Dr K S Krisnan:

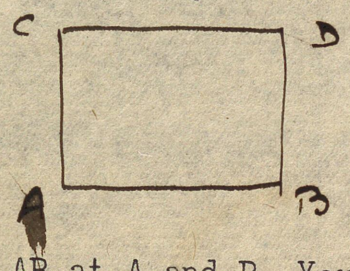
Mr President and gentlemen, it is a great pleasure to me and I feel a privilege to take part in the jubilee celebration of this great college. About a year ago I had occasion to address a gathering of medical men in this pandal. I then chose as my subject Virus(?), a subject of great interest to the medical men and one to which physics has recently contributed ~~also~~ much. But it being a very special occasion I think I must choose a subject of more general interest. I shall speak about Geometry that underlies the physical universe.

There must be a general impression among ~~xxxxxxxinxx~~ non-scientific men that what is generally called non-euclidian geometry is something ~~ixxx~~n very hypobole, something which is created hypermetrical mathematicians out of sheer love of logic; but nothing can be f~~xxx~~ farther from it. What is generally described as non-euclidian ~~ge~~ is really a more natural geometry by which I mean the geometry that lies at the basis of ~~x~~ nature. What is more it is simpler to understand than common geometry with which we are all familiar. There is a wellknown saying attributed to a great mathematican that no mathematics is good mathematics unless you can explain it to the first man in the street. I do not knonw if much of the mathematics that we know can answer this stringent test. But noneuclidian geometry which I referred to as the basis of the physical universe does stand the test and that is why I have chosen that as a subject for popular exposition before a nonmathematic audience.

It is also connected with an age old problem. Is the universe finite or infinite ? Bruno, the famous astronomer stood for an infinite universe; on the other hand Kepler, an equally distinguished Astronomer stood for finite universe and Galilio had no opinion to offer and he said that evidence that was available was not sufficient to determine whether the universe is finite or determined or infinite or undetermined. But today owing to scientific work that has been carried on for the last three centuries we think we know a little more about the structure of the universe to depend upon for an answer.

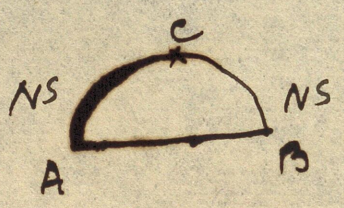
Before I give the answer I shall start with our good old euclid. For twenty thousand years the teaching of geometry in every part of the civilized world has been dominated by Euclid. It is certainly a great compliment to the great mathematician that a text book written in the third century B.C. should have been used continuously for twenty centuries. It is however very doubtful how much of the geometry which Euclid gave us is his own or how much is just a compilation from earlier works that were known at his time. However that may be, there is no doubt that the arithmetical arrangements of the propositions as following from a set of axioms are certainly due to euclid and he will be remembered not so much for having supplied the geometry which is being taught in all colleges for twenty centuries but more for certain postulates which are now called euclidian postulates. As he went on developing his geometry starting with a very simple self-evident axiom he could prove 27 postulates and when he came to the 28th he got stuck up. He obtained results which might be expressed in different ways. One way of expressing it is to say that through a given point you can draw only one straight line parallel to a given straight line and no more. It looks very plausible. You are given a point and a straightline. The postulate says that from that point you can draw only one straight line parallel to the given straight line. But euclid in his intelligence and honesty recognised that it was not self-evident and he needed proof. Probably he tried to supply a proof; he did not succeed but was honest enoughx to state it as an axiom which is now very xxxxxxxxx wellknown as euclidian parallel postulate. For nearly 20 centuries after that almost all great geometricians and also mathematicians tried to prove that xxxxxxx euclidd's postulate was either right or wrong. They did not succeed in proving either the validity or falsity of that postulate. It was really a very intriguing situation; mathematician with highest calibre trying to prove what looks like a very plausible postulate and not succeeding either in proving it or disproving it, and Tambridge (?) the great mathematicianx pronounced it as almost a scandal that problem of such a magnitude should remain still unsolved. Gows (?) who is generally regarded as one of the world's greatest mathematicians, the other two being Aristotole and Newton, tried also to prove the postulate and he spent, I think, about 15 years and finally convinced himself that it was impossible to prove either the truth or the falsity of the proposition. That at any rate was not a satisfactory conclusion. Why was it left very vague as a postulate which may be true or false ? It was now realised that it was one of those propositions about which it was impossible to say anything dxxxxxxx definitely, and with a boldness characteristic to Gows (?) he xxxx proceeded to investigate the significance of that conclusion. For twenty centuries after they assumed that euclid was right, that there can be only one straight line that can be drawn through a point parallel to a given straight line. Arithmetically there is one thing left open to him, namely, to take the very alternative assumption that euclid is definitely wrong and proceeded on a denial in that postulate. He was led to some extremely interesting result. Meanwhile other mathematicians of whom some were his contemporaries had been at the problem without knowing that Gows (?) had been at it for sometime, particularly a Russian mathematician and later Reymond one of the German mathematicians If euclid's postulate is wrong you can have one or two alternatives given, either you may say there can be no straight line that can be drawn through a point parallel to a given straight line or you can say there can be more than one straight lixxsx

lines parallel to the given straight line. They tried these alternatives ; they got a curious type of geometry. They got results at variance with euclid's results. Take for example the well known proposition that the sum of the three angles of a triangle is equal to two right angles. In that new geometry,- according to one alternative,- the result was that the sum exceeded two right angles or was less than two right angles. I shall mention just now another result. You take a circle; you know that the area of the circle must be given by π r2 where r is the radius. If you have two circles, one a foot in diameter and the other two feet in diameter you can definitely say that the area of one must be four times the area of the first. If you have a circle three ft. in diameter you know its area must be nine times of the first one. I shall now mention just one more :-



You take a line AB at A and B. You draw perpendiculars

AC and BD of equal length, join the perpendiculars Cd. If you deny the ~~xxxxidix~~ euclid's postulate you can definitely say that the angles at C and D should be equal. But you can not say anything further regarding the magnitude of these angles. Euclid's postulate, if granted, would lead us to the conclusion that the angles should be equal to two right angles. But if you deny euclid's postulate, then both the angles are either acute or obtuse according to the two alternative assumptions that I have mentioned. You can now ask me,- it looks a very strange geometry to think of three angles of a triangle different from ~~two~~ two right angles. I shall put to you this question. Suppose we have not learnt geometry at schools from euclid, suppose we are suffering from what ~~is~~ they call wabbling and wandering from earth to earth. Suppose we ~~txx~~ take three straight lines, say in the stellar region which are very far removed, and suppose we find a straight line joining ~~them~~ any two of them We will define the straight line as the least distance of passage. If you do that I can immediately convince you the kind of geometry that would have developed, say the geometry of traingles would not have been the geometry of euclid. I shall take two places for the sake of convenience on the equator separated by a very large distance. At each one of the places you draw straight lines, north-south meridian and east-west ~~meridian~~ direction across the equator. At a point A you have two directions exactly north-south or exactly east-west.



The angle at A is only a right angle, similarly the angle at B is also a right angle and Ac is the north-south direction of the straight line and Bc is the ~~xxxxxxxxx~~ south-north direction of B . They meet at C. The sum of the three angles must be greater than two right angles. So the conclusion which they draw from the denial of euclid's postulate, namely, the three angles of a traingle are not equal to two right angles is not such an unacceptable theory after all. It is even more a natural geometry which we would have developed if we took any three points on the surface of the equator and join them and call the shortest distance or the shortest passage between any two points a straight line. And you ~~xx~~ sum the three angle you have then less than two right angles and even greater than that I can just try to corelate with euclid's geometry in this way that the geometry that would be obtained on the surfance of ~~xx~~ a sphere,- and a sphere has no plane surface,you say it is a curvature. But in order to appease those who have been brought up ith euclidian tradition we may grant that the geometry ~~xwhich~~ which we have developed will be geometry within euclid's postulate provided you take his geometry on the~~xsurfac~~ surface of the sphere . If we can show that just ~~thexxxxx~~ because curvature has space then we get the ~~xxx~~c excess of the sums of the three angles over two right angles, depends upon the curvature of the space. We may ~~xx~~ go further and to satisfy people who still insists on euclid's geometry, if they ask us what do you mean by curvature . We may shelve the ~~quxxti~~ question by saying that I merely define surface as being the curvature that we find on experiment for the sum that the angles of a traingle constructed on the surface exceeds two right angles, the excess will give you the measure of the curvature. I am at liberty to take surface as just I am at liberty to draw a traingle on the surface. I may add the three angles of a traingle, if I find it exceedds two right angles, I say it is the curvature and the excess will be the measure of the curvature. If you so choose you can explain it with reference to a solid. We know that in any space of solid dimension, you take a sphere,- provided the ~~geox~~ geometry is the geometry of euclid,- then the volume of the solid is the cube of its diameter. Supposing you do find on experiment in some space where you can make measurements of volume you construct two spheres with one million miles scale, because for the purpose of logic it ~~do~~ does not matter how long the distance you take. You take two spheres, one of them having a diamter~~x~~ double that of the other. If you do find that the second sphere has not wight times volume of the first sphere, has larger volume or smaller volume then you can say that the space has curvature and the measure of the curvature will be the defects or excess of the angles according to the ~~wellknownn~~ well knonwn cube law. Then you may ask: what do you mean by the curvature of the space ? I may answer it by a counter question. After all in the general sense formerly we used to regard space as something which has certain intrinsic properties, some geometrical structural ~~propertixs~~ properties. We ~~intrdx~~ introduce matter into it as though we a filling the flats in a modern house with people. The space is is not altered by putting materials in that. But Reymond asked the question: "Are you quite sure that the matric properties of a space will not alter if you put in matter inside or you take out matter from inside ?". It can be very well conceded that the matric properties of the space will change very slightly which may be neglected for our purpose. Some people may like to have space well kxxxxx,xxxxx xxxxxx known, determined and independent of matter. Obviously it looks purely a question of sentiment for individual taste. For, today we have mechanism by which we do kxx know definitely that the proposition that matters in a space affects its metric properties is preferable. You must all have heard of the famous experiment suggested by Einestein to find out from the time of the total solar eclipse whether the light of a star which happens to be very near the perpendicular will bend or not bend. Experiments show that the light of the star do bend. Anywhere in the solar universe any material particle would be attracted by the sun due to the law of gravitation Before I discuss the signifiaance of this I may mention that even before we had any idea as to how the curvature xxxxx can be produced in the sense I have mentioned, people xx thought of curvature because sheer logic for the denial of the euclid's postulate did require a curvature of that type.

They look for some experimental observation which woul would decide one way or the other. Many of you probably are

familiarxwithxwhatxisxknown

familiar with what is known as the paralax method of finding the distance of stars. The earth is moving round the sun, so there is no place which is contant in the earth and no star can be viewed from the same place all round the year. Supposing a star is more or less stationery with reference to the solar system. If you find,- in one part of the year when the earth is at one end of its orbit,- that the star is making a particular angle at a particularly selected part of the year xnxohenxitxx and when it is on the other end of the orbit of the earth,you are viewing it from a different place, it makes a different angle. If you make xxxxx observation from two extrem extreme positions in the orbit you will only get a small x deviation in the angle and that angle will give you an idea of the distance. It is called the paralax method. In that way assuming that light travels in straight line in space where there is no matter to disturb its path we can definitely find the distance of the stars by paralax observations. I think it was Poincoire, the great mathematician and not Reymond the politician stated: You look for paralax, if you do find that any star ix in the sky gives you a negative paralax, that is to say, you look at A and suppose by observation you find that it is a little in the opposite direction, something is certainly happening which we do not anticipate by euclidian geometryxx . And here is a clear case for catching the criminal. We know definitely that the space of nature is not following euclid's geometry but is following a geometry of non-euclidian type So you find ffxxmx from the distance of the stars shxxldx it should not show any appreciable paralax at all. Here again is a case for catching the geometry of the earth as misbehaving, because it will prove that it is not the geometry of euclidian type. It was Poincoire who having been x brought up in euclidian influence was bold enoughto experiment it which goes against euclid. Today all the world is prepared to have a straight line propagation of light in absolute space even whe we say there is a large gravitating matter like the sun and the path of the light is different from the path we expected from the old geometry. We still say it is not the fault of the light but the space through which it travels. You may now ask me: have you got any other basis by which you can decide whether the geometry by which you can decide whether the geometry is really the geometry of the x physical universe. There is a unique basis. We know that what is called the milky way is just one of the curious islandx universe. If you will look xxxx up to the sky any night you will find a cloudy patch extending from the horizon to the other end and most of the stars that we can see are concentrated in that space in that direction. We generally talk about xxx the impossibility of counting thestars, they are so numerous. I do not know whether you realise that the total number of stars that nakes eye at any time can see does not exceed two thousand orthree thousand. It is not such a tremendous job or uncountable. In one hemisphere there is altogether three billion stars bxxxxxxx bright enough to be seen in the naked eye and then about five hundred have to be eliminated because xxxxxxx they lie so close to the universe that a mere particle in the universe will prevent them xx to be seen. But if you will look through a telescope you will find. them. So according to our present estimate there

35

are about forty thousand billions (I am using billion in the American way) in our galaxy of ~~xxxxx~~ or milky way which look like a lense. In millions of such galaxies each containing 20, 30 40 billion stars, you may regard each one of them as a unit, the unique test I mentioned for finding out whether the geometry of space is of euclidian type or non euclidian is very simple . You take a small telescope. There is a certain minimum of stars which come within its view from an approximate distance of ten million light yards depending upon the aperture, count all the neubulii which are included in the sphere of that particular radius. You take a bigger telescope, count the total number, ~~naturallyxx~~ which naturally will be much larger. You take still bigger telescope and count. Now we have so many spheres of various distance. If we find that we have reasons to believe from other considerations that irrespective of the neubulii o~~fx~~ in any space since they are so numerous the total ..... Making that assumption we can ~~fixx~~ now find whether the total number included in the various spheres is just proporitionate to the volume, because we ~~kxx~~ know the distance. If it is found that ~~theyyarexx~~ they are proportional to the cube of the distance then we know we are operating with euclidian space. But if you find that the number if not proportional to the cube but follows a different law then you know that the geometry is not euclidian geometry. By observation I am not going to elaborate the experiments it has been found that the number is no pro ortional to the cube but deviates very considerably. So the euclidian really does not apply practically to ~~thex~~ nature itself. That is to say that non euclidian proves more natural in the sense that it

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is the geometry that underlies the work of nature herself.

I think I have given you some idea of the geometrial function of the physical universe and I believe have been able to convey to you something of the principle that underlies the modern research regarding the construction of the universe.