28 57
75 82 43

41253786
4 63176 8
8 871634
5

41253786
4 17 8
8 71 34

4,1,5,2,8,6

4,1,5,8

41253786
42 17 38
83 71 24

41253786
28
82

4,1,5,3,7,6

41735286
24137856
65873142

new 13/8/74

6,2,1,5,8,4

1,3,5,7,8,4

3,5,8,4

62135874
13 26 57
75 62 31

62135874
37
73

6,1,5,4

6,2,1,3,5,7

6,2,3,5

62135874
1 48 37
7 84 31

62153874
17
71

62153874
28
82

6,1,5,3,7,4

6,2,5,3,8,4

6,1,2,5,8,4

6,1,2,3,5,7

2,7,6,8

2,3,1,7

7,1,2,8

6,5,3,4

61235784
26
62

61235784
37
73

61235784
48
84

231768
68
86

1,3,5,7,8,4

261738
68
86

2,1,7,3

831762
13
31

8,7,6,2

712683
12 38
83 21

712683
36
63

613584
48
84

6,1,3,5

673524
36
63

2,7,3,2,4

6,5,2,4

413586
48
84

1,3,5,6

2,7,6,8

683514
48
84

6,3,5,1

312687
36
63

1,2,8,7

382617
36
63

8,2,1,7

231768
13
31

1,2,5,9

123579
37
73

1,2,3,7

4

261738
6

261738
68
86

2,1,7,3

123579
59
95

123579
15
51

2,3,7,9

125379
153972
277451
93

2,5,3,9

1,2,5,7

142659
25
52

2,6,5,9

1,4,6,9

1,4,6,5

5,2,6,9

1,6,4,9

1,5,2,4

142659
69
96

1,4,3,5

146259
29
92

152649
36
63

152649
29
96

5,3,2,9

1,7,5,2

175329
37
93

156249
412965
569214

1,5,6,4

173529
59
95

1,5,2,9

1,7,3,2

4,1,5,2,8,6

4,5,2,6

495
499
994

4,5,3,7,1,6,3,7

41735286
34126857
75862143

1,7,3,5,2,6

4,1,7,3,5,8

41735286
24137856
65873142

41753286
28
82

41735286
48
84

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41235786
37
73 (12)

4,1,3,5,7,8
~~4156~~
4378

41253786
42 17 38
83 71 24

4,2,5,3,8,6

42135876

~~15~~ 9286
~~84~~

2,1,3,5,7,6

~~4,1,3,5,8,7~~
4,2,1,5,8,6

4,~~1,2,5,8,6~~
1,2,3,5,76

42153876

4,1,5,3,7,6

4,2,5,3,8,6 ~~1,2,3,5,6,7~~

4,~~2,8,3,~~76
4,1,5,8,2,6 | 4,7,1,3,3,8

42835176
47135826
31426587
78562413

47135826
48
84

7,1,3,5,2,6
4,7,3,8,2,6

31562487
34218756
65781243
new 14/8

3,5,6,2,4,7

47135826
21437586
68573412
new 14/8/74

No more search from (C).

47~~835126~~

31426587 x
31462587 x

31526487
26314875
57841362

3,2,6,5,8,7
3,1,4,5,8,7
1,4,2,5,8,7
3,1,4,6,5,8
3,4,6,2,5,7
3,5,2,6,8,7

3,1,6,4,8,7
3,1,6,5,2,8
3,1,2,4,6,7

34826157 X

3,5,2,6,8,7
3,4,8,6,1,7
3,5,1,2,6,8
3,1,2,6,4,7

35126847
13625784
48752631
new 14/8

35126847
51247368
86374215
new 14/8

3,5,6,2,4,7
6,2,3,7,8,4
1,3,5,7,8,4
61235784
57
41
75
6,1,2,5,8,4
6,1,2,3,5,7

new 14/8

31526487
16325874
47852361
new 14/8

31526487
42158637
73685124
new 14/8

35126847
41258367
76385214
new 14/8

31526487
52147638
83674125
new 14/8

35162847
32418576
67581423
new 14/8

No more from (B) — Thank God!

Back again to (E) & (D). Of course (F) is out — Thank God!

From (E) only the 4 cases in 1st col., 1st two in each of 2nd & 3rd cols. need be examined by our new technique (ie starting from the middle two digits)

75126843
12436578
87563421
new 14/8

5,1,2,6,8,3 x From (D).
7,1,2,6,4,3 x

7,5,6,2,4,3

~~75162843~~ x
74126853 x
74162853 x
71426583 x
71462583 x
71526483 x
71562483 x

7,2,6,8,5,3
7,4,1,6,8,3
7,5,1,2,8,4

5,1,6,2,8,3
7,4,6,2,5,3

7,4,6,2,5,3 x

7,5,2,6,8,3
7,1,6,4,8,3
7,1,5,2,6,8,3
7,5,6,2,4,3

67153824 x
67135824 x
62135874 x
62153874 x
61235784 x
61253784 x
61735284 x
8
8 1
61753284 x

None from (D) but one from (E) rather surprising

~~6,5,3,8,2,4~~ x
~~7,1,3,5,8,4~~

6,2,3,8,7,4
1,3,5,8,7,4
6,2,1,5,8,4
6,2,1,3,5,7
6,2,5,3,8,4
6,1,5,3,7,4
6,2,5,3,8,4
6,1,5,3,7,4
~~7,3,5,4~~
6,7,3,2,8,4
1,7,3,5,8,4
6,1,5,2,8,4
6,1,7,3,5,2

6,2,3,7,8,4
1,3,5,7,8,4
6,1,2,5,8,4
6,1,2,3,5,7

2,3,4,1,7,5

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From (A): 23417568 ×
~~234 864~~
~~468 432~~
~~2,3,4,1,6,8~~
23471568 ×

2,4,7,5,6,8
3,1,7,5,6,8
2,4,1,7,6,8
2,3,1,7,5,8
2,4,1,7,6,8

2,4,6,8
2,7,5,8
234168
2,3,4,1,6,8
2,3,1,6

23517468
45213786
68731254 new

~~2,5,7,4,6,8~~ | 2,3,5,1,7,8
~~3,5,1,7,6,8~~ | 2,3,1,4,6,8
~~3,1,7,4,6,8~~ | 2,3,5,1,7,4

23571468 × None of 71's in (A) work.

~~24317568~~
~~95 64~~
~~4 32~~

(3,1) → 2,4,7,5,6,8
(4,2) → 1,3,7,5,6,8
(5,3) → 1,2,4,7,6,8
(6,4) → 1,3,5,6,7,8
(7,5) → 1,2,3,4,6,8
(8,6) → 1,2,3,4,5,7

Common when 1,7 are middle digits

23517468
15246783
38764251 new

23517468
31268745
54786213 new

~~23417568~~
~~231 564~~
~~465 132~~
24317568 ×

23517468
61235748
84753216 Same as on p. 5

24617358 ×

25317648
42513876
67831524 new 16/8

25317648
62135478
87453126 new 16/8

25317648
32168475
57486123 new 16/8

26417538 ×

~~2641~~ 26517438 ×

25617348 ×

16/8/74 That finishes the search (as far as humanly possible) for 8 digit cases. Anyway got plenty more than originally found.

Let us now try the 6 digit cases (where we have only 3 cases) by the new "centrifugal method" (!) I think this has already been done on p. 11. Anyway do it again. From (G) in this case there are none. From (F) only cases to be considered are 712683, 716283, 782613, 786213. By above method (7,3) → 6,1,2,8, 712683 × } Also the other two inadmissible
(8,1) → 7,2,6,3, 716283 ×

From (E) none admissible. From (D) middle (3,5) → (5,1) & (8,4) ⟺ So 413586 × (1,3,5,6) (4,3,8,6)
423576
473526 ×
483516 ×

(5,3) → no possibility

423576 ×, (7,3) & (6,2) → 4,2,5,6 ; 4,3,5,7. | From (D) none

From (C) ~~(2,6)~~ 312687 (2,6) → (6,3) → (1,2,8,7) ×
316287 (6,2) → (8,1) → 3,6,2,7 ×

342657: (2,6) → (6,3) & (5,2), (7,4)
(6,3) → (4,2,5,7) ×
(5,2) → (3,4,6,7) ×
(7,4) → (3,2,6,5) ×
(6,2) → ×

~~No other (62)'s since~~ 352647, (2,6) → (6,3), (7,4), (5,2)
(6,3) → (5,2,4,7) ×
(7,4) → (3,5,2,6) ×
(5,2) → (3,6,4,7) ×

356247 ×

(14) 382617 → (6,3) → (8,2,1,7) ×

386217 ×

From (B) 231768 → (3,1), (8,6) → (2,7,6,8); ~~2131~~ (2,3,1,7)
× ×

237168 ×

241758 → (4,2), (7,5) → (1,7,5,8), (2,4,1,8), No (71)
× ×

~~2420~~ 251748 → (4,2), ~~(5,2), (7,4), (2,5)~~ → (2,5,7,8), ~~(1,7,4,8)~~,
×

261738 → (3,1), (8,6) → (2,6,7,8), (2,1,7,3)
× ×

None from (B). From (A) 123579 → (7,3), (9,5) → (1,2,5,9), (1,2,3,7)
× ×

(5,1) → (2,3,7,9) → 123579
✓ 271593
395172 already got

125379 → (7,1), (9,3)
↓ ↓
(2,5,3,9) (1,2,5,7)
× ✓

125379
153972 ✓ already got.
279351

146259 → (9,2) → (1,4,6,5)
×

142659 → (5,2), (4,1), (9,6)
↓ ↓ ↓
(1,4,6,9) (2,6,5,9) (1,4,2,5)
× × ×

152649 → (4,1), (5,2), (9,6)
↓ ↓ ↓
(2,6,5,9) (1,4,6,9) (1,4,2,5)
× × ×

156249 → (9,2) → (1,5,6,4) ✓
412965
569214 already got ✓

173529 → (7,3), (9,5), (5,1)
↓ ↓ ↓
(1,5,2,9) (1,7,3,2) (7,3,2,9)
× × ×

175329 → (9,3), (7,1)
↓ ↓
(1,7,5,2) (5,3,2,9)
× ×

So what was obtained earlier was correct viz only 3 Cass.

Re. 5 & 7 digits 9 comes as middle digit & we have to consider digits before & after 9.

~~85912~~

5 digits 21978 → (9,7) → (2,1,8) ×

23958 → (9,5) → (2,3,8) ×

25938 → (9,3), (8,2) → (2,5,8), (5,9,3).
× ✓

32985
58923

~~32967~~ → (9,6) → (3,2,7), 36927 → (9,2) → (3,6,7)
× 32796
69723

72963 → (7,3), (9,6)
23769
96732 (2,9,6) (7,2,3)
×

41976 → (6,4), (9,7) → (1,9,7), (4,1,6)
× ×

43956 → (9,5), → (4,3,6)
×

81972 → (9,7) → (8,1,2)
×

45936 → (9,3) → (4,5,6)
×

83952 → (9,5) → (8,3,2) ×

47916 → (9,1) → (4,7,6)
×

85932 → (8,2), (9,3)
↓ ↓
(5,9,3) ×
×

87912 → (9,1) → (8,7,2)
×

So here again what is done earlier viz 4 Cass is correct

(3) ~~Similarly from~~ $x = 3$, we get

314296587	no cases for $x=3, y=2$	no cases for $x=3, y=3$	no cases for $x=3, y=4$
314692587			
315296487			
315692487			

From (B)

314296587	315296487	341296857	348296157
582417698	421798638	238164975	341275986
896714285	736895124	579461832	689572143

351296847	358296147	384296517	385296417
328154976	321548976	412538697	312548796
679451823	679845123	796835214	697845213

These are just 8 sample cases & we have not taken the trouble to find more as we have done for the cases (C) to (F).

From (A).

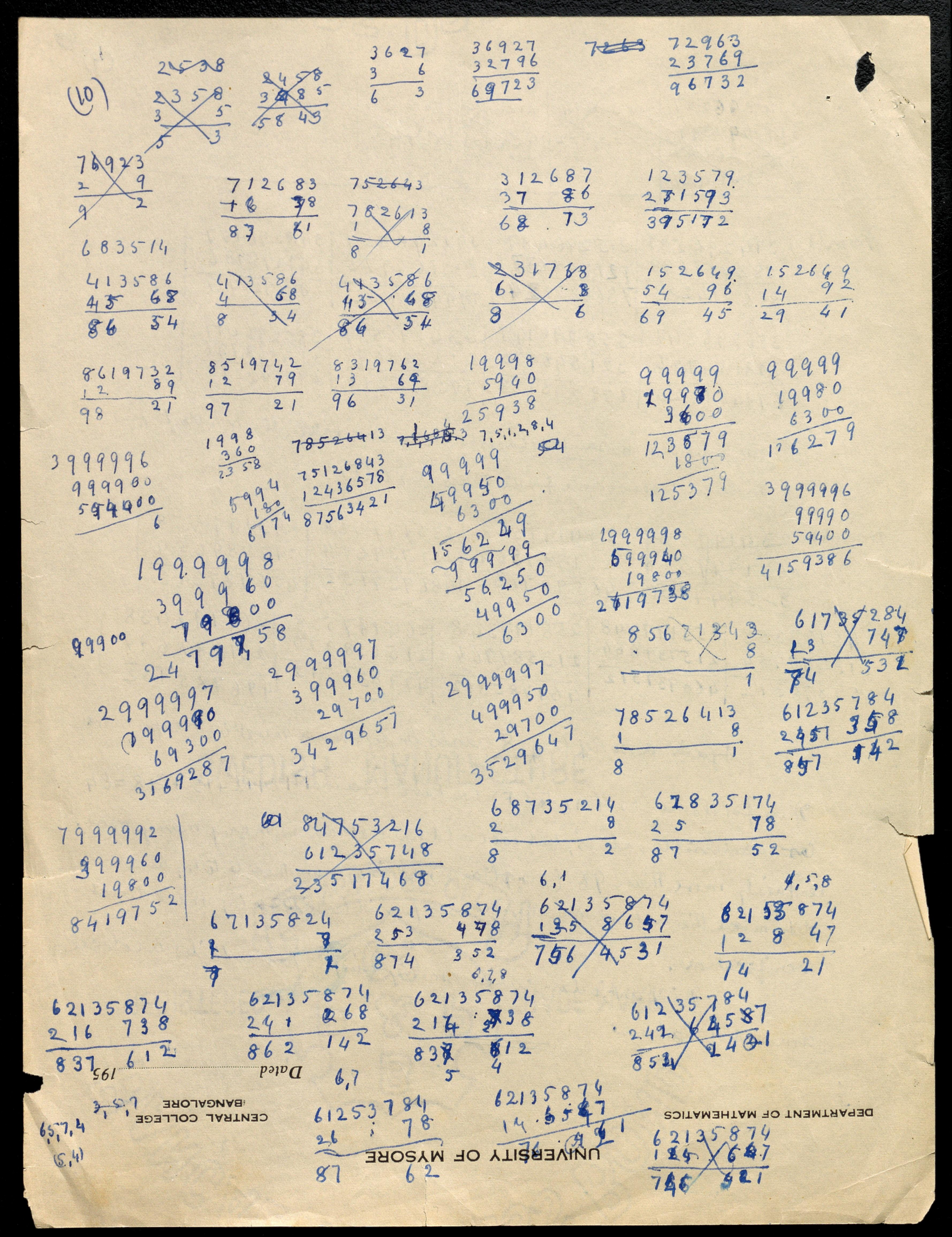
234197568	234197568	235197468	243197658
631784952	231789564	251739684	213789654
865947136	465987132	486937152	456987312

246197358	253197648	256197348	264197538	265197438
213678954	215739864	213587964	215638974	231587694
459876312	468937512	469785312	479836512	496785132

Again a sample of 8 cases only. There must be heaps more

Thus for the cases (A) to (F) we have obtained $9+12+12+15+8+8 = 64$ admissible values, but we could have obtained many many more, certainly more than 96 (no. obtained). It is not worthwhile to try and determine the actual number of cases possible without by mere direct verification.

This particular curio starting simply has entailed a colossal amount of labour.



(10)

2538
2358
3 5
5 3

2458
3485
5843

3627
3 6
6 3

36927
32796
69723

72963
23769
96732

76923
2 9
9 2

712683
16 78
87 61

752643
782613
1 8
8 1

312687
37 86
68 73

123579
271593
395172

683514

413586
45 68
86 54

413586
4 68
8 34

413586
45 68
86 54

231768
6 8
8 6

152649
54 96
69 45

152649
14 92
29 41

8619732
12 89
98 21

8519742
12 79
97 21

8319762
13 69
96 31

19998
5940
25938

99999
19980
3600
123579
1800
125379

99999
19980
6300
126279

3999996
999900
594000
6

1998
360
2358

5994
180
6174

75126843
12436578
87563421

99999
49950
6300
156249
99999
56250
49950
6300

1999998
599940
19800
2719738

3999996
99990
59400
4159386

1999998
399960
79800
24797,58

99900

2999997
399960
29700
3429657

2999997
499950
29700
3529647

2999997
199980
69300
3169287

85621343
8
1

61735284
13 747
74 532

78526413
1 8
8 1

61235784
2457 358
857 142

7999992
399960
19800
8419752

84753216
61235748
23517468

68735214
2 8
8 2

62835174
25 78
87 52

6,1

4,5,8

67135824
1 7
7 1

62135874
253 478
874 352

6,2,8

62135874
135 8657
756 4531

62135874
12 8 47
74 21

62135874
216 738
837 612

62135874
241 268
862 142

62135874
214 738
837 612
5 4

6,7

61235784
242 6457
854 2431

3,5,7

6,5,7,4

(5,4)

61253784
26 78
87 62

62135874
14 5547
76 421

62135874
125 647
746 521

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(F)

from (D) we have

853197642	846197352	843197652	843197652	843197652
125436879	132456879	135264879	132648579	143527689
978634521	978654231	978462531	975846231	986725341

835197462	834197562	834197562	834197562
152436789	153426789	142385679	123658759
987634251	987626351	976583241	957856321

9 Cases. (perhaps many more)

from (E)

7512968 43	7512968 43	7512968 43	7412968 53	7412968 53
2361 54789	1236594 78	2135784 69	2173468 59	1346295 78
9874541532	8748563 21	9648753 12	9586437 12	8759264 31

715296483	715296483	715296483	715296483	714296583
263154879	268~~1~~4879	231578649	132659748	271346589
978451362	973 1362	946875132	847956231	985643172

714296583	714692583	714692583
125467938	243168759	143269758
839764521	957961342	857962341

12 Cases (perhaps many more).

From (D).

682395714	671395824	671395824	628395174	621395874
214357698	314276589	125438697	241357968	216549738
896753412	985672413	796834521	869753142	837945612

621395874	621395874	621593874	617395284	617395284
243519468	351468279	342157369	341276859	152438967
864915342	972864153	963751243	958672143	769834251

612395784	612593784
124589637	346127859
736985421	958721643

12 cases (perhaps many more)

From (C)

482395716	482395716	482593716	471395826	471395826
215438796	415236798	314258697	314296587	426135798
697834512	897632514	796852413	785692413	897531624

471593826	428395176	428395176	428593176	421395876
214379586	451236978	251438976	341258967	341589267
685973412	879632154	679834152	769852143	762985143

421395876	421593876	417395286	417395286	412593786
273185496	451369278	241397856	452317968	524187639
694581372	872963154	658793142	869713254	936781425

and 15 cases (obviously many many more)

I wonder why I undertook this nuisance of a job! 6/8/74

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(3) For $x=3$, we have similarly

314296587	341296857	351296847	384296517
314692587	341692857	351692847	384692517
315296487	348296157	358296147	385296417
315692487	348692157	358692147	385692417

(16 cases) (B)

(4) For $x=4$.

412395786	421395876	471395826	482395716
412593786	421593876	471593826	482593716
417395286	~~427395816~~	478395126	487395216
417593286	428395176	478593126	487593216
	428593176		

(16) cases (C)

(5) For $x=5$, no cases

(6) For $x=6$, we have

612395784	621395874	671395824	682395714
612593784	621593874	671593824	682593714
617395284	628395174	678395124	687395214
617593284	628593174	678593124	687593214

(16) Cases (D)

(7) For $x=7$, we have, just reversing the digits in the case for $x=3$ from bottom to top.

714296583	741296853	751296843	784296513
714692583	741692853	751692843	784692513
715296483	748296153	758296143	785296413
715692483	748692153	758692143	785692413

(16) Cases (E)

(8) For $x=8$, similarly using the case $x=2$

834197562	843197652	853197642	864197532
834791562	843791652	853791642	864791532
835197462	846197352	856197342	865197432
835791462	846791352	856791342	865791432

(16) cases (F).

Thus there are altogether 96 cases. (A) & (F), (B) & (E), (C) & (D) are reverses of each other from top to bottom. (F) – (A) gives no admissible values. (E) – (B) gives just the two values

$$\begin{array}{r} 785692413 \\ -\ 314296587 \\ \hline = 471395826 \end{array}$$

which is one of the number in (C)

$$\begin{array}{r} 758692143 \\ 341296857 \\ \hline 417395286 \end{array}$$

again found in (C)

Again (D) – (F) gives no admissible value. Thus there are just two admissible values obtained by this method of subtracting reverses. But in the obvious case

$$987654321 - 123456789 = 864197532$$

[this is in Table (F)], there is no use of the reversion subtraction method and we have to use the direct method of summation to the R.H.S. This appears easy in this case of 864197532 viz

$$\begin{array}{r} 864197532 \\ 123456789 \\ \hline 987654321 \end{array}$$

since at each step the digits to be added are determined uniquely. Let us verify this true of all the 96 cases

$99999x + 9990y + 900z$

$11111x + 1110y + 100z$

$1101x + 1010y + 100(x+y+z)$

$1000x$

199998
19980
3600
23578

9 digits – Although we have the simple solution 987654321 + 123456789 = 864197532

we might attempt to find other solutions if any, since as in the case of 8 digits, only four variables x, y, z, u are involved. The diophantine eqn is

$$99999999x + 9999990y + 999900z + 99000u$$

99999999	·9999999·
9999990	·99999·0
999900	·9999·00
99000	·999·000

(1) $x = 1$, $y = 1$, $z = 1, \ldots 8$

111 · 9 · 889	121 · 9 · 879
112 · 9 · 789	
113 · 9 · 689	
114 · 9 · 589	
115 · 9 · 489	
118 · 9 · 389	
· 289	
9 · 189	

No further need to consider the case $x = 1$ since 9 appears as the middle and last digit.

$x = 2, y = 1$

211 · 9 · 885
212 · 9 · 788
· · ·
· · ·
· · ·
8 · 9 · 188

$x = 2, y = 2$

221 · 9 · 878
222 · 9 · 778
· · ·
· · ·
228 · 9 · 178

$x = 2, y = 3$

231 · 9 · 868 ×
232 · 9 · 768 ×
233 · 9 · 668 ×
234 · 9 · 568 ✓
235 · 9 · 468 ✓
236 · 9 · 368 ×
237 · 9 · 268 ×
238 · 9 · 168 ×

$x = 2, y = 4$

241 · 9 · 858 ×
242 · 9 · 758 ×
243 · 9 · 658 ✓
244 · 9 · 558 ×
245 · 9 · 458 ×
246 · 9 · 358 ✓
247 · 9 · 258 ×
248 · 9 · 158 ×

$x = 2, y = 5$

251 · 9 · 848 ×
252 · 9 · 748 ×
253 · 9 · 648 ✓
254 · 9 · 548 ×
255 · 9 · 448 ×
256 · 9 · 348 ✓
257 · 9 · 248 ×
258 · 9 · 148 ×

$x = 2, y = 6$

261 · 9 · 838 ×
262 · 9 · 738 ×
263 · 9 · 638 ×
264 · 9 · 538 ✓
265 · 9 · 438 ✓
266 · 9 · 338 ×
267 · 9 · 238 ×
268 · 9 · 138 ×

$x = 2, y = 7$

1 · 9 · 828 ×
2 · 9 · 728
3 · 9 · 628
· · ·
· · ·
· · ·
78 · 9 · 128

$x = 2, y = 8$

281 · 9 · 828 ×
· · ·
· · ·
288 · 9 · 128

So for the case $x = 2$, we have only to consider 4 cases $y = 3, 4, 5, 6$ giving and taking in the places marked · 9 · the values for $(u = 1, \ldots 8)$ viz $(0,8), (1,7), (2,6), (3,5), (4,4), (5,3), (6,2), (7,1)$ respectively except the admissible numbers.

234197568	243197658	253197648	264197538
234791568	243791658	253791648	264791538
235197468	246197358	256197348	~~12 cases~~
235791468	246791358	256791348	(A)
			265197438
			265791438

(A) (16) cases

(5) For $x=5$, no numbers are obtained since as per schemes for $x=1, -4$ (5) etc, the numbers begin with 5 and end with 5.

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(6) For $x=6$, we have the 16 cases.

61235784	62135874	67135824	68235714	} (D)
61253784	62153874	67153824	68253714	
61735284	62835174	67835124	68735214	
61753284	62853174	67853124	68753214	

(7) For $x=7$,

71426583	74126853	75126843	78426513	} (E)
71462583	74162853	75162843	78462513	
71526483	74826153	75826143	78526413	
71562483	74862153	75862143	78562413	

(8) For $x=8$,

83417562	84317652	85317642	86417532	} (F)
83471562	84371652	85371642	86471532	
83517462	84617352	85617342	86517432	
83571462	84671352	85671342	86571432	

We notice that (A) and (F) are reverses (from top to bottom) of each other

& " (B) & (E) "

& (C) & (D) "

Consider now (E) − (B) we get in two cases viz

$$78562413 - 31426587 = 47135826$$
$$\text{and } 75862143 - 34126857 = 41735286$$

} two required solutions.

But (F) − (A) and (D) − (C) give no such cases. Thus without attempting the direct method of attack we get two solutions. However, it does not follow that these are the only two possible ones, specially in view of the fact that we have found 14 solutions for the case of 7 digits!

From (A) we have,

23517468	25317648
61235748	62135478
84753216	87453126

(B) we have the "two solutions" obtained by the indirect method viz { 31426587, 34126857 }

(C) we have the same two solutions viz { 41735286, 47135286 }

(D) the same "two solutions" given in (A) above.

(E) " (A) "

Thus there are really 4 solutions!

~~No point in trying to find other solns~~

Suffp

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(1)

$x=1, y=1$ ($z=1,2\cdots 8$)	$x=1, y=2$	$x=1, y=3$	$x=1, y=4$	$x=1, y=5$	$x=1, y=6$	$x=1, y=7$	$x=1, y=8$
× 111 .. 889	121 .. 879 ×	131 .. 869 ×	141 .. 859 ×	151 .. 849 ×	161 .. 839 ×	171 .. 829 ×	181 .. 819 ×
× 112 .. 789	122 .. 779 ×	132 .. 769 ✓	142 .. 759 ✓	152 .. 749 ✓	162 .. 739 ✓	172 .. 729 ×	182 .. 719 ×
× 113 .. 689	123 .. 679 ✓	133 .. 669 ×	143 .. 659 ✓	153 .. 649 ✓	163 .. 639 ×	173 .. 629 ✓	183 .. 619 ×
× 114 .. 589	124 .. 579 ✓	134 .. 569 ✓	144 .. 559 ×	154 .. 549 ×	164 .. 539 ✓	174 .. 529 ✓	184 .. 519 ×
× 115 .. 489	125 .. 479 ✓	135 .. 469 ✓	145 .. 459	155 .. 449 ×	165 .. 439 ✓	175 .. 429 ✓	185 .. 419
× 116 .. 389	126 .. 379 ✓	136 .. 369 ×	146 .. 359 ✓	156 .. 349 ✓	166 .. 339 ×	176 .. 329 ✓	186 .. 319 ×
× 117 .. 289	127 .. 279 ×	137 .. 269 ✓	147 .. 259	157 .. 249 ✓	167 .. 239 ✓	177 .. 229 ×	187 .. 219 ×
× 118 .. 189	128 .. 179 ×	138 .. 169 ×	148 .. 159 ×	158 .. 149 ×	168 .. 139 ×	178 .. 129 ×	188 .. 119 ×

discarding the cases where digits are repeated. Now considering each of these separately and taking for the middle two digits in each of the tables the values (0,8), (1,7), (2,6), (3,5), (4,4), (5,3), (6,2), (7,1) we get, considering the cases marked ✓, from the six tables (1st & last being discarded), the result that there is no number without repeated digits for the case $x=1$.

(2)

$x=2, y=1$ ($z=1,2,\cdots 8$)	$x=2, y=2$	$x=2, y=3$	$x=2, y=4$	$x=2, y=5$	$x=2, y=6$	$x=2, y=7$	$x=2, y=8$
211 .. 888 ×	221 .. 878 ×	231 .. 868 ×	241 .. 858 ×	251 .. 848 ×	261 .. 838 ×	271 .. 828 ×	281 .. 818 ×
212 .. 788 ×	22 .. 778 ×	232 .. 768 ×	242 .. 758 ×	252 .. 748 ×	262 .. 738 ×	272 .. 728 ×	282 .. 718 ×
. . .	. . .	233 .. 668 ×	243 .. 658 ✓	253 .. 648 ✓	263 .. 638 ×	273 .. 628 ×	— — ×
		234 .. 568 ✓	244 .. 558 ×	254 .. 548 ×	264 .. 538 ✓	. . . ×	— — ×
		235 .. 468 ✓	245 .. 458 ×	255 .. 448 ×	265 .. 438 ✓	. . . ×	— — ×
188 ×	228 .. 178 ×	236 .. 368 ×	246 .. 358 ✓	256 .. 348 ✓	266 .. 338 ×	— — ×	
		237 .. 268 ×	247 .. 258 ×	257 .. 248 ×	267 .. 238 ×	— — ×	
		238 .. 168 ×	248 .. 158 ×	258 .. 148 ×	268 .. 138 ×	278 .. 128 ×	288 .. 118 ×

...e the permissible numbers

23417568	24317658	25317648	26417538	(A)
23471568	24371658	25371648	26471538	
23517468	24617358	25617348	26517438	
23571468	24671358	25671348	26571438.	

...ilarly from $x=3$, we get

31426587	34126857	35126847	38426517	(B)
31462587	34162857	35162847	38462517	
31526487	34826157	35826147	38526417	
31562487	34862157	35862147	38562417	

For $x=4$, we get.

41235786	42135876	47135826	48235716	(C)
41253786	42153876	47153826	48253716	
41735286	42835176	47835126	48735216	
41753286	42853176	47853126	48753216	

giving the cases

x=2	x=3	x=4	x=6	[illegible]	x=8
2319768	3129687	4139586	6139584	7129683	8319762
2379168	3169287	4159386	6159384	7169283	8379162
2419758	3429657	4239576	6239574	7429653	8419752
2479158	3469257	4259376	6259374	7469253	8479152
2519748	3529647	4739526	6739524	7529643	8519742
2579148	3569247	4759326	6759324	7569243	8579142
2619738	3829617	4839516	6839514	7829613	8619732
2679138	3869217	4859316	6859314	7869213	8679132
(A)	(B)	(C)	(D)	(E)	(F)

giving in all 48 cases

(A) & (F) are reverses top to bottom
(B) & (E) " "
(C) & (D) " "

Easily verified that none of the differences F−A, E−B or D−C give required cases

So only the direct method of verification is available.

from (A)

2419758	2479158	2619738
2178954	5412987	7216389
4598712	7892145	9836127

" (B)

3169287	3429657	3529647
3721986	6321579	2437695
6891273	9751236	5967342

" (C)

4159386	4239576	4259376
4531968	5431769	5213749
8691354	9671345	9473125

" (D)

6139584	6239574
3541869	3524679
9681453	9764253

giving all together 14 cases.

" (E)

7129683	7169283
1267938	2613879
8397621	9783162

" (F)

8419752
1452789
9872541

8 digits: $9999999x + 999990y + 99900z + 9000u$.

9999999	writing in the form ·999999·
999990	·9999·0
99900	·99·00
9000	··000

where the dots in the four columns from the top make allowance for values of x, y, z, u respectively from 1 to 8.

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$x = 5 \to 51\ 85\times$

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6 digits. $99999x + 9990y + 900z$

	$x=1$	$x=2$	$x=3$	$x=4$	$x=6$	$x=7$	$x=8$
·99999	110889 ×	21 ~~1888~~ ×	31 ~~0~~887 ✓	41 86 ✓	61 84 ✓	71 83 ✓	81 82 ×
·9990	120879 ✓	22 78 ×	32 77 ×	42 76 ✓	62 74 ✓	72 73 ×	82 72 ×
·900	13 69 ×	23 68 ✓	33 67 ×	43 66 ×	63 64 ×	73 63 ×	83 62 ✓
	14 59 ✓	24 58 ✓	34 57 ✓	44 56 ×	64 54 ×	74 53 ✓	84 52 ✓
	15 49 ✓	25 48 ✓	35 47 ✓	45 46 ×	65 44 ×	75 43 ✓	85 42 ✓
	16 39 ×	26 38 ✓	36 37 ×	46 36 ×	66 34 ×	76 33 ×	86 32 ✓
	17 29 ✓	27 28 ×	37 27 ×	47 26 ✓	67 24 ✓	77 23 ×	87 22 ×
	18 19 ×	28 18 ×	38 17 ✓	48 16	68 14 ✓	78 13 ✓	88 12 ×

3579	231768	312687	413586	613584	712683	831762
3 79	237168	316287	415386	615384	716283	837162
659	241758	342657	423576	623574	742653	841752
259	247158	346257	425376	625374	746253	847152
649	251748	352647	473526	673524	752643	851742
49	257148	356247	475326	675324	756243	857142
529	261738	382617	483516	683514	782613	861732
5329	267138	386217	485316	685314	786213	867132
(A)	(B)	(C)	(D)	(E)	(F)	(G)

ie $7 \times 8 = 56$ cases as already found

(B) & (G) are reverses top to bottom.
(C) & (F) " "
(D) & (E) " "

Subtraction between the several of (B) & (G) yields no result. only (A) gives 3 possible solutions viz

(1) $123579 + 271593 = 395172$

(2) $125379 + 153972 = 279351$

(3) $156249 + 412965 = 569214$

(3) was not noticed in the earlier work

digits. $999999x + 99990y + 9900z$

	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$
999999	11 9 89 ×	21 9 88 ×	31 9 87 ✓	41 9 86 ✓	51 9 85 ×
99990	12 9 79 ×	22 9 78 ×	32 9 77 ×	42 9 76 ✓	5 - - 5 ×
9900	13 9 69 ×	23 9 68 ✓	33 9 67 ×	43 9 66 ×	- - - ×
	14 9 59 ×	24 9 58 ✓	34 9 57 ✓	44 9 56 ×	- - - ×
	15 9 49 ×	25 9 48 ✓	35 9 47 ✓	45 9 46 ×	
	16 9 39 ×	26 9 38 ✓	36 9 37 ×	46 9 36 ×	
	17 9 29 ×	27 9 28 ×	37 9 27 ×	47 9 26 ✓	5 - - 5 ×
	18 9 19	28 9 18 ×	38 9 17 ✓	48 9 16 ✓	

$x=8$	$x=7$	$x=6$
81 9 82 ×	71 9 83 ✓	61 9 84 ✓
82 9 72 ×	72 9 73 ×	62 9 74 ✓
83 9 62 ✓	73 9 63 ×	63 9 64 ×
84 9 52 ✓	74 9 53 ✓	64 9 54 ×
85 9 42 ✓	75 9 43 ✓	65 9 44 ×
86 9 32 ✓	76 9 33 ×	66 9 34 ×
87 9 22 ×	77 9 23 ×	67 9 24 ✓
88 9 12 ×	78 9 13 ✓	68 9 14 ✓

9x + 90y.

9998	10	89✓	1089×	2088×	3087×	4086×	6084×	7083×	8082×
90	2	8✓	1179×	2178✓	3177×	4176✓	6174✓	7173×	8172✓
	3	7✓	1269✓	2268×	3267✓	4266×	6264×	7263✓	8262×
	4	6✓	1359✓	2358✓	3357×	4356✓	6354✓	7353×	8352✓
	5	5×	1449×	2448×	3447×	4446×	6444×	7443×	8442×
	6	4✓	1539✓	2538✓	3537×	4536✓	6534✓	7533×	8532✓
	7	3✓	1629✓	2628×	3627✓	4626×	6624×	7623✓	8622×
	8	2✓	1719×	2718✓	3717×	4716✓	6714✓	7713×	8712✓

ie

1269	2178	3267	4176	6174	7263	8172
1629	2718	3627	4716	6714	7623	8712
1359	2358	(C)	4356	6354	(F)	8352
1539	2538		4536	6534		8532
(A)	(B)		(D)	(E)		(G)

ie altogether 24 cases.

Note that: (B) & (G), (C) & (F), (D) & (E) are reverses of each other.

while (A) has no reverse.

(G) → 8712 / 2178 / ✗ 6534 | 8532 / 2358 / 6174 | ~~8172 / 2718 / 5454~~ | ~~8352 / 2538 / 5814~~

(E) → 6714 / 4176 / ✗ 2538 | ~~6174 / 4716 / 1458~~ | 6534 / 4356 / 2178 | ~~6354 / 4536 / 1818~~

(F) → 7623 / 3267 / ✗ 4356 | ~~7263 / 3627 / 3636~~

So the method used for eight digits does not work here & the direct method giving the 3 cases has to be adopted.

---

5 digits 9999x + 990y

9999	1	9✓	10989	20988×	30987×	40986×	50985	60984×	70983×	8
990	2	8✓	11979	21978✓	31977×	41976✓	51975	61974✓	71973×	8
	3	7✓		22968×	32967✓	42966×	52	62964×	72963✓	8
	4	6✓		23958✓	33957×	43956✓		63954✓	73953×	8
	5	5×		24948×	34947×	44946×		64944×	74943×	8
	6	4✓		25938✓	35937×	45936✓		65934✓	75933×	8
	7	3✓		26928×	36927✓	46926×		66924×	76923✓	8
	8	2✓	18929	27918✓	37917×	47916✓		67914✓	77913×	87912

21978	32967	41976	61974	72963	81972
23958	36927	43956	63954	76923	83952
25938		45936	65934		85932
27918		47916	67914		87912
(A)	(B)	(C)	(D)	(E)	(F)

ie altogether 20 cases

(A) & (F) reverses
(B) & (E) "
(C) & (D) "

72963	76923
36927	32967
45936	43956

~~61974~~ ~~47916~~ The method as in 8 digits does not work. The direct method gives the four cases.

25938 + 32985 = 58923
36927 + 32796 = 69723
61974 + 14967 = 76941
72963 + 23769 = 96732

The direct method is useful for cases of 4 & 5 because of only 64 cases to be examined.