EVOLUTION OF STARS

By

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Introduction

The first notable success of astrophysics was the theory of ionisation of stellar atmospheres. Later work related to constitution of stars dealing with problems of equilibrium and energy transport leading to the mass—luminosity relation, which can be taken to characterise the second stage of this development. As long as considerations relating to energy-production were not tackled rigorously, there was no hope of proceeding further and reaching the third stage of explaining the mysteries of the Russell-Hertzsprung diagram. The development of nuclear physics in the last few years has made it possible to obtain definite results regarding energy—generation. Just as in the first stage it was the theory of atomic structure that helped in the development, it is appropriate that in this third stage we should invoke the help of nuclear structure. Some of the achievements of these latest ideas, their bearing on stellar evolution and the difficulties still to be surmounted are indicated in this article.

It is a great pleasure to offer this as my humble contribution to the volume commemorating the 61st birthday of one whose ideals in founding this University have been as lofty and sublime in conception as the subject of this article.

I. *Internal constitution of stars*

A general theory of the internal constitution of stars has been shown to be possible on the basis of the laws of gravitation, of radiation, of atomic structure and of simple gas laws. The theory is not too complicated mainly on account of the fact that the properties of matter in its gaseous and highly ionised state in the interior due to the enormous pressures and temperatures ruling there are much simpler than in any other state.

The principle of mechanical equilibrium permits the calculation of the pressure P at any point of a star if one knows the way in which the density ϱ varies with the distance from the centre in other words if the "model" be known.

In the simple gaseous ionisated state the mean molecular weight μ can be calculated from atomic theory, and the equation of state for the perfect gas is also valid. For a given model therefore the temperature T at any point can also be calculated.

The next important consideration is that at the high temperatures in the interior, radiation pressure is as important as gas pressure. Taking this into account and using the fact that radiation pressure varies as the fourth power of the temperature, one could calculate the internal temperature of a star for any given model. The calculations become particularly simple on Eddington's model for which $\varrho_c=28$ that of water,

$P_c=36\times10^9$ atm; $T_c=2{\cdot}9\times10^7$ K for the sun.

To relate the above quantities with conditions at the surface, one has next got to calculate the escape of radiation from the interior. On general principles it is evident that the heat will flow, inside the star, from regions where the radiation pressure is greater to those where it is smaller. This flow of heat however meets with a resistance due to the opacity of the gas, and the co-efficient of opacity $\varkappa$ can be calculated as a function of P, μ and T by applying the general methods of the quantum theory of the interaction of matter and radiation. It is thus possible, starting with pure theory, to calculate the luminosity of a star of given mass and radius and built on a given model. It is found that the luminosity increases very rapidly with the star's mass-rather faster than its fourth power on the average. For the same mass it changes but slowly with the star's size (inversely as $\sqrt{r}$). Differences in the model make surprisingly little difference in the luminosity. The chemical composition makes little difference too except for the abundance of hydrogen, the luminosity of a star of almost pure hydrogen being less by a factor of 300. Applying this to the sun, an agreement between calculated and observed luminosities is obtained if hydrogen forms 35% by weight of the interior mass, the rest being heavy elements.

This conclusion that the luminosity of a star depends mainly upon its mass is in effect Eddington's well known "mass-luminosity relation," and it will be shown later that it is really a consequence of the fact that the hydrogen content of a star does not vary at random for a given mass.

2. *The Russell-Hertzsprung diagram.*

The theory of constitution of stars described above accounts for the close correlation between luminosities and masses, but it gives no explanation at all of the equally conspicuous relations connecting luminosity and spectral class as is brought out clearly in the Russell—Hertzsprung diagram, R.H.D. in brief. Experience has shown that, up to a certain approximation, all stars are characterised by two numbers which might be chosen in general as the luminosity L and the surface temperature T. These at the same time also define the radius of the star R. In the R.H.D. these co-ordinates are plotted as log L and log T; alternatively one could also plot log $(R/R_{\odot})$ and log $(L/L_{\odot}$ and call this the modified R.H.D. or the R-L plane. The diagram shows (Fig. I) that stars favour only certain regions of the plane. The great majority of the stars belong to the so called *main-sequence;* their luminosities and radii increase rather regularly with their mass, as also the effective temperature. So the stars of this group range from hot and luminous blue giants down to the cool and faint *red dwarfs.* According to the best observations it is practically a sharp line, and the stars belonging to it therefore form a one-parameter group. Besides the stars of the main sequence,

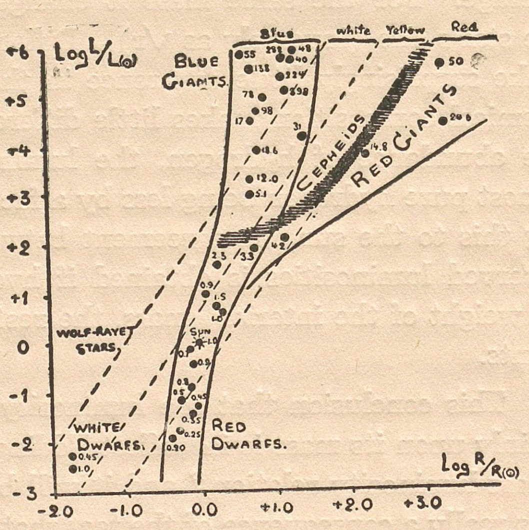

Fig. 1. Showing the relation between masses, radii and luminosities of various stars and the division of stars into the normal stars or the stars of the main sequence, red giants (including Cepheid variables) and white dwarfs (including probably Wolf-Rayet-Stars).

Fig. I

and to the right of them above lie the *red giants* (L large T small) with L and R larger than for stars of the same mass in the main sequence. In the R-L diagram these form a separate branch (giant branch) branching off near the middle of the main sequence. It should also be noted that some particular stars located in this region possess a property of periodic luminosity changes (Cepheid variables and others of long period) and represent the upper boundary, in respect of L, of the giant branch. Again to the left and below the main sequence are the *white dwarf* states (T large, L small) corresponding to smaller luminosities and radii than stars of corresponding masses in the main sequence. Probably related to the white dwarfs are the central stars of planetary nebulae (Wolf-Rayet stars) which also possess small radii for given luminosities.

A proper understanding of this distribution of stars in the R-L diagram is of fundamental importance for questions of stellar evolution and it can be seen from very general considerations that this understanding depends on a knowledge of the mechanism of energy production in stars. In consonance with the theorem of Vogt and Russell one must expect theoretically that under certain assumptions the state of a star is completely characterised by two parameters and accordingly by its position in the R.H.D. Further the matter which a star consists of is determined by specifying its total mass and its chemical composition. If now the original chemical composition of stellar material be universally the same (and our knowledge of the abundance relations of chemical elements makes this assumption plausible) a difference in the chemical composition of stars can only be a result of the energy-generating nuclear reactions which on their part are determined by the state of the star. Therefore there remain, besides, only the mass and the age of the star as independent parameters.

The calculation of the empirical parameters L and T from the mass and the parameter of chemical composition assumes a theory of the internal constitution of the star. On the theory of Eddington sketched in § 1 which assumes the conditions of equilibrium and energy-transport as fundamental but not the energy-generation, the mass-luminosity relation is obtained as a relation between two parameters, the stars of different luminosities in the main sequence being also stars of different masses. But it is obvious however that the mass-luminosity relation merely describes their uniform chemical composition. The principal problem of the theory of nuclear reactions in stars is to derive the dependence of energy-generation on chemical composition, and thereby elucidate the structure of the R.H.D.

3. *Stellar nuclear reactions*

The magnitude of the problem of energy-generation inside a star can be best illustrated by considering the Sun, a typical star. The Sun radiates 2 ergs per second per gram of its mass which corresponds to a loss of 4,200,000 metric tons per second, and since there is equilibrium between generation and loss of energy, energy of the same order must have continued to be generated throughout geological times during the last $2 \cdot 10^9$ years. Besides the Sun there are stars which throw out nearly thousand times as much energy. The question naturally arises: where does this energy come from? According to the ideas of modern Physics, there are four possible sources:

(i) Contraction of a star without change of chemical constitution—the energy liberated is gravitational energy.

(ii) The building up of heavy atomic nucleii out of lighter ones—the energy liberated is nuclear energy.

(iii) Contraction by transformation of a part of the matter into densely packed neutrons.

(iv) Complete annihilation of matter—energy liberated is the rest energy of matter.

Of these the last source can be left out of account in view of the fact that it has not been so far observed in the laboratory, and even from a theoretical point of view the discovery of the neutron and positron has shown that by the equalisation of positive and negative charges only the electron mass is transformed into radiation while the proton mass is unaltered. On very general thermodynamic arguments it can be shown that the third source postulated is improbable for normal stars but might be invoked for explaining catastrophic phenomena. Thus we have to make the assumption that during the life time of a star, in so far as it is subject to our observation, only the first two sources need be considered. Of these the first alone is not sufficient to explain the production of energy as for example, in the case of the Sun whose present rate of radiation would exhaust this source in $4 \cdot 10^7$ years. One is led almost by a process of exhaustion to the second as the most likely one. Although this had been surmised some years ago it is only the progress of nuclear physics in the last few years that has made it possible to prove this surmise and decide rather definitely which process can and which cannot occur in the interior of stars. A careful analysis by Bethe of all the possible processes has shown that the only thermonuclear reactions which can occur at sufficiently large rates at the temperatures of stellar interiors are those between protons and the light nucleii. In general terms one might say that the energy production of stars is due entirely to the combination of four protons and two electrons into an α particle. As can be seen from Table I this formation of four atoms of hydrogens into one of helium results in a diamution of the combined masses of the interacting nucleii by 1 part in 135. This simplifies the discussion of stellar evolution in as much as the amount of heavy matter, and therefore the opacity, does not change with the time.

TABLE ***Corrected and additional nuclear masses, and binding energies.***

NUCLEUS	MASS	BINDING ENERGY (mMU)	REFERENCE
n^1	1.008 93		19
He^3	3.016 99	5.87	18
H^4	4.025 4	0.6 ±1	
He^4	4.003 86		29
Li^4	4.026 9	−1 ±1	
He^5	5.013 7	−0.9 ±0.2	23
Li^5	5.013 6	−1.6 ±0.3	
Be^6	6.021 9	−1.8 ±0.8	21
Be^7	7.019 28	5.7	26
Be^8	8.007 80	−0.08±0.04	28
B^8	8.027 4	0.0 ±0.4	21
B^9	9.016 4	−0.5 ±0.2	21
C^{10}	10.020 2	3.8	21
N^{12}	12.022 5 −24 3	0.0 ±0.9	21
N^{13}	13.010 08	2.03	19
O^{14}	14.013 1	5.1	21

Table I

These reactions of hydrogen with the lighter nucleii are shown in Table II which gives the energy evolution Q of the reaction, its probability per second and also the life time, all calculated for a temperature of 2×10^7 degrees.

As has been shown by Bethe no elements heavier than helium can be built up to any appreciable extent permanently in the interior of stars under present conditions. An extract from table II of reactions leading to He^4 is given in Table III, along with the average energy produced in ergs/gm per second. As can be seen at once from this table it is the nitrogen reaction alone which gives energy generation in consonance with the observed data for the sun. We can divide these reactions into three classes:

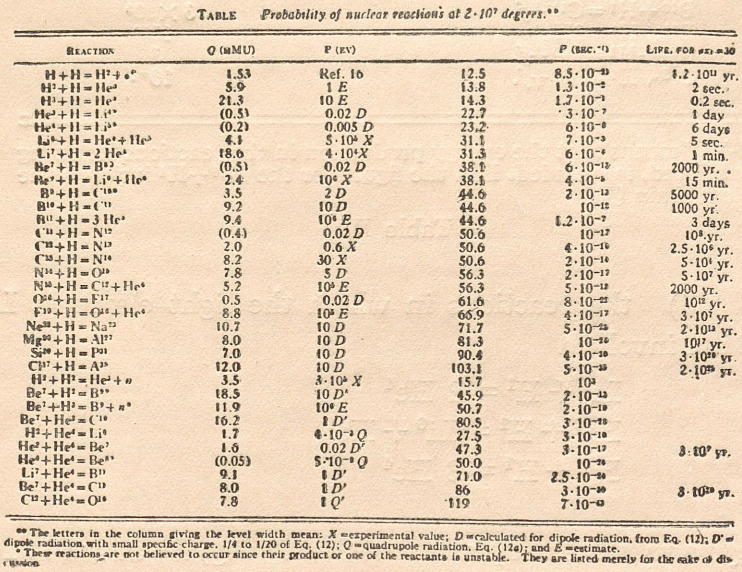

TABLE *Probability of nuclear reactions at $2\cdot10^7$ degrees.***

REACTION	Q (MMU)	Γ (EV)		p (SEC.⁻¹)	LIFE, FOR ρx = 30
$H+H=H^2+\epsilon^+$	1.53	Ref. 16	12.5	$8.5\cdot10^{-20}$	$1.2\cdot10^{11}$ yr.
$H^2+H=He^3$	5.9	1 *E*	13.8	$1.3\cdot10^{-2}$	2 sec.
$H^3+H=He^4$	21.3	10 *E*	14.3	$1.7\cdot10^{-1}$	0.2 sec.
$He^3+H=Li^{4*}$	(0.5)	0.02 *D*	22.7	$3\cdot10^{-7}$	1 day
$He^4+H=Li^{5*}$	(0.2)	0.005 *D*	23.2	$6\cdot10^{-8}$	6 days
$Li^6+H=He^4+He^3$	4.1	$5\cdot10^5$ *X*	31.1	$7\cdot10^{-3}$	5 sec.
$Li^7+H=2\,He^4$	18.6	$4\cdot10^4$ *X*	31.3	$6\cdot10^{-4}$	1 min.
$Be^7+H=B^{8}?$	(0.5)	0.02 *D*	38.1	$6\cdot10^{-13}$	2000 yr.
$Be^9+H=Li^6+He^4$	2.4	10^6 *X*	38.1	$4\cdot10^{-5}$	15 min.
$B^9+H=C^{10*}$	3.5	2 *D*	44.6	$2\cdot10^{-13}$	5000 yr.
$B^{10}+H=C^{11}$	9.2	10 *D*	44.6	10^{-12}	1000 yr.
$B^{11}+H=3\,He^4$	9.4	10^6 *E*	44.6	$1.2\cdot10^{-7}$	3 days
$C^{11}+H=N^{12}$	(0.4)	0.02 *D*	50.6	10^{-17}	10^8 yr.
$C^{12}+H=N^{13}$	2.0	0.6 *X*	50.6	$4\cdot10^{-16}$	$2.5\cdot10^6$ yr.
$C^{13}+H=N^{14}$	8.2	30 *X*	50.6	$2\cdot10^{-14}$	$5\cdot10^4$ yr.
$N^{14}+H=O^{15}$	7.8	5 *D*	56.3	$2\cdot10^{-17}$	$5\cdot10^7$ yr.
$N^{15}+H=C^{12}+He^4$	5.2	10^5 *E*	56.3	$5\cdot10^{-13}$	2000 yr.
$O^{16}+H=F^{17}$	0.5	0.02 *D*	61.6	$8\cdot10^{-22}$	10^{12} yr.
$F^{19}+H=O^{16}+He^4$	8.8	10^5 *E*	66.9	$4\cdot10^{-17}$	$3\cdot10^7$ yr.
$Ne^{22}+H=Na^{23}$	10.7	10 *D*	71.7	$5\cdot10^{-23}$	$2\cdot10^{13}$ yr.
$Mg^{26}+H=Al^{27}$	8.0	10 *D*	81.3	10^{-26}	10^{17} yr.
$Si^{30}+H=P^{31}$	7.0	10 *D*	90.4	$4\cdot10^{-30}$	$3\cdot10^{20}$ yr.
$Cl^{37}+H=A^{38}$	12.0	10 *D*	103.1	$5\cdot10^{-35}$	$2\cdot10^{25}$ yr.
$H^2+H^2=He^3+n$	3.5	$3\cdot10^5$ *X*	15.7	10^3	
$Be^7+H^2=B^{9*}$	18.5	10 *D′*	45.9	$2\cdot10^{-13}$	
$Be^7+H^3=B^9+n^*$	11.9	10^6 *E*	50.7	$2\cdot10^{-19}$	
$Be^7+He^3=C^{10}$	16.2	1 *D′*	80.5	$3\cdot10^{-28}$	
$H^2+He^4=Li^6$	1.7	$4\cdot10^{-3}$ *Q*	27.5	$3\cdot10^{-18}$	
$He^3+He^4=Be^7$	1.6	0.02 *D′*	47.3	$3\cdot10^{-17}$	$3\cdot10^7$ yr.
$He^4+He^4=Be^{8*}$	(0.05)	$5\cdot10^{-9}$ *Q*	50.0	10^{-26}	
$Li^7+He^4=B^{11}$	9.1	1 *D′*	71.0	$2.5\cdot10^{-26}$	
$Be^7+He^4=C^{11}$	8.0	1 *D′*	86	$3\cdot10^{-30}$	$3\cdot10^{20}$ yr.
$C^{12}+He^4=O^{16}$	7.8	1 *Q′*	119	$7\cdot10^{-43}$	

** The letters in the column giving the level width mean: *X* = experimental value; *D* = calculated for dipole radiation, from Eq. (12); *D′* = dipole radiation with small specific charge, 1/4 to 1/20 of Eq. (12); *Q* = quadrupole radiation, Eq. (12*a*); and *E* = estimate.

* These reactions are not believed to occur since their product or one of the reactants is unstable. They are listed merely for the sake of discussion.

Table II

(i) $H+H=D+\varepsilon^+$

with the deuteron being next transferred into He^4 by further capture of protons. From the life time value in Table II and energy generation value in Table III, this appears a probable reaction, but there is a possibility that this reaction itself may be forbidden by selection rules.

TABLE *Energy production in the sun for several nuclear reactions.*

REACTION	AVERAGE ENERGY PRODUCTION ϵ(erg/g sec.)
$H^1+H^1=H^2+\epsilon^++f.$*	0.2
$H^2+H^1=He^3$	3×10^{16}
$Li^7+H^1=2He^4$	4×10^{14}
$B^{10}+H^1=C^{11}+f.$	3×10^{5}
$B^{11}+H^1=3He^4$	10^{10}
$N^{14}+H^1=O^{15}+f.$	3
$O^{16}+H^1=F^{17}+f.$	10^{-4}

* "$+f.$" means that the energy production in the reactions following the one listed, is included. E.g. the figure for the $N^{14}+H^1$ includes the complete chain (1).

Table III

(ii) the reactions in which the light elements Li, Be, B are involved

$$Li^7 + H^1 = 2\ He^4$$
$$Be^9 + H^1 = Li^6 + He^4$$
$$B^{11} + H^1 = 3\ He^4$$

Li begins to be used up at about 2×10^6 deg, Be at $3{\cdot}5\times10^6$ deg and the isotopes of B at about 9×10^6 degrees. As seen from Table II these light elements would "burn" in a very short time, and moreover they are destroyed permanently and will not be replaced. Thus for example, Be would act in the following way

$$Be^9 + H^1 = Li^6 + He^4$$
$$Li^6 + H^1 = Be^7$$
$$Be^7 + E^- = Li^7$$
$$Li^7 + H^1 = 2\ He^4$$

(iii) $N^{14}\ H^1 = 0^{15}$

which written out fully as a chain reaction is given in Table IV. This is in fact the most important source of stellar

$$
\begin{aligned}
C^{12}+H^{1} &= N^{13},\\
N^{13} \qquad &= C^{13}+\epsilon^{+},\\
C^{13}+H^{1} &= N^{14},\\
N^{14}+H^{1} &= O^{15},\\
O^{15} \qquad &= N^{15}+\epsilon^{+},\\
N^{15}+H^{1} &= C^{12}+He^{4}.
\end{aligned}
$$

Table IV

energy and in it carbon and nitrogen isotopes serve merely as catalysts for the combination. It can conveniently be called the carbon-nitrogen cycle. As seen from Table II, a given C^{12} nucleus will, at the centre of the sun, capture a proton once in $2{\cdot}5 \times 10^{6}$ years, a given N^{14} once in 5×10^{7} years. These times are short compared with the age of the sun, and therefore the cycle will have repeated itself many times in the history of the sun so that statistical equilibrium has been established between all the nucleii occuring in the cycle. Another important point about this cycle is its very strong dependence on temperature viz. T^{18} and this has important astrophysical consequences.

The one thing that is common to all the above reactions is the end product He^{4}, the α-particle. Obviously nothing can happen to it since the reaction $He^{4} + H = Li^{5}$ is unstable because of the non-existence of Li^{5}. The

α-particle appears to be the only thing stable in this microcosm of changes, and if hydrogen be the "fuel of the stars" helium is the ashes.

4. *The Sun*

As has already been remarked in connection with Table II it is the carbon-nitrogen cycle that keeps the sun shining. This can be brought out in a more striking way by answering the following question. Neglecting all nuclear considerations regarding the cycle, which nucleus will give us the right energy evolution in the sun ? or conversely ; given an energy evolution of 20 ergs/g-sec at the centre, and 2 ergs/g.sec at the surface, which nuclear reaction will give us the right central temperature ($\sim 19 \times 10^6$ degrees)?

TABLE *Central temperatures necessary for giving observed energy production in sun, with various nuclear reactions.*

REACTION	T (MILLION DEGREES)
$H^2+H=He^3$	0.36
$He^4+H=Li^5$	2.1
$Li^7+H=2He^4$	2.2
$Be^9+H=Li^6+He^4$	3.3
$B^{10}+H=C^{11}$	9.2
$B^{11}+H=3He^4$	5.5
$C^{12}+H=N^{13}$	15.5
$N^{14}+H=O^{15}$	18.3
$O^{16}+H=F^{17}$	32
$Ne^{22}+H=Na^{23}$	37

Table V

This calculation has been carried out in Table V. It has been assumed that the density is 80, the hydrogen-concentration 35% that of the other reactant 10% by weight. It is seen from the table that all nucleii up to boron require extremely low temperatures in order not to give too much energy-production; these temperatures ($<10^7$ degrees) are quite irreconcilable with the equations of hydrostatic and radiation equilibrium. On the other hand, oxygen and neon would require much too high temperatures. Only carbon and nitrogen require nearly, and nitrogen in fact exactly, the central temperature obtained from the Eddington integrations (19×10^6 degrees). Thus from stellar data alone we could have predicted that the carbon-nitrogen cycle is the process responsible for the energy production.

TABLE *Comparison of the carbon-nitrogen reaction with observations.*

STAR	LUMINOSITY ERG/G SEC.	CENTRAL DENSITY	*H* CONTENT (PER-CENT)	CENTRAL TEMPERATURE (MILLION DEGREES)	
				INTEGRATION	ENERGY PRODUCTION
Sun	2.0	76	35	19	18.5
Sirius A	30	41	35	26	22
Capella	50	0.16	35	6	32
U Ophiuchi (bright)	180	12	50	25	26
Y Cygni (bright)	1200	6.5	80	32	30

Table VI

5. *The main sequence*

The theory that the main sequence stars owe their energy generation chiefly to the carbon-nitrogen reaction is very satisfactorily verified from observational data. In table VI a comparison of the theory with observation is made in the case of five stars for which the data are sufficiently well-known. The last column in the table is calculated as the necessary central temperature to give the correct energy evolution as observed. In the calculations the N^{14} content is taken as 10%. The last column but one gives the temperatures as calculated on Eddington's theory. The agreement between the two columns is highly satisfactory, the only exception being the star Capella which cannot really be considered as belonging to the main sequence.

Russell had suggested long ago that the central temperatures of all stars of the main sequence are nearly the same although the luminosities of these stars varied by factors of the order 10^6. This is easily understood on the present theory if we assume that in general all these stars have the same energy source. In fact the very strong dependence of the N-C cycle on temperature $(\sim T^{18})$ shows that a small variation of the central temperature brings about a large change in the luminosity.

As pointed out by Von Weiszsäcker it is also possible on this theory to understand the bend in the R·H·D (See Fig. 1) in the region of the red dwarfs. The reaction $H + H = D + \varepsilon^+$ already considered before plays a role in this connection. Due to its weak dependence on temperature this reaction is not of much importance for the major part of stars in the main sequence whose central temperature are $\geqslant 2 \times 10^7$ degrees. In the region of smaller temperatures of the order 15×10^6 degrees and less, this reaction appears to be concurrent with the N—C reaction and as shown by Fig. 2. even of greater importance. The bend in the main sequence is to be attributed to the weak dependence of luminosity on central temperature in this region of red dwarfs of mass $(1/10) M_\odot$ nearly.

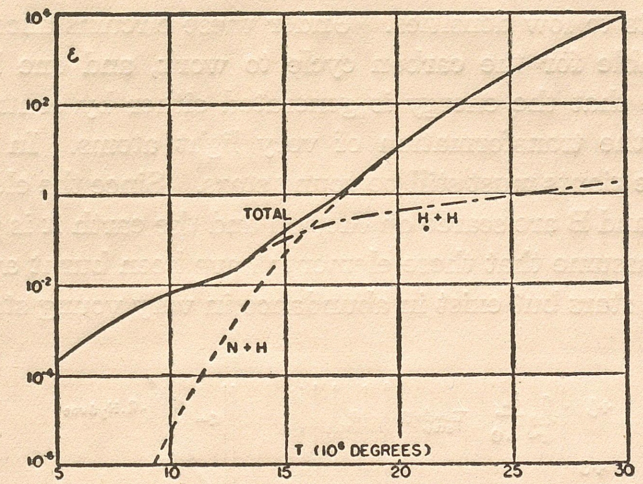

FIG. . The energy production in ergs/g sec. due to the proton-proton combination (curve H+H) and the carbon-nitrogen cycle (N+H), as a function of the central temperature of the star. Solid curve: total energy production caused by both reactions. The following assumptions were made: central density = 100, hydrogen concentration 35 percent, nitrogen 10 percent; average energy production 1/5 of central production for H+H, 1/10 for N+H.

Fig. II

The narrow width of the main sequence can be understood if we observe that its stars are prescribed to lie in a region which corresponds to certain allowed variations in their chemical composition. These stars must satisfy both the following conditions (a) they must not be so young that their energy-generation is due either to contraction or the burning of elements lighter than carbon and (b) on the other hand they must not fail to possess hydrogen. We describe gaints as those stars which do not satisfy (a) and the white dwarfs as those which do not satisfy (b).

6. *Giants and Variable Stars.*

The central temperatures of these stars are less by a factor 10 than those of main sequence stars, which also amounts to low densities. Under these circumstances it is impossible for the carbon cycle to work, and one has to assume that the energy is generated either by contraction or by the transformation of very light atoms. In either case the giants must still be young stars. Since the elements Li, Be and B are scarce on the sun and the earth it is plausible to assume that these elements have been burnt away in normal stars but exist in abundance in very young stars.:

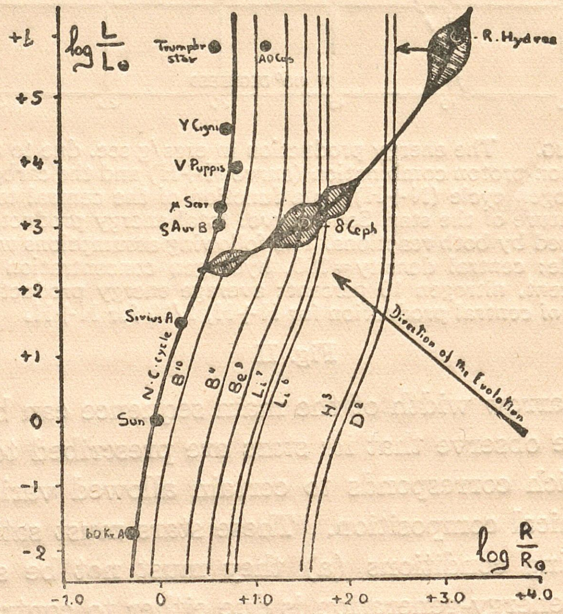

FIG Pulsating variables and different nuclear reactions.

Fig. III

On the assumption that the energy generation in giants is due to reactions of the lighter elements, Gamow and Teller have drawn in the R—L diagram calculated curves for each reaction parallel to the main sequence (See Fig. 3). A star which contains all these nucleii in large quantities would stay along one of these curve as long as the corresponding isotope was completely burnt out, and then make a transition to the curve of next higher temperature and finally land in the main sequence. On this picture such a star should spend a comparatively long time within each of these bands and undergo a more rapid gravitational contraction during the transition from one such region to another. As is well known these variable stars from a one-parameter sequence, all their characteristics being dependent on the vibration period. Thus the knowledge of this period fixes the position of the star on the R—L diagram. In fig 3. the region of pulsating stars in shown by the shaded area, the width of each area being proportional to the number of stars observed. It is seen that there are definite concentrations of the stars near the regions where the nuclear reactions of light elements become important. The three regions corresponding to cluster, Cepheid and long-period variables might be associated with the B^{10} and Li, Be, and perhaps the D—reactions respectively. Gamow goes even further in explaining the line of the pulsating stars as a limit to the distribution of red-giants in the R—L diagram. According to him this line is to be interpreted as the limit above which the evolution is purely gravitational (until the star gets into the main sequence) and below which it is due to nuclear reactions. Because of the short time scale of gravitational contraction the number of stars observed above this line must be statistically small, and this explains the gap between this line and the main sequence. The pulsative instability of the stars near this limiting line can be explained as due to the conditions existing during the transition from the state of thermonuclear evolution into the state of purely gravitational contraction.

This theory, charming as it is, meets with the difficulty that the abundance of the lighter elements in red giants does not appear to be sufficient to retard the process of contraction suitably, and it may be still necessary to assume that either pure contraction and some other unknown source of energy plays a part in the evolution of giants.

7. *White dwarfs and Novae*

In connection with the Vogt-Russell theorem it has already been remarked that the mass and a parameter denoting chemical composition can be chosen as independent numbers characterising a star. For stars in the region to the left of the main sequence we can take the hydrogen content as the parameter of this chemical composition in so far as nuclear reactions are concerned. From the theory of nuclear reactions it follows at once that a star to the left of the main sequence can contain little or no hydrogen, for if it did the state of high temperature and density would, inspite of gravitation, induce sufficient energy generation to prevent contraction.

Before understanding the evolutionary significance of white dwarfs it is necessary to get some theoretical ideas about them which, thanks to the work of Chandrasekhar, are very satisfactory. They represent senility, almost the approach to the final state of a contracting star in which all the energy, gravitational, nuclear or what not, has been exhausted and radiated away into space, and nothing more can happen to it. Within them the electrons are degenerate jammed together as closely as the quantum laws permit. It has been shown that the radius and density of a star in this state are determined by its mass (and H-content if any). If the mass of a star does not exceed the value $M_0 = 5 \cdot 7 M_\odot / \mu^2$

(μ=molecular weight and equal to 2 for no hydrogen) i.e. $1{\cdot}4\,M_\odot$, the final state by contraction will be a sphere of completely degenerate (partially relativistic) electron gas. For such masses less than $M_\odot$, each mass gives a definite value R_{min} for the final radius the least value 0 or R_{min} corresponding to M_0 itself. For masses larger than M_0 the critical conditions will not be reached, and as far as present knowledge goes such a star might contract indefinitely. Another interesting point in connection with stars of mass $\geqslant M_0$ might also be noticed. For the mass lying between $5{\cdot}7\,M_\odot/\mu^2$ and $6{\cdot}6\,M_\odot/\mu^2$, the degeneracy of an electron gas will always begin at a certain stage while for still heavier stars the electrons will always remain in the state of an ideal gas. The evolutionary significance of these ideas will be discussed in the last section.

As intermediate states between the main sequence and white dwarfs are the novae, according to ideas put forth by Biermann. This theory is based on the following facts:

(i) For a normal nova outburst the energy generated is small as compared with the thermal energy content of the stars.

(ii) The luminosity of a nova before and after the outburst is the same within the limit of errors of observation.

(iii) As far as the best observations go, the final state after an outburst is intermediate between the main-sequence and white dwarf states. The first two observations which are mutually compatible show that the outburst does not materially alter the inner structure of the star, and this shows that, in view of (iii) the star was also in the intermediate state before the outburst.

The origin of the outburst itself has been ascribed by Vogt to the fact that the onset of degeneracy would automatically liberate the great quantity of radiant energy previously trapped in the gas, since degenerate gas has very small opacity.

8. *Evolution of Stars*

If we accept the evolutionary hypothesis and postulate the energy sources as in section, 3 it follows that small and large masses should have a rather different evolutionary history, since the mass of a star during its whole life history is almost invariant changing by less than 1 per cent.

Consider first a star of small mass. This would start from the main sequence, and for its further evolution the H-content might be taken as the parameter. The energy-generation would be due to the N-C cycle and the luminosity would increase by nearly a factor of 100 as the H-content is decreasing. The existence of the empirical mass-luminosity relation can be interpreted as a statistical correlation intrinsically due to the fact that the star spends most of its life time in the low luminosity part of its evolutionary track. This track based on the N-H reaction is shown schematically in Fig. 4 for the sun. After the hydrogen content has fallen below a certain limit the star will start a contraction which steadily increases in speed. When the H-contents falls to nearly 0·002 per cent the nuclear energy liberation becomes negligible as compared with the gravitational. The evolutionary track due to contraction, is shown further in the same figure, and gives rise to a continuous increase in luminosity during a comparatively long period of time. The last stage of contraction will now essentially depend on the mass. For masses $< 1{\cdot}4\,M_\odot$ the contractive evolution begins to deviate because of the beginning of the formation of a degenerate electron gas in the central region. The rate of contraction will considerably slow down, and the star reaches the white dwarf stage where it acquires a long lease of life. Going still further in evolution after the white dwarf stage, the increasing exhaustion would result in the outer non-degenerate layers becoming thinner, and the star would shrink and grow fainter and cooler becoming "a yellow dwarf" and ending as a "black dwarf." For stars with masses larger than $1{\cdot}4\,M_\odot$ (but small) the process of gravitational contraction is not limited by any

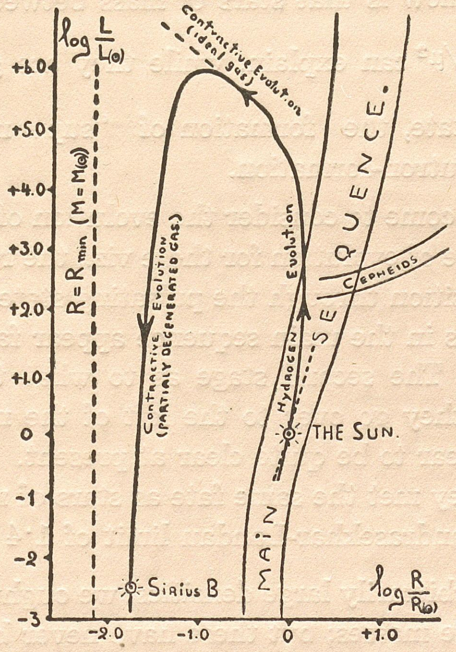

FIG. Evolutionary track of a light star.

Fig. IV

maximum density and such stars are apparently destined to unlimited contraction with central density and temperature rising above any given value. Fig. 4 also shows, according to Gamow, that white dwarfs are at present far from the finite stage of contraction, as the difference between the actual track for a star of mass $M_\odot$ and the dotted track $R=R_{min}$ indicates. Another very interesting suggestion made by Gamow is that stars of mass between $5{\cdot}7M_\odot/\mu^2$ and $6{\cdot}6\,M_\odot/\mu^2$ can explain, while they are getting into a degenerate state, the formation of "super-novae" by the process of neutron-formation.

We now come to consider the evolution of large masses. The first stage of evolution for these viz., the red giant state, and the transition through the pulsating state to the region of blue giants in the main sequence appear fairly simple to understand. The second stage as to what to happens to these when they go over to the left of the main sequence does not appear to be quite clear at present. If one postulated that they met the same fate as stars of masses greater than the Chandrasekhar-Landau limit of $1{\cdot}4\,M_\odot$ viz. contraction to arbitrarily large densities, we ought to find dense states of large masses; but these have never been observed. Two ways, perhaps not mutually exclusive, have been suggested to meet this difficulty Gamow has pointed out that such contraction cannot take place indefinitely because, on account of the angular momentum of the stars, the centrifugal forces soon become large and cause the breaking of such a massive star into several small pieces (see Fig. 5) with the masses below the critical value. These pieces will then continue to exist indefinitely in the form of white dwarfs. Such an explanation would amount to the drastic assumption that existing white dwarfs do not represent a finite stage of evolution of a single star but are fragments of the explosion of heavy stars. The other way is based on the suggestion of Chandrasekhar that all stars of large mass when they come near the region of white dwarfs actually cast off their masses on account of excessive radiation pressure, as is observed in the Wolf-Rayet stars. After casting off their mass, these stars would reach the white dwarf stage. This suggestion like that of Gamow also makes the white dwarf stage not a finite one but the result of a catastrophic change.

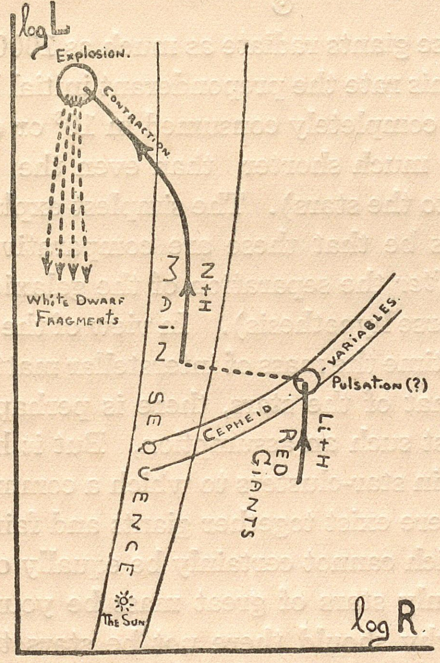

Fig. . Evolutionary track of a heavy star

Fig. V

While the results based on the carbon cycle energy-generation can be considered quite satisfactory for the main-sequence stars, the above considerations relating to giants and white dwarfs are not quite satisfactory and there appear some contradictions which will now be pointed out. The first difficulty is presented by the existence of stars of very high luminosity like the blue giant γ-Cygni near the top of the main sequence $(M=17_{\odot})$ and the red super giant) ζ-Aurigae. These giants radiate as much as 1000 erg/g. sec. or more, and at this rate the preponderant initial hydrogen content would be completely consumed in 10^8 or even 10^7 years (i.e. in a time much shorter than even the age 10^9 years usually given to the stars). The simplest explanation of this would perhaps be that these are comparative young stars formed long *after* the separation of the galaxies (on the expanding Universe hypothesis). In view of the fact that even at the present time the mass of interstellar matter is commensurate with that of the stars, there is perhaps intrinsically nothing against such an assumption, But it has to face the difficulty that in star-clusters to which a common origin is to be ascribed there exist together giants and faint main- sequence stars which cannot certainly be equally old. Moreover why should only stars of great mass be younger than the galaxy, and why should there not be stars to the right of the main sequence having low luminosity and going over into stars of the solar type or fainter types? The second difficulty relates to the white dwarfs. Such a star having the mass of the sun, and negligible hydrogen content would require for its formation, through the process of normal evolution, at least 10^{11} years i.e. periods longer than the age of the galaxies. The suggestion of Gamow that white dwarfs known at present do not represent the finite stages of normal evolution of smaller masses but fragments of larger stars broken into pieces would no doubt remove this difficulty, but it would be hard to assume this unless it can be shown independently that the present white dwarfs *are not* the result of the normal evolution of a star of mass $< 1{\cdot}4\ M_{\odot}$ starting from the main sequence. Another way of escape out of the difficulty suggested by DeSitter is to assume that the white dwarfs are really older than the galaxies, and being dense "hard nuts to crack" they actually came through the period when the galaxies were all together and had not begun to separate. This again appears difficult to understand if the idea were applied to Sirius A and Sirius B, components of a double star and the latter a white dwarf.

In conclusion we might say that while we know why the main sequence stars are there shining, we do not know why the giant stars still shine, and why the white dwarfs are already there.

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