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Geometry and Universe

[Professor V.V. Narlikar Memorial Lecture]

P. C. Vaidya

1. Introduction

I am indeed grateful to INSA for inviting me to deliver the first Professor Vishnu Vasudev Narlikar Memorial Lecture. Prof. V.V. Narlikar was my teacher. But our student-teacher relationship was peculiar. I was not a student of Banaras University where he worked. I was his "personal student". He initiated me to mathematical research. I used to meet him at his home twice a week and that established a link ~~to~~ between Professor VVN's family and my little family. (My wife and my little daughter were with me at Banaras.) For Narlikars I am almost a senior family member and ~~was~~ in my family Prof. VVN was Narlikar Dada (Grand Pa Narlikar). Thus

The feelings of gratefulness towards INSA that I expressed in the beginning came from the bottom of my heart.

The mathematics section of the 1953 science congress was presided over by ~~the~~ Prof VVN. At that session there was a symposium on teaching of Mathematics. Speaking at that symposium, Prof. S. N. Bose described two characteristics of a good teacher. ~~He~~ One characteristic, he described, in the following words. "After all, what does a teacher do? He presents himself as a model for his students to imitate." And I was very lucky to have the model of my teacher Prof. VVN to imitate and to mould my teaching and research career on his model. Again in this talk I shall try to follow his way of ~~way of~~ exposition of mathematical aspects of physical situations. If I succeed, you will have an idea of the expository techniques of Prof. VVN. And if I do not appear convincing, the fault lies with the imitator and not with the model.

2. Spaces of Constant Curvature.

We begin with the familiar equation

$x^2 + y^2 = R^2$. We know it represents a circle drawn on

the x - y plane. We say it is a one dimensional curve of constant curvature drawn in a 2-dimensional flat plane. Well, we also know that

$x^2 + y^2 + z^2 = R^2$ represents a sphere in space. We say, it is a 2-dimensional surface of constant curvature immersed in 3-dimensional flat space.

~~We proceed further in the same manner and~~

We proceed further in the same manner and say that $x^2 + y^2 + z^2 + w^2 = R^2$ represents a 3-dimensional space of constant curvature immersed in a 4-dimensional flat space. We are interested in these 3-spaces of constant curvature because of 2 simple assumptions of Einstein: (1) A gravitating object produces a curvature in the space surrounding that object and (2) if we are interested not

in the gravitational field of isolated objects but in the average field pervading the whole universe, we must assume the matter in the universe to be smoothly and uniformly spread in the universe, producing a uniform curvature in space. Thus the 3-space in our universe taken as a whole must be of constant curvature.

Another feature of Einstein theory is that gravitation is not an "action at a distance" phenomena as in Newton's theory, but it is a field phenomena. Thus the effect is transmitted from a given point to a neighbouring point. So the equations specifying the law of gravitation are differential equations and the corresponding geometry is differential geometry. Therefore we shall try to translate the analytic geometry of 3-space of constant curvature to the corresponding differential geometry.

We again begin with the familiar case of a circle $x^2 + y^2 = R^2$. Its parametric equations are $x = R \cos \phi$, $y = R \sin \phi$. The corresponding differential form is $ds^2 = dx^2 + dy^2$, or in terms of the parameter ϕ , $ds^2 = R^2 d\phi^2$.

For a sphere $x^2 + y^2 + z^2 = R^2$, the parametric equations are

$$\begin{aligned}x &= R \sin \theta \cos \phi \\y &= R \sin \theta \sin \phi \\z &= R \cos \theta.\end{aligned}$$

The corresponding differential form is $ds^2 = dx^2 + dy^2 + dz^2$ or in terms of parameters θ and ϕ ,

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2.$$

We can generalize this procedure for 3-spaces of constant curvature (or in short, for 3-spheres). The analytic equation for a 3-sphere is

$$x^2 + y^2 + z^2 + w^2 = R^2.$$

We now need 3 parameters χ, θ, ϕ and write the parametric equations as

$$\begin{aligned}x &= R \sin \chi \sin \theta \cos \phi \\y &= R \sin \chi \sin \theta \sin \phi \\z &= R \sin \chi \cos \theta \\w &= R \cos \chi.\end{aligned}$$

One can now verify that the differential form $d\sigma^2 = dx^2 + dy^2 + dz^2 + d\omega^2$, when expressed in terms of the parameters x, θ, φ takes the form

$$d\sigma^2 = R^2 dx^2 + R^2 \sin^2 x (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (2.1)$$

3. Einstein Universe

In an epoch making paper of ~~1918~~ 1917,

Einstein suggested that the space-time measurements in the gravitational field pervading the universe would follow the 4-dimensional space-time geometry given by $ds^2 = c^2 dt^2 - d\sigma^2$, where $d\sigma^2$ is given by (2.1). Thus

$$ds^2 = c^2 dt^2 - R^2 [dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta d\varphi^2)] \quad (3.1)$$

Here c is the velocity of light, t is the time and the 3-dimensional physical space is not the usual Euclidean flat space, but a 3-sphere of radius R . In (3.1), t is, of course, the time-like coordinate and the 3 space-like coordinates are given by the three angles x, θ, φ . We can

replace one of these by a distance-coordinate r .

In (3.1) set $R \sin x = r$, so that $R \cos x dx = dr$

(3.1) then takes the form

$$ds^2 = c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{R^2}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (3.2)$$

(3.2) is the standard form in which Einstein's 1917 model of the universe is expressed.

On looking at the metric (3.2) it will be seen that the constant R^2 can be taken as positive or negative. But R^2 being negative needs a reconsideration of our strategy leading to (2.1). If R^2 is negative R is to be replaced by iR . But $r = R \sin x$ implies that if R is replaced by iR , x needs to be replaced by ix so that $r = -R \sinh x$ continues to be real. Thus in the parametric equations of the 3-space, one has to replace $\sin x$ by $\sinh x$ and $\cos x$ by $\cosh x$. We do not go into details here but note that the differential metric

of Einstein's Universe with 3-space of constant negative curvature will be

$$ds^2 = c^2 dt^2 - R^2 [dx^2 + \sinh^2 x (d\theta^2 + \sin^2 \theta d\phi^2)] \quad (3.3)$$

Again setting $R \sinh x = r$, we shall get

$$ds^2 = c^2 dt^2 - \frac{dr^2}{1 + \frac{r^2}{R^2}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (3.4)$$

4. Expanding Universe

Einstein's model given by (3.2) or (3.4) was the first mathematical model of the universe. The physical properties of this model universe were worked out using Einstein's field equations in the background of Geometry given by (3.2) or (3.4). It was found that this model admits presence of matter but does not admit any large scale motion. By about the same time (in 1918) another mathematical model of the universe

was worked out by de Sitter. (We are going to give a simple mathematical derivation of this model later.) This de Sitter's model does not admit presence of matter density but admits motion in the sense that a test particle introduced in the model will acquire motion! In 1924, Hubble observed large scale motion of extra-galactic nebulae, and thus it became clear that in the physical universe, on a large scale, both matter and motion coexist. A mathematical model worked out by Friedman ^{way back} in 1921 was found to incorporate this physical requirement. This model was later improved by several workers and is now known as the expanding universe model or Robertson-Walker model. The geometry of this model is again very easy to understand.

Let us turn back to the 3-space of ~~positive~~ constant positive curvature so that Einstein's universe is given by

$$ds^2 = c^2 dt^2 - R^2 [dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta d\phi^2)] \quad (3.1)$$

Here R is the radius of the spherical 3-space.

~~Now~~ now R is a constant in relation to the geometrical variable x, θ, ϕ of the 3-space. But in shifting

shifting from geometry to physics, we introduced a new time parameter t , and R need not be constant in relation to t . If we take R as a function of t , nothing changes in section 2 above and we get (3.1) with $R = R(t)$. But

now the radius of the 3-sphere (which is our physical space) becomes a function of time and if this function is an increasing function, we have an "expanding universe".

$$ds^2 = c^2 dt^2 - R^2(t) [dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta d\phi^2)]$$

In order to introduce a radial co-ordinate r , we rewrite $R^2(t) = \frac{R^2(t)}{R_0^2} R_0^2$

R_0 being a constant, so that our metric takes

the form

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{R_0^2} \left[R_0^2 dx^2 + R_0^2 \sin^2 x (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$

Now

Now put $R_0 \sin x = r$ and we get

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{R_0^2} \left[\frac{dr^2}{1 - \frac{r^2}{R_0^2}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (4.1)$$

Similarly beginning with the metric (3.3) of Einstein Universe with negative constant curvature, we get the metric of the corresponding expanding universe as

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{R_0^2} \left[\frac{dr^2}{1 + \frac{r^2}{R_0^2}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (4.2)$$

In (4.1) and (4.2) one can replace r by a new coordinate $\bar{r} = \frac{r}{R_0}$ and then the

two cases can be combined in the form

$$ds^2 = c^2 dt^2 - R^2(t) \left[\frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (4.3)$$

with

$k = +1$	for 3-space of uniform positive curvature
$k = -1$	" " " " negative "
$k = 0$	" " " " zero "

(4.3) is the well-known Robertson-Walker metric of expanding universe.

5. de Sitter- Universe

Let us revert to spaces of constant curvature. We shall use the single word "sphere" to denote these spaces of constant curvature, irrespective of the dimensions.

Thus we would say that

$x^2 + y^2 = R_0^2$ gives a 1-sphere immersed in 2-flat

$x^2 + y^2 + z^2 = R_0^2$ gives a 2-sphere immersed in 3-flat ;

$x^2 + y^2 + z^2 + w^2 = R_0^2$ gives a 3-sphere immersed in 4-flat.

Can we extend this and have a ~~4~~ⁿ-sphere immersed in 5-flat? Well, this is what de Sitter did and that is what we are going to do now!

To construct a geometrical model of the universe Einstein assumed that the physical 3-space of the universe is a sphere and then constructed its differential line-element ds^2 (given as (2.1) above). He then introduced an orthogonal time t and wrote down the line-element

of 4-dimensional space-time as $ds^2 = c^2 dt^2 - ds^2$.

In place of Einstein's assumption of only the 3-space being spherical, de Sitter assumed that the entire 4-dimensional space-time is spherical, i.e. he assumed that the entire space-time is a 4-sphere immersed in 5-flat. But for the 4-dimensional space-time which we use in physics, the metric ds^2 is not positive definite but indefinite. Therefore when we consider a 4-sphere, our interest will be in a 4-sphere with an indefinite metric. This will be reflected in the 5-flat in which it is immersed. We take

$$x^2 + y^2 + z^2 + w^2 - v^2 = R_0^2$$

as the 4-sphere immersed in 5-flat. Following the previous procedure, we write the parametric equations of this 4-sphere in terms of 4 angular parameters ψ, χ, θ, ϕ as follows:

$$x = R_0 \cosh \psi \sin \alpha \sin \theta \cos \phi$$

$$y = R_0 \cosh \psi \sin \alpha \sin \theta \sin \phi$$

$$z = R_0 \cosh \psi \sin \alpha \cos \theta$$

$$w = R_0 \cosh \psi \cos \alpha$$

$$v = R_0 \sinh \psi$$

Setting $ds^2 = dv^2 - dx^2 - dy^2 - dz^2 - dw^2$, one gets

$$ds^2 = R_0^2 d\psi^2 - R_0^2 \sinh^2 \psi [dx^2 + \sin^2 \alpha (d\theta^2 + \sin^2 \theta d\phi^2)]$$

Putting $R_0 d\psi = c dt$ and $R_0 \sin \alpha = r$, we get the completely spherical 4-space-time given by the

$$\text{metric} \quad ds^2 = c^2 dt^2 - \sinh^2 \frac{ct}{R_0} \left[\frac{dr^2}{1 - \frac{r^2}{R_0^2}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (5.1)$$

This is the metric of de Sitter model of the universe. Comparing with the metric (4.1) of the expanding universe metric, we see that it is of the form (4.1) when the undetermined function

$$\frac{R(t)}{R_0} \text{ becomes } \sinh \frac{ct}{R_0} \text{ i.e. when } R(t) = R_0 \sinh \frac{ct}{R_0}.$$

In that case it is easy to see that $R(t)$ is an increasing function and so de Sitter's model

becomes a particular case of the expanding universe model.

The metric form (5.1) is time-dependent. But time-dependent metric form for describing the geometry of the universe was first developed by Friedman in 1921. It is, therefore, clear that in 1918 de Sitter did not describe his universe by (5.1). It is ~~easier~~ easy to derive de Sitter's original time-independent (static) form of the metric. Of course, we begin with the 4-sphere $x^2 + y^2 + z^2 + w^2 - v^2 = R_0^2$, but write the parametric equations as follows:

$$\begin{aligned} x &= R_0 \sin \chi \sin \theta \cos \phi \\ y &= R_0 \sin \chi \sin \theta \sin \phi \\ z &= R_0 \sin \chi \cos \theta \\ w &= R_0 \cos \chi \cosh \psi \\ v &= R_0 \cos \chi \sinh \psi \end{aligned}$$

Setting $ds^2 = dv^2 - dx^2 - dy^2 - dz^2 - dw^2$ one gets

$$ds^2 = R_0^2 \cos^2 \chi d\psi^2 - R_0^2 d\chi^2 - R_0^2 \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)$$

Put $R_0 \sin \chi = \bar{r}$ and $R_0 d\psi = c \bar{t}$, we find

$$ds^2 = \left(1 - \frac{\bar{r}^2}{R_0^2}\right) c^2 d\bar{t}^2 - \frac{d\bar{r}^2}{1 - \frac{\bar{r}^2}{R_0^2}} - \bar{r}^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (5.2)$$

(5.2) is the standard form of de Sitter metric (5.1) or (5.2) give the differential form for the same 4-sphere $x^2 + y^2 + z^2 + w^2 - v^2 = R_0^2$. So any one of them is a transform of the other.

6. Gödel's Rotating Universe

In 1949 K. Gödel gave a model of the universe exhibiting large-scale rotation of the matter in the universe. It is possible to derive the geometry of this universe from that of Einstein's universe, by using a geometric trick. We first introduce a rotation in the co-ordinate axes in our 3-space and then develop a criterion that will convert this spurious geometrical rotation into a physical rotation of matter, and thus get the model of the rotating universe. We carry out this plan in 3 stages.

(A) Spheroidal Polar Coordinates

In the usual ~~geom~~ Euclidean geometry

Cartesian coordinates (x, y, z) and the spherical polar co-ordinates (r, θ, φ) are connected by relations

$$x = r \sin \theta \cos \varphi, \quad y = r \sin \theta \sin \varphi, \quad z = r \cos \theta \quad (6.1)$$

In place of these spherical polar coordinates, we now introduce new coordinates (r, θ, φ) by the relations

$$\begin{aligned} x &= r \sin \theta \cos \varphi - a \sin \theta \sin \varphi \\ y &= r \sin \theta \sin \varphi + a \sin \theta \cos \varphi \\ z &= r \cos \theta \end{aligned} \quad (6.2)$$

So that we have $\frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1$. Thus the

surfaces $r = \text{const.}$ are ellipsoids of revolution. So we call (r, θ, φ) defined by (6.2) as spheroidal polar coordinates. It can be verified that (6.2) can be obtained from (6.1) by imparting a rotation to the x - and y -axes through an angle $\tan^{-1} \left(\frac{a}{r} \right)$ round the z -axis.

We now take the Minkowski's space-time of special relativity

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2.$$

Here we have chosen the units in such a way that the velocity of light becomes 1; Replacing x, y, z by spheroidal coordinates (r, θ, φ) of (6.2) we get

$$ds^2 = dt^2 - (r - a \sin^2 \theta)^2 - (r^2 + a^2 \cos^2 \theta) (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (6.3)$$

Using the null coordinate $u = t - r$ in place of the space-like coordinate r we rewrite (6.3) as

$$ds^2 = 2 (du + a \sin^2 \theta d\varphi) dt - (du + a \sin^2 \theta d\varphi)^2 - (r^2 + a^2 \cos^2 \theta) (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (6.4)$$

with $r = t - u$.

(6.4) is the metric form for the usual Euclidean geometry of space-time in absence of gravitation. To describe the uniform gravitational field pervading the universe we take the corresponding Riemannian geometry of space-time as given by

$$ds^2 = 2 (du + g \sin \theta d\varphi) d\xi - L (du + g \sin \theta d\varphi)^2 - M^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (6.5)$$

$g = g(\theta), \quad L = L(u, \theta, \xi) \quad M = M(u, \theta, \xi)$

Here, in place of the usual t , we have used ξ to

denote the time coordinate.

(B) ~~Method of shift~~ The geometric

(B) The Geometric Trick

The metric (6.5) is an indefinite metric with signature $(-, -, -, +)$. The coordinates used are u, θ, φ, ξ . We take them as x^1, x^2, x^3, x^4 respectively. u is a null coordinate, θ, φ are space-like, but ξ is time-like or space-like according as the coefficient L is positive or negative

Consider now a time-like ^{unit} vector v_i and an orthogonal space-like vector V_i

$$v_i v^i = +1, \quad V_i V^i = -1, \quad v_i V^i = 0$$

It is easy to find the components of these vectors in terms of the metric (6.5). It will be found that

$$v_i = \left[\frac{1}{2}(1-L), 0, \frac{1}{2}(1-L)g \sin \theta, 1 \right]$$

$$V_i = \left[\frac{1}{2}(1+L), 0, \frac{1}{2}(1+L)g \sin \theta, -1 \right]$$

We now choose $L = +1$ or $L = -1$.

When $L = +1$, v_i becomes $(0, 0, 0, 1)$ and V_i becomes $(1, 0, g \sin \theta, -1)$. It can now be verified that $v_{i;k} - v_{k;i} = 0$ but $V_{i;k} - V_{k;i} \neq 0$ if $g \sin \theta$ is not constant. So vector v_k has no twist, while V_k is a rotating vector field. However when $L = -1$, v_k becomes $(1, 0, g \sin \theta, 1)$ and V_k becomes $(0, 0, 0, -1)$ so that now v_k has rotation while V_k becomes rotation-free. Thus by changing the parameter L from $+1$ to -1 , we have, so to speak shifted the rotation from space-like V_k to time-like v_k .

Now if v_k and V_k happen to be eigenvectors of the Einstein's matter tensor G_{ik} , then the shifting of the twist from V_k to v_k will imply a shift of twist from a space-like eigenvector to a time-like eigenvector. But a twisting time-like eigen-vector implies a rotating matter distribution. In this way we are able to relate a non-rotating material distribution to a rotating one. This is the geometrical trick of shifting the twist.

(c) Einstein - Gödel Metric

In the metric (6.5) we have introduced 3 unknown functions L , M and g . ~~Of~~ Our geometrical method of shifting the twist implies that L is either $+1$ or -1 . We have to use physical properties of Einstein's Universe to find explicitly the other two functions. We do not go into details of their derivation but write down the final solution in the form

$$L = +1 \text{ or } -1$$

~~$$g \sin \theta = 1 + L - [4/(1+3L)] e^{-(1+\theta)}$$~~

$$g \sin \theta = 1 + L - [4/(1+3L)] e^{-(1+3L)\gamma/2R}$$

$$M \sin^2 \theta = g \sin \theta e^{-(1+3L)\gamma/2R}$$

where γ is defined by $g d\theta = d\gamma$

It will be easier to rewrite metric (6.5) using γ as a coordinate in place of θ . We also replace u by the coordinate z given by

$$z = u + (1+L)\varphi - L\xi$$

(6.5) will then change to

$$ds^2 = L d\xi^2 - L dr^2 + 2LY dr d\varphi - Y d\varphi^2 [1+L - (1-L)Y/4] - \frac{1}{4}(1+3L) Y dy^2 / (1+L-Y) \quad (6.6)$$

with $Y = \frac{4}{1+3L} e^{-(1+3L)r/2R}$

(6.6) is If in (6.6) we put $L=1$, we find that

$$ds^2 = d\xi^2 - dr^2 + 2e^{-2r/R} dr d\varphi - 2e^{-2r/R} d\varphi^2 - e^{-2r/R} dy^2 / [2 - e^{-2r/R}] \quad (6.7)$$

It can be verified that (6.7) is a transform of the standard metric of the Einstein Universe, the coordinate ξ being the cosmic time t .

On putting $L=-1$ in (6.6), one finds

$$ds^2 = dr^2 - d\xi^2 + 4e^{r/R} dr d\varphi + 2e^{2r/R} d\varphi^2 - \frac{1}{2} dy^2 \quad (6.8)$$

This is the ~~standard form~~ of Gödel's Universe metric, the coordinate r being the time coordinate.

In (6.7) the space-like vector dr had a spurious (co-ordinate-generated) rotation. In (6.8) dr becomes time-like and so the rotation which was ^{the vector}

geometrical in (6.7) has now become physical in (6.8).

Thus (6.8) gives the geometry of a rotating universe. It is not customary to use z for a time-like coordinate. So we replace z by τ in (6.8) and write

$$ds^2 = d\tau^2 - d\xi^2 + 4e^{y/R} d\tau d\varphi + 2e^{2y/R} d\varphi^2 - \frac{1}{2} dy^2 \quad (6.9)$$

This is the standard form of the metric describing Gödel's universe.

7. Concluding Remarks

In this talk, I have tried to discuss the geometry of certain well-known mathematical models of the universe. The physical contents of these models as suggested by Einstein's field equations form an equally (or perhaps more) important aspect of the study of cosmology. But I thought, I should limit myself to the area with which I was more familiar.

However, this discussion would be

incomplete if I do not mention a very unphysical consequence of Gödel's rotating universe. Mathematically ~~It was~~^{is} found that this model allows closed time-like geodesics. In physical terms this implies that in this universe, observers exist whose world-lines are closed curves. But this is highly unphysical. The world-line of an observer is the 4-dimensional path of an observer in space and time. Now physically time always increases and in natural motion one cannot go backward in time and so the time-like tangent to the world-line can never reverse its direction. ~~Thus world-lines can never reverse its direction~~
Thus world-lines can never turn round in time to form a closed curve. In popular language we can say that if world-lines are closed curves and if my life follows one such world line, then at some age I shall reach the starting point of my world-line, i.e. I shall be able to witness my own birth!

That Gödel's rotating ~~model~~ model possesses this highly unphysical feature

was proved mathematically by Prof. S. Chandrasekhar in 1966. And that reminds me of an episode narrated by Prof. VVN to me in 1942. I shall end my talk with that episode.

Prof VVN and Prof. Chandra were colleagues at Cambridge. These were the triumphant days of Eddington's Internal Constitution of Stars. On return to India VVN thought that the equations of stellar structure were all based on Newton's law of gravitation. But with the more satisfactory description of gravitation by Einstein, one must recast the entire theory of stellar structure on the basis of Einstein's theory. In 1938 VVN carried out that exercise, wrote out his results in the form of a paper and sent it to Prof. Chandra for his opinion. Within 3 months Chandra wrote back congratulating VVN for his excellent mathematical work but added at the end, "At present there are no observations in Stellar Astronomy which cannot be explained by the simple Newtonian Theory, so why go in for the complicated Einstein's Theory?" So the matter ended there!

This was the episode narrated to me

by Prof VUN in 1942. In 1966, I read a ~~news~~ news-item in which it was reported that one fine evening Mrs. Chandrashekhar finds that the Professor returns from the university with a big pile of books. On inquiry from his wife regarding this pile of books Professor Chandrashekhar replied, "Now there have been observations in ~~ast~~ astronomy which cannot be understood on simple Newtonian Theory, but require the use of Einstein's theory. So I must now study GR or General Relativity.!"

Or One of the first works of Prof. Chandra in general relativity was ~~the one~~ about the unphysical property of Gödel's universe mentioned earlier. And as is the practice with him his studies in General Relativity ultimately led him to write the famous volume on "Theory of Black-Holes."

But this story has another moral. It is said that one should not be late in his scientific work and should not be

behind the times. But then there is a disadvantage also if one is too early, earlier than the times. And Professor Vishnu Vasudev Narlikar was too early, too much ahead of his times - more than 25 years ahead - in the application of General Relativity to stellar astronomy. !!



PROFESSOR V V NARLIKAR MEMORIAL LECTURE – 1994[®] GEOMETRY AND UNIVERSE

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(Received 08 September 1997)

1. Introduction

I am indeed grateful to INSA for inviting me to deliver the first Professor Vishnu Vasudev Narlikar Memorial Lecture. Professor Narlikar was my teacher. But our student-teacher relationship was peculiar. I was not a student of Banaras University where he worked. I was his "personal student". He initiated me to mathematical research. I used to meet him at his home twice a week and that established a link between Professor VVN's family and my little family. (My wife and my little daughter were with me at Banaras). For Narlikars, I am almost a senior family-member and in my family Professor VVN was Narlikar Dada (Grand Pa Narlikar). Thus the feeling of gratefulness towards INSA that I expressed in the beginning came from the bottom of my heart.

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We begin with the familiar equation $x^2+y^2 = R^2$. We know it represents a circle drawn on the x - y plane. We say it is a one dimensional curve of constant curvature drawn in a 2-dimensional flat plane. Well; we also know that $x^2+y^2+z^2 = R^2$ represents a sphere in space. We say, it is a 2-dimensional surface of constant curvature immersed in 3-dimensional flat space.

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We proceed further in the same manner and say that $x^2+y^2+z^2+w^2 = R^2$ represents a 3-dimensional space of constant curvature immersed in a 4-dimensional flat space. We are interested in these 3-spaces of constant curvature because of 2 simple assumptions of Einstein: (1) A gravitating object produces a curvature in the space surrounding that object and (2) if we are interested not in the gravitational field of isolated objects but in the average field pervading the whole universe, we must assume the matter in the universe to be smoothly and uniformly spread in the universe, producing a uniform curvature in space. Thus the 3-space in our universe taken as a whole must be of constant curvature.

Another feature of Einstein theory is that gravitation is not an "action at a distance" phenomena as in Newton theory, but it is a field phenomena. Thus the effect is transmitted from a given point to a neighbouring point. So the equations specifying the law of gravitation are differential equations and the corresponding geometry is differential geometry. Therefore, we shall try to translate the analytic geometry of 3-space of constant curvature to the corresponding differential geometry.

We again begin with the familiar case of a circle $x^2+y^2=R^2$. Its parametric equations are $x = R \cos\phi$, $y = R \sin\phi$. The corresponding differential form is $d\sigma^2 = dx^2 + dy^2$, or in terms of the parameter ϕ , $d\sigma^2 = R^2 d\phi^2$.

For a sphere $x^2+y^2+z^2 = R^2$, the parametric equations are

$$x = R \sin\theta \cos\phi,$$

$$y = R \sin\theta \sin\phi$$

$$z = R \cos\theta.$$

and

The corresponding differential form is $d\sigma^2 = dx^2 + dy^2 + dz^2$ or in terms of parameters θ and ϕ

$$d\sigma^2 = R^2 d\phi^2 + R^2 \sin^2\theta d\theta^2$$

We can generalize this procedure for 3-spaces of constant curvature (or in short, for 3-spheres). The analytic equation for a 3-sphere is

$$x^2+y^2+z^2+w^2 = R^2.$$

We now need 3 parameters X, θ, ϕ and write the parametric equations as

$$x = R \sin X \sin\theta \cos\phi,$$

$$y = R \sin X \sin\theta \sin\phi,$$

$$z = R \sin X \cos\theta$$

and $w = R \cos X$.

One can now verify that the differential form

$d\sigma^2 = dx^2 + dy^2 + dz^2 + dw^2$, when expressed in terms of the parameters X, θ, ϕ takes the form

$$d\sigma^2 = R^2 dX^2 + R^2 \sin^2 X (d\theta^2 + R^2 \sin^2 \theta d\phi^2) \quad \dots(2.1)$$

3. Einstein Universe

In an epoch making paper of 1917, Einstein suggested that the space-time measurements in the gravitational field pervading the universe would follow the 4-dimensional space-time geometry given by $ds^2 = c^2 dt^2 - d\sigma^2$, where $d\sigma^2$ is given by (2.1). Thus

$$ds^2 = c^2 dt^2 - R^2 [dX^2 + \sin^2 X (d\theta^2 + \sin^2 \theta d\phi^2)], \quad \dots(3.1)$$

Here c is the velocity of light, t is time and the 3-dimensional physical space is not the usual Euclidean flat space, but a 3-sphere of radius R . In (3.1), t is, of course, the time-like coordinate and the 3 space-like coordinates are given by the three angles X, θ, ϕ . We can replace one of these by a distance-coordinate r . In (3.1), set $R \sin X = r$, so that $R \cos X dX = dr$, (3.1) then takes the form

$$ds^2 = c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{R^2}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad \dots(3.2)$$

(3.2) is the standard form in which Einstein's 1917 model of the universe is expressed.

On looking at the metric (3.2) it will be seen that the constant R^2 can be taken as positive or negative. But R^2 being negative needs a reconsideration of our strategy leading to (2.1). If R^2 is negative R is to be replaced by iR . But $r = R \sin X$ implies that if R is replaced by iR , X needs to be replaced by iX so that $r = -R \sin X$ continues to be real. Thus in the parametric equations of the 3d-space, one has to replace $\sin X$ by $\sinh X$ and $\cos X$ by $\cosh X$. We do not go into details here but note that the differential metric of Einstein's Universe with 3-space of constant negative curvature will be

$$ds^2 = c^2 dt^2 - R^2 [dX^2 + \sinh^2 X (d\theta^2 + \sin^2 \theta d\phi^2)]. \quad \dots(3.3)$$

Again setting $R \sinh X = r$, we shall get

$$ds^2 = c^2 dt^2 - \frac{dr^2}{1 + \frac{r^2}{R^2}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad \dots(3.4)$$

4. Expanding Universe

Einstein's model given by (3.2) or (3.4) was the first mathematical model of the universe. The physical properties of this model universe were worked out using Einstein's field-equations in the background of geometry given by (3.2) or (3.4). It was found that this model admits presence of matter but does not admit any large scale motion. By about the same time (in 1918) another mathematical model of the universe was worked out by de Sitter. (We are going to give a simple mathematical derivation of this model later). This de Sitter's model does not admit presence of matter density but admits motion in the sense that a test particle introduced in the model will acquire motion. In 1924, Hubble observed large scale motion of extragalactic nebulae, and thus it became clear that in the physical universe, on a large scale, both matter and motion coexist. A mathematical model worked out by Friedman, way back in 1921 was found to incorporate this physical requirement. This model was later improved by several workers and is now known as the expanding universe model or Robertson-Walker model. The geometry of this model is again very easy to understand.

Let us turn back to the 3-space of constant positive curvature so that Einstein's Universe is given by

$$ds^2 = c^2 dt^2 - R^2 [dX^2 + \sin^2 X (d\theta^2 + \sin^2 \theta d\phi^2)], \quad \dots(3.1)$$

Here R is the radius of the spherical 3-space. Now R is a constant in relation to the geometrical variable X, θ, ϕ of the 3-space. But in shifting from geometry to physics, we introduced a new time parameter t , and R need not be constant in relation to t . If we take R as a function of t , nothing changes in section 2 above and we get (3.1) with $R = R(t)$. But now the radius of the 3-sphere (which is our physical space) becomes a function of time and if this function is an increasing function, we have an "expanding universe".

$$ds^2 = c^2 dt^2 - R^2(t) [dX^2 + \sin^2 X (d\theta^2 + \sin^2 \theta d\phi^2)].$$

In order to introduce a radial co-ordinate r , we rewrite $R^2(t) = R_0^2 (R(t)/R_0)^2$, R_0 being a constant, so that our metric takes the form

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{R_0^2} [R_0^2 dX^2 + R_0^2 \sin^2 X (d\theta^2 + \sin^2 \theta d\phi^2)].$$

Now put $R_0 \sin X = r$ and we get

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{R_0^2} \left[\frac{dr^2}{1 - \frac{r^2}{R_0^2}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad \dots(4.1)$$

Similarly, beginning with the metric (3.3) of Einstein universe with negative constant curvature, we get the metric of the corresponding expanding universe as

$$ds^2 = c^2 dt^2 - \frac{R^2(t)}{R_0^2} \left[\frac{dr^2}{1 + \frac{r^2}{R_0^2}} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad \dots(4.2)$$

In (4.1) and (4.2) one can replace r by a new coordinate $\bar{r} = r/R_0$ and then the two can be combined in the form

$$ds^2 = c^2 dt^2 - R^2(t) \left[\frac{d\bar{r}^2}{1 - k\bar{r}^2} + \bar{r}^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad \dots(4.3)$$

with

$k = +1$ for 3-space of uniform positive curvature

$k = -1$ for 3-space of uniform negative curvature

$k = 0$ for 3-space of uniform zero curvature.

Eq. (4.3) is the well-known Robertson-Walker metric of expanding universe.

5. de Sitter- Universe

Let us revert to spaces of constant curvature. We shall use the single word "sphere" to denote these spaces of constant curvature, irrespective of the dimensions. Thus we would say that

$x^2 + y^2 = R_0^2$ gives a 1-sphere immersed in 2-flat;

$x^2 + y^2 + z^2 = R_0^2$ gives a 2-sphere immersed in 3-flat;

$x^2 + y^2 + z^2 + w^2 = R_0^2$ gives a 3-sphere immersed in 4-flat.

Can we extend this and have a 4-sphere immersed in 5-flat? Well, this is what de-Sitter did and that is what we are going to do now!

To construct a geometrical model of the universe Einstein assumed that the physical 3-space of the universe is a sphere and then constructed its differential line-element $d\sigma^2$ (given as (2.1) above). He then introduced an orthogonal time t and wrote down the line-element of 4-dimensional space-time as $ds^2 = c^2 dt^2 - d\sigma^2$. In place of Einstein's assumption of only the 3-space being spherical, de Sitter assumed that the entire 4-dimensional space-time is spherical, i.e. he assumed that the entire space-time is a 4-sphere immersed in 5-flat. But for the 4-dimensional space-time which we use in physics, the metric ds^2 is not positive definite but indefinite. Therefore,

when we consider a 4-sphere, our interest will be in a 4-sphere with an indefinite metric. This will be reflection in the 5-flat in which it is immersed. We take

$$x^2 + y^2 + z^2 + w^2 - v^2 = R_0^2$$

as the 4-sphere immersed in 5-flat. Following the previous procedure, we write the parametric equations of this 4-sphere in terms of 4 angular parameters ψ, X, θ, ϕ as follows:

$$x = R_0 \cosh \psi \sin X \sin \theta \cos \phi,$$

$$y = R_0 \cosh \psi \sin X \sin \theta \sin \phi$$

$$z = R_0 \cosh \psi \sin X \cos \theta,$$

$$w = R_0 \cosh \psi \cos X$$

$$\text{and } v = R_0 \sinh \psi.$$

Setting $ds^2 = dv^2 - dx^2 - dy^2 - dz^2 - dw^2$, one gets

$$ds^2 = R_0^2 d\psi^2 - R_0^2 \sinh^2 \psi [dX^2 + \sin^2 X (d\theta^2 + \sin^2 \theta d\phi^2)].$$

Putting $R_0 d\psi = c dt$ and $R_0 \sin X = r$, we get the completely spherical 4-space-time given by the metric

$$ds^2 = c^2 dt^2 - \sinh^2 \frac{ct}{R_0} \left[\frac{dr^2}{1 - \frac{r^2}{R_0^2}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \quad \dots(5.1)$$

This is the metric of de Sitter model of the universe. Comparing with the metric (4.1) of the expanding universe metric. We see that it is of the form (4.1) when the undetermined function $R(t)/R_0$ becomes $\sinh ct/R_0$ i.e., when $R(t) = R_0 \sinh ct/R_0$. In that case it is easy to see that $R(t)$ is an increasing function and so de Sitter's model becomes a particular case of the expanding universe model.

The metric form (5.1) is time-dependant. But time-dependant metric form for describing the geometry of the universe was first developed by Friedman in 1921. It is, therefore, clear that in 1918 de Sitter did not describe his universe by (5.1). It is easy to derive de Sitter's original time independent (static) form of the metric. Of course, we begin with the 4-sphere $x^2 + y^2 + z^2 + w^2 - v^2 = R_0^2$, but write the parametric equations as follows:

$$x = R_0 \sin X \sin \theta \cos \phi,$$

$$y = R_0 \sin X \sin \theta \sin \phi,$$

$$z = R_0 \sin X \cos \theta,$$

$$w = R_0 \cos X \cosh \psi$$

$$\text{and } v = R_0 \cos X \sinh \psi.$$

Setting $ds^2 = dv^2 - dx^2 - dy^2 - dz^2 = dw^2$ one gets

$$ds^2 = R_0^2 \cos^2 X d\psi^2 - R_0^2 dX^2 - R_0^2 \sin^2 X (d\theta^2 + \sin^2 \theta d\phi^2).$$

Put $R_0 \sin X = \bar{r}$ and $R_0 d\psi = c\bar{t}$, we find

$$ds^2 = (1 - \bar{r}^2/R_0^2) c^2 d\bar{t}^2 - \frac{d\bar{r}^2}{1 - \frac{\bar{r}^2}{R_0^2}} - \bar{r}^2 (d\theta^2 + \sin^2 \theta d\phi^2) . \quad \dots(5.2)$$

Eq. (5.2) is the standard form of de Sitter metric (5.1) or it gives, the differential form for the same 4-sphere

$$x^2 + y^2 + z^2 + w^2 - v^2 = R_0^2.$$

So any one of them is a transform of the other.

6. Gödel's Rotating Universe

In 1949, K. Gödel gave a model of the universe exhibiting large-scale rotation of matter in the universe. It is possible to derive the geometry of this universe from that of Einstein's universe, by using a geometric trick. We first introduce a rotation in the coordinate axes in our 3-space and then develop a criterion that will convert this spurious geometrical rotation into a physical rotation of matter, and thus get the model of the rotating universe. We carry out this plan in 3 stages.

(A) Spheroidal Polar Coordinates

In the usual Euclidean geometry cartesian coordinates (x, y, z) and the spherical polar coordinates (r, θ, ϕ) are connected by relations

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta. \quad \dots(6.1)$$

In place of these spherical polar coordinates, we now introduce new coordinates (r, θ, ϕ) by the relations

$$x = r \sin \theta \cos \phi, \quad - a \sin \theta \sin \phi,$$

$$y = r \sin \theta \sin \phi, \quad + a \sin \theta \cos \phi$$

and

$$z = r \cos \theta,$$

so that we have $\frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1$. Thus the surface $r = \text{constant}$ are ellipsoids of the

revolution. So we call (r, θ, ϕ) defined by (6.2) as spheroidal polar coordinates. It can be verified that (6.2) can be obtained from (6.1) by imparting a rotation to the x - and y -axes through an angle $\tan^{-1}(a/r)$ round the z -axis.

We now take the Minkowski's space-time of special relatively $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$. Here we have chosen the units in such a way that the velocity of light becomes 1. Replacing x, y, z by spheroidal coordinates (r, θ, ϕ) of (6.2) we get

$$ds^2 = dt^2 - (dr - a \sin^2 \theta d\phi)^2 - (r^2 + a^2 \cos^2 \theta) d\theta^2 + \sin^2 \theta d\phi^2. \quad \dots(6.3)$$

Using the null coordinate $u = t - r$ in place of the space like coordinates we rewrite (6.3) as

$$ds^2 = 2(du + a \sin^2 \theta d\phi) dt - (du + a \sin^2 \theta d\phi)^2 - (r^2 + a^2 \cos^2 \theta) (d\theta^2 + \sin^2 \theta d\phi^2). \quad \dots(6.4)$$

with $r = t - u$.

(6.4) is the metric form for the usual Euclidean geometry of space-time in absence of gravitation. To describe the uniform gravitational field pervading the universe we take the corresponding Riemannian geometry of the space-time as given by

$$ds^2 = 2(du + g \sin \theta d\phi) d\zeta - L (du + g \sin \theta d\phi)^2 - M^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad \dots(6.5)$$

$$g = g(\theta), \quad L = L(u, \theta, \zeta), \quad M = M(u, \theta, \zeta).$$

Here, in place of the usual t , we have used u to denote the time coordinate.

(B) The Geometric Trick

The metric (6.5) is an indefinite metric with signature $(-, -, -, +)$. The coordinates used are u, θ, ϕ, ζ . We take them as x^1, x^2, x^3, x^4 respectively. u is a null coordinate θ, ϕ are space-like, but ζ is time-like or space-like according as the coefficient L is positive or negative.

Consider now a time-like unit vector v_i and an orthogonal space-like vector V_i .

$$v_i v^i = 1, \quad V_i V^i = -1, \quad v_i V^i = 0.$$

It is easy to find the components of these vectors in terms of the metric (6.5). It will be found that

$$v_i = [(1/2) (1-L), 0, (1/2) (1-L) g \sin \theta, 1]$$

$$V_i = [(1/2) (1+L), 0, (1/2) (1+L) g \sin \theta, -1]$$

We now choose $L = +1$ or $L = -1$

When $L = +1$, v_i becomes $(0,0,0,1)$ and V_i becomes $(1,0, g \sin \theta, -1)$. It can now be verified that $v_{i;k} - v_{k;i} = 0$ but $V_{i;k} - V_{k;i} \neq 0$ if $g \sin \theta$ is not constant. So vector v_k has no twist, while V_k is a rotating vector-field. However, when $L = -1$, v_k becomes $(1,0, g \sin \theta, 1)$ and V_k becomes $(0,0,0,-1)$ so that now v_k has rotation while V_k becomes rotation-free. Thus by changing the parameter L from $+1$ to -1 , we have shifted the rotation from space-like V_k to time-like v_k .

Now if v_k and V_k happen to be eigenvectors of the Einstein's matter tensor G_{ik} , then the shifting of the twist from V_k to v_k will imply a shift of twist from a space-like eigenvector to a time-like eigenvector. But a twisting time-like eigenvector implies a rotating matter distribution. In this way, we are able to relate a non-rotating material distribution to a rotating one. This is the geometrical trick of shifting the twist.

(C) *Einstein-Gödel Metric*

In the metric (6.5) we have introduced 3 unknown functions L, M and g . Our geometrical method of shifting the twist implies that L is either $+1$ or -1 . We have to use physical properties of Einstein's universe to find explicitly the other two functions. We do not go in to details of their derivation but write down the final solution in the form

$$L = +1 \text{ or } -1,$$

$$g \sin \theta = 1+L - [4/(1+3L)] e^{-(1+3L)y/2R}$$

$$\text{and } M^2 \sin^2 \theta = g \sin \theta e^{-(1+3L)y/2R},$$

where y is defined by $g d\theta = dy$.

It will be easier to rewrite metric (6.5) using y as a coordinate in place of θ . We also replace u by the coordinate r given by

$$r = u + (1+L) \phi - L \zeta$$

(6.5) will then change to

$$ds^2 = L d\zeta^2 - L dr^2 + 2LY d\zeta dr d\phi - Y d\phi^2 [1+L - (1-L) Y/4] - (1/4) (1+3L) Y dy^2 / (1+L-Y) \dots(6.6)$$

$$\text{with } Y = \frac{4}{1+3L} e^{-(1+3L)y/2R}$$

If in (6.6) we put $L = 1$, we find

$$ds^2 = d\zeta^2 - dr^2 + 2e^{2y/R} drd\phi - 2e^{2y/R}d\phi^2 - e^{-2y/R}dy^2/[2-e^{2y/R}]. \quad \dots(6.7)$$

It can be verified that (6.7) is a transform of the standard metric of the Einstein Universe. The coordinate ζ being the cosmic time t .

On putting $L = -1$ in (6.6), one finds

$$ds^2 = dr^2 - d\zeta^2 + 4e^{y/R} drd\phi + 2e^{y/R}d\phi^2 - (1/2) dy^2. \quad \dots (6.8)$$

This is Gödel's Universe metric the coordinate r being the time coordinate.

In (6.7), the space-like vector dr had a spurious (coordinate-generated) rotation. In (6.8) the vector $d\tau$ becomes time-like and so the rotation which was geometrical in (6.7) has now become physical in (6.8). Thus (6.8) gives the geometry of a rotating universe. It is not customary to use r for a time-like coordinate. So we replace r by τ in (6.8) and write

$$ds^2 = d\tau^2 - d\zeta^2 + 4e^{y/R} d\tau d\phi + 2e^{y/R}d\phi^2 - (1/2) dy^2. \quad \dots (6.9)$$

This the standard form of the metric describing Gödel's universe.

7. Concluding Remarks

In this talk, I have tried to discuss the geometry of certain well-known Mathematical models of the universe. The physical contents of these models as suggested by Einstein's field equations form an equally (or perhaps more) important aspect of the study of cosmology. But I thought, I should limit myself to the area with which I was more familiar.

However, this discussion would be incomplete if I do not mention a very unphysical consequence of Gödel's rotating universe. It is found that this model allows closed time-like geodesics. In physical terms, this implies that in this universe, observers exist whose world lines are closed curves. But this is highly unphysical. The worldline of an observer is the 4-dimensional path of an observer in space and time. Now physically time always increases and in natural motion one can not go backwards in time and so the time-like tangent to the world-line can never reverse its direction. Thus world-lines can never turn round in time to form a closed curve. In popular language we can say that if world-lines are closed curves and if my life follows one such world-line then at some age I shall reach the starting point of my world-line, i.e., I shall be able to witness my own birth!

That Gödel's rotating model possesses this highly unphysical feature was proved mathematically by Professor S Chandrashekhar in 1966. That reminds me of an episode narrated by Professor VVN to me in 1942. I shall end my talk with that episode.

Professor VVN and Professor Chandra were colleagues at Cambridge. Those were the triumphant days of Edington's Internal Constitution of stars. On return to India VVN thought that the equations of stellar structure were all based on Newton's law of gravitation. But with the more satisfactory description of gravitation by Einstein, one must recast the entire theory of stellar structure on the basis of Einstein's theory. In 1938, VVN carried out that exercise, wrote out his results in the form of a paper and sent it to Professor Chandra for his opinion. Within three months Chandra wrote back congratulating VVN for his excellent mathematical work but added at the end "At present there are no observations in Stellar Astronomy which can not be explained by the simple Newtonian theory. So why go in for the complicated Einstein's theory?" So the matter ended there.

This was the episode narrated to me by Professor VVN in 1942. In 1966, I read a news-item in which it was reported that one fine evening Mrs. Chandrashekhar finds that the Professor returns from the university with a big pile of books. On inquiry from his wife regarding this pile of books Professor Chandrashekhar replied "Now there have been observations in Astronomy which cannot be understood on simple Newtonian theory, but require the use of Einstein's theory. So I must now study General Relativity."

One of the first works of Professor Chandra in General Relativity was about the unphysical property of Gödel's universe mentioned earlier. As is the practice with him his studies on General Relativity ultimately led him to write the famous volume on "Theory of Black-Holes."

But this story has another moral. It is said that one should not be late in his scientific work and should not be behind the times. But then there is a disadvantages also if one is too early, earlier than the times Professor Vishnu Vasudev Narlikar was too early, too much ahead of his times – more than 25 years ahead – in the application of General Relativity to stellar astronomy.