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F. 708
M.Sc. (Physics) I
Mathematical Physics
[Home Assignments]

i) If $\bar{a}, \bar{b}$ & $\bar{c}$ are coplanar vectors, and $\bar{b} \nparallel \bar{c}$ then it is possible to write

$$\bar{a} = \alpha \bar{b} + \beta \bar{c}$$

From the starting pt. of $\bar{a}$, say O, draw $\overline{ON} \parallel \bar{b}$ & $\overline{OM} \parallel \bar{c}$, such that ONAM is a parallelogram.

Now $\overline{ON} \parallel \bar{b} \Rightarrow \exists$ scaler α such that $\overline{ON} = \alpha \bar{b}$

$\overline{OM} \parallel \bar{c} \Rightarrow \exists$ scaler β such that $\overline{OM} = \beta \bar{c}$

Now by law of parallelogram of forces,

$\bar{a} = \overline{OA} = \overline{OM} + \overline{ON}$ but $\overline{ON} = \alpha \bar{b}$, $\overline{OM} = \beta \bar{c}$

$\therefore \bar{a} = \alpha \bar{b} + \beta \bar{c}$ q.e.d.

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ii) If $\bar{a}, \bar{b}, \bar{c}$ are coplanar, $\begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$.

$\bar{a}, \bar{b}, \bar{c}$ coplanar $\Rightarrow \exists$ scalers α, β ~~st~~ such that

$\bar{a} = \alpha \bar{b} + \beta \bar{c}$

$\Rightarrow a_1 = \alpha b_1 + \beta c_1$, $a_2 = \alpha b_2 + \beta c_2$, $a_3 = \alpha b_3 + \beta c_3$

$$\therefore \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} c_1 & c_2 & c_3 \\ \alpha b_1 + \beta c_1 & \alpha b_2 + \beta c_2 & \alpha b_3 + \beta c_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \alpha \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{vmatrix} + \beta \begin{vmatrix} c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$= 0$ since two rows are identical in both the determinants.

(iii) Show that $\bar{a}\cdot\bar{b}$ in invariant under transformation of co-ordinates

i.e. $\bar{a}\cdot\bar{b} = \underline{\bar{a}}\cdot\underline{\bar{b}}$.

We have $\underline{\bar{a}}\cdot\underline{\bar{b}} = \sum_p \underline{a}_p \underline{b}_p$

$$\sum_{i,j,p} a_i l_{ip} b_j l_{jp}$$

$$= \sum_{i,j} a_i b_j \left(\sum_p l_{ip} l_{jp} \right)$$

$$= \sum_{i,j} a_i b_j \delta_{ij}$$

$$= \sum_i a_i b_i = \bar{a}\cdot\bar{b}. \quad \text{q.e.d.}$$

(iv) $\bar{a}\times(\bar{b}\times\bar{c}) = \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b})$

LHS $= \bar{a}\times(\bar{b}\times\bar{c})$

$$= \sum_l a_l e_{(l)} \times \sum_{ijk} \epsilon_{ijk} b_i c_j e_{(k)}$$

$$= \sum_{ijkl} \epsilon_{ijk} a_l b_i c_j [e_{(l)} \times e_{(k)}]$$

$$= \sum_{ijklmnp} \epsilon_{ijk} \epsilon_{mnp} a_l b_i c_j e_{(l)m} e_{(k)n} \hat{e}_{(p)}$$

$$= \sum_{ijklmnp} \epsilon_{ijk} \epsilon_{mnp} a_l b_i c_j \delta_{lm} \delta_{kn} \hat{e}_{(p)}$$

$$= \sum_{ijklp} \epsilon_{ijk}\epsilon_{lkp}\, a_l b_j c_j\, e_{(p)}$$

kpl

$$= \sum_{ijklp} (\delta_{ip}\delta_{jl} - \delta_{il}\delta_{jp}) a_l b_j c_j\, e_{(p)}$$

$$= \sum_{ij} a_j b_i c_j e_{(i)} - \sum_{ij} a_i b_i c_j e_{(j)}$$

$$= \left[\sum_i b_i e_{(i)}\right] \sum_j a_j c_j - \left[\sum_j c_j e_{(j)}\right] \sum_i a_i b_i$$

$$= \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b}) = RHS \quad \text{q.e.d.}$$

(v) $\bar{a}\times(\bar{b}\times\bar{c}) + \bar{b}\times(\bar{c}\times\bar{a}) + \bar{c}\times(\bar{a}\times\bar{b}) = 0$

$$\begin{aligned} LHS &= \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b}) \\ &+ \bar{c}(\bar{b}\cdot\bar{a}) - \bar{a}(\bar{b}\cdot\bar{c}) \\ &+ \bar{a}(\bar{b}\cdot\bar{c}) - \bar{b}(\bar{c}\cdot\bar{a}) \\ &= 0 = R.H.S. \quad \text{q.e.d.} \end{aligned}$$

(vi) $(\bar{a}\cdot\hat{e})\hat{e} + \hat{e}\times(\bar{a}\times\hat{e}) = \bar{a}$

Now, we have

$$\hat{e}\times(\bar{a}\times\hat{e}) = \bar{a}(\hat{e}\cdot\hat{e}) - \hat{e}(\bar{a}\cdot\hat{e})$$

$$\begin{aligned} \therefore LHS &= (\bar{a}\cdot\hat{e})\hat{e} + \hat{e}(\bar{a}\times\hat{e}) \\ &= (\bar{a}\cdot\hat{e})\hat{e} + \bar{a}(\hat{e}\cdot\hat{e}) - (\bar{a}\cdot\hat{e})\hat{e} \\ &= \bar{a}(\hat{e}\cdot\hat{e}) = \bar{a} = RHS \end{aligned}$$

since $\hat{e}$ is a unit vector $\Rightarrow \hat{e}\cdot\hat{e} = 1$.

(vii) $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = \begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d} \\ \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d} \end{vmatrix}$

$L.H.S. = (\bar{a} \times \bar{b}) \cdot \bar{m}$ where $\bar{m} = \bar{c} \times \bar{d}$)

$= \bar{a} \cdot (\bar{b} \times \bar{m})$

$= \bar{a} \cdot [\bar{b} \times (\bar{c} \times \bar{d})]$

$= \bar{a} \cdot [\bar{c}(\bar{b}\cdot\bar{d}) - \bar{d}(\bar{b}\cdot\bar{c})]$

$= (\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{d}) - (\bar{b}\cdot\bar{c})(\bar{a}\cdot\bar{d})$

$= \begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d} \\ \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d} \end{vmatrix} = R.H.S.$

(viii) $\{\bar{a} \times \bar{b}\} \times [\bar{c} \times \bar{d}] = [\bar{a}\cdot(\bar{b} \times \bar{d})]\,\bar{c} - [\bar{a}\cdot(\bar{b} \times \bar{c})]\,\bar{d}$

$LHS = \cancel{\bar{a} \times \{\bar{b} \times (\bar{c} \times \bar{d})\}}$

$= \cancel{\bar{b}\{\bar{a}\cdot(\bar{c} \times \bar{d})\} - (\bar{c} \times \bar{d})\{}$

$L.H.S. = [\bar{a} \times \bar{b}] \times [\bar{c} \times \bar{d}]$

$= \bar{c}\,[(\bar{a} \times \bar{b})\cdot\bar{d}] - \bar{d}\,[(\bar{a} \times \bar{b})\cdot\bar{c}]$

$= [\bar{a}\cdot\bar{b} \times \bar{d}]\,\bar{c} - [\bar{a}\cdot\bar{b} \times \bar{c}]\,\bar{d}$

$= RHS$

(ix)
$$L = \begin{vmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{vmatrix} = 1$$ using orthonormality of vectors.

$$L = l_{11}(l_{22} l_{33} - l_{23} l_{32}) + l_{12}(l_{23} l_{31} - l_{21} l_{33}) + l_{13}(l_{21} l_{32} - l_{22} l_{31}) \quad \text{———} \quad (d)$$

Now $e_{(1)1} = [\hat{e}_{(2)} \times \hat{e}_{(3)}] \cdot \hat{e}_{(1)}$

$$= \left\{ \sum_{ijk} \epsilon_{ijk} \, e_{(2)i} \, e_{(3)j} \, e_k \right\} \cdot \hat{e}_{(1)} \qquad k=1$$

$$= e_{(2)2} \cdot e_{(3)3} - e_{(2)3} \, e_{(3)2} \quad \text{———} \quad (a)$$

Hence $l_{11} = l_{22} l_{33} - l_{23} l_{32}$

Similarly, we have

$$e_{1(2)} = [\hat{e}_{(2)} \times \hat{e}_{(3)}] \cdot \hat{e}_{(2)}$$

$$= \left\{ \sum_{ijk} \epsilon_{ijk} \, e_{(2)i} \, e_{(3)j} \, e_k \right\} \cdot \hat{e}_{(2)} \qquad k=2$$

$$= e_{(2)3} \, e_{(3)1} - e_{(2)1} \, e_{(3)3}$$

$\therefore l_{12} = l_{23} l_{31} - l_{21} l_{33}$ ——— (b)

Simly $l_{13} = l_{21} l_{32} - l_{22} l_{31}$ ——— (c)

substituting (a), (b), (c) in (d)

$L = l_{11}^2 + l_{12}^2 + l_{13}^2 = 1 = $ R.H.S.

(x) If $(m_1\ m_2\ m_3)$ and $(\mu_1\ \mu_2\ \mu_3)$ are such that $\sum_i a_i m_i = \sum_p a_p \mu_p$ then $m_1 m_2 m_3$ are components of a vector.

Now,

$$\sum_p a_p \mu_p = \sum_p \left[\sum_i a_i l_{ip} \right] \mu_p$$

$$= \sum_{ip} a_i l_{ip} \mu_p$$

Now $\sum_p a_p \mu_p = \sum_i a_i m_i$

$$\therefore \sum_{ip} a_i l_{ip} \mu_p = \sum_i a_i m_i$$

$$\therefore \sum_i \left(\sum_p \mu_p l_{ip} - m_i \right) a_i = 0 .$$

$$a_i \neq 0 \Rightarrow m_i = \sum_p \mu_p l_{ip}$$

Hence $(m_1\ m_2\ m_3)$ ~~is~~ a ~~vector~~ are components of a vector.

(xi) (a) $\nabla \, 1/r$

$$= \sum_i e_{(i)} \frac{\partial}{\partial x_i} (1/r)$$

$$= -1/r^2 \sum e_{(i)} \frac{\partial}{\partial x_i} \left(\sqrt{x_1^2 + x_2^2 + x_3^2} \right)$$

$$\therefore \nabla \frac{1}{r} = \frac{1}{r^2} \sum e_{(i)} \frac{x_i}{r}$$
$$= \bar{r}/r^3$$

b) $$\nabla r^n = \sum_i \frac{\partial}{\partial x_i} r^n e_{(i)}$$
$$= n r^{n-1} \sum \frac{\partial (r)}{\partial x_i} e_{(i)}$$
$$= n r^{n-2} \bar{r}$$

c) $\nabla(\bar{e}\cdot\bar{r})$ where $\bar{e}$ is a constant vector

$$\nabla(\bar{e}\cdot\bar{r}) = \nabla\left(\sum_i e_i x_i\right)$$
$$= \sum_{ij} \hat{e}_{(j)} \frac{\partial}{\partial x_j}(e_i x_i)$$
$$= \sum_{ij} \hat{e}_{(j)} e_i \frac{\partial x_i}{\partial x_j} \quad \text{since } \frac{\partial e_i}{\partial x_j} = 0$$
$$= \sum_{ij} \hat{e}_j e_i \delta_{ij}$$
$$= \sum_i \hat{e}_{(i)} e_i$$
$$= \bar{e}$$

(xii) Show that $\nabla . \bar{a}$ is invariant under the rotation of axes.

$$\underline{\nabla} . \bar{a} = \sum_p \frac{\partial}{\partial x_p} a_p$$

$$= \sum_{pij} \frac{\partial a_j}{\partial x_i} l_{ip} l_{jp}$$

$$= \sum_{pij} \frac{\partial a_j}{\partial x_i} \delta_{ij}$$

$$= \sum_i \frac{\partial a_i}{\partial x_i} = \nabla . \bar{a} \quad \text{q.e.d.}$$

(xiii) For two dimentional motion.

$$\bar{\omega} = \frac{1}{2}\left(\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2} \right)$$

$$\nabla \times \bar{V} = \sum_{ijk} \epsilon_{ijk} \frac{\partial}{\partial x_i} v_j e_{(k)}$$

But for two dimentional motion, only one component of $\bar{\omega}$ exists.

$$\therefore |\nabla \times \bar{V}| = \sum_{ij} \epsilon_{ij3} \frac{\partial v_j}{\partial x_i}$$

$$= \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}$$

but $|\nabla \times \bar{V}| = |\nabla \times (\bar{\omega} \times \bar{r})| = 2\omega$

$$\therefore 2\omega = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}$$

$$\therefore \omega = \frac{1}{2}\left[\frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2} \right] \quad \text{— q.e.d.}$$

(xiv) Show that

$$\nabla \times (\phi \bar{A}) = \nabla \phi \times \bar{A} + \phi (\nabla \times \bar{A})$$

$$L.H.S. = \sum_{ijk} \epsilon_{ijk} \frac{\partial}{\partial x_i} (\phi A_j) \hat{e}_k$$

$$= \sum_{ijk} \left(\frac{\partial \phi}{\partial x_i} A_j + \phi \frac{\partial A_j}{\partial x_i} \right) \epsilon_{ijk} \hat{e}_{(k)}$$

$$= \sum_{ijk} \epsilon_{ijk} \frac{\partial \phi}{\partial x_i} A_j \hat{e}_k + \phi \sum_{ijk} \epsilon_{ijk} \frac{\partial A_j}{\partial x_i} \hat{e}_{(k)}$$

$$= \nabla \phi \times A + \phi (\nabla \times \bar{A}) \quad \text{q.e.d.}$$

(xv) $\nabla \cdot (f \bar{V}) = \nabla f \cdot \bar{V} + f (\nabla \cdot \bar{V})$

$$L.H.S. = \sum_i \frac{\partial}{\partial x_i} f(V_i)$$

$$= \sum_i \left[\frac{\partial f}{\partial x_i} V_i + f \frac{\partial V_i}{\partial x_i} \right]$$

$$= \sum_i \frac{\partial f}{\partial x_i} V_i + f \sum_i \frac{\partial V}{\partial x_i}$$

$$= \nabla f \cdot \bar{V} + f (\nabla \cdot \bar{V})$$

Thus

$$\nabla \cdot (f \bar{V}) = \nabla f \cdot \bar{V} + f (\nabla \cdot \bar{V}) \quad \text{q.e.d.}$$

(xvi) $\overline{A} = \bar{\alpha} \times \bar{\beta}$ $\bar{\alpha}$ is a constant vector

Find $\nabla \cdot \overline{A}$

$$\nabla \cdot \overline{A} = \sum_l \hat{e}_{(l)} \frac{\partial}{\partial x_l} \cdot \sum_{ijk} \epsilon_{ijk} \alpha_i \beta_j \hat{e}_{(k)}$$

$$= \sum_{ijkl} \frac{\partial}{\partial x_l} (\alpha_i \beta_j) \delta_{lk} \epsilon_{ijk}$$

$$= \sum_{ijk} \epsilon_{ijk} \alpha_i \frac{\partial \beta_j}{\partial x_k}$$

$$= \sum_{ijk} \epsilon_{ijk} \alpha_i \frac{\partial \beta_j}{\partial x_k} \hat{e}_{(i)} \cdot \hat{e}_{(i)}$$

$$= \sum_i \alpha_i \hat{e}_{(i)} \cdot \sum_{ijk} \epsilon_{ijk} \frac{\partial \beta_j}{\partial x_k} \hat{e}_{(i)}$$

$$= -\bar{\alpha} \cdot \sum_{ijk} \epsilon_{jki} \frac{\partial \beta_j}{\partial x_k} \hat{e}_{(i)}$$

$$= -\bar{\alpha} \cdot (\nabla \times \bar{\beta})$$

(xvii) $\overline{A} = \psi \nabla \phi - \phi \nabla \psi$

To show that

$$\iiint_V [\psi \nabla^2 \phi - \phi \nabla^2 \psi] \, dV$$

$$= \oiint \left(\psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n} \right) \cdot d\bar{s}$$

By Green's thm -

$$\iiint_V \nabla\cdot\bar{A}\, dV = \oiint_S \bar{A}\cdot d\bar{S}$$

$$\therefore \iiint_V \nabla\cdot(\psi\nabla\phi - \phi\nabla\psi)\, dV = \oiint_S (\psi\nabla\phi - \phi\nabla\psi)\cdot d\bar{S}$$

$$LHS = \iiint_V (\psi\nabla^2\phi + \nabla\psi\cdot\nabla\phi - \nabla\phi\cdot\nabla\psi - \phi\nabla^2\psi)\, dV$$

$$= \iiint_V [\psi\nabla^2\phi - \phi\nabla^2\psi]\, dV$$

$$= \oiint [\psi\nabla\phi - \phi\nabla\psi]\cdot d\bar{S} = RHS \quad \text{q.e.d.}$$

xviii Show that $V = \frac{1}{3}\oiint \bar{r}\cdot d\bar{S}$

$$\oiint \bar{r}\cdot d\bar{S}$$

$$= \iiint_V \nabla\cdot\bar{r}\, dV \quad \text{by Green's thm.}$$

$$= \iiint_V 3\, dV$$

$$= 3\iiint dV = V$$

$$\therefore V = \frac{1}{3}\oiint \bar{r}\cdot d\bar{S}$$

q.e.d.

(xix) Show that

$$\oint\!\!\oint_S \{\bar{A}(\bar{A}\cdot\bar{n}) - \tfrac{1}{2}A^2\bar{n}\}\cdot d\bar{s}$$

$$= \iiint_V \{\bar{A}(\nabla\cdot\bar{A}) - \bar{A}\times\nabla\times\bar{A}\}\, dV$$

Now

$$\oint\!\!\oint_S \bar{A}(\bar{A}\cdot\bar{n})\cdot d\bar{s}$$

$$= \cancel{\oint\!\!\oint A^2 \bar{A}\cdot d\bar{s}} \quad \oint\!\!\oint A^2\, d\bar{s}$$

$$= \iiint \nabla\cdot A^2\, dV$$

$$= \iiint 2\,\nabla\cdot\bar{A}\, dV$$

$$\oint\!\!\oint \frac{1}{2}A^2\,\bar{n}\cdot d\bar{s} = \oint\!\!\oint \frac{A^2}{2}\, d\bar{s}$$

$$= \iiint_V \nabla\cdot\left(\frac{A^2}{2}\right) dv$$

=

(xix) J = heat flowing / unit area / unit time $= -K\nabla T$

To prove that eqn. of heat conduction

$$K\nabla^2 T = c\rho \frac{\partial T}{\partial t} \quad \text{where} \quad c = \text{sp. heat}, \quad \rho = \text{density}.$$

Heat inflow in volume V enclosed by $d\bar{S}$

$$= \iint_S (-K\nabla T)\cdot d\bar{S}$$

$$= \iiint_V \nabla\cdot(-K\nabla T)\, dV$$

$$= \iiint_V -K\nabla^2 T \, dV.$$

Now, if ρ is the density & c is the sp. heat

heat absorbed by unit volume / unit time

$$= \cancel{\rho\, c\, dV \cdot \frac{\partial T}{\partial t}} \qquad \rho\, c\, dV \frac{\partial T}{\partial t}$$

$\therefore$ Total heat absorbed / unit time

$$= \iiint_V \rho c \frac{\partial T}{\partial t}\, dV$$

$$\therefore \quad K\nabla^2 T = c\rho \frac{\partial T}{\partial t}$$

(xx) If $A = \bar{\alpha}\phi$ where $\bar{\alpha}$ is a constant

Show that $\iint_S (\hat{n} \times \nabla\phi)\, d\bar{s} = \oint_C \phi\, d\bar{l}$

$$\nabla \times \bar{\alpha}\phi = \bar{\alpha} \times \nabla\phi$$

$$\therefore \iint (\nabla \times \bar{\alpha}\phi)\, d\bar{s} = \iint (\alpha \times \nabla\phi) \cdot \bar{n}\, ds$$

$$= \iint_S \vec{\alpha} \cdot (\nabla\phi \times \bar{n})\, ds$$

$$= \vec{\alpha} \iint (\nabla\phi \times \bar{n})\, ds$$

Applying Stoke's thm.

$$\iint_S (\nabla \times \bar{\alpha}\phi) \cdot d\bar{s} = \oint_C (\bar{\alpha}\phi) \cdot d\bar{l}$$

$$= \bar{\alpha} \oint_C \bar{\phi}\, dl$$

$$\therefore \bar{\alpha} \iint_S (\nabla\phi \times \hat{n})\, ds = \bar{\alpha} \oint \phi\, d\bar{l}$$

$$\therefore \iint_S (\nabla\phi \times \bar{n})\, ds = \oint_C \phi\, dl$$

q.e.d.

(xxi) $\bar{\alpha} \times \bar{\beta} = \bar{A}$

To prove that $\iint_S (dS \times \nabla) \times \bar{\beta} = \oint_C (d\bar{l} \times \bar{\beta})$

Now

$$\iint \nabla \times \bar{A} \cdot ds = \oint \bar{A} \cdot dl$$

$$\therefore \iint \nabla \times (\bar{\alpha} \times \bar{\beta})\, ds = \oint_C (\bar{\alpha} \times \bar{\beta})\, d\bar{l}$$

Now $\nabla \times (\bar{\alpha} \times \bar{\beta}) = \bar{\alpha} \times (\nabla \times \bar{\beta})$

$$\therefore \iint_S [\bar{\alpha} \times (\nabla \times \beta)] \cdot d\bar{s} = \oint (\bar{\alpha} \times \bar{\beta}) \cdot d\bar{l}$$

$$\therefore \iint_S \bar{\alpha} \cdot [(\nabla \times \beta) \times d\bar{s}] = \oint \alpha \cdot (\bar{\beta} \times d\bar{l})$$

$$\therefore \iint_S (\nabla \times \beta) \times d\bar{s} = \oint (\bar{\beta} \times d\bar{l})$$ q.e.d.

V. Good

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(24) Prove that the Jacobian of transformation for a volume element

$$J\left(\frac{x_1\ x_2\ x_3}{q_1\ q_2\ q_3}\right) = \sum_{ijk} \epsilon_{ijk} \frac{\partial x_i}{\partial q_1} \frac{\partial x_j}{\partial q_2} \frac{\partial x_k}{\partial q_3}$$

We have, the infiniteomal volume element

$$dV = d\bar{x}_1 \cdot (d\bar{x}_2 \times d\bar{x}_3)$$

Using $d\bar{x}_1 = \frac{\partial x_1}{\partial q_1} d\bar{q}_1 + \frac{\partial x_1}{\partial q_2} d\bar{q}_2 + \frac{\partial x_1}{\partial q_3} d\bar{q}_3$

We get $dV = d\bar{x}_1 \cdot (d\bar{x}_2 \times d\bar{x}_3)$

$$= \begin{vmatrix} \frac{\partial x_1}{\partial q_1} dq_1 & \frac{\partial x_1}{\partial q_2} dq_2 & \frac{\partial x_1}{\partial q_3} dq_3 \\ \frac{\partial x_2}{\partial q_1} dq_1 & \frac{\partial x_2}{\partial q_2} dq_2 & \frac{\partial x_2}{\partial q_3} dq_3 \\ \frac{\partial x_3}{\partial q_1} dq_1 & \frac{\partial x_3}{\partial q_2} dq_2 & \frac{\partial x_3}{\partial q_3} dq_3 \end{vmatrix}$$

Since all the elements in colomn 1 are multiplied by dq_1, we can take it outside the determinant. Same holds for dq_2 & dq_3

$$\therefore dV = \begin{vmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \frac{\partial x_1}{\partial q_3} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \frac{\partial x_2}{\partial q_3} \\ \frac{\partial x_3}{\partial q_1} & \frac{\partial x_3}{\partial q_2} & \frac{\partial x_3}{\partial q_3} \end{vmatrix} dq_1\, dq_2\, dq_3$$

$$\therefore [dx_1\, dx_2\, dx_3] = J\left(\frac{x_1\ x_2\ x_3}{q_1\ q_2\ q_3}\right) [dq_1\, dq_2\, dq_3] \quad \text{q.e.d.}$$

(25) Determine the volume element in cylindrical co-ordinates

$x_1 = r\cos\theta \qquad x_2 = r\sin\theta \qquad x_3 = x_3$

we have

$dx_1\, dx_2\, dx_3 = h_1 h_2 h_3\, dq_1\, dq_2\, dq_3$

$h_1^2 = \sum_i \left(\frac{\partial x_i}{\partial q_1}\right)^2 = \sum_{i=1}^{3}\left(\frac{\partial x_i}{\partial r}\right)^2 = \cos^2\theta + \sin^2\theta + 0 = 1$

$\therefore h_1 = 1.$

$h_2^2 = \sum_{i=1}^{3}\left(\frac{\partial x_i}{\partial \theta}\right)^2 = r^2\sin^2\theta + r^2\cos^2\theta + 0 = r^2 \Rightarrow h_2 = r$

$h_3^2 = \sum_{i=1}^{3}\left(\frac{\partial x_i}{\partial x_3}\right)^2 = 0 + 0 + 1 = 1 \Rightarrow h_3 = 1$

$\therefore dV = 1 \times r \times 1\, dq_1\, dq_2\, dq_3$

$= r\, dr\, d\theta\, dx_3$

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(26) Determine the volume element in spherical polar co-ords.

$x_1 = r\sin\theta\cos\phi \qquad x_2 = r\sin\theta\sin\phi \qquad x_3 = r\cos\theta.$

we have, as explained in (25)

$h_1^2 = \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta = 1 \Rightarrow h_1 = 1$

$h_2^2 = r^2\cos^2\phi\cos^2\theta + r^2\sin^2\phi\cos^2\theta + r^2\sin^2\theta = r^2 \Rightarrow h_2 = r$

$h_3^2 = r^2\sin^2\theta\sin^2\phi + r^2\sin^2\theta\cos^2\phi + 0 = r^2\sin^2\theta \Rightarrow h_3 = r\sin\theta$

Now

$dV = h_1 h_2 h_3\, dq_1\, dq_2\, dq_3$

$\therefore dV = 1 \times r \times r\sin\theta\, dr\, d\theta\, d\phi$

$= r^2 \sin\theta\, dr\, d\theta\, d\phi.$

(27) Prove that

$$\nabla \times V = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1 V_1 & h_2 V_2 & h_3 V_3 \end{vmatrix}$$

We have

$$\nabla \times V = \nabla \times \left(\Sigma V_i \hat{e}_i \right) \text{ in cartesian}$$

$$= \nabla \times \left(\sum_p \underline{V}p \, \underline{\hat{e}}_{(p)} \right) \text{ in o.c.c.}$$

Now taking components

$$\nabla \times \bar{V}_1 = \nabla \times \left(v_1 \hat{e}_{(1)} \right)$$

$$= \nabla \times \left(V_1 h_1 \nabla q_1 \right)$$

$$= \nabla (V_1 h_1) \times \nabla q_1 + v_1 h_1 \nabla \times \nabla q_1$$

$$= \nabla (V_1 h_1) \times \frac{\underline{\hat{e}}_{(1)}}{h_1}$$

$$= \cancel{\sum_i \left(\frac{\hat{e}_i}{h_i} \frac{\partial}{\partial q_i} \right.}$$

$$= \sum_i \frac{\hat{e}_i}{h_i} \frac{\partial}{\partial q_i} (v_1 h_1) \times \frac{\hat{e}_{(1)}}{h_1}$$

$$= \sum_i \frac{1}{h_i h_1} \frac{\partial}{\partial q_i} (V_1 h_1) \left(\hat{e}_{(i)} \times \hat{e}_{(1)} \right)$$

On expanding, we get

$$\nabla \times \bar{V}_1 = \frac{1}{h_1 h_3} \frac{\partial}{\partial q_3} (V_1 h_1) \hat{e}_{(2)} - \frac{1}{h_1 h_2} \frac{\partial}{\partial q_2} (v_1 h_1) \hat{e}_{(3)}$$

$$= \frac{1}{h_1 h_2 h_3} \left[h_2 \frac{\partial (V_1 h_1)}{\partial q_3} \hat{e}_{(2)} - h_3 \frac{\partial}{\partial q_2} (V_1 h_1) \hat{e}_{(3)} \right]$$

similarly

$$\nabla \times \bar{V}_2 = \frac{1}{h_1 h_2 h_3} \left[h_3 \frac{\partial (V_2 h_2)}{\partial q_1} \hat{e}_{(3)} - h_1 \frac{\partial}{\partial q_3} (v_2 h_2) \hat{e}_{(1)} \right]$$

Adding together similar expression for $\nabla \times \bar{V}_3$ we get

$$\nabla \times V = \begin{vmatrix} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1 V_1 & h_2 V_2 & h_3 V_3 \end{vmatrix} \quad \text{q.e.d.}$$

(28) Find an expression for $\nabla \psi$ in the parabolic co-ordinate system.

$x_1 = uv \qquad x_2 = \frac{1}{2}(u^2 - v^2) \qquad x_3 = w.$

We have

$$\nabla \psi = \sum_p \frac{\hat{e}_{(p)}}{h_p} \frac{\partial \psi}{\partial q_p}$$

$$h_1^2 = \sum_{i=1}^{3} \left(\frac{\partial x_i}{\partial u} \right)^2 = v^2 + u^2 \Rightarrow h_1 = \sqrt{u^2 + v^2}$$

Similarly, $h_2 = \sqrt{u^2 + v^2}$ & $h_3 = 1$.

$$\therefore \nabla \psi = \frac{1}{(u^2+v^2)^{1/2}} \frac{\partial \psi}{\partial u} \hat{e}_u + \frac{1}{(u^2+v^2)^{1/2}} \frac{\partial \psi}{\partial v} \hat{e}_v + \frac{\partial \psi}{\partial v} \hat{e}_w$$

(29) Find an expression for $\nabla \cdot \bar{a}$ in parabolic co-ordinate system.

$x_1 = uv, \qquad x_2 = \frac{1}{2}(u^2 - v^2) \qquad x_3 = w$

We have

$$\nabla \cdot \bar{a} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial q_1}(a_1 h_2 h_3) + \frac{\partial}{\partial q_2}(h_1 a_2 h_3) + \frac{\partial}{\partial q_3}(h_1 h_2 a_3) \right]$$

As proved in the previous example

$h_1 = h_2 = (u^2+v^2)^{1/2}$, $h_3 = 1$

$$\therefore \nabla \cdot \bar{a} = \frac{1}{u^2+v^2}\left[\frac{\partial}{\partial u}(u^2+v^2)^{1/2} a_1 + \frac{\partial}{\partial v}(u^2+v^2)^{1/2} a_2 + \frac{\partial (u^2+v^2) a_3}{\partial w}\right]$$

$$= \frac{1}{u^2+v^2}\left[\frac{2a_1 u}{2(u^2+v^2)^{1/2}} + (u^2+v^2)^{1/2}\frac{\partial a_1}{\partial u} + \frac{2va_2}{2(u^2+v^2)^{1/2}} + (u^2+v^2)^{1/2}\frac{\partial a_2}{\partial v} + (u^2+v^2)\frac{\partial a_3}{\partial w}\right]$$

$$\therefore \nabla \cdot \bar{a} = \frac{1}{(u^2+v^2)^{3/2}}[a_1 u + a_2 v] + \frac{1}{(u^2+v^2)^{1/2}}\left[\frac{\partial a_1}{\partial u} + \frac{\partial a_2}{\partial v} + (u^2+v^2)^{1/2}\frac{\partial a_3}{\partial w}\right]$$

(30) Using $V = \nabla\psi$ derive the expression for $\nabla^2\psi$ in curvilinear co-ordinates.

As we have already proved

$$\nabla \cdot \bar{V} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(V_1 h_2 h_3) + \frac{\partial}{\partial q_2}(h_1 V_2 h_3) + \frac{\partial}{\partial q_3}(h_1 h_2 V_3)\right]$$

If $\bar{V} = \nabla\psi$, we have in O.C.C.

$$V_1 = \frac{1}{h_1}\frac{\partial\psi}{\partial q_1}, \quad V_2 = \frac{1}{h_2}\frac{\partial\psi}{\partial q_2}, \quad V_3 = \frac{1}{h_3}\frac{\partial\psi}{\partial q_3}$$

Substituting

$$\nabla \cdot \bar{V} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\psi}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\psi}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\psi}{\partial q_3}\right)\right]$$

Since $\bar{V} = \nabla\psi$ we get $\nabla \cdot \bar{V} = \nabla^2\psi$

$$\therefore \nabla^2\psi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\psi}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\psi}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\psi}{\partial q_3}\right)\right]$$

(31) If (r, θ, ϕ) are polar co-ordinates and $\psi(r,\theta,\phi)$ is a ~~pt~~ scalar pt. function, show that

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

We have already obtained an expression for $\nabla^2\psi$ in the previous problem, viz. (30)

Again, refering to problem (26) we have

$h_r = 1$, $h_\theta = r$ and $h_\phi = r\sin\theta$

Substituting these values and simplifying

$$\nabla^2\psi = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{r\sin\theta}{r}\frac{\partial\psi}{\partial\theta}\right) + \frac{\partial}{\partial\phi}\left(\frac{r}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\right)\right]$$

$$\therefore \nabla^2\psi = \frac{1}{r^2\sin\theta}\left[2r\sin\theta\frac{\partial\psi}{\partial r} + r^2\sin\theta\frac{\partial^2\psi}{\partial r^2} + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{\partial\psi}{\partial\phi}\right)\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

which is the required result.

(32) Show that $\psi = \frac{1}{r} e^{ikr}$ where k is a const. is a sdn. of the eqn.

$$\nabla^2 \psi + k^2 \psi = 0.$$

Since ψ is a f^n of r alone, we have

$$\nabla^2 \psi = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right)$$

Now $\psi = \frac{1}{r} e^{ikr} \Rightarrow \frac{\partial \psi}{\partial r} = -\frac{e^{ikr}}{r^2} + \frac{ike^{ikr}}{r}$

$$\therefore r^2 \frac{\partial \psi}{\partial r} = -e^{ikr} + rike^{ikr}$$

$$\therefore \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) = -ike^{ikr} + ike^{ikr} + ri^2k^2e^{ikr}$$

$$= -rk^2e^{ikr} = -r^2 \frac{k^2e^{ikr}}{r} = -r^2k^2\psi$$

$$\therefore \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) = \frac{-1}{r^2} r^2k^2\psi = -k^2\psi$$

$$\therefore \nabla^2 \psi = -k^2 \psi$$

$$\therefore \nabla^2 \psi + k^2 \psi = 0$$

Thus, $\psi = \frac{1}{r} e^{ikr}$ satisfies the eqn. $\nabla^2 \psi + k^2 \psi = 0$ which is the required result.

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(33) Using $\nabla q_1 = \hat{e}_{(1)}/h_1$ show that

$$\nabla \times \hat{e}_{(1)} = -\frac{\hat{e}_{(1)}}{h_1} \times \nabla h_1$$

We have $\hat{e}_{(1)} = h_1 \nabla q_1$

$$\therefore \quad \nabla \times \hat{e}_{(1)} = \nabla \times (h_1 \nabla q_1)$$

$$= \nabla h_1 \times \nabla q_1 + h_1 (\nabla \times \nabla q_1)$$

but $\nabla \times \nabla q_1 = 0$.

$$\therefore \quad \nabla \times \hat{e}_{(1)} = - \nabla q_1 \times \nabla h_1$$

$$= - \frac{\hat{e}_{(1)}}{h_1} \times \nabla h_1$$

since $\nabla q_1 = \hat{e}_{(1)} / h_1$

Thus $\nabla \times \hat{e}_{(1)} = - \dfrac{\hat{e}_{(1)}}{h_1} \times \nabla h_1$ q.e.d.

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(34) Prove that

$$\sum_t h^{rt} h_{tp} = \delta^r_p$$

We have

$$h_p = \sum_t h^t h_{pt}$$

$$\therefore \quad h^r \sum_t h^t h_{pt} = \sum_t h^r h^t h_{pt} = \sum_t h^{rt} h_{pt}$$

$$\therefore \quad \sum_t h^{rt} h_{pt} = h^r \cdot h_p = \delta^r_p$$

$$\therefore \quad \sum_t h^{rt} h_{pt} = \delta^r_p \quad \text{q.e.d.}$$

---

(35) Use $h_{pr} = \overline{h}_p \cdot \overline{h}_r$ and $\overline{h}_p = \dfrac{\partial x}{\partial q_p}$

to show that

$$[pr : t] = \overline{h}_t \cdot \frac{\partial \overline{h}_p}{\partial q_r}$$

P.T.O.

We have,

$$[pr:t] = \frac{1}{2}\left[\frac{\partial \bar{h}_{pt}}{\partial q_r} + \frac{\partial \bar{h}_{rt}}{\partial q_p} - \frac{\partial \bar{h}_{pr}}{\partial q_t}\right]$$

$$= \frac{1}{2}\left[\frac{\partial}{\partial q_r}(\bar{h}_p \cdot \bar{h}_t) + \frac{\partial}{\partial q_p}(\bar{h}_r \cdot \bar{h}_t) - \frac{\partial}{\partial q_t}(\bar{h}_p \cdot \bar{h}_r)\right]$$

$$= \frac{1}{2}\left[\bar{h}_p \cdot \frac{\partial \bar{h}_t}{\partial q_r} + \bar{h}_t \cdot \frac{\partial \bar{h}_p}{\partial q_r} + \bar{h}_r \cdot \frac{\partial \bar{h}_t}{\partial q_p} + \bar{h}_t \cdot \frac{\partial \bar{h}_r}{\partial q_p} - \bar{h}_p \cdot \frac{\partial \bar{h}_r}{\partial q_t} - \bar{h}_r \cdot \frac{\partial \bar{h}_p}{\partial q_t}\right]$$

Now $\frac{\partial \bar{h}_r}{\partial q_t} = \frac{\partial}{\partial q_t}\left(\frac{\partial \bar{x}}{\partial q_r}\right) = \frac{\partial}{\partial q_r}\left(\frac{\partial \bar{x}}{\partial q_t}\right) = \frac{\partial \bar{h}_t}{\partial q_r}$

Hence, we get the only terms remaining as

~~$2[pr:t] =$~~ $\bar{h}_t \cdot \frac{\partial \bar{h}_p}{\partial q_r}$ and $\bar{h}_t \cdot \frac{\partial \bar{h}_r}{\partial q_p}$

which are again equal.

Hence,

$$[pr:t] = \frac{1}{2}\left[2\bar{h}_t \cdot \frac{\partial \bar{h}_r}{\partial q_p}\right]$$

$$= \bar{h}_t \cdot \frac{\partial \bar{h}_r}{\partial q_p}$$

or again applying the result

$$[pr:t] = \bar{h}_t \cdot \frac{\partial \bar{h}_p}{\partial q_r} \qquad \text{q.e.d.}$$

36 (36) Prove that $\dfrac{\partial \bar{h}_j}{\partial q_k} = \sum_i [jk:i]\,\bar{h}^i$

We have, using the result of the previous problem,

$$\bar{h}_p \cdot \frac{\partial \bar{h}_j}{\partial q_k} = [jk:p]$$
$$= \sum_i [jk:i]\,\delta^i_p$$
$$= \sum_i [jk:i]\,\bar{h}_p \cdot \bar{h}^i$$
$$= \bar{h}_p \cdot \sum_i [jk:i]\,\bar{h}^i$$

Hence

$$\frac{\partial \bar{h}_j}{\partial q_k} = \sum_i [jk:i]\,\bar{h}^i$$

37 (37) Show that

$$\frac{\partial \bar{h}_j}{\partial q_k} = \sum_l \left\{ {l \atop j\;k} \right\} \bar{h}_l$$

We have $\displaystyle \sum_l \left\{ {l \atop j\;k} \right\} \bar{h}_l = \sum_l \Big[\sum_i [jk:i]\, h^{il} \Big] \bar{h}_l$

$$= \sum_{l\,i} [jk:i]\, h^{il}\,\bar{h}_l \quad \text{but } \sum_l h^{il}\,\bar{h}_l = \bar{h}^i$$

$$\therefore \cancel{\frac{\partial \bar{h}_j}{\partial q_k}} = \sum_i [jk:i]\,\bar{h}^i$$

but we have, by problem (36)

$$\sum_i [jk:i]\,\bar{h}^i = \frac{\partial \bar{h}_j}{\partial q_k}$$

$$\therefore \sum_l \left\{ {l \atop j\;k} \right\} \bar{h}_l = \frac{\partial \bar{h}_j}{\partial q_k} \qquad \text{q.e.d.}$$

(38) ~~Prove that the value of a determinant is unaltered if to~~

(38) Show that

$$\sum_i \sum \left\{ {i \atop i\ k} \right\} = \frac{1}{2} \frac{\partial \ln h}{\partial q_k}$$

We have $\left\{ {i \atop j\ k} \right\} = \sum_p h^{ip} [jk : p]$

$$= \frac{1}{2} \sum_p h^{ip} \left[\frac{\partial h_{pj}}{\partial q_k} + \frac{\partial h_{kp}}{\partial q_j} - \frac{\partial h_{jk}}{\partial q_p} \right]$$

$\therefore$ ~~$\left\{ {i \atop j\ k} \right\} =$~~

$$\therefore \left\{ {i \atop i\ k} \right\} = \frac{1}{2} \sum_{ip} h^{ip} \left[\frac{\partial h_{pi}}{\partial q_k} + \frac{\partial h_{kp}}{\partial q_i} - \frac{\partial h_{ik}}{\partial q_p} \right]$$

$$= \frac{1}{2} \sum_{ip} h^{ip} \left[\frac{\partial h_{pi}}{\partial q_k} + \frac{\partial h_{kp}}{\partial q_i} - \frac{\partial h_{pk}}{\partial q_i} \right]$$

As interchange of i & p is permitted in the last term.

$$\therefore \left\{ {i \atop i\ k} \right\} = \frac{1}{2} \sum_{ip} h^{ip} \frac{\partial h_{ip}}{\partial q_k}$$

we have $h^{ip} = \delta_{ip} / h_i^2$

$$\therefore \left\{ {i \atop i\ k} \right\} = \frac{1}{2} \sum_i \frac{1}{h_i^2} \frac{\partial h_{ii}}{\partial q_k}$$

$$= \frac{1}{2} \frac{1}{h} \frac{\partial h}{\partial q_k}$$ On summing up. as $h = h_1^2 h_2^2 h_3^2$

$$= \frac{1}{2} \frac{\partial \ln h}{\partial q_k}$$

$$\therefore \left\{ {i \atop i\ k} \right\} = \frac{1}{2} \frac{\partial \ln h}{\partial q_k}$$ q.e.d.

(39) If $B_i = \sum_{jk} \epsilon_{ijk} \frac{\partial A_j}{\partial q_k}$

Show that $\sum_k \epsilon_{ijk} B_k = \frac{\partial A_i}{\partial q_j} - \frac{\partial A_j}{\partial q_i}$

Now, we have

$$B_k = \sum_{lm} \epsilon_{klm} \frac{\partial A_l}{\partial q_m}$$

$$\therefore \sum_k \epsilon_{ijk} B_k = \sum_{klm} \epsilon_{klm} \frac{\partial A_l}{\partial q_m} \epsilon_{ijk}$$

$$= \sum_{lm} \frac{\partial A_l}{\partial q_m} \left(\sum_k \epsilon_{ijk} \epsilon_{klm} \right)$$

$$= \sum_{lm} \frac{\partial A_l}{\partial q_m} \left(\delta_{li} \delta_{mj} - \delta_{lj} \delta_{mi} \right)$$

$$= \sum_{lm} \frac{\partial A_l}{\partial q_m} \delta_{li} \delta_{mj} - \sum_{lm} \delta_{lj} \delta_{mi} \frac{\partial A_l}{\partial q_m}$$

Thus $\sum_k \epsilon_{ijk} B_k = \frac{\partial A_i}{\partial q_j} - \frac{\partial A_j}{\partial q_i}$ q.e.d.

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(40) Representing the second rank covariant tensor ~~by~~ $h_{ij} = h_i h_j$, show that

$$h_{ij,k} = \frac{\partial h_{ij}}{\partial q_k} - \sum_l \left\{ {l \atop i\ k} \right\} h_{lj} - \sum_l \left\{ {l \atop j\ k} \right\} h_{il}$$

hence deduce the result $h_{ij,k} = 0$.

Now, we already have

$$h_{i,k} = \frac{\partial h_i}{\partial q_k} - \sum_l \left\{ {l \atop i\ k} \right\} h_l$$

P.T.O.

And since $h_{ij} = h_i h_j$

$$\therefore h_{ij,k} = h_{i,k} h_j + h_i h_{j,k}$$

$$\therefore h_{ij,k} = h_j \left\{ \frac{\partial h_i}{\partial q_k} - \sum_l \left\{ {l \atop i\;k} \right\} h_l \right\} + h_i \left\{ \frac{\partial h_j}{\partial q_k} - \sum_l \left\{ {l \atop j\;k} \right\} h_l \right\}$$

$$= h_j \frac{\partial h_i}{\partial q_k} + h_i \frac{\partial h_j}{\partial q_k} - \sum_l \left\{ {l \atop i\;k} \right\} h_{lj} - \sum_l \left\{ {l \atop j\;k} \right\} h_{il}$$ ~~As other terms cancel.~~

$$\therefore h_{ij,k} = \frac{\partial h_{ij}}{\partial q_k} - \sum_l \left\{ {l \atop i\;k} \right\} h_{lj} - \sum_l \left\{ {l \atop j\;k} \right\} h_{il}$$

which is the required result.

Now

$$\left\{ {l \atop i\;k} \right\} = \sum_p [ik:p]\, h^{pl}$$

$$\therefore \sum_l \left\{ {l \atop i\;k} \right\} h_{lj} = \sum_{p,l} [ik:p]\, h^{pl} h_{lj}$$

$$= \sum_p [ik:p]\, \delta_j^p = [ik:j]$$

Similarly, we get $\sum_l \left\{ {l \atop j\;k} \right\} h_{il} = [jk:i]$

Now we have

$$[ik:j] = h_j \frac{\partial h_i}{\partial q_k} \quad \& \quad [jk:i] = h_i \frac{\partial h_j}{\partial q_k}$$

$$\therefore h_{ij,k} = h_j \frac{\partial h_i}{\partial q_k} + h_i \frac{\partial h_j}{\partial q_k} - h_j \frac{\partial h_i}{\partial q_k} - h_i \frac{\partial h_j}{\partial q_k}$$

$$\therefore h_{ij,k} = 0 \quad \text{q.e.d.}$$

(41) If any two rows of a determinant are proportional, then the determinant vanishes.

Let the l^{th} row be K times the m^{th} row

we have

$$D_n = \sum_p \delta_p P[a_{11} \ldots a_{ll} \cdots a_{mm} \cdots a_{nn}]$$

$$= \sum_p \delta_p P \{ a_{11} \cdots K a_{mm} \cdots a_{mm} \; a_{nn}]$$

$$= K D_n' \quad \text{where } D_n' = \sum_p \delta_p P \{ a_{11} \cdots \overset{a_{mm}}{\wedge} a_{mm} \cdots a_{nn}]$$

Since, if two rows & a determinant are equal then its value is zero,

we have $D_n' = 0$.

$\therefore D_n = K D_n' = 0$ q.e.d.

(42) The value of a determinant is unchanged if ~~to~~ to each element of any row is added K times the corrossponding element of some other row, where K is a const.

Let $D_n = \sum_p \delta_p P[a_{11} \cdots a_{ii} \cdots a_{jj} \cdots a_{nn}]$

Let $D_n' = \sum_p \delta_p P[a_{11} \cdots a_{ii} + K a_{jj} \cdots a_{jj} \cdots a_{nn}]$

$$= \sum_p \delta_p P \{ a_{11} \ldots a_{nn}] + K \sum_p \delta_p P [a_{11} \cdots a_{jj} \, a_{jj} \, a_{nn}]$$

$$= D_n + 0$$

$\therefore D_n' = D_n$ q.e.d.

(43) Given that $\bar{B}$ is a vector and C_{ij} are the components of a second rank tensor and that $\sum A_{ijk} B^k = C_{ij}$ holds in all frames of reference show that A_{ijk} is a third rank tensor. Assume that B is independent of A_{ijk}

Now, we have

Since $\sum_k A_{ijk} B^k = C_{ij}$ holds in all frames

$$\therefore \sum_n A'_{lmn} B'^n = C'_{lm}$$

Now $$C'_{lm} = \sum_n A'_{lmn} B'^n$$

$$= \sum_{ij} \frac{\partial q_i}{\partial q'_l} \frac{\partial q_j}{\partial q'_m} C_{ij}$$

$$= \sum_{ij} \frac{\partial q_i}{\partial q'_l} \frac{\partial q_j}{\partial q'_m} \sum_k A_{ijk} B^k$$

As B^k is independent of A_{ijk}, $B^k = \sum_n \frac{\partial q_k}{\partial q'_n} B'^n$ since B^k are contravariant components.

$$\therefore C'_{lm} = \sum_{ijk} \frac{\partial q_i}{\partial q'_l} \frac{\partial q_j}{\partial q'_m} A_{ijk} \sum_n \frac{\partial q_k}{\partial q'_n} B'^n$$

$$\therefore \sum_n A'_{lmn} B'^n = \sum_n \sum_{ijk} \frac{\partial q_i}{\partial q'_l} \frac{\partial q_j}{\partial q'_m} \frac{\partial q_k}{\partial q'_n} A_{ijk} B'^n$$

$$\therefore \sum_n \left[A'_{lmn} - \sum_{ijk} \frac{\partial q_i}{\partial q'_l} \frac{\partial q_j}{\partial q'_m} \frac{\partial q_k}{\partial q'_n} A_{ijk} \right] B'^n = 0$$

$$\therefore A'_{lmn} = \sum_{ijk} \frac{\partial q_i}{\partial q'_l} \frac{\partial q_j}{\partial q'_m} \frac{\partial q_k}{\partial q'_n} A_{ijk}$$

which is according to the law for transformations of the covariant components of a tensor of rank 3.

$\therefore$ A_{ijk} is a covariant tensor of rank 3 q.e.d.

(44) $\sum_{ij} K_{ij} A^i B^j$ is invariant for any two vectors A^i, B^j prove that K_{ij} is a second rank covariant tensor.

Now, since we have

$$\sum_{ij} K_{ij} A^i B^j \text{ invariant}$$

$$\therefore \sum_{pr} K'_{pr} A'^p B'^r = \sum_{ij} K_{ij} A^i B^j$$

since A^i & B^j are contravariant vectors,

$$A^i = \sum_p \frac{\partial q_i}{\partial q'_p} A'^p \qquad B^j = \sum_r \frac{\partial q_j}{\partial q'_r} A'^r$$

$$\therefore \sum_{pr} K'_{pr} A'^p B'^r = \sum_{ij\,pr} K_{ij} \frac{\partial q_i}{\partial q'_p} \frac{\partial q_j}{\partial q'_r} A'^p B'^r$$

$$\therefore \sum_{pr} A'^p B'^r \left[K'_{pr} - \sum_{ij} \frac{\partial q_i}{\partial q'_p} \frac{\partial q_j}{\partial q'_r} K_{ij} \right] = 0$$

$$\therefore K'_{pr} = \sum_{ij} \frac{\partial q_i}{\partial q'_p} \frac{\partial q_j}{\partial q'_r} K_{ij}$$

which is the transformation law for second rank covariant tensor.

$\therefore$ K_{ij} is a second rank covariant tensor. q.e.d.

---

(45) If $T_{ij\cdots n}$ is a tensor of rank n, then show that $\sum_j \frac{\partial}{\partial x_j} (T_{ij\cdots n})$ is a tensor of rank $n-1$.

We have, since $T_{ij\cdots n}$ is an n'th rank tensor

$$T'_{\alpha\beta\cdots\eta} = \sum_{ijk\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \frac{\partial x'_\beta}{\partial x_j} \cdots \frac{\partial x'_\eta}{\partial x_n} T_{ij\cdots n}$$

$$\therefore \sum_{\beta} \frac{\partial T'_{\alpha\beta\cdots n}}{\partial x'_\beta} = \sum_{ij\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \cdots\cdots \frac{\partial T_{ij\cdots n}}{\partial x'_\beta}$$

$$= \sum_{i\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \frac{\partial x'_\beta}{\partial x_j} \cdots \frac{\partial T_{ij\cdots n}}{\partial x'_\beta}$$

$$= \sum_{i\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \frac{\partial x_j}{\partial x'_\beta} \frac{\partial T}{\partial x_j} \cdots \cancel{\sum_{i\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \frac{\partial x_j}{\partial x'_\beta} \frac{\partial T_{i\cdots n}}{\partial x_j} \frac{\partial x'_\beta}{\partial}}$$

$$= \sum_{ik\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \cdots \frac{\partial x'_n}{\partial x_n} \sum_j \frac{\partial x'_\beta}{\partial x_j} \frac{\partial x_j}{\partial x'_\beta} \frac{\partial T\cdots}{\partial x_j}\Big]$$

$$\therefore \sum_{\beta} \frac{\partial T'_{\alpha\cdots n}}{\partial \beta'} = \sum_{ik\cdots n} \frac{\partial x'_\alpha}{\partial x_i} \cdots \left(\sum_j \frac{\partial T}{\partial x_j} \right)$$

which is the transformation law for tensors of rank $n-1$.

Hence $\sum_j \frac{\partial T_{i\ldots n}}{\partial x_j}$ is a tensor of rank $n-1$.

(46) Show that, a negative ~~scalar~~ field is connected to the negative vector.

Now, we have to prove that

$$(-1)\bar{U} = -\bar{U}$$

we have

$$(-1) + 1 = 0$$

$$\therefore [(-1) + 1]\bar{U} = 0\bar{U} = 0$$

$$\therefore (-1)\bar{U} + \bar{U} = 0$$

$$\therefore (-1)\bar{U} = -\bar{U} \quad \text{q.e.d.}$$

(47) Show that a set of n tuples of complex no.s forms a vector space over a field of complex no.s.

$u = (u_1 \dots u_n) \quad v = (v_1 \dots v_n)$

$\bar{u} + \bar{v} = (u_1+v_1, \dots u_n+v_n) \qquad a(\bar{u}) = (au_1 \dots au_n)$

Now

Since closure under addition holds for complex no.s, it also holds for this set.

Since for every u_1 & v_1 $\exists$ unique u_1+v_1, for every $\bar{u}, \bar{v}$ $\exists$ unique $\bar{u}+v$.

$(u_1+v_1)+w_1 = u_1+(v_1+w_1)$ for complex no.s

$\Rightarrow (\bar{u}+\bar{v})+\bar{w} = \bar{u}+(\bar{v}+\bar{w})$ holds for the above set.

Similarly $u_1+v_1 = v_1+u_1 \Rightarrow \bar{u}+\bar{v} = \bar{v}+\bar{u}$.

$\exists\ 0 = (0\ 0 \dots 0)$ s.t. $\bar{u}+\bar{0} = \bar{0}+\bar{u} = \bar{u}$

$\exists\ -\bar{v} = (-v_1, -v_2 \dots -v_n)$ for every $\bar{v}$ s.t. $\bar{v}+(-\bar{v}) = 0$.

Since closure under multiplication holds for complex no.s, it also holds for the above set.

Similarly commutativity under scaler multiplication follows along with assossiativity also.

$\exists\ 0$ s.t. $0(\bar{u}) = 0$.

Thus, all the postulates for a vector space are satisfied by the set of n^{+}tuples & complex no.s.

$\therefore$ It is a vector space.

(48) If the spaces $l_1 \ldots l_k$ are linearly independent, show that every vector X contained in $l_1 \ldots l_k$ can be written uniquely as

$\bar{X} = \Sigma x_i$ where $\bar{x}_i \in l_i$

Now, we have

$$\bar{X} = \sum_i x_i = \sum_i \bar{x}'_i \quad \text{(say}$$

$$\therefore \bar{X} - \bar{X} = 0 = \sum_i (x_i - \bar{x}'_i)$$

$x_i, x'_i \in l_i \Rightarrow \bar{x}_i - \bar{x}'_i \in l_i$

Hence $\bar{x}_i - \bar{x}'_i = 0 \Rightarrow x_i - x_i' = 0$ for every component seperately. [due to linear independence.]

$\therefore x_i = x'_i$

$\therefore$ The representation $\sum_i x_i = \bar{X}$ is unique.

---

(49) Show that the two dimentional subspace L_2 of E_3 is a proper subspace.

Now, we have

in E_3, basis vectors i, j, k.

We consider L_2 with a basis $i\ j$.

Now orthogonality of $i, j\ k \Rightarrow L_2$ is closed under all the operations on i, j.

Similarly ~~L_2 be~~ since i, j are the basis vectors all the properties of a vector space viz.

assossiativity & commutativity under addition and scalar multiplication hold.

$\exists\ \bar{0} = 0i + 0j$ s.t. $\bar{0} + \bar{v} = \bar{v}$

$\exists\ 0$ s.t. $0\bar{v} = 0$. $\exists\ -\bar{v} = -v_1\hat{i} + -v_2\hat{j}$

Hence L_2 is a vector space.

Now $\hat{k} \notin L_2$, $\hat{k}, \hat{i}, \hat{j} \in E_3$

$\therefore L_2$ is a proper subspace of E_3.

---

(50) Are the vectors

$\bar{x}_1 = (0,-1,0)$, $\bar{x}_2 = (0,1,-1)$ & $\bar{x}_3 = (1-2\,1)$ linearly independent? Express $(-2,1,-3) = \bar{x}$ as a combination (linear) of $\bar{x}_1\ \bar{x}_2\ \bar{x}_3$ if possible.

Now, consider

$a\bar{x}_1 + b\bar{x}_2 + c\bar{x}_3 = 0$

$\therefore$ Considering the components.

$0a + 0b + c = 0 \Rightarrow c = 0$

$-a + b - 2c = 0 \Rightarrow -a + b = 0 \Rightarrow a = b$

$\cancel{0a - 0b}$ $0a - b + c = 0 \Rightarrow -b = 0 \Rightarrow b = 0 \Rightarrow a = 0$

Thus $a\bar{x}_1 + b\bar{x}_2 + c\bar{x}_3 = 0 \Rightarrow a = b = c = 0$

$\therefore \bar{x}_1, \bar{x}_2, \bar{x}_3$ are linearly idepedent.

Let $\bar{x} = m\bar{x}_1 + n\bar{x}_2 + p\bar{x}_3$

$\therefore 0m + 0n + p = -2 \Rightarrow p = -2$

$-m + n - 2p = 1 \Rightarrow n - m = -3$

$0m - n + p = -3 \Rightarrow -n = -1 \Rightarrow n = 1 \Rightarrow m = 4$

$\therefore \bar{x} = 4\bar{x}_1 + \bar{x}_2 - 2\bar{x}_3$

(51) Using $(u,v) = (vu)^*$, $(au\ v) = a^*(u,v)$
show that $(u, av) = a(uv)$

we have

$$(u\ av) = (av\ u)^* = \{a^*(vu)\}^*$$
$$= a(vu)^* = a(uv) \quad q.e.d.$$

(52) Prove that $(u+v, w) = (u,w) + (v\,w)$

Now we have

$$(u+v, w) = (w, u+v)^*$$
$$= [(w,u) + (w,v)]^*$$
$$= (w\,u)^* + (w\,v)^*$$
$$= (u\,w) + (v\,w) \quad q.e.d.$$

(53) Prove that $\|a\bar{v}\| = |a|\,\|\bar{v}\|$

we have

$$\|a\bar{v}\| = \sqrt{(a\bar{v}\ a\bar{v})}$$
$$= \sqrt{a(a\bar{v}\ \bar{v})}$$
$$= \sqrt{aa^*(\bar{v}\ \bar{v})}$$
$$= \sqrt{|a|^2\,\|\bar{v}\|^2}$$

$\therefore \|a\bar{v}\| = |a|\,\|\bar{v}\| \quad q.e.d.$

(54) Prove Schwartz's inequality

$$|(\bar{u}\ \bar{v})| \le \|\bar{u}\|\,\|\bar{v}\|$$

Now, consider $\bar{w} = \bar{u} - a\bar{v}$ $\quad a = \dfrac{(\bar{v}\ \bar{u})}{(\bar{v}\ \bar{v})}$

$$0 \le (\bar{w}\ \bar{w})$$

$$\therefore\ 0 \le (\bar{u} - a\bar{v},\ \bar{u} - a\bar{v})$$

$$\therefore\ 0 \le (\bar{u}\ \bar{u}) - (\bar{u}\ a\bar{v}) - (a\bar{v}\ \bar{u}) + (a\bar{v}\ a\bar{v})$$

$$\therefore\ 0 \le (\bar{u},\bar{u}) - a(\bar{u}\ \bar{v}) - a^*(\bar{v}\ \bar{u}) + aa^*(\bar{v}\ \bar{v})$$

Substituting the value of a

$$0 \le (\bar{u}\ \bar{u}) - \frac{(\bar{v}\ \bar{u})(\bar{u}\ \bar{v})}{(\bar{v}\ \bar{v})} - \frac{(\bar{u}\bar{v})(\bar{v}\bar{u})}{(\bar{v}\bar{v})} + \frac{(\bar{u}\bar{v})(\bar{v}\bar{u})}{(\bar{v}\ \bar{v})}$$

$$\therefore\ 0 \le (\bar{u}\ \bar{u})(\bar{v}\ \bar{v}) - (\bar{u}\bar{v})(\bar{v}\ \bar{u})$$

$$\therefore\ (\bar{v}\ \bar{u})(\bar{u}\ \bar{v}) \le (uu)(vv)$$

$$\therefore\ |(\bar{u}\ \bar{v})|^2 \le \|\bar{u}\|^2\,\|\bar{v}\|^2$$

$$\therefore\ |(\bar{u}\ \bar{v})| \le \|\bar{u}\|\,\|\bar{v}\| \qquad \text{q.e.d.}$$

---

(55) ~~Prove triangle inequality~~

$$\|\bar{u} + \bar{v}\| \le \|\bar{u}\| + \|\bar{v}\|$$

Now we have

$$\|\bar{u} + \bar{v}\|^2 = (\bar{u} + \bar{v},\ \bar{u} + \bar{v})$$

$$= (\bar{u}\bar{u}) + (\bar{u}\ \bar{v}) + (\bar{v}\ \bar{u}) + (\bar{v}\ \bar{v})$$

$$= \|\bar{u}\|^2 + \|\bar{v}\|^2 +$$

(55) Show that the n dimentional complex vector space satisfies the usual rules regarding the scalar product.

We have $(\bar{U}\bar{V}) = \sum_{ij} (u_i^* \hat{e}_{(i)}) \cdot (v_j \hat{e}_{(j)})$

$= \sum_{ij} u_i^* v_j \delta_{ij}$

$= \sum_i u_i^* v_i$

Now ~~e~~ we see

(i) $(\bar{V}\bar{U}) = \sum_i v_i^* u_i$

$= \sum_i (v_i u_i^*)^*$

$= \sum_i (u_i^* v_i)^*$

$= (\bar{U}\bar{V})^*$

$\therefore (\bar{V}\bar{U}) = (\bar{U}\bar{V})^*$.

(ii) $(\bar{U}, \bar{V}+\bar{W}) = \sum_i u_i^* (v_i + w_i)$

$= \sum_i u_i^* v_i + \sum_i u_i^* w_i$

$\therefore (\bar{U}\ \bar{V}+\bar{W}) = (\bar{U}\bar{V}) + (\bar{U}\bar{W})$

(iii) $(a\bar{U}\bar{V}) = \sum_i (a u_i)^* v_i$

$= \sum_i a^* u_i^* v_i = a^* \sum_i u_i^* v_i$

$= a^* (\bar{U}\bar{V})$

Thus the scalar product as defined above ~~satifi~~ satisfies the usual rules.

(56) Express Schwartz's inequality in terms of the components of the complex vector space C_n

We have Schwartz's inequality as

$$|(\bar{u}\,\bar{v})| \leq \|\bar{u}\|\|\bar{v}\|$$

Hence for C_n

$$|(\bar{u}\,\bar{v})| = \left|\sum_i u_i^* v_i\right|$$

& $\|\bar{u}\| = \sqrt{|(\bar{u}\,\bar{u})|} = \sqrt{\sum_i u_i^* u_i}$

$$\therefore \left|\sum_i u_i^* v_i\right| \leq \sqrt{\sum_i u_i^* u_i}\,\sqrt{\sum_i v_i^* v_i}$$

(57) Express the triangle inequality in terms of the components of C_n.

We have

$$\|\bar{u}+\bar{v}\| \leq \|\bar{u}\| + \|\bar{v}\|$$

We have

$$\|\bar{u}+\bar{v}\| = \sqrt{(\bar{u}\,\bar{v})(\bar{u}\,\bar{v})} = \sqrt{\sum_{ij}(u_i^*+v_j^*)(u_i+v_j)}$$

$$\|\bar{u}\| = \sqrt{\sum_i u_i^* u_i}$$

$$\therefore \sqrt{\sum_{ij}(u_i^*+v_j^*)(u_i+v_j)} \leq \sqrt{\sum_i u_i^* u_i} + \sqrt{\sum_j v_j^* v_j}$$

(58) If f and g are bounded then $f^* g$ and $f g^*$ are also bounded. Prove this. Also prove that $|f+g|$ exists in the same space.

Consider $|f+g|^2$

$$|f+g|^2 = |f|^2 + |g|^2 + |(f^* g)| + |(f g^*)|$$

Now, we have

$$|(f g^*)| \le |f|\,|g^*|$$

$$\therefore \quad |(f g^*)| \le |f|\,|g| \qquad \text{since } |g^*| = |g|$$

$\therefore$ $f g^*$ is bounded, as ~~$|f|$ & $|g|$~~ f & g are bounded.

Similarly $f^* g$ is also bounded.

Hence the RHS of

$$|f+g|^2 = |f|^2 + |g|^2 + |(f^* g)| + |(f g^*)|$$

is bounded

$\therefore$ $f+g$ is also bounded.

$\therefore$ $|f+g|$ belongs to the same space.

---

(59) Prove that

$$\|\bar{u}+\bar{v}\|^2 + \|\bar{u}-\bar{v}\|^2 = 2\|\bar{v}\|^2 + 2\|u\|^2$$

we have

$$\|\bar{u}+\bar{v}\|^2 = (\bar{u}+\bar{v}, \bar{u}+\bar{v}) = (\bar{u}\bar{u}) + (\bar{u}\bar{v}) + (\bar{v}\bar{u}) + (\bar{v}\bar{v})$$

$$\|\bar{u}-\bar{v}\|^2 = (\bar{u}-\bar{v}, \bar{u}-\bar{v}) = (\bar{u}\bar{u}) - (\bar{u}\bar{v}) - (\bar{v}\bar{u}) + (\bar{v}\bar{v})$$

Adding

$$\|\bar{u}+\bar{v}\|^2 + \|\bar{u}-\bar{v}\|^2 = 2\|\bar{u}\|^2 + 2\|\bar{v}\|^2 \quad \text{q.e.d.}$$

(60) Prove the Pythogoras theorem

i.e. $\|\bar{U} - \bar{V}\|^2 = \|U\|^2 + \|\bar{V}\|^2$ if $(U, V) = 0$.

Now

$$\|\bar{U} - \bar{V}\|^2 = (\bar{U}-\bar{V} \;\; \bar{U}-\bar{V}) = \cancel{(\bar{U}\bar{U})}$$

$$= (\bar{U}\,\bar{U}) - (\bar{U}\,\bar{V}) - (\bar{V}\,\bar{U}) + (\bar{V}\,\bar{V})$$

$$(\bar{U}\,\bar{V}) = 0 \Rightarrow (\bar{V}\,\bar{U}) = (\bar{U}\,\bar{V})^* = 0.$$

$$\therefore \|\bar{U} - \bar{V}\|^2 = \|\bar{U}\|^2 + \|\bar{V}\|^2 \quad \text{q.e.d.}$$

---

(61) In Gram Schmidt Process of orthonormalization we ~~have~~ have

$$X_K = \frac{d_K}{\|d_K\|} \quad \text{where } d_K = \begin{vmatrix} (Y_1 Y_1) & \cdots & (Y_1 Y_K) \\ \vdots & & \\ (Y_1 Y_{K-1}) & \cdots & \cdot \\ Y_1 & \cdots & Y_K \end{vmatrix}$$

Verify this for $K=1$ & 2

(a) For $K=1$

$$\bar{X}_1 = \frac{Y_1}{\|Y_1\|} \quad \text{which is obviously true}$$

& consistent with the previous results of G.S.O process

(b) for $K=2$

$$\bar{X}_2 = \frac{\begin{vmatrix} (Y_1 Y_1) & (Y_1 Y_2) \\ Y_1 & Y_2 \end{vmatrix}}{\left\| \begin{vmatrix} (Y_1 Y_1) & (Y_1 Y_2) \\ Y_1 & Y_2 \end{vmatrix} \right\|} = \frac{Y_2 (Y_1 Y_1) - Y_1 (Y_1 Y_2)}{\| Y_2 (Y_1 Y_1) - Y_1 (Y_1 Y_2) \|}$$

$$\therefore \bar{X}_2 = \frac{Y_2 - \frac{Y_1}{(Y_1 Y_1)}(Y_1 Y_2)}{\| Y_2 - Y_1 (Y_1 Y_2)/(Y_1 Y_1) \|} = \frac{Y_2 - (X_1 Y_2) X_1}{\| Y_2 - (X_1 Y_2) X_1 \|} \quad \text{q.ed}$$

(62) Starting from the linearly independent square integrable polynomial basis
$1, x, x^2 \dots x^n$ defined on the interval
-1 to $+1$ deduce an othonormal basis.

Now let the basis be

$\psi_0 \dots\dots \psi_{n-1}$

Now we have

$\psi_0 = \frac{1}{\|1\|} = $ But we have

$\|f\|^2 = \int_{-1}^{1} |f|^2 dx \quad \therefore \|1\|^2 = [1-(-1)] = 2$

$\therefore \|1\| = \sqrt{2}$

$\psi_0 = 1/\sqrt{2}$

Now going + ψ_1, we have

$\psi_1 = \frac{d_2}{\|d_2\|}$ where $d_2 = \begin{vmatrix} (1\ 1) & (1\ x) \\ 1 & x \end{vmatrix}$

$\therefore d_2 = 2x$

$\therefore \|d_2\|^2 = \int_{-1}^{1} 4x^2 dx = \left[\frac{4x^3}{3}\right]_{-1}^{+1} = \frac{8}{3}$

$\therefore \|d_2\| = \sqrt{8/3}$

$\therefore \psi_1 = \frac{2x}{\sqrt{8/3}} = \sqrt{\frac{3}{2}}\, x$

Proceeding along similar lines, we can find $\psi_3 \dots \psi_n$.

(63) Test these vectors for linear idependence.

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad X_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \quad X_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \quad x_4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

Consider $\sum c_i X_i = 0$

$\therefore$

$$c_1 + c_2 + c_4 = 0$$
$$-c_2 + c_3 - c_4 = 0$$
$$c_1 - c_3 + c_4 = 0$$
$$c_2 + c_3 - c_4 = 0$$

Now For $c_1 c_2 c_3 c_4$ to have nonzero solns

$$\begin{vmatrix} 1 & 1 & 0 & -1 \\ 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{vmatrix}$$ must be equal to zero.

but this is equal to -2.

$\therefore \; c_1 = c_2 = c_3 = c_4 = 0.$

Thus

$\sum c_i X_i = 0 \Rightarrow$ all $c_i = 0$.

$\therefore$ The vectors are linearly independent.

---

(64) Consider two sets of rectangular co-ord for $E=2$ space with common origin unit vectors being along the co-ord. axes. In terms of cosines of the angles between the two sets of axes, find

the matrix A relating the two co-ord systems.
Now, we have
If the axes turn at an angle θ.

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta.$$

$$\therefore \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ gives the transformation matrix.

And, rotating through $(-\theta)$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

but $A^{+} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$\therefore A^{+} = A^{-1}$$

$\therefore$ A is orthogonal.

---

(65) Given two basis Σx_i, Σy_i such that $x_i = \sum_j a_{ij} y_j$ prove that the transformation matrix is nonsingular.

Now, since Σx_i, Σy_i are both basis, we can have

$$Y_k = \sum_l b_{kl} X_l.$$

$$\therefore \quad X_i = \sum_j a_{ij} y_j = \sum_j a_{ij} \left\{ \sum_l b_{jl} X_l \right\}$$

$$\therefore \quad X_i = \sum_{jl} a_{ij} b_{jl} X_l.$$

but linear independence gives

$$x_i = \cancel{\sum_l x_{il} \delta_{il}} \quad \sum_l x_l \delta_{il}$$

$$\therefore \quad \sum_l \cancel{x} \delta_{il} X_l = \sum_{jl} \cancel{x} a_{ij} b_{jl} X_l.$$

$$\therefore \quad \sum_l \left[\delta_{il} - \sum_j a_{ij} b_{jl} \right] X_l = 0.$$

$$\therefore \quad \sum_j a_{ij} b_{jl} = \delta_{il}.$$

$$\therefore \quad \sum_j a_{ij} b_{jl} = I$$

$$\therefore \quad AB = I$$

This shows that ~~the~~ A can not be a singular matrix.

$$\therefore \quad A^{-1} = B.$$

$$\therefore \quad \{x_i\} = A\{Y_i\} \qquad \{y_i\} = A^{-1}\{x_i\}$$

which give the transformation relations.

(66) Find c_i , $i = 1 \ldots m$ which ~~minimises~~ minimizes $\| \bar{U} - \sum_{i=1}^{m} (c_i x_i) \|^2$ where $\bar{U}$ is a given vector in an n dimentional vector space and x_i is a set of orthonormal vectors.

~~When~~ Now, we have

$$0 \le \| \bar{U} - \sum_{i=1}^{m} c_i \bar{X}_i \|^2$$

$$= \left(\bar{U} - \sum_{i=1}^{m} c_i \bar{X}_i \, , \, \bar{U} - \sum_{j=1}^{m} c_j \bar{X}_j \right)$$

$$= (\bar{U}, \bar{U}) - \left(\bar{U}, \sum_{j=1}^{m} c_j \bar{X}_j \right) - \left(\sum_{i}^{m} c_i \bar{X}_i \, \bar{U} \right) + \left(\sum_i c_i x_i \, \sum_j \right.$$

$$= \| \bar{U} \|^2 - \sum_j c_j (\bar{U} x_j) - \sum_{j} c_j^* (\bar{X}_i \bar{U}) + \sum_{ij} c_i^* c_j (x_i x_j)$$

Adding and subtracting $\sum_{i}^{m} |(x_i \bar{U})(\bar{U} x_i)|^2$ gives

$$0 \le \| U \|^2 - \sum_{i=1}^{m} | c_i - (x_i \bar{U}) |^2 - \sum_{i=1}^{m} | (x_i \bar{U}) |^2$$

Since all the quantities on RHS are +ve

For RHS to be minimum

$c_i = (\bar{X}_i \bar{U})$ which is the required result.

---

(67) Prove Parseval's eqn.

$$(\bar{U} \bar{V}) = \sum_{i=1}^{n} (\bar{X}_i \bar{U})^* (\bar{X}_i \bar{V})$$

Now, consider RHS.

$$(\bar{X}_i \bar{U}) = \left(\bar{X}_i \, , \sum_j u_j \bar{X}_j \right)$$

$$= (\bar{X}_i \, , u_1 \bar{X}_1) + (\bar{X}_i \, , u_2 \bar{X}_2) \ldots \ldots$$

$$= \sum_j u_j (\bar{X}_i \bar{X}_j)$$

$$= \quad = u_i$$

$\therefore (\bar{X}_i \ \bar{U}) = U_i$

similarly $(\bar{X}_i \bar{V}) = V_i$

$$\therefore RMS = \sum_i (\bar{X}_i \ U)^* (\bar{X}_i \ \bar{V})$$

$$= \sum_i U_i^* V_i$$

$$= (\bar{U} \ \bar{V})$$

$$\therefore (\bar{U} \ \bar{V}) = \sum_{i=1}^{n} (\bar{X}_i \ \bar{U})^* (\bar{X}_i \ \bar{V})$$

$_jX_j)$

(68) If $\bar{U} = LU \quad \bar{V} = LV$

$|L| \neq$ and $V = AU$

$\bar{V} = B\bar{U}$ then what is B ?

Now, we have

$\bar{V} = LV = LAU$

but $\bar{V} = B\bar{U}$ gives $\bar{V} = BLU$

$\therefore LAU = BLU$

$\therefore LA = BL$

$\therefore B = \cancel{LAA^{-1}}\ LAL^{-1}$ which is the read. relation.

(69) Show that a similarity transformation preserves Algebraic operations.

i.e. if $A' = UAU^{-1}$, $B' = UBU^{-1}$

P.T.O.

Then $A'B' = U(AB)U^{-1}$

& $(A'+B') = U(A+B)U^{-1}$

Now we have

$$A'B' = (UAU^{-1})(UBU^{-1})$$
$$= UA(U^{-1}U)BU^{-1})$$
$$= U(AB)U^{-1}$$
$$A'+B' = UAU^{-1} + UBU^{-1}$$
$$= U(AU^{-1}+BU^{-1})$$
$$= U(A+B)U^{-1}$$

Which are the required results.

---

(70) Show that the trace ~~and the~~ of ~~the~~ a matrix is invariant under similarity transformation.

Now, Let $A' = UAU^{-1} = BU^{-1}$ say

$\therefore \text{Tr } A' = \text{Tr } BU^{-1}$

$\therefore \text{Tr } A' = \text{Tr } (U^{-1}B)$ since $\text{Tr}(PQ) = \text{Tr}(QP)$

$= \text{Tr}(U^{-1}UA)$

$= \text{Tr } A$

$\therefore \text{Tr } A' = \text{Tr } A$

Thus the trace of a matrix is invariant under similarity transformation.

(71) Show that the determinant of a matrix is invariant under similarity transformations.

Let $A' = UAU^{-1}$

$$\therefore \det A' = \det (UAU^{-1})$$
$$= \det U \det A \det U^{-1}$$
$$= \det (UU^{-1}) \det A$$
$$= \det I \det A$$

$\therefore \det A' = \det A$

$\therefore$ the determinant of a matrix is invariant under similarity transformations.

---

(72) Determine A^{-1} for

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 4 \end{bmatrix}$$

Now we have

$$A^{-1} = A^{+}/|A|$$

Now $A^T = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$

And A^{+} = matrix of cofactors $= \begin{vmatrix} 4 & -3 \\ 3 & 2 \end{vmatrix}$

$$|A| = 8 + 9 = 17$$

$$\therefore A^{-1} = \begin{vmatrix} 4/17 & -3/17 \\ 3/17 & 2/17 \end{vmatrix}$$

(73) Find B^{-1} for $B = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -3 \\ 5 & -4 & 1 \end{vmatrix}$

Now we have

$$B^T = \begin{vmatrix} 1 & 2 & 5 \\ 1 & 4 & -4 \\ 1 & -3 & 1 \end{vmatrix}$$

$$\therefore B^+ = \begin{vmatrix} -8 & -5 & -7 \\ -17 & -4 & +5 \\ -28 & 9 & 2 \end{vmatrix}$$

$$|B| = -8 - 15 + 4 - 20 - 12 - 2 = \cancel{-53}\ -53$$

$$\therefore B^{-1} = \frac{B^+}{|B|} = \begin{vmatrix} 8/53 & 5/53 & 7/53 \\ 17/53 & 4/53 & -5/53 \\ 28/53 & -9/53 & -2/53 \end{vmatrix}$$

---

(74) Prove that A^{-1} is unique.

Let $AB = I$ $AC = I$ & $B \neq C$.

Now $B \neq C \Rightarrow AB \neq AC$.

but $AB = I$ & $AC = I$

give $I \neq I$ which is absurd.

This contradiction is due to assumption $B \neq C$.

$\therefore B = C$

$\therefore$ Inverse is unique.

(75) Show that if

$C = AB$ then $C^{-1} = B^{-1}A^{-1}$

we have

$$(AB)^{-1}(AB) = I$$

$$\therefore (AB)^{-1}(AB)B^{-1} = IB^{-1}$$

$$\therefore (AB)^{-1}A(BB^{-1}) = B^{-1}$$

$$\therefore (AB)^{-1}A = B^{-1}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1} \quad \text{q.e.d.}$$

(76) Show that

$$(\tilde{A})^{-1} = \widetilde{(A^{-1})}$$

we have

$$(\tilde{A})^{-1}\tilde{A} = I$$

$\therefore$ multiplying by $\widetilde{(A^{-1})}$

$$(\tilde{A})^{-1}[\tilde{A}\widetilde{(A^{-1})}] = \widetilde{(A^{-1})}$$

$$\therefore (\tilde{A})^{-1}[\widetilde{A^{-1}A}] = \widetilde{(A^{-1})}$$

Since $\cancel{(\tilde{A})^{-1}}\ \widetilde{(A^{-1}A)} = \tilde{I} = I$

we get

$$(\tilde{A})^{-1} = \widetilde{(A^{-1})}$$

(77) Show that $(I+S)(I-S)^{-1}$ is orthogonal if S is a real skew symmetric matrix.

Now, consider

$$[(I+S)(I-S)^{-1}]\,\widetilde{[(I+S)(I-S)^{-1}]}$$

$$= (I+S)(I-S)^{-1}\,(\widetilde{I-S})^{-1}\,(\widetilde{I+S})$$

$$= (I+S)(I-S)^{-1}\,(I-\tilde{S})^{-1}\,(I+\tilde{S})$$

$$= (I+S)(I-S)^{-1}(I+S)^{-1}(I-S)$$

Due to skew symmetry.

$$= (I+S)\,[(I+S)(I-S)]^{-1}(I-S)$$

$$= I$$

$\therefore$ $(I+S)(I-S)^{-1}$ is idempotent.

(78) Prove that a singular matrix must have at least one eigenvalue zero.

~~The~~

Now, we have, the eigen value eqn.

Det. $[A - \lambda I] = 0.$

which is a polynomial eqn. of the n^{th} degree in λ.

$$\lambda^n + (-1)a_1\lambda^{n-1} + (-1)^2\lambda^{n-2} \ldots = 0.$$

where $a_n = |A| = 0$ for a singular matrix.

$\therefore$ The eqn. becomes

$$\lambda\left[\lambda^{n-1} + (-1)a_1\lambda^{n-2} + \cdots\right] = 0.$$

which has at least one root $\lambda = 0$.

which is the required result.

---

(79) If A is a symmetric matrix show that $(\widetilde{A^{-1}}) = A^{-1}$

Now we have

$$(\widetilde{A^{-1}}) = (\tilde{A})^{-1}$$

But for a symmetric matrix

$$\tilde{A} = A$$

$$\therefore (\widetilde{A^{-1}}) = A^{-1} \quad \text{q.e.d.}$$

---

(80) Find the rank of

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & -3 & 9 \\ 3 & 5 & -2 & 11 \end{pmatrix}$$

Now

$$A \xrightarrow[C_4-C_1]{\substack{C_2-C_1 \\ C_3-C_1}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & -5 & 7 \\ 3 & 2 & -5 & 8 \end{pmatrix} \xrightarrow{R_2-R_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \\ 3 & 2 & -5 & 8 \end{pmatrix}$$

Hence the rank of the matrix is ~~2~~ 3. [~~Non~~ Largest Nonzero determinant]

(81) Solve the characteristic eqn. of the matrix

$$A = \begin{pmatrix} 8 & -12 & 5 \\ 15 & -25 & 11 \\ 24 & -42 & 19 \end{pmatrix}$$

The eigenvalue eqn. is

$$AX = \lambda X$$

& Charact. eqn is $\text{Det.}(A - \lambda I) = 0$

$$\begin{vmatrix} 8-\lambda & -12 & 5 \\ 15 & -25-\lambda & 11 \\ 24 & -42 & 19-\lambda \end{vmatrix} = 0.$$

$$= (8-\lambda)\{(-25-\lambda)(19-\lambda) + 462\}$$
$$-12\{15(19-\lambda) - 264\}$$
$$+5\{-630 + 24(-25-\lambda)\}$$

$$= \lambda^3 - 2\lambda^2 + \lambda - 2 = 0.$$
$$= (\lambda^2 - 1)(\lambda - 2) = 0.$$

$\therefore \lambda = +1, -1$ or $+2$.

Now for $\lambda = 2$ the eqn. for components of eigenvectors become

$6 - 36 + 30$
~~$15 - 81 + 66$~~

$$6x_1 - 12x_2 + 5x_3 = 0$$
$$15x_1 - 27x_2 + 11x_3 = 0$$
$$24x_1 - 42x_2 + 17x_3 = 0.$$

On solving the eqn. we get

~~$x_1 = \frac{1}{\sqrt{46}}, \; x_2 = \frac{3}{\sqrt{46}}$~~

$x_1 = 1 \quad x_2 = 3 \quad \& \quad x_3 = 6.$

And normalising, we get the eigenvector

$$\begin{pmatrix} 1/\sqrt{46} \\ 3/\sqrt{46} \\ 6/\sqrt{46} \end{pmatrix}$$

Similarly we can proceed to determine the eigenvectors corresponding to $\lambda = \pm 1$

Thus we can solve the characteristic eqn & a matrix.

$120x_1 - 240x_2 + 100x_3 = 0$
$120x_1 - 216x_2 + 88x_3 = 0$ $\Big\}$ $= -24x_2 + 12x_3 = 0$

$-6x_2 + 3x_2 = 0$

$120x_1 - 210x_2 + 85x_3 = 0$

$x_3 - 2x_2 = 0$

X	Y	Z			X	Y	Z
43.88	24.34	27.81	1 ALA N	5 Val	49.72	24.86	17.47
44.24	25.82	29.77	C^{β}		49.00	24.08	15.14
44.24	25.73	28.19	C^{α}		49.99	24.16	16.21
45.58	26.34	27.69	C		51.25 ~~54.22~~	25.04	15.89
45.85	27.48	27.88	O		51 ~~54~~.16	26.26	15.96
46.30	25.47	26.99	2 ASP N	6 Ala	52.33	24.42	15.45
48.82	25.99	27.37	C^{β}		54.85	24.86	15.96
47.56	25.91	26.49	C^{α}		53.57	25.12	15.14
47.92	24.86	25.35	C		54.22	24.69	13.81
48.73	23.99	25.61	O		54.13	23.46	13.43
47.29	25.04	24.22	3 THR N	7 Val	54.85	25.65	13.18
46.12	23.55	22.83	C^{β}		55.11	25.99	10.66
47.47	24.16	23.08	C^{α}		55.56	25.30	11.92
48.01	24.95	21.89	C		56.91	25.73	12.49
47.65	26.08	21.70	O		57.18	26.87	12.74
48.82	24.34	21.07	4 Ile N	8 GLU	57.81	24.69	12.61
50.71	25.65	20.25	C^{β}		59.43	23.90	14.25
49.36	24.95	19.88	C^{α}		59.16	24.95	13.12
49.72	24.16	18.61	C		60.42	24.77	12.30
50.08	22.94	18.67	O		60.51	23.90	11.42

X	Y	Z		X	Y	Z
61.41	25.65	12.55	9 Leu 13 PRO N	69.32	19.10	15.26
63.21	26.87	11.10	C^{β}	69.14	17.10	14.06
62.67	25.56	11.79	C^{α}	68.69	17.79	15.39
63.66	24.95	12.74	C	68.87	16.84	16.59
64.11	25.56	13.69	O	69.86	16.14	16.71
64.[illegible] 02	23.73	12.42	10 ASP 14 ASN N	67.88	16.92	17.41
64.38	~~20.85~~ 21.63	13.75	C^{β}	67.16	16.75	19.74
65.00	22.94	13.24	C^{α}	67.88	16.05	18.61
66.44	22.85	12.80	C	67.34	14.65	18.35
66.71	22.24	11.79	O	66.17	14.39	18.48
67.25	23.55	13.50	11 THR 15 THR N	68.24	13.78	17.91
69.32	25.04	13.24	C^{β}	69.14	11.51	17.34
68.69	23.64	13.18	C^{α}	67.88	12.39	17.60
69.41	22.51	13.88	C	66.89	11.78	18.60
70.13	21.72	13.24	O	65.90	11.17	18.29
69.23	22.42	15.20	12 TYR 16 ASP N	67.25	12.04	19.88
70.31	21.55	17.47	C^{β}	67.07	~~13.61~~ 11.78	22.33
69.86	21.37	16.02	C^{α}	66.44	11.43	20.94
69.14	19.98	16.21	C	64.92	11.69	20.94
68.42	~~19.98~~ 19.80	17.22	O	64.20	10.99	21.63

X	Y	Z			X	Y	Z
64.56	12.56	20.06	17 Ile N	21 SER	68.24	13.26	10.85
63.03	14.48	19.87	C^{β}		70.58	12.39	10.72
63.12	12.91	19.93	C^{α}		69.68	13.52	10.91
62.40	12.21	18.73	C		70.40	14.83	10.85
61.41	11.51	18.92	O		71.66	14.92	11.10
62.94	12.39	17.60	18 GLY N	22 TYR	69.68	15.70	10.28
					70.40	16.66	08.83
62.40	11.78	16.34	C^{α}		70.31	17.01	09.84
62.67	12.65	15.14	C		69.14	17.97	10.03
62.13	12.39	14.06	O		68.06	17.53	10.47
63.57	13.61	15.26	19 ASP N	23 PRO	69.41	19.19	09.71
64.56	15.88	14.44	C^{β}		69.05	21.55	09.59
63.93	14.57	14.13	C^{α}		68.33	20.24	09.90
65.00	13.78	13.37	C		67.07	19.89	09.08
65.81	13.08	13.88	O		67.16	19.54	07.95
64.92	13.96	12.05	20 PRO N	24 HIS	65.90	20.15	09.78
65.45	13.70	09.78	C^{β}		64.38	18.40	9.65
65.90	13.35	11.16	C^{α}		64.65	19.80	9.08
67.25	13.61	11.67	C		63.48	20.67	9.52
67.43	14.13	12.80	O		63.48	21.37	10.53

62.40	20.67	8.64	25 Ile N	29 Ile	51.79	19.19	16.02
60.78	22.33	7.76	C^{β}		49.36	18.41	15.70
61.23	21.37	8.89	C^{α}		50.35	19.28	16.46
60.06	20.41	8.89	C		50.35	18.75	17.85
60.06	19.37	8.33	O		50.44	17.53	18.10
58.98	20.85	9.59	26 GLY N	30 LYS	50.17	19.63	18.79
		~~9.71~~			49.09	18.14	20.43
57.81	19.98	9.71	C^{α}		50.17	19.28	20.25
56.64	20.59	10.60	C		51.43	18.93	20.94
56.82	21.63	11.16	O		51.52	19.10	22.20
55.56	19.80	10.66	27 Ile N	31 ser	52.51	18.60	20.18
53.23	19.89	10.53	C^{β}		53.68	16.75	20.88
54.40	20.24	11.42	C^{α}		53.77	18.32	20.75
54.31	19.63	12.80	C		55.02	18.49	19.93
54.31	18.41	12.93	O		55.02	18.84	18.73
54.04	20.41	13.81	28 ASP N	32 val	56.19	18.23	20.56
54.94	20.41	~~16.15~~ 16.27	C^{β}		58.62	17.97	20.81
53.94	19.98	15.20	C^{α}		57.45	18.41	19.93
52.51	20.24	15.64	C		57.54	17.36	18.73
52.06	21.37	15.64	O		58.08	17.71	17.66

			33 ARG 37 THR			
56·91	16·22	18·98	N	57·90	15·70	8·39
56·19	~~15·88~~ 13·96	18·54	C^{β}	60·42	15·61	9·27
56·82	15·18	17·97	C^{α}	59·25	16·22	8·39
56·19	15·61	16·65	C	59·79	16·40	7·00
55·11	15·18	16·33	O	59·25	15·88	6·05
			34 SER 38 ALA			
56·91	16·40	15·89	N	60·87	17·18	6·87
57·45	17·62	13·81	C^{β}	60·51	18·32	4·73
56·37	16·92	14·63	C^{α}	61·50	17·36	5·55
55·74	15·79	13·81	C	62·85	17·97	5·74
56·28	16·40 14·65	13·81	O	63·21	18·67	6·75
			35 LYS 39 LYS			
54·58	16·05	13·18	N	63·75	17·79	4·73
52·51	15·44	12·36	C^{β}	65·99	17·18	4·23
53·95	15·00	12·42	C^{α}	65·09	18·23	4·79
54·49	14·65	11·04	C	65·54	19·54	4·04
54·40	13·52	10·53	O	65·45	19·54	2·84
			36 LYS 40 TRP			
55·02	15·70	10·41	N	65·90	20·59 24·95	4·79
54·76	16·05	07·88	C^{β}	65·27	22·68 24·42	4·98
55·56	15·53	09·08	C^{α}	66·26	21·81	4·23
57·00	16·22	09·15	C	67·79	22·16	4·41
57·18	17·18	09·90	O	68·33	21·89 (22·77)	5·42

68.33	22.68	3.34	41 ASN N	45 GLY	74.27	32.36	8.39
70.04	22.77	1.83	C^{β}				
69.77	22.94	3.34	C^{α}		74.63	33.76	8.07
70.31	24.25	3.97	C		74.00	34.28	6.81
70.~~89~~ 49	25.30	3.28	O		74.27	35.42	6.37
70.49	24.25	5.30	42 Met N	46 LYS	73.10	33.50	6.24
71.21	24.77	7.44	C^{β}		72.65	32.71	4.04
70.94	25.38	6.05	C^{α}		72.47	33.85	4.98
72.02	26.26	5.42	C		71.03	34.28	5.23
73.01	25.73	4.79	O		70.40	33.85	6.24
71.84	27.56	5.55	43 GLN N	47 VAL	70.49	35.07	4.35
72.11	28.87	3.72	C^{β}		68.60	36.99	4.23
72.83	28.52	4.98	C^{α}		69.05	35.50	4.48
73.10	29.48	6.12	C		68.06	34.46	3.97
72.47	30.53	6.24	O		68.15	33.93	2.90
74.09	29.13	6.94	44 ASP N	48 GLY	67.07	34.19	4.86
75.43	~~31.40~~ 29.31	9.02	C^{β}		~~65.99~~		
74.45	30.01	8.07	C^{α}		65.99	33.23	4.54
74.90	31.40	7.69	C		64.56	33.76	4.54
75.70	31.58	6.81	O		64.29	34.89	4.92

63.66	32.89	4.16	49 THR N	53 Ile N	50.53	28.09	4.35
61.77	33.85	2.78	C^{β}	C^{β}	49.09	28.09	2.59
62.22	33.23	4.10	C^{α}	C^{α}	49.09	28.18	4.10
61.23	32.10	4.16	C	C	48.10	27.04	4.41
61.14	31.23	3.22	O	O	48.55	25.91	4.54
60.42	32.10	5.17	50 Ala N	54 TYR N	46.84	27.39	4.48
59.34	30.79	6.87	C^{β}	OH	41.99	23.55	9.02
59.34	31.05	5.30	C^{α}	CD2	43.61	26.08	7.06
57.99	31.40	4.86	C	CE2	42.62	25.38	7.76
57.54	32.54	4.86	O	CZ	42.98	24.16	8.39
57.18	30.36	4.54	51 HIS N	CE1	44.24	23.73	8.39
55.65	30.44	2.52	C^{β}	CD1	45.22	24.42	7.69
55.83	30.53	4.04	C^{α}	CG	44.96	25.65	7.06
54.94	29.40	4.60	C	C^{β}	46.03	26.34	~~6.30~~ 6.31
55.38	28.26	4.86	O	C^{α}	45.76	26.43	4.79
53.68	29.75	4.73	Ile N	C	44.33	26.95	4.79
52.87	28.87	6.75	C^{β}	O	44.06	28.09	4.67
52.89	28.79	5.23	C^{α}	55 ASN N	43.43	25.99	4.86
51.16	29.05	5.05	C	C^{β}	41.72	26.95	3.47
50.62	30.00	5.49	O	C^{α}	41.99	26.34	4.86

				C	43.70	20.76	5.68
41.09	25.12	5.05	C	O	43.88	19.71	6.31
41.54	23.90	4.79	O	60 ARG N	44.60	21.37	4.92
39.92	25.38	5.49	56 SER N	Cβ	46.12	20.15	3.41
37.85	25.99	7.06	OG	Cα	45.94	20.76	4.79
37.94	25.38	5.74	Cβ	C	47.11	21.72	5.11
38.93	24.25	5.68	Cα	O	47.47	22.59	4.35
38.48	23.38	4.54	C	61 Leu N	47.65	21.55	6.31
38.48	22.16	4.67	O	Cβ	49.18	22.24	8.26
38.12	23.99	3.41	57 VAL N	Cα	48.82	22.33	6.75
37.58	23.73	0.82	Cβ	C	50.08	22.24	5.93
37.76	23.29	2.21	Cα	O	50.80	21.28	5.99
38.75	22.07	2.27	C	62 SER N	50.35	23.29	5.11
38.30	20.94	2.52	O	Cβ	50.98	23.64	2.90
40.01	22.42	2.02	58 ASP N	Cα	51.52	23.29	4.23
42.26	21.98	1.39	Cβ	C	52.60	24.34	4.35
41.00	21.37	2.08	Cα	O	52.24	25.56	4.68
41.63	20.94	3.34	C	63 ALA N	53.86	23.99	4.23
42.08	19.71	3.47	O	Cβ	55.74	24.42	5.55
41.72	21.~~80~~81	4.35	59 LYS N	Cα	54.94	24.95	4.35
41.27	20.50	6.24	Cβ	C	56.10	24.95	3.34
42.35	21.46	5.61	Cα	O	56.37	23.90	2.71
					627	~~274~~ 274	~~43~~ 43

0·08991 0·08723 0·06307

X		Y			Z	
630	56·64	300	64 VAL N	26·17	50	3·15
635	57·09	~~299~~ 307	C^{β}	26·78	15	0·95
643	57·81	302	C^{α}	26·34	35	2·21
655	58·89	312	C	27·22	44	2·78
653	58·71	326	O	28·44	46	2·90
667	59·97	305	65 VAL N	26·61	49	3·09
685	61·59	307	C^{β}	26·78	78	4·92
680	61·14	314	C^{α}	27·39	57	3·59
691	62·13	315	C	27·48	39	2·46
699	62·85	304	O	26·52	33	2·08
693	62·31	329	66 SER N	28·70	31	1·96
694	42·40	340	C^{β}	29·66	−2	−0·13
703	63·21	332	C^{α}	28·96	13	0·82
717	64·47	340	C	29·66	18	1·14
719	64·65	348	O	30·36	35	2·21
728	65·45	339	67 TYR N	29·57	3	0·19
755	67·88	337	C^{β}	29·40	12	0·76
742	66·71	347	C^{α}	30·27	6	0·38
746	67·07	353	C	30·79	−16	−1·01
742	66·71	345	O	30·09	−31	−1·96

X		Y			Z		Z
			68 PRO				
752	67.61	366	N		31.93	-17	-1.07
760	68.33	389	C^{β}		33.93	-33	-2.08
756	67.97	373	C^{α}		32.54	-38	-2.40
768	69.05	364	C		31.75	-50	-3.15
780	70.13	361	O		31.49	-42	-2.65
			69 ASN				
764	68.69	360	N		31.40	-70	-4.41
791	71.12	357	C^{β}		31.14	-82	-5.17
775	69.68	352	C^{α}		30.70	-84	-5.30
773	69.50	335	C		29.22	-77	-4.86
774	69.59	325	O		28.35	-91	-5.74
			70 ALA				
770	69.23	333	N		29.05	-57	-3.59
771	69.32	317	C^{β}		27.65	-25	-1.58
768	69.05	317	C^{α}		27.65	-49	-3.09
753	67.70	311	C		27.13	-54	-3.41
742	66.71	320	O		27.91	-58	-3.66
			71 ASP				
751	67.52	296	N		25.82	-54	-3.41
741	66.62	274	C^{β}		23.90	-72	-4.54
737	66.26	288	C^{α}		25.12	-59	-3.72
726	65.27	286	C		24.95	-41	-2.59
730	65.63	285	O		24.86	-22	-1.39

	X	Y	Z		X	Y	Z
72 ALA N	712 64.02	286 24.95	-46 -2.90	76 SER N	599 53.86	218 19.02	65 4.10
Cβ	685 61.59	287 25.04	-41 -2.59		576 51.79	217 18.93	46 2.90
Cα	700 62.94	285 24.86	-30 -1.89		583 52.42	220 19.19	67 4.23
C	697 62.67	270 23.55	-17 -1.07		575 51.70	207 18.06	80 5.05
O	698 62.76	257 22.42	-25 -1.58		577 51.88	194 16.92	77 4.86
73 THR N	693 62.31	273 23.81	2 0.13	77 TYR	566 50.89	213 18.58	95 5.99
Cβ	702 63.12	262 22.85	34 2.14		566 50.89	201 17.53	131 8.26
Cα	690 62.04	260 22.68	17 1.07		558 50.17	202 17.62	109 6.87
C	674 60.60	262 22.85	25 1.58		541 48.64	205 17.88	110 6.94
O	670 60.24	274 23.90	35 2.21		536 48.19	218 19.02	110 6.94
74 SER N	665 59.79	251 21.89	21 1.32	78 ASP	533 47.92	192 16.75	110 6.94
Cβ	641 57.63	253 22.07	9 0.57		507 45.58	180 15.70	101 6.37
Cα	649 58.35	252 21.98	28 1.77		517 46.48	193 16.84	110 6.94
C	643 57.81	237 20.67	40 2.52		511 45.94	193 16.84	133 8.39
O	650 58.44	225 19.63	38 2.40		513 46.12	182 15.88	145 9.15
75 Val N	631 56.73	239 20.85	51 3.22	79 Val	504 45.31	206 17.97	138 8.70
Cβ	631 56.73	225 19.63	85 5.36		511 45.94	215 18.75	171 10.~~8~~78
Cα	625 56.19	227 19.80	63 3.97		498 44.78	~~200~~ 208 18.14	160 10.09
C	608 54.67	230 20.06	65 4.10		484 43.52	218 19.02	161 10.15
O	603 54.22	243 21.20	66 4.16		482 43.34	228 19.89	148 9.33

80 ASP	476	216	178	84 Val	457	244	261
N	42.80	18.84	11.23		41.09	21.28	16.46
	450	214	189		485	245	266
C^{β}	40.46	18.67	11.92		43.61	21.37	16.78
	462	225	181		470	247	276
C^{γ}	41.54	19.63	11.42		42.26	21.55	17.41
	464	237	199		472	263	286
C	41.72	20.67	12.55		42.44	22.94	18.04
	462	233	218		473	264	306
O	41.54	20.32	13.75		42.53	23.03	19.30
81 Leu	469	250	193	85 ~~Ala~~ Leu	471	275	273
	42.17	21.81	12.17		42.35	23.99	17.22
	480	274	194		477	300	261
	43.16	23.90	12.24		42.89	~~[illegible]~~ 26.17	16.46
	472	262	208		473	291	281
	42.44	22.85	13.12		42.53	25.38	17.72
	457	266	218		459	297	291
	41.09	23.20	13.75		41.27	25.91	18.35
	456	270	237		446	293	285
	41.00	23.55	14.95		40.10	25.56	18.04
82 ASN	445	264	205	86 PRO	461	309	305
	40.01	23.03	12.93		41.45	26.95	19.24
	417	261	199		456	323	335
	37.49	22.77	12.55		41.00	28.18	21.13
	430	267	213		446	316	316
	38.66	23.29	13.43		40.10	27.56	19.93
	449	257	234		442	325	298
	40.37	22.42	14.76		39.74	28.35	18.79
	426	263	250		449	331	284
	38.30	22.94	15.77		40.37	28.87	17.91
83 ASP	424	243	232	87 Glu	427	337	299
	38.12	21.20	14.63		38.39	29.40	18.86
	434	216	245		401	335	286
	39.02	18.84	15.45		36.05	29.22	18.04
	434	233	251		419	335	283
	39.02	20.32	15.83		37.67	29.22	17.85
	445	236	268		426	351	278
	40.01	20.59	16.90		38.30	30.62	17.53
	444	232	287		426	356	261
	39.92	20.24	18.10		38.30	31.05	16.46

88 TRP	433	357	295	92 GLY	577	395	266
N	38.93	31.14	18.61	N	51.88	34.46	16.78
	434	383	308				
C^β	39.02	33.41	19.43	C^β			
	441	372	293		592	401	273
C^α	39.65	32.45	18.47	C^α	53.23	34.98	17.22
	457	369	299		606	394	264
C	41.09	32.19	18.86	C	54.49	34.37	16.65
	460	364	317		605	385	249
O	41.36	31.75	19.99	O	54.40	33.58	15.70
89 VAL	467	372	284	93 Leu	618	398	274
N	41.99	32.45	17.91	N	55.56	34.72	17.28
	488	359	271		640	403	251
C^β	43.88	31.32	17.09	C^β	57.54	35.15	15.83
	483	370	289		633	392	266
C^α	43.43	32.28	18.23	C^α	56.91	34.19	16.78
	493	384	289		642	385	284
C	44.33	33.50	18.23	C	57.72	33.58	17.91
	488	397	286		644	392	301
O	43.88	34.63	18.04	O	57.90	34.19	18.98
90 ARG	507	381	292	94 Ser	649	372	280
N	45.58	33.23	18.42	N	58.35	32.45	17.66
	522	392	316		650	351	305
C^β	46.93	34.19	19.93	C^β	58.44	30.62	19.24
	518	394	293		658	364	295
C^α	46.57	34.37	18.48	C^α	59.16	31.75	18.61
	532	389	279		673	359	285
C	47.83	33.93	17.60	C	60.51	31.32	17.97
	535	375	277		674	356	266
O	48.10	32.71	17.47	O	60.60	31.05	16.78
91 Val	539	400	269	95 ALA	685	359	298
N	48.46	34.89	16.97		61.59	31.32	18.79
	546	403	234		712	367	289
C^β	49.09	35.15	14.76		64.02	32.01	18.23
	551	398	256		699	354	290
C^α	49.54	34.72	16.34		62.85	30.88	18.29
	566	405	263		707	348	310
C	50.89	35.33	16.59		63.57	30.36	19.55
	567	414	266		702	353	328
O	50.98	36.55	16.78		63.12	30.79	20.69

96 SER	718	339	307	100 TYR	816	287	350
N	64.56	29.57	19.36		73.37	25.04	22.07
C^β	718 64.56	317 27.65	331 20.88		818 73.55	281 24.51	318 20.06
C^α	725 65.18	332 28.96	326 20.56		822 73.91	294 25.65	335 21.13
C	742 66.71	329 28.70	327 20.62		818 73.55	310 27.04	330 20.81
O	750 67.43	327 28.52	311 19.61		806 72.47	316 27.56	336 21.19
97 THR N	748 67.25	329 28.70	347 21.89	101 LYS	828 74.45	318 27.74	320 20.18
C^β	771 69.32	342 29.83	351 22.14		843 75.79	339 29.57	317 19.99
C^α	763 68.60	326 28.44	351 22.14		827 74.36	334 29.13	314 19.80
C	765 68.78	317 27.65	372 23.46		820 73.73	336 29.31	292 18.42
O	755 67.88	317 27.65	385 24.28		823 74.00	326 28.44	278 17.53
98 GLY N	777 69.86	309 26.95	374 23.59	102 GLU	813 73.10	349 30.44	288 18.16
C^β	8				795 71.48	339 29.57	263 16.59
C^α	780 70.13	300 26.17	393 24.79		806 72.47	352 30.70	267 16.84
C	796 71.57	297 25.91	400 25.23		798 71.75	367 32.01	270 17.03
O	802 72.11	303 25.43	415 26.17		794 71.39	372 32.45	287 18.10
99 LEU N	803 72.20	286 24.95	388 24.47	103 THR	796 71.57	375 32.71	252 15.89
C^β	815 73.28	266 23.20	381 24.03		797 71.66	402 35.07	241 15.20
C^α	818 73.55	281 24.51	393 24.79		788 70.85	389 33.93	251 15.83
C	825 74.18	288 25.12	373 23.53		772 69.41	386 33.67	243 15.33
O	837 75.25	295 25.73	373 23.53		770 69.23	376 32.80	230 14.51

11.80

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104 ASN			
N	762 68.51	395 34.46	251 15.86
C^{β}	736 66.17	382 33.32	256 16.15
C^{α}	746 67.07	394 34.37	244 15.39
C	740 66.53	410 35.76	241 15.20
O	731 65.72	415 36.20	254 16.02
THR 105 N	744 66.89	417 36.37	224 14.12
C^{β}	754 67.79	439 38.29	217 13.69
C^{α}	739 66.44	432 37.68	219 13.81
C	728 65.45	432 37.68	201 12.68
O	729 65.54	424 36.99	185 11.67
106 ILE N	716 64.38	441 38.47	203 12.80
C^{β}	690 62.04	439 38.29	208 12.61
C^{α}	704 63.30	432 37.68	187 11.79
C	706 63.48	458 39.95	176 11.10
~~107 O~~	~~708 63.66~~	470 41.00	186 11.73
N	706 63.48	457 39.86	155 9.78
C^{β}	720 64.74	467 40.74	130 8.20
C^{α}	708 63.66	471 41.09	142 8.96
C	692 62.22	477 41.61	137 8.64
O	688 61.86	489 42.66	143 9.02

107 Leu

SER 108	684	467	126
N	61.50	40.74	7.95
C^{β}	670 60.24	470 41.00	97 6.12
C^{α}	669 60.15	471 41.09	120 7.57
C	658 59.16	459 40.04	129 8.14
O	662 59.52	445 38.82	133 8.39
TRP 109 N	644 57.90	464 40.47	132 8.33
C^{β}	632 56.82	451 39.34	164 10.34
C^{α}	633 56.91	453 39.52	140 8.83
C	618 55.56	458 39.95	131 8.26
O	613 55.11	471 41.09	132 8.33
SER 110 N	610 54.85	447 38.99	122 7.69
C^{β}	597 53.68	451 39.34	88 5.55
C^{α}	596 53.59	449 39.17	112 7.06
C	583 52.42	438 38.21	117 7.38
O	586 52.69	424 36.99	116 7.32
PHE 111 N	570 51.25	444 38.73	122 7.69
C^{α}	555 49.90	433 37.77	151 9.52
C^{β}	557 50.08	434 37.86	127 8.01
C	543 48.82	438 38.21	117 7.38
O	537 48.28	451 39.34	120 7.57

112 THR N C^{β} C^{α} C O SER N OG C^{β} C

0123

112 THR	537	427	106
N	48.28	37.25	6.69
	526	436	71
Cβ	47.29	38.03	4.48
	522	429	95
Cα	46.93	37.42	5.99
	511	417	102
C	45.94	36.37	6.43
	516	403	104
O	46.39	35.15	6.56
SER 113	498	421	106
N	44.78	36.72	6.69
	482	414	151
OG	43.34	36.11	9.52
	490	406	136
Cβ	44.06	35.42	8.58
	486	411	113
Cα	43.70	35.85	7.13
	470	415	112
C	42.26	36.20	7.06
	465	427	121
O	41.81	37.25	7.63
114 LYS	462	406	100
N	41.54	35.42	6.31
	445	415	74
Cβ	40.01	36.20	4.68
	446	410	97
Cα	40.10	35.76	6.12
	433	398	100
C	38.93	34.72	6.31
	436	384	99
O	39.20	33.50	6.24
115 LEU	420	404	104
N	37.76	35.24	6.56
	404	398	131
Cβ	36.32	34.72	8.26
	407	394	107
Cα	36.59	34.37	6.75
	393	396	94
C	35.33	34.54	5.93
	385	407	96
O	34.62	35.50	6.05

116 LYS	390	385	81
N	35.06	33.58	5.11
	384	375	49
Cβ	34.53	32.71	3.09
	377	385	67
Cα	33.90	33.58	4.23
	362	380	76
C	32.55	33.15	4.79
	360	366	82
O	32.37	31.93	5.17
SER 117	351	390	76
N	31.56	34.02	4.79
	334	400	98
Cβ	30.03	34.89	6.18
	336	386	84
Cα	30.21	33.67	5.30
	324	382	69
C	29.13	33.32	4.35
	326	381	50
O	29.31	33.23	3.15
ASN 118	310	381	78
N	27.87	33.23	4.92
	283	371	78
Cβ	25.44	32.36	4.92
	297	377	66
Cα	26.70	32.89	4.16
	290	391	56
C	26.07	34.11	3.53
	285	391	37
O	25.62	34.11	2.33
SER 119	289	403	68
N	25.98	35.15	4.29
	292	429	71
Cβ	26.25	37.42	4.48
	282	417	60
Cα	25.35	36.37	3.78
	283	421	37
C	25.44	36.72	2.33
	271	421	26
O	24.37	36.72	1.64

3

Residue	Atom						
THR120	N	296	26·61	423	36·90	29	1·83
	C^{β}	294	26·43	442	38·56	1	0·06
	C^{α}	299	26·88	426	37·16	6	0·38
	C	315	28·32	422	36·81	2	0·13
	O	318	28·59	408	35·59	−3	−0·19
HIS121	N	325	29·22	433	37·77	5	0·32
	C^{β}	348	31·29	432	37·68	−20	−1·26
	C^{α}	341	30·66	430	37·51	1	0·06
	C	351	31·56	442	38·56	10	0·63
	O	347	31·20	455	39·69	17	1·07
GLN122	N	365	32·82	438	38·21	11	0·69
	C^{β}	380	34·17	464	40·47	13	0·82
	C^{α}	377	33·90	448	39·08	20	1·26
	C	374	33·63	452	39·43	43	2·71
	O	375	33·72	465	40·56	49	3·09
THR123	N	369	33·18	441	38·47	55	3·47
	C^{β}	359	32·28	428	37·23	83	5·23
	C^{α}	366	32·91	443	38·64	78	4·92
	C	378	33·99	450	39·25	91	5·74
	O	383	34·44	463	40·39	88	5·55

Residue	Atom						
ASP124	N	384	34·53	441	38·47	106	6·69
	C^{β}	390	35·06	448	39·08	142	8·96
	C^{α}	396	35·60	446	38·90	120	7·57
	C	413	37·13	444	38·73	116	7·32
	O	417	37·49	432	37·68	107	6·75
ALA125	N	422	37·94	454	39·60	124	7·82
	C^{β}	440	39·56	457	39·86	98	6·18
	C^{α}	438	39·38	453	39·52	122	7·69
	C	450	40·46	464	40·47	129	8·14
	O	448	40·28	478	41·70	132	8·33
LEU126	N	463	41·63	457	39·86	132	8·33
	C^{β}	477	42·89	462	40·30	162	10·22
	C^{α}	476	42·80	466	40·65	138	8·70
	C	493	44·33	465	40·56	131	8·26
	O	498	44·78	453	39·52	127	8·01
HIS127	N	500	44·96	478	41·70	129	8·14
	C^{β}	516	46·39	483	42·13	99	6·24
	C^{α}	515	46·30	479	41·78	122	7·69
	C	526	47·29	488	42·57	136	8·58
	O	523	47·02	500	~~9·15~~ 43·62	145	9·15

128 Phe	N	540 48.55	482 42.04	138 8.70
	C^β	550 49.45	488 42.57	174 10.97
	C^α	552 49.63	489 42.66	150 9.46
	C	566 50.89	489 42.66	138 8.70
	O	572 51.43	476 41.52	133 8.39
Met129	N	572 51.43	502 43.79	134 8.45
	C^β	585 52.60	510 44.49	100 6.31
	C^α	586 52.69	503 43.88	123 7.76
	C	601 54.04	509 44.40	134 8.45
	O	601 54.04	521 45.45	143 9.02
Phe130	N	612 55.02	500 43.42	132 8.33
	C^β	630 56.64	495 43.18	162 10.22
	C^α	627 56.37	504 43.96	142 8.96
	C	641 57.63	503 43.88	129 8.14
	O	645 57.99	492 42.92	119 7.51
ASN131	N	650 58.44	515 44.92	131 8.26
	C^β	659 59.25	527 45.97	101 6.37
	C^α	664 59.70	516 45.01	120 7.57
	C	676 60.78	523 45.62	135 8.51
	O	689 61.95	524 45.71	129 8.14

GLN 132	N	671 60.33	528 46.06	153 9.65
	C^β	687 61.77	550 47.98	163 10.28
	C^α	681 61.23	535 46.67	169 10.66
	C	672 60.42	538 46.93	189 11.92
	O	661 59.43	546 47.63	189 11.92
Phe133	N	678 60.96	532 46.41	207 13.06
	C^β	674 60.60	519 45.27	239 15.07
	C^α	671 60.33	534 46.58	227 14.32
	C	677 60.87	548 47.80	238 15.01
	O	689 61.95	554 48.33	232 14.63
SER134		669 60.15	554 48.33	254 16.02
		661 59.43	579 50.51	263 16.59
		674 60.60	567 49.64	265 16.71
		678 60.96	566 49.37	289 18.23
		677 60.87	553 48.24	298 18.79
LYS135		683 61.41	578 50.42	298 18.79
		696 62.58	592 51.64	329 20.75
		687 61.77	578 50.42	320 20.18
		672 60.42	578 50.42	329 20.75
		669 60.15	573 49.98	347 21.89

Residue	Atom						
ASP 136	N	661	59.43	584	50.94	316	19.93
	C^{β}	640	57.54	600	52.34	313	19.74
	C^{α}	646	58.08	585	51.03	323	20.37
	C	639	57.45	569	49.63	319	20.12
	O	637	57.27	560	48.85	334	21.07
GLN 137	N	634	57.00	567	49.46	300	18.92
	C^{β}	635	57.02	538	46.93	288	18.16
	C^{α}	627	56.37	552	48.15	294	18.54
	C	616	55.38	550	47.98	312	19.68
	O	619	55.65	541	47.19	327	20.62
LYS 138	N	603	54.22	558	48.67	310	19.55
	C^{β}	587	52.78	572	49.90	335	21.13
	C^{α}	592	53.23	556	48.50	326	20.56
	C	578	51.97	546	47.63	322	20.31
	O	570	51.25	543	47.37	336	21.19
ASP 139	N	577	51.88	541	47.19	302	19.05
	C^{β}	558	50.17	539	47.02	276	17.41
	C^{α}	565	50.80	531	46.32	296	18.67
	C	572	51.43	515	44.92	294	18.54
	O	568	51.07	506	44.14	280	17.66
LEU 140	N	582	52.33	512	44.66	308	19.43
	C^{β}	605	54.40	501	43.70	299	18.86
	C^{α}	590	53.05	497	43.35	308	19.43
	C	591	53.14	491	42.83	331	20.88
	O	593	53.32	500	43.62	345	21.76
ILE 141	N	590	53.05	476	41.52	333	21.00
	C^{β}	579	52.06	458	39.95	362	22.83
	C^{α}	592	53.23	469	40.91	354	22.33
	C	606	54.49	459	40.04	355	22.39
	O	606	54.49	446	38.90	347	21.89
LEU 142	N	617	55.47	466	40.65	364	22.96
	C^{β}	643	57.81	471	41.09	366	23.08
	C^{α}	632	56.82	458	39.95	365	23.02
	C	633	56.91	449	39.17	386	24.35
	O	627	56.37	454	39.60	403	25.42
GLN 143	N	640	57.54	435	37.95	385	24.28
	C^{β}	632	56.82	411	35.85	404	25.48
	C^{α}	641	57.63	426	37.16	404	25.48
	C	658	59.16	424	36.99	408	25.73
	O	667	59.97	425	37.07	394	24.85

GLY 144			
N	662 59.52	422 36.81	428 26.99
Cα	677 60.87	420 36.64	435 27.44
C	690 62.04	430 37.51	429 27.06
O	692 62.22	443 38.64	437 27.56
ASP 145			
N	700 62.94	424 36.99	415 26.17
Cβ	728 65.45	425 37.07	413 26.05
Cα	713 64.11	433 37.77	408 25.73
C	713 64.11	440 38.38	386 24.35
O	724 65.09	444 38.73	377 23.78
ALA 146			
N	700 62.94	440 38.38	377 23.78
Cβ	681 61.23	446 38.90	352 22.20
Cα	698 62.76	446 38.90	355 22.39
C	702 63.12	463 40.39	357 22.52
O	701 63.03	470 41.00	374 23.59
147 THR			
N	708 63.66	469 40.91	340 21.44
Cβ	729 65.54	489 42.66	343 21.63
Cα	712 64.02	486 42.39	339 21.38
C	707 63.57	493 43.00	319 20.12
O	709 63.75	486 42.39	301 18.98

148 THR			
N	700 62.94	506 44.14	320 20.18
Cβ	678 60.96	519 45.27	300 18.92
Cα	694 62.40	514 44.84	302 19.05
C	703 63.21	529 46.14	300 18.92
O	706 63.48	536 46.76	316 19.93
149 GLY			
N	707 63.57	533 46.49	281 17.72
Cα	716 64.38	574 50.07	277 17.47
C	733 65.90	545 47.54	279 17.60
O	741 66.62	556 48.50	274 17.28
150 THR			
N	738 66.35	532 46.41	287 18.10
Cβ	757 68.06	512 44.66	287 18.10
Cα	754 67.79	529 46.14	290 18.29
C	762 68.51	537 46.84	272 17.16
O	772 69.41	547 47.71	276 17.41
151 ASP			
N	759 68.24	532 46.41	252 15.89
Cβ	777 69.86	527 45.97	223 14.06
Cα	767 68.96	539 47.02	234 14.76
C	754 67.79	542 47.28	219 13.81
O	756 67.97	543 47.37	199 12.55

152 GLY	N	741	66.62	545	47.54	228	14.38
	C^{α}	728	65.45	548	47.80	215	13.56
	C	721	64.83	533	46.49	208	13.12
	O	715	64.29	532	46.41	190	11.98
153 ASN	N	722	64.92	522	45.53	221	13.94
	C^{β}	730	65.63	496	43.27	211	13.31
	C^{α}	717	64.47	506	44.14	216	13.62
	C	709	63.75	499	43.53	235	14.82
	O	711	63.93	504	43.96	253	15.96
Leu 154	N	701	63.03	486	42.39	231	14.57
	C^{β}	679	61.05	470	41.00	245	15.45
	C^{α}	694	62.40	478	41.70	248	15.64
	C	706	63.48	466	40.65	251	15.83
	O	710	63.84	457	39.86	236	14.88
155 GLU	N	712	64.02	465	40.56	270	17.03
	C^{β}	736	66.17	464	40.47	287	18.10
	C^{α}	724	65.09	454	39.60	275	17.34
	C	717	64.47	444	38.73	292	18.42
	O	717	64.47	448	39.08	311	19.61
156 LEU	N	712	64.02	430	37.51	286	18.04
	C^{β}	697	62.67	407	35.50	289	18.23
	C^{α}	706	63.48	420	36.64	301	18.98
	C	716	64.38	415	36.20	319	20.12
	O	712	64.02	414	36.11	337	21.25
157 THR	N	730	65.63	412	35.94	312	19.68
	C^{β}	752	67.61	395	34.46	318	20.06
	C^{α}	741	66.62	407	35.50	328	20.69
	C	753	67.70	418	36.46	336	21.19
	O	756	67.97	430	37.51	326	20.56
158 ARG	N	759	68.24	415	36.20	355	22.39
	C^{β}	781	70.22	426	37.16	383	24.16
	C^{α}	770	69.23	425	37.07	364	22.96
	C	785	70.58	429	37.42	354	22.33
	O	795	71.48	419	36.55	352	22.20
159 VAL	N	787	70.76	443	38.64	348	21.95
	C^{β}	795	71.48	453	39.52	316	19.93
	C^{α}	800	71.93	448	39.08	338	21.32
	C	810	72.83	459	40.04	350	22.07
	O	805	72.38	468	40.82	364	22.96

160 SER N	825	74.18	459	40.04	345	21.76
Cβ	850	76.42	461	40.21	356	22.45
Cα	835	75.07	469	40.91	356	22.45
C	838	75.34	484	42.22	342	21.57
O	831	74.72	486	42.39	326	20.56
161 SER N	848	76.24	493	43.00	350	22.07
Cβ	857	77.05	520	45.36	353	22.26
Cα	852	76.60	507	44.23	339	21.38
C	862	77.50	506	44.14	319	20.12
O	864	77.68	517	45.10	307	19.36
162 ASN N	868	78.04	493	43.00	316	19.93
Cβ	893	80.29	484	42.22	308	19.43
Cα	878	78.94	490	42.74	298	18.79
C	871	78.31	478	41.70	283	17.85
O	876	78.76	475	41.43	265	16.71
163 GLY N	859	77.23	471	41.09	291	18.35
Cα	851	76.51	459	40.04	279	17.60
C	850	76.42	446	38.90	296	18.67
O	840	75.52	446	38.90	309	19.49

164 SER N	862	77.50	437	38.12	297	18.73
Cβ	872	78.40	431	37.60	329	20.75
Cα	862	77.50	424	36.99	312	19.68
C	847	76.15	418	36.46	320	20.18
O	841	75.61	423	36.90	336	21.19
165 PRO N	841	75.61	407	35.50	307	19.36
Cβ	824	74.09	390	34.02	295	18.61
Cα	827	74.36	401	34.98	313	19.74
C	829	74.54	394	34.37	335	21.13
O	840	75.52	385	33.58	339	21.38
166 GLU N	818	73.55	397	34.63	349	22.01
Cβ	809	72.74	401	34.98	385	24.28
Cα	819	73.64	391	34.11	370	23.34
C	813	73.10	375	32.71	370	23.34
O	806	72.47	370	32.28	354	22.33
167 GLY N	815	73.28	367	32.01	387	24.41
Cα	810	72.83	351	30.62	389	24.53
C	799	71.84	349	30.44	408	25.73
O	796	71.57	360	31.40	420	26.49

Residue	Atom			
168 SER	N	793 71.30	336 29.31	410 25.86
	Cβ	789 70.94	329 28.70	448 28.26
	Cα	782 70.31	332 28.96	427 26.93
	C	769 69.14	344 30.01	427 26.93
	O	764 68.69	349 30.44	444 28.00
169 SER	N	765 68.78	349 30.44	408 25.73
	Cβ	760 68.33	~~389~~ 375 32.71	~~403~~ 396 24.98
	Cα	~~760~~ 753 67.70	~~375~~ 361 31.49	~~396~~ 406 25.61
	C	~~753~~ 738 66.35	~~384~~ 357 31.14	~~406~~ 397 25.04
	O	736 66.17	346 30.18	386 24.35
170 VAL	N	727 65.36	367 32.01	402 25.35
	Cβ	706 63.48	352 30.70	407 25.67
	Cα	712 64.02	365 31.84	394 24.85
	C	700 62.94	378 32.97	397 25.04
	O	697 62.67	383 33.41	415 26.17
171 GLY	N	693 62.31	382 33.32	379 23.90
	Cα	682 61.32	394 34.37	380 23.97
	C	673 60.51	393 34.28	359 22.64
	O	677 60.87	386 33.67	344 21.70

Residue	Atom			
172 PRO	N	661 59.43	402 35.07	359 22.64
	Cβ	639 57.45	391 34.11	343 21.63
	Cα	651 58.53	403 35.15	340 21.44
	C	641 57.63	417 36.37	339 21.38
	O	641 57.63	426 37.16	354 22.33
173 ALA	N	633 56.91	418 36.46	322 20.31
	Cβ	632 56.82	445 38.82	309 19.49
	Cα	623 56.01	431 37.60	319 20.12
	C	610 54.85	429 37.42	304 19.17
	O	611 54.94	421 36.72	288 18.16
174 Leu	N	597 53.68	435 37.95	311 19.61
	Cβ	574 51.61	421 36.72	310 19.55
	Cα	584 52.51	433 37.77	298 18.79
	C	575 51.70	448 39.08	294 18.54
	O	575 51.70	458 39.95	307 19.36
175 Phe	N	568 51.07	448 39.08	276 17.41
	Cβ	550 49.45	461 40.21	249 15.70
	Cα	559 50.26	462 40.30	270 17.03
	C	547 49.18	464 40.47	286 18.04
	O	541 48.64	453 39.52	295 18.61

	C	458 41.18	400 34.89	239 15.07
	O	455 40.91	393 34.28	255 16.08
176 TYR	N	544 48.91	479 41.78	291 18.35
	Cβ	534 48.01	500 43.62	311 19.61
	Cα	532 47.83	488 42.57	307 19.36
	C	517 46.48	475 41.43	303 19.11
	O	512 46.03	466 40.65	315 19.87
177 ALA	N	511 45.94	480 41.87	285 17.97
	Cβ	487 43.79	484 42.22	264 16.65
	Cα	496 44.60	474 41.35	279 17.60
	C	495 44.51	457 39.86	277 17.47
	O	503 45.22	449 39.17	265 16.71
178 PRO	N	484 43.52	450 39.25	287 18.10
	Cβ	~~469~~ 469 42.17	431 37.60	302 19.05
	Cα	481 43.25	433 37.77	286 18.04
	C	478 42.98	430 37.51	263 16.59
	O	476 42.80	440 38.38	250 15.77
179 VAL	N	476 42.80	415 36.20	259 16.34
	CG2	499 44.87	410 35.76	277 17.47
	CG1	483 43.43	391 34.11	209 13.18
	Cβ	486 43.70	400 34.89	229 14.44
	Cα	472 42.44	410 35.76	238 15.01
180 HIS	N	450 40.46	399 34.80	221 13.94
	Cβ	421 37.85	396 34.54	215 13.56
	Cα	437 39.29	390 34.02	220 13.88
	C	437 39.29	376 32.80	206 12.99
	O	435 39.11	377 32.89	187 11.79
181 ILE	N	440 39.56	363 31.66	216 13.62
	Cβ	452 40.64	338 29.48	217 13.69
	Cα	440 39.56	349 30.44	205 12.93
	C	427 38.39	343 29.92	192 12.11
	O	427 38.39	342 29.83	172 10.85
182 TRP	N	415 37.31	340 29.66	203 12.80
	Cβ	399 35.87	320 27.91	207 13.06
	Cα	401 36.05	334 29.13	193 12.17
	C	388 34.89	342 29.83	203 12.80
	O	389 34.97	348 30.36	221 13.94
183 GLU	N	375 33.72	341 29.75	193 12.17
	Cβ	363 32.64	364 31.75	194 12.24
	Cα	361 32.46	347 30.27	201 12.68
	C	346 31.11	341 29.75	195 12.30
	O	344 30.93	331 28.87	181 11.42

184 SER	335	347	205
N	30.12	30.27	12.93
	306	351	209
C^{β}	27.51	30.62	13.18
	319	343	201
C^{α}	28.68	29.92	12.68
	315	340	177
C	28.32	29.66	11.16
	312	327	171
O	28.05	28.52	10.78
185 SER	316	353	165
N	28.41	30.79	10.41
	302	366	139
C^{β}	27.15	31.93	8.77
	312	352	143
C^{α}	28.05	30.70	9.02
	325	352	127
C	29.22	30.70	8.01
	323	356	108
O	29.04	31.05	6.81
186 ALA	338	348	135
N	30.39	30.36	8.51
	365	347	136
C^{β}	32.82	30.27	8.58
	351	347	122
C^{α}	31.56	30.27	7.69
	353	331	112
C	31.74	28.87	7.06
	352	320	124
O	31.65	27.91	7.82
187 THR	354	331	92
N	31.83	28.87	5.80
	340	318	70
C^{β}	30.57	27.74	4.41
	356	316	80
C^{α}	32.01	27.56	5.05
	370	316	67
C	33.27	27.56	4.23
	374	304	58
O	33.63	26.52	3.66

188 VAL	377	329	66
N	33.90	28.70	4.16
	384	337	34
C^{β}	34.53	29.40	2.14
	391	331	54
C^{α}	35.15	28.87	3.41
	402	341	65
C	36.14	29.75	4.10
	401	356	63
O	36.05	31.05	3.97
189 SER	412	335	77
N	37.04	29.22	4.86
	432	351	124
OG	38.84	30.62	7.82
	424	340	112
C^{β}	38.12	29.66	7.06
	424	344	88
C^{α}	38.12	30.01	5.55
	440	342	81
C	39.56	29.83	5.11
	444	331	71
O	39.92	28.87	4.48
190 ALA	449	354	86
N	40.37	30.88	5.42
	464	361	57
C^{β}	41.72	31.49	3.59
	464	354	79
C^{α}	41.72	30.88	4.98
	476	362	91
C	42.80	31.58	5.74
	474	375	101
O	42.62	32.71	6.37
191 PHE	490	356	91
N	44.06	31.05	5.74
	504	359	125
C^{β}	45.31	31.32	7.88
	502	363	102
C^{α}	45.13	31.66	6.43
	516	364	88
C	46.39	31.75	5.55
	520	353	78
O	46.75	30.79	4.92

192 GLU	524	377	89
N	47.11	32.89	5.61
Cβ	537 48.28	383 33.41	52 3.28
Cα	538 48.37	379 3.06	76 4.79
C	550 49.45	388 33.85	87 5.49
O	547 49.18	401 34.98	96 6.05
193 ALA			
N	563 50.62	382 33.32	87 5.89
Cβ	580 52.15	387 33.76	121 7.63
Cα	576 51.79	389 33.93	97 6.12
C	591 53.14	386 33.67	87 5.49
O	594 53.41	373 32.54	81 5.11
194 THR			
N	600 53.95	398 34.72	86 5.42
Cβ	613 55.11	405 35.33	57 3.59
Cα	615 55.29	396 34.54	77 4.86
C	625 56.21	404 35.24	94 5.93
O	619 55.66	414 36.11	106 6.69
195 PHE			
N	639 57.45	401 34.98	93 5.87
Cβ	648 58.26	407 35.50	132 8.33
Cα	649 58.35	409 35.68	108 6.81
C	666 59.88	406 35.42	103 6.50
O	670 60.24	392 34.19	100 6.31

196 ALA	674	418	103
N	60.60	36.46	6.50
Cβ	697 62.69	431 37.60	89 5.61
Cα	690 62.04	417 36.37	98 6.18
C	697 62.67	415 36.20	120 7.57
O	692 62.22	423 36.90	135 8.51
197 PHE			
N	709 63.75	406 35.42	122 7.69
Cβ	712 64.02	388 33.85	150 9.46
Cα	716 64.38	403 35.15	142 8.96
C	733 65.90	403 35.15	138 8.70
O	738 66.35	400 34.89	121 7.63
LEU 198			
N	741 66.62	406 35.42	155 9.78
Cβ	766 68.87	422 36.81	153 9.65
Cα	758 68.15	406 35.42	154 9.71
C	765 68.78	398 34.72	172 10.85
O	766 68.87	404 35.24	190 11.98
199 ILE			
N	771 69.32	384 33.50	169 10.66
Cβ	772 69.41	359 31.32	182 11.48
Cα	778 69.95	375 32.71	183 11.54
C	795 71.48	376 32.80	181 11.42
O	801 72.02	373 32.54	163 10.28

200 LYS	803	380	198	204 SER	886	294	227
N	72.20	33.15	12.49	N	79.66	25.65	14.32
C^{β}	824 74.09	398 34.72	201 12.68	C^{β}	895 80.47	275 23.99	205 12.92
C^{α}	819 73.64	381 33.23	197 12.42	C^{α}	881 79.21	284 24.77	209 13.18
C	828 74.45	372 32.45	214 ~~13~~ 13.50	C	868 78.04	275 23.99	219 13.81
O	825 74.18	373 32.54	233 14.70	O	860 77.32	268 23.38	208 13.12
201 SER N	838 75.34	363 31.66	206 12.99	205 HIS N	867 77.95	276 24.08	240 15.14
C^{β}	842 75.70	337 29.40	221 13.94	C^{β}	862 77.50	256 22.33	264 16.65
C^{α}	847 76.15	353 30.79	220 13.88	C^{α}	855 76.87	268 23.38	251 15.83
C	864 77.68	353 30.79	218 13.75	C	843 75.79	279 24.34	259 16.34
O	870 78.22	348 30.36	202 12.74	O	841 75.61	281 24.51	278 17.53
202 PRO N	871 78.31	359 31.32	234 14.76	206 PRO N	835 75.07	287 25.04	244 15.39
C^{β}	891 80.11	370 32.28	253 15.96	C^{β}	818 73.55	303 26.43	229 14.44
C^{α}	888 79.84	360 31.40	234 14.76	C^{α}	824 74.09	298 25.99	250 15.77
C	892 80.20	343 29.92	234 14.76	C	811 72.92	293 25.56	265 16.71
O	904 81.28	339 29.57	226 14.25	O	805 72.38	280 24.42	265 16.71
203 ASP N	883 79.39	334 29.13	245 15.45	207 ALA N	806 72.47	304 26.52	278 17.53
C^{β}	879 79.03	313 27.30	268 16.91	C^{β}	786 70.67	317 27.65	297 18.73
C^{α}	886 79.66	318 27.74	247 15.58	C^{α}	794 71.39	302 26.34	293 18.48
C	880 79.12	308 26.87	228 14.38	C	782 70.31	290 25.30	290 18.29
O	871 78.31	314 27.39	215 13.56	O	784 70.49	276 24.08	291 18.35

208 ASP	769	296	287
N	69.14	25.82	18.10
Cβ	747 67.16	285 24.86	305 ~~29~~ 19.24
Cα	756 67.97	286 24.95	284 17.91
C	747 67.16	289 25.21	264 16.65
O	743 66.80	279 24.34	251 15.83
209 GLY	743	304	261
N	66.80	26.52	16.46
Cα	735 66.08	309 26.95	242 15.26
C	723 65.00	321 28.00	246 15.52
O	723 65.00	329 28.70	262 16.52
210 Ile	714	322	231
N	64.20	28.09	14.57
Cβ	702 63.12	346 30.18	216 13.62
Cα	700 62.94	333 29.05	233 14.70
C	685 61.59	324 28.26	232 14.63
O	684 61.50	310 27.04	231 14.57
211 ALA	673	333	233
N	60.51	29.05	14.70
Cβ	657 59.07	318 27.74	253 15.96
Cα	658 59.16	327 28.52	232 14.63
C	645 57.99	337 29.40	232 14.63
O	643 57.81	348 30.36	245 15.45

212 Phe	635	335	217
N	57.09	29.22	13.69
Cβ	617 55.47	346 30.18	191 12.05
Cα	622 55.92	344 30.01	214 13.50
C	609 54.76	335 29.22	224 14.13
O	605 54.40	323 28.18	215 13.56
213 Phe	603	340	241
N	54.22	29.66	15.20
Cβ	599 53.86	329 28.70	273 17.22
Cα	591 53.14	332 28.96	252 15.89
C	575 51.70	339 29.57	257 16.21
O	574 51.61	353 30.79	263 16.59
214 Ile	564	329	256
N	50.71	28.70	16.15
CD1	529 47.56	335 29.22	207 13.06
CG1	542 48.73	330 28.79	221 13.94
Cβ	538 48.37	327 28.52	244 15.39
CG2	523 47.02	334 29.13	252 15.89
Cα	549 49.36	334 29.13	260 16.40
C	544 48.91	327 28.52	281 17.72
O	546 49.09	313 27.30	285 17.97

Residue	Atom			
215 SER	N	537 48.28	336 29.31	295 18.61
	C^{β}	545 49.00	332 28.96	329 20.75
	C^{α}	532 47.83	331 28.87	315 19.87
	C	519 46.66	341 29.75	323 20.37
	O	512 46.03	349 30.44	311 19.61
216 ASN	N	517 46.48	341 29.75	344 21.70
	C^{β}	498 44.78	339 29.57	370 23.34
	C^{α}	550 49.45	350 30.53	354 22.33
	C	511 45.94	366 31.93	359 22.64
	O	524 47.11	368 32.10	362 22.83
217 Ile	N	500 44.96	377 32.89	360 22.71
	C^{β}	488 43.88	399 34.80	365 23.02
	C^{α}	504 45.31	393 34.28	365 23.02
	C	513 46.12	392 34.19	387 24.41
	O	521 46.84	403 35.15	392 24.72
218 ASP	N	510 45.85	379 33.06	398 25.10
	C^{β}	509 75.76	368 32.10	435 27.44
	C^{α}	518 46.57	377 32.89	418 26.36
	C	533 47.92	370 32.28	419 26.43
	O	540 48.55	369 32.19	436 27.50

Residue	Atom			
219 SER	N	538 48.37	364 31.75	401 25.29
	C^{β}	557 50.08	353 30.79	377 23.78
	C^{α}	552 49.63	356 31.05	400 25.23
	C	566 50.89	363 31.67	411 25.92
	O	578 51.97	376 32.80	408 25.73
220 SER	N	572 51.43	353 30.79	425 26.80
	C^{β}	581 52.24	359 31.32	461 29.08
	C^{α}	585 52.60	358 31.23	437 27.56
	C	598 53.77	346 30.18	436 27.50
	O	595 53.50	332 28.96	435 27.44
221 Ile	N	611 54.94	351 30.62	437 27.56
	C^{β}	639 57.45	350 30.53	438 27.62
	C^{α}	624 56.10	341 29.75	436 27.50
	C	625 56.19	331 28.87	455 28.70
	O	626 56.28	336 29.31	473 29.83
222 PRO	N	626 56.28	316 27.56	451 28.44
	C^{β}	623 56.01	290 25.30	459 28.95
	C^{α}	627 56.37	305 26.61	469 29.58
	C	643 57.81	304 26.52	478 30.15
	O	653 58.71	313 27.30	473 29.83

223 SER		645		293		493	
	N		57.99		25.56		31.09
	C^{β}	657	59.07	288	25.12	527	33.24
	C^{α}	659	59.25	291	25.38	503	31.72
	C	671	60.33	283	24.69	490	30.90
	O	670	60.24	270	23.55	482	30.40
224 GLY		683		292		487	
	N		61.41		25.47		30.72
	C^{α}	696	62.58	286	24.95	475	29.96
	C	690	62.04	285	24.86	452	28.51
	O	694	62.40	274	23.90	441	27.81
225 SER		682		296		445	
	N		61.32		25.82		28.07
	C^{β}	665	59.79	309	26.95	424	26.74
	C^{α}	676	60.78	297	25.91	424	26.74
	C	691	62.13	298	25.99	412	25.98
	O	692	62.22	297	25.91	393	24.79
226 SER		702		301		425	
	N		63.12		26.26		26.80
	C^{β}	731	65.72	306	26.69	430	27.12
	C^{α}	717	64.47	303	26.43	416	26.24
	C	722	64.92	289	25.21	405	25.54
	O	714	64.20	277	24.16	406	25.61
227 GLY		735		289		395	
	N		66.08		25.21		24.91
	C^{α}	740	66.53	276	24.08	383	24.16
	C	732	65.81	272	23.73	363	22.89
	O	729	65.54	282	24.60	350	22.07
228 ARG		729		257		360	
	N		65.54		22.42		22.71
	C^{β}	724	65.09	234	20.41	343	21.63
	C^{α}	722	64.92	252	21.98	340	21.44
	C	705	63.39	254	22.16	344	21.70
	O	696	62.58	253	22.07	329	20.75
229 Leu		701		258		363	
	N		63.03		22.51		22.89
	C^{β}	684	61.50	261	22.77	393	24.79
	C^{α}	686	61.68	261	22.77	369	23.27
	C	680	61.14	276	24.08	358	22.58
	O	668	60.06	281	24.51	362	22.83
230 Leu		689		282		344	
	N		61.95		24.60		21.70
	C^{β}	679	61.05	292	25.47	311	19.61
	C^{α}	685	61.59	296	25.82	333	21.00
	C	673	60.51	306	26.69	342	21.57
	O	666	59.88	315	27.48	331	20.88

231 GLY	671	304	363
N	60.33	26.52	22.89
Cα	659 59.25	313 27.30	374 23.59
C	644 57.90	312 27.22	363 22.89
O	635 57.09	323 28.18	363 22.89
232 Leu	641	299	355
N	57.63	26.08	22.39
Cβ	627 56.37	293 25.56	320 20.18
Cα	626 56.28	296 25.82	344 21.70
C	619 55.65	283 24.69	357 22.52
O	606 54.49	283 24.69	361 22.77
233 Phe	628	272	363
N	56.46	23.73	22.89
Cβ	624 56.10	245 21.37	360 22.71
Cα	623 56.01	259 22.59	375 23.65
C	630 56.64	255 22.24	396 24.98
O	643 57.81	255 22.24	399 25.16
234 PRO	620	252	412
N	55.74	21.98	25.98
Cβ	612 55.02	249 21.72	447 28.19
Cα	626 56.28	248 21.63	433 27.31
C	633 56.91	233 20.32	431 27.18
O	635 57.09	227 19.80	413 26.05

235 ASP	636	226	449
N	57.18	19.71	28.32
Cβ	652 58.62	211 18.41	471 29.71
Cα	643 57.81	211 18.41	449 28.32
C	630 56.64	200 17.45	449 28.32
O	623 56.01	197 17.18	465 29.33
236 ALA	627	193	430
N	56.37	16.84	27.12
Cβ	599 53.86	190 16.57	429 27.06
Cα	615 55.29	183 15.96	428 26.99
C	617 55.47	176 15.35	406 25.61
O	617 55.47	185 16.14	390 24.60
237 ASN	618	161	405
N	55.56	14.04	25.54
Cβ	636 57.18	159 13.87	377 23.78
Cα	621 55.83	154 13.43	384 24.22
C	622 55.92	136 11.86	385 24.28
O_1	610 54.85	128 11.17	385 24.28
O_2	635 57.09	131 11.43	385 24.28

10^{th}

Circuit →

~~8~~ Certification →

Binding →

Su~~a~~bmissi 2

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