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$$K(\underset{\sim}{x}) = \int \frac{d\underset{\sim}{k}}{(2\pi)^3} e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} \exp\left[-\tfrac{1}{2} i \frac{mn}{\hbar} \int_0^t dt' \left(\underset{\sim}{v} - \frac{\hbar \underset{\sim}{k}}{mn}\right)^2\right]$$

If we say that the largest contribution comes from $\underset{\sim}{k} = mn\underset{\sim}{v}/\hbar$, then

$$K(\underset{\sim}{x}) = e^{i\underset{\sim}{k}(mn\underset{\sim}{v})/\hbar \cdot \underset{\sim}{x}}$$

$$\hat{\Psi}(\underset{\sim}{x}) = \int d\underset{\sim}{x}' \, \hat{\Psi}(\underset{\sim}{x} - \underset{\sim}{x}', n, t) \, e^{i(mn\underset{\sim}{v}\cdot\underset{\sim}{x}')/\hbar}$$

$$= \int d\underset{\sim}{x}' \, e^{i mn\underset{\sim}{v}\cdot(\underset{\sim}{x}-\underset{\sim}{x}')/\hbar} \hat{\Psi}(\underset{\sim}{x}')$$

$$= e^{imn\underset{\sim}{v}\cdot\underset{\sim}{x}/\hbar} \int d\underset{\sim}{x}' \, e^{-imn\underset{\sim}{v}\cdot\underset{\sim}{x}'/\hbar} \hat{\Psi}(\underset{\sim}{x}')$$

$$\tilde{\Psi}(\underset{\sim}{x}) = e^{i m\underset{\sim}{v}\cdot\underset{\sim}{x}/(\hbar/n)} \tilde{\Psi}(\underset{\sim}{k} - m\underset{\sim}{v}/(\hbar/n))$$

Thus $\Psi(x)$ acts as a plane wave with ~~momentum~~ $m\underset{\sim}{v}$ wave number $[m\underset{\sim}{v}/(\hbar/n)]$

$$\Psi(x) = \int d\underset{\sim}{x}' \, e^{\frac{1}{2} i \frac{(\underset{\sim}{x}-\underset{\sim}{x}')^2}{t} \frac{mn}{\hbar^2}} \hat{\Psi}(\underset{\sim}{x}')$$

Change $k \to k'$

$$= \left(k - \frac{mn\underset{\sim}{v}}{\hbar}\right)$$

$$e^{-i\left(\underset{\sim}{k}' + \frac{mn\underset{\sim}{v}}{\hbar}\right)\cdot\underset{\sim}{x}}$$

$$\exp\left[-\tfrac{1}{2} i \frac{mn}{\hbar} \underset{\sim}{k}'^2 t \frac{\hbar}{mn}\right.$$

$$\left. - i\tfrac{1}{2} \frac{\hbar n t}{mn} \left(\underset{\sim}{k}' - \frac{mn\underset{\sim}{x}}{\hbar n t}\right)^2\right.$$

$$\left. + i\tfrac{1}{2} \left(\frac{mn}{\hbar n} \frac{\underset{\sim}{x}^2}{t}\right)\right]$$

$$\sin^2 nkx \Rightarrow \sin^2 nk\left(x - \int_0^t v\,dt\right)$$

$$\sum_{n=0}^{\infty} \sin^2 \frac{2\pi n}{L}\left(x - \int_0^t v\,dt\right)$$

$$= \sum_{n=0}^{\infty}\left[\left(1 - \cos\frac{4\pi n}{L}\left(x - \int v\,dt\right)\right)\right]$$

$$= \psi \sim \frac{e^{ik\left|\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right|}}{\left|\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right|}$$

$$\psi\dot{\psi} = = \frac{1}{\left|\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right|^2}$$

Let $\Psi(\underset{\sim}{x}, n, t) = A \sin \underset{\sim}{k_0} \cdot \underset{\sim}{x}$

$$\tilde{\Psi}(\underset{\sim}{k}) = \int e^{i \underset{\sim}{k} \cdot \underset{\sim}{x}} A \sin \underset{\sim}{k_0} \cdot \underset{\sim}{x} \, d\underset{\sim}{x} = \frac{A}{2i} \left[\delta(\underset{\sim}{k} - \underset{\sim}{k_0}) - \delta(\underset{\sim}{k} + \underset{\sim}{k_0}) \right]$$

$$\tilde{\Psi}(\underset{\sim}{k}, t) = \frac{1}{2i} \left[\delta(\underset{\sim}{k} - \underset{\sim}{k_0}) - \delta(\underset{\sim}{k} + \underset{\sim}{k_0}) \right] \exp\left[\frac{1}{2} i m \int_0^t dt \left(\underset{\sim}{v} - \frac{\hbar \underset{\sim}{k}}{m} \right)^2 / \hbar \right]$$

$$\therefore \hat{\Psi}(\underset{\sim}{x}) = \int \frac{d\underset{\sim}{k}}{(2\pi)^3} e^{i \underset{\sim}{k} \cdot \underset{\sim}{x}} \frac{1}{2i} \left[\delta(\underset{\sim}{k} - \underset{\sim}{k_0}) - \delta(\underset{\sim}{k} + \underset{\sim}{k_0}) \right] \exp\left[\frac{1}{2} i m \int_0^t \left(\underset{\sim}{v} - \frac{\hbar \underset{\sim}{k}}{m} \right)^2 dt / \hbar \right]$$

$$= e^{i \underset{\sim}{k_0} \cdot \underset{\sim}{x}} \exp\left[\frac{1}{2} i m n \int dt \left(\underset{\sim}{v} - \frac{\hbar}{m} \underset{\sim}{k_0} / m \right)^2 / \hbar \right]$$

$$- e^{-i \underset{\sim}{k_0} \cdot \underset{\sim}{x}} \exp\left[\frac{1}{2} i m \int \left(d\underset{\sim}{v} + \frac{\hbar}{m} \underset{\sim}{k_0} / m \right)^2 / \hbar \right]$$

$$= e^{i \underset{\sim}{k_0} \cdot \underset{\sim}{x}} \; e^{-i \underset{\sim}{k_0} \cdot \int_0^t \underset{\sim}{v} dt} \exp\left[\frac{1}{2} i \frac{m^2}{\hbar} \int dt \left(v^2 + \frac{(\hbar k_0)^2}{m^2} \right) \right]$$

$$- e^{-i \underset{\sim}{k_0} \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v} dt \right)} \exp\left[\frac{1}{2} i \frac{n}{\hbar} m \int dt \left(v^2 + \frac{(\hbar k)^2}{n^2 m^2} \right) \right]$$

$$= \exp\left[\frac{1}{2} i m n \int_0^t dt \left(v^2 + \left(\frac{\hbar k}{n} / m \right)^2 \right) \right] \sin\left[\underset{\sim}{k_0} \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v} dt \right) \right]$$

$$\therefore \hat{\Psi}^*(\underset{\sim}{x}) \hat{\Psi}(\underset{\sim}{x}) = A^2 \sin^2 \left[\underset{\sim}{k_0} \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v} dt \right) \right]$$

$e^{iEt/\hbar}$ $A \sin kx$ has physical significance

$e^{-iEt/\hbar} e^{ikx} - e^{-iEt/\hbar} e^{-ikx}$

This is a superposition of wave propagating in two different directions.

We should therefore use as a soln

$$\Psi = A e^{-iEt/\hbar} \sin kx$$

Then $$\tilde{\Psi}(k) = \frac{A e^{-iEt/\hbar}}{2i} \left[\delta(k - k_0) - \delta(k + k_0)\right]$$

$$\hat{\Psi}(x) = \int \frac{dk}{(2\pi)^3} \tilde{\Psi}(k) e^{ik\cdot x} = \int \frac{A e^{-iEt/\hbar}}{2i} e^{\frac{1}{2}im\int dt (v - \hbar k/m)^2/\hbar} \left[\delta(k - k_0) - \delta(k + k_0)\right] e^{ik\cdot x}\, dk$$

$$= \frac{A}{2i} e^{-iEt/\hbar} \left[e^{ik_0 x_0} e^{\frac{1}{2}im\int dt (v - \hbar k_0/m)^2/\hbar} - e^{-ik_0 x_0} e^{\frac{1}{2}im\int dt (v + \hbar k_0/m)^2/\hbar} \right]$$

$$\psi(x, S, t) = \left[e^{-iEt/\hbar} \left[\frac{A}{2i} e^{-iS} e^{\frac{1}{2}im\int dt (v^2 + (\hbar k_0/m)^2)} \right] \left\{ e^{ik_0 (x - \int v\, dt) - ik_0 (x)} - e \right\} \right.$$

$$\left. + \frac{iS}{-2i} e^{+iEt/\hbar} \right] + \text{c.c.}$$

But $\psi(\underset{\sim}{x}, S, t) = \left\{ e^{-inS/\hbar} \exp\left[\frac{1}{2} i m n \int_0^t dt \left(v^2 + \left(\frac{\hbar k_0}{m n}\right)^2\right)\right] \right.$

$\left. + e^{+iS/\hbar} \exp\left[-\frac{1}{2} i \frac{m}{\hbar} \int_0^t dt \left(v^2 + \left(\frac{\hbar k_0}{n m}\right)^2\right)\right] \right\} A \sin\left[\underset{\sim}{k}_0 \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$

~~$\frac{\partial}{\partial t} = \frac{1}{2} i m \left(v^2 + \left(\frac{\hbar k_0}{m}\right)^2\right) e^{iS/\hbar}$~~

~~$S = \int_0^t dt \left(\frac{1}{2} m v^2 - V\right)$~~ $\cos\left[\frac{nS}{\hbar} - \frac{n}{\hbar} \int_0^t dt \left(\frac{1}{2} m v^2 + \frac{(\hbar k)^2}{2 m n^2}\right)\right]$

$\exp\left[-\frac{i}{\hbar} \int_0^t dt \left(\frac{1}{2} m v^2 - V\right)/\hbar + \frac{1}{2} i m \int_0^t dt \left(v^2 + \left(\frac{\hbar k_0}{m}\right)^2\right)\right]$.

$\left\{ \exp\left[+\frac{i}{\hbar} \int dt \left\{\left(\frac{\hbar k_0}{m}\right)^2 \frac{1}{2m} + V\right\}\right] + \text{c.c} \right\} A \sin\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$

$= A \cos\left(E t/\hbar\right) \sin\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$

$-\frac{E}{\hbar} A \sin(Et/\hbar) \sin\left[\underset{\sim}{k}_0 \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right] - \overbrace{A \cos Et/\hbar \cos\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]}^{(\underset{\sim}{k}_0 \cdot \underset{\sim}{v})}$

$+ A(\underset{\sim}{v} \cdot \underset{\sim}{k}_0) \cos Et/\hbar \cos\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$

$\psi(\underset{\sim}{x}, S, t) = \sum_n A \cos\left[\frac{nS}{\hbar} - \frac{n}{\hbar} \int_0^t dt' \left(\frac{1}{2} m v^2 + \frac{(\hbar k_0)^2}{2 m n^2}\right)\right] \sin\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$

$+ A \sin\left[S/\hbar - \cdots\right] \left(\frac{1}{2} m v^2 + \frac{(\hbar k_0)^2}{2m}\right) \sin\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right] + \left(\frac{1}{2} m v^2 - V\right) A \sin\left[\frac{S}{\hbar} - \cdots\right]$

$= A \cdot E \sin\left[S/\hbar - \cdots\right] \frac{t}{\hbar} \sin\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$ $\sin\left[\underset{\sim}{k}_0 \cdot \left(\underset{\sim}{x} - \int \underset{\sim}{v}\, dt\right)\right]$

BOSE AND 20TH CENTURY PHYSICS
S N BOSE NATIONAL CENTRE FOR BASIC SCIENCES, CALCUTTA

$$\therefore \psi(x,s,t)$$

$$= A \sin\left[\underset{\sim}{k_0}\cdot\left(\underset{\sim}{x} - \int_0^t \underset{\sim}{v}\,dt\right)\right] \cos\left[\Phi + Et/\hbar + \tfrac{1}{2}m\int dt\left(v^2 + \left(\tfrac{\hbar k_0}{m}\right)^2\right)/\hbar\right]$$

$$- \left(\frac{E}{\hbar} - \tfrac{1}{2}m\left(v^2 + \left(\frac{\hbar k_0}{m}\right)^2\right)/\hbar\right) A \sin(\quad)\sin(\quad)$$

$$- \frac{L}{\hbar} A \sin(\quad)\sin(\quad)$$

$$- A\cos(\quad)\cos(\quad)\,\underset{\sim}{k_0}\cdot\underset{\sim}{v} + A\,\underset{\sim}{v}\cdot\underset{\sim}{k_0}\cos(\quad)\cos(\quad)$$

$$= -\left[E + L - \tfrac{1}{2}m\left(v^2 + \left(\frac{\hbar k_0}{m}\right)^2\right)\right] A \sin(\quad)\sin(\quad)$$

$$\left[\left(\tfrac{1}{2}mv^2 + V\right) + \left(\tfrac{1}{2}mv^2 - V\right) - \tfrac{1}{2}m\left(v^2 + \left(\tfrac{\hbar k_0}{m}\right)^2\right)\right]$$

Hence the eqn is satisfied provided we identify $\frac{1}{2}mv^2 = \frac{(\hbar k)^2}{2m}$

$$\therefore \psi(x,s,t) = A \sin\left(\underset{\sim}{k_0}\cdot\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)\right) \cos\left(\Phi + Et/\hbar \quad \cdots\right)$$

$$\psi^2 = A^2 \sin^2\left[\underset{\sim}{k_0}\cdot\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)\right] \cos^2\left[\Phi + Et/\hbar \cdots\right]$$

$$= A^2 \sin^2\left[\underset{\sim}{k_0}\cdot\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)\right] \cos^2\left[\Phi - \int L\,dt/\hbar\right]$$

$$\left[\Phi + \int dt\,(E - \underset{\sim}{p}\cdot\underset{\sim}{v})\right]$$

$$\psi^2 = A^2 \sin^2\left[k_0\cdot\left(x - \int v\,dt\right)\right] \cos^2\left[\Phi + \frac{1}{\hbar}\int_0^t dt\,(E - p\cdot v)\right]$$

$\psi^*(x)\psi(x)$ (Average over Φ) $\Rightarrow A^2 \sin^2\left[k_0\cdot\left(x - \int v\,dt\right)\right]$.

If the starting point were the plane wave, then the $\psi(x, S, t)$ would be:

$$\psi(x, S, t) = A\, e^{-iS/\hbar} e^{-iEt/\hbar}\, e^{ik_0\cdot\left(x - \int v\,dt\right)}$$

$$\exp\left[\tfrac{1}{2} im \int dt\left(v^2 + \left(\frac{\hbar k_0}{m}\right)^2\right)\right] + \text{c.c.}$$

$$= \cos\left[\frac{S}{\hbar} + \frac{Et}{\hbar} - \int dt\left(v^2 + \left(\frac{\hbar k_0}{m}\right)^2\right) + k_0\cdot\left(x - \int v\,dt\right)\right]$$

Thus it has the form

$$\cos(\Phi - \ldots)\cos(k_0\cdot(x - \ldots)) - \sin(\Phi - \ldots)\sin(k_0\cdot(x - \int v\,dt))$$

provided again that $v = \hbar k_0/mn$.

Note that in this case

$$\int d\Phi\, \psi^2 = \tfrac{1}{2}(2\pi) A^2$$ (as should be the case for a plane wave)

while in the earlier case it is

$$\int d\Phi\, \psi^2 = \tfrac{1}{2} A^2 \sin^2\left[k_0\cdot\left(x - \int v\,dt\right)\right] \qquad k_0 = \frac{mnv}{\hbar}$$

If we ~~w~~ so now put n at appropriate places & sum over n we get

$$\delta\left(\cancel{\Phi - \int L\,dt + \left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)}\right)$$

$$\delta\left[\Phi - Et/\hbar + \frac{1}{2}\frac{m}{\hbar}\int \underset{\sim}{v}^2\,dt + \frac{\underset{\sim}{p}\cdot\underset{\sim}{x}}{\hbar}\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right) + \frac{1}{2}\int dt\,\frac{p^2}{2m}\right]$$

$$= \delta\left[\Phi - \left(\frac{Et}{\hbar} - \frac{\underset{\sim}{p}\cdot\underset{\sim}{x}}{\hbar}\right) + \frac{1}{\hbar}\underbrace{\left(\frac{1}{2}m\int v^2\,dt + \int\frac{p^2}{2m}\,dt - \underset{\sim}{p}\cdot\int \underset{\sim}{v}\,dt/\hbar\right)}_{=0}\right]$$

Hence this gives, simply

$$\frac{\partial}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial}{\partial x} + L/\hbar\,\frac{\partial}{\partial\Phi}$$

$$(\Phi - \ldots) \pm \left(\underset{\sim}{p}\cdot\left(x - \int \underset{\sim}{v}\,dt\right)\right) \qquad -E + \underset{\sim}{p}\cdot\underset{\sim}{v} - L = 0$$

Satisfied for plane waves

which is the classical relation

However, for stationary waves we do not get this. We get instead the equation satisfied with the combination of various terms

$$\frac{\partial}{\partial t} + \frac{L}{\hbar}\frac{\partial}{\partial\Phi} \quad \text{for} \quad \cos\left(\Phi - \int L/\hbar\right)$$

$$\frac{\partial}{\partial t} + \underset{\sim}{v}\cdot\frac{\partial}{\partial x} = \quad \text{for} \quad \sin\left(\underset{\sim}{k_0}\cdot\left(x - \int \underset{\sim}{v}\,dt\right)\right)$$

We see that the "classical" δ-function trajectory comes only on summing over n modes

---

$$\psi(\underset{\sim}{x}, S, t) = \sum_n \cos\left[n\left(\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(v^2 + \left(\tfrac{\hbar k_0}{mn}\right)^2\right)\right)\right] \sin\left[\underset{\sim}{k_0}\cdot\left(x - \int \underset{\sim}{v}\,dt\right)\right]$$

$k_0 = \left(\frac{2\pi\nu}{L}\right)$ ~~k_0~~

If we write

$$\underset{\sim}{k_0} = n\underset{\sim}{p}/\hbar$$

$p = (\hbar k_0/n)$

$$\left(\frac{\hbar}{n}\right)^2 \nabla^2\psi = E\psi$$

$$\frac{1}{2m}\left(\frac{\hbar k_0}{n}\right)^2 = E$$

Hence

$$\psi(\underset{\sim}{x}, S, t) = \sum_n A_n \cos\left[n\left(\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(v^2 + \frac{k^2}{m^2}\right)\right)\right] \sin\left[n\frac{\underset{\sim}{p}}{\hbar}\cdot\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)\right]$$

$$= \sum_n A_n \exp\left[n\left(\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(v^2 + p^2/m\right)\right) \pm n\frac{\underset{\sim}{p}}{\hbar}\cdot\left(x - \int \underset{\sim}{v}\,dt\right)\right]$$

$$= \delta\left[\left(\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(v^2 + p^2/m\right)\right) \pm \underset{\sim}{p}\cdot\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)\right]$$

$$+ \delta\left[\left(\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(v^2 + p^2/m\right)\right) - \underset{\sim}{p}\cdot\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)\right]$$

On putting it in the equation of motion, it gives

$$\frac{\partial}{\partial t}\left(E - \tfrac{1}{2}m\left(v^2 + p^2/m\right) \mp \underset{\sim}{p}\cdot\underset{\sim}{v}\right) \pm \underset{\sim}{v}\cdot\underset{\sim}{p} + L$$

$L = \frac{1}{2}mv + V$

BOSE AND 20TH CENTURY PHYSICS
S N BOSE NATIONAL CENTRE FOR BASIC SCIENCES, CALCUTTA

This gives

$$E - \tfrac{1}{2}mv^2 - p^2/2m + \left(\tfrac{1}{2}mv^2 - V\right) = 0$$

$$E - \left(p^2/2m + V\right) = 0$$

This gives the energy ~~value~~ expression.

We thus have the "superposition principle" clearly holding here. That is

$$\psi_1 = \delta\left[\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(mv^2 + p^2/m^2\right) - p\left(x - \int v\,dt\right)\right]$$

$$\psi_2 = \delta\left[\Phi + Et/\hbar - \tfrac{1}{2}m\int_0^t dt\left(v^2 + p^2/m^2\right) + p\cdot\left(x - \int v\,dt\right)\right]$$

are clearly solutions each. Then clearly

$\psi = \psi_1 + \psi_2$ is also a solution.

Now $f = \psi^2 = \psi_1^2 + \psi_2^2 - 2\psi_1\psi_2$

These are highly singular functions.

Integration over Φ gives

$$\int d\Phi\, f = 2\delta(\epsilon) - 2\delta\left[p\cdot\left(x - \int v\,dt\right)\right]$$

BOSE AND 20TH CENTURY PHYSICS
S N BOSE NATIONAL CENTRE FOR BASIC SCIENCES, CALCUTTA

If we consider the damped solution for ψ

$$\psi = A e^{-Kx}, \qquad \int_{-\infty}^{+\infty} dx\, e^{-Kx} e^{ikx} = \frac{1}{K+ik} \; e^{ikx}_{k=+iK}$$

then $\tilde{\psi}(k) = \frac{A}{k^2+K^2}$, where $K = \sqrt{(V-E)(2m)}$.

$$\psi(x) = \int \tilde{\psi}(k) e^{ikx} = \int \frac{A}{k^2+K^2} \exp\left[i\tfrac{1}{2} m \int_0^t dt' \left(v + \frac{\hbar k}{m} \right)^2 \right] e^{ikx}\, dk$$

$$= A \exp\left[i\tfrac{1}{2} m \int_0^t dt \left(v - \frac{i\hbar K}{m} \right)^2 \right] e^{-Kx}$$

$$= A e^{-Kx} \exp\left[i\tfrac{1}{2} m \int_0^t dt' \left(v + \frac{i\hbar K}{m} \right)^2 \right]$$

$$= A e^{-Kx} \exp\left[\tfrac{1}{2} m i \int_0^t dt' \left(v^2 - 2i \frac{\hbar K}{m} v - \frac{\hbar^2 K^2}{m^2} \right) \right]$$

$$\therefore \psi(x, S, t) = \sum_n \tilde{A}(n)\, e^{-inEt/\hbar}\, e^{-i\Phi n}\, e^{-nKx}$$

$$\exp\left[\tfrac{1}{2} i \frac{mn}{\hbar} \int_0^t dt' \left(v^2 - 2i \frac{\hbar K}{mn} v - \frac{\hbar^2 K^2}{m^2 n^2} \right) \right]$$

$$= \sum_n \tilde{A}(n)\, e^{-inEt/\hbar}\, e^{-in\Phi} \exp\left[\tfrac{1}{2} i \frac{mn}{\hbar} \int_0^t dt' \left(v^2 - \frac{\hbar^2 K^2}{m^2 n^2} \right) - nK\left(x - \int_0^t v\, dt \right) \right].$$

$$= e^{-nK\left(x - \int v\, dt\right)} \cos\left[nEt/\hbar + n\Phi + \tfrac{1}{2} \frac{mn}{\hbar} \int_0^t dt \left(v^2 - \frac{\hbar^2 K^2}{m^2 n^2} \right) \right]$$

$$\left[nEt/\hbar + \frac{n}{\hbar} \int v\, dt + \tfrac{1}{2} \frac{mn}{\hbar} \int_0^t dt \left(\frac{\hbar^2 K^2}{m^2 n^2} \right) \right]$$

$\Phi = \int \tfrac{1}{2} m v^2 - V$

Now to check, whether this satisfies the Liouville eqn. for ψ:

$$\frac{\partial}{\partial t} = nKv \cos(\quad) \cdot \left[-\frac{nE}{\hbar} + \frac{1}{2}\frac{mn}{\hbar}\left(v^2 - \frac{\hbar^2 K^2}{m^2 n^2}\right)\right] \sin(\quad)$$

$$\underline{v}\cdot\frac{\partial}{\partial \underline{x}} = -\underline{v}\, nK \cos(\quad)$$

$$L\frac{\partial}{\partial \phi} = -\frac{n}{\hbar}\sin(\quad) = -\frac{n}{\hbar}\left(\frac{1}{2}mv^2 - V\right)\sin(\quad)$$

$$\therefore \left(\frac{\partial}{\partial t} + \underline{v}\cdot\frac{\partial}{\partial x} + L\frac{\partial}{\partial \phi}\right)\psi = \left[-\frac{nE}{\hbar} + \frac{1}{2}\left(\frac{\hbar K}{mn}\right)^2 \frac{mn}{\hbar} + \frac{nV}{\hbar}\right]\sin(\quad)$$

This gives:

$$E = -\frac{1}{2m}\left(\frac{\hbar K}{n}\right)^2 + V$$

which is gives:

$$\left(\hbar K/n\right)^2 = 2m(V - E)$$

This is how K is anyhow defined.

Hence the solution satisfies the eqn. for ψ.

$$\int d(\Delta x)\, e^{\frac{1}{2} i m \frac{(\Delta x)^2}{\tau \hbar}} \approx \left(\frac{2\pi\hbar\tau}{im}\right)^{3/2} \qquad -\frac{\hbar^2}{2mn^2}\nabla^2\psi + V\psi = E\psi$$

$$\int \exp\left[i\frac{1}{2}\frac{nm}{\hbar}\int_0^t dt' v^2 + i\underset{\sim}{k}\cdot\left(\underset{\sim}{x} - \int_0^t \underset{\sim}{v}\, dt\right) + i\frac{\hbar k^2}{2mn} t \right] \frac{dk}{(2\pi)^3}$$

$$\Psi(\underset{\sim}{k}, n, t)\; e^{-iEt/\hbar}$$

$$\hat{\Psi}(\underset{\sim}{x}, n, t) = \exp\left[i\frac{1}{2}\frac{nm}{\hbar}\int_0^t dt' v^2 \right] e^{-i\frac{\hbar t}{2mn}\nabla'^2 - iEt/\hbar}\, \Psi(\underset{\sim}{x}, n, t)$$

$$\psi(\underset{\sim}{x}, S, t) = \exp\left[-i\Phi n + i\frac{1}{2}nm\int_0^t v^2 dt \right] e^{-i\frac{\hbar t}{2mn}\nabla'^2 - inEt/\hbar}\, \Psi(\underset{\sim}{x}, n, t)$$

$$= \exp\left[-in\Phi + \frac{1}{2}\frac{inm}{\hbar}\int_0^t v^2 dt \right] \exp\left[-i\frac{n}{\hbar}\int_0^t V dt \right] \Psi(\underset{\sim}{x}, n, t)$$

$$\left(\underset{\sim}{x} - \int_0^t v\, dt\right)$$

$$= \cancel{\exp[}$$

$$\frac{\partial}{\partial t} = \cancel{\left(\frac{1}{2}inm\int_0^t v^2 - \frac{in}{\hbar}\int V dt\right)\Psi - \underset{\sim}{v}\cdot\frac{\partial\Psi}{\partial x}}$$

$$\underset{\sim}{v}\cdot\frac{\partial\Psi}{\partial x} = \cancel{+\underset{\sim}{v}\cdot\frac{\partial\Psi}{\partial x}}$$

$$\frac{L}{\hbar}\frac{\partial\Psi}{\partial\Phi} = \cancel{-\frac{1}{2}n\left(\frac{1}{2}mv^2 - V\right)}$$

Consider :

$$\int_0^t d\underset{\sim}{k}\; e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} \exp\left[i\tfrac{1}{2}\frac{mn}{\hbar}\int_0^t v^2\,dt - i\underset{\sim}{k}\cdot\int_0^t \underset{\sim}{v}\,dt + \frac{i(\hbar k)^2}{2mn}t\right]$$

$$= \int \exp\left[i\tfrac{1}{2}\frac{mn}{\hbar}\int_0^t dt\, v^2 + i\underset{\sim}{k}\cdot\left(\underset{\sim}{x} - \int_0^t \underset{\sim}{v}\,dt\right) + i\frac{\hbar k^2 t}{2mn}\right] d\underset{\sim}{k}$$

$$= \exp\left[\tfrac{1}{2}i\frac{mn}{\hbar}\int dt\, v^2\right] \int \frac{d\underset{\sim}{k}}{(2\pi)^3} \exp\left[i\frac{\hbar t}{2mn}\left\{\underset{\sim}{k} + \frac{\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)}{t}\frac{mn}{\hbar}\right\}^2 - \frac{i\hbar}{2mn}\frac{\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)^2 m^2 n^2}{\hbar^2}\right]$$

$$= \exp\left[\tfrac{1}{2}i\frac{mn}{\hbar}\left\{\int dt\, v^2 - \frac{\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)^2}{t}\right\}\right] \int \frac{d\underset{\sim}{k}}{(2\pi)^3} \exp\left[\frac{i\hbar t}{2mn}\left\{\underset{\sim}{k} + \cdots\right\}^2\right]$$

$$= \exp\left[\tfrac{1}{2}i\frac{mn}{\hbar}\left\{\int v^2\,dt - \frac{\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)^2}{t}\right\}\right] \left[\frac{\pi \cdot 2mn}{i\hbar t}\right]^{3/2}$$

$$\therefore K(x) = \left(\frac{2\pi mn}{i\hbar t}\right)^{3/2} \exp\left[\tfrac{1}{2}i\frac{mn}{\hbar}\left\{\int_0^t v^2\,dt - \frac{\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)^2}{t}\right\}\right]$$

$$\therefore \hat{\psi}(x) = \int d\underset{\sim}{x}'\, K(\underset{\sim}{x}')\, \tilde{\psi}(\underset{\sim}{x} - \underset{\sim}{x}')$$

$$= \int d\underset{\sim}{x}'\, K(\underset{\sim}{x} - \underset{\sim}{x}')\, \tilde{\psi}(\underset{\sim}{x}')$$

$$= \int d\underset{\sim}{x}''\, K(\underset{\sim}{x} - \underset{\sim}{x}'')\, \tilde{\psi}\left(\underset{\sim}{x}'' - \int_0^t \underset{\sim}{v}\,dt\right)$$

$v^2 = x^2$

$x - x'$

$x - \left(x' + \int_0^t v\,dt\right)$

$x'' + \int_0^t v\,dt$

$x' = x''$

$$\psi(\underset{\sim}{x}, S, t) = \sum_n \exp\left[-in\Phi + \frac{1}{2}\frac{in}{\hbar}\int dt\left(\frac{1}{2}mv^2 - V\right)\right]$$

$$\underset{\sim}{\Psi}\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt, nt\right) + \text{c.c.} \longrightarrow \underset{\sim}{\Psi}(\underset{\sim}{x})$$

$$= \underset{\sim}{\Psi}\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt, \underset{\sim}{\Phi} - \frac{1}{\hbar}\int L\,dt, t\right)$$

Solution of the Schrödinger eqn.

This $\underset{\sim}{\Psi}$ is a solution of the Schrödinger eqn.

Consider

$$\Psi = \frac{e^{ik|\underset{\sim}{r}+\underset{\sim}{d}|}}{|\underset{\sim}{r}+\underset{\sim}{d}|} + \frac{e^{ik|\underset{\sim}{r}-\underset{\sim}{d}|}}{|\underset{\sim}{r}-\underset{\sim}{d}|}$$

then

$$\underset{\sim}{\Psi}\cdot\psi(\underset{\sim}{x}, S, t) = \left[\frac{e^{ik\left|\underset{\sim}{r} - \int \underset{\sim}{v}\,dt + \underset{\sim}{d}\right|}}{\left|\underset{\sim}{r} - \int \underset{\sim}{v}\,dt + \underset{\sim}{d}\right|} + \frac{e^{ik\left|\underset{\sim}{r} - \int \underset{\sim}{v}\,dt - \underset{\sim}{d}\right|}}{\left|\underset{\sim}{r} - \int \underset{\sim}{v}\,dt - \underset{\sim}{d}\right|}\right]$$

$$e^{-i\left(\underset{\sim}{\Phi} - \int \frac{L}{\hbar}\,dt\right)} + \text{c.c.}$$

$$+$$

$$\psi^*\psi = \underset{\sim}{\Psi}^*\underset{\sim}{\Psi}\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)$$

$$f = \psi^2 = \sum_{n,n'}\left[e^{i(n+n')\Phi'}\right.$$

$$\underset{\sim}{\Psi}(\underset{\sim}{x}', n, t)\,\underset{\sim}{\Psi}(\underset{\sim}{x}', n', t)$$

$$\downarrow$$

$$\left(\underset{\sim}{x} - \int \underset{\sim}{v}\,dt\right)$$

Prob Problems of Scince Education

Elements of Education

1. Teacher
2. Tought
3. Medium → Curriculum

Can a student be its own teacher of Scince?

What we require in an Natural Environment for the learning

The de. concept of "curriculum" must go drastic conceptual change

Concept of examination must change

We do not need "kits"

News Watch
P. Box 772
GPO Bombay

Things to do:

II' → *** 1. A.O.U Interview. (Soon)

~~*** 2. Write paper for Olympiad Conference Soon 31~~

I → *** 3. ~~Send names to INSA for COSPAR. (Soon)~~

~~Talk to Daniel & Kasturirangan.~~

4. Send letters of invitation to

~~(i) A.K. Raychaudhary~~

? (ii) A. Leggett (v) P.K Kabir.

(iii) Gautam

(iv) Behera.

5. Finalise INSA report (Diamond Jubilee).

6. Arrange INCOSPAR National Committee. ←

II → *** 7. ~~Prepare Faculty Meeting a Talk~~

8. Suggest Next Committee for INCOSPAR. ←

✓9. ~~Ask MRD about telescope.~~

✓10. ~~Reply to Tahir Shah e-mail~~

11. ~~Call MRA. Hassan.~~

12. Apply for Sabbatical leave.

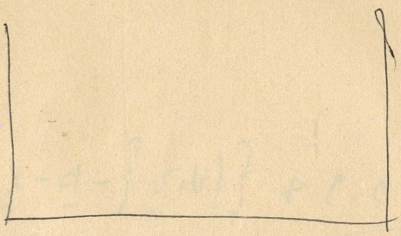
13. Complete QM papers

(i) Schrödinger Eqn

(ii) Bell inequality

$$\sin kx = \sin\left[k\left(x - \int v\,dt\right)\right]$$

$$v = \left[2\frac{(E-e\phi)}{m}\right]^{1/2} = \left[\frac{2(E-e\phi)}{m}\right]^{1/2}$$



$$k = \frac{2\pi n}{L}$$

$$\sin\left[\frac{2\pi n}{L}\left(x - \int v\,dt\right)\right]$$

$$e^{i\frac{2\pi n}{L}x}\; e^{-\frac{2\pi i n}{L}\int_0^t v\,dt} - e^{-i\frac{2\pi n x}{L}}\; e^{+\frac{2\pi i n}{L}\int_3^t v\,dt}$$

This gives zero at the point $x = \int v\,dt = 0$

Thus the point of zero probability (or any maximum) thus oscillates with the classical velocity. But at a given time oscillates with x at a given x oscillates with time

$$\frac{e^{ik|x-\int v\,dt|}}{|x - \int v\,dt|}$$

This amounts to a spherical wave propagating with the particle velocity v in all direction

$$\frac{e^{ik|x+a-\int v\,dt|}}{|x+a-\int v\,dt|} + \frac{e^{ik|x-a-\int v\,dt|}}{|x-a-\int v\,dt|}$$

$$\frac{1}{|\underset{\sim}{x}+\underset{\sim}{d}-\int \underset{\sim}{v}\,dt|^2} + \frac{1}{|\underset{\sim}{x}-\underset{\sim}{d}-\int \underset{\sim}{v}\,dt|^2}$$

$$+ \exp\left[\frac{ik\{|\underset{\sim}{x}+\underset{\sim}{d}-\int \underset{\sim}{v}\,dt| - |\underset{\sim}{x}-\underset{\sim}{d}-\int \underset{\sim}{v}\,dt|\} + c.c}{|\underset{\sim}{x}+\underset{\sim}{d}-\int \underset{\sim}{v}\,dt||\underset{\sim}{x}-\underset{\sim}{d}-\int \underset{\sim}{v}\,dt|}\right.$$

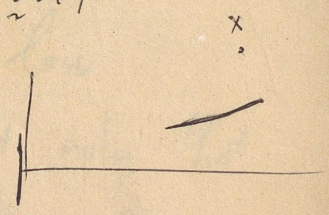
$$= (\quad) + (\quad) + \cos\left[\frac{k\{|\underset{\sim}{x}+\underset{\sim}{d}-\int \underset{\sim}{v}\,dt| - |\underset{\sim}{x}-\underset{\sim}{d}-\int \underset{\sim}{v}\,dt|\}}{|\underset{\sim}{x}+\underset{\sim}{d}-\int \underset{\sim}{v}\,dt||\underset{\sim}{x}-\underset{\sim}{d}-\int \underset{\sim}{v}\,dt|}\right]$$

$$= (\quad) + (\quad) + \cos\left[\frac{k\{(d^2+(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt)^2+2\underset{\sim}{d}\cdot(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt))^{1/2} - (d^2+(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt)^2-2\underset{\sim}{d}\cdot(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt))^{1/2}\}}{|\quad||\quad|}\right]$$

Assume $\underset{\sim}{x} \gg \int \underset{\sim}{v}\,dt$, then

$$= (\quad) + (\quad) + \cos\left[\cancel{k|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|} + \frac{\underset{\sim}{d}(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt)}{|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|^2} - \left\{\cancel{k|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|} - \frac{\underset{\sim}{d}\cdot(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt)}{|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|^2}\right\}\right] \Big/ |\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|^2$$

$$= (\quad) + (\quad) + \cos\left[\frac{k\,\frac{\underset{\sim}{d}\cdot(\underset{\sim}{x}-\int \underset{\sim}{v}\,dt)}{|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|}}{|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|^2}\right]$$



$$\frac{p}{\hbar}\,\frac{\underset{\sim}{d}\cdot(\overset{\Delta x}{\underset{\sim}{x}-\int \underset{\sim}{v}\,dt})}{|\underset{\sim}{x}-\int \underset{\sim}{v}\,dt|} = 2n\pi$$

$\underset{\sim}{x} < \Delta$

$\underset{\sim}{x} - \Delta\underset{\sim}{x}(t)$

Wave-like Behaviour and ~~existence~~ observations of discrete forbidden states in classical mechanical domain

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Present Scientific Scenario & Futuristic Perspectives
My own predelictions !
Possible future scenarios.

I shall like to share with you some thoughts which are not necessarily confined to the immediate concerns relating to the Laboratory, but ~~somewhat~~ ~~the~~ go somewhat beyond these concerns.

~~It is perhaps~~

If there as a matter of fact, both short and long term concerns that an organization faces at any given ~~instant of time~~ point in time. ~~Sometime~~ ~~immediate~~ Most of the time, unfortunately, the immediate concerns dominate, sidelining the long term ~~concerns~~ and important concerns. It is not to

~~Let me list what may be regarded as~~
~~short term~~

suggest that the short term concerns are are less important than the long term ones, but only that ~~long term~~ while short term concerns pertain to day to day existence of an institution, ~~the long~~

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which is important, the long term ones relate to

I would also like to extend my ~~consideration~~ concerns towards the end of my talk to the question of ~~the~~ the relationship of ~~the~~ ~~as~~ we scientists to the society that we live in.

What is the sort of relationship that we should have. Should the society be concerned about what we do and should we be concerned about society and ~~its need~~ what we can do.

But Society is not something different from us. ~~Sc~~ We are the society, particularly when we realise that our children are going to be the members of the society.

So far we have running in this laboratory a standard course: (i) in administrative functions, (ii) personnel management, & (iii) in the carrying out scientific activity

the long term scientific viability of an institution.

It may be emphasized that inasmuch as an institution is part of a society and a nation where it resides and indeed obtains its sustenance from, it is embedded in its socioeconomic environment and cannot be divorced from it. The concern of its socioeconomic environment also necessarily becomes or ought to become, in some measure at least, its own concern. It has an instrinsic obligation to its society*. What form a discharge of that obligation takes is, or should take is a different matter which requires careful discussion. To be sure, the society socio-economic environment around us today does not inspire much enthusiasm and indeed the life & quality of life for the honest has deteriorated greatly. The fact, nevertheless remains that ~~we are in~~ this is our country, our society and our children are going to live here after we depart, and we cannot ~~run away~~ afford to run away from this society whatever it be

* which it derives its sustenance from

So far we have been running in this laboratory a standard course in terms of

(i) administrative functions

(ii) personnel management

& even (iii) carrying out scientific activities

But the various scenarios around us the social scenario, the economic scenario have been changing so fast that there seems a need to have a long term look at the various issues

Let me therefore list both the short term and long term concerns that an institution such as P.R.L has or ought to have:

Short term:

1. The various subsystems of an institution work smoothly and efficiently. This may include
   - General administration and personnel management.
   - Technical facilities: Workshop; library computer etc
   - General maintenance and upkeepment.

This is a very important short term concern. If an institution can carry out take care of these points, then to even 90% satisfaction the result will be great. But obviously it is not so simple. As most of us here who have been involved in these tasks in some way or another know only too well, that these are the present the most exasperating issues that keep us tied down most of the time.

The reasons are that these issues

Problems of getting work out of your people

1. State of work ethics outside of the institution.
2. Problems associated with promotions etc. evaluation
3. Problems arising from human dynamics
4. Problem of personnel development

Can we evolve a better system of administration and management which breaks away from the standard approaches.?

(i) A more cooperative approach.?

(ii) Greater decentralization of responsibility?

(iii) But human dynamics? Problems Can they be surmounted?

Towards a New Paradigm in the administrative functions!

~~the most difficult of the human problems faced by the society since ancient~~ work

are being constantly influenced by the state of ethics ~~outside the~~ in the society outside of the confines of the institution and ~~the~~ are moreover governed by the with which it is constantly interacting, and are moreover plagued by problems involving "human dynamics" among the individuals belonging to the subsystems.

My concern is that these problems will only become more acute with the passage of time as the mores of the society ~~undergo~~ continue to change in an ~~unfavourable~~ manner.

unless we effect a change in our attitudes & perception

The state of PRL today is ~~far from~~ may not be perfect in this respect, but we I ~~could~~ ~~fear that~~ should be happy if we can keep it at the least at the present level, though we must strive constantly to improve it.

But to be able to do so, I believe that a more cooperative approach has to be evolved. It should not be "we" vs "they". It should be only "we". The "we" may have various

sub as "we's", who should appreciate each other tasks and difficulties and help each other. I think we are in for difficult times ahead, and we need all the cooperation of each other in carrying out these tasks. It cannot be anymore, 'we' need this and "they" must provide these.

The situation, I am afraid, may become more acute in this respect, as the senior administrators of PRL retire in about 3-4 years.

What I am suggesting, in other words is a paradigm shift in our attitudes. I cannot define all the elements of such a paradigm shift. But because I have not myself thought it through. But I can only state what I consider its central tenet, namely (a) to continue to identify the interest of the self with that of the institution. which essentially means the common interests of other colleagues. as well.

This is easier said than done. But one can at least try. I would suggest that we should have some serious discussions on these

aspects and evolve a methodology whereby the functions of the Laboratory can be run more cooperatively. ~~in~~

Another short term concerns of the scientists and a vital one is

2. Budgetary requirement and allocation.

This aspect presents an altogether different dimension and is a matter of vital concerns for the scientists.

The budgetary requirement is such a topic that ~~has evoked~~ may evoke great passions, particularly among the experimentalists.

It appears to follow what has come to known as "Parkinsons Law". Projects will expand so as to engulf any budgetary allocation. Even then there is no satisfaction.

[Around 1986-87, the budget of each area was roughly 9-10 lakhs each. Today, it is about 30-40 lakhs each with additional money coming to each area from Plan Component ~~2 crore~~, ~~and~~ DST and

other agencies amounting to 1–1.5 Crore each other. [Even then there is dissatisfaction and demand for more].

Of course, one could argue that after all greater demand only reflects a desire to do greater amount of research work. ~~It is we know greater amount of research does not necessarily mean better research~~. This cannot of course, be accepted without qualification. On the other hand, the budgetary scene in the country has been changing so dramatically, that old assumptions are no longer valid.

This is one area where I would like to share my serious concerns with you.

I have listed the "budgetary allocation" as a short term concern. But it is not confined to short term. It has long term implications and serious one at that. and they spill over into long term concerns - this a bit

Let me elaborate at this juncture.

So far we have led a rather comfortable existence, compared to scientists in Universities and

also, if I may say so, in the U.S. in terms of the efforts involved in getting funds, though of course, the amount of funds is not commensurate to the funds available in the U.S. But by Indian standards these ~~are not~~ have not been inconsiderable ~~bad~~, notwithstanding the dissatisfaction that may exist.

~~recent~~

However, the economic scenario ~~and perception~~ as well as governmental perceptions regarding the science and technology in the country has been changing very ~~so~~ fast ~~that it a it now calls for urgent what~~ as a result of the so-called "globalization of economy" and the ~~unbrindled and almost unthinking~~ rush to invite the multinational in all spheres of industry and of technology.

The present state of affairs ~~does lead~~ and the governmental perception that ~~they point to~~ these developments point to, ~~leads one to ought to~~ does ~~one~~ make one somewhat uneasy about the future of science in the country. ~~This trend of perceptions~~ The present trend of events and perceptions seem to cast a dark shadow over the future

of science in the country which the science community of the country does not appear to be seized with.

Can one continue to do science in the same manner as before? Can one be assured of the same kind of flow of funds as ~~before~~ at present? Can one continue to carry out the science in the same style and direction as before? Can one/I even do the same science ~~I~~ as before and what one likes to do? Will science now be sponsored and guided more closely by the demands of technology & and ~~weighed~~ judged in the balance of "relevance"?

These questions are real questions and not ~~hypothetical~~ hypothetical ones. Fortunately, so far PRL has been spared of coming face to face with these questions as we are under the umbrella of the Department of Space which has done so well in terms of ~~science~~ space-technological benefits to the country, so that ~~we~~ our funds have not suffered any cuts. But there is no room for complacency. We should not be caught unawares.

The question is: "What should our response be to this changing scenario?" Certainly not sit and wait for things to happen.

There are two ~~possible~~ the responses:

A. Either accept things as they come and change yourself, your activity, your mode of work and mode of thinking accordingly

B. Or defend your ~~a~~ manner of activity and your science values against the onslaught of change and resist this onslaught.

The first response (A) ~~is not a~~ may not appear to be a very self-worthy response self-respecting response. But if there is no alternative there may be one can ~~ex~~ accept it under extenuating circumstances.

The response (B) is ~~another~~ the diametrically opposite response ~~other side~~ and may prove to be ~~too~~ difficult a proposition ~~so as~~ if one is ranged against a ~~natural~~ tide of ~~change~~ ideological metamorphosis and a lack of ~~sympathy~~ ~~ron~~ the ~~ron~~ almost romantic sympathy for science that the Nehru-Indira Gandhi era was characterized by in terms of the support for science

On the other hand ~~science~~ science can in my personal opinion, science can hardly be regarded to have ~~covered~~ covered itself with glory on the whole. ~~We have~~ have had and I am not referring here to technology, space or atomic energy or any other. We have had ~~about~~ more than 40 years of liberal funding and ~~what is it that~~ what and how much we have to show in terms of original ideas and unique results.

Therefore when we have to defend "our activities and our scientific values" we have not really much material to defend with. We have not much to show. Most of our papers have ~~had~~ been the continuation of ideas originating in the West We have to ask the question why?

C. There is therefore a third response to the changing scenario. That is that we should not ignore the changing scenario nor resist it, but study and analyse it closely.

We ~~are~~ should consider this as a challenge and as an opportunity. Challenge to meet the onslaught squarely and an opportunity to put one's

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our house in order.

We have to be able to demonstrate the importance and relevance of science in the changing scenario, and to convince the people, the tax payers, of the need for the continued support for pure science. On the other hand, we may ~~must~~ also have to abandon our "ivory towerish" attitude to our profession. It is not enough that we publish papers in ~~international~~ prestigeous international journals and present them at conferences. It is important and "necessary" to do so. But it is not sufficient. We have to be able to interact with "people", interested people, students and teachers so as to be able to convey at least the excitement of science that we do; so as to be able to help teachers and students of science to help them in their jobs and thereby to raise standards of science at least

But the most important task that we must undertake to do rather urgently is to introspect our own style and manner of doing research so as

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to ensure its greater creativity at home.

We ought to have some seminars and discussions ~~on these issues~~ in the faculty to on the entire gamut of these issues. Specifically we should discuss

1. The implication for the Science and Technology Development in the Country of the economic and industrial ~~glo~~ globalization and the multinational ~~en~~ entry into the Indian Market

2. A possible restructuring of Science in the country including education and research.

— An introspection and redefinition of aims and objectives ~~in view~~ to meet national ~~aspirations~~ developmental aspirations.

~~Can soon~~

Quite obviously these are important urgent issues which the scientists of the country will have to face ultimately if not today certainly tomorrow. Therefore why not face them today why not think of them today and give lead to the country

3. Scientific Personnel & Their Evolution

An important issue which is intimately related to all the foregoing issues is the question of the ~~early~~ identification, and nurturing of what has been termed as the "Science Talent" which is a much touted term. We have been talking about it a lot, but nothing much has happened. Unfortunately, we have never been ~~either~~ bold or imaginative to tackle ~~Nor have~~ this important issue. Nor have we cared to leave the comforts of our office to venture out to search, ~~and~~ identify and nurture. ~~May be we~~ or We have tried to use the instrument of allurement rather than that of inspiration to attract talent. ~~Talent may be still be lying wrapped in rags rather than glitter~~ May be talent is lying where we are not looking. May be talent needs an inspiration to unearth itself which the scientists of today are certainly not providing.

All organizations including DRL are

following the standard course of advertizing for ready made products, rather than contributing to ~~the making~~ of the product and participating in the preparation of the product. Even though it has been mentioned time and again here that all support is available to visit Universities.

It is important that we attempt some non standard procedures, carry out non standard educational and training procedures to promote creativity, to inspire creativity among the young. Prior to that we must do something to inspire the young to science.

But for that to happen we have to come out of "ivory towers" and be able to share our time with the young so as to interact with them meaningfully, and to inspire them. I am sure the joy of discovery a discovering a or inspiring a another talent would far exceed that of writing a paper.

Perhaps, I should mention in this connection that the discovery, inspiration and nurturing

of a talent is both an "individualistic" and a "cooperative" venture. While the individual is one who inspires others must also pitch in to ~~gr~~ impart a wholesome training in a matter of fact manner. Different individuals may have a different ways of inspiring, but the whole collective of teachership is involved in producing and creating the next generation of scholarship

Unfortunately this sacred task has got somewhat rudely interrupted and the tradition sadly broken. The teacher in us who used to take pleasure in simply being able to transfer knowledge has been hijacked and appropriated by ~~is~~ our self-centric pursuits. With the University environment being what it is the ~~trade~~ sacred tradition of transfer of knowledge has been disrupted. It is necessary to restore these traditions with a certain amount of farsighted ~~see~~ devotion to the cause so that science tradition and healthy institutions ~~prevail~~ are built and prevail

On the other hand the ~~students have also seem to have lost to some~~ extent teachers have also suffered

a loss over the years of some of traditional respect and status in society. A multitude of factors are responsible for this. But at least we can begin by giving the appropriate regard and respect to teachers in us and teachers in others. Which means to give regard to the process of teaching and not consider it a task of secondary importance

I would therefore, like to suggest to hold a third seminar on

3. Upbringing of science talent: a search and nurturing — and new paradigms in science teaching and training.

It is quite clear that a consideration of all these aspects pointed out above is no mean task and it would really involve a considerable amount of time and effort of those who wish to undertake it seriously. Some may say that they are too much preoccupied with their own personal tasks.

That certainly may be true, but these tasks mentioned above

of no less importance ~~if~~ even for one's own long term future if, ~~let alone~~ not ~~that~~ of the future of science in the country.

But much more than that, it relates to the future of our very existence in this country, not just as scientists but as decent human beings, not just for ourselves, but for ~~our~~ the millions of our countrymen and for the future of our children and ~~others~~ those of others.

~~What seems alarming today is the manner in which the values and institutions are being eroded, the manner in which communism~~

Why I say so? Why I am seeking to relate the future of science in our country to the future of our children, future of our countrymen and the future of our country itself. The reason is clear. Science stands for the search for truth. Science is for the abolition of superstition. Science stands for the welfare of society and a better quality of life. Science moreover stands for simple truths

not camouflages. The message of science must percolate to the society through simple truths rather than through technologies. Technologies do no doubt help, but they must not be allowed to carry away the society.

While technologies allow a section of the society to be carried away by it, by its advances, both desirable and undesirable, a dominant section of the society remains mired in ignorance and poverty. Worse still, the wrong and undesirable aspects of technology also tend to influence the ignorant through its glitter in a wrong manner.

Can science help in this? ~~Perhaps~~. May be not entirely. But it certainly can take a lead, by spreading the message of simple truths. Others must join of course.

What is it that I am suggesting? What I am suggesting is a proliferation of science culture in the society, by aiming at creating a whole continuum of science messengers, with

the science teacher being the focal point.

We must generate a ~~band~~ community of science teachers who should interact with their senior counterparts at one end and people at large at the other. For this to happen the dignity of teachers must be restored.

May be when this happens, science talent will unearth itself automatically. But more than that the society will begin to be more anchored in ~~natural~~ nature's truth, away from superstition, more self sufficient, ~~away from~~ and less exploitable by unscrupulous politicians.

That is the hope!

But if nothing is done, we may not be able to ~~d~~ even hope.

Some of us may ask - is it not too formidable an undertaking for the poor scientists to engage.? It certainly is. But it is an agenda that no body else has an ability to do. Only scientists can. But ~~must~~ it can be done!!

$$N(B) = \int_0 d\mu\ \delta(\mu - \mu_0)\,\delta(Q(\mu) - n\mu)$$

$$N(B) = \delta(Q(\mu_0 B) - n\mu_0)$$

Thus we have ~~max~~ non-zero result whenever $Q(\mu_0 B) = n\mu_0$.

If on the other hand, we have:

$$N(B) = \int d\mu\ e^{-\alpha(\mu-\mu_0)^2}\,\delta(Q(\mu_0 B) - n\mu)$$

$$= \int d\mu\ e^{-\alpha(\mu-\mu_0)^2}\,\frac{\delta(\mu-\mu_0)}{[Q'(\mu_0 B) - n]}$$

$$= \frac{1}{[Q'(\mu_0 B) - n]}$$

This gives ~~or~~ maxima whenever $Q'(\mu_0 B) = n$ which means $\bar{\Omega} T = 2\pi n$

$$2\pi n = \int \frac{\Omega\, dx}{\sqrt{E - \mu_0 \Omega}} = \int \frac{\Omega\, dx}{\sqrt{E - E\sin^2\theta_0 \dfrac{\Omega}{\Omega_0}}}$$

$$= \frac{1}{\sqrt{E}} \int \frac{\Omega\, dx}{\sqrt{1 - \sin^2\theta \,\Omega/\Omega_0}}$$

unless θ_0 is close to the loss cone

And if Ω/Ω_0 is small the change in θ_0 may not make too much difference

~~and as an~~ It is my earnest hope that ~~a~~ not only as an esteemed member of the IRL Council of Management and the head of its primary funding agency, namely DOS, ~~you will~~ but also as one who has been so intimately related to IRL you will give a very serious thought to what I would like to say.

** There may have occured because of a variety of historical reasons which we may not go into.

There may have been some lapses on the part of PRL administration as I know there were in handling the situation as it arose in the fateful years 1978-79.** But the undeniable fact remains that in most our minds that the re classification of PRL as an "industry" and consequent which puts it brought it under the purview of the Industrial Disputes Act and consequent formation of PRL employes union, have played havoc with the vitals of PRL coupled with the Gujarat University Act. No inquiry could be peacefully conducted and no action could therefore be taken against the erring employees The PRL administration thus stands paralysed, though not completely yet. has been showing signs of at least a partial paralysis, which will only deteriorate with time if some corrective measures are not taken. Fortunately the vital administration components of PRL, like Accounts, Stores and Purchase are still functioning alright under these trying conditions, But I am only afraid the infection may catch on if the staff begins to feel agrieved for some real or imaginary reasons.

EXCELLENCE IN SCIENCE

It is against this backdrop that I would like

to discuss the question of achieving "excellence in science" which we all ~~wish to~~ talk about and wish to achieve but have by and large failed to do except in some isolated cases. I, for one, am convinced that so long as autonomous institutions like PRL continue to be classified as "industry", the environment in these institutions will be industry-like, where staff members will stage dharnas, gheraos, shout slogans and abuses to ~~try to~~ achieve their legitimate or illegitimate demands. This completely undermines the position and dignity of a director who ~~and senior admins~~ is directly exposed to these acts. In a real industry it may be business as usual after the bout is over, because after all it is the production in terms of numbers and quantities which counts. In a research institute which wishes to strive for excellence, it cannot be business as usual after the bout because it does leave its impact on the minds of the scientists and affects their creativity,* This points cannot be overemphasized if one is serious about excellence in science. It is

* and it leads to strained relations with the staff members with whom they have to work day to day technical

~~PRL today is poised for~~ a

From a scientific point ~~of~~ view, PRL is today for great scientific ventures: We have the infrared telescope which will be installed shortly*. The 3-station IPS is also almost ready and the ~~Ahm~~ Thaltey station has already produced ~~an~~ important results on the ~~the~~ plasma density irregularities in the Halley's comet tail (* ~~A highly productive and interesting activi~~ If ~~ha~~ properly organized it has a tremendous potential for a highly interesting and productive activity ~~centrin~~ relating to star formation ~~can~~ growing around it) ~~The full~~ When fully operational it will give results on the structure of solar wind at various heliolatitude and its microstructures. In addition, ~~the~~ various investigations ~~on~~ using the SROSS series of satellites ~~is~~ are already ~~on~~ planned. If all

~~The serious question before~~

these activities proceed as envisaged, the PRL ~~will be~~ can ~~be visualized as an institution with~~ look forward to another period of

absolutely imperative therefore that something has to be done to take PRL and other such autonomous institution out of the definition of "industry" if we are serious about our goals in science. I am saying this with the full realization that it is a political decision But as you know, a bill to this effect was already passed by the previous parliament and so has been waiting only to be gazetted

The question before us is whether PRL can be salvaged or be allowed to go on like this until it reaches a point of no return. What I am absolutely convinced is that we needs some drastic solutions if PRL is to be saved unless one can find a superman as a Director who besides being a good scientist is a good administrator, and be able to hear abuses shouted at and without it affecting his creativity, efficiency and mental state.

great scientific output of a high standard.

The serious question before us is whether with the present state of affairs as it obtains at PRL it can cope with the level of activity that the various programmes promise. The answer to my mind, is very emphatically ~~and unhesitatingly~~ in the negative, unless there is a drastic change in the structure of PRL and in its status, ~~was a one~~ as an "industry." If allowed to continue as at present the future of all the programmes will be in jeopardy, unless one is prepared to be continuously blackmailed by the Union and accede to its exaggerated demands or unless one is able to find a superman as a Director who besides being a good scientist is a tough administrator who can withstand being shouted at, and abused without affecting his creativity, efficiency and mental state.

PRL is now 38 years old, and is showing signs of old age. PRL has had a rather conservative promotion policy for all its staff compared to the other ISRO units. As a result the technical staff have developed a sense of grievance over the years, as they have compared their upward motion with those of their counterparts in ISRO, and have found themselves unfavorably placed. The upward mobility of the administrative staff is even slower as they compare themselves with the technical staff and have developed a growing sense of grievance and frustration. The desirability of keeping the total PRL strength within limits further dictates that the promotions be stricter, which in the technical and administrative cadres seems to be strict which unfortunately would lead to dissatisfaction among the staff.

In the initial stages of an Institute such problems of ~~few~~ do not arise as there appears to be enough room at the top. As

the Institute grows older and as the vacancies at the top begin to get filled, the frustrations begin to manifest themselves. Also the enthusiasm of the large body of workers as they reach middle age begins to decline in favour of the security of the and the comfort of the job other comforts of life. At that point people begin to Somewhere along the line it is not the challenge of the job that keeps them excited and busy occupied. Rather the attention is drawn towards intercomparision among themselves of the dates of promotions, and other distinctions in various Career points as at that point one would say that the Institute has begun to show signes of old age. All variety of grievances then begin to arise, which can get converted to collective representations from different groups of employees. The institute then has taken a different direction away from the The efficiency goes down, the focus of attention is no more the challenge of the job on hand. but rather

The situation is ripe for a trade union kind of activity to flourish. The only way to keep them keep people efficiently on the job is to keep promoting them. What is most distressing however is that merit as a criterion for promotion gets obliterated as more and more staff members ask for more or less uniform promotions. If it is conceded as it must be, no incentives would remain for the brighter ones. [The merit promotions of the brighter and the more devoted ones are branded as favouritism].

What must one do, when one reaches such a state, when such a state gets exploited by the "trade unions" which have unfortunately been allowed to operate in academic institutions, when a very large fraction of the time of the scientific management of the institution is wasted in dealing with the trade unions and the problems created

by them and in trying to redress grievances most of which may frankly irredressible within the framework that one has to work in.

Is the grand objective of achieving scientific excellence compatible with a state of affairs and scenario outlined above? The answer is obvious.

What are the alternatives? One is to continue somehow in the state that obtains, and not try to make the best of it. The Director then consumes most of his energies doing only non-scientific administration and has little time and energy for directing science which ought to be his main job. Such an alternative should not be acceptable.

What is then the remedy? Either the academic institutions be freed from the clutches of trade unionism or through a legislation or alternative strategies must be found. ~~What~~ If

I would like to make ~~pose this question~~ to the ~~PRL Council of Management~~ with a most earnest and urgent plea that the Council give serious consideration to ~~this~~ these ~~problem~~ questions, ~~and~~ and try to find out ways and means of resurecting PRL.

Scientifically PRL is today poised for ~~great~~ taking ~~off~~ up great scientific ventures: We will have the IR telescope which should be commissioned shortly. ~~If properly~~ It has a tremendous potential for a highly interesting and ~~fro~~ productive activity relating to star formation growing around it. The 3-station IPS is also almost ready and the Thaltej station has already produced intresting observations on the Halley's plasma tail when fully operational it will give results on the structure of solar wind on at various heliolatitudes and its microstructure. In addition, various investigations using the SROSS series of satellites are already planned

If all these activities are to flourish to achieve excellence, then it is imperative that PRL must be able to direct most of its energies toward these scientific objectives. Unfortunately with the state of PRL today, ~~in relation to~~ and ~~that it~~ more importantly its prospects in future as ~~a energy~~ because of unionism, these scientific objectives appear to be in jeopardy. ~~The objectives of the Union leaders~~ ~~What it seems~~ A "total" solution to the problem seems to be called for because patch work and stop gap ~~sales~~ measures would not do. ~~PRL~~ All the new activities of PRL must be protected ~~from becoming a part of~~ against the spreading cancer

The PRL Council ~~must address~~ urgently is most earnestly urged to consider these questions seriously.

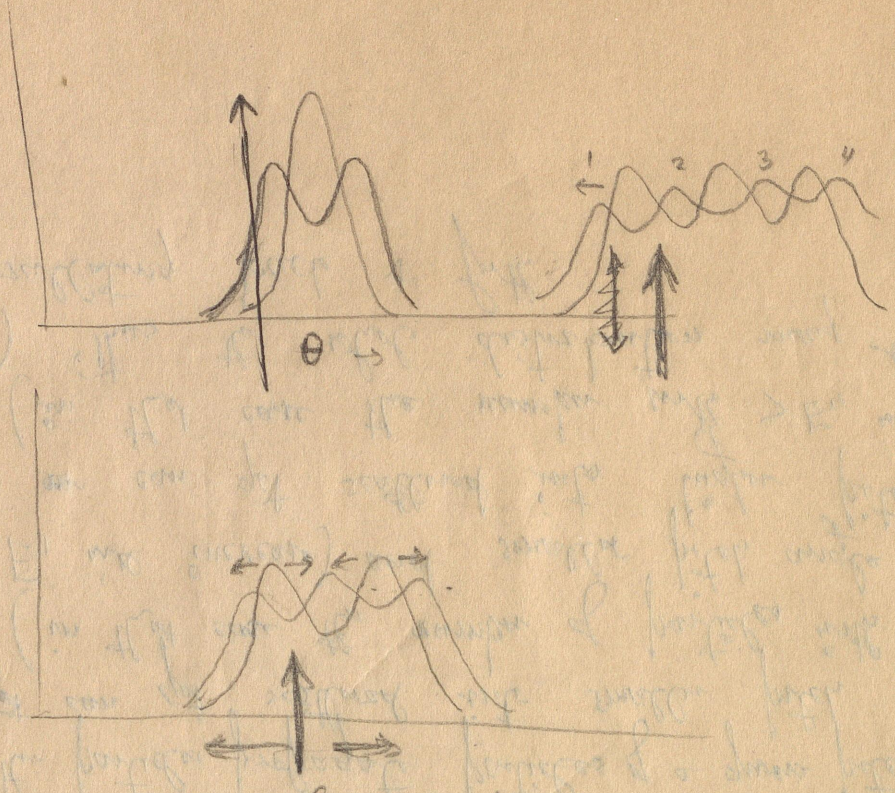
What happens when the particles are injected at one end at $\simeq$ 42-45°-50°?

Why do they show maximum interaction with $\theta \simeq 40°$?

→ Of course with $\theta \simeq 45°$, there is equipartition of energy between $\parallel$ & $\perp$ components. One may argue that most particles through scattering by the magnetic field settle down to $\theta \simeq 45°$, and thus the maximum effect will be visible at that angle simply because the maximum number of particle belong to that angle

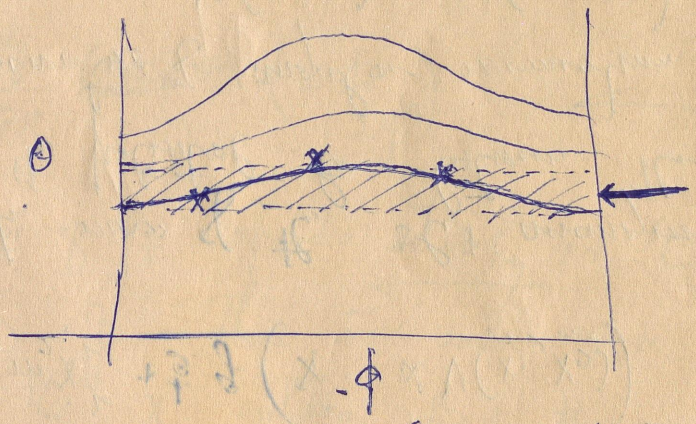
---

As the particles propagate particles of a given pitch angle can get scattered into smaller pitch angle (in that case the number of particles with greater $E_\parallel$ will increase) and smaller pitch angle particles can get scattered into higher pitch angle (in that case the number with $> E_\parallel$ will decrease) Thus the whole distribution may be oscillating back & forth



As the magnetic field changes, the distribution ~~of the phase~~ & initial phase angle changes and the above distribution with respect θ may oscillate back & forth with the magnetic field.

As the particle crosses the mid plane, there will be a jump in both ϕ and θ, the phase angle and the pitch angle. Each member of the ensemble carries the signature of this jump.



As particles go to smaller or larger values of θ the parallel energy increases and decreases, the RP current with suitable KP ~~would~~ (as shown for instance by the arrow) would show undulatory behaviour with ϕ. The ~~change of~~ ϕ ~~can be effected by a change of B~~. as it obtains at the mid plane ~~can be a can be~~, would change as the magnetic field ~~is~~ strength changes. and hence the emerging θ value would change with ϕ or B

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} g (\dot{x} - \alpha V)^2$$

$$p_x = \frac{\partial L}{\partial \dot{x}} = m_1 \dot{x}$$

$$p_x = g(\dot{x} - \alpha V)$$

$$H = p_x \cdot \dot{x} + p_x \cdot \dot{x} - L = p_x \cdot p_x / m + p_x \left[\frac{p_x}{g} + \alpha V \right]$$

$$- p_x^2 / 2m - p_x^2 / 2g$$

$$= \frac{p_x^2}{2m} + \alpha p_x V + p_x^2 / 2g$$

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} g (\dot{x} - \alpha V(x_1, x_2))^2$$

Let m_2 be the mass of the slit arrangement and $V(x_1, x_2)$ the potential between them

On conservation of angular momentum holds

~~$m_1 \dot{x} + m_2$~~ . if $V(x_1, x_2) = V(|x_1 - x_2|)$

$$m_1 \dot{x}_1 + m_2 \dot{x}_2 = 0$$

$$m\ddot{x}_1 = -\alpha p_x \frac{\partial V}{\partial x_1}$$

$$m\ddot{x}_2 = -\alpha p_x \frac{\partial V}{\partial x_2}$$

$$\dot{x} = p_x/g - \alpha V$$

$$\dot{p}_x = -\alpha p_x^2 \frac{\partial g}{\partial x} \left(+\frac{1}{2g^2}\right)$$

Thus the evolution of p_x depends entirely on the x-dependence of g. Thus in some sense it is uncoupled to the x motion. But the evolution of x_1 & x_2 depends on the evolution of p_x. Also the ~~value of x at t~~, evolution of x involves x_1 & x_2 through $V(x_1, x_2)$. In this sense therefore the evolution of x_1 affects that of x_2

From conservation of momentum we have across the slab

$$m\dot{x}_1^{(+)} = m\dot{x}_1^{(-)}$$

$$\ddot{x}_2 \quad \ddot{x}_2 \dot{x}_2 = \alpha p_x \frac{\partial V}{\partial x_2}$$

$$\frac{1}{2}(\Delta \dot{x}_2)^2 = \frac{\Delta V}{\Delta x} \Delta x$$

$$H = \frac{p_1^2}{2m} + \frac{p_r^2}{2m} + \alpha p_2 V + \frac{p_x^2}{2g}$$

If p_x, p_1, x_1, x_2 are same for all particles, then p_2 ~~[illegible]~~ must have a spread if g is a function of x.

$$0 = \frac{p_2 \Delta p_2}{m} - \frac{p_x^2}{2g^2} \Delta g$$

$$\frac{p_2^2}{2m} - \frac{p_r^2}{2g^2} \frac{\Delta g}{\Delta x} \Delta x \quad 2\pi$$

$$p_2 = \hbar\, 2\pi \frac{\partial g}{\partial x}$$

$$\nabla^2 \Phi = V(x, y, z)$$

$$q\Phi$$

$$g\,\Phi = \Phi_0 + a^2 \nabla^2 \Phi$$

$$\frac{\partial \Phi}{\partial x}$$

Let the Lagrangian be

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} g(x)\left(\dot{x} - \alpha \Phi\right)^2$$

where we define potential V through

$$\nabla^2 \Phi = -4\pi V.$$

Now Φ can be expanded

$$\Phi_0 + \frac{1}{2}\rho^2 \nabla^2 \Phi \quad \rho \cdot \nabla \Phi + \frac{1}{2} \rho^2 \nabla^2 \Phi.$$

where $\rho \cdot \nabla \Phi$ would average to zero. so that we have:

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} g(x)\left(\dot{x} - \alpha \Phi_0 + \frac{1}{2}\rho^2 \nabla^2 \Phi\right)$$

we may introduce a new x'

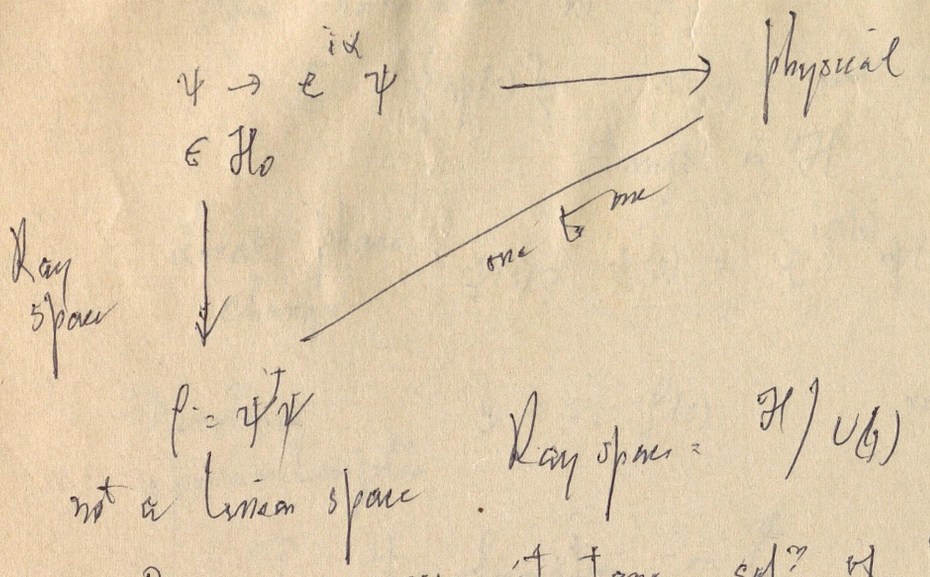
$$x' = x - \int \alpha \Phi_0 \, dt$$

Then we have:

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} g\left(x' - \int \alpha \Phi_0 \, dt\right)$$
$$\left(\dot{x}' + \frac{1}{2}\alpha \rho^2 \nabla^2 \Phi\right)^2.$$

If the particle be represented by a closed string

and of the points on the string intersect via the pulses



$\rho = \psi^\dagger\psi$ Ray space $= \mathcal{H}/U(1)$

not a linear space.

Berry: $U(t)$ instantaneous sol^n of $H(t)U(t) = E(t)U(t)$

$\psi(t) \approx e^{-\frac{i}{\hbar}\int E(t')dt'} U(t)$ $\approx$ approx. sol^n

If $H(T) = H(0)$

$\psi(T) = e^{-i\Phi_{tot}}\psi(0)$

$\Phi_{tot} = \int_0^T E(t)dt + \Phi_{geom}$

	Adiab	Cyclic	Unitary	Schr
Berry	✓	✓	✓	✓
Aharonov	✗	✓	✓	✓
Samuel Bhandari	✗	✗	✗	✓
Conrad Mackenzie	✗	✗	✗	✓

Unit sphere in H.S.

Open smooth curves in $\mathcal{H}$

$\mathcal{C} = \{\psi(s)\}$ $\psi(s)$ = unit vector

strings in $\mathcal{H}$

Local phase change $\psi(s) \to \psi'(s) = e^{i\alpha(s)}\psi(s) \Rightarrow \mathcal{C} \to \mathcal{C}'$

Monotonic reparameterization $s \to s' = f(s)$ reassignment of $\mathcal{C}$

Two transformations on $\mathcal{C}$

$\varphi_g = \arg(\psi(s_1), \psi(s_2)) - \int_{s_1}^{s_2} ds\, \mathrm{Im}(\dot{\psi}(s), \dot{\psi}(s))$

= doubly invariant

$\varphi_g(\mathcal{C})$ = Pancharatnam phase of $\mathcal{C}$ - dynamical phase of $\mathcal{C}$

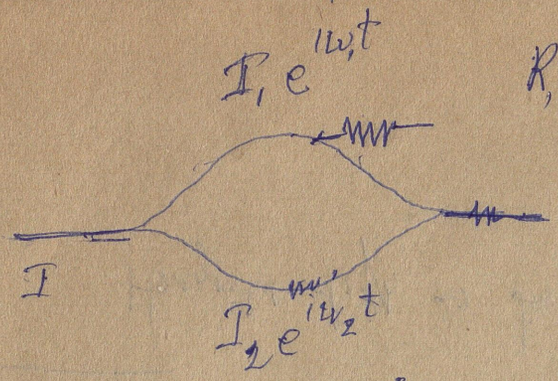
$\varphi_p[\mathcal{C}] = \arg.(\psi(s_1), \psi(s_2))$

$\varphi_{dyn}[\mathcal{C}] = \int_{s_1}^{s_2} ds\, \mathrm{Im}(\psi(s), \dot{\psi}(s))$

$\mathcal{C} \to \mathcal{C}$

Horizontal lift $\Rightarrow \mathrm{Im}(\psi, \dot{\psi}) = 0 \to \varphi_{dyn} = 0$

Properties of Hilbert space



R $\quad R|I_1 + I_2|^2$

$R_1(|I_1|^2 + |I_2|^2)$ $\quad = R[|I_1|^2 + I_1^2 + I_1^* I_2 + I_2^* I_1]$

Causal Quantum Mechanics.

Classical or QM

Unitary app

Complete description of events.

Partha Ghose:

$$\frac{\partial f}{\partial t} + \underline{v}\cdot\nabla f + \frac{L}{\hbar}\frac{\partial f}{\partial \Phi} = 0$$

$$f = \sum h^n f^{(n)} e^{in\Phi}$$

$$\frac{\partial f^{(n)}}{\partial t} + \underline{v}\cdot\nabla f^{(n)} + \frac{L}{\hbar}\frac{\partial f^{(n)}}{\partial \Phi} = 0$$

$$i\frac{Ln}{\hbar}\frac{\partial f^{(1)}}{\partial \Phi} f^{(n)} = 0$$

whence $f^{(n)} =$

$$\left(\frac{\partial}{\partial t} + \underline{v}\cdot\nabla\right)\underbrace{\left(f^{(n)} e^{in\int_{t_0}^{t}\frac{L\,dt}{\hbar}}\right)}_{\hat{f}^{(n)}} = 0$$

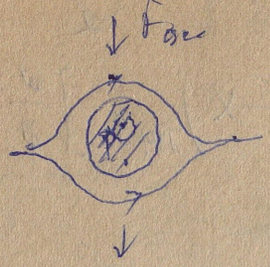
$$\frac{d}{dt}\left[f^{(n)} e^{in\int_{t_0}^{t} L\,dt/\hbar}\, e^{i\int^{t} \underline{v}\cdot dx}\right] = 0$$

$$\frac{d}{dt}\left[f^{(n)}\left(x - \int_{t'}^{t}\underline{v}\,dt,\ t\right) e^{in\int_{t_0}^{t} L\,dt/\hbar}\right] = 0$$

$$\frac{\partial}{\partial t} = -\underline{v}\,dt + \underline{v}\,dt$$

$$f^{(n)}\left(x - \int_{t'}^{t}\underline{v}\,dt'',\ t\right) e^{in\int_{t'}^{t} L\,dt} = \text{const}$$

$$f^{(n)}(k, t)\, e^{\frac{in}{\hbar}\int L\,dt' - k\int \underline{v}\cdot dt} = \text{const}$$



Current produces a magnetic field

$$\underset{\sim}{B}_1 = \int \frac{\underset{\sim}{J}\, d^3x'}{|\underset{\sim}{r}-\underset{\sim}{r}'|}$$

Total field at P, $B = \underset{\sim}{B}_0 + \underset{\sim}{B}_1$

Field energy: $|B|^2 = |B_0|^2 + |B_1|^2 + 2\,\underset{\sim}{B}_0 \cdot \underset{\sim}{B}_1$

If the flux were changed,

The energy of the field

$$\frac{1}{8\pi}\left(\underset{\sim}{E}^2 + \underset{\sim}{B}^2\right) + \frac{e}{c}\,\underset{\sim}{J}\cdot\underset{\sim}{A}$$

$$\frac{1}{8\pi}\,|\mathrm{curl}\,\underset{\sim}{A}|^2 + \frac{e}{c}\,\underset{\sim}{J}\cdot\underset{\sim}{A}$$

$$\frac{1}{8\pi}\int d^3x\,|B|^2 + I_1 \oint \underset{\sim}{A}\cdot d\underset{\sim}{\ell}$$

For a solenoid if current is in opposite direction

The $\underset{\sim}{J}\cdot\underset{\sim}{A}$ term is

$$I_1 \int_1^2 \underset{\sim}{A}\cdot d\underset{\sim}{\ell} - I_2 \int_1^2 \underset{\sim}{A}\cdot d\underset{\sim}{\ell}$$

$$= I_1 \oint \underset{\sim}{A}\cdot d\underset{\sim}{\ell} = \quad \text{, if } I_1 = I_2$$

$$+ I_1 \Phi$$

Expression in terms of density matrix

$$\varphi_g[C] = -$$

Geometric + Bargmann invariant:

C in $\mathcal{H}$ → image C in ray space → $\mathcal{L}[C]$

Key result $\boxed{\varphi_g[\text{geodesic}] = 0}$.

Bargmann invariant

$$\Delta_n = (\psi_1, \psi_2)(\psi_2, \psi_3)\cdots(\psi_n, \psi_1)$$
$$= \mathrm{Tr}(\rho_1 \rho_2 \cdots \rho_n)$$

$$\varphi_g[\text{n-sided polygon in ray space}] = -\arg \Delta_n$$

Unitary Representations of Lie groups:

Framework: Lie group G, unitary representation

ψ_0 = chosen initial vector

If however, the current distribution is to be such that it should minimise the total energy then all the current would tend to flow only along one path, namely where $\underline{J} \cdot \int \underline{A} \cdot d\underline{l}$ is < 0.

Large scale structures in

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Liberalization and Globalization: Implications for ~~Science~~ in ~~India~~ Science and Technology

Some concerns and dilemmas

Many types of Reactions:

1. Initial reaction was euphoric particularly on the part of business community and industrialists -
   - As if they have been released from chains of permit raj
   - As if they can now do what they could not do before
     - No waiting in Qs
     - No greasing of palms
     - No harassment by beaurocrats

- Multinationals in Big Business in | Small scale industries – future uncertain

- Privatization in — state intervention out

↳ the state abdicating its social responsibility by abandoning the concept of state intervention ?

So is it perceived to be doing.

Has the stat thrown out the baby with the bath tub ?

So is it perceived to have done

Reaction of the Industrialists.

Most industrialists are, of course, happy. They are euphoric about the tie-up with multinationals and the profits that will roll in.

They never had it so good !!

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- As if they can now ~~roll in profits~~, expand capacities, roll in profit in an unprecedented manner.

~~So far fair enough~~

~~But unfortunately the thinking has been~~

~~[illegible]~~

Thus "permit ray" — Out ↑

"Market economy" — in

~~So far fair enough.~~

→ Reaction of the Common Man:

One of a state of confusion

He does not know what it means to his

- price of bread + butter and other necessities of life
- Employment opportunities

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He is told that ~~he can get~~ there will be many jobs as

- capacities expand
- ~~As~~ and as multinationals troop in to save him.

He is made to feel happy.

~~But he has many gun question marks~~ ?

He ~~a~~ is told that ~~as there~~ now since ~~as~~ the prices will be determined by market forces ~~and~~ there will be competition among ~~industrialists~~ the manufacturers of a given consumer product, the prices will ~~be~~ actually go down. Likewise he is is told that quality will also go up, by the same token.

But he does not see that happening. In fact he finds that ~~pro~~ prices ~~be~~ are getting ~~slowly~~ dollarised rapidly. ⟹

- It has brought in foreign brands into the market at international prices

But who needs the foreign brands of everything - from tooth paste shoes shaving cream etc - to suit cases, 'packed' food stuffs, Kentucky Chicken Mc Donalds Hamburger or Kellogg's Corn cereals at dollar prices ??

Most Indians don't ! Think of those who live in the villages and the stupidity of it all will be quite evident.

Even mango prices are getting dollarized in sympathy ! An Alphonso will pretty soon cost a dollar a ~~piece~~ piece.

The Common man

has several questions which have remained largely unanswered

? Why are multinationals coming here to make money from the Indian market and the Indian public ? They obviously are.

? If multinationals indulge in crass consumerism, as they are doing in other countries. what would happen to our

- physical environment
- Social environment
- cultural values.

No Indian business in his euphoria has time and inclination to ask these questions, let alone addressing them

Government perception:

Government believes that its NIEP & NIP has been highly successful because it has attracted foreign exchange to the time of $ 20 billion, as compared to the Feb 8 near bankruptcy as obtained in 1991.

However as most economists agree not only is this reserve highly volatile it is highly inflationary. Unless this reserve integrates and entangles the economy, it does not do anything than produce a false sense of security besides fuelling inflation.

The Government seems to have forgotten that unless you increase the honest purchasing power of the populace (as against the black purchasing power), globalization leading to dollarization of prices will only either enhance corruption, or create vast disparities, or both, leading to unavoidably to social tension and increase of crime rate. This has already been happening if the series of kidnappings in Delhi is any indication.

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Reaction of the ~~Scientist~~ Economists

Economists have universally acclaimed these changes ~~saying~~ stating that they had been arguing for them ~~so~~ for a long time. The reason being that these are economists pet themes.

Reaction of the Scientists and Technologists

~~Their reactions should have been favourable~~

No reaction of the scientists and technologists is available, at least not in the new paper columns.

Most reactions are that have been available are essentially off the cuff. and mostly relate to the issue of their survival only on the short term. I have not seen any in depth analysis with any long term perspective.

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60
DIAMOND JUBILEE

What I would like to discuss today ~~is not a~~ does not pretend to be any in depth analysis with any perspective, but I shall only make a few observations of some relevance.

One of the fundamentals of the NEP + NPP is the ~~invitation to the multi.~~

~~the multinationals to~~

- invitation to multinationals to come in with joint-ventures (in India as well as TW countries).
- Transfer of ~~technology~~ foreign (latest or not so latest ?) technology.

"Transfer of Technology in"

— Self reliance "out" ?

— Indegeneous Development of technology "out" ?

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Assumptions are :

- Multinational will bring new technology
- Will develop country's infra structure
- Will expand industrial base of the country

⇓

implies more jobs for the people.

If that is really the compulsion of the government. Then let us analyse its consequences for Science + technology

First, will the Multinationals ~~really~~ bring really new technology ?

The answer is "doubtful"

Even if they do — the whole concept is bad news for the indegeneous Science + Technology.

Globalization:

~~Implications~~:

Welcome

Good! ← · Market economy in, ~~social~~ permit controls out

· [Multinationals in | Small scale industries: future uncertain ?
[Big business in

~~Good points are~~

~~This is supposed to~~

~~Prev~~ Privatization is a catch word

· Privatization in — ~~social int~~ in a big way state (social) intervention out

~~Is the~~ Is the state "~~abjuring~~ abdicating" its social responsibility by abandoning the concept of ~~responsibility~~ state intervention?

So ~~it~~ is it perceived to be doing?

~~It was~~

~~Is the~~

Has the state thrown out the baby with the bath tub?

So ~~it~~ is it perceived to have done.

Result: Total public disenchantment with the NEP – New Economic Policy

Consequence: Thrashing of Congress Government in the recent elections: So it is perceived to be reason of its defeat

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Let us come to the associated New Industrial Policy: (NIP)

Multinationals in: local industries (used to protection) must compete.

Perceived results:

(A) Competition will improve quality & lower prices

An assumption not vindicated so far

- Prices have only soared
- It has brought in foreign brands into the market at international prices

Most prices have been ~~dollar~~ happily + merrily dollaring.

~~Most~~ But who needs the foreign brands of everything — from toothpaste shoes, shaving cream — to suit cases; and now Kelloggs Corn flakes (besides Coke).

Most Indians don't !!

Even mango prices are getting dollarized in sympathy. — An alphonso will pretty soon cost a dollar a mango — if it ~~has~~ does not already do so.

Most industrialist are of course, happy: They are rolling in lots of profits — they never had it so good !!

Assumption (B) Multinational ↓ will bring new technology
↓ will develop country infrastructure
↓ will expand industrial base of the Country
↓
more jobs for the people

implications:

- Joint-venture will proliferate

But will it bring really new technology?

Answer is "doubtful"

What will be the fallout for indegeneous development of technology?

I see "bad news" for indegenous development of technology.

Protagonists of the NIP say:

The (a) local R&D efforts did not cover themselves with glory.

True, But the industry was never interested in R&D and new innovations, as they were operating in sellers market

But now with the "irreversibility" of the NEP + NIP announced to the world.
~~that is the~~ will there be any incentive for indegenous development of R+D for industrial applications.

It is bad news for ID of R&D.

for technology - in the short run at least.

And if it is so in the short run - it will also be in the long run because given the ~~mind~~ present mind set - long run is generally made up of short runs. unless one thinks differently - + changes the mind set.

Now how about <u>science</u>:

In the "good old days", science was done ~~so~~ for its own sake

<u>Keplers laws</u>: – to synthesize the observations of the motion of planets by T. Brahe

– There was no technological benefits to be derived from these.

<u>Newtons laws</u>: to understand Keplers' laws at a deeper level.

Even ~~with~~ Michael Faraday ~~to~~ did not have any applications in mind when he studied the behaviour of electricity

Laws of QM were invented to understand Atomic spectra which were studied for their own sake.

During the 20th century and more particularly during and after the 2nd World war, the technological applications of science assumed unparalled importance.

In course of time - technology became more or less self-driven. - One form of technology feeding into another technology.

To be sure, science still played a role, but only a certain kind of science. Only that which was demanded by technology.

What about the other kinds of science

→ They either became "piggy back" on the "relevant" science or on more charity, not as a matter of right unless you have the clout of a "Club-Class" like the particle physicists had or the "astronomers" and club had in getting their accelerators or Hubble telescope etc

its pre-eminent states

Then Science ends up losing to technology even though it is regarded as a 'mother' to technology. unless it is a Science which is relevant to technology.

Thus the coordinate axes have by and large turned by 90° in terms of funding. which is now more or less determined by "relevant" vs "irrelevant" science.

If your Science can only be tolerated you will get only tolerable amount of funding not as a matter of right, but more or less as a charity.

Within that amount of money, of course, you are free to classify your science as "exciting" or "non-exciting" and fund the only the exciting one. That privilege is generously granted to us quite

The Question is whether we are tending to a state in pure Science where it is simply tolerated but not appreciated. for its own sake. ?

India vs Japan

Having given this background, I shall now go back to the scenario created by ~~the NIP~~ NIP - where the tie-ups with the Multinationals determine the order of the day.

~~Is there any~~

Will any space be left ~~in~~ for indigenous development of technology ? Frankly I do not yet see any space that will be so left, as the scenario provides little ~~room~~ incentive for such an R+D effort.

If there is no R+D for technology there will be no space ~~for science~~ even for "relevant" science let alone "irrelevant" science

Perhaps one should compare and contrast the case of the Evolution of Science + Technology in India and Japan (without any participation of multinationals) or for that matter South Korea or Singapore which are quoted today as shining examples of success stories.

Japan had a tradition in Basic Sciences but not as strong as in India; if we compare the ~~pre-independ~~ ~~pre-World War~~ pre-independence names:

India:		Japan
Raman	Physics	Yukawa
Bose		Tomonaga
Saha		
Ramanujan		

Technology:

India: ← almost same as → Japan

(Remember the quality of products "Made in Japan".

Then in the fifties:

Japan ~~became~~ began to be

"innovative copiers" - but without "multinationals"
which did not exist at that time

without any significant either scientific or technological base.

Industrialization through innovative copying

⇓

This led to the motivation for the development of technology appropriate to the demands of the industry: ⇒ ~~Technology Development~~

Industrial demand defined the choice of technology to be developed.

The Industrial demand on the other hand was determined by the ~~Market~~ Export Market whose importance the Japanese realized fully

In fact "export" is the life line of the Japanese who have absolutely no natural

resources — barring the ~~Earth an~~ long and thin stretch of land and the Earth Quakes

~~The~~ In fact, the bombing of the Pearl Harbour was a part of the war of attrition between ~~the~~ ~~U.S.~~ Japan and the U.S. over the Markets of the East. ~~[illegible]~~

~~Dep~~ Expansion of the Technological Base

Having thrived on the technology ~~development~~ developed essentially in the West, Japan has now expanded its own technological base phenomenally — from Optics to Supercomputer. They have now a technological base which is an enemy if not an ~~eye sore~~ eye sore to the West.

Thus Japan moved from

Innovative Copiers → Technology Absorber → Technology Creators.

→ All determined by Market demands.

Expansion of Scientific Base:

Innovation of Technology has it own dynamism & it must eventually lead to technological research which in turn demands basic research.

Eventually Japan has expanded its basic research base to meet the demands of the innovating technology.

→ But of course, only the "relevant" basic research.

But the G.E. funds so generated provide enough space for the "blue-sky" (read "irrelevant" basic research)

Japan thus today is in a happy situation of being a leader in technology, a great economic power (among the G-7) and an ever expanding scientific base.

Contrast the case of India:

- We had no built-in motivation for exporting our products.
- We expanded our scientific base ~~enormously~~ without any reference to the
- Our industry was always more interested in making quick profits in a sellers market — This led to hackneyed technology being perpetrated through decade.
  - Remember the Ambassador Car & Fiat — Technology of the 40's & 50's still being dished out to the consumers
- We went on expanding our scientific base without any reference to application of technology.
- We set up (CSIR) Laboratories without any clear mandate or accountability. Any technology that did get developed had no buyers, because it was

not motivated by the demand of the industry. There was no relationship between the technology developed and technology needed at a given point of time.

- Our attitude to research has been "Brahmanical". "Applied Research" has belongs to lesser mortals - the lower stratum of scientists. Technology belongs to the "Blue Collar".
- No recognition for the Applied Physicists or Technologyes Innovators

Thus we have sought to build our Industrial Base in Reverse Order compared to the Japanese: But now determined by any free: Market or not any other incentive

Base Research → Applied Research →

Technology → Industry

Return to the Present Scenario

Now with the entry of the Multinationals what is the likely scenario?

There is a tie up between imported technology & Industry.

This threatens to leave our Basic as well as Applied Research in the cold.

~~There will be new~~

The pressure will be exerted & by the interests of the Multinationals and the local industrial partners.

Both the local and the global consumer will determine the direction of development in this so-called Market Economy

They

This character of development threatens to start short around the areas of Basic & Applied Research.

If that is so then in the long run we either become dependent on "Charity" -- that is often or lose our position altogether. or an ~~concept~~ underlying innovative force for the future technology.

This is bad news for the Country.

Do we then end up surrendering to the multi nationals & ~~their~~ foreign imported technology forever?

If the present government continues to encourage such a flow of events and our industrialist continue to suffer from myopia and blind because of their quick profit motivation, the MNC will only be too happy to continue to explit us to the hilt.

And we shall become slaves again in a different form — intellectually!!

What should be the reaction of scientists and Technologists together:

One ought to take a serious note of these portents:

- One must develop strong linkages with the industry and try to improve technologies in response to the industry's requirement.

~~Technology must develop linkages with~~

Unless one can show that you have a better technology than the imported one ~~you~~ ~~to~~ one cannot persuade the industry to

- Technology must develop strong linkages with Basic science where basic research must be carried out as determined by the demands of technology development.

- Finally we must shed our Brahmanical ~~attitude~~ mind set which endows more prestige to Blue sky

as against the Blue Collar research

& This had led to the Applied Researchers ~~being~~ wanting to be counted as Basic researchers.

In future, ~~Basic res~~ I do not see, basic research being regarded as a matter of right ~~in a~~ because of the Market system where the value of work is determined by its value in the market place. Basic research will be supported either as a matter of good will, or charity, or at best piggy back on technology.

None of the alternatives represents a happy augury for basic research. It ought to have its rightful place.

On the other hand, it must also be stated that we have had it rather good so far. But on the whole, country as a whole does not have too many spectacular things to show as their achievements

I think there is a lot of introspection ~~to be~~ that is ~~necessary~~ required to be done on the part of scientists.

Continuous flow of funds can no longer be taken for granted in the long run. More than that, it is the very status of the Community is at stake.

$$\frac{\partial f}{\partial t} + \underset{\sim}{v} \cdot \frac{\partial f}{\partial \underset{\sim}{x}} + \frac{L}{\tau} \frac{\partial f}{\partial \underset{\sim}{x}} = 0$$

Introduce a variable

$$\underset{\sim}{\Phi}' = \underset{\sim}{\Phi} - \int \underset{\sim}{v} \cdot d\underset{\sim}{x} + \int \frac{L}{\tau} dt$$

~~$f = f\left(\underset{\sim}{\Phi} - \int \underset{\sim}{v}\, dt - \int \frac{L}{\tau} dt\right)$~~

1. Introduction:

Ever since the advent of quantum mechanics the question of it and classical and quantum mechanics has been a longstanding issue which has defied a complete understanding. while quantum mechanics is certainly a more general theory the manner in which it goes over into classical mechanical limit and the conditions under which the latter is an appropriate description are not entirely clear. Another enigmatic point concerning the relationship between the two theories is that the more general quantum theory is known to be probabilistic while the classical mechanics which is a limiting theory is deterministic. The former is, moreover, regarded to be intrinsically probabilistic.

Quantum mechanics as we know, was constructed seventy years ago,

~~in two apparently different forms by Heisenberg and Schrödinger.~~
~~Almost seven Ever since these constructions, seven~~

Because of the probabilistic nature it has been speculated whether one could introduce a set of the so-called "hidden variables" to render the ~~pro~~ system deterministic.

In view of the above remarks one is led to ask the question whether it is possible to ~~found~~ give a ~~bridg~~ covering formalism which provides a somewhat more logical relationship between classical and quantum mechanics. ~~In the process,~~ it may be also be possible to ~~give a "derivation" of quantum formalism. This is~~ One also seeks in the process, to give a derivation of quantum formalism starting from some generalized classical framework. This is what ~~we~~ I propose to do in what follows.

A Generalized Schrödinger Formalism as a Hilbert Space Representation of a Generalized Liouville Equation

1. Introduction:

Quantum mechanics has since its advent revealed itself to be as enigmatic as it is novel. While it is supposed to be a more general theory, supposed to be containing classical mechanics as a limit it really is not just a generalization of the latter, but represents a fundamentally distinct theory and indeed a paradigm shift from the conceptual framework of classical mechanics.

Attempts have continued since the inception of QM to forge a conceptual unity and continuity between the two theories through a possible unifying formalism. The present author has been concerned with the development of such a formalism for a number of years. Clearly the most distinctive feature of the quantum mechanical description is the probability amplitude nature of its description and the interference effects resulting therefrom. This has no classical analogue, nor a classical limit.

~~There is obviously a close relationship between classical mechanics~~

While there is obviously a genetic relationship between classical and quantum mechanics there exist differences in their mathematical form and content which make them appear so distinct from each other. Classical mechanics for example, admits the whole continuum of ~~all~~ initial ~~values as allowed states~~ and the resulting dynamical states as "allowed states," which are propagated as allowed states by the equation of motion. Quantum mechanics ~~on the other hand~~, by contrast being a boundary value theory, ~~omits~~ permits, for a bounded system, only a discrete set of states of motion determined by the boundary conditions. Quantum mechanical states are thus determined by a set of global constraints (boundary conditions), while there are no such constraints for a classical mechanical system: A classical particle is not affected by a distant boundary, and ~~describes~~ represents

~~The question that presents itself is~~

~~One may ask the question whether quantum mechanics may be regarded as a~~

only a local evolution. One may therefore, ask the question whether classical and quantum mechanics may be regarded as two different manifestations of the same underlying dynamics, described by two different mathematical representations: global vs local.

2. ~~The Hamilton – Jacobi formalism~~:

~~As is well known the H – J formalism of classical mechanics bears a close relationship to with the quantum formalism mechanics à la Schrödinger.~~

There are two crucial facts about QM that need to be recalled. One, that it bears a close relationship with the Hamilton – Jacobi formalism of classical mechanics. The second one is that it is a probabilistic theory and ~~as~~ ought to have its origin in a probabilistic framework. In classical statistical mechanics, the probability arises, from a lack of knowledge about the initial data†. ~~In quantum mechanics on the other as well it has been~~ argued that the probability (may be considered to) arises because of a lack of knowledge about certain "hidden variables" which when specified could make an event deterministic

† While the orthodox (Copenhagen) point of view proclaims that quantum mechanics is intrinsically probabilistic, it has also been

We shall make two stipulations. One, that the probability in QM is has the same origin as in classical statistical mechanics. ~~(and is no~~ Second that the quantum events can be described by a classical Liouville equation, but in a suitably generalized form. To see the manner in which it is generalized we consider ~~here~~ in Sec 2 the Hamilton-Jacobi formalism as the theory of first order partial differential equation.. In Sec. 3 we give the generalized ~~equation~~ Liouville equation and in Sec 4 a generalized Schrödinger theory is obtained as a Hilbert space representation of the former. It will be seen that this formalism covers both classical and quantum mechanics ~~in a~~ suitably.

A fascinating consequence of this formalism namely obtaining Schrödinger theory as a Hilbert space representation of the classical (generalized) equation is that this may be applied to a suitably defined classical mechanical system as well. This means that if the particular classical mechanical system would be predicted to exhibit ~~quantum like~~ properties*. This would

* characteristic of a probability amplitude description.

1. R K Varma Phys. Rev Lett. 26, 417 (1971).
2. " " Phys. Rev A 31, 3951 (1985)

appear to be quite heretical. The interesting fact is that such a classical mechanical system had indeed been identified by the present author more than two decades ago and a set of Schrödinger-like equations were obtained then (through a heuristic derivation)[1]. Its quantum-like observable consequences have indeed been observed over the years. Some of these observed effects will be presented in Sec. . In some sense these results are more fascinating than the derivation of the quantum mechanic Schrödinger equation, because they are something entirely new and unexpected and, in fact, cannot be understood in the framework of the standard classical mechanical paradigm: if the latter is indeed the case as it seems to be, then we have an entirely new physics in the classical mechanical domain of parameters: What it means that the standard classical mechanical paradigm need to be supplemented by constraints delimiting the allowed values of the initial conditions.

→

* Later they were derived as a Hilbert space representation of the Liouville equation for the system. (Ref. 1975)

It may be emphasized here that the predictions and subsequent observations of a set of entirely new phenomena which cannot be understood even post-facto in terms of the standard classical mechanical paradigm in whose domain they fall, do testify to the physical significance and reality of the Schrödinger-like description which led to these predictions. This may also serve as a validation of the procedure for obtaining the quantum mechanic Schrödinger formalism as a Hilbert space representation of a generalized Liouville equation which has been presented here and elsewhere earlier [3,4].

3. R.K Varma Had. Jr Supp. 1, 437 (1985)
4. R K Varma in Microphysical Reality and Quantum Formalism A van der Merwe (eds), p. 437 (1988) Kluwer Academic Publishers p. 175 (1988).
5. R. Courant and D. Hilbert. Methods of Mathematical Physics Vol II p. 107. John Wiley.

2. The Hamilton-Jacobi formalism and a Generalized Liouville Equation in an extended phase space.

The standard manner of arriving at the Hamilton-Jacobi formalism given in most standard text books on classical mechanics, is via the canonical transformation. A generating function S ~~defining a canonical transformation~~ which makes the new (canonically transformed) Hamiltonian vanish is essentially the Hamilton principal function and the defining equation for the new Hamiltonian is the Hamilton-Jacobi equation.

Following Courant and Hilbert[5] (for example) we introduce ~~at first~~ the Hamilton-Jacobi equation as a first order partial differential equation, whose characteristics are the Hamiltons equations of motion. Thus consider a partial differential equation of the form:

$$F(x_1, x_2 \cdots x_N, t, S; p_1, p_2 \cdots p_N, p_t)$$
$$\equiv p_t + H(x_1, x_2 \cdots x_N; t; p_1, p_2 \cdots p_N) = 0 \qquad 1.$$

where

$$p_i = \frac{\partial S}{\partial x_i}, \qquad p_t = \frac{\partial S}{\partial t} \qquad 2.$$

and when t corresponds to the H-J equation, F has been taken to be independent of S, and the dependence on p_t to be linear.

With the above (first order) partial differential equation $F=0$, one associates the following system of ordinary differential equations for the $(2N+1)$ functions $(x_i, p_i = \partial S/\partial x_i, S)$ of a parameter τ.

$$\frac{dx_i}{d\tau} = \frac{\partial F}{\partial p_i} = \frac{\partial H}{\partial p_i}, \quad \frac{dt}{d\tau} = \frac{\partial F}{\partial p_t} = 1. \tag{2a}$$

$$\frac{dp_i}{d\tau} = -\frac{\partial F}{\partial x_i} - p_i \frac{\partial F}{\partial S} = -\frac{\partial H}{\partial x_i}. \tag{2b}$$

$$\frac{dS}{d\tau} = \sum_i p_i \frac{\partial F}{\partial p_i} + p_t \frac{\partial F}{\partial p_t}$$

$$= \sum p_i \frac{\partial H}{\partial p_i} - H = L. \tag{2c}$$

~~This system~~ is where L is the Lagrangian.

This system is referred to as the system of "characteristic differential equations" belonging to the partial differential equation (1). Using the equation $dt/d\tau = 1$, the two sets of these equations (2a) and (2b) will be recognized as the Hamilton equations with H as the Hamiltonian

function, while (2c) gives the time rate of change of the ~~Lagrangian~~ action S as the Lagrangian on the right. The partial differential equation (the H-J equation) and the characteristic equations are thus equivalent to each other. It is this equivalence which was exploited by Jacobi to formulate his theory.

There is an advantage in the above manner of introduction of the H.-J. equation: It leads naturally to a complete set of characteristic equations: In addition to the standard Hamilton equations (2a) and (2b), one also finds Eqn. (2c), as one of the characteristic equations. This provides a basis for generalizing augmenting the phase space of a dynamical system, and writing down a generalized Liouville equation. As we shall see later, a Hilbert space representation of this generalized Liouville equation yields a generalized Schrödinger theory.

3. The set of "characteristic differential equations" (2a, b, c) belonging to the quantities (x_i, p_i, S) define trajectories in the space of the $(2N+1)$ variables. One can

then construct a $(2N+1)$ dimensional phase space corresponding to these variables and a probability (phase) function f defined over this phase space. (Usually one defines a phase space of the $2N$ variables (x_i, p_i). But we include here also the variable S to correspond to the $(2N+1)$ characteristic equations (2a, b, c)). The "equation of continuity" for $f(x_i, p_i, S)$ can then be written down as:

$$\frac{\partial f}{\partial t} + \sum_i \frac{\partial}{\partial x_i}(\dot{x}_i f) + \sum_i \frac{\partial}{\partial p_i}(\dot{p}_i f) + \frac{\partial}{\partial S}(\dot{S} f) = 0 \qquad (3)$$

Using the expressions for (2a) and (2b) for $\dot{x}_i$ and $\dot{p}_i$ and the fact that $\dot{S} = L$ is independent of S, so that $\partial \dot{S}/\partial S = 0$, Eq (3) becomes:

$$\frac{\partial f}{\partial t} + \sum \frac{\partial H}{\partial p_i}\frac{\partial f}{\partial x_i} + \sum \frac{\partial H}{\partial x_i}\frac{\partial f}{\partial p_i} + L\frac{\partial f}{\partial S} = 0 \qquad (4)$$

This is a generalization ~~in the extension~~ of the classical Liouville equation, in the extended phase space of (x_i, p_i, S) ~~to include~~ resulting in the inclusion of the last term, $L\,\partial f/\partial S$.

This, as usual, describes the conservation of the probability measure along the characteristics described by (2a, b, c).

6. PAM Dirac Zeit. für Physik der Sowjetunion
3, 63 (1933)

The underlying dynamics is of course, the classical Hamiltonian flow. Since all the "flow" components ~~are~~ $(\partial H/\partial p_i \; , \; -\partial H/\partial x_i)$ are independent of S, an integration with respect to S yields the standard classical Liouville equation:

$$\frac{\partial \bar{f}}{\partial t} + \frac{\partial H}{\partial p_i}\frac{\partial \bar{f}}{\partial x_i} - \frac{\partial H}{\partial x_i}\frac{\partial \bar{f}}{\partial p_i} = 0 \qquad (5)$$

with

$$\bar{f} = \int ds\, f \; . \qquad (6)$$

Classical analogue of the state of motion in quantum mechanics.

Dirac, ~~more than any~~ it will be ~~noted~~ noticed as the readers would already know, was ~~constantly~~ quite ~~concerned~~ occupied with the question of the nature of relationship between classical and quantum mechanics, and with the question of classical analogues of quantum objects. In his 1933 paper[6] for example he ~~also~~ very beautifully ~~elucidated~~ elucidated the role of classical Lagrangian in quantum mechanics ~~and~~ which later

7. R. P. Feynman, Rev. Mod. Phys. 20, 367 (1948).
8. P.A.M. Dirac, Canadian J. of Math. 3, 1, (1951).

culminated ~~into~~ into the path integral formulation at the hands of Feynman[7]. Another important remark made by Dirac[8] was in ~~connection~~ relation to the significance of the Hamilton principal function as a solution of the Hamilton-Jacobi equation. A principal function, ~~according to Dirac~~, satisfying the Hamilton-Jacobi equation, defines a "family". ~~§~~ "The family does not have any significance from the point of view of Newtonian mechanics; but it is a family which corresponds to one state of motion in the quantum ~~mechanics~~ theory, & so presumably the family has some deep significance in nature, not yet properly understood". (quoted from Dirac[8]).

~~For following the above~~

Coming to the ~~question~~ of generalized Liouville equation (4) the Liouville density function f can be any function of x_i, p_i and S, which solves (4). In my earlier ~~derivation~~ papers, however, f was chosen to be a δ-function in all the initial momenta including the ~~global~~ global integrals of motion. ~~If~~

This was done to make the distribution f correspond closely to a quantum state where the momenta (in terms of which the distribution is a δ-function) represent ~~controllable~~ integrals of motion with respect to which the state is prepared. I was not aware of Dirac's remarks then, but it turns out that such a distribution represents what Dirac has called a "family," ~~which in turn corresponds to~~ defined by a principal function, and which according to him corresponds to one state of motion in the quantum theory.

In accordance with the above considerations f is regarded as a function of the initial momenta values α_i, rather than of the current momenta p_i. Eq.(4) is then transformed accordingly (from the current momenta p_i to the initial momenta α_i). This yields,

$$\frac{\partial \hat{f}}{\partial t} + v_i \frac{\partial \hat{f}}{\partial x_i} + \frac{L}{\eta} \frac{\partial \hat{f}}{\partial \Phi} = 0 \qquad (7)$$

where now $\hat{f} = \hat{f}(x_i, \Phi, t; \alpha_i)$ and v_i is not to be regarded as independent of the x_i, but as a function of the x_i and the α_i, while the α_i appear as parameters in the argument of $\hat{f}$. Here, we have also

introduced a constant η of the dimensions of action to non-dimensionalize the action S, so that

$$S = \eta \Phi \,, \qquad (8)$$

or to measure S in units of η. Φ is then a dimensionless quantity which may be termed as "action-phase". The meaning of $\hat{f}$ is then also analogous to the meaning of probability in quantum mechanics: $\hat{f}$ gives the probability of finding the system at x_i at time t with a given value of S, if it had initially the momenta α_i.

A Hilbert-Space Representation

Eq (7) with the definition (8) and the underlying characteristic equations & (2a,b,c), it may be recalled, are essentially the governing equations representing classical dynamics. Though the Liouville equation has been generalized to include a term corresponding to the action variable S. It may be recalled that Eq (5) for Φ & $\hat{f}$ the action-averaged distribution is essentially the classical Liouville equation.

9. ~~R K Varma Hadronic Tr.~~ Supp 1,

One seeks a Hilbert space representation of Eqn (7) and writes accordingly:

$$\hat{f} = \psi^2 . \tag{9}$$

where ψ is a real quantity. (One could let ψ be complex and write $\hat{f} = \psi^* \psi$, but there is no loss of generality if ψ is taken to be real). Next a finite time representation of Eq (7) is written as:

$$\hat{f}\left(x_i, \Phi, t; \alpha_i\right) = \hat{f}\left[x_i - \int_{t'}^{t} v_i\left(x_i(t''), \alpha_i\right) dt'' \right.$$

$$\left. \Phi - \frac{1}{\eta} \int_{t'}^{t} L\left(x_i(t''), v_i(t''), t'; \alpha_i\right) \right] . \tag{10}$$

Using (9) and taking the square root of (10), choosing the positive sign at all space-time points:

$$\psi\left(x_i, \Phi, t; \alpha_i\right) = \psi\left(x_i - \int_{t'}^{t} v_i(t'')\, dt'', \Phi - \frac{1}{\eta} \int_{t'}^{t} L\, dt'', t', \alpha_i \right) \tag{11}$$

I shall not repeat the procedure given elsewhere[3,4] which yields, starting from (11), a generalized set

of Schrödinger equations

$$\frac{i\hbar}{n}\frac{\partial \Psi(n)}{\partial t} = -\left(\frac{\hbar}{n}\right)^2 \frac{1}{2m}\nabla^2 \Psi(n) + V\Psi(n). \qquad (12)$$

with the total probability density

$$G(x,t) = \sum_{n=1,2,3,\ldots} \Psi^*(n)\Psi(n) \qquad (13)$$

It is, however, pertinent to point out some crucial steps, which ~~lead to the above set of Schrödinger equations~~, represent a point of departure from classical mechanics and which lead to the above set of Schrödinger equations.

The first is clearly the generalization of the Liouville equation to include the action S as a ~~dynamical~~ variable. However, as the action-averaged Liouville equation (5) shows (which is the standard classical Liouville equation) shows, the classical mechanics is clearly contained in (4).

The second step is the Eq (9) writing $f = \Psi^2$, seeking a Hilbert space representation of Eq (7). While it may be an unusual proposition to seek a Hilbert space representation of (7), clearly this

by itself could not make it depart in its content from that of classical mechanics, for one could in principle solve for ψ and reconstruct $f = \psi^2$ and action-average it to obtain $\bar{f}$ the classical distribution $\bar{f}$.

The third step is the assumption of ψ being periodic in $\xi = S/\eta$ with the period 2π. It is this assumption (where η is identified with $\hbar$) that enables ψ to be Fourier decomposed as a series (rather than an integral) and the subsequent procedure to be followed leading up to the derivation of the Schrödinger equations which are second order in space. One should follow the ~~closely~~ derivation closely to see how the second order operator ∇^2 arises. It is clear that this operator could not have arisen if the action S were not included as a variable.

The important consequence of this structure (second order differential operator) is that the functions $\Psi(n)$ of Eq (12) ~~are now~~ have now to be specified through appropriate boundary conditions.

† ~~What needs to~~ It ought to be pointed out that this particular derivation yields ~~has~~ a generalized formalism in the form of an infinite set of Schrödinger equations (12) with $(\hbar/n)$, $n=1, 2, 3 \ldots$ in the place of $\hbar$ and a correspondingly generalized probability current (13). Some of the direct observable consequences of these ~~equations~~ additional modes $n=2, 3 \ldots$ have been discussed in Ref 4. They have not been so far identified nor has there been any attempt made to check their existence experimentally.

This amounts to, apart from other consequences, to delimiting the allowed values of certain integrals of motion to a discrete set. - that is to "quantization".

↑ ‡ → That the limit $\hbar \to 0$ of the Schrödinger equation yields classical mechanics via the WKB approximation is well known. One obtains the Hamilton-Jacobi equation of classical mechanics. It is, however, instructive to see what happens to the generalized equation in the limit $\hbar \to 0$. Introducing $\Phi = S/\hbar$ in (4) one obtains:

$$\frac{\partial f}{\partial t} + \sum_i \frac{\partial H}{\partial p_i}\frac{\partial f}{\partial x_i} - \sum_i \frac{\partial H}{\partial x_i}\frac{\partial f}{\partial p_i} + \frac{L}{\hbar}\frac{\partial f}{\partial \Phi} = 0.$$

One may now consider an asymptotic expansion of f in the form:

$$f = \sum \hbar^n f^{(n)}$$

Define a variable

$$\Phi = \int p \cdot dx$$

This gives to the lowest order:

$$\frac{L}{\hbar}\frac{\partial f^0}{\partial \Phi} = 0.$$

Define

$$\Phi' = \left(\Phi - \int p \cdot dx/\hbar\right)$$

$$\therefore \frac{\partial f}{\partial x} \cdot \frac{\partial \Phi}{\partial \Phi} \cdot \frac{\partial \Phi'}{\partial x}$$

~~which means that $f^{(0)}$ be independent of Φ. In the next order one obtains.~~

$$\cancel{\frac{\partial f^{(0)}}{\partial t} + \sum_i \frac{\partial H}{\partial p_i}\frac{\partial f^{(0)}}{\partial x_i} - \sum \frac{\partial H}{\partial x_i}\frac{\partial f^{(0)}}{\partial p_i} + \frac{L}{\hbar}\frac{\partial f^{(1)}}{\partial \Phi} = 0}$$

L (analogous to $\hbar$)

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3. A Classical Mechanical system with the Schrödinger-like description:

There is nothing special in the procedure given above (where we obtained the Schrödinger-like description ~~star~~ as a Hilbert space representation of the Liouville equation) which could not be applied to any suitable classical mechanical system. As pointed out above perhaps the most important step where the quantization really came in was the assumption of periodicity of ψ with respect to Φ ($\equiv S/\hbar$) with the period 2π. What is of course required for the classical mechanical system* is ~~an appropriate action in the system which serves as a unit (analogous to $\hbar$)~~.

Such a classical mechanical system was identified by the present author more than two

* the justification of ~~the assumption of~~ periodicity with respect to the action phase and an appropriate unit of action L belonging to the system in terms of which to define the latter

decades back[1]. The dynamical system of charged particles in a magnetic field is such a system. The action in the system which can serve as a unit is the gyroaction $\mu = \frac{1}{2} m v_\perp^2 / \Omega$ where $v_\perp$ is the component of velocity perpendicular to the magnetic field at the point of injection and $\Omega = eB/mc$ is the gyro-frequency also at the point of injection so that μ is essentially an initial value which is an exact constant of motion.

The Liouville equation for the system is given by

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \Omega \frac{\partial f}{\partial \phi} = 0 \qquad (14)$$

where ϕ is the gyro-phase.

Transforming the equation to momenta initial values of the total energy $\mathcal{E}$, the canonical angular momentum P_ϕ, and the gyro-action μ, as well as transforming ϕ to the action phase

$$\Phi = \frac{1}{\mu} \int \tfrac{1}{2} m v_\parallel^2 \, dt + \phi$$

$$= \frac{1}{\mu} \int_0^t dt \left(\tfrac{1}{2} m v_\parallel^2 - \mu \Omega \right) = S_A / \mu , \qquad (15)$$

where S_A is the time integral of the adiabatic

* The periodicity of $\psi = \sqrt{f}$ with respect to Φ which is crucial for obtaining the Schrödinger-like equation (17) is guaranteed through the periodicity with respect to ϕ (the gyro-phase) which is an additive part of Φ. as per Eq(15).

Lagrangian $L_A = \left(\frac{1}{2} m v_{\parallel}^2 - \mu \Omega\right)$ one obtains

$$\frac{\partial f}{\partial t} + v_{\parallel} \frac{\partial f}{\partial x_{\parallel}} + \frac{L}{\mu} \frac{\partial f}{\partial \xi} = 0 \qquad (16)$$

This equation has a structure similar to that of Eq (7) except that μ takes the place of η (later identified as h). A Hilbert space representation of this equation, which can be sought without any prejudice to QM, yields in a similar manner the following set of Schrödinger like equations:

$$i\hbar \frac{\partial \hat{\Psi}(n)}{\partial t} = -\frac{(\hbar/n)^2}{2m} \frac{\partial^2 \hat{\Psi}(n)}{\partial x_{\parallel}^2} + (\mu \Omega)\hat{\Psi}(n), \qquad (17)$$

$n = 1, 2, 3 \ldots$

again with the total probability density

$$G(x,t) = \sum_n \hat{\Psi}^*(n)\hat{\Psi}(n) \qquad (18)$$

It may be mentioned that $(\mu\Omega)$ which appears in the place of "potential" is in fact the adiabatic potential which describes the effective parallel motion along the magnetic field through the equation of motion (as in the adiabatic approximation).

$$m \frac{dv_{\parallel}}{dt} = -\nabla_{\parallel}(\mu\Omega) \qquad (19)$$

9 (a) D. Bora et al Phys. Lett

(b) D. Bora et al Plasma Physics.

(c) D. Bora et al Phys. Fluids.

9 J. L. Synge. in Principles of Mechanics and Field Theory

Handbuch der Physik Vol III/1 Springer-Verlag

(Berlin, Gottingen, Heidelberg) p. 121.

A. Existence of multiplicity of residence times in an adiabatic trap.

8. Quantum-like Consequences of this description for the classical mechanical system:

Even though the above mentioned system of charged particles in a magnetic is a classical mechanical one, with macroscopic dimensions, the above mentioned amplitude description governed by the Schrödinger-like equations entails that this system exhibits quantum-like properties for the ensemble of trajectories (- the coherent set of trajectories, à la Synge[9] or a "family" à la Dirac[8]), for which these equations are obtained. Whether this is indeed the case can be ascertained only through appropriate experimentation.

Two kinds of predictions were identified for experimental verification. One relates to the residence times of charged particles trapped in an adiabatic potential well as described by the adiabatic equation of motion (19). Charged particles can be confined in a potential well if the potential $\mu\Omega$, if the total energy $\mathcal{E}$ of the particle is $\mathcal{E} < (\mu\Omega)_{max}$. But the trapping is not for ever, because the equation of motion (19), which describes this trapping

† Leaving aside for the moment the question of the interpretation ~~vis a vis the quantum mechanic~~ of the amplitude functions $\hat{\Psi}(n)$ vis a vis the quantum mechanic ~~wave~~ Schrödinger wave functions,

‡ This as already pointed ~~out~~ out, is also true of the generalized Schrödinger formalism for quantum mechanics as contained in the set of equations (12) and (13).

only an approximate equation. Particles so trapped have indeed been found experimentally to have leaked out of the trap, with characteristic residence times. No satisfactory theory for the determination of these residence life-times existed when these experimental results were reported during 1968-69. The Schrödinger-like equations (17), along with (18) which were first given by the author in 1971, afford a possible description of the leakage of particles through a quantum-tunnelling like process. The possibility of determining the life times afforded

† What is different here from the standard quantum formalism is that we have here not one but a set of equations for the $\hat{\Psi}(n)$ with $n = 1, 2, 3 \ldots$

‡ Correspondingly, one predicts here the existence of a multiplicity of residence times as determined by the equations corresponding to $n = 1, 2, 3, \ldots$, for the injection of particles with the same energy and pitch angle. It should perhaps be pointed out that no

‡ The experimental observations of these modes $n=1, 2, 3$, and consequent validation of the ~~Schrödinger procedure leading to~~ the generalized Schrödinger like formalism and the procedure leading to it, ~~makes one~~ allows me to believe that the additional mode $n \neq 1$, may also have observational consequences.

10 D. Bora, P. I. John, V. C. Saxena and R. K. Varma
(a) Phys. Lett A 75 60, (1979)
(b) Plasma Phys. 22, 653 (1980)
(c) Phys. & Fluids 25, 2284 (1982)