Quimby & Siegel Phys Rev. 54, 293 (1938)

The fundamental frequency, f, of free longitudinal ~~and~~ or torsional vibration of a cylinder of a cubic crystal, is ~~are~~ given by

$$2fl = (M/\rho)^{1/2}, \qquad \text{--- (1)}$$

where l = length of cyl.r along axis of cyl.r, ρ density, M elastic modulus [Young's E or torsional G, as the case may be]

--- (2)

$$1/E = s_{11} - 2s\lambda \qquad \text{--- (3)}$$

$$s = s_{11} - s_{12} - \tfrac{1}{2}s_{44}$$

$$\lambda = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\delta^2 \qquad \text{--- (4)}$$

$\alpha\ \beta\ \gamma$ direction cosines of the axis of cylr with ~~ref cubic~~ reference to the cubic axes of the crystal [100]

$$1/G = s_{44} + 4s\lambda \qquad \text{--- (5)}$$

if bending is not prevented, whereas if bending is prevented

$$1/G = s_{44} + 4s\lambda - 2s^2(\lambda - 4\lambda^2 + 3\alpha^2\beta^2\gamma^2)/(s_{11} - 2s\lambda) \qquad \text{--- (6)}$$

For high frequencies ~~¢~~ (6) probably holds instead of (5).

The elastic moduli s's are related to the elastic constants c's by the ~~rel~~ formulae

$$c_{11} = (s_{11} + s_{12}) / (s_{11} - s_{12})(s_{11} + 2s_{12})$$

$$c_{12} = -s_{12} / (s_{11} - s_{12})(s_{11} + 2s_{12})$$

$$c_{44} = 1/s_{44}.$$

P.T.O

The Compressibility ~~of~~ K is given by

$$K = 3(s_{11} + 2s_{12})$$
$$= 3/(c_{11} + 2c_{12}).$$

The isothermal & adiabatic constants are related as follows

$$s_{11a} - s_{11i} = s_{12a} - s_{12i} = -T\Delta^2/\rho C_p$$
$$c_{11a} - c_{11i} = c_{12a} - c_{12i} = T\Delta^2/(\rho C_p)(s_{11} + 2s_{12})^2$$

T = Abs temp, ~~α~~ Δ coeff of linear thermal expansion

Adiabatic and isothermal values of c_{44} are same; so also of s_{44}.

In cubic crystals the elastic anisotropy is given by

$$\epsilon = c_{11} - c_{12} - 2c_{44}$$

The velocities of propagation of the longitudinal waves, when the wave-vector $\vec{K}$ is along some of the principal directions of the cubic crystal are as follows:-

Direction of $\vec{K}$	[100]	[110]	[111]
ρV_L^2	c_{11}	$c_{11} - \epsilon/2$	$c_{11} - 2\epsilon/3$

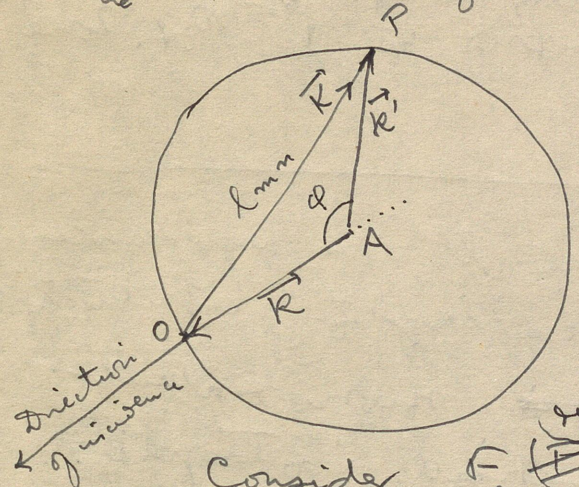
The Principal Electrical resistivities of graphite

1. Phys. Rev. 71 622 (1947) and 72 258 (1947)
P. R. Wallace, Natl Research Council of Canada, Chalk River Lab., Chalk River, Ontario.

2. J. Gibson Structure of G. Nature, 158 752 (1946)

3. Gibson + others: Amorphous C JCPS (1946) 456

Is the attenuation coefficient μ of a cubic crystal for long waves, due to scattering, independent of the direction of incidence? [For these long waves $e^{-2M} \simeq 1$; S_α is indept of α]



Let AQ be the direction of incidence,
AP of scattering, in a monatomic crystal.
Let lmn be the direction cosines of $\vec{K}$ with reference to the cubic axes of the crystal.

Consider F_{lmn} (defined by (1) (see attached sheets)), a scalar, and a function of lmn alone. Let α be the angle, and θ the azimuthal angle, of scattering.

Is the value of F_{lmn} averaged over all positions of P on the surface of the sphere of propagation independent of the direction of $\vec{K}$; i.e. indept of the direction of the diameter through O?

For $[\bar{1}00]$ direction of incidence the computation can be done numerically; and the value known ($\bar{F} \approx 0.72/c_{11}$).
Await Vij's reply, and see if numerical computation can be undertaken for other directions of incidence.

Instead of averaging F_{lmn}, if we average $F_{lmn}{}^*(1-\cos\alpha)$, over the sphere of propagation, will this average be independent of the direction of incidence?
In other words will the mean free path of long elastic waves ~~incident along di~~ be indept of direction of incidence in a cubic crystal?

For incidence along $[\bar{1}00]$ $\bar{F}(1-\cos\alpha) = 0.7$...

* In an elastically isotropic crystal, since F_{lmn} is indept of l, m, n ($= 1/c_{11}$), both μ and l will be independent of the direction of incidence.

For short* waves, of course, both μ, the attenuation coefft, & l the free path, are known to be fns of direction of incidence. This will be so even in an elastically isotropic crystal.

* i.e. when P extends beyond the 1st Br. Zone, in a monatomic crystal.

Calculate mean free path l for incidence of electrons of Fermi wave-length in the special case when the ~~crystal is~~ monovalent metal crystal is elastically isotropic. i.e. $\epsilon = 0$, F_{lmn} is indept of lmn and equal to $1/c_{11}$. [This will be roughly the case when the temp. is close to m. pt of the crystal. * c_{44} tends to zero, and c_{12} tends to approach c_{11}; ~~= 1/β~~. so that at m.pt we may take $c_{44} \approx 0$, $c_{12} \approx c_{11} \approx 1/\beta$.

This calculation will be needed for the paper on change of ϱ of alkali metals on melting.

Regarding the series $\sum_{n=-\infty}^{+\infty} \frac{\sin^2(n\alpha+\theta)}{(n\alpha+\theta)^2} = \pi/\alpha$

$0 < \alpha \leq \pi$, n is an integer & θ a const.; is Wiener's proof given above, of wider application? i.e. to fns for which the Fourier Transforms are different from zero over only a finite range.

See also
1) Ramanujan's collected papers, nos 22 + 27
2) Titchmarsh F. Integr. Chap 7, p. 177
3) P. Roy Soc. Edin $\underline{\underline{51}}$ 116–126.

For an extensive list of Fourier Transforms see G. A. Campbell & R. M. Foster "Fourier Integrals for practical applications" Bell Tel. Sys. Tech. Pub. Monograph B-584, 1931 [F.T. are of importance in analysis of elec. impulses, in quantum mechanics in transformation from coord. to momentum space] See also Karman + Biot Math. methods in Engineering.

A photoelectric Fourier transformer By Born, Furth + Pringle, Nature $\underline{156}$ 756 (1945) They promised a longer paper. where was it published?

$$\sum_{n=-\infty}^{+\infty} \frac{\sin(n\alpha+\theta)}{n\alpha+\theta} = \frac{\pi}{\alpha}, \quad 0 < \alpha \leq 2\pi$$ [as against the upper limit of π in the previous case]

Let $f(u) = \sin(u+\theta)/(u+\theta)$.

Its Fourier transform is

$$g(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{\sin(u+\theta)}{u+\theta} e^{iuv}\, du$$

$$= \frac{1}{\sqrt{2\pi}} \times \pi e^{i\theta v} \quad \text{if } -1 \leq v \leq +1$$

and $= 0$ otherwise [i.e. if $v < -1$ or $> +1$]

$$\therefore \sum_{n=-\infty}^{+\infty} f(u) = \sum_{n=-\infty}^{+\infty} \frac{\sqrt{2\pi}}{\alpha}\, g\left(\frac{2\pi n}{\alpha}\right) = \frac{\sqrt{2\pi}}{\alpha} \times \text{~~[illegible]~~}\, g(0)$$

$$= \frac{\sqrt{2\pi}}{\alpha} \times \sqrt{\frac{\pi}{2}} = \frac{\pi}{\alpha}$$

and is indept of θ.

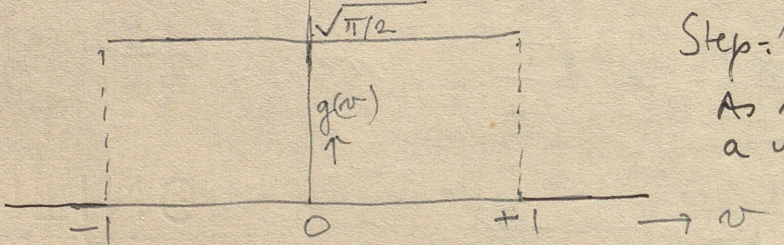
$$\sum_{-\infty}^{+\infty} \frac{\text{~~sin~~}\ 1}{(n\pi+\theta)^2} = \operatorname{cosec}^2\theta$$ is a known series.

Hence for both the functions $y = \frac{\sin^2 x}{x^2}$, and $y = \frac{\sin x}{x}$, the area subtended between the curve and the x-axis can be obtained by simple quadrature: i.e. by taking ordinates at equal intervals α [$\leq \pi$, $\leq 2\pi$ resp'y for the two cases], and multiplying by the interval α.

1) Are there other fns of this type?

2) For the above fns study in detail ~~the~~ i.e. term by term, the contribution from successive intervals α, to the area.

Fourier transform of $f(u) = \frac{\sin u}{u}$ is given by $g(v)$



Step-fn. Let $f = \frac{\sin kl}{kl}$. As $l \to 0$ $g \to \infty$. It is then called a unit impulse fn* or a δ-fn. [$f=1$]

* of Heaviside?

for $\frac{\sin(u+\theta)}{u+\theta}$, the ordinate has to be multiplied by $e^{i\theta v}$, which at $v=0$ is independent of θ

The F transform of (Titchmarsh p. 177)

$$\frac{1}{\sqrt{2\pi}}\left\{\frac{\sin a(1-x)}{1-x} + \frac{\sin a(1+x)}{1+x}\right\} = \cos x\ (0,a),\ 0\ (a,\infty)$$

$$\frac{1}{\sqrt{2\pi}}\left\{\cancel{\sin}\ \frac{1-\cos a(1-x)}{1-x} + \frac{1-\cos a(1+x)}{1+x}\right\} = \sin x\ (0,\alpha),\ 0\ (a,\infty)$$

Considering the series [See Bromwich 1931 edn p. 371]

$$\sum_{-\infty}^{+\infty} \frac{\sin (n-a)\theta}{n-a} = \pi \quad , \quad 0 < \theta < 2\pi$$

are there any limits or restrictions on the value of a? In particular, is a = 0 permitted? Yes.

NO

If so $\sum_{-\infty}^{+\infty} \frac{\sin n\theta}{n} = \pi$, ~~where it should be $= \pi - \theta$.~~ (See p. 369)

~~How do we reconcile the two?~~

Since $\frac{\sin n\theta}{n} \to \theta$ as $n \to 0$ $\quad \sum_{1}^{\infty} \frac{\sin n\theta}{\theta} = \frac{1}{2}(\pi - \theta)$

In the series

$$\sin\theta + \frac{\sin 2\theta}{2} + \frac{\sin 3\theta}{3} + \cdots = \frac{1}{2}(\pi - \theta), \quad 0<\theta<2\pi$$

according to Br. p. 356 the sum of the odd terms $= \frac{\pi}{4}$ if $0 < \theta < \pi$
& $-\frac{\pi}{4}$ if $\pi < \theta < 2\pi$

and of the even terms $= \frac{\pi}{4} - \frac{\theta}{2}$ if $0 < \theta < \pi$.

Is this correct? $= \frac{3\pi}{4} - \frac{\theta}{2}$ if $\pi < \theta < 2\pi$

See Edwards II p. 212

$$\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2} \; ; \quad \int_0^\infty \frac{\sin^2 x}{x^2}dx = \frac{\pi}{2} \; ; \quad \int_0^\infty \frac{\sin^3 x}{x^3}dx = \frac{3\pi}{8} \; ; \quad \int_0^\infty \frac{\sin^4 x}{x^4}dx = \frac{\pi}{3}$$

$0<\alpha<2\pi \qquad 0<\alpha<\pi \qquad 0<\alpha<\frac{2\pi}{3} \checkmark \qquad 0<\alpha<\frac{\pi}{2}?$

1) Can the latter two ~~series~~ be replaced by a series

$$\sum_{-\infty}^{+\infty} \frac{\sin^3 n\alpha}{(n\alpha)^3} \; \& \; \sum_{-\infty}^{+\infty} \frac{\sin^4 n\alpha}{(n\alpha)^4}$$ respectively? Yes

2) In that case can $n\alpha$ be replaced by $n\alpha + \theta$?

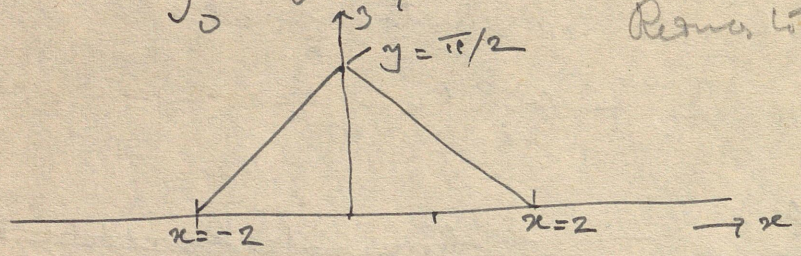
For the purposes of Poisson's ~~theor~~ formula Titchmarsh p. 60, in $g(v) = \frac{1}{\sqrt{2\pi}}\int f(u)\, e^{iuv}\, du$ can we put $f(u) = \frac{\sin(u+\theta)}{(u+\theta)}$ without introducing it in the exponential term also? Yes.

Edwards: Int. Cal. II p. 202

$$y = \frac{1}{2}\int_0^\infty \frac{\sin\theta}{\theta^2}\left\{\sin(x+1)\theta - \sin(x-1)\theta\right\} d\theta$$

Reduces to $y = \int \frac{\sin^2\theta}{\theta^2} e^{ix\theta} d\theta$

F.T. of $\frac{\sin^2\theta}{\theta^2}$ =



$y = 0$ if $x \geq 2$, $x \leq -2$;

if ~~x is <~~ $-2 < x < 2$ $y = \frac{\pi}{4}(2-x)$

what will be its Four. transform?

If $F(z)$ is any fn of z wh can be expanded in a convergent series of + integral values of z

$$\int_{-\infty}^{+\infty} \frac{\sin^2 x}{x^2} F(\sin^2 x)\, dx = \int_0^\pi F(\sin^2 x)\, dx.$$

Wolstenholme's result — See Ed. II p. 207

Titchmarsh p. 177 Some simple examples are given of F transforms.

Edw. II p. 204

$$\int_0^\infty \frac{\sin^5 x}{x^5} dx = \frac{115}{384}\pi \qquad \int_0^\infty \frac{\sin^6 x}{x^6} = \frac{11\pi}{40}.$$

$$\int_0^\infty \frac{\sin^7 x}{x^7} dx = \frac{5887}{23040}\pi.$$

Courant-Hilbert I p65 (Poisson's summation-formula.)

$$\sum_{n=-\infty}^{+\infty} \varphi(2\pi n + t) = \frac{1}{2\pi} \sum_{\nu=-\infty}^{\infty} e^{i\nu t} \int_{-\infty}^{+\infty} \varphi(\tau) e^{-i\nu\tau} d\tau;$$

$0 \leq t < 2\pi$. (why shd t be restricted to this interval? – it does not actually restrict it)

corresponds to $\alpha = 2\pi$, $\beta = 1$.

It appears that the diffce between the Fourier transforms of $f(u)$ and $f(u+\theta)$ is that the latter has a multiplying factor $e^{i\nu\theta}$

~~i.e. $g(\nu)$~~ where $g(\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(u) e^{iu\nu} du$

We are adopting our previous notation, of Titchmarsh.
In Cour. Hil. $e^{iu\nu}$ is replaced by $e^{-iu\nu}$.

Titchmarsh p. 90. For a fixed t, the transform of $f(u+t)$ is

$$\lim_{a\to\infty} \frac{1}{\sqrt{2\pi}} \int_{-a}^{+a} f(u+t) e^{ixu} du$$

$$= \lim_{a\to\infty} \frac{e^{-ixt}}{\sqrt{2\pi}} \int_{-a+t}^{a+t} f(v) e^{ixv} dv.$$

Jeffreys p. 432

Ex 10. Find a soln of the integral eqn. $\int_0^\infty g(x) \cos \alpha x \, dx = f(\alpha)$
where $f(\alpha) = 1 - \alpha^2$ $(0 < \alpha < 1)$ & $= 0$ $(\alpha > 1)$

On the propagation of waves ~~in~~ through a stratified medium = Lord Rayl. Sc. Papers VI, 111.

$$\int_0^{\pi/2} \frac{\sin x}{x}\,dx = \pi/2 \qquad \int_0^{\pi} \frac{\sin x}{x}\,dx = 1.8519$$

$$\int_0^{\infty} = \frac{\pi}{2} \qquad \int_{\pi}^{\infty} = -\cdot 2811$$

Let $I_n = \int_{n\pi}^{\infty} \frac{\sin x}{x}\,dx$

$$= \frac{(-1)^n}{x}\left(1 - \frac{\lfloor 2}{x^2} + \frac{\lfloor 4}{x^4} - \ldots\right)$$

where ~~x~~ $x = n\pi$.

$I_0 = +\pi/2$
$= +1.5708$

$I_1 = -\cdot 2811$

$I_2 = +$

$I_3 = -\cdot 1040$

$I_4 = +\cdot 0786$

$I_5 = -\cdot 0631$

$I_6 = +\cdot 0528$

See Glaisher Phil Trans <u>160</u>, 387 (1870)

See Bromwich p. 339

The remainder is less than twice the following term in the ~~each~~ series.

For the values of $\int_0^{r\pi} \frac{\sin x}{x}\,dx$ Carslaw refers to Annals of Maths (2) <u>7</u> (1906) 129.

Some empirical results with the series

$$\sum_{n=-\infty}^{+\infty} \frac{\sin^m(n\alpha+\theta)}{(n\alpha+\theta)^m} = S_m \qquad I_m = \int \frac{\sin^m x}{x^m}\,dx$$

$m=1$ $\quad S_1 = \pi$, $= I_1$ $\quad$ $0<\alpha\le 2\pi$, θ any value. (check for $\alpha = 2\pi$)

$m=2$ $\quad S_2 = \pi$, $= I_2$ $\quad$ $0<\alpha\le\pi$

$m=3$ $\quad S_3 = 3\pi/4$ $\quad$ $0<\alpha\le \frac{2\pi}{3}$ & $I_3 = \frac{3\pi}{4}$.

when $\alpha = \frac{\pi}{2}$ $\quad S_3 = \cdot 749768\,\pi$ (up to $x = 720°$)

& $\theta = 0$ $\quad 2\pi/3$ $\quad \cdot 749705\,\pi$

$3\pi/4$ $\quad = \cdot 777385\,\pi \gg 3\pi/4$.

With $\alpha = 2\pi/3$ & $\theta = 10°$ $\quad = \cdot 749797\,\pi$.

$m=4$ $\quad \theta = 0$ $\quad \alpha = \pi/2$ $\quad S_4 = \cdot 666615\,\pi$ $\quad (2/3\cdot\pi)$

$2\pi/3$ $\quad 1\cdot 06244 \times \frac{2\pi}{3}$

$3\pi/4$ $\quad 1\cdot 14806 \times 2\pi/3$ $\quad \gg \frac{2}{3}\pi$

$0<\alpha\le\frac{\pi}{2}$.

$\alpha = \pi/2$ $\quad \theta = 10°$ $\quad S_4 = \cdot 666617\,\pi$.

What about $m>4$?

$m=7$ $\quad \theta = 0$ $\quad \alpha = \pi/6$ $\quad S = \cdot 5110495\,\pi$.

$I = \cdot 5110\,\pi$ $\quad \pi/3$ $\quad = \cdot 5110669\,\pi$

$\theta = 30°$ $\alpha = \pi/3$ gives $\cdot 510979\,\pi$ $\quad \pi/2$ $\quad \cdot 5423608\,\pi \gg I$

The upper lt of α $> \pi/3$ & $< \pi/2$. fix it. (or $< \pi/3$ also?)

$m=6$ $\quad I = \pi \times 11/20$ ~~$2\pi \times 275$~~ $0\cdot 55\,\pi$.

$\theta = 0$ $\quad \alpha = \pi/6$ $\quad S = \cdot 549996\,\pi$

$\pi/3$. $\quad S =$ " "

$\pi/2$ $\quad \cdot 5665 \gg I$.

$\theta = 30°$ $\quad \alpha = \pi/3$ $\quad S = \cdot 5499617\,\pi$

$m = 5$ $\quad I = \frac{115}{192}\pi = .5989583\,\pi$

$\theta = 0$ $\quad \alpha = \pi/6$ $\quad S = .5989\,3692\,\pi$ + [illegible]

" $.5674\,\pi$ + [illegible]

[illegible] $2\pi/5$.
i.e. $0 < \alpha \leq 2\pi/5$

$= \pi/3$ $\quad .60406559\,\pi$: too high.

$\pi/2$

$\theta = \pi/6$ $\quad \alpha = \pi/3$. $\quad .5989558\,\pi$.

$$\int_0^\infty \frac{\sin a\theta \, \sin b\theta}{\theta^2} \, d\theta = \frac{\pi}{2} b \; ; \quad b < a$$ Edwards II 202.

Let $y = \int_0^\infty \frac{\sin^2\theta}{\theta^2} \cos x\theta \, d\theta$

$$= \frac{1}{2}\int_0^\infty \frac{\sin\theta}{\theta^2} \cdot \sin(x+1)\theta \, d\theta - \frac{1}{2}\int_0^\infty \frac{\sin\theta \, \sin(x-1)\theta}{\theta^2} \, d\theta.$$

whence fig 1.

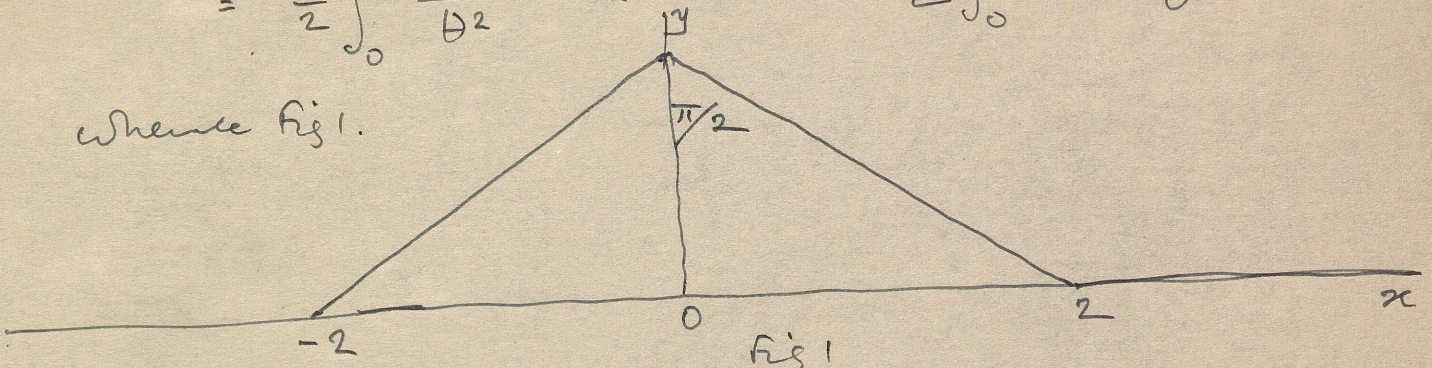
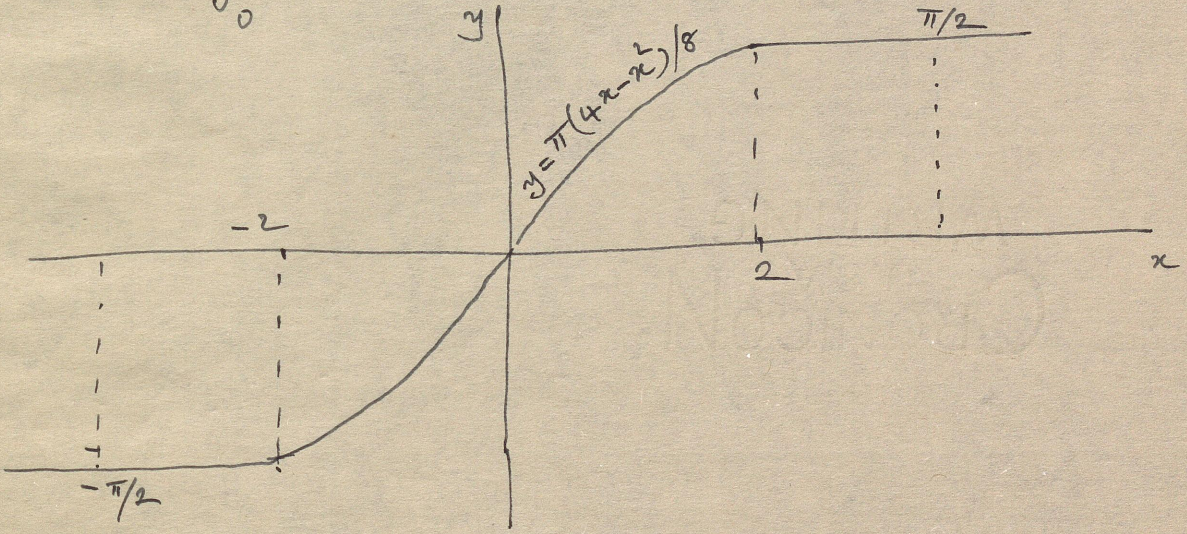


Fig 1

Let $y = \int_0^\infty \frac{\sin^2\theta \, \sin x\theta \, d\theta}{\theta^2}$ Edw. II 203-a.



~~Let $y = \int_0^\infty \frac{\sin^3 x}{x^3} \cdot \cos xv \, dx$ be the cosine transf. of $\frac{\sin^3 x}{x^3}$.~~

~~dy/dv By writing y in the form~~ $\int_0^\infty \frac{\sin a\theta \sin b\theta}{\theta^2} d\theta = \frac{\pi}{2} b$ where $b < a$.

~~$\frac{1}{2} \int \frac{\sin\theta}{\theta^2} \sin$~~

Let $g(v) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{\sin^3 u}{u^3} \cos uv \, du$ be the cosine transform of $\sin^3 u / u^3$.

$$\sqrt{2\pi} \times \frac{dg}{dv} = -\frac{1}{2}\int_0^\infty \frac{\sin u}{u^2} \sin uv \, du + \frac{1}{4}\int_0^\infty \frac{\sin u}{u^2} \sin \overline{(v+2)} u \, du + \frac{1}{4}\int_0^\infty \frac{\sin u}{u^2} \sin \overline{(v-2)} u \, du$$

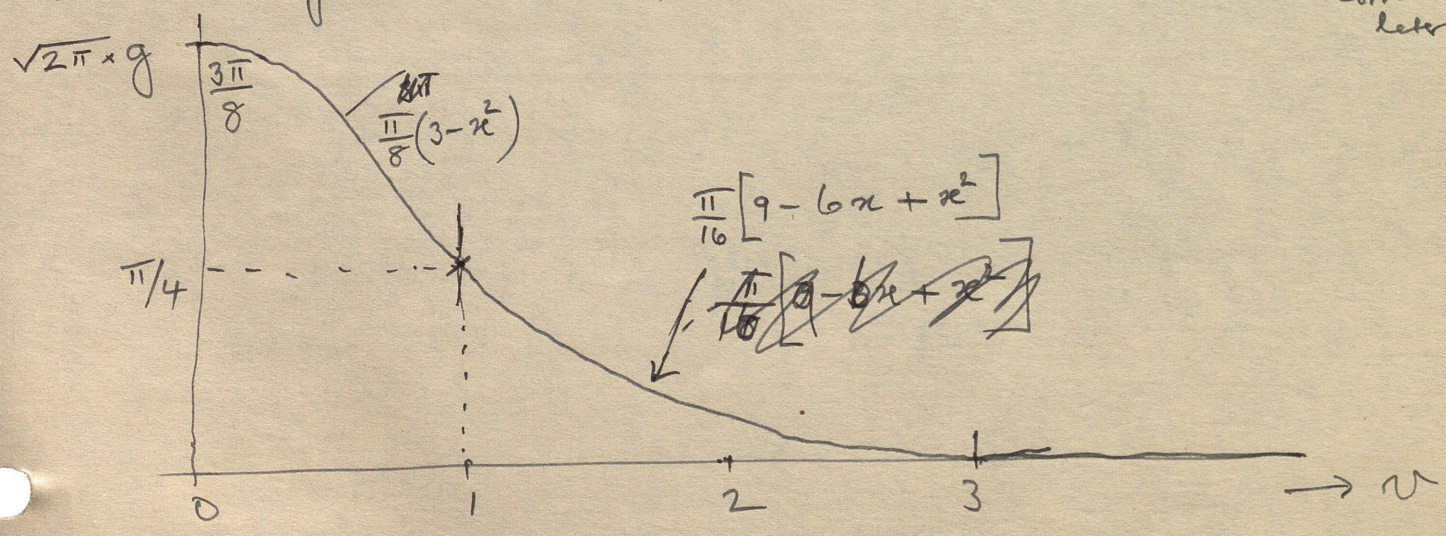
$= 0 \qquad 3 \leq v < \infty$

$= -\frac{\pi}{8}(3-v) \qquad 1 \leq v \leq 3$

$= -\frac{\pi}{4} v \qquad 0 \leq v \leq 1$

whence g is probably ~~somewhat~~ as follows :- (check)

Correct. See a later p.



$$\therefore \sum_{n=-\infty}^{+\infty} \frac{\sin^3(n\alpha+\theta)}{(n\alpha+\theta)^3} = \frac{\sqrt{2\pi}}{\alpha} g(0) = \frac{\sqrt{2\pi}}{\alpha} \times \frac{1}{\sqrt{2\pi}} \times \frac{3\pi}{8}$$

$$= \frac{3\pi}{8\alpha} \qquad 0 < \alpha \leq 2\pi/3$$

$(2\pi \times 1/\alpha = 3)$

$\sqrt{2\pi}\,\frac{dg}{dv}$ can also be put in the form

$$-\frac{3}{4}\int\frac{\sin u\,\sin uv}{u^2}+\frac{1}{4}\int\frac{\sin 3u\,\sin uv}{u^2}$$

(See Edw. II p. [illegible])

& the same results for dg/dv deduced.

Some useful formulae

$$\cos x = \frac{1}{2}\left(y + \frac{1}{y}\right)$$

$$\cos nx = \frac{1}{2}\left(y^n + \frac{1}{y^n}\right)$$

where $y = e^{ix}$

$$\sin x = \frac{1}{2i}\left(y - \frac{1}{y}\right)$$

~~2i~~ $\sin nx = \frac{1}{2i}\left(y^n - \frac{1}{y^n}\right)$

Edwards Int. Cal. I p. 78.

$$\sinh x = -i \sin ix$$

$$\cosh x = \cos ix$$

$$\tanh x = -i \tan ix$$

$$\coth x = i \cot ix$$

Supplement to Campbell + Foster.

	$F(p)$	$G(g)$	
	$1/p$	± 1,	$0 < \pm g$
From 620	$\frac{\cosh(ap)}{p}$	$\pm \frac{1}{2}$	$0 < \pm g < a$
622	$\frac{\sinh(ap)}{p}$	$\frac{1}{2}$	$\lvert g \rvert < a$
623	$\frac{\sinh(ap)}{p^2} - \frac{a}{p}$	$\frac{1}{2}(g \mp a)$	$0 < \pm g < a$
$\therefore$	$\frac{\sinh(ap)}{p^2}$	$\frac{1}{2}(g \pm a)$	"
from 624.5	$-\frac{a\cosh(ap)}{p}$	$\mp \frac{a}{2}$	same as 620.

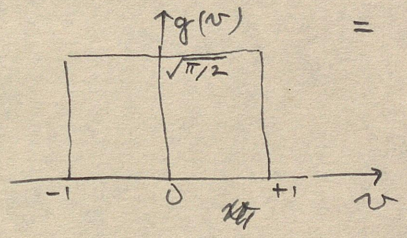
Steward (Advanced Calculus, Methuen p. 477) gives the following:-

1) $$\int_0^\infty \frac{\sin x}{x} \cos ax \, dx = \frac{1}{2}\pi \quad (0 \le a < 1)$$
$$= 0 \quad (a > 1)$$

from which it follows that the F.T. of $\frac{\sin x}{x}$, namely

$$g(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{\sin x}{x} e^{ixv} \, dx = \frac{\pi}{\sqrt{2\pi}} = \sqrt{\frac{\pi}{2}} \quad 0 \le |v| < 1$$

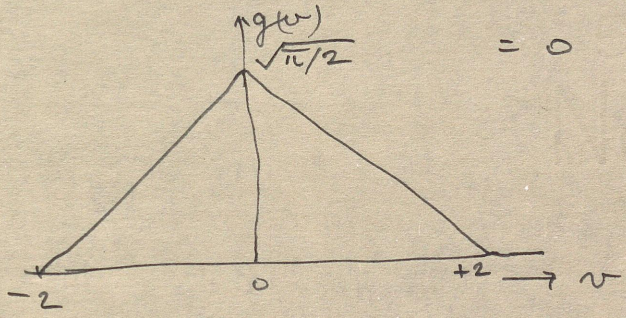
$$= 0 \quad |v| > 1.$$



(same as before)

2) $$\int_0^\infty \left(\frac{\sin x}{x}\right)^2 \cos ax \, dx = \frac{\pi}{4}(2-a) \quad 0 \le a \le 2$$
$$= 0 \quad a > 2$$

whence $$g(v) = \frac{1}{\sqrt{2\pi}} \frac{\pi}{2}(2-|v|) = \sqrt{\frac{\pi}{2}}\left(1 - \frac{|v|}{2}\right) \quad 0 \le v \le 2$$
$$= 0 \quad v > 2$$



3) $$\int_0^\infty \left(\frac{\sin x}{x}\right)^3 \cos ax \, dx = \frac{\pi}{8}(3-a^2) \quad 0 \le a \le 1$$
$$= \frac{\pi}{16}(3-a^2)^2 \quad 1 \le a < 3$$
$$= 0 \quad a > 3$$

$$\sqrt{2\pi}\, g(v) = \frac{\pi}{4}(3-v^2) \quad 0 \le |v| \le 1$$
$$= \frac{\pi}{8}(3-v^2)^2 \quad 1 \le |v| < 3$$
$$= 0 \quad |v| > 3.$$

Verify the following result wh follows from 624.6

p. 73 of Camp. + Foster:

$$\sum_{n=-\infty}^{+\infty} \frac{\sin(n\alpha+\theta)}{(n\alpha+\theta)^2} = \sum_{n=-\infty}^{+\infty} \frac{\cos(n\alpha+\theta)}{n\alpha+\theta}$$

$0 < \alpha \leq 2\pi$

$\Big\updownarrow = ?$

$$\nexists \int_{-\infty}^{+\infty} \frac{\sin x}{x^2}\,dx = \int_{-\infty}^{+\infty} \frac{\cos x}{x}\,dx$$

Edw. I, 113. $$\int \frac{x\,dx}{\sin x} = \frac{2x^2-1}{4}\sin^{-1}x - \frac{1}{4}x\sqrt{1-x^2}.$$

Parseval's Formula in Fourier Series

$$\frac{1}{\pi}\int_0^{2\pi} f(x)^2\,dx = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin nx\,dx.$$

EDT (Ethylene diamin tartarate) crystal can be used for piezo oscillation in place of quartz (Bell Teleph advt in J A Phys. Oct 1947)

Electron Microscope

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✓ Ackermann, A.S.E., Open Packing of Spheres. Nature 155 82 (1945)

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✓ Lacey: the Mixing of Solid particles. Chem. Age 53 119 and 145 (1945)

Marin, E.I., A practical graphic method for mixtures of materials. Rev. Caminos. 17, 376 (1943)

✓ Melmore, S: Densest packing of equal spheres Nature 159, 817 (1947) || Open packing; Nature 154, 708 (1944)

✓ Smith, Foote and Busang. Phys. Rev. 94 1272 (1929)

✓ Verman & Banerjee: Effect of container walls on packing density of particles. Nature 157. 584 (1946)

✓ White, H.E. and Walton, S.F., Jour. Am. Ceramic Soc. 20 155 (1937)

Supplementary list.

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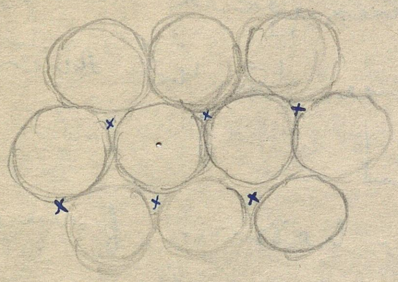
Densest packing of equal spheres

Consider close packed arrangement of spheres in a plane. There will obviously be twice as many interspaces between spheres as there are spheres. The 2nd layer of spheres will have the centres of spheres above every alternate gap in the 1st layer. ~~Their~~ projections of these centres are marked ×.



What is face-centred hexagonal packing?

Hexag. close packing has axial ratio c:a = 1.632:1.

Now the 3rd layer may be disposed in either of the following ways:

1) centres exactly above the centres in layer I

Hexag^l close packing - consisting 2 hexag^l lattices corresponding to the 2 layers

2) ~~above~~ " " " 2nd set of alternate gaps in the I layer (centres of spheres in II layer are above the 1st set of alternate gaps)

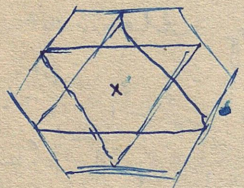
Cubic face-centred lattice the normal to the layers being 3-fold axis

~~3)~~ In arrangement ~~I/A~~ (1) period is 2 layers and in (2) 3 layers — i.e. in (2) 4th layer is identical with the first.

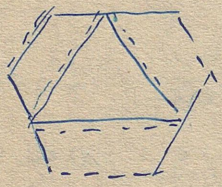
In both (1) and (2) ~~the~~ each sphere makes contact with 12 others — 6 in the same ~~layer~~ layer, 3 in layer above and 3 in layer below. These 12 pts of contact form a 14-hedron. In (2) this 14-hedron is a cubooctahedron consisting of 6 sqq (100) and 8 equilat. Δ's. [which is really the Archim. solid obtained by inversion of the dodecahedron. [Indeed if the spheres were of dough and are made to expand, each sphere will become a rhombic dodecahedron R, in (2) and trapezorhombic dodecahedron T if the arrangt is (1)] (1) ? (2) ?

These two solids have the same vol, same surface, ~~and~~ and are both space fillers, and one can be obtained from the other by cutting half way ⊥ to the axis of hexagonal prism and rotating one half with reference to the other, about the above axis, by 60°]

Coming back to the 14-hedrons formed by the 12 pts of contact ~~with ...~~ as vertices:- in (2) the ~~far~~ 14-hedron has ~~2~~ 3 of the vertices at the corners of an equil. Δ, at the top, 3 at the ~~bott~~ bottom, and 6 at ~~the~~ the corners of a hexagon in the middle.

(2) 

~~In~~ In arrangement (1) the ~~trapezoidal~~ 14-hedron is obtained from the 14-hedron of (2) by cutting by the plane of the ~~hex~~ hexagon and rotating one of the halves by 60°.

(1) 

According to Melmore, Nature 159, 817, 1947

There are 4 homogeneous mixtures of (1) and (2), i.e. of dodecahedra: trapezo-rhombic T and rhombic R as building units:-

I TRR, TRR,

II RT, RT, ~~RTT~~

III RRTT, RRTT,

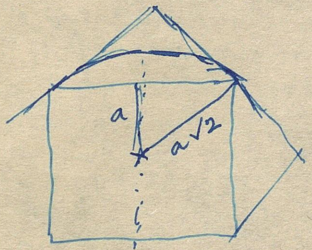
IV RTT, RTT.

In I 7th layer overlies and has same orientation as 1st, in II 5th, in III 13th, and in IV 10th.

check.

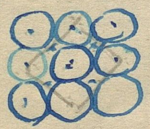
If the density of the material of the sphere is unity, the density of closest packing will be given by

$$d = \frac{\text{Vol of sphere inscribed in a rhombic dodecahedron}}{\text{Vol of dodecahedron.}}$$



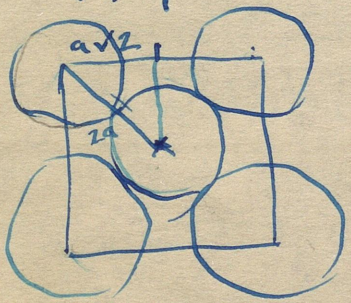
$$= \frac{4\pi}{3} \times \frac{1}{\sqrt{2}^3} a^3 \Big/ \frac{16}{\sqrt{2}} a^3 = \frac{\pi}{3\sqrt{2}}$$

As we shall see in the next page another type of 12 contact close packing gives exactly the same density $d = \pi/(3\sqrt{2})$



these 2 are identical, one can be obtained from the other by rotation about an axis ⊥ to plane of paper by $\pi/4$.

A face-centred cubic packing, with 12 pt contact for face centre + 12 pt-c. for corner spheres, also gives again



$$d = \frac{\frac{4\pi}{3} a^3}{(2a\sqrt{2})^3/4} = \pi/(3\sqrt{2}).$$

the sphere at the centre of a cube face touches not only the 4 at the corners, but 4 below at centre of side faces of the cube, and similarly 4 above.

A body centred packing however gives a lower density, namely

$$d = \frac{\frac{4\pi}{3} a^3}{\left(\frac{2a}{\sqrt{3}} \times 2\right)^3 / 2} = \frac{\pi\sqrt{3}}{8}$$

which is $\sqrt{27/32}$ of the density of closest packing.

See Elmes & Francis on next page for arrangement of equal spheres:

An open 6-pt contact packing is described by Melmore for which $d = 0.370$
$\rho = 0.630$

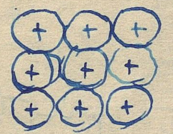
and a 4 pt diamond like tetrahedral packing for which $d = 0.338$
$\rho = 0.662$

~~Body centered~~

Elmes and Francis refer in addition to the simple cubic ($p = 47.64\%$) and the hexag. close packing ($p = 25.95\%$) which they refer to as tetrahedral system of packing, to 3 intermediate ones.

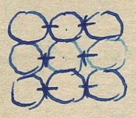
+ indicates positions of centre of sphere in next layer

no of contacts

6 

$$d = \frac{4\pi/3\, a^3}{8a^3} = \pi/6 \qquad p = 1 - \frac{\pi}{6} = .4764$$

Simple cubic

8 

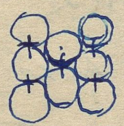
Hutson calls this tetragonal wh is probably the correct description

Single stagger (cubical tetrahedral)

Brown + Hawksley call this Orthorhombic.

The separation between 2 layers in this arrangement ⊥ʳ to plane of paper is obviously $\sqrt{3}/2$ of the separation in simple cubic.

$$\therefore d = \pi/6 \times \frac{2}{\sqrt{3}} = \frac{\pi}{3\sqrt{3}} \qquad p = .3955$$

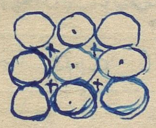
10 

Hutson calls this rhombohedral. probably the correct description.

Double stagger

Brown + Hawksley call this tetragonal

$$d = \pi/6 \times \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{2\pi}{9} \qquad p = .3019$$

12 

face centred cubic.

Pyramidal

~~$d = \pi/6 \times$ 9a / ht of a sq. pyramid of edge lengths a~~

~~$= \pi/6 \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\pi}{3\sqrt{2}}$~~

#

$$d = \pi/6 \times \frac{2a}{\text{ht of a sq pyramid of edge lengths } 2a}$$

$= \pi/(3\sqrt{2})$, same as for ~~[?]~~ close packed hexagonal arrangement ($p = .2595$)

Theoretically in order to produce a dense aggregate of ~~with~~ spherical particles of different sizes, successive grades may be so chosen that the 2nd fraction will just fit into the largest ~~[illegible]~~ interstices betw the particles. For tetrahedral (hexagonal close-) packing they quote White and Walton (J. Am. Ceram. Soc, 20 155, 1937) as giving the following radii for successive particles:

$r \times$ 1 ~~r~~, 0.41, 0.225, 0.177, 0.116. (r_1, r_2, r_3, r_4, r_5)

With these 5 grades p can be brought down from 25.95% to 14.9%. If the further voids are filled by fine particles p may be made as low as 3.9%. Such a reduction in voidage can not of course occur in practice. For ~~[illegible]~~ graded particles p ~ 40%. (according to Westman + Hugill (J Am ~~[illegible]~~ Cer. Soc. 13, 767, 1930)

For higher densities "continuous" grading or "gap" grading may be used, either alone or in combination (i.e. fairly wide size ranges may be used in gap grading, and the fractions so produced may be continuously graded — a method much ~~used~~ for exptal purposes)

— (Irregular shape and frictional effects main cause)

Suitable Gap grading gives higher bulk densities.

By mixing 3 grades W + H obtain

	U.S. stand sieve	
Coarse (c)	4-5	·185 - ·168"
medium (m)	20-30	·0328 - ·0232"
fine f	140-200	·0041 - ·0029"

these ~~[illegible]~~ gradings were used in pairs

cf, mf, mc

and curves were drawn plotting the vol. of each particular mixt against its composition. Then from these curves a solid prismatic diagram was constructed which predicted the position of max. bulk density.

$f = 9\%$
$m = 25$
$c = 66$
Total 100

$$\frac{\text{Bulk density of mixt (max)}}{\text{" of one of the compts}} = \frac{1.605}{1.057} = 1.518$$

Taking $p = .40$ or $d = 0.60$ for any of the graded specimens max $d = .91$ ‖ which is quite a dense packing
min $p = 9\%$.

Properties affected by density of packing
1) porosity 2) mechanical strength 3) thermal + elec. conductivity 4) Resist. to torsional stress 5) ... to abrasion 6) permeability.

Why is hexag.l close packing called tetrahedral?

	1	2	3	4	5	fine particles as fillers.
Radius of successive spheres	$r_1 = r$	$r_2 = 0.414 r$	$0.225 r$	$0.177 r$	$0.116 r$	small
their relative number. no having r_1 as 1	1	1	2	8	8	∞
Total vol. of all the spheres	$4.189 r^3$	$4.487 r^3$	$4.582 r^3$	$4.762 r^3$	$4.815 r^3$	$5.437 r^3$
Voids in mixt. = p.	.2595	.207	.190	.158	.149	.039

For ~~same~~ single stagger packing of cylinders

$p = 1 - \frac{\pi}{2\sqrt{3}} = .0940$ | With secondary cylinders $r_{11} = \frac{2}{\sqrt{3}} r - r = .155 r$
Resulting void $= .0721$.

Nature Dec 10 1960 p. 908

Random close packing of spheres corresponds to densities varying from 0.63(7) to 0.60(1) as against $\rho = \frac{\pi}{6}\sqrt{2} = 0.7405$ for hexagl or f.c.c. ~~close~~ packing — which is closest packing.

$\frac{\pi}{3\sqrt{2}}$

The change in density of rare gas liquids on melting is 10% wh corresponds to change from ·74 to random packing

Refces to Coxeter, Bernal, ~~& Rice~~ Westman are given in this note.

Chem & Industry 4 3/61

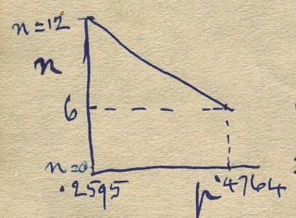
W. A. Gray: The packing of real spherical particles [wall effect, effect of energy input, coeffts of restitution etc]

Macrae & Gray: Br J Ap Phys. 12 164 (1961)

Ackermann refers to the experimental fact that "graded to size coal weighs the same per unit volume whatever the size of the lumps"; shows this result is to be expected on the basis of cubical packing of spherical particles, since voidage 0.476 is independent of size of particles; states, however, incorrectly that for closest packing voidage decreases as the size of spheres decreases; the voidage figures which he gives for 8 pt. contact packing - which presumably he regards as closest packing - are probably incorrect.

Foord shows that the experimental fact referred to by Ackermann is applicable to any symmetrical arrangement of spheres, including closest packing; draws attention to Ackermann's error and points out that the closest packing corresponds to 12 pt. contact, the lines joining the centres of the spheres forming a system of regular tetrahedra; thinks this is a stable arrangement (unlike other~~s~~ arrangements), which is taken up automatically (?)

Smith and others: Porosity ~~and~~ or voidage for packing of spherical particles without any care is about 0.40, but with special care may be varied over the range 0,35 to 0.45 (for close packing p = 0.26; for 6 cont. packing p = 0.48) By filling with acetic acid and draining the acid, a small ring of liquid is retained in the regions of contact which gives white lead acetate; the points of contact are thus identified and counted. For a particular porosity observed, he determines (a) the average no. n of contacts of a sphere and (b) the %age distribution of the numbers of contact. ~~Highest~~ Smallest porosity observed is 0.359 for which n = 9.14. Max. of distribution curve is at about n = 8. ~~Minimum~~ Max. porosity observed was p = 0.447 for which n = 6.92, and the distribution is almost Gaussian. n plotted against p almost a straight line* and same as that connecting the theoretical points n = 12, p = 0.2595; and n = 6, p = 0.4764. Suggests that any observed packing having a particular p may be regarded as due to



statistical occurrence of regions of close hexag. and simple cubic packings in appropriate proportion.

<u>Verman and Banerji</u> point out that the effect of the geometry of the boundary (i.e. assuming the distribution is not affected) is much larger than the earlier references suggest. For a cubical box of side equal to n x diam. of sphere

$$d_{obs} = d\left(\frac{n-1}{n}\right)^3 \quad \text{- - - - - - - - - - - - - -} \quad (1)$$

and for a ~~regular~~ rectangular box

$$d_{obs} = d\left(\frac{n_1-1}{n_1}\right)\left(\frac{n_2-1}{n_2}\right)\left(\frac{n_3-1}{n_3}\right) \quad \text{- - - - - - - - -} \quad (2)$$

If further the boundary affects the distribution, and assuming the effect to be confined to one layer of atoms only, (1) may be generalized by adding to the right hand side a term

$$\left\{1-\left(\frac{n-2}{n}\right)^3\right\}\Delta d$$

which would make (1) correspond to

$$y = d + x\Delta d \quad \dots (2)$$

where $y = d_{obs.}\left(\frac{n}{n-1}\right)^3 \dots (3)$ and $x = \left(\frac{n}{n-1}\right)^3 - \left(\frac{n-2}{n-1}\right)^3 \dots (4)$

Empirically, from some observational data, <u>Brown and Hawksley</u> in a supplement to the above note, find that equations (2) and (3) hold if x ~~has~~ is taken to have the value

$$x = \left(\frac{n}{n-1}\right)^3 - \left(\frac{n-3}{n-1}\right)^3$$

instead of (4).

Further, from the same observational data, they find

Δd = 0.28 for both tight and loose packing
d = 0.58 for tight packing
0.57 for loose packing.

With non spherical particles (discs) - n is now equal to side of cube ÷ the diameter of the equivalent sphere (i.e. sphere of same volume as the disc).

Δd = 0.28 as before.
d = 0.60 for tight, and
0.54 for loose packing.

For coal particles d = 0.59, $\Delta d \leq$ 0.28.

P. M. C. Lacey <u>The mixing of solid particles</u>,

Chemical Age <u>53</u> 119 and 145 1945

If α is the fraction of one of the solid component A in a binary ~~mix~~ mixture of solid particles, all of same size, shape and vol., of A and B

$1 - \alpha = \beta$ = fraction B.

~~$\sigma = \sqrt{\alpha - \bar{\alpha}}$~~ σ = root mean sq of the deviation ~~for~~ of α from its mean value $\bar{\alpha}$ in an element of vol. containing altogether n particles

$$= \sqrt{\frac{\bar{\alpha}\,\bar{\beta}}{n}}$$

can be deduced as follows:-

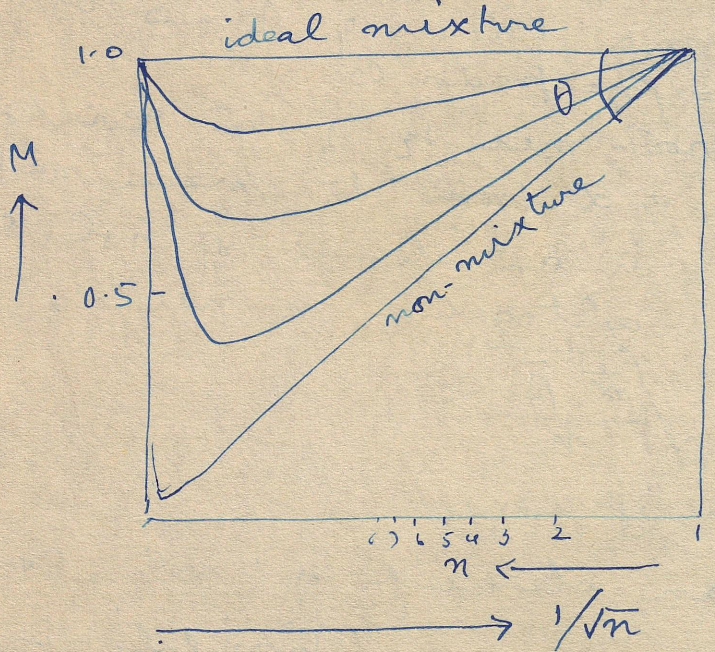
If we take an element of vol. containing n particles there are $n+1$ possibilities: none, 1, 2, - r, - - n ~~qu~~ particles of type A. Let P_r be the probability associated with r of them being of type A. $\alpha_r = r/n$ [i.e. of n particles selected r of type A, $n-r$ of B

$$P_r = \frac{\left(\frac{1-\bar{\alpha}}{\bar{\alpha}}\right)^{n-n\alpha_r} n!\,(\bar{\alpha})^n}{(n\bar{\alpha})!\;(n-n\bar{\alpha})!}$$

$$\sigma = \sqrt{\sum_{r=0}^{n} \left(\alpha_r - \bar{\alpha}\right)^2 P_r} = \sqrt{\frac{\bar{\alpha}\bar{\beta}}{n}}.$$

Provided we handle altogether large volumes, for an ideal mixt $\sigma\sqrt{\frac{n}{\bar{\alpha}\bar{\beta}}}$ = unity

For an imperfect mix, taking the reciprocal of the above quantity namely $M = \frac{1}{\sigma}\sqrt{\frac{\bar{\alpha}\bar{\beta}}{n}}$ as a measure of imperfection ($0 < M \leq 1$, the upper limit corresponding to ideal mix) Lacey finds explicitly the following :



For an non-mix $M \propto 1/\sqrt{n}$.
$\theta = 45°$

For an ideal mixture $M = \text{const} = 1$ $\theta = 0$

What is Osborne Reynolds's Principle of Dilatancy
Collected Sc. papers II 203, 207 (1901) referred to by
Brown and Hawksley Nat. 156, 421 (1945)?

Brown + Hawksley refer to some expts
of his on voidage p for packing ~~graded~~ coal
and gives p = 0.47 to 0.43. Quotes spheres ~~Furnas + Bussey~~
who give p = 0.47 to 0.54 depending on size (the
latter limit of p corresponding to ~~higher or for~~ lower particle size.

Bennet + Brown Tests on starch particles for frequency
of no. of contacts with nearest neighbours J. Inst.
Fuel 13 232, 1940.

P. G. H. Boswell Br. Sc. News. 2 342 (1949) defines

n = void-ratio as vol. of empty space (i.e. of liquid needed to fill) ÷ vol. of solid

p = porosity = " ÷ total vol.

$$p = \frac{n}{n+1} \qquad \text{e.g. when } n = \tfrac{1}{2} \quad p = \tfrac{1}{3}.$$

Thixotropy weaker ~~at~~ in sandy sediments, just sufficient clay or ferric oxide to coat the sand grains n as ~~l~~low as 28%.

With pure quartz sand 18 – 24%

Gives n for several clay minerals.

Consider a cubic crystal which is elastically also isotropic; i.e. one for which

$$\epsilon = c_{11} - c_{12} - 2c_{44} = 0$$

or $c_{11} = c_{12} + 2c_{44}$.

For such a crystal the no. of elastic constants reduces to 2, which are

$$c_{11} = \lambda + 2\mu$$

$$c_{12} = \lambda$$

where λ is ~~the~~ Young's modulus, and μ the mod. of rigidity.

Bulk mod. $k = 1/\beta = \lambda + \frac{2}{3}\mu$.

The elastic waves, whatever the direction of the waves, will split into longit^{l}. & transverse, or condensational & distortional, whose velo: V_l and V_t resp. are given by

~~$V_l^2 \rho = \mu = c_{11} - c_{12}4$~~

~~$V_t^2 \rho = k + \frac{4}{3}\mu = c_{11}$~~

$$V_l^2 \rho = k + \frac{4}{3}\mu = c_{11}$$

$$V_t^2 \rho = \mu = \frac{1}{2}(c_{11} - c_{12}).$$

In eqn. (12) of R.S. paper on Diffuse scattering if we substitute ~~for~~ $A_i = l$, $B_i = m$ and $C_i = n$ we get $V_l^2 \rho = c_{11}$.

Similarly by putting $A_i l + B_i m + C_i n = 0$, we obtain

$$V_t^2 \rho = c_{44} = \frac{1}{2}(c_{11} - c_{12})$$

11 elementary particles known :-

the familiar electrons, neutrons, protons, photons

positron, neutrino

positive & negative mesotrons

$\sim 200\,m \quad \tau = 2 \times 10^{-6}$ sec.

One Product of decay of mesotron is electron, which however has only 1/4 of the energy of mesotron. Other products may be either 2 neutrinos or single particle ~~so~~ ~ 140 m. If single particle, ± electron's charge of energy const. Otherwise variable. Rossi's expts suggest former.

Powell gets evidence at high altitudes of heavy mesotron 320 m, as against light M's of 200 m. These heavy M's ~~decompose~~ decay into light M's — which have always the same energy — & one other single particle, neutral & ~ 90 m. Heavy M's probably responsible for nuclear binding forces.

Even with α particles 380 Mev. from Berkeley's 184" cyclotron, both + & − heavy ~~mes~~ M's observed. − ones attracted by nucleii of of photogr. plate used for their detection and produce stars. +ve ones repelled : & decay.

light M's also produced by direct impact of α- in Berkeley C, with nuclei.
wh is surprising since light M's do not interact easily with nuclei, & not at all with light nuclei — & if capture is not likely production from the nucleus will also be —

Thermionic constant

Some metals wh can be studied conveniently -

1) Hf. Hagstrum J A Ph. 28 323 1957

$\varphi = 3.60$ ev $A = 22.9$ 1250° to 1820° K.

2nd sp.., not-so reliable $\varphi = 3.91$ $A = 20.5$

m.p. 2400° K.

2) Mo ibid $\varphi = 4.38$ $A = 88.5$

In the calculation of the electrical resistivity of a metal one comes across the integral

$$\int_{-\infty}^{+\infty} \frac{d\eta}{(e^{\eta}+1)(1+e^{-(\eta+z)})} \quad \text{where } \eta = (E-\zeta)/kT, \quad z = h\nu/kT$$

which refers to a given temp T, ~~and given ν~~.

The integral can be shown to be equal to

$$z/(1-e^{-z}).$$

Finally one also comes across the integral

$$\int_0^{\Theta/T} \frac{z^5\,dz}{(e^z-1)(1-e^{-z})} = J_5(\Theta/T) \text{ say}$$

Sommerfeld & Bethe (Handbuch d. Physik 24₂ p528) have shown

that $J_5(\infty) = -\int_0^\infty z^5\,dz \frac{d}{dz}\left(\frac{1}{e^z-1}\right) = -\left[\frac{z^5}{e^z-1}\right]_0^\infty + 5\int \frac{z^4\,dz}{e^z-1}$

$\therefore J_5(\infty) = 5\int_0^\infty \frac{z^4\,dz}{e^z-1} = 5\sum_{n=1}^{\infty}\int z^4\,dz\,e^{-nz} = 5!\sum_{n=1}^{\infty}\frac{1}{n^5}$

At high temps $T \gg \Theta$ $\quad J_5(\Theta/T) = \int_0^{\Theta/T = 124'4} z^3\,dz = \frac{1}{4}\left(\frac{\Theta}{T}\right)^4$

order - disorder

$CuAu_3$ X-ray study J A Ph. 28 556, 1957

$S = 0.87 \pm 0.04$ at a temp slightly below Ⓐ ~

~~20~~ $199°C \sim 472°K$ Ⓗ increases with increasing Au

DEPARTMENT OF SCIENTIFIC RESEARCH

GOVERNMENT OF INDIA

ATOMIC ENERGY COMMISSION

13 FEB 1950

Commissioners
DR. H. J. BHABHA, F.R.S.
DR. K. S. KRISHNAN, F.R.S.
DR. S. S. BHATNAGAR, F.R.S

No.G.IV(2)- 240
~~CENTRAL SECRETARIAT~~
10-King George's Avenue,
NEW DELHI 11th February, 1950.

My dear Dr. Krishnan,

Apropos of our telephonic conversation yesterday I send you herewith a few of the typical analyses of Indian monazite which I collected while in Ceylon. These may be of some use to you. The figures are averages of 5 or 6 samples.

Thorium oxide	9	to	10.75	p.c.
Cerium oxide	26	to	27.50	"
Lanthanum & Didymium oxides	29	to	30	"
Yttrium & allied oxides	1.50	to	3.9	"

These analyses were done in England, most probably at the Imperial Institute.

Yours sincerely,

D N Wadia

(D.N.WADIA).

Dr. K.S.Krishnan, F.R.S.,
Director, National Physical Laboratory,
Hillside Road,
New Delhi.

DNW/SBL.

10 MAR 1950

D.O. No. Dm2(III)/19/50
DIRECTORATE GENERAL OF INDUSTRIES & SUPPLIES
DEVELOPMENT OFFICE (METALS)
Shahjahan Road, New Delhi.
Dated the 8th March, 1950.

/9

Dear Dr. Krishnan,

Reference our discussion yesterday, I give below the current British quotation of cerium metal:

Cerium Metal 97/98%	£7¼ per lb.
Cerium Alloy 52%	Sh.35/- per lb.

Unfortunately it is not mentioned what are the other ingredients of the cerium alloy. I feel that this must be the iron alloy.

Yours sincerely,

D. P. Antia

(D.P. Antia)

Dr. K. S. Krishnan,
Director,
National Physical Laboratory,
New Delhi.

Ultrasonics

Phys. Today. Aug. 1950 p. 8.

Sonic $\nu = 20$ to 20,000 cycles/sec.

Distinction between super sonic vel > vel. of sound
ultra sonic freq > that of audible sound.

Super heterodyne receiver sh'd be called ~~ut~~ ultraheterodyne r.

Suppose vibrating O^r piston is used to generate sound waves. At high frequencies wave energy concentrated in a truncated cone. $\angle^{le}$ of cone $f(\lambda/d)$ where d is the diameter of piston.
$\therefore$ higher the frequency smaller the spreading.

~~H~~ ν 500 meghacycles has been produced: corresponding λ in air = λ of red light.

At 20 K.C. sufficiently directional for use in submarine detection.

Ultrasonic sources have been made to generate as much as 50W per cm² and beams of radiation have been focussed so as to give 5000 W/cm².

At a dist. of 2 metres from a trumpeter sound intensity = 10^{-6} W/cm²
10^{-4} W/cm² threshold of pain for human ear.

If a million people were to speak at the same time power generated ~ 10 W.

Forced-air whistle constructed in 1883 which ~~gives~~ reaches 25 K.C.

Bat can generate pulses 2×10^{-6} sec. duration at ~~20~~ 30 per sec. N in each pulse from 30 KC to 100 KC.

Tuning forks with tines some mm in length can produce N ~ 90 KC.

French apothecary Pierre de la Seignette, of La Rochelle, in 1672 discovered the crystal called R salt & in 1880 Pierre and Jacques Curie discovered their piezo ele. property. Inverse, appln. of voltage producing change in thickness predicted by Lippmann in 1881, & verified by the Curies.

Up to 60 KC magnetostriction generators developed by G.W. Pierce (1925) can be used.

For N > 60 K.C. only piezo oscillators.

In ultrasonic diffraction of light, as the intensity of U.so~~und~~nic wave is increased, more & more of light forced from 0 order to side orders of diffraction spectra. & at a certain ultrasonic intensity, i.e. a certain carrier voltage zero order is absent. Hence the amount of light passing thro' a slit can be controlled in this manner, or modulated, with double the frequency of the ultrasonic waves. Better than Kerr cell. The modulation frequency can not, however, be varied continuously since it is determined by the quartz crystal.

Lucite lens for refracting & focussing U.S. waves.

<u>For detection of flaws</u>: A ~~pulse~~ spot is

A crystal is used to send a short pulse of U.S. waves: Same crystal used by direct piezo effect to receive reflections. The amplified elec. output portrayed on the screen of an oscilloscope.

For producing true colloidal solns & for making fine emulsions. e.g. photographic emulsions of improved homogeneity, stability and sensitivity. [Molten iron & lead, not ordinarily miscible in liquid state can be mixed & alloys ~~formed~~ made]. New bearing materials have been made in this way.] Similarly homogenization of milk.

Coarse crystals of sulphathiazole broken down to form a creamy emulsion.

On aerosols on the other hand (as against ~~effects~~ dispersion in hydrosols) effect is one of coagulation. Solid + liq. particles in mist + smoke precipitated in this manner. At an installation in Kingsmill, Texas, this technique is used to recover carbon black from a smoke stack.

Similarly used to run heavy fog to rain.

Certain chemical reactions influenced. chain molec. of starch broken down into several fragments ~~of~~ to produce dextrine.

Gum arabic + gelatine have been decomposed.

aging of whiskey

Bactericidal ~~value~~ effects given on p. 15.

Solid glass marbles suspended in air by U. sound waves reflected from a ~~st~~ board.

Science in general

New Era of Science

Br. Ch. Engg / Jan 1958. p. 7 "Sc. as a liberal study" ~~p. 7~~ refers to Linstead's 9th Hinchley Memorial Lecture An education for our times"

"Civilization is the humanization of man in Society" Arnold

may not the study of any subject — not necessarily humanities — be humanizing ~~if it~~ provided that it is closely connected with man's every-day life?

the methodology of science is itself a powerful humanizing factor.

Science & the Citizen. by Warren Weaver Sc. 126, 1225 Dec. 13, 1957

Each thursday afternoon 300 yrs ago a group of gentlemen ~~we~~ gathered at the Bull-Head Tavern in Cheapside London / to carry out expts, to eat & drink together & to discuss Sc.: Among them Sir Christopher Wren, who was ~~primarily~~ primarily professor of Astronomy at Oxford, Robert Boyle, Lord Brounker a patron of all types of learning, Bishops Wilkins master of Trinity College, Sir William Petty, Political Economist & Prof of Anatomy at Oxford, & Prof of Music at Gresham's College: Samuel Pepys, Sec. of the Admiralty, and at a later time the 2 American Benjamin Franklin & Thomson later known as Count Rumford

This was the beginning of the Royal Society club, which, together with others, received the ~~Roy~~ charter of the Royal Soc. on 15 July 1662. The resolution that the Soc. be formed is dated Nov. 28, 1660.

To think of science as a set of special tricks, to see the scientist as the manipulator of outlandish skills — this is the root of the poison man-drake which flourishes rank in the comic strips. — Bronowski.

~~The~~ Human life is far too broad, deep, subtle and rich to be exhausted by anything the scientist would find out in his own field — Oppenheimer.

There is a likeness between the creative acts of the mind in art and in science. . . The scientist or the artist takes two facts or experiences that are separate. He finds in them a likeness which had not been seen before, and he creates a unity by showing the likeness — Bronowski

This discovery of unity is at the centre of science, and it is also at the centre of art. Whenever Coleridge tried to define beauty, he returned to a central deep thought. Beauty he said is unity in variety. — Weaver.

Earth's Magnetic field

1. Halley Phil Trans 13 208, (1683) 17 468 (1692)

2. Sir Edward Bullard's art. on Halley on the occⁿ of tercentenary of his birth (in 1956)
   Endeavour Oct 1956 p. 189

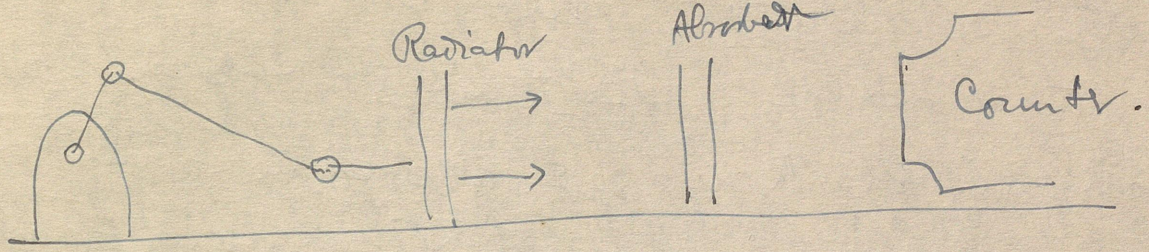
3. Ring current in space =
   Discovery 1959 p. 400.

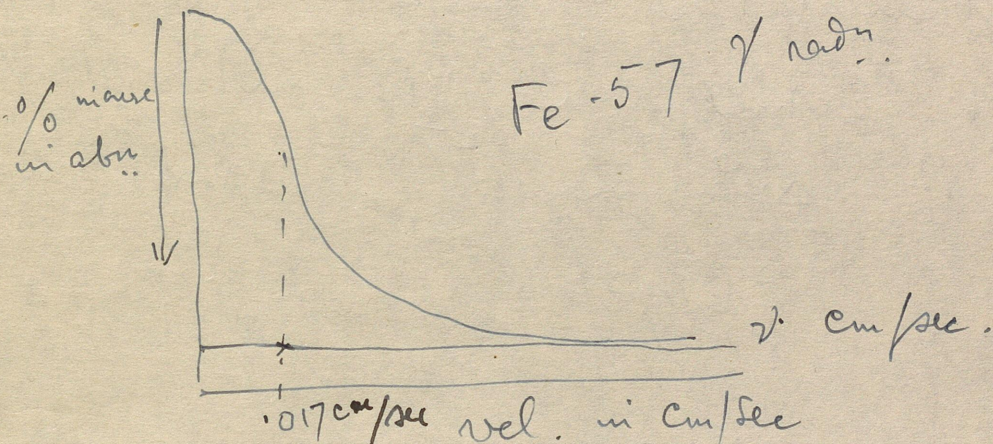
Transistors

Research XI 135, 381, 425 XII 21

See Sc. 27/5/60 p. 1588 **Gravitational Red Shift**

R.L. Mossbauer Zs Phys. 151, 124 (1958)





Temp effect very important

$3\times10^{10}/.017 = 2\times10^{12}$ ∴ line width ~~to~~ $\frac{\Delta\nu}{\nu} = 10^{-12}$

In a quartz ~~of~~ clock regulation capability $= 10^{-3}$ of line width

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Theoretical 2000 miles above the surface of the earth wd make a diffce ~~of~~ in clock rate of 1 sec in 500 yrs.

Pound + Rebka Phys Rev Letters 3 439, (1959), 4 163, 337 (1960)

At 10 cycles/sec., the .017 cm/sec. vel. wh was equiv to a half line width would correspond to peak to peak amp. of 9 microns. ~~At~~ At 1 megha cycle/sec

" " " " = 0.9 A.U. only little greater than rad of H atom.

In the case of optical resonance, the lines would be sharper in the gaseous state, than in the solid, since the interaction of the electrons with the environment is stronger in the latter. If there had been no such interactions, the resonance lines wd be much sharper than γ- resonance lines.

In the optical case the interaction corresponds to $\Delta\nu/\nu \sim 10^{-7}$ Hence duration

Prodn of temps below 1°K.

London confce Dec 1959 Brit. J App Phy. 11 449 1960.

^{3}He extends the liquid He range down to 0.3°

Vap. press curves of both given.

^{3}He still costs $1000 per litre at NPT.

^{4}He II 2 interpenetrating liquids normal & superfluid capable of frictionless motion

betn wh no ~~ent~~ exchange of momentum,

Surfaces of vessel above He II ~~have a~~ covered by a film of He

100 Å ~~atoms~~ thick = F. Superfluid component flows

thro' the film at a rate ∝ nal to circumference of the

vessel & indept of length of film

The rougher & dirtier the surface the greater the flow.

At 1° vap pres of ^{3}He = 10 mm.

With both liquids 3, 4 He there a boundary

heat resistce betn a solid body & the liquid surface

For 3 $\frac{130}{T^2}$ & for 4 $45/T^2$ in °K cm^2 W^{-1}.

$$C_p = dQ/dT = T\,dS/dT \qquad dQ/dS = T$$

Sc. Teaching & training in Research

~~Br J. App Phys.~~

If we in industrial Res. had a choice as between abbreviated formal course work and full research training on the one hand, & full formal course work & a resultant shorter period of research training on the other hand, I think we would choose the latter – # C G ~~[illegible]~~ Suits / vice Pres. & director of Res. GEC Schenectady in Bull. Inst of Physics 1960 p 291

Also certain classical problems of graduate training in US.

i Using him in team work

ii inheriting problem & equip. from a previous student – one additional res. in an extended series

iii in constructing an inst: mechanical but necessarily technical task.

creative research vs problem solving –

There should be opportunities to recognize creative traits & aptitudes ~~in the~~ student. At present one is not sure from the reports even of Ph.D.'s

In 1958 among physicists employed (largest among scientists) 47%, ~~37%~~ in industry, 35% in Educational Instns, 13% Govt. Employment. The % in Industry – much higher now

A large fraction of this 47% going to Industry, viz 18% of this 47% are engaged in management of research & development –

C2

Lattice vacancies in thermal eqm at high temps. in noble metals.

M. Doyama & J.S. Koehler Phys. Rev. 119, 939 (1960)

99.999 % Ag. 2mils diam wire.

Increase in resistivity due to lattice vacy $= 1.3\ \mu\Omega\,\mathrm{cm}$ for 1% vacancy.

$\Delta\rho$ due to vacancies alone $= A\,e^{-E_F/kT}$

Entropy term $A = 4.5 \times 10^{-4}$ ohm cm. $E_F = 1.10$ eV.

E_F is energy of formation of lattice vacancy.

Activation energy for vacancy motion 0.81 eV.

Res. resistivity when well annealed $\ll 3 \cdot 10^{-9}\ \Omega\,\mathrm{cm}$.

R.O. Simmons & R.W. Balluffi Bull Am Phys Soc. 5 181 (1960)

$\Delta L/L$ change in length $\Delta a/a$ change in lattice const.

No of vacancies $\Delta N/N = 3(\Delta L/L - \Delta a/a)$ at m.p. of Ag estimated 1.7×10^{-4}

$E_F = 1.08$ eV. which agrees with prev. 1.10.

According to Jin-Ichi Takamura Bull. above p 182

~~$\Delta V/V$~~ $\Delta V/V = \alpha \times$ Concentn of vacancies $= \alpha\,\Delta N/N$

where $\alpha \neq 1$, $= 0.43$: Also E_F & A fns of dimensions of the wire.

~~Oct~~ 27

C.T. Tomizuka & E. Sonder Phys. Rev. 103, 1182 (1956)

From self diffusion in Ag, he finds activn. energy for diffn 44·1 K Cal./mole.

Self diffusion of Cl^- ion in NaCl

Neal Lawrance Phys. Rev. 120, 57 (1960)

$$D = 56 \cdot e^{-2.12 eV/kT}$$

Diff^n coeff in crystals containing Ca is smaller

2.5 eV. actn energy now.

$$D = 1280 \cdot e^{-2.49 eV/kT}$$

Earth's magnetosphere

Including whistlers, Allen belts, Argus expts

J. Geoph. Res. 64 865 (1959) Symp. on Argus

Argus explosions (1-2 kilotons) at 480 Km, over

I. 12° W 38° S	Explorer IV	Aug. 27 1958	pp. 880-1
II 8° W 50° S	250 transits	30	
III 10° W 50°		Sep 6	

p 886 // By the basic Poincare - Störmer - Alfven theory the magc moment of a spiralling particle is an adiabatic invariant of the motion.

$$\mu = \frac{1}{2} m v_{\perp}^2 / B = \text{const.} \qquad B = \text{magc vector}$$

Turning pts in the path correspond to B = const.

Rosenbluth & Longmire, Ann Phys 1 120 1957

$$I = \int_{T_1}^{T_2} v_{\parallel}\, dl \qquad T_1, T_2 \text{ turning pts.}$$

$$= v \int \sqrt{1 - B/B_r}\, dl$$

(action or integral of momentum along the magc lines) is an adiabatic invariant (under an important class of physical conditions) of its motion where B_r is the scalar value of B at the turning pts.

p. 872 Gives the various invariants.

T. Northrup & E Teller = Univ Calif - Radn Lab report - 5615 :

Martin Kruskal Princeton Univ ;

C. O. Hines wavepackets, Pointing $\vec{v}$ + energy flow J Geop. Res. ~~61 139-4~~ 56 (1951) nos 1, 2, 4 (4 parts.)

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theory of trapping of whistlers = J Geop. Res. 65 815 (1960)

Newell & Naugle = Radn Environment in Space Sc. (1960) 1465 –

Space Res. Nice Sympm 1960 in particular pp 780 – 795. reg. motion of particles in the belts.

Optical properties of single crystals (organic)

1. Lyons & Mackie Photo & Semiconductance
~~VII~~ J.C.S Parts VII – IX in Dec. 1960 issue p 5186.

2. Bree & Lyons: Intensity of Ultraviolet absn.
J.C.S Part IV ibid p. 5206, 5213