SUMMARY OF TALK

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The subject of Magneto-hydrodynamics, including Magneto-gas-dynamics has been vigorously pursued in recent years, and literature pertaining to the same has therefore been expanding very rapidly. I wish to present briefly some of these developments by dividing the subject into three categories specified respectively by the macroscopic, normal, and microscopic scales of the phenomena concerned.

It need hardly be said that much of the advance in all the three regions has been due to recent intensive study, to a great extent initiated by S. Chandrasekhar, of the solutions of the fundamental equations themselves. For example, the systematic discussion of the hydromagnetic equations for axi-symmetric fields in case of fluids of finite electrical conductivity, based on the decomposition of the magnetic and velocity fields into their poloidal and toroidal components has yielded valuable results. Further generalization by dropping the assumption of a static field, and using instead the condition that the undisturbed $\vec{v}$ and $\vec{H}$ fields are stationary, and that the $\vec{v}$ field is everywhere proportional to the $\vec{H}$ field, a great freedom can be obtained in the choice of the undisturbed field. By such a suitable choice, expressions can be derived for the periods of small oscillations about the equilibrium configuration, and lower bounds deduced for the periods of the lowest modes of vibration. Other types of work consist in the discussion of wave motion in compressible and incompressible media, thermal instability in the presence of a magnetic field, and the propagation and structure of magneto-hydrodynamic shock waves. Consideration of the stability of the simplest solutions of the fundamental equations has also given significant results.

At the macroscopic level, lie the results of an astrophysical significance. It has been possible to discuss from the point of view of the hydromagnetic equations, the question of generation and maintenance of cosmic magnetic fields. Some recent Russian work in the field of relativistic magneto-gas-dynamics also falls in this category, but it is still in the introductory stage of setting up the equations of motion in the Minskowski 4-space, there being, however no derivation so far of results of astrophysical interest. Attempts at obtaining the possible equilibrium configurations of magnetic stars, and the nature of dissipation of magnetic energy in interstellar space are other examples.

In the second category, the most striking result recently obtained is that due to S. Chandrasekhar about the decay of the Earth's magnetic field. Considering the effect of internal motions on the decay of a magnetic field in a fluid conductor, he has applied the general theory relating to axi-symmetric fields to this particular problem, and shown by numerical analysis that reasonable motions, if they occur appropriately, can lengthen the decay time of the Earth's magnetic field from 17,000 years—the value it would have in the absence of internal motions—to 500,000 years, a value suggested by recent experimental results. Theories have also been set up to serve as a basis for explaining the origin of the Earth's magnetic field. One such assumes a non-uniform rotation which generates a toroidal magnetic field from an initial poloidal one, next a succession of " cyclones " creating out of this toroidal field loops of flux in the meridian planes, and finally these coalescing and regenerating a poloidal field. Mention must also be made of recent work done at the Japanese Earthquake Research Institute about the Earth's magnetic field based on the model of a self-exciting dynamo, and investigating the stability of the Earth's dynamo. This institute has also done work on the explanation of the origin of earthquakes based on magneto-hydrodynamic theory.

On the microscopic level, the outstanding recent contribution is the development of the stellarator which is an experimental device employing a twisted magnetic field to contain an ionized gas, and electric fields to heat it, and which is perhaps the first step towards the deriving of useful power from thermo-nuclear reactions, a suggestion first made by H. J. Bhabha. When two nuclei of heavy hydrogen collide at high energies, they interact forming a new nucleus, and liberating either a proton or neutron of high energy. To obtain a useful power yield from this reaction, a gas of deuterium (H_2) or of mixed deuterium and tritium (H_3) must be brought to an enormously high temperature—equal to about 10^8 degrees absolute. The problem then is to confine this gas within a limited region away from contact with any solid matter, as otherwise it will cool and the solid material will evaporate. Magnetic forces seem to offer the only way to contain a thermo-nuclear reaction using the *pinch effect*, viz. the flow of a heavy electric current through the hot gas, thereby generating a strong magnetic field which at once compresses the gas and brings it up to a high temperature. In the stellarator, the gas is originally contained in a magnetic field produced by the above process thereby reaching a temperature of about 10^6 degrees, and then the ultra-high temperatures are reached through an effect called *magnetic pumping* induced by a second extremely rapidly pulsating magnetic field. Besides the stellarator, recent work done which belongs to this third category relates to the oscillations of an electron plasma in a magnetic field, and plasma losses by fast electrons in thin films.

Magneto-hydrodynamics is a very young and rapidly expanding subject, and by its very nature a meeting ground of several disciplines of theoretical physics. A consequence of this is that at each step in the development of the subject, unexpected problems arise which need to be solved, and exact solutions are not, in general, possible because of the mathematical complexity of the fundamental equations. When, however, solutions are obtained under reasonable assumptions and approximations, the interpretation of these solutions leads to results as unexpected as the problems that arise. Surely there are exciting possibilities for research in magneto-hydrodynamics.

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