

NATIONAL INSTITUTE OF ADVANCED STUDIES

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*By Courier*

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My dear Sreekantan,

I am sorry I have not been able to write to you as I did not have your new fax number in California.

I have sent some reprints by mail today which I received only a few days ago. In the paper I had noticed the fact that out of the 32 particles, 23 of them fall at points where  $-\log n/2En$  is a Prime Number. So it looks as though the choice of  $n$  is in a sense related to Prime Numbers. It would be too much to believe that it is connected with Partition Theory. Anyway, I have received no comments from all the people to whom I have sent the original paper, but there is a good review of my book on music, a copy of which I am sending you.

We are all greatly looking forward to your return in December.

With best wishes,

Yours sincerely,

*Raja Raman*

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# Music of the spheres

**THE STRUCTURE OF MUSIC IN RAGA AND WESTERN SYSTEM:** By Raja Ramanna. Bharatiya Vidya Bhavan, Kulapati Munshi Marg, Bombay-400 007. Rs. 400.

An eminent scientist, Raja Ramanna in this beautifully produced book seeks to enlighten with an incisive scientific mind the Carnatic raga system and its structure with the help of Western staff notation. The book dedicated to Mysore Veena Vidwan V. Doraswamy Iyengar deals mainly with the fundamentals of Carnatic music, bringing out the similarities with and essential differences that distinguish it from the Hindustan and Western systems. The author's abiding interest in Western music, having had lessons on the piano, and his deep roots in Sanskrit, particularly the Advaita philosophy of Sankara have conferred on him the probing mind of a philosopher and the enjoyment of an aesthete. As an expression of his personal experience Raja Ramanna, as the publisher's note says, has realised that music is a yoga which can take the mind and therefore one's consciousness to a higher level of perception.

As for the choice of the Western staff notation to explain the basics of the raga scales and talas, he has said that by this means one "can be trained to hear 'visually' the written sounds." Carnatic music being a gamaka-oriented expression, no one system of writing could do full justice, for gamaka differs from vidwan to vidwan according to his tonal strength and inflexions. He has given in staff notation the Malahari gita "Lambodara Lakumikara," the Suddha Bangala song of Tyagaraja "Rama Bhakti Samrajya," Sarasara Samaraikasura (Kuntelavarali), Raghuvamsa-sudambudichandra (Katanakuthohalam), Sri Chamundeswari (Bilari), a kirti of Vasudevachar, Singara Lahari (Nilambari) a composition of Lingaraj Urs, Vatapi Ganapatim (Hamsadhvani), to mention a few illustration to express the kirtana structure and the tala system. In fact, Chinnaaswamy Mudaliar, as early as in the last decade of the last century in his work "oriental music in European notation," has tried this method and excerpts from Mayamalavagaula raga for different talas and jatis find a place in the book. Similarly, Raja Ramanna has given the list of the Melakarta ragas and the janyas derived from them and expressed them in staff notation. He has also made the perceptive remark that some ragas are very similar and differ more in the spirit in the way they are sung. For example, Darbar-Nayaki, Bhairavi-Manji. Through charts he has highlighted the formation of ragas by tonic shifts and has given a few examples. All about the mela ragas and their derived scales are tellingly and briefly explained. The 108 talas and their angas are also given.

Towards the end, in the chapter on "Aesthetics of the raga systems" he has concisely analysed how and why Carnatic music has maintained the vitality though some changes have taken place by the passage of Time. Any art is a matter of personal intuitive perception, but appreciation of a great art like Carnatic music depends to a large extent on years of constant hearing of musical exposition by great masters. This Raja Ramanna has emphasised by contrasting the Carnatic music and the way the Western system has developed. To quote him: "The consolidation of the raga system by Venkatamakhin in the 17th century is the culmination of the develop-

ments over the previous centuries." The firm foundations and the well-developed theory explain how Carnatic music has survived retaining its purity down the centuries.

As for the Western system Raja Ramanna has this to say: "Counterpoint had been developed to a high degree of achievement and was greatly used in Church music. It permitted modulation and a common tuning for all orchestral instruments leading to the development of a modern orchestra. After Bach, harmony became the pre-eminent occupation of the musicians. The piling of notes of various pitch and timbre to lead to symphonic climaxes reached its high point in the works of Mozart and Beethoven. In course of time the orchestras became too big and tonality began to lose its meaning and magnificence and bombast took over." In these words he has pinpointed the state of the art of carnatic music and western orchestral pattern. Along with Venkatamakhin's contribution he has made reference to the relevance of 22 srutis. One particular remark about present day music appreciation deserves mention. "Unfortunately, with the coming of the large audiences, bad acoustics and bad electronics, our ears have become less sensitive and appreciation of the finer quality of pitch is now considerably lost to us." Installation of mikes and loud speakers even in small, compact halls has become the order of the day to the detriment of listening pleasure. Talking of Bharatanatyam he says that "only in the Thirties of the present century classical Bharatanatyam found its release from caste chains and today it is recognised as one of the greatest contributions to the world art, both in the realms of dance and rhythm." Finally a chapter is devoted to musical notations and definitions. The theoretical basis of Carnatic music is succinctly dealt with in every chapter. It will certainly open new vistas of understanding of our musical system for the Western artistes.

SVK //

# Concept of discreteness, continuity and the Cantor continuum theory as related to the life-time and masses of elementary particles

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It is shown that with the use of the Cantor continuum theory it is possible to derive the systematics of the lifetime and masses of elementary particles.

The theory gives a new meaning to the foundations of quantum theory as Cantor's theory deals with discreteness and continuity, which is the essence of quantum mechanics.

LET  $A_1, A_2, \dots, A_n$  be the various possible physical events and  $a_{11}, a_{12}, a_{13}, \dots$ , etc. be the various conditions which generate the events, e.g.  $A_1$  takes place when conditions  $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$  exist. We write all possibilities in a matrix form as follows:

Events	Sequence of conditions					
$A_1$	$a_{11}$	$a_{12}$	$a_{13}$	$\dots$	$a_{1n}$	$\dots$
$A_2$	$a_{21}$	$a_{22}$	$a_{23}$	$\dots$	$a_{2n}$	$\dots$
—	—	—	—	—	—	—
$A_n$	$a_{n1}$	$a_{n2}$	$a_{n3}$	$\dots$	$a_{nm}$	$\dots$ , etc.

(I)

The values of the  $a$ 's can be 1 or 0 depending on whether the condition is relevant or not to that particular event.

In order to convert the above matrix to represent actual physical quantities and be able to interpret them as physical laws, we invoke the Cantor diagonal construction which Cantor used in showing the difference between the various types of transfinite numbers, particularly between countable and non-countable infinities. The theory in itself is a mathematical differentiation between continuous and discrete quantities and can also be made the starting point for the foundations of quantum mechanics, since it deals with physical quantities, which under certain conditions are discrete and in others continuous.

It will be recalled that Cantor proved that if we measure all the points on a continuous line, there are not enough integers to count them all. The Cantor diagonal method consists of writing all the points on a line lying say between 0 and 1 (or in any continuous interval) in decimal notation<sup>1</sup>. Let  $S_0, S_1, \dots, S_n, \dots$  be points on a straight line. We can presume that we can continuously count all the points by taking the decimals to an infinite number.

$$\begin{matrix}
 S_0 = & 0.X_{00} & X_{01} & X_{02} & X_{03} & \dots & X_{0n} & \dots \\
 S_1 = & 0.X_{10} & X_{11} & X_{12} & X_{13} & \dots & X_{1n} & \dots \\
 S_2 = & 0.X_{20} & X_{21} & X_{22} & X_{23} & \dots & X_{2n} & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 S_n = & 0.X_{n0} & X_{n1} & X_{n2} & X_{n3} & \dots & X_{nn} & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots
 \end{matrix}$$

It will, however, be noticed that a number, say  $S$ , will not be found in the enumeration if  $S$  is created such that

$$S \neq 0.X_{00} \cdot X_{11} \cdot X_{22} \cdot X_{33} \cdot \dots \cdot X_{nn} \cdot \dots$$

and each diagonal element is replaced.

This is because the particular diagonal element will always be different by definition. This demonstrates that if one tries to enumerate all the points continuously, there are an infinite number of missing entries showing that a countable  $\infty$  is different from a non-countable  $\infty$ .

This method can be used to determine the missing event from matrix (I) for each diagonal. If, for some reason, the events are all ordered, there will be only one such sequence.

After having shown the existence of two kinds of infinities, Cantor went on to show that an infinite number of infinities can be formed by what is known as the Cantor continuum. The method consists of showing that from a set of positive integers, it is possible to construct another set of integers with a different cardinal number<sup>2</sup>. If a set  $A$  consists of three integers (1, 2, 3), a set  $B$  can be constructed {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, and 0, i.e. with 8 different elements. As shown later, this continuum property can be used to get the levels of atoms if a proper correspondence is made. Cantor has in general shown that a set with  $n$  elements, can generate another set with  $2^n$  elements of a different cardinal number.

From these very general considerations, we proceed to consider the special case of atomic spectra and elementary particles.

In classical and relativistic theories, we can say that an event either happens or does not happen. In quantum mechanics, we have a very different situation where events are described by abstract wave functions whose interpretation depends on probability considerations.

The Uncertainty Principle further adds to complexity of defining the *conditions* for an *event*. Even this is not established until a measurement is made and digested by the *observer* with a consciousness.

In this present matrix formulation, because of the existence of quantized parameters, wave particle duality and of virtual states, it is necessary to assume that the matrix elements could have values continuously between 0 and 1. This could come about due to averaging effect over time or as a result of the influence of other parameters.

In general, the differential equation provides causal relations in classical and quantum mechanics. In quantum mechanics, the solution of the Schrodinger equation depends on the type of boundary conditions one imposes on it. Some quantities get quantized while others remain in a continuum. For example, for a particle in a box, energy is quantized while position remains in a continuum. Thus their cardinality is different. We have from Cantor's theory of the continuum that  $n$  entries from a lower cardinality will have for the next higher cardinality  $2^n$  entries.

Cardinality is used essentially to differentiate between the discrete and continuous. We consider an actual example, from atomic spectroscopy to apply the Continuum theory to quantum mechanics.

It is known that there is a certain natural width  $\Delta E_0$  corresponding to an intensity distribution of a spectral line emitted by an atom. In classical theory this is due to the reaction force of the emitted radiation on the emitting source. In quantum mechanics, following the perturbation theoretic treatment due to Weisskopf and Wigner (see ref. 3), one gets a formula for  $\Delta E_0$  which is consistent with the time - energy uncertainty relation where  $\Delta E_0$  is interpreted as an estimate of the accuracy with which an energy level is known, as  $\Delta t$  is the lifetime of the excited state of the atom. The continuum of the line width thus coexists with the discrete structure of the levels. At higher energies the levels will begin to overlap and a *continuum* of nonenumerable infinite number of states will come into existence.

Let us assume that the ratio of the energy width in the continuum state which is riding on the discrete structure and the energy of the level, be proportionate to the ratio of the number of discrete states ( $n$ ) and the next cardinal number given by  $2^n$ .

$$\text{i.e. } \rho (n / 2^n) = \Delta E_0 / E, \quad (1)$$

where  $\rho$  is some constant of conversion from cardinal states to energy states and  $\Delta E_0 \approx (h / \Delta t)$  is the spread of the level of energy  $E$  and is related to the time an atom can retain excess energy before re-emitting it in the form of photons.

We now apply the above formula to the case of a free electron falling to the various levels of the hydrogen atom. Table 1 gives the *cardinal levels*, the simplest series of the levels of the hydrogen atom and the energy widths.

**Table 1.** Cardinality and the width of spectral lines of hydrogen for constant  $\rho$

Card No.	Quantum No.	$2^n$	$n/2^n$	$E$ (ergs) Energy level	$\Delta E_0$ width (ergs)
1	8	2	0.500	$3.37 \times 10^{-13}$	$1.7 \times 10^{-19}$
2	7	4	0.500	4.39	2.2
3	6	8	0.375	5.99	2.2
4	5	16	0.250	8.61	2.2
5	4	32	0.156	13.45	2.1
6	3	64	0.094	24.00	2.25
7	2	128	0.055	53.90	3
8	1	256	0.031	214.00	6

The ratio of the values of  $n/2^n$  and  $\Delta E_0/E$  is nearly the same provided we allow for the variation of the value of  $\Delta E_0$ .  $\rho$  is some constant which has to have the value of  $10^{16}$ . It will be noticed that the three levels which are nearer the ground state could belong to a separate family having a different cardinality. These first three levels could be of cardinality 3 which in turn can give rise to 8 levels of higher cardinality.

From equation (1) it was shown that the natural width of the spectral lines of the hydrogen spectrum could be obtained using the Continuum Theory of Cantor. Here we show that the level spacing given by the theory is consistent with the quantum theory of the hydrogen atom.

Let  $D_1$  be the level spacing of the hydrogen atom as given by the Bohr formula.

$$D_1 = \text{Const } (1/p^2 - 1/q^2), \quad (2)$$

where  $p$  and  $q$  are the quantum numbers and the constant is the Rydberg constant. From equation (1) we have  $D_2$ , i.e.

$$D_2 = \rho \left( \frac{n}{2^n} - \frac{m}{2^m} \right) = \Delta E_0 \left( \frac{E_n - E_m}{E_m E_n} \right) \quad (3)$$

Table 2 gives the values of  $D_1$  and  $D_2$ . Here the cardinal number 8, which is also the number of discrete levels in hydrogen, corresponds to the quantum number  $q=1$ , and the ordering of the cardinal numbers is in the reverse order to the quantum numbers. The last column gives  $\rho/\Delta E_0$ .

It is known that the hydrogen lines are grouped in various series such as Lyman, Balmer, Paschen, etc. It, therefore, suggests that  $\rho$  and  $\Delta E_0$  need not necessarily be constant but can have quantized values. In Figure 1,  $n / 2En$  is plotted against  $-\text{Log } E$ . The degeneracy of the hydrogen lines makes it difficult to bring out the changes in the values of  $\rho$  and  $\Delta E_0$ . But the changes in the slopes are clearly visible as we move from series to series.

Using the equations  $\rho/\Delta E_0 = 0.06 \times 10^{14}$  (Table 2);  $\rho (8/2^8) = \Delta E_0/E$ ;  $\Delta E_0/E = \Delta \lambda_0/\lambda_0$ , where  $\Delta \lambda_0$  is the natural line width,  $\Delta \lambda_0 = 1.17 \times 10^{-4} \text{ \AA U}$ , and  $E \lambda = hc$ ,  $\rho$  takes the value of  $1 \times 10^6$ .

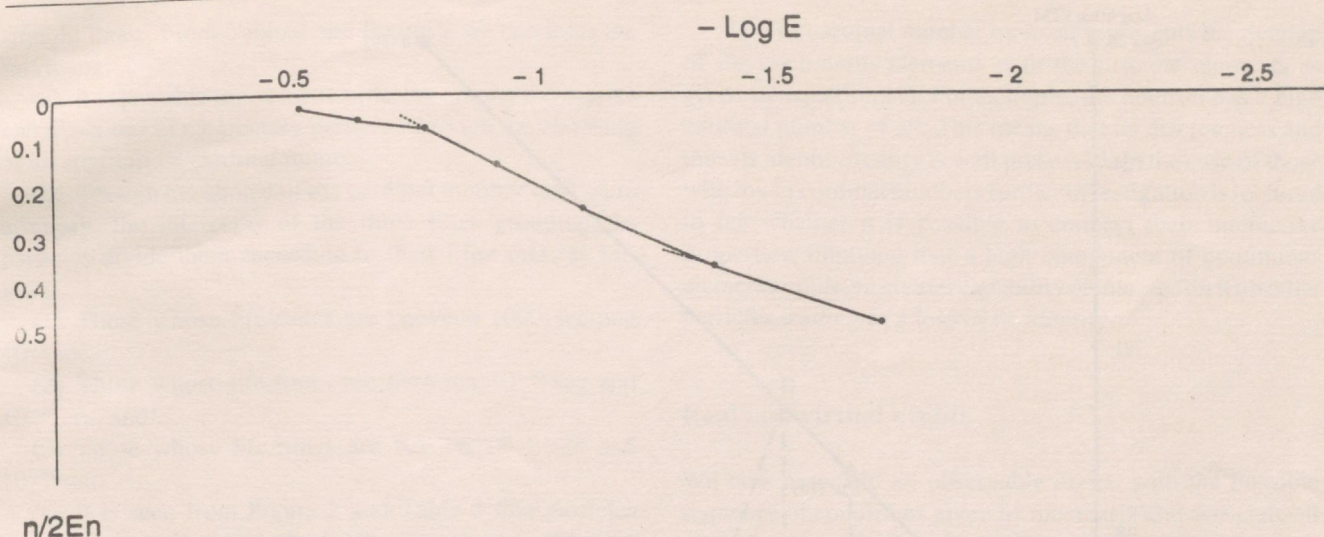


Figure 1.  $-\text{Log } E$  vs.  $n/2En$  showing the change in the values of  $\rho$  due to the formation of Lyman, Balmer and Paschen series. The changes in the slopes are clearly visible.

Table 2. Cardinality and the Bohr theory of the hydrogen atom

1	2	3	4	5	6	7	8
		$-\left(\frac{n}{2^n} - \frac{n-1}{2^{n-1}}\right) \times 2^8$				$D_1 = -R_H \frac{hc}{\rho^2 - \frac{1}{q^2}} \times 10^{12}$	
$n$	$n-1$	$(= D_2 / \rho)$	$E_n \quad E_{n-1} \times 10^{26}$	$10^{26} \times 2^8$	$(3) \times (4) \times 10^{26} \times 2^8$	$p \quad q$	$(\rho/\Delta E_0) \times 10^{-14}$ erg <sup>-1</sup>
8	7	6	11615		69690	2 1	16.5
7	6	10	1293		12930	3 2	3.06
6	5	16	324		5184	4 3	1.07
5	4	24	116		2784	5 4	0.495
4	3	32	51.57		1650	6 5	0.269
3	2	32	26.29		841	7 6	0.162
2	1	0	14.93		0	8 7	0.105

For  $\rho = 1 \times 10^{-6}$  and cardinality 8, i.e. quantum number 1,  $\Delta\lambda$  the spectral width comes out to be  $1.17 \times 10^{-4}$  ÅU, which is the value given by quantum mechanics.

This value establishes a consistency between Table 1 and 2 and also establishes a consistency between the Bohr theory of the hydrogen atom and the present work.

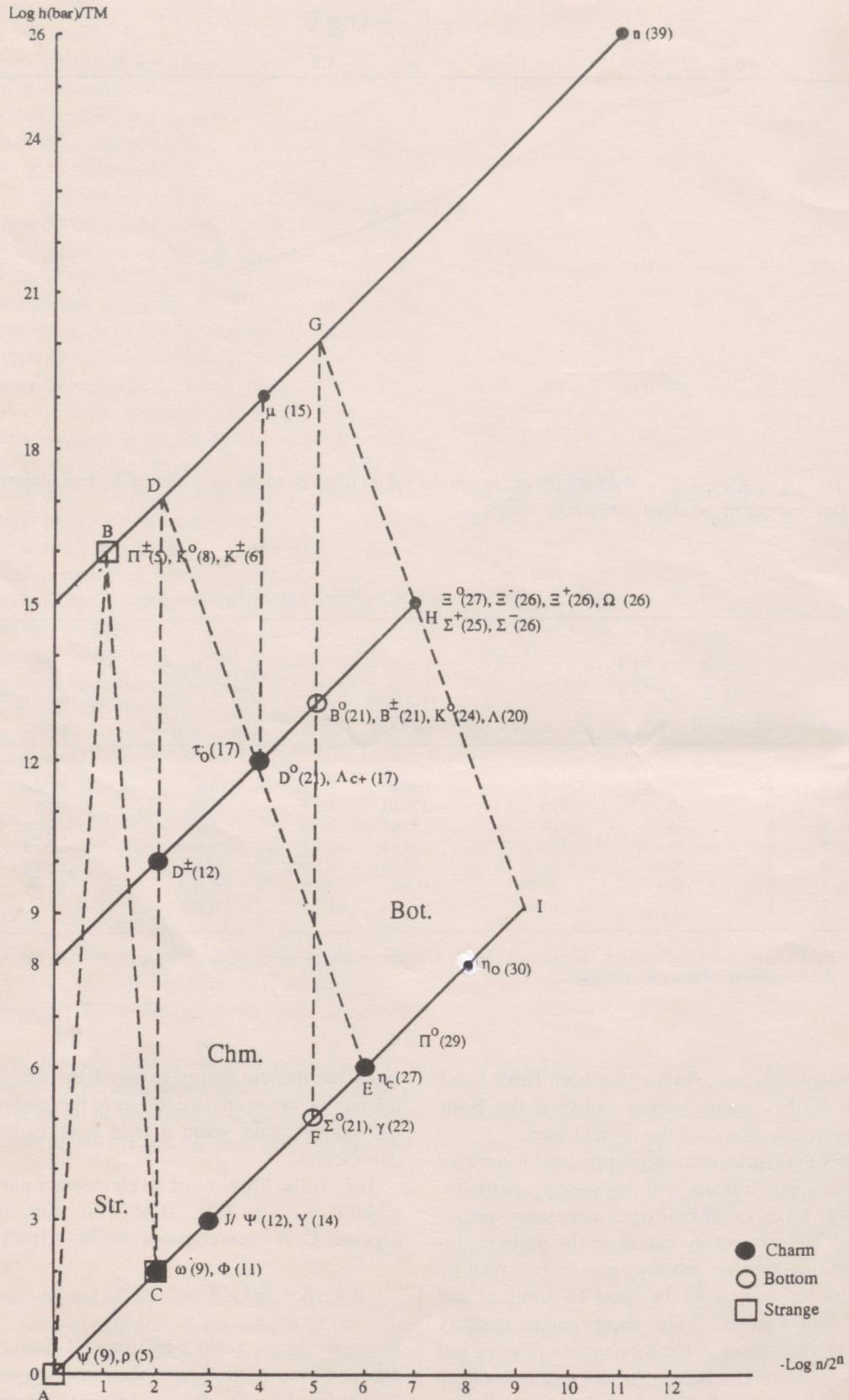
We now apply the Cantor continuum principle to another branch of physics which deals with elementary particles. Earlier the energy levels of the hydrogen atom were considered because of the elementary nature of the particles involved, and the fact that the problem had been solved by quantum mechanics and could be used to compare the results of the two theories. The theory of elementary particles concerns problems in the forefront of physics and the solution is far from complete. An attempt is made to derive some order in the distribution of masses of elementary particles as it is essentially a problem of energy levels. The Cantor continuum principle is so general that it can be used in principle to the study of all physical problems involving discreteness and continuity. It is not possible to

apply the present theory to problems where there is close interaction between particles as in the case of nuclear physics, but it could stand a trial with isolated elementary particles.

If  $T_0$  is the life-time of an elementary particle, its energy width is given by  $\hbar/T_0$ . If  $M$  is the mass of the particle in ergs and  $C$  is some constant, we have from equation (1)

$$\Delta E_0/E = \hbar/T_0 M = C \cdot n/2^n.$$

Since we know from experimental data the lifetimes of many of these particles and their masses, we can determine the values of  $C \cdot n/2^n$ . By considering different values for the constant  $C$  and by selecting the values of the cardinal number for various values of  $C$ , it can be shown that for values of  $C = 10E - 1$ ,  $C = 10E - 8$  and  $C = 10E - 15$ , the values of  $\log \hbar/TM$  when plotted against  $\log n/2^n$  give



**Figure 2.** Families of elementary particle and cardinal number determining discreteness and continuity. (Figures in brackets denote cardinal numbers.) For representing the information from Table 3 in this figure the nearest integers have been used for the Log. values.

straight lines. From Table 3 and Figure 2 we can infer the following:

(a) It is possible to see some ordering among the masses and life-times of elementary particles through the choosing of an appropriate cardinal number.

(b) Though the choice of the cardinal number may seem arbitrary, the intercepts of the three lines grouping the particles divide them according to their life-times, as follows:

(1) Those whose life-times are between 1000 sec and  $10^{-8}$  sec.

(2) Those whose life-times are between  $10^{-10}$  sec and  $10^{-13}$  sec, and

(3) Those whose life-times are beyond  $10^{-17}$  sec and  $10^{-24}$  sec.

(c) It is seen from Figure 2 and Table 3 that particles described as strange (Str), charm (Chm) and bottom (Bot) are found in the sides of triangles ABC, CDE and FGI respectively. It suggests that  $\Psi'$  should be a strange particle. All particles found at H are hyperons. One could expect new particles at points marked D, G and I.

(d) The cardinal number by itself represents the overlap of the continuous elements with the discrete elements as given by equation (1). For example, the neutron has a high cardinal number of 39. This means that its discreteness and thus its identity/reality is well preserved. In the case of those with lower cardinal numbers further investigation is required to see whether it is possible to connect their interactive properties, implying that a high component of continuous elements implies a greater capability of interaction with other particles leading to a loss of its identity.

**Real and virtual events**

We now associate all observable *events* with the possible sequence of conditions given by the matrix and associate all *virtual events*, which as we know are predicted by quantum theory, with an infinite number of the missing sequences, i.e. those different from the diagonal elements of the matrix.

If all the observable physical *events* are described by the elements of the matrix and if all virtual *events* are associ-

**Table 3.** Cardinality and ordering of elementary particles

1	2	3	4	5	6	7
Particle	Mass ( <i>M</i> ) (in ergs)	Decay time ( <i>T</i> ) (secs)	<i>h</i> (bar)/ <i>TM</i>	<i>n</i>	<i>n</i> /2 <sup>n</sup>	<i>C</i>
<b>Mesons and baryons</b>						
1. $\Pi^{\pm}$ Pion	0.2243E-3	2.6E-08	1.8E-16	5	1.5E-1	E-15
2. $\Pi^0$ Pion	0.216E-3	8.7E-17	0.56E-7	29	0.54E-7	E-0
3. $\rho$ Meson	1.232E-3	4.3E-24	0.198E-0	5	0.156E-0	E-0
4. $\omega$ Meson	1.25E-3	0.67E-22	1.25E-2	9	1.82E-2	E-0
5. $\mu$ Meson	0.169E-3	2.197E-6	2.98E-19	15	3.1E-4	E-15
6. $\tau$ Meson	2.854E-3	0.3E-12	1.2E-12	17	1.3E-4	E-8
<b>Strange particles</b>						
7. $K^{\pm}$ Kaon	0.789E-3	1.24E-8	1.07E-16	6	0.94E-1	E-15
8. $\phi$ Meson	1.019E-3	1.5E-22	6.869E-3	11	5.0E-3	E-0
9. $K^0$ Kaon	0.789E-3	5.2E-8 or 8.9E-11	0.25E-16 0.15E-13	8 24	0.31E-1 1.43E-6	E-15 E-8
<b>Charm particles</b>						
10. $D^0$ Meson	2.97E-3	4.4E-13	0.8E-12	21	1.29E-4	E-8
11. $D^{\pm}$ Meson	2.99E-13	9.2E-13	0.38E-10	12	0.29E-2	E-8
12. $\eta_c$ Meson	0.876E-3	6.0E-18	0.2E-6	27	0.19E-6	E-0
13. $J/\Psi$ Meson	3.097E-3	1.0E-20	3.4E-3	12	2.9E-3	E-0
14. $\Psi'$ Meson	3.77E-3	0.26E-22	0.0107E-0	9	0.018E-0	E-0
<b>Bottom</b>						
15. $B^{\pm}$ Meson	8.43E-3	14.2E-13	0.88E-13	21	1.0E-5	E-8
16. $B^0$ Meson	8.44E-3	14.2E-13	0.38E-13	21	1.0E-5	E-8
17. $\gamma$ Meson	15.13E-3	1.5E-20	0.46E-5	22	0.52E-5	E-0
18. $\eta_0$ Meson	5.48E-3	6.0E-18	3.19E-8	30	2.79E-8	E-0
19. <i>n</i> Neutron	1.504E-3	898	7.8E-26	39	7.0E-11	E-15
<b>Hyperons</b>						
20. $\Lambda$	1.784E-3	2.6E-10	2.2E-13	20	1.91E-5	E-8
21. $\Sigma^+$	1.902E-3	0.8E-10	6.9E-15	25	7.45E-7	E-8
22. $\Sigma^0$	1.907E-3	5.8E-20	0.9E-5	21	1.00E-5	E-0
23. $\Sigma^-$	1.915E-3	1.48E-10	3.7E-15	26	3.8E-7	E-8
24. $\Xi^0$	2.103E-3	2.9E-10	1.7E-15	27	2.0E-7	E-8
25. $\Xi^-$	2.113E-3	1.642E-10	3.0E-15	26	3.8E-7	E-8
26. $\Omega$	2.675E-3	0.822E-10	4.7E-15	26	3.8E-7	E-8
$\Xi^+$						
27. $\Lambda_{cs}$	3.649E-3	2.3E-13	1.2E-12	17	1.29E-4	E-8

ated with the missing sequences, which now form a set by themselves, it provides for a 'duality'. This arises because for every *real event*  $A$ , there is a *virtual event*  $A'$ , opposite in character, since a one-to-one correlation exists between  $A$  and  $A'$ . Every sequence in the missing set arises from an element denied in the diagonal set.

Whether the wave and particle nature of matter which are both observable, are found in  $A$  or  $A'$ , depends on whether *events* belong to either  $(A)$ 's or  $(A')$ 's respectively. It also depends on the validity of the Complementarity Principle of Bohr. If particle and wave behaviour are described as both belonging to  $A$  only, then the wave and particle property can be observed simultaneously which is contrary to that predicted by Bohr (see ref. 4). If one is from *event* set  $A$  and the other from set  $A'$ , their detection will be mutually exclusive.

### Conclusion

This work is based on Buddhist Logic of Conditional Reality, Kshana and Nothingness<sup>5</sup>. In the matrix I, if the causes of events are dependent on each other it leads to what is known as 'conditioned reality'. While in classical physics, space and time are separate, in reality this is not so and it

leads to new physics. In quantum mechanics, the wave nature is merged with particle nature and leads to many paradoxes but this is all a part of 'conditioned reality'. 'Kshana' is the theory of time averages and 'nothingness' is the existence of virtual states. These are described in a forthcoming paper entitled 'Causality, cardinality and conditioned reality' in the Project of History of Indian Science, Philosophy and Culture, Calcutta.

1. Ramanna, R., *Sci. Cult.*, 969, **35**, 649 - 660.
2. Courant, R. and Robbins, H., *What is Mathematics: An Elementary Approach to Ideas and Methods*, Oxford University Press, 1978, pp. 84.
3. Heitler, W., *The Quantum Theory of Radiation*, 2nd edn., Oxford University Press, 1944.
4. Ghose, P., Home, D. and Agarwal, G. S., *Phy. Lett. A*, 1992, **168**, 95-99.
5. Stcherbatsky, F. Th., *Buddhist Logic*, Oriental Books, New Delhi, 1984; Originally printed in *Bibliotheca Buddhica Series*, 1930, vol. 26, Parts 1 and 2.

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# Concept of discreteness, continuity and the Cantor continuum theory as related to the life-time and masses of elementary particles

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It is shown that with the use of the Cantor continuum theory it is possible to derive the systematics of the lifetime and masses of elementary particles.

The theory gives a new meaning to the foundations of quantum theory as Cantor's theory deals with discreteness and continuity, which is the essence of quantum mechanics.

LET  $A_1, A_2, \dots, A_n$  be the various possible physical events and  $a_{11}, a_{12}, a_{13}, \dots$ , etc. be the various conditions which generate the events, e.g.  $A_1$  takes place when conditions  $a_{11}, a_{12}, a_{13}, \dots, a_{1n}$  exist. We write all possibilities in a matrix form as follows:

Events	Sequence of conditions					
$A_1$	$a_{11}$	$a_{12}$	$a_{13}$	$\dots$	$a_{1n}$	$\dots$
$A_2$	$a_{21}$	$a_{22}$	$a_{23}$	$\dots$	$a_{2n}$	$\dots$
—	—	—	—	—	—	—
$A_n$	$a_{n1}$	$a_{n2}$	$a_{n3}$	$\dots$	$a_{nm}$	$\dots$ , etc.

(I)

The values of the  $a$ 's can be 1 or 0 depending on whether the condition is relevant or not to that particular event.

In order to convert the above matrix to represent actual physical quantities and be able to interpret them as physical laws, we invoke the Cantor diagonal construction which Cantor used in showing the difference between the various types of transfinite numbers, particularly between countable and non-countable infinities. The theory in itself is a mathematical differentiation between continuous and discrete quantities and can also be made the starting point for the foundations of quantum mechanics, since it deals with physical quantities, which under certain conditions are discrete and in others continuous.

It will be recalled that Cantor proved that if we measure all the points on a continuous line, there are not enough integers to count them all. The Cantor diagonal method consists of writing all the points on a line lying say between 0 and 1 (or in any continuous interval) in decimal notation<sup>1</sup>. Let  $S_0, S_1, \dots, S_n, \dots$  be points on a straight line. We can presume that we can continuously count all the points by taking the decimals to an infinite number.

$$\begin{matrix}
 S_0 = 0.X_{00} & X_{01} & X_{02} & X_{03} & \dots & X_{0n} & \dots \\
 S_1 = 0.X_{10} & X_{11} & X_{12} & X_{13} & \dots & X_{1n} & \dots \\
 S_2 = 0.X_{20} & X_{21} & X_{22} & X_{23} & \dots & X_{2n} & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 S_n = 0.X_{n0} & X_{n1} & X_{n2} & X_{n3} & \dots & X_{nm} & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots
 \end{matrix}$$

It will, however, be noticed that a number, say  $S$ , will not be found in the enumeration if  $S$  is created such that

$$S \neq 0.X_{00} \cdot X_{11} \cdot X_{22} \cdot X_{33} \cdot \dots \cdot X_{nn} \cdot \dots$$

and each diagonal element is replaced.

This is because the particular diagonal element will always be different by definition. This demonstrates that if one tries to enumerate all the points continuously, there are an infinite number of missing entries showing that a countable  $\infty$  is different from a non-countable  $\infty$ .

This method can be used to determine the missing event from matrix (I) for each diagonal. If, for some reason, the events are all ordered, there will be only one such sequence.

After having shown the existence of two kinds of infinities, Cantor went on to show that an infinite number of infinities can be formed by what is known as the Cantor continuum. The method consists of showing that from a set of positive integers, it is possible to construct another set of integers with a different cardinal number<sup>2</sup>. If a set  $A$  consists of three integers (1, 2, 3), a set  $B$  can be constructed {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, and 0, i.e. with 8 different elements. As shown later, this continuum property can be used to get the levels of atoms if a proper correspondence is made. Cantor has in general shown that a set with  $n$  elements, can generate another set with  $2^n$  elements of a different cardinal number.

From these very general considerations, we proceed to consider the special case of atomic spectra and elementary particles.

In classical and relativistic theories, we can say that an event either happens or does not happen. In quantum mechanics, we have a very different situation where events are described by abstract wave functions whose interpretation depends on probability considerations.

The Uncertainty Principle further adds to complexity of defining the *conditions* for an *event*. Even this is not established until a measurement is made and digested by the *observer* with a consciousness.

In this present matrix formulation, because of the existence of quantized parameters, wave particle duality and of virtual states, it is necessary to assume that the matrix elements could have values continuously between 0 and 1. This could come about due to averaging effect over time or as a result of the influence of other parameters.

In general, the differential equation provides causal relations in classical and quantum mechanics. In quantum mechanics, the solution of the Schrodinger equation depends on the type of boundary conditions one imposes on it. Some quantities get quantized while others remain in a continuum. For example, for a particle in a box, energy is quantized while position remains in a continuum. Thus their cardinality is different. We have from Cantor's theory of the continuum that  $n$  entries from a lower cardinality will have for the next higher cardinality  $2^n$  entries.

Cardinality is used essentially to differentiate between the discrete and continuous. We consider an actual example, from atomic spectroscopy to apply the Continuum theory to quantum mechanics.

It is known that there is a certain natural width  $\Delta E_0$  corresponding to an intensity distribution of a spectral line emitted by an atom. In classical theory this is due to the reaction force of the emitted radiation on the emitting source. In quantum mechanics, following the perturbation theoretic treatment due to Weisskopf and Wigner (see ref. 3), one gets a formula for  $\Delta E_0$  which is consistent with the time - energy uncertainty relation where  $\Delta E_0$  is interpreted as an estimate of the accuracy with which an energy level is known, as  $\Delta t$  is the lifetime of the excited state of the atom. The continuum of the line width thus coexists with the discrete structure of the levels. At higher energies the levels will begin to overlap and a *continuum* of nonenumerable infinite number of states will come into existence.

Let us assume that the ratio of the energy width in the continuum state which is riding on the discrete structure and the energy of the level, be proportionate to the ratio of the number of discrete states ( $n$ ) and the next cardinal number given by  $2^n$ .

$$\text{i.e. } \rho (n / 2^n) = \Delta E_0 / E, \tag{1}$$

where  $\rho$  is some constant of conversion from cardinal states to energy states and  $\Delta E_0 \approx (h / \Delta t)$  is the spread of the level of energy  $E$  and is related to the time an atom can retain excess energy before re-emitting it in the form of photons.

We now apply the above formula to the case of a free electron falling to the various levels of the hydrogen atom. Table 1 gives the *cardinal levels*, the simplest series of the levels of the hydrogen atom and the energy widths.

**Table 1.** Cardinality and the width of spectral lines of hydrogen for constant  $\rho$

Card No.	Quantum No.	$2^n$	$n/2^n$	$E$ (ergs) Energy level	$\Delta E_0$ width (ergs)
1	8	2	0.500	$3.37 \times 10^{-13}$	$1.7 \times 10^{-19}$
2	7	4	0.500	4.39	2.2
3	6	8	0.375	5.99	2.2
4	5	16	0.250	8.61	2.2
5	4	32	0.156	13.45	2.1
6	3	64	0.094	24.00	2.25
7	2	128	0.055	53.90	3
8	1	256	0.031	214.00	6

The ratio of the values of  $n/2^n$  and  $\Delta E_0/E$  is nearly the same provided we allow for the variation of the value of  $\Delta E_0$ .  $\rho$  is some constant which has to have the value of  $10^{-6}$ . It will be noticed that the three levels which are nearer the ground state could belong to a separate family having a different cardinality. These first three levels could be of cardinality 3 which in turn can give rise to 8 levels of higher cardinality.

From equation (1) it was shown that the natural width of the spectral lines of the hydrogen spectrum could be obtained using the Continuum Theory of Cantor. Here we show that the level spacing given by the theory is consistent with the quantum theory of the hydrogen atom.

Let  $D_1$  be the level spacing of the hydrogen atom as given by the Bohr formula.

$$D_1 = \text{Const } (1/p^2 - 1/q^2), \tag{2}$$

where  $p$  and  $q$  are the quantum numbers and the constant is the Rydberg constant. From equation (1) we have  $D_2$ , i.e.

$$D_2 = \rho \left( \frac{n}{2^n} - \frac{m}{2^m} \right) = \Delta E_0 \left( \frac{E_n - E_m}{E_m E_n} \right) \tag{3}$$

Table 2 gives the values of  $D_1$  and  $D_2$ . Here the cardinal number 8, which is also the number of discrete levels in hydrogen, corresponds to the quantum number  $q=1$ , and the ordering of the cardinal numbers is in the reverse order to the quantum numbers. The last column gives  $\rho/\Delta E_0$ .

It is known that the hydrogen lines are grouped in various series such as Lyman, Balmer, Paschen, etc. It, therefore, suggests that  $\rho$  and  $\Delta E_0$  need not necessarily be constant but can have quantized values. In Figure 1,  $n / 2En$  is plotted against  $-\text{Log } E$ . The degeneracy of the hydrogen lines makes it difficult to bring out the changes in the values of  $\rho$  and  $\Delta E_0$ . But the changes in the slopes are clearly visible as we move from series to series.

Using the equations  $\rho/\Delta E_0 = 0.06 \times 10^{14}$  (Table 2);  $\rho (8/2^8) = \Delta E_0/E$ ;  $\Delta E_0/E = \Delta\lambda_0/\lambda$ , where  $\Delta\lambda_0$  is the natural line width,  $\Delta\lambda_0 = 1.17 \times 10^{-4}$  ÅU, and  $E\lambda = hc$ ,  $\rho$  takes the value of  $1 \times 10^{-6}$ .

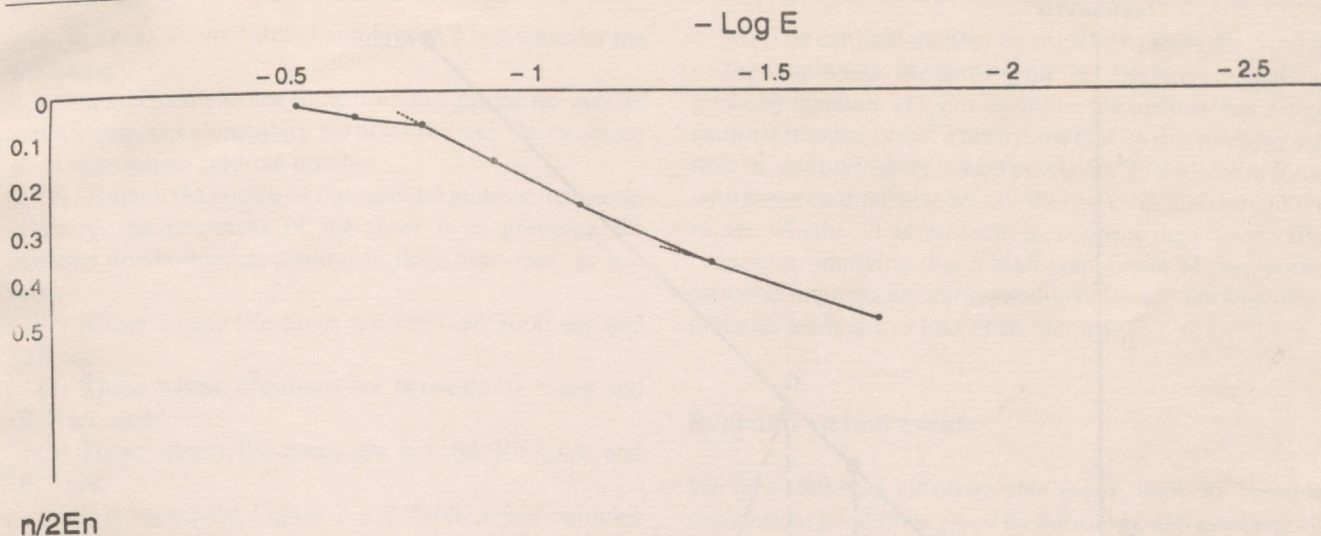


Figure 1. - Log E vs.  $n/2En$  showing the change in the values of  $p$  due to the formation of Lyman, Balmer and Paschen series. The changes in the slopes are clearly visible.

Table 2. Cardinality and the Bohr theory of the hydrogen atom

1	2	3	4	5	6	7	8	
		$-\left(\frac{n}{2^n} - \frac{n-1}{2^{n-1}}\right) \times 2^8$						
$n$	$n-1$	$(= D_2 / \rho)$	$E_n E_{n-1} \times 10^{26}$	$(3) \times (4) \times 10^{26} \times 2^8$	$p$	$q$	$D_1 = -\frac{R}{n} \frac{hc}{\rho}$ $\left(\frac{1}{p^2} - \frac{1}{q^2}\right) \times 10^{12}$	$(\rho/\Delta E_0) \times 10^{-14}$ erg <sup>-1</sup>
8	7	6	11615	69690	2	1	16.5	0.06
7	6	10	1293	12930	3	2	3.06	0.06
6	5	16	324	5184	4	3	1.07	0.052
5	4	24	116	2784	5	4	0.495	0.045
4	3	32	51.57	1650	6	5	0.269	0.041
3	2	32	26.29	841	7	6	0.162	0.049
2	1	0	14.93	0	8	7	0.105	—

For  $\rho = 1 \times 10^{-6}$  and cardinality 8, i.e. quantum number 1,  $\Delta\lambda$  the spectral width comes out to be  $1.17 \times 10^{-4}$  ÅU, which is the value given by quantum mechanics.

This value establishes a consistency between Table 1 and 2 and also establishes a consistency between the Bohr theory of the hydrogen atom and the present work.

We now apply the Cantor continuum principle to another branch of physics which deals with elementary particles. Earlier the energy levels of the hydrogen atom were considered because of the elementary nature of the particles involved, and the fact that the problem had been solved by quantum mechanics and could be used to compare the results of the two theories. The theory of elementary particles concerns problems in the forefront of physics and the solution is far from complete. An attempt is made to derive some order in the distribution of masses of elementary particles as it is essentially a problem of energy levels. The Cantor continuum principle is so general that it can be used in principle to the study of all physical problems involving discreteness and continuity. It is not possible to

apply the present theory to problems where there is close interaction between particles as in the case of nuclear physics, but it could stand a trial with isolated elementary particles.

If  $T_0$  is the life-time of an elementary particle, its energy width is given by  $\hbar/T_0$ . If  $M$  is the mass of the particle in ergs and  $C$  is some constant, we have from equation (1)

$$\Delta E_0/E = \hbar/T_0 M = C \cdot n/2^n.$$

Since we know from experimental data the lifetimes of many of these particles and their masses, we can determine the values of  $C \cdot n/2^n$ . By considering different values for the constant  $C$  and by selecting the values of the cardinal number for various values of  $C$ , it can be shown that for values of  $C = 10E - 1$ ,  $C = 10E - 8$  and  $C = 10E - 15$ , the values of  $\log \hbar/TM$  when plotted against  $\log n/2^n$  give

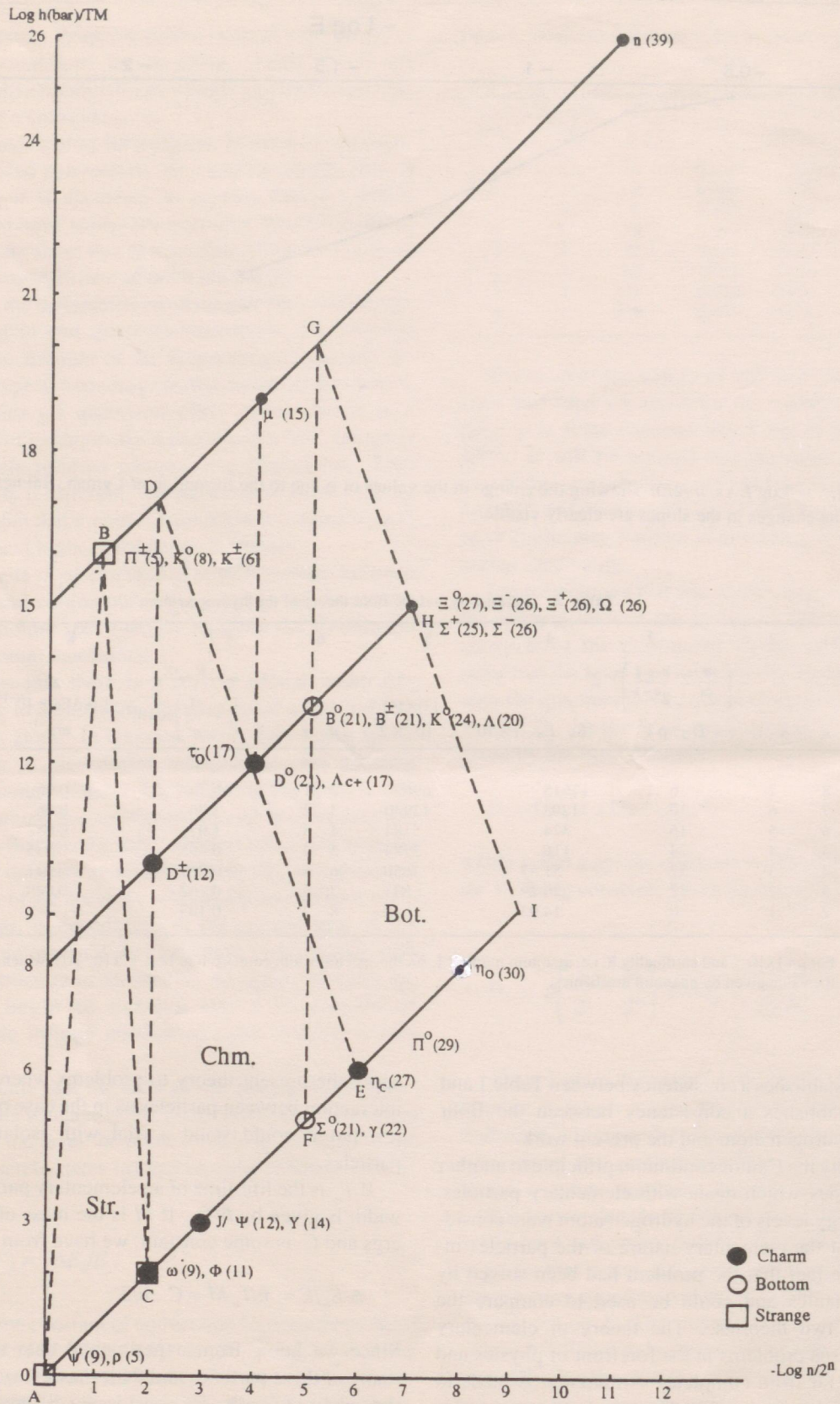


Figure 2. Families of elementary particle and cardinal number determining discreteness and continuity. (Figures in brackets denote cardinal numbers.) For representing the information from Table 3 in this figure the nearest integers have been used for the Log. values.

straight lines. From Table 3 and Figure 2 we can infer the following:

(a) It is possible to see some ordering among the masses and life-times of elementary particles through the choosing of an appropriate cardinal number.

(b) Though the choice of the cardinal number may seem arbitrary, the intercepts of the three lines grouping the particles divide them according to their life-times, as follows:

(1) Those whose life-times are between 1000 sec and  $10^{-8}$  sec.

(2) Those whose life-times are between  $10^{-10}$  sec and  $10^{-13}$  sec, and

(3) Those whose life-times are beyond  $10^{-17}$  sec and  $10^{-24}$  sec.

(c) It is seen from Figure 2 and Table 3 that particles described as strange (Str), charm (Chm) and bottom (Bot) are found in the sides of triangles ABC, CDE and FGI respectively. It suggests that  $\Psi'$  should be a strange particle. All particles found at H are hyperons. One could expect new particles at points marked D, G and I.

(d) The cardinal number by itself represents the overlap of the continuous elements with the discrete elements as given by equation (1). For example, the neutron has a high cardinal number of 39. This means that its discreteness and thus its identity/reality is well preserved. In the case of those with lower cardinal numbers further investigation is required to see whether it is possible to connect their interactive properties, implying that a high component of continuous elements implies a greater capability of interaction with other particles leading to a loss of its identity.

**Real and virtual events**

We now associate all observable *events* with the possible sequence of conditions given by the matrix and associate all *virtual events*, which as we know are predicted by quantum theory, with an infinite number of the missing sequences, i.e. those different from the diagonal elements of the matrix.

If all the observable physical *events* are described by the elements of the matrix and if all virtual *events* are associ-

**Table 3.** Cardinality and ordering of elementary particles

1	2	3	4	5	6	7
Particle	Mass (M) (in ergs)	Decay time (T) (secs)	$h$ (bar)/TM	$n$	$n/2^n$	C
<b>Mesons and baryons</b>						
1. $\pi^\pm$ Pion	0.2243E-3	2.6E-08	1.8E-16	5	1.5E-1	E-15
2. $\pi^0$ Pion	0.216E-3	8.7E-17	0.56E-7	29	0.54E-7	E-0
3. $\rho$ Meson	1.232E-3	4.3E-24	0.198E-0	5	0.156E-0	E-0
4. $\omega$ Meson	1.25E-3	0.67E-22	1.25E-2	9	1.82E-2	E-0
5. $\mu$ Meson	0.169E-3	2.197E-6	2.98E-19	15	3.1E-4	E-15
6. $\tau$ Meson	2.854E-3	0.3E-12	1.2E-12	17	1.3E-4	E-8
<i>Strange particles</i>						
7. $K^\pm$ Kaon	0.789E-3	1.24E-8	1.07E-16	6	0.94E-1	E-15
8. $\phi$ Meson	1.019E-3	1.5E-22	6.869E-3	11	5.0E-3	E-0
9. $K^0$ Kaon	0.789E-3	5.2E-8 or 8.9E-11	0.25E-16 0.15E-13	8 24	0.31E-1 1.43E-6	E-15 E-8
<i>Charm particles</i>						
10. $D^0$ Meson	2.97E-3	4.4E-13	0.8E-12	21	1.29E-4	E-8
11. $D^\pm$ Meson	2.99E-13	9.2E-13	0.38E-10	12	0.29E-2	E-8
12. $\eta_c$ Meson	0.876E-3	6.0E-18	0.2E-6	27	0.19E-6	E-0
13. $J/\Psi$ Meson	3.097E-3	1.0E-20	3.4E-3	12	2.9E-3	E-0
14. $\Psi'$ Meson	3.77E-3	0.26E-22	0.0107E-0	9	0.018E-0	E-0
<i>Bottom</i>						
15. $B^\pm$ Meson	8.43E-3	14.2E-13	0.88E-13	21	1.0E-5	E-8
16. $B^0$ Meson	8.44E-3	14.2E-13	0.38E-13	21	1.0E-5	E-8
17. $\gamma$ Meson	15.13E-3	1.5E-20	0.46E-5	22	0.52E-5	E-0
18. $\eta_b$ Meson	5.48E-3	6.0E-18	3.19E-8	30	2.79E-8	E-0
19. $n$ Neutron	1.504E-3	898	7.8E-26	39	7.0E-11	E-15
<b>Hyperons</b>						
20. $\Lambda$	1.784E-3	2.6E-10	2.2E-13	20	1.91E-5	E-8
21. $\Sigma^+$	1.902E-3	0.8E-10	6.9E-15	25	7.45E-7	E-8
22. $\Sigma^0$	1.907E-3	5.8E-20	0.9E-5	21	1.00E-5	E-0
23. $\Sigma^-$	1.915E-3	1.48E-10	3.7E-15	26	3.8E-7	E-8
24. $\Xi^0$	2.103E-3	2.9E-10	1.7E-15	27	2.0E-7	E-8
25. $\Xi^-$	2.113E-3	1.642E-10	3.0E-15	26	3.8E-7	E-8
26. $\Omega$	2.675E-3	0.822E-10	4.7E-15	26	3.8E-7	E-8
27. $\Lambda_{cs}$	3.649E-3	2.3E-13	1.2E-12	17	1.29E-4	E-8

ated with the missing sequences, which now form a set by themselves, it provides for a 'duality'. This arises because for every *real event*  $A$ , there is a *virtual event*  $A'$ , opposite in character, since a one-to-one correlation exists between  $A$  and  $A'$ . Every sequence in the missing set arises from an element denied in the diagonal set.

Whether the wave and particle nature of matter which are both observable, are found in  $A$  or  $A'$ , depends on whether *events* belong to either ( $A$ )'s or ( $A'$ )'s respectively. It also depends on the validity of the Complementarity Principle of Bohr. If particle and wave behaviour are described as both belonging to  $A$  only, then the wave and particle property can be observed simultaneously which is contrary to that predicted by Bohr (see ref. 4). If one is from *event* set  $A$  and the other from set  $A'$ , their detection will be mutually exclusive.

### Conclusion

This work is based on Buddhist Logic of Conditional Reality, Kshana and Nothingness<sup>5</sup>. In the matrix I, if the causes of events are dependent on each other it leads to what is known as 'conditioned reality'. While in classical physics, space and time are separate, in reality this is not so and it

leads to new physics. In quantum mechanics, the wave nature is merged with particle nature and leads to many paradoxes but this is all a part of 'conditioned reality'. 'Kshana' is the theory of time averages and 'nothingness' is the existence of virtual states. These are described in a forthcoming paper entitled 'Causality, cardinality and conditioned reality' in the Project of History of Indian Science, Philosophy and Culture, Calcutta.

1. Ramanna, R., *Sci. Cult.*, 1969, **35**, 649 - 660.
2. Courant, R. and Robbins, H., *What is Mathematics: An Elementary Approach to Ideas and Methods*, Oxford University Press, 1978, pp. 84.
3. Heitler, W., *The Quantum Theory of Radiation*, 2nd edn., Oxford University Press, 1944.
4. Ghose, P., Home, D. and Agarwal, G. S., *Phys. Lett. A*, 1992, **168**, 95-99.
5. Stecherbatsky, F. Th., *Buddhist Logic*, Oriental Books, New Delhi, 1984; Originally printed in *Bibliotheca Buddhica Series*, 1930, vol. 26, Parts 1 and 2.

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November 22, 1993

My dear Sreekantan,

It was nice to hear from you again after a gap of some weeks. I am glad Ratna enjoyed reading my book on music. It also can give rise to controversies.

We have tried our best to group the other 9 particles also into the scheme of prime numbers. But instead of playing around too much with the numbers, I have included another paragraph to the paper which is to appear in "The Project of History of Indian Science, Philosophy and Culture," Calcutta. It reads as follows:-

"It is interesting to note that out of 32 particles, 23 of them fall at points where  $-\text{Log}_2 n/2En$  (implying  $-\text{Log}_2 \frac{n}{2En}$ ) is a prime number. The association of sets of particles with prime numbers leads to a clustering of cardinal numbers around the prime number concerned. Thus, cardinal numbers between 5 & 8, 8 & 12, 14 & 16, 21 & 24, 25 & 30 and 39 & 42 correspond to 1, 2, 3, 5, 7 and 11 respectively. (It expresses the same idea without disturbing the classification of Strange, Charm & Bottom particles.)

The significance of these fundamental connections is yet to be investigated."

We are eagerly looking forward to your return,

With kind regards,

Yours sincerely,

*R. Ramanna*

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