Good Luck

A.G. BOOK MANUFACTURER

BOMBAY 3

LETTER BOOK

100

Name B. S. Madhava Rao

Class – Subject [illegible]

School

I   Pauli & Fierz. Proc. Roy. Soc A., 173, 1939, p. 211-232.

~~Spin 1/2~~

Rest ~~mass~~ zero. Spin 2

A symmetrical ~~tensor~~ tensor A_{ik} whose trace ΣA_{ii} vanishes & field eq^ns for spin 2 are

(a) $\square A_{ik} = \kappa^2 A_{ik}$

(b) $\dfrac{\partial A_{ik}}{\partial x_i} = 0.$ (for total energy to be +ve definite).

$$\delta \int L \, d\Omega = 0.$$

Introduce arbitrary scalar field C in L besides A_{ik} such that variation of C gives subsidiary condition.

~~$\kappa^2 A_{ik} A_{ik}$~~ + Introduce constants a_1, a_2, a_3 in L such that $\partial A_{ik}/\partial x_i$ & C vanish as a consequence of the variational eq^ns obtained by varying A_{ik} & C.

Thus one is led to

$$L = \kappa^2 A_{ik} A_{ik} + \frac{\partial A_{ik}}{\partial x_l}\frac{\partial A_{ik}}{\partial x_l} - 2\frac{\partial A_{rk}}{\partial x_r}\frac{\partial A_{sk}}{\partial x_s}$$

$$- \frac{3}{4}\kappa^2 C^2 - \frac{3}{8}\frac{\partial C}{\partial x_l}\cdot\frac{\partial C}{\partial x_l} + \frac{\partial A_{rk}}{\partial x_r}\cdot\frac{\partial C}{\partial x_k} \quad \text{———} \quad (c)$$

& the corresponding field eq^ns

From (1) & (2) we can derive (a) & (b) & also C = 0.

$$2\varkappa^2 A_{ik} - 2\Box A_{ik} + 2\left\{\frac{\partial^2 A_{sk}}{\partial x_s \partial x_i} + \frac{\partial^2 A_{si}}{\partial x_s \partial x_k} - \frac{1}{2}\delta_{ik}\frac{\partial^2 A_{rs}}{\partial x_r \partial x_s}\right\}$$

$$- \frac{\partial^2 C}{\partial x_i \partial x_k} + \frac{1}{4}\delta_{ik}\Box C = 0 \quad \text{—— (1)}$$

$$-\frac{3}{2}\varkappa^2 C + \frac{3}{4}\Box C - \frac{\partial^2 A_{rk}}{\partial x_r \partial x_k} = 0 \quad \text{—— (2)}$$

$\varkappa$

This belongs to spin value $f = 2$ & gives $2f+1 = 5$ states for a given dirⁿ & frequency

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<u>Zero-rest mass: spin 2</u> : Put $\varkappa = 0$ & get eqⁿˢ from (1) & (2). From these (1') & (2') say it does not follow that C & $\partial A_{ik}/\partial x_i = 0$. Nevertheless there are four identities following from them by differentiating (1') & (2') w.r.t x_i & x_k & in <u>either case</u>

$$\frac{3}{4}\Box\frac{\partial C}{\partial x_k} - \frac{\partial}{\partial x_k}\frac{\partial^2 A_{rl}}{\partial x_r \partial x_l} \quad (k = 1, 2, 3, 4) \quad \text{—— (3)}$$

& by subtracting we get identically zero for each value of k. As a result of these 'identities' it is possible to construct

A^0_{ik}, C^0 from an arbitrary vector field f_i satisfying the field eqns identically. Writing

$$\left.\begin{aligned} A^0_{ik} &= \frac{\partial f_i}{\partial x_k} + \frac{\partial f_k}{\partial x_i} - \frac{1}{2}\delta_{ik}\frac{\partial f_l}{\partial x_l} \\ C^0 &= 2\frac{\partial f_l}{\partial x_l} \end{aligned}\right\} \quad (4)$$

The Lagrange fn to which (C) reduces when $x \to \infty$ is unaltered by the "guage transformation"

$$\left.\begin{aligned} A^1_{ik} &= A_{ik} + A^0_{ik} \\ C^1 &= C + C^0 \end{aligned}\right\} \quad (5)$$

Einstein's eqns for weak gravitational fields

$$g_{ik} = \delta_{ik} + r_{ik}, \quad r_{ii} = r$$

(r_{ik} are small quantities of 1st order).

Write $r_{ik} = A_{ik} + \frac{1}{4}\delta_{ik}C$, $r = C$

We obtain the following diff. eqns for r_{ik}

$$(6)\quad \begin{cases} -\Box r_{ik} - \dfrac{\partial^2 r}{\partial x_i \partial x_k} + \dfrac{\partial^2 r_{lk}}{\partial x_l \partial x_i} + \dfrac{\partial^2 r_{li}}{\partial x_l \partial x_k} + \dfrac{1}{2}\delta_{ik}\left\{\Box r - \dfrac{\partial^2 r_{lr}}{\partial x_l \partial x_r}\right\} = 0 \\ \Box r - \dfrac{\partial^2 r_{lr}}{\partial x_l \partial x_r} = 0 \end{cases}$$

$$-i\partial_k \psi^* \beta^k + \chi \psi^* = 0$$

These eqns are the same as those that Einstein gave for space containing no matter.

"Guage transformation" (5) appears in gravitational theory as an infinitesimal Coord. transformation. When interactions with matter occur & it is no longer sufficient to restrict oneself to the linear terms the guage group is altered. This keeps the dimensionality of the possible transformations unchanged; four fns of position always remain arbitrary. It is well-known that the existence of the energy-mom. tensor is closely connected with the invariance of gravitational theory under these transformations.

II Harish-Chandra – Particle & wave aspects of the meson & the photon.

Proc. Roy. Soc. A. 186, 1946, 502–525.

$$i\beta^k \partial_k \psi + \chi\psi = 0 \qquad (1)$$

$\exists\ \eta_k$ such that $\psi^* \eta_k$ transforms as a vector

Eqns in Harish-Chandra from p. 502 – 507 apply to all spins in general (without using Kemmer-Duffin Commutation rule).

From prescriptions for T_k etc, Γ_k etc. given there we can find β & $\gamma = \beta$ can be taken for rest mass zero –

* Multiply (1') by $(1-\gamma)$ on the left. Then

$$~~\partial_m \beta~~ \; i \beta^k \partial_k (\gamma \psi) = 0.$$

Relations connecting γ_k with the I^{kl} (matrices of inf. transfs of Lorentz group). Hence γ_k's connected with β's – different relns for different representations –

Defn of $\beta = \gamma_k \Gamma^{*k}$ $\quad (\Gamma^*_k = \gamma^+_k \Lambda)$

$(\Lambda \beta^k \Lambda^{-1} = \beta^{+k})$

If $\gamma = \frac{1}{2} \beta_k \beta \beta^k$

then we have $\gamma = \beta$ or $1-\beta$

For zero rest-mass, instead of (1) write

$i\beta^k \partial_k \psi + \gamma\psi = 0$ where γ is a matrix — (1')

satisfying $\gamma^2 = \gamma$, $\gamma\beta^k + \beta^k\gamma = \beta^k$. Then it follows that *

$\gamma\psi$ satisfies (1) with $\chi = 0$. & also second order

wave eqn $\partial_k \partial^k (\gamma\psi) = 0$ follows.

If ϕ be any soln of $\beta^k \partial_k \phi = 0$

& since $\gamma(1-\gamma)\phi = 0$

$(1-\gamma)\phi$ satisfies (1')

Thus $\psi + (1-\gamma)\phi$ is also a soln of (1').

(1') can also be obtained from a Lagrangian formulation

Freq. symmetric energy-momentum tensor is

$$\Theta_{kl} = \psi^* (g_{kl} - \beta_k \beta_l - \beta_l \beta_k) \gamma \psi$$

Θ_{kl} is gauge-inv. because of factor γ.

$\gamma\psi$ is " but not $(1-\gamma)\psi$

The Lagrangian is $L = \frac{1}{2i} \left\{ \partial_k \psi^* \beta^k \psi - \psi^* \beta^k \partial_k \psi \right\} + \psi^* \gamma \psi$

Either $\gamma = 1 - \beta$ or β.

In meson case for 10-row repn eqn (1') corresponds to usual ~~meson~~ Maxwell eqns for $\gamma = 1-\beta$, and for $\gamma = \beta$ the eqns are those of the usual scalar theory.

For the 5-row repn: $\gamma = 1-\beta$ has no physical interest

$\gamma = \beta$ gives the usual scalar theory.

III Harish-Chandra: Prcd. 192, 1948, 195-217.

– nothing much relevant in this

Bergmann, p. 178

(i) Field eq^{ns} are ten diff. eq^{ns} of the second order in $g_{\mu\nu}$.

(ii) The ten diff. eq^{ns} for the $g_{\mu\nu}$ cannot be fully independent of each other, but must satisfy four identities.

15/12/50. — 9.45 – 12.45 — H. J. Bhabha (Chair)

Rosenfeld – Joint paper with Bohr. (Bohr told him nothing to communicate)

(1) $\Delta p . \Delta q \gtrsim \hbar$ as a consequence of $[p, q] = \hbar/i$

& in general $\Delta A . \Delta B \gtrsim C$ when $[A, B] = iC$

(2) Opposition of Einstein led Bohr recently to a deeper analysis of complementarity

(3) Analysis of concepts of electromagnetism in the same spirit as that of classical mechanics – This occurred to Pierls & Landau.

(4) Recent work of Schwinger, Tomanaga, Dyson & Feynman has shown that quantum electrodynamics is a consistent system & that the divergences arise because of an imperfect understanding of the physical significance of the commutation relations

(5) For relations like $[E_x(P), E_x(P')] \neq 0$ etc are there reciprocal relations like $\Delta p . \Delta q \geq h$. — Also quantities here are continuous fns of x, y, z, t — one has to proceed by approximation. with new theory of Schwinger we can go to a higher approximation without however any new principle arising

(6) Approximations:

(1) Charges classical $(\hbar, c)$ — $(\hbar/mc)$

(2) $e^2/\hbar c$ order effects considered — Equivalent to assuming that fields producing pairs is not quantised ie fields classical & charges quantised

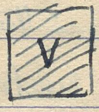
(3) — — — Both quantised ?

—

(7) commutation rules like $[E_x(P), E_x(P')] = i D(P, P')$ where D is a singular fn of P − P' has really no meaning, but by itself but becomes meaningful when integrated.

(8) Use of electrons as test-bodies for measuring averages generally

$E_x(V,T) = \frac{1}{VT}\int E_x(P,t)\,dv\,dt$ $\begin{cases} T = \text{time interval} \\ V = \text{volume} \end{cases}$



$\Delta p . \Delta q \gtrsim \hbar$ & the restriction on momentum of this area =

$= \bar{E}_x \rho V T$

& uncertainty in the momentum $= \Delta \bar{E}_x \rho V T$

$$\therefore \Delta \bar{E}_x \rho V T > \hbar / \Delta x$$

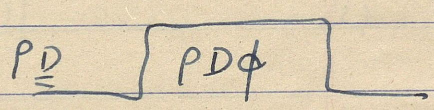
$$\Delta \bar{E}_x > \frac{\hbar}{\rho \Delta x \cdot VT}$$

If $\Delta x \to 0$ but $\rho \to \infty$, $\rho \Delta x$ can be $\to \infty$ and $\Delta \bar{E}_x \to 0$ ie if ρ can be made as large as we like (ρ = density of charge). but ρ can't be made as large as possible ie infinite accuracy in $\bar{E}_x$ is not possible. if we cannot approxn (2). In approxn (1) ρ can be $\to \infty$ since there is no restriction on scale of space & time dimensions. In approxn 2) for space regions less than Compton

[News received that Vallabhai died today at 9.40 A M
15/12/50 from Bombay House in the middle of the meeting].

$$E_x = \delta \phi_\cdot + e_x$$ (any soln of homogeneous eq.)

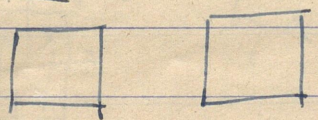
wave length $\hbar/mc$. At this stage take into consideration the field due to the test body.



We can compensate for displacement of the ~~elastic~~ test body. But we arrive at a contradiction that ΔE_x cannot $\rightarrow 0$ even taking into consideration the fact that the field can be accurately determined by causing fluctuations due to emission of photons due to motion of test bodies. This however is no contradiction but is just due to the fact that a quantized oscillator has a zero-point value at which its amplitude $\neq 0$.

The fluctuating field is always present & is a consequence of the theory

2nd problem – two field quantities simultaneously



Consider the perturbation of the first measurement on the itself & on the other test-body & vice versa. The conclusions are about the same. The Commutator here has the form

$$i\left[A(R,R') - A(R',R)\right] \qquad (1)$$

[The $A(R,R')$ & $A(R',R)$ are intimately connected with the the mutual fields: thus

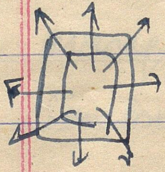
$$\Phi(RR') = \rho_1 D_1 A(RR')$$

[It took nearly 2 months to get the minus sign in (1) above after calculating perturbations]

The − sign is obtained as an extra information obtained by back & forth signals between the two bodies.

[$A(R,R')$ are the Green's functions & the connections between them as given in the Commutation rules are remarkable].

For charge Commutation rules, enclose volume by a shell of test-bodies



A detailed consideration leads to the conclusion that charge can be classically considered in the first approximation. In second ~~appr~~ approx displacement of test body causes polarisation of vacuum & change of no. of electrons of $-^{ve}$ energy. This creation changes only distn of charge & not total charge & estimation of the average polarisation is again $\propto$ to displacement of test-body i.e. $\rho D B$ (B can be calculated by quantum method)

i.e. average $= \overline{\rho D B}$ & actual fluctuation $\propto$ polarisation is an atomic phenomenon. Hence there is a fluctuation of charge. This circumstance is as before a consequence of the commutation rule. Same thing holds good for case of two distributions & their mutual perturbation.

question of thickness 'd' of the shell leads to interesting points. This thickness introduces a new averaging process. If we take pair creation into consideration we cannot let $d \to 0$.

fluctuation: $\log\left(\frac{\hbar/mc}{d}\right)$ shows that $d \not\to \infty$.

This is consistent with the fact that in electrodynamics we cannot attach meaning to $d \to 0$ but the logarithmic form shows that d cannot be small fractions of $\hbar/mc$. But going to limit of smaller dimensions raises the question of test-bodies i.e. if we go to regions of magnitude e^2/mc^2 a consistent theory becomes possible only if we ~~go down to~~ consider forces of non-electromagnetic origin. ~~This is only a~~

The same theory is possible for mesons (electromagnetic properties of mesons if they be of spin 0, but for spin 1, fluctuations are more much worse & quantum electrodynamics cannot be reasonably applied.

Schwinger's method of removing divergences is to compensate e by $e+\delta e$ or m by $m+\delta m$.

ie the renormalisation ie $\int \log x \, dx$ associated to . . .

Jost has recently obtained the following answer to the question raised by Dyson about finiteness of δe & δm.

$$\left\langle \left[\varphi_\mu^{(n)}(x), \varphi_\nu^{(n)}(x') \right] \right\rangle_{vac}$$ ie average value for vacuum state

[(n) indicates renormalisation]

$$= i \delta_{\mu\nu} D(x - x').$$
$$+ (\text{correction term}) \iint dx^1 dx^2 \, \bar{D}(x, x^1) \, \bar{D}(x^2, x') \left\langle \left[j_\mu^{(0)}(x), j_\nu^{(0)}(x') \right] \right\rangle_{vac}$$

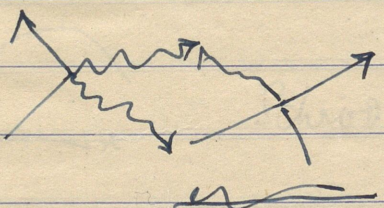
$\bar{D}$ = Green's fn]

Due to renormalisation the second term is the additional uncertainty depending only on the given current distribution. The first is the uncertainty of the non-renormalised case.

Bhabha's question – What happens in first case if in addition to $\hbar$ & c we consider a neutral meson of mass χ or μ i.e. dimension of a length is introduced.

Prof. Wentzel :- gave an example of a new divergence which cannot be removed by renormalisation of charge or mass

$$\left(\frac{e^2}{\hbar c}\right)^2 : \quad e^2 \left|\vec{\varphi}\right|^2 \psi^* \psi .$$



Rosenfeld replied that Dyson has confirmed that those divergence is removed by renormalisation.

Prof. Peierls mentioned work of Salam & Mathews in Cambridge & said that point raised by Wentzel had been worked out

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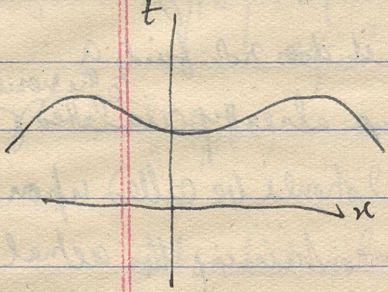
16/12/50 – Chairman Möller

Peierls on commutation ~~rules~~ laws of relativistic field ~~equations~~ theory.

$$[p,q] = -i\hbar$$

$$\psi(\phi, t)$$

$$[\phi_\alpha(r,t), \phi_\beta(r',t)] =$$



$$H = H_0 + H_1$$

Schrödinger & Heisenberg representations.

$$[\phi_\alpha(r,t), \phi_\beta(r',t')]$$

Spoke of the reversal of the procedure of Schwinger & Tomonaga.

$$\mathcal{L}' = \iint L\,d\omega + \lambda\phi_\alpha(x_0) \left(\text{one of the field quantities}\right)$$

Abstract of lecture

if called upon. on 16/12/50.

At the outset let me say how thankful I am to Prof. Bhabha for allowing me to present before this distinguished gathering ~~some small~~ a piece of work I have recently been doing on ~~..~~ the relativistic wave equations for a particle of spin 2, although it does not find a place in the typed programme already circulated to the members. In fact, I never expected that I should be called upon to do so & ~~..~~ my note books containing the actual details of the work are not with me just now. I shall therefore content myself by presenting the broad outlines of the work done.

As is well-known Pauli & Fierz have developed a theory for particles of arbitrary spin and in particular have proposed that a gravitational particle might be considered

[Peierls further referred to his paper with Landau as having excited Bohr into his recent work — made fundamental remarks about energy of electron.

—

Bhabha made some remarks on Peierls remarks]

16/12/50 (contd).

equivalent to replacing $L \longrightarrow \ldots + \lambda \phi_\alpha(x_0)\,\delta(x - x_0)$

If $\phi_\beta(x) \longrightarrow \phi_\beta(x) + \lambda D^{\phi_\beta(x)}_{\phi_\alpha(x_0)}$. Take the

retarded soln. $D^{\phi_\beta(r,t)}_{\phi_\alpha(r,t_0)} \to 0$ as $t \to -\infty$

advanced soln $\mathbb{D} \to 0$ when $t \to +\infty$

Sofar terminology. The statement is:

For any two A, B

$$[A, B] = -i\hbar\left\{D^B_A - \mathbb{D}^B_A\right\} \quad \text{——} \; \text{(A)}$$

a perfectly general statement.

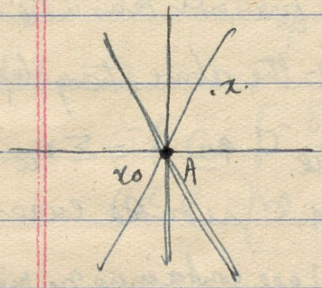
an elementary particle of spin 2 with zero rest-mass. Their method consists in taking for the field quantities the symmetric tensor A_{ik} with $\Sigma A_{ii} = 0$ & the further divergence condn $\partial A_{ik}/\partial x_i = 0$ – meaning of these condns & setting up the field eqns

$$\left.\begin{aligned} \Box A_{ik} &= \varkappa^2 A_{ik} \\ \partial A_{ik}/\partial x_i &= 0 \end{aligned}\right\}. \qquad (1)$$

These can also be derived from a ~~Lagrangian~~ variational principle $\delta \int L\, d\Omega = 0$ where, however, the Lagrangian contains besides second order derivatives of A_{ik} also those of an auxiliary scalar C so that variation of ~~both~~ C gives the subsidiary condition. A consistent Lagrangian can be set up for the case $\varkappa \neq 0$ & also $\varkappa = 0$, and it is the latter case with which we are interested. In this case the field eqns derived from the Lagrangian does not directly lead to $\partial A_{ik}/\partial x_i = 0$ & $C = 0$, but it can be shown that the Lagrange fn is now unaltered by a "guage transformation"

$A'_{ik} = A_{ik} + A^0_{ik}, \; C' = C + C^0$

~~Inthe~~ To deduce some properties of this ~~to~~ rule :—



Let $[A, B]$ say. $A \to x_0$, $B \to x$

If x be outside light cone of x_0, R.H.S is obviously zero.

Both sides of (A) have singularities

Make a transformation $e^{-iS} A e^{iS}$. This does not alter the eqn. motion i.e. physical properties are not changed.

Choose $S = \lambda B$. , $e^{-iS} A e^{iS} = A + i\lambda [A, B]$

D & C satisfy the same inhomogeneous eqn as ~~that~~ satisfied by ~~φ~~ modified $\phi_\beta(x)$.

To be sure that the laws have the right properties at singularities we have to make the following considerations

Let A be one of the field quantities $A \to \phi_\alpha(x_0)$

Eqn. motion is: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\phi}_\beta}\right) - \dfrac{\partial L}{\partial \phi_\beta} = \lambda \delta_{\alpha\beta}\, \delta(r - r_0)\, \delta(t - t_0) = 0.$

(π_β conj of ϕ_β) $\quad \pi'_\beta(t+0) = \pi'_\beta(t_0 - 0)$ retarded

$+ \lambda \delta_{\alpha\beta}\, \delta(r - r_0)$

For Einstein's ~~relat~~ weak gravitation fields specified by γ_{ik} where $g_{ik} = \delta_{ik} + \gamma_{ik}$ (only first order terms in γ_{ik} retained) we make the correspondence with the above theory by putting $\gamma_{ik} = A_{ik} + \frac{1}{4}\delta_{ik} C$ and $\gamma = \Sigma \gamma_{ii} = C$.
and the field eq^ns in the A_{ik} & C reduce exactly the eq^ns given by Einstein for space containing no matter.^{x}

Gauge tr^ns correspond to inf. Coord. tr^ns

~~There~~ Recently the alternative way has been obtained by considering ~~a~~ particles of higher spin as specified by a first order wave equation of the Dirac form

$$i\beta^{\mu}\partial_{\mu}\psi + \chi\psi = 0 \qquad \left[\partial_{\mu} = \partial/\partial x_{\mu}\right. \quad (2)$$

– β_{μ}, ψ explained – In 1942, I gave the theory for particles of spin 3/2 & 2, and this has been generalised by Bhabha for ~~arbitrary spin~~ elementary particles of arbitrary spin. This theory is based on the postulate that in the general case the spin operator $t_{\mu\nu} \equiv i S_{\mu\nu}$
$\equiv (\beta_{\mu}, \beta_{\nu})$ ——— (3)
The consequence of this assumption have been fully investigated by Bhabha who has shown that the

$$D_\alpha^\beta \begin{cases} = 0, & t < t_0 \\ \lambda\,\delta_{\alpha\beta}\,\delta(r - r_0), & t = t_0 + 0 \end{cases}$$

For Advanced soln

$$C_\alpha^\beta = \begin{cases} -\delta_{\alpha\beta}\,\delta(r - r_0), & t = t_0 - 0 \\ 0, & t > t_0 \end{cases}$$

$$t = t_0, \quad [\pi_\beta, \phi_\alpha] = -i\hbar\,\delta_{\alpha\beta}\,\delta(r - r_0)$$

$$[\phi_\alpha, \phi_\beta] = 0$$

are the same as the ~~commutation~~ canonical commutation laws when they exist & are independent of particular coord. system.

If $D_A^B - C_A^B$ shall be $= -D_B^A + C_B^A$

then $D_A^B = C_B^A$ giving the inversion theorem for point functions

This theorem holds provided we can ignore order problem of finding all irreducible eqns of the form (2) ~~are~~ is connected with the problem of finding all irreducible representations of the Lorentz group in five dimensions. Thus for a particle of maximum spin 3/2 there are only two representations $R_5(3/2, 3/2)$ & $R_5(3/2, 1/2)$ of orders 20 and 16 respectively. In a paper published in the Roy. Soc A. 187, 1946, p. 385 myself & two collaborators have independently investigated the algebra generated by the β_μ & the unit element governed by commutation rules which I had previously obtained in this case based on (2) & the further assumption that the eigenvalues of $S_{\mu\nu}$ are $(3/2, 1/2, -1/2, -3/2)$ - Show this on slide - we show that this algebra is the direct product of the Dirac algebra & another algebra which we call the $\mathfrak{F}$-algebra. By counting the linearly independent elements of this algebra we get its order as $42 = 1^2 + 4^2 + 5^2$ giving the three irreducible representations of order 1, 4, 5 leading in the general case to three repns of orders of factors. -

What is the use of all this (i) Helps us to talk about field equations, (ii) technique developed is convenient for cases where there are complications requiring orientation of curved surfaces.

In the whole formulation there is no mention of Hamiltonian & can therefore be applied to theories where no Hamiltonian can be set up. In these cases it is difficult to see if the inversion theorem still holds good. We want to investigate these cases still further.

These considerations can also be generalised to Fermi-Statistics

Möller raised the question if (A) is consistent with

$$[AB, C] = A[BC] + [AC]B.$$

Peierls mentioned this is true for $[A, BC]$ leading to

$$D_A^{BC} = B D_A^C + D_A^B C.$$

$3C_2 = 3$.

4, 16 & 20, the last two being intrinsic to spin 3/2. This splitting up of the ~~3/2 algebra~~ spin 3/2-algebra into a direct product makes it very simple to obtain the matrices explicitly, but such a simplification does not appear possible for the case of spin 2. In this case there are three representations $R_5(2,0)$, $R_5(2,1)$, $R_5(2,2)$ of orders 10, 35 & 35, but looking at the commutation rules for this case – show slide – a direct algebraic approach as for spin 3/2 appears hopeless. In this case the algebra is not a direct product & in fact the ξ-algebra defined by the symbols $\xi_\mu = \eta_\mu \beta_\mu = \beta_\mu \eta_\mu$ (η_μ being the metric operator D of Bhabha) is identical with the β_μ-algebra & this automorphism shows no possibility of simplification. Although the commutation rules are very complicated particular cases corresponding to some indices being equal & so on are easier, and I thought a direct attack on the ~~algebras~~ matrices of the 10-order representation suitably chosen so as to satisfy these rules might be made.

I asked the question as to whether

$$\Delta A \cdot \Delta B \geqq \hbar$$

can be deduced from (A) & if so the inversion theorem has ~~been~~ to be applied.

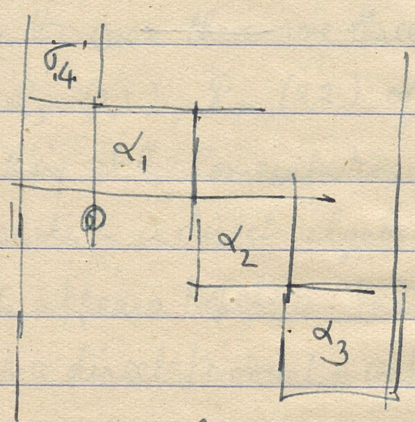
I have found that 14 x 14 matrices in which the Pauli & Dirac matrices appear as sub-matrices ~~are the~~ ~~yet~~ yield the required representation. I do not remember the details but ~~for~~ the representation matrix for the β_4 is given by

$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

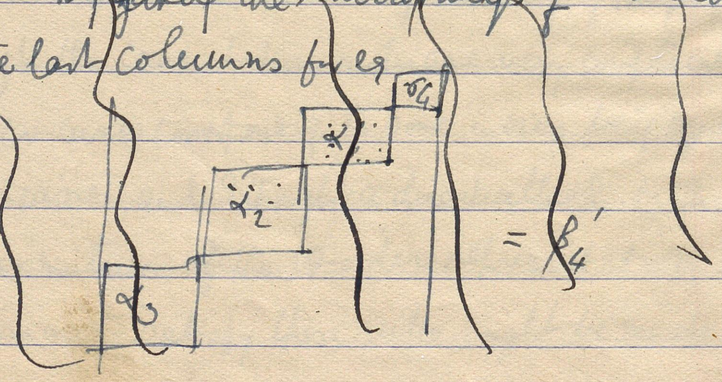
$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$



with $(\sigma_1, \alpha_2, \alpha_3, \alpha_4)$, $(\sigma_2, \alpha_3, \alpha_1, \alpha_4)$, $(\sigma_2, \alpha_1, \alpha_2, \alpha_4)$ suitably inserted in $\beta_1, \beta_2, \beta_3$ matrices.

By taking the mirror image of these matrices in the last columns for eg



The two of representations 35, 35 have baffled all efforts at construction & it appears impossible to attach any physical ~~gives another inequivalent representation also of order 14. The one of order 35 has baffled all my efforts at construction.~~ significance to them.

Since the ~~idea with which the~~ work was undertaken ~~was~~ with a view to examine if support could be lent to the idea of the gravitational particle being an elem. particle of spin 2 & rest mass zero, we have to examine how eqn (2) is to be modified so as to apply it to the case of zero rest mass. Instead of (2) we write the eqn as

$$i\beta^\mu \partial_\mu \psi + \gamma\psi = 0 \text{ where } \gamma \text{ is a} \quad \text{——} (3)$$

matrix satisfying $\gamma^2 = 1$, $\gamma\beta^\mu + \beta^\mu\gamma = \beta^\mu$. Then it follows by multiplying (3) on the left by $(1-\gamma)$ that

$$i\beta^\mu \partial_\mu (\gamma\psi) = 0$$

ie $\gamma\psi$ satisfies (2) with $\chi = 0$. This formalism has been consistently applied by Harish-Chandra (Proc. Roy. Soc. A. 186, 1946, 502-525) for the case of a particle of spin 1 & $\chi = 0$ (photon) and can be extended to this case also with slight modifications and we can find γ in the case of spin 2 & rest mass 0 for the given representation.

The next step is to put the ~~field~~ eqns of a weak gravitational field in the first order form. These eqns made up of ten diff. eqns ~~in the~~ A_{ik} ~~and~~ four ~~conditions~~ coordinate conditions are given as follows:-

Putting $g_{\rho\sigma} = \epsilon_{\rho\sigma} + h_{\rho\sigma}$ with $\epsilon_{\rho\sigma} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$
these eqns can be written

(Bergmann, p. 184)

, for the case of no matter present, ~~in~~ the form

$$\left.\begin{aligned} \epsilon^{\rho\sigma}\gamma_{\mu\nu,\rho\sigma} &= 0 \\ \epsilon^{\rho\sigma}\gamma_{\mu\rho,\sigma} &= 0 \end{aligned}\right\} \quad \text{———} (5)$$

where $\gamma_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu} h$, with $(h = \epsilon^{\rho\sigma} h_{\rho\sigma})$, and the commas denote ordinary (& not covariant) partial derivatives.

As in the meson case ~~where we write the equations~~

$$U_{ik} = \frac{\partial U_k}{\partial x_i} - \frac{\partial U_i}{\partial x_k}$$

$$\frac{\partial U_{ik}}{\partial x_k} + \chi^2 U_i = 0.$$

We can write these equations in the Dirac form

$$i\beta^\mu \partial_\mu (\gamma\psi) = 0. \qquad (8)$$

provided the β^μ's are ~~the~~ 14×14 matrices of the ~~first~~ type

given above (~~not the images~~). Alternatively taking the matrices for the β^μ & γ as given by this 14-order representation we set up the eq^ns (6) and then show ~~that~~ eq^ns (5) can be brought to the same form by taking suitable linear combinations of $\partial h_{\mu\nu}/\partial x_\sigma$, & $h_{\mu\nu}$ as the fourteen ψ-functions. ~~This fails when the image representation~~ is considered. ~~This justifies the~~ Thus one ~~is justified in treating the gravitation~~ gets a sort of confirmation of Pauli's idea of a gravitational particle having spin 2.

We have so far been dealing ~~so~~ only with the c-number theory & made no attempts at double quantisation to reach the particle picture. This has not been undertaken since one is not sure, even at the beginning, that the quantisation will not lead to a contradiction between spin & statistics. In fact, Prof. Pauli to whom I had sent the work on the spin $3/2$ particle was very doubtful, nay sure, that the connection between spin & statistics would not hold in the theory. In fact his characteristic remark about that work was "very good algebra but doubtful Physics". It is a pity that Prof. Pauli is not here today, but I have no doubt his reactions to this proposed quantisation of the spin 2 theory would have been any different.

Irrespective of double quantisation, one cannot but help being struck by this c-number theory relating the fundamental quantum

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Meaning: $R_5(2,0) \to R_4(2,0) + R_4(1,0) + R_4(0,0)$

$$14 = 9 + 4 + 1$$

ie symmetric tensor G_{KL} satisfying $G_K^K = 0$ breaking up under restriction to the Lorentz group in 4 dimensions into a symmetric tensor G_{kl}, a vector G_{k4} & scalar G_{44}.

In the scheme $R_5(n,0)$ there is no non-relativistic approach in which the particle displays spin $\underline{n}$.