Geometry in Nature

Due to mechanical necessity

Elliptic path of the earth round the sun

spherical shape of the rain drop ; Soap bubble

Symmetrical pattern of the snow flake.

Spiral forms in certain shells.

Pigeon st path.

Another example where great credit has been given for geometrical sagacity is found in the ~~st~~ cell structure of the honey bee. The cleverness of the bee excited the admiration of Pappus some 1600 yrs ago!—

Hexagonal form: fill up space, & contains most honey.

the least of the 3 shapes.

Close packed bees act as cylinders; ~~the~~ due to warmth wax melts & takes this shape.

Ablest geometer among the animals is the spider: eg. epeiridae.

Lays out a rim, the radii, centre wd.

R.W.G. Hingston: A naturalist in Hindusthan, London 1923. p.3.

2 amazing features

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2 amazing features

D'Arcy Thomson "On Growth &

Euler, contemporary of Linnaeus, carried the subject of regular solids beyond where Pythagoras, Plato, Euclid & Archimedes had left it. He drew up a binomial nomenclature based on the no. of their corners or vertices & sides or faces. eg. 8 corners & 7 faces Octogonum heptaedrum (analogy betw this & Linnaeus's botanical classifn. & nomenclature (eg. Hexandria trigynia) is very close & curious

Simple Theorem of wh Euler, acc. to T, was vastly proud,

Analogy with Phase rule of Gibbs.

$$C + F = E + 2$$

C = no. of corners
F " faces
E " edges

In a regular polygon each $\angle^{le} = 2 - \frac{4}{C}$ rt $\angle^{s}$.

(The sum of the plane $\angle^{s}$ at each corner of a regular or isogonal polyhedron $= 4 - \frac{8}{C}$ rt $\angle^{s}$.

i.e. determined by the no. of its corners, ~~rather than~~ & not by the no. of its sides. — a result wh delighted Euler, & led him to base his primary or generic classifn of polys on no. & not on their corners rather than their sides.

(See de Morgan, article on Polyhedron in Penny Cyclopaedia)

Platonic solids or regular solids (tetrahedron, octa, icosa, cube, reg. dodeca.) have all their corners & all faces alike, are isogonal & isohedral.

Semi-regular solids (a) Archimedean isogonal 13 in no.
eg. solid formed of six sq & 8 hexagons ~~—~~ (cubo octahedron)
Encloses a given vol. with min. area = space filler.

(b) Catalan(ic) or isohedral also 13 in no eg. rhombic dodecahedron. Known to ancients ~~Rest~~ discovered by Catalan 75 yrs ago.

A sphere can be circumscribed about Ar. solid containing all the corners
" inscribed in Cat. " touching ~~sides~~ faces

Method of constructing the Cat. solids is to divide &

into reqd no of similar ≠ segments, ~~If~~ ~~Hence~~ requires knowledge of Spherical trigonometry & hence discovered so late. [Chinese carving spheres of ivory]

Ar solids obtained by truncating reg. solids

Ca " " (not solids) by adding . to " "

~~Re~~ rhombic dodecahedron also a space filler – used at the base of the bee's cell.

Archimedean or isogonal bodies seldom occur in nature or at least play no conspicuous part in crystallography whereas the Catalanic ones are the characteristic forms of well-known minerals.

Cubo octahedron (Archimedean) however occurs in Alum.

$\left(6F_4, 8F_6\right)$

Euler No polyhedron can exist with 7 edges.

(Tetrahedron, cube and dodecahedron have trihedral corners.) For ~~the~~ any polyhedron with trihedral or three way corners, the following formula holds (f_n is the no of faces which are n-gonal polygons)

for trihedral corners only.

$$4f_2 + 3f_3 + 2f_4 + f_5 \pm 0 \times f_6 - f_7 - 2f_8 - \cdots = 12$$

$$\Sigma (6-n) F_n = 12$$

The zero coeff^t for f_6 indicates that there can be no polyhedron with all its faces hexagons.

If the corners are not all trihedral

$$f \quad (f_3 + c_3) \boxed{\pm 0 (f_4 + c_4)} - (f_5 + c_5) - 2(f_6 + c_6) - 3(f_7 + c_7) - \cdots = 8$$

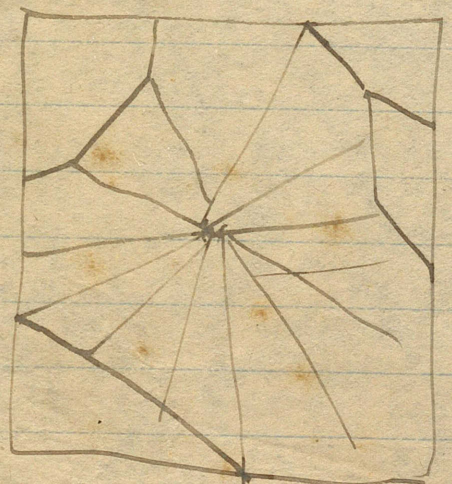
around and around in one directn, sometimes she reverses the directn to pile more chords on one side thereby reducing a web that has a ~~has~~ a very unsymmetrical contour to near symmetry at the centre. When near the middle she destroys the temporary spiral as she goes along. Two pts of interest:–

1) How does the spiral manage to make the ∠ openings of the radii almost = ?

2) How is the spiralling effected?

Radii laid out in a haphazard manner. The spider does not even see the web as a whole from a distance: outer frame so irregular. May be the chords of the temporary spiral are uniform in length.

Also of the snaring spiral "



Looks an Archimidean spiral

not equiangular as False thought.

gaseous + Crystal States as
well amenable to math. treatment
10^5/sec. × 10^6 yrs. statistics / molecules.

Cryst. perfection : { There is order even in imperfection
perfectly imperfect.

External form merely a reflex of internal order.
1) Planes 7 Å mica // 1/10' 3 pleg at

External form naturally fit to attract : Polyhedra : one of the most fascinating branches of math :
Plato ; regular solids : Let no man ignorant of Geometry enter" wd no more have indicated the geometry contents than a warning to the guests not to forget to bring a packet of sandwiches wd indicate Toss promise of a good dinner : However that may be Aware 5 & only 5 solids

Polygon :
Regular polyhedra

$2-\frac{4}{n}$
4) 180 − 90 = 90°
5) 180 − 360/5 = 108
6 120 − 360/6 = 120.

Euler's Law

	F	C	E
Cube.	6	8	12
Tetragon	4	4	6
Octa.	8	6	12
of wh he was vastly proud			
Dodeca	12	20	30
Icosahed.			

Sum of the solid L's. $4-\frac{8}{C}$ 8 corners.
Cube 3 rt. L^{les}. 720
Duodeca $4-\frac{8}{20}$ = 360 / 36 / 324/3 120

Octa $4-\frac{8}{6}$ 360 / 120 / 240 / 4
−4. # 60°

Semi Irregular (1) Archimedes. All corners eq. inscribed
~~All~~ Isogonal = (faces regular polygons not similar.
(2) Isohedral :
Catalan also 13 of them 13 of them

i.e. $\Sigma\,(4-n)\,(f_n + c_n)$

$\therefore\ (f_3 + c_3)$ should be at least eight.

$$\Sigma\, n f_n = 2E \qquad\qquad \Sigma\, n c_n = 2E$$

Each edge separates 2 faces. Each edge joins 2 corners.

$$3f_3 + 2f_4 + f_5 = 12 + 0\cdot c_3 + 2c_4 + 4c_5 \;-- + 0\cdot f_6 + f_7 + 2f_8$$

& $$3c_3 + 2c_4 + c_5 = 12 + 0\cdot f_3 + 2f_4 + 4f_5 \; + 0\cdot c_6 + c_7 + 2c_8$$

i.e. ~~If every~~ No polyhedron can exist wh has not a certain no. of triangles, sqq or pentagons in its composition.

Lucretius: There are things wh are possible and things wh are impossible even to Nature herself.

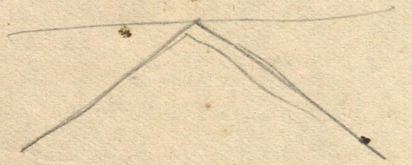
In a polyhedron whose corners are all trihedral (Tetrahed, cube, Dodeca.) $\quad 3C = 2E \quad \boxed{E = \frac{\Sigma n c_n}{2}}$

since $F + C = E + 2 \qquad F = E/2 + 2$

Euler's polyhedra

There can not be more than 5 regular polyhedra

Each face n-gonal;
r of them meet at a vertex



Each polygonal $\angle^{le} = \dfrac{(n-2)\pi}{n}$

$$r \times \frac{n-2}{n}\pi < 2\pi$$

$$r - \frac{2r}{n} < 2 \; ; \quad 1 - \frac{2}{n} < \frac{2}{r} \qquad \frac{1}{n} + \frac{1}{r} > \frac{1}{2}$$

$$n > 2 \qquad r > 2$$

Dayal

Possible values are

n	3	3	4	5	3
r	3	4	3	3	5
	tetrahed.	octa.	cube	dodeca	icosahedron

8

16

$\cancel{325}$

$$\frac{\overset{1.05}{\cancel{525}} \times 55}{16 \times \cancel{500}}$$ reas.

$$\frac{57\frac{1}{2}}{16}$$

650

4