of the 8 different combinations of three digits having sum equal to 15 viz

(1, 5, 9), (1, 6, 8), (2, 4, 9), (2, 5, 8), (2, 6, 7), (3, 4, 8), (3, 5, 7), (4, 5, 6)

we have to choose three triplets containing all the numbers 1 to 9. there are just two viz:

[(1, 5, 9), (2, 6, 7), (3, 4, 8)]; [(3, 5, 7), (2, 4, 9), (1, 6, 8)];

consider the second [ ] write them as 3 2 1 5 4 6 7 9 8 (a)

or as 4 7 1 2 5 8 9 3 6 (b) (4, 2, 9), (7, 5, 3), (1, 8, 6)

(5 3 7)(2 4 9)(6 1 8) or as 5 2 6 3 4 1 7 9 8 (c)

i.e. there are 6 × 6 × 6 choices of writing these three triplets & for each such choice we can form a series of 9 numbers (1 to 9) of which the first three digits are the first terms of each triplet, the next three are the second terms of each triplet, and the last three are the third terms of this triplet as for eg. (a), (b), (c) above. Moreover there are ~~six~~ 6 ways in which these three triplets can be taken & for each one we have 6 × 6 × 6 ways of choices of the series of 9 numbers. of these 216 choices we can take any one in the middle row of A and any other (or the same) (expressed in corresp. odd nos) as middle row of B.

Then steps, (6, 3) or (3, 6) and (3, 6) or (6, 3) carried out in A & B & also in C & D gives

From the second [ ], (2,4,9), (3,5,7), (1,6,8) → 231456978 symmetric

(2,4,9), (3,5,7), (6,1,8) → 236451978 non-sym.

8	1	3	7	9	8	2	5	4
7	9	8	2	5	4	6	1	3
2	5	4	6	1	3	7	9	8
6	1	3	7	9	8	2	5	4
7	9	8	2	5	4	6	1	3
2	5	4	6	1	3	7	9	8
6	1	3	7	9	8	2	5	4
7	9	8	2	5	4	6	1	3
2	5	4	6	1	3	7	9	8

~~[illegible]~~ ~~[illegible]~~, A_6 (n) ✓

(7,2,6), (9,5,1), (8,4,3)

9	36	18	45	0	63	54	72	27
54	72	27	9	36	18	45	0	63
45	0	63	54	72	27	9	36	18
9	36	18	45	0	63	54	72	27
54	72	27	9	36	18	45	0	63
45	0	63	54	72	27	9	36	18
9	36	18	45	0	63	54	72	27
54	72	27	9	36	18	45	0	63
45	0	63	54	72	27	9	36	18

B_6 (n ✓)

(7,2,6), (9,5,1), (4,3,8)

C_6 (n) (no need to add)

To put it more simply

(1) Form series of numbers 1 to 9 such that the sum of the 1st, 4th, 7th terms, of the 2nd, 5th, 8th terms, and 3rd, 6th, 9th terms are each equal to 15.

Any two such series taken in the middle rows of A and B (expressed in root numbers) and the steps (6,3), (3,6) or (3,6), (6,3) are used for A and B, both these will be nasik & hence the sum C = A + B is also nasik.

(2) If, in addition, the series be symmetric, corresponding A & B will be both associated and nasik.

—

Is it possible to get ~~non~~symmetric series not satisfying condns in (1)?

Take for eg: 439258176 which is symmetric but the triplets (4,2,1), (3,5,7), (9,8,6) do not all add to 15.

(2) nasik squares. Of these 216 choices, those that are symmetric about the central

one viz. 5 ~~can be~~ any two (or the same) can be chosen for A & B & add to give associate nasik

square. A similar procedure with the first set of triplets [ ].

What we done on p. 69 of Bk. 17 is to take the series

27, 54, 0, 9, 36, 63, 72, 18, 45 corresponding to 4 7 1 2 5 8 9 3 6 i.e. derived from the

triplet series [(4, 2, 9), (7, 5, 3), (1, 8, 6)] & putting it in middle row of B &

~~putting~~ operating with the step (3, 6). Next in A we have taken the same series

4 7 1 2 5 8 9 3 6 & put it in middle Col of A & operated with step (3, 6) again

More systematically we could have put it in the first row of A of and operated with step (6, 3).

In this case the series 4 7 1 2 5 8 9 3 6 is symmetric i.e. nos equidistant from the

central number 5 add up to 10.

Let us illustrate by some choices from the first [ ] wrote at as

(2, 6, 7), (1, 5, 9), (3, 4, 8) leading to 2 1 3 6 5 4 7 9 8 (a symmetric series

or (7, 2, 6), (9, 5, 1), (8, 4, 3) → ~~726~~ 7 9 8 2 5 4 6 1 3 (not symmetric although 5 is the central)

(7, 2, 6), (9, 5, 1), (4 8 3) → 7 9 4 2 5 8 6 1 3 → a symmetric series.

(4 3 8) → 7 9 4 2 5 3 6 1 8 → not sym

1+9
8+1
9
17 - 879
18 - 8+98
44 - 8736

This splitting of Moreau's 9×9 associated square, and

the fact that $A_5' + B_5' = \underline{n}$ on p. 70 of Bk. 17 brings up other possibilities i.e.

that we need not ~~consider only~~ restrict choice only to (6,3) & (3,6) for the 9×9 squares.

Let us call the two types of series (i) nasik series & (ii) associated (symmetric) series.

(i) Take a nasik series & try if (8,1), (1,8) leads to a nasik square. Try.

3	5	6	4	9	7	8	1	2
5	6	4	9	7	8	1	2	3
6	4	9	7	8	1	2	3	5
4	9	7	8	1	2	3	5	6
9	7	8	1	2	3	5	6	4
7	8	1	2	3	5	6	4	9
8	1	2	3	5	6	4	9	7
1	2	3	5	6	4	9	7	8
2	3	5	6	4	9	7	8	1

no – Sq not magic in all after diagonals & also not nasik " right ones.
(8,1). ✗

3	5	6	4	9	7	8	1	2
6	4	9	7	8	1	2	3	5
9	7	8	1	2	3	5	6	4
8	1	2	3	5	6	4	9	7
2	3	5	6	4	9	7	8	1
5	6	4	9	7	8	1	2	3
4	9	7	8	1	2	3	5	6
7	8	1	2	3	5	6	4	9
1	2	3	5	6	4	9	7	8

nasik series (7, 2) ✓ is $\underline{n}$.

~~2 [illegible] 3 6 5 4 7 9 8~~

7	9	4	2	1	3	6	5	8
1	3	6	5	8	7	9	4	2
8	7	9	4	2	1	3	6	5
2	1	3	6	5	8	7	9	4
5	8	7	9	4	2	1	3	6
4	2	1	3	6	5	8	7	9
6	5	8	7	9	4	2	1	3
9	4	2	1	3	6	5	8	7
3	6	5	8	7	9	4	2	1

nasik series (5, 4) ✓ ($\underline{n}$)

6	5	8	2	1	3	7	9	4
3	7	9	4	6	5	8	2	1
5	8	2	1	3	7	9	4	6
7	9	4	6	5	8	2	1	3
8	2	1	3	7	9	4	6	5
9	4	6	5	8	2	1	3	7
2	1	3	7	9	4	6	5	8
4	6	5	8	2	1	3	7	9
1	3	7	9	4	6	5	8	2

nasik series (4,5) ✓ ($\underline{n}$)

1	7	6	4	3	9	2	5	8
4	3	9	2	5	8	1	7	6
2	5	8	1	7	6	4	3	9
1	7	6	4	3	9	2	5	8
4	3	9	2	5	8	1	7	6
2	5	8	1	7	6	4	3	9
1	7	6	4	3	9	2	5	8
4	3	9	2	5	8	1	7	6
2	5	8	1	7	6	4	3	9
21	45	69						

not magic in cols.

9	36	63	0	54	45	27	18	72
27	18	72	9	36	63	0	54	45
0	54	45	27	18	72	9	36	63
9	36	63	0	54	45	27	18	72
27	18	72	9	36	63	0	54	45
0	54	45	27	18	72	9	36	63
9	36	63	0	54	45	27	18	72
27	18	72	9	36	63	0	54	45
0	54	45	27	18	72	9	36	63
108	324							

not magic in cols.

X

4 2 9 7 5 3 1 8 6 X This method does not give a square a but n; but the ordinary D. D. M does & its variants do. Ditto Fig. 24. Andrews, p. 13 & split

8	4	9	5	1	6	2	7	3
9	5	1	6	2	7	3	8	4
1	6	2	7	3	8	4	9	5
2	7	3	8	4	9	5	1	6
3	8	4	9	5	1	6	2	7
4	9	5	1	6	2	7	3	8
5	1	6	2	7	3	8	4	9
6	2	7	3	8	4	9	5	1
7	3	8	4	9	5	1	6	2

(A) step (7,2)

72	54	36	18	0	63	45	27	9
0	63	45	27	9	72	54	36	18
9	72	54	36	18	0	63	45	27
18	0	63	45	27	9	72	54	36
27	9	72	54	36	18	0	63	45
36	18	0	63	45	27	9	72	54
45	27	9	72	54	36	18	0	63
54	36	18	0	63	45	27	9	72
63	45	27	9	72	54	36	18	0

(B) step (5,4)

middle row: 384951627 is sym.

$\underset{18}{(3,9,6)}$, $\underset{15}{(8,5,2)}$, $\underset{12}{(4,1,7)}$ not nasik

27, 9, 72, 54, 36, 18, 0, 63, 45 not sym
4 2 9 7 5 3 1 8 6

$\underset{81}{(27,54,0)}$, $\underset{108}{(9,36,63)}$, $\underset{135}{(72,18,45)}$ not n

~~Unless the series for A & B are identical.~~

~~This restriction is not necessary; we can also apply (2,3) to each of A & B and also (3,2) to each of A & B.~~

Similar remarks apply to $n = 7$ also.

4 (ii) Re associated squares of 9×9. The splitting of Andrews square suggests that we need only take an associated series (not necessarily nasik) and apply to A & B, any two of (7,2), (6,3), - - (2,7) ~~including repetitions~~.

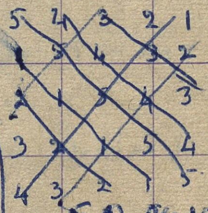
For $n = 5$ we have seen that for a-squares (4,1) & (1,4) are also admissible, ~~in fact, this is Narayana's example~~. Let us see if this holds for $n = 9$ also.

3	4	8	1	5	9	2	6	7
7	3	4	8	1	5	9	2	6
6	7	3	4	8	1	5	9	2
2	6	7	3	4	8	1	5	9
9	2	6	7	3	4	8	1	5
5	9	2	6	7	3	4	8	1
1	5	9	2	6	7	3	4	8
8	1	5	9	2	6	7	3	4
4	8	1	5	9	2	6	7	3

(1, 8)

5	4	3	2	1
1	5	4	3	2
2	1	5	4	3
3	2	1	5	4
4	3	2	1	5

3	2	1	5	4
4	3	2	1	5
5	4	3	2	1
1	5	4	3	2
2	1	5	4	3



5	0	20	15	10
0	20	15	10	5
20	15	10	5	0
15	10	5	0	20
10	5	0	20	15

If we take the top row an associated series, all the other rows are not associated series unlike as in the case of Nasik series. This is true even if we take middle row an associated series (true even for $n = 5$ with ~~(4,1) & (1,4)~~ any object).

5	4	3	2	1
2	1	5	4	3

So let us take an associated series in middle row (for A) ($n = 9$). & apply (8,1), ~~and another associated row series in middle row for B~~ and apply (8,1) and add.

(6)

[Hereafter I shall work in rough without drawing the actual sq lines to make up the square]

2	1	3	6	5	4	7	9	8		2	1	3	6	5	4	7	9	8
9	8	2	1	3	6	5	4	7		8	2	1	3	6	5	4	7	9
4	7	9	8	2	1	3	6	5		9	8	2	1	3	6	5	4	7
6	5	4	7	9	8	2	1	3		7	9	8	2	1	3	6	5	4
1	3	6	5	4	7	9	8	2		4	7	9	8	2	1	3	6	5
8	2	1	3	6	5	4	7	9		5	4	7	9	8	2	1	3	6
7	9	8	2	1	3	6	5	4		6	5	4	7	9	8	2	1	3
5	4	7	9	8	2	1	3	6		3	6	5	4	7	9	8	2	1
3	6	5	4	7	9	8	2	1		1	3	6	5	4	7	9	8	2

nasik series (2,7) ✓ ($\underline{n}$) | nasik series (1,8). ✗ not $\underline{n}$

So there appears to be nothing special about choosing only (6,3), (3,6) for 9 × 9 squares to get nasiks: only we have to choose nasik series (primary nos and root nos) and apply to A and B <u>any</u> two of (7,2), (6,3), (5,4), (4,5), (3,6), (2,7) one to ~~(including same breaks provided the nasik series for A & B are not the same)~~, each (ie excepting (8,1) and (1,8). In fact this is just what we (no repetition) had for n = 5. in which case we had only 4 possible steps ie (4,1), (3,2), (2,3), (1,4) & discarding (4,1) & (1,4) we had only two choices (3,2), (2,3), ~~but earlier in Book 17 I restricted myself to using (2,3) to A & (3,2) to B or vice versa, but~~

centres, we get magic squares for (4,1) & (1,4) operating on A & B & adding A & B.

But are they nasik just as in case of (2,3) & (3,2)?

3	2	5	4	1
1	3	2	5	4
4	1	3	2	5
5	4	1	3	2
2	5	4	1	3

A (1, 4)

20	0	15	5	10
0	15	5	10	20
15	5	10	20	0
5	10	20	0	15
10	20	0	15	5

B (4,1)

23	2	20	9	11
1	18	7	15	24
19	6	13	22	5
10	14	21	3	17
12	25	4	16	8

C (magic all right) but not nasik.

If we take A (2,3) ~~(1,4)~~ & B (1,4) ~~(3,2)~~ in the above ~~we get a semi-nasik square~~.

5	4	1	3	2
3	2	5	4	1
4	1	3	2	5
2	5	4	1	3
1	3	2	5	4

10	20	0	15	5
5	10	20	0	15
15	5	10	20	0
0	15	5	10	20
20	0	15	5	10

15	24	1	18	7
8	12	25	4	16
19	6	13	22	5
2	20	9	11	23
21	3	17	10	14

magic but and semi-nasik.

Suppose we take middle rows arbitrary (with 3 & 10 not in centres)

3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4
2	4	1	3	5

(2,3)

2	1	3	4	5
5	2	1	3	4
4	5	2	1	3
3	4	5	2	1
1	3	4	5	2

5	0	10	15	20
20	5	0	10	15
15	20	5	0	10
10	15	20	5	0
0	10	15	20	5

(1,4)

8	5	12	19	21
24	6	3	15	17
20	22	9	1	13
11	18	25	7	4
2	14	16	23	10

X

only semi-magic

1	3	5	2	4
3	5	2	4	1
5	2	4	1	3
2	4	1	3	5
4	1	3	5	2

(1,4)

6	3	15	17	24
23	10	2	14	16
20	22	9	1	13
12	19	21	8	5
4	11	18	25	7

not magic but magic only in rows & cols.

(8)

5	9	2	6	7	3	4	8	1
1	5	9	2	6	7	3	4	8
8	1	5	9	2	6	7	3	4
4	8	1	5	9	2	6	7	3
3	4	8	1	5	9	2	6	7
7	3	4	8	1	5	9	2	6
6	7	3	4	8	1	5	9	2
2	6	7	3	4	8	1	5	9
9	2	6	7	3	4	8	1	5

(1,8) <u>a</u>

1	2	7	6	4	3	8	9	5
2	7	6	4	3	8	9	5	1
7	6	4	3	8	9	5	1	2
6	4	3	8	9	5	1	2	7
4	3	8	9	5	1	2	7	6
3	8	9	5	1	2	7	6	4
8	9	5	1	2	7	6	4	3
9	5	1	2	7	6	4	3	8
5	1	2	7	6	4	3	8	9

(8,1) <u>a</u>

Expressing the 2nd rows in both rows we again get an <u>a</u>. Adding we get an <u>a</u>. This is just what happens for $n = 5$. In this case, Narayana's example shows that if we have <u>3</u> in the centre of the middle row of A, and 10 in the centre of the middle row of B, we get a magic square for (4,1) & (1,4) applied to A & B (If <u>3</u> & <u>10</u> be not in the centre, we do not get magic squares). Further with <u>3</u> & <u>10</u> in centres the middle rows are symmetric, we have an <u>a</u>. This is just what has happened above for $n = 9$, where <u>5</u> and <u>36</u> are in middle rows of A & B & the rows are symmetric.

Whatever be the middle rows of A, B, (2,3) & (3,2) give nasiks. Suppose we take middle rows of A & B arbitrary except for having 3 & 10 in the

$$\begin{array}{ccccc} 3 & 1 & 2 & 4 & 5 \\ 5 & 3 & 1 & 2 & 4 \\ 4 & 5 & 3 & 1 & 2 \\ 2 & 4 & 5 & 3 & 1 \\ 1 & 2 & 4 & 5 & 3 \end{array} + \begin{array}{ccccc} 20 & 0 & 15 & 5 & 10 \\ 0 & 15 & 5 & 10 & 20 \\ 15 & 5 & 10 & 20 & 0 \\ 5 & 10 & 20 & 0 & 15 \\ 10 & 20 & 0 & 15 & 5 \end{array} = \begin{array}{ccccc} 23 & 1 & 17 & 9 & 15 \\ 5 & 18 & 6 & 12 & 24 \\ 19 & 10 & 13 & 21 & 2 \\ 7 & 14 & 25 & 3 & 16 \\ 11 & 22 & 4 & 20 & 8 \end{array}$$

magic neither a nor n

2(b)

$$\underset{(1,4)}{\begin{array}{ccccc} 3 & 1 & 5 & 4 & 2 \\ 2 & 3 & 1 & 5 & 4 \\ 4 & 2 & 3 & 1 & 5 \\ 5 & 4 & 2 & 3 & 1 \\ 1 & 5 & 4 & 2 & 3 \end{array}} + \underset{(2,3)}{\begin{array}{ccccc} 20 & 10 & 0 & 15 & 5 \\ 15 & 5 & 20 & 10 & 0 \\ 10 & 0 & 15 & 5 & 20 \\ 5 & 20 & 10 & 0 & 15 \\ 0 & 15 & 5 & 20 & 10 \end{array}} = \begin{array}{ccccc} 23 & 11 & 5 & 19 & 7 \\ 17 & 8 & 21 & 15 & 4 \\ 14 & 2 & 18 & 6 & 25 \\ 10 & 24 & 12 & 3 & 16 \\ 1 & 20 & 9 & 22 & 13 \end{array}$$

magic & not nasik

$$\underset{(2,3)}{\begin{array}{ccccc} 5 & 1 & 2 & 3 & 4 \\ 3 & 1 & 5 & 4 & 2 \\ 4 & 2 & 3 & 1 & 5 \\ 1 & 5 & 4 & 2 & 3 \\ 2 & 3 & 1 & 5 & 4 \end{array}} \quad \underset{(1,4)}{\begin{array}{ccccc} 15 & 5 & 20 & 10 & 0 \\ 0 & 15 & 5 & 20 & 10 \\ 10 & 0 & 15 & 5 & 20 \\ 20 & 10 & 0 & 15 & 5 \\ 5 & 20 & 10 & 0 & 15 \end{array}} = \begin{array}{ccccc} \cancel{17} & \cancel{8} & \cancel{21} & \cancel{15} & 4 \\ 20 & 9 & 22 & 13 & 1 \\ 3 & 16 & 10 & 24 & 12 \\ 14 & 2 & 18 & 6 & 25 \\ 21 & 15 & 4 & 17 & 8 \\ 7 & 23 & 11 & 5 & 19 \end{array}$$

Semi-magic

4(b). (1,4), (2,3) — magic, ~~neither~~ a not a and semi-nasik

(c) (2,3), (3,2) — magic, not a but n.

(5) Middle rows both symmetric

(a) (1,4), (4,1) give magic square a but not n

(b) (1,4), (3,3) " a and semi-nasik

(c) (2,3), (3,2) " both a and n

covers altogether 14 cases completely are quite complicated.

Essential cases are (i) middle rows with 3 or 10 in centre otherwise arbitrary

(a) (1,4), (4,1) → magic square neither a nor n – Narayana's example

(b) with 3 & 10 not in centre no magic square.

~~(10) These results for $n=5$ can be generalised to $n \geq 9$ as follows:~~

~~(1) middle rows arbitrary (1,4) & (4,1) → no magic square – magic only in rows & cols. & not in the 2 diagonals~~

~~(2) middle rows arbitrary (1,4) or (4,1) and any of the other steps → no magic square – not magic in just one diagonal~~

(3) middle rows arbitrary in one & arbitrary in the other except for 3 or 10 in the centre

Results for $n = 5$.

(1) Middle rows of A & B arbitrary (without 3 & 10 in centres)

(a) (1,4) & (4,1) or (4,1) & (1,4) do not give magic squares (only rows & cols. magic) i.e. semi-magic

(b) (1,4) or (4,1) with one of (2,3) or (3,2) in A or B again leads to semi magic squares not magic in one diagonal

(c) (2,3) or (3,2) in A & B (not same for both). Leads to nasik magic squares.

(2) Middle rows one with central no. 3 or 10 & the other without

(a) (1,4) & (4,1) lead to semi-magic square (one diagonal not magic)

$$\underset{(1,4)}{\begin{vmatrix} 3 & 4 & 2 & 1 & 5 \\ 5 & 3 & 4 & 2 & 1 \\ 1 & 5 & 3 & 4 & 2 \\ 2 & 1 & 5 & 3 & 4 \\ 4 & 2 & 1 & 5 & 3 \end{vmatrix}} + \underset{(4,1)}{\begin{vmatrix} 20 & 0 & 5 & 10 & 15 \\ 0 & 5 & 10 & 15 & 20 \\ 5 & 10 & 15 & 20 & 0 \\ 10 & 15 & 20 & 0 & 5 \\ 15 & 20 & 0 & 5 & 10 \end{vmatrix}} = \begin{vmatrix} 23 & 4 & 7 & 11 & 20 \\ 5 & 8 & 14 & 17 & 21 \\ 6 & 15 & 18 & 24 & 2 \\ 12 & 16 & 25 & 3 & 9 \\ 19 & 22 & 1 & 10 & 13 \end{vmatrix}$$

(b) (1,4), (2,3) lead to magic or semi-magic square acc. as (1,4) is applied to square with centre 3 or 10 or otherwise.

(3) Middle rows both having 3 or 10 in centres

(a) (1,4) & (4,1) give magic square which is not nasik

(b) $\begin{matrix}(1,4)\\(4,1)\end{matrix}$ & $\begin{matrix}(2,3)\\(3,2)\end{matrix}$ " which is semi-nasik

(c) (2,3) & (3,2) give nasik squares [as in 1(c)]

(4) Middle rows both having 3 or 10 in centres & one of the series symmetric

(a) (1,4) & (4,1)

(2) Intermediate steps lead to n magic squares whatever the middle rows

central or otherwise, and if ~~they~~ the middle rows be symmetric we get magic

(3) squares both a and n.

(3) End step applied to a central series & an intermediate step to a central or non-central series

leads to a magic square neither a nor n.

These conclusions can be applied to all cases where n = prime, 7, 11, — only pay any two intermediate steps.

For composite case take only $n = 9$:

5	8	2	1	6	7	4	3	9
9	5	8	2	1	6	7	4	3
3	9	5	8	2	1	6	7	4
4	3	9	5	8	2	1	6	7
7	4	3	9	5	8	2	1	6
6	7	4	3	9	5	8	2	1
1	6	7	4	3	9	5	8	2
2	1	6	7	4	3	9	5	8
8	2	1	6	7	4	3	9	5

(1, 8) central series

+

72	45	27	9	0	63	54	18	36
45	27	9	0	63	54	18	36	72
27	9	0	63	54	18	36	72	45
9	0	63	54	18	36	72	45	27
0	63	54	18	36	72	45	27	9
63	54	18	36	72	45	27	9	0
54	18	36	72	45	27	9	0	63
18	36	72	45	27	9	0	63	54
36	72	45	27	9	0	63	54	18

(8, 1) central series

✓ magic

5

77	53	29	10	6	70	58	21	45 ✓
54	32	17	2	64	60	25	40	75
30	18	5	71	56	19	42	79	49
13	3	72	59	26	38	73	51	34

✓ is magic

no need to get this merely get sums of rows & cols & diagonals of A & B and add

5	4	2	3	9	1	7	8	6
1	7	8	6	5	4	2	3	9
4	2	3	9	1	7	8	6	5

72	45	9	18	54	63	27	0	36
~~36~~	~~72~~	~~45~~	~~9~~	~~18~~	~~54~~	~~63~~	~~27~~	~~0~~
54	63	27	0	36	72	45	9	18
36	72	45	9	18	54	63	27	0
~~0~~	~~36~~	~~72~~	~~45~~	~~9~~	~~18~~	~~54~~	~~63~~	~~27~~

consecutive digits

(12)

(2) middle row one with centre 3 or 10 & other arbitrary – (1,4), (2,3) lead to magic or non-magic square according as (1,4) is applied to square with centres 3 or 10 or otherwise

(3) middle rows arbitrary. (2,3) & (3,2) lead to n and not a

(4) (a) middle rows symmetric (1,4), (4,1) lead to a but not n
(b) " (2,3), (3,2) lead to a & n

Let us simplify still further calling series with 3 or 10 in centre as central series

(1) ~~central series~~ one or Both Middle rows not ~~&~~ central series
(a) (1,4), (4,1) – no magic square.
(b) (2,3), (3,2) – magic squares n.

~~(c)~~ one central & other non-central – If (1,4) be applied to the central
(c) & (2,3) to the non-central series we get a magic square not n.

(2) Both middle rows central symmetric
(a) (1,4), (4,1) lead to magic square a not n
(b) (2,3), (3,2) " Both a and n.

~~in just five cases~~. Still further simplification needed. Use terms

end ~~extreme~~ steps & intermediate steps. for (1,4), (4,1) and (2,3), (3,2).

~~(1). Both middle rows non-central~~
~~(a) end steps lead to semi-magic squares~~
~~(b) intermediate steps lead to ~~ possible magic square~~
~~(c) one end & one intermediate~~
~~(2) one middle row central & the other non-central – magic square or semi-magic according as the end step to the former or not.~~
~~(3) Both rows central~~
~~(a) end steps leads to magic square not n~~
~~(b)~~ ~~symm~~ only. (or squares neither a nor n)

(i) End steps lead to magic square if both rows are central; & if further the ~~centre~~ rows are also symmetric, the magic square is a ~~not n~~ ~~Squares~~ These squares are not n

by two ~~steps~~ diff steps x.

9	0	63	54	36	18	72	45	27
45	27	9	0	63	54	36	18	72
18	72	45	27	9	0	63	54	36
54	36	18	72	45	27	9	0	63
0	63	54	36	18	72	45	27	9
27	9	0	63	54	36	18	72	45
72	45	27	9	0	63	54	36	18
36	18	72	45	27	9	0	63	54
63	54	36	18	72	45	27	9	0

(7,2)

63	54	36	18	72	45	27	9	0
36	18	72	45	27	9	0	63	54
72	45	27	9	0	63	54	36	18
27	9	0	63	54	36	18	72	45
0	63	54	36	18	72	45	27	9
54	36	18	72	45	27	9	0	63
18	72	45	27	9	0	63	54	36
45	27	9	0	63	54	36	18	72
9	0	63	54	36	18	72	45	27

(2,7)

27	9	0	63	54	36	18	72	45
36	18	72	45	27	9	0	63	54
9	0	63	54	36	18	72	45	27
18	72	45	27	9	0	63	54	36
0	63	54	36	18	72	45	27	9
72	45	27	9	0	63	54	36	18
63	54	36	18	72	45	27	9	0
45	27	9	0	63	54	36	18	72
54	36	18	72	45	27	9	0	63

(4,5)

54	36	18	72	45	27	9	0	63
45	27	9	0	63	54	36	18	72
63	54	36	18	72	45	27	9	0
72	45	27	9	0	63	54	36	18
0	63	54	36	18	72	45	27	9
18	72	45	27	9	0	63	54	36
9	0	63	54	36	18	72	45	27
36	18	72	45	27	9	0	63	54
27	9	0	63	54	36	18	72	45

(5,4)

(14)

(1) holds for $n = 9$ also

(2) Modify by saying "nasik central series"

Re (3)

5	8	2	1	6	7	4	3	9
9	5	8	2	1	6	7	4	3
3	9	5	8	2	1	6	7	4
4	3	9	5	8	2	1	6	7
7	4	3	9	5	8	2	1	6
6	7	4	3	9	5	8	2	1
1	6	7	4	3	9	5	8	2
2	1	6	7	4	3	9	5	8
8	2	1	6	7	4	3	9	5

(A) (1, 8)

9	0	63	54	36	18	27	45	72
45	72	9	0	63	54	36	18	27
18	27	45	72	9	0	63	54	36
54	36	18	27	45	72	9	0	63
0	63	54	36	18	27	45	72	9
72	9	0	63	54	36	18	27	45
27	45	72	9	0	63	54	36	18
36	18	27	45	72	9	0	63	54
63	54	36	18	27	45	72	9	0

(B) (2, 7)

153 Rows & cols add to 45 and 324 → 369 ie magic in rows & cols.

r.d is 45 + 459 = 504 ✗, l.d is 45 + 324 = 369.

So ~~this square~~ $\mathcal{C}$ would be magic if B were magic. B fails only in r.d & would be magic if 2nd, 5th & 8th terms of the triplets forming the central row added to 108 ie if they were mid terms of a nasik series.

Andrews Fig. 277, p. 173.

(4,0) (4,15) (5,20) (5,10) (2,0)

~~(1,0), (1,5) (1,10), (1,15), (1,20)~~
~~(2,0), (2,5) (2,10) (2,15), (2,20)~~
~~(3,0), (3,5), (3,10), (3,15), (3,20)~~
~~(4,0), (4,5), (4,10), (4,15), (4,20)~~
~~(5,0) (5,5), (5,10), (5,15), (5,20)~~

25	1	23	6	10
12	14	3	20	16
2	24	13	8	18
11	7	21	9	17
15	19	5	22	4

B.M: (1,-1), (1,2)
Due to Planck.
(-1, -2), (2,1), (2,2), (-2,0)
(-2, 2), (2,1), (0,-2), (1,-2)

Seems to be another bizarre one

like the Chinese & bordered squares

Another example of a non-associative square with 13 in centre

Pandiagonal

5	1	3	1	5
2	4	3	5	1
2	4	3	3	3
1	2	1	4	2
5	4	5	2	4

20	0	20	5	5
10	10	0	15	15
0	20	10	5	15
10	5	20	5	15
15	15	0	20	0

(5,20), (1,0), (3,20), (1,5), (5,5)
(2,10) (4,10), (3,0), (5,15), (1,15)
(2,0), (4,20), (3,10), (3,5), (3,15)
(1,10), (2,5), (1,20), (4,5), (2,15)
(5,10), (4,15), (5,0), (2,20), (4,0)

1, 6, 11, 16, 21
2, 7, 12, 17, 22
3, 8, 13, 18, 23
4, 9, 14, 19, 24
5, 10, 15, 20, 25

1 3 5 5 1
1 4 4 1 5
2 2 5 2 4
2 3 4 4 2
3 3 3 1 5

3

25	1	23	6	10
12	14	3	20	16
2	24	13	8	18
11	7	21	9	17
15	19	5	22	4

(2,0), (2,2), (-1,0), (0,1).
~~(0,2), (2,0), (-2,2), (2,2) (-1,-1)~~
(1,-1), (-1,2), (-1,-2), (-1,-1) -2
(1,-1), (-1,0), (2,0), (-2,2)
(-1,1), (-2,2), (0,2), (0,2)
(2,-1), (1,1), (-2,-2). (2,1)

1 2 3 4 5

1 3 5
1 4 4
2 2 5
2 3 4

1 5
2 4
3 3

3 3 3

2 3 4 1 5
1 3 5 2 4
1 4 4 3 3
2 2 5 1 5
3 3 5 2 4

(16)

36	18	72	45	27	~~9~~	0	63	54
0	63	54	36	18	72	45	27	9
45	27	9	0	63	54	36	18	72
36	18	72	45	27	9	0	63	54
0	63	54	36	18	72	45	27	9
45	27	9	0	63	54	36	18	72
36	18	72	45	27	9	0	63	54
0	63	54	36	18	72	45	27	9
45	27	9	0	63	54	36	18	72

~~requ~~

$36 + 0 + 45 =$ Sum of 1st terms of triplets
$18 + 63 + 27 =$ " 2nd "
$72 + 54 + 9 =$ " 3rd "

Hence for cond^n (3) holding for $n = 9$

(3,6)

is that ~~the~~ any intermediate step ~~should~~ be applied to a nasik series (instead of central or non-central).

27	9	0	63	54	36	72	~~18~~	~~45~~
36	72	18	45	27	9	0	63	54
9	0	63	54	36	72	18	45	27
72	18	45	27	9	0	63	54	36
0	63	54	36	72	18	45	27	9
18	45	27	9	0	63	54	36	72
63	54	36	72	18	45	27	9	0
45	27	9	0	63	54	36	72	18
54	36	72	18	45	27	9	0	63

(4,5) & (5,4) ~~also~~ require sum of ~~[illegible]~~ 1st terms of triplets should equal 108.

just as (2,7) & (7,2)

but (3,6), (6,3) require ~~[illegible]~~ series to be nasik

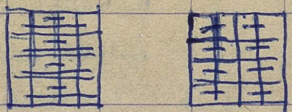
(4,5)

4	4	5	5	2	0	15	20	10	0
5	5	5	3	2	15	5	0	15	10
3	2	3	4	3	0	15	10	5	20
4	3	1	1	1	10	5	20	15	5
4	1	1	2	2	20	10	0	5	20

Chemise 5×5. No need to generalise them to $n = 7, 9$ etc. So also the Brierson's schemes.

~~Take~~ Put them under chapter on squares of $n = 5$.

Narayana's method for even squares.



$$\begin{matrix} 1 & 3 & 2 & 4 \\ 4 & 2 & 3 & 1 \\ 4 & 2 & 3 & 1 \\ 1 & 3 & 2 & 4 \end{matrix}$$

(1) Andrews. Fig. 72, p. 35

$$\underset{C}{\begin{bmatrix} 1 & 15 & 14 & 4 \\ 12 & 6 & 7 & 9 \\ 8 & 10 & 11 & 5 \\ 13 & 3 & 2 & 16 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 1 & 3 & 2 & 4 \\ 4 & 2 & 3 & 1 \\ 4 & 2 & 3 & 1 \\ 1 & 3 & 2 & 4 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 0 & 12 & 12 & 0 \\ 8 & 4 & 4 & 8 \\ 4 & 8 & 8 & 4 \\ 12 & 0 & 0 & 12 \end{bmatrix}}$$

$$\begin{matrix} \cdot & \cdot & \cdot & 1 \\ \cdot & 2 & 2 & \cdot \\ \cdot & 3 & \cdot & 3 \\ 4 & & & 4 \end{matrix} \qquad \begin{vmatrix} 1 & 4 & 4 & 1 \\ 3 & 2 & 2 & 3 \\ 2 & 3 & 3 & 2 \\ 4 & 1 & 1 & 4 \end{vmatrix}$$

N. a

Alg A, B, C

(obvious. rows ↔ cols)

$$\begin{bmatrix} 5 & 8 & 9 & 12 \\ 14 & 15 & 2 & 3 \\ 11 & 10 & 7 & 6 \\ 4 & 1 & 16 & 13 \end{bmatrix} \quad \underset{(A)}{\begin{bmatrix} 1 & 4 & 1 & 4 \\ 2 & 3 & 2 & 3 \\ 3 & 2 & 3 & 2 \\ 4 & 1 & 4 & 1 \end{bmatrix}} \quad \underset{B}{\begin{bmatrix} 4 & 4 & 8 & 8 \\ 12 & 12 & 0 & 0 \\ 8 & 8 & 4 & 4 \\ 0 & 0 & 12 & 12 \end{bmatrix}}$$

~~non-N (diagonals)~~

X

not a magic square in ordinary sense

(2) A. Fig. 185, p. 91, Franklin square

X

(2) is not a magic square

$$\begin{matrix} 2 & 3 & 1 & 4 \\ 1 & 4 & 2 & 3 \\ 4 & 1 & 3 & 2 \\ 3 & 2 & 4 & 1 \end{matrix} \Bigg\}$$

1st col, 2nd col → 2nd row, 1st row

3rd col ↔ 4th col → 4th row & 3rd row

(3) Jaina square, Fig. 220, p. 125 Andrews

X

$$\begin{bmatrix} 7 & 12 & 1 & 14 \\ 2 & 13 & 8 & 11 \\ 16 & 3 & 10 & 5 \\ 9 & 6 & 15 & 4 \end{bmatrix} = \underset{A}{\begin{bmatrix} 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 4 & 8 & 0 & 12 \\ 0 & 12 & 4 & 8 \\ 12 & 0 & 8 & 4 \\ 8 & 4 & 12 & 0 \end{bmatrix}}$$

N. not a

N – Ime g Lat A & B

Andrews Chap VIII, p. ~~169~~ 179 – Twelve plans of Frierson & Dudeney which is said " to cover probably all types of ~~n~~ 4 × 4 magic squares.

$$\begin{vmatrix} 2 & 3 & 1 & 4 \\ 1 & 4 & 2 & 3 \\ 4 & 1 & 3 & 2 \\ 3 & 2 & 4 & 1 \end{vmatrix}$$

(18) Andrews, I p. 489, p. 263

0, 1, . 4
3.4.5 10.5 $\frac{1}{2}n^2(n-1)$

24	3	9	4	25
21	6	11	8	19
12	16	13	10	14
7	18	15	20	5
1	22	17	23	2

(C)

4	3	4	4	5
1	1	1	3	4
2	1	3	5	4
2	3	5	5	5
1	2	2	3	2

(A)

20	0	5	0	20
20	5	10	5	15
10	15	10	5	10
5	15	10	15	0
0	20	15	20	0

(B)

This is called non-la Hireian, I wonder what this means. Does it mean that in A & B numbers are repeated in rows & cols & diagonals - If this be so the even squares constructed in Andrews p. 34 must also be non-la Hireian, but Andrews says that they are constructed by de la Hire's method - In Frierson's terminology, the above squares in A & B are not "pure" magic squares, in fact ~~not~~ not even magic squares - I think this gives an idea of what is meant by a non-la Hireian square - i.e. "although not pure the sums of rows, cols & diagonals in A & B should each be $\frac{1}{2}n(n+1)$ and $\frac{1}{2}n^2(n-1)$ respectively". Thus the squares (4) to Andrews, Chinese & the above two of Planck are non-la Hireian.

1	1	1	3	4
2	2	2	1	3
3	3	3	4	2
4	4	4	1	2
5	5	5	5	5

It appears futile to deal with these non-Narayanean squares.

(21)

(3) rows → cols → interchange quarters. obvious

Jaina.

3	4	1	2
2	1	4	3
4	3	2	1
1	2	3	4

→

3	2	4	1
4	1	3	2
1	4	2	3
2	3	1	4

→

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

Jaina 4 3 2 1

1 2 4 3
3 4 2 1
2 1 3 4
4 3 1 2

6 (4)

4	1	1	4
3	2	2	3
2	3	3	2
1	4	4	1

4	1	4	1
2	3	2	3
1	4	1	4
3	2	3	2

4 3 2 1
1 2 3 4
1 2 3 4
4 3 2 1

3 2 2 3
4 1 1 4
1 4 4 1
2 3 3 2

4 3 2 1
1 2 3 4
1 2 3 4
4 3 2 1

4	1	1	4
3	2	2	3
1	4	4	1
2	3	3	2

4	1	4	1
3	2	3	2
1	4	1	4
2	3	2	3

4	1	4	1
2	3	2	3
1	4	1	4
3	2	3	2

not obvious

look for for fetcher.

non-N in diags

$3^{rd} \leftrightarrow 4^{th}$ rows 3^{rd} col $\leftrightarrow$ 4^{th} col 2^{nd} row $\leftrightarrow$ 4^{th} row

3 2 2 3
4 1 1 4
1 4 4 1
2 3 3 2

4 1 1 4
2 3 3 2
3 2 2 3
1 4 4 1

1 4 1 4
2 3 2 3
3 2 3 2
4 1 4 1

4 3 1 2
1 2 4 3
1 2 4 3

4 1 4 1
2 3 2 3
3 2 3 2
4 1 4 4

(5)

4	1	13	16
14	15	3	2
11	10	6	7
5	8	12	9

4	1	1	4
2	3	3	2
3	2	2	3
1	4	4	1

+

0	0	12	12
12	12	0	0
8	8	4	4
4	4	8	8

1	1	4	4
4	4	1	1
3	3	2	2
2	2	3	3

not obvious

(6)

1	13	4	16
8	12	5	9
14	2	15	3
11	7	10	6

1	1	4	4
4	4	1	1
2	2	3	3
3	3	2	2

0	12	0	12
4	8	4	8
12	0	12	0
8	4	8	4

1	4	1	4
2	3	2	3
4	1	4	1
3	2	3	2

1	1	4	4
2	2	3	3
4	4	1	1
3	3	2	2

→

1	4	1	4
2	3	2	3
4	1	4	1
3	2	3	2

✓ obvious 502

2^{nd} row $\leftrightarrow$ 3^{rd} row 2^{nd} col $\leftrightarrow$ 3^{rd} col.

1 2 3 4 1 2 3 4
2 1 4 3 3 4 1 2
3 4 1 2 2 1 4 3
4 3 2 1 4 3 2 1

(7)

x x
x x

1	7	14	12
10	16	5	3
15	9	4	6
8	2	11	13

→

1	3	2	4
2	4	1	3
3	1	4	2
4	2	3	1

+

0	4	12	8
8	12	4	0
12	8	0	4
4	0	8	12

1	2	4	3
3	4	2	1
4	3	1	2
2	1	3	4

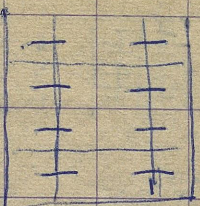
3 2 4 1
1 4 2 3
2 3 1 4
4 1 3 2

not obvious

(20)

Plan 1.

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	4



No mention is made how to place the numbers in the several cols so that the plan may lead to a magic square.

Choose any 4 adding up to S, ~~say~~ ~~1, 7, 10~~, 16 not containing pairs complementary

1, 9, 10, 14

Then 4 others are nos 16, 8, 7, 3

2, 4, 5, 6, 11, 12, 13, 16.

1	16	15	2
9	8	11	6
10	7	13	5 4
14	3	5	12

1	16	6	11
9	8	2	15
10	7		
14	3		

No method is given as to how enables to construct at least one square with a prescribed plan.

1	2	3	4
0	4	8	12

only 12 plans given for purposes of classification. Since one are just nos.

Considering Narayana's method for $n = 4$, let us take one of each plan given in this chapter & see how the plans are related to the respective A & B.

(4)

16	1	13	4
7	10	6	11
2	15	3	14
9	8	12	5

A v.285, p.179

4	1	1	4
3	2	2	3
2	3	3	2
1	4	4	1

A

12	0	12	0
4	8	4	8
0	12	0	12
8	4	8	4

B

Same plan for A & B also.

N

neither a nor n

A is a but not B.

1	4	2	3	4	1
2	3	1	4	3	2
3	2	4	1	4	1
4	1	3	2	3	2

1	4	4	1
3	2	4	1
3	2	4	1
3	2	2	3

1	4	1	4
3	2	1	4
3	2	3	2
3	2	1	4

no use trying to find ~~method~~ number possible under each plan

(13)

$$\begin{array}{cccc|cccc|cccc|cccc}
12 & 4 & 13 & 5 & 4 & 4 & 1 & 1 & 8 & 0 & 12 & 4 & 3 & 1 & 4 & 2 \\
1 & 9 & 16 & 8 & 1 & 1 & 4 & 4 & 0 & 8 & 12 & 4 & 1 & 3 & 4 & 2 \\
15 & 7 & 2 & 10 & 3 & 3 & 2 & 2 & 12 & 4 & 0 & 8 & 4 & 2 & 1 & 3 \\
6 & 14 & 3 & 11 & 2 & 2 & 3 & 3 & 4 & 12 & 0 & 8 & 2 & 4 & 1 & 3
\end{array}$$

$$\begin{array}{cccc|cccc}
4 & 1 & 3 & 2 & 2 & 3 & 1 & 4 \\
4 & 1 & 3 & 2 & 2 & 3 & 1 & 4 \\
1 & 4 & 2 & 3 & 3 & 2 & 4 & 1 \\
1 & 4 & 2 & 3 & 3 & 2 & 4 & 1
\end{array}$$

not obvious

(14)

$$\begin{array}{cccc|cccc|cccc|cccc}
1 & 2 & 16 & 15 & 1 & 2 & 4 & 3 & 0 & 0 & 12 & 12 & 1 & 1 & 4 & 4 \\
13 & 14 & 4 & 3 & 1 & 2 & 4 & 3 & 12 & 12 & 0 & 0 & 4 & 4 & 1 & 1 \\
12 & 7 & 9 & 6 & 4 & 3 & 1 & 2 & 8 & 4 & 8 & 4 & 3 & 2 & 3 & 2 \\
8 & 11 & 5 & 10 & 4 & 3 & 1 & 2 & 4 & 8 & 4 & 8 & 2 & 3 & 2 & 3
\end{array}$$

diagonally non-N

$$\begin{array}{cccc|cccc}
1 & 2 & 4 & 3 & 1 & 4 & 2 & 3 \\
4 & 3 & 1 & 2 & 4 & 1 & 3 & 2 \\
1 & 2 & 4 & 3 & 1 & 4 & 2 & 3 \\
4 & 3 & 1 & 2 & 4 & 1 & 3 & 2
\end{array}$$

not obvious

(15)

$$\begin{array}{cccc|cccc|cccc|cccc}
2 & 15 & 1 & 16 & 2 & 3 & 1 & 4 & 0 & 12 & 0 & 12 & 1 & 4 & 1 & 4 \\
11 & 10 & 8 & 5 & 3 & 2 & 4 & 1 & 8 & 8 & 4 & 4 & 3 & 3 & 2 & 2 \\
14 & 3 & 13 & 4 & 2 & 3 & 1 & 4 & 12 & 0 & 12 & 0 & 4 & 1 & 4 & 1 \\
7 & 6 & 12 & 9 & 3 & 2 & 4 & 1 & 4 & 4 & 8 & 8 & 2 & 2 & 3 & 3
\end{array}$$

not obvious

~~non-N~~

diagonally non-N

Squares from Kraitchik. p.191.

(16)

$$\begin{array}{cccc|cccc|cccc|cccc}
1 & 8 & 12 & 13 & 1 & 4 & 4 & 1 & 0 & 4 & 8 & 12 & 1 & 2 & 3 & 4 \\
14 & 11 & 7 & 2 & 2 & 3 & 3 & 2 & 12 & 8 & 4 & 0 & 4 & 3 & 2 & 1 \\
15 & 10 & 6 & 3 & 3 & 2 & 2 & 3 & 12 & 8 & 4 & 0 & 4 & 3 & 2 & 1 \\
4 & 5 & 9 & 16 & 4 & 1 & 1 & 4 & 0 & 4 & 8 & 12 & 1 & 2 & 3 & 4
\end{array}$$

obvious

rows ↔ cols

1	2	3	4	1	2	3	4	1	2	3	4	12	8	4	0			
3	4	1	2	4	3	2	1	3	4	1	2	4	0	12	8	(22)		
2	1	4	3	2	1	4	3	2	1	4	3	8	12	0	4	13	10	7	4
4	3	2	1	3	4	1	2	4	3	2	1	0	4	8	12	7		

(8)

1	8	10	15	1	4	2	3	0	4	8	12	1	2	3	4	
14	11	5	4	2	3	1	4	12	8	4	0	4	3	2	1	
7	2	16	9	3	2	4	1	4	0	12	8	2	1	4	3	obvious
12	13	3	6	4	1	3	2	8	12	0	4	3	4	1	2	

mere change of rows & cols.

(9)

1	15	14	4	1	3	2	4	0	12	12	0	1	4	4	1	identical with Andrews, Fig 72
12	6	7	9	4	2	3	1	8	4	4	8	3	2	2	3	p. 35
8	10	11	5	4	2	3	1	4	8	8	4	2	3	3	2	obvious
13	3	2	16	1	3	2	4	12	0	0	12	4	1	1	4	

change of rows & cols

(10)

16	1	12	5	4	1	4	1	12	0	8	4	4	1	3	2	non-N diagonals
2	11	6	15	2	3	2	3	0	8	4	12	1	3	2	4	
7	14	3	10	3	2	3	2	4	12	0	8	2	4	1	3	not obvious
9	8	13	4	1	4	1	4	8	4	12	0	3	2	4	1	

(11)

11	14	3	6	4	2	3	1	3	2	3	2	8	12	0	4	3	4	1	2	not
8	9	16	1	1	3	2	4	4	1	4	1	4	8	12	0	2	3	4	1	obvious
10	7	2	15	4	2	3	1	2	3	2	3	8	4	0	12	3	2	1	4	N
5	4	13	12	1	3	2	4	1	4	1	4	4	0	12	8	2	1	4	3	

(12)

5	1	12	16	1	1	4	4	4	0	8	12	2	1	3	4 ?	
10	14	3	7	2	2	3	3	8	12	0	4	3	4	1	2	not obvious
15	11	6	2	3	3	2	2	12	8	4	0	4	3	2	1	
4	8	13	9	4	4	1	1	0	4	12	8	1	2	4	3	diagonals non-N

	1	8	13	12	1	4	1	4	0	4	12	8	1	2	4	3	obvious
	14	11	2	7	2	3	2	3	12	8	0	4	4	3	1	2	rows & cols
(21)	4	5	16	9	4	1	4	1	0	4	12	8	1	2	4	3	
	15	10	3	6	3	2	3	2	12	8	0	4	4	3	1	2	

	1	4	16	13	1	4	4	1	0	0	12	12	1	1	4	4	
	14	15	3	2	2	3	3	2	12	12	0	0	4	4	1	1	
(22)	7	6	10	11	3	2	2	3	4	4	8	8	2	2	3	3	not obvious
	12	9	5	8	4	1	1	4	8	8	4	4	3	3	2	2	

	1	6	16	11	1	2	4	3	0	4	12	8	1	2	4	3	
(23)	12	15	5	2	4	3	1	2	8	12	4	0	3	4	2	1	not obvious
	7	4	10	13	3	4	2	1	4	0	8	12	2	1	3	4	
	14	9	3	8	2	1	3	4	12	8	0	4	4	3	1	2	

	1	7	16	10	1	3	4	2	0	4	12	8	1	2	4	3	not obvious
(24)	12	14	5	3	4	2	1	3	8	12	4	0	3	4	2	1	
	6	4	11	13	2	4	3	1	4	0	8	12	2	1	3	4	
	15	9	2	8	3	1	2	4	12	8	0	4	4	3	1	2	

	1	4	13	16	1	4	1	4	0	0	12	12	1	1	4	4	
(25)	14	15	2	3	2	3	2	3	12	12	0	0	4	4	1	1	not obvious.
	8	5	12	9	4	1	4	1	4	4	8	8	2	2	3	3	
	11	10	7	6	3	2	3	2	8	8	4	4	3	3	2	2	

	1	6	11	16	1	2	3	4	0	4	8	12	1	2	3	4	
	12	15	2	5	4	3	2	1	8	12	0	4	3	4	1	2	not obvious
(26)	8	3	14	9	4	3	2	1	4	0	12	8	2	1	4	3	
	13	10	7	4	1	2	3	4	12	8	4	0	4	3	2	1	

(24)

$$(17)\quad \begin{array}{cccc} 1 & 8 & 14 & 11 \\ 12 & 13 & 7 & 2 \\ 15 & 10 & 4 & 5 \\ 6 & 3 & 9 & 16 \end{array} \quad \left|\begin{array}{cc|cc} \multicolumn{2}{c}{A} & \multicolumn{2}{c}{B} \\ 1 & 4 & 2 & 3 \\ 4 & 1 & 3 & 2 \\ \hline 3 & 2 & 4 & 1 \\ 2 & 3 & 1 & 4 \\ \multicolumn{2}{c}{C} & \multicolumn{2}{c}{D} \end{array}\right| \quad \left|\begin{array}{cccc} 0 & 4 & 12 & 8 \\ 8 & 12 & 4 & 0 \\ 12 & 8 & 0 & 4 \\ 4 & 0 & 8 & 12 \end{array}\right| \quad \left|\begin{array}{cccc} 1 & 2 & 4 & 3 \\ 3 & 4 & 2 & 1 \\ 4 & 3 & 1 & 2 \\ 2 & 1 & 3 & 4 \end{array}\right|$$

(A_1) (B_1) not obvious

Note these are the same.

$$(18)\quad \begin{array}{cccc} 1 & 8 & 15 & 10 \\ 12 & 13 & 6 & 3 \\ 14 & 11 & 4 & 5 \\ 7 & 2 & 9 & 16 \end{array} \quad \left|\begin{array}{cccc} 1 & 4 & 3 & 2 \\ 4 & 1 & 2 & 3 \\ 2 & 3 & 4 & 1 \\ 3 & 2 & 1 & 4 \end{array}\right| \quad \left|\begin{array}{cccc} 0 & 4 & 12 & 8 \\ 8 & 12 & 4 & 0 \\ 12 & 8 & 0 & 4 \\ 4 & 0 & 8 & 12 \end{array}\right| \quad \left|\begin{array}{cccc} 1 & 2 & 4 & 3 \\ 3 & 4 & 2 & 1 \\ 4 & 3 & 1 & 2 \\ 2 & 1 & 3 & 4 \end{array}\right|$$

(A_2) (B_2) not obvious

In these there is some order in picking up 2 × 2 squares.

In A_1 squares A & D are $\begin{vmatrix} 1 & 4 \\ 4 & 1 \end{vmatrix}$ & $\begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix}$ & B & C are $\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}$, $\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}$

In B_1 they are $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$, $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ and $\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$, $\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$.

In A_2 squares A & D are same as in A_1, B & C are interchanged. B_1 & B_2 are identical re. 2 × 2 squares. Mention this under Narayana's method for even squares.

$$(19)\quad \begin{array}{cccc} 1 & 8 & 10 & 15 \\ 12 & 13 & 3 & 6 \\ 7 & 2 & 16 & 9 \\ 14 & 11 & 5 & 4 \end{array} \quad \left|\begin{array}{cccc} 1 & 4 & 2 & 3 \\ 4 & 1 & 3 & 2 \\ 3 & 2 & 4 & 1 \\ 2 & 3 & 1 & 4 \end{array}\right| \quad \left|\begin{array}{cccc} 0 & 4 & 8 & 12 \\ 8 & 12 & 0 & 4 \\ 4 & 0 & 12 & 8 \\ 12 & 8 & 4 & 0 \end{array}\right| \quad \left|\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \end{array}\right|$$

not obvious

$$(20)\quad \begin{array}{cccc} 1 & 8 & 11 & 14 \\ 12 & 13 & 2 & 7 \\ 6 & 3 & 16 & 9 \\ 15 & 10 & 5 & 4 \end{array} \quad \left|\begin{array}{cccc} 1 & 4 & 3 & 2 \\ 4 & 1 & 2 & 3 \\ 2 & 3 & 4 & 1 \\ 3 & 2 & 1 & 4 \end{array}\right| \quad \left|\begin{array}{cccc} 0 & 4 & 8 & 12 \\ 8 & 12 & 0 & 4 \\ 4 & 0 & 12 & 8 \\ 12 & 8 & 4 & 0 \end{array}\right| \quad \left|\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \end{array}\right|$$

not obvious

4	4	2	2	2	1	4	3	4	8	8	4	6	9	12	7
1	1	3	3	3	4	1	2	0	12	12	0	3	16	13	2
1	1	3	·	3	4	1	2	12	0	0	12	15	4	1	14
4	4	2	2	2	1	4	3	8	4	4	8	10	5	8	11

3	·	·	1	2	·	·	4	1	·	·	2	4	3	2	1	12	0	0	12
·	4	2	4	·	1	3	1	·	4	4	·	1	2	3	4	8	4	4	8
·	4	2	4	·	1	3	1	·	3	4	·	1	2	3	4	4	8	8	4
3	0	0	1	2	·	·	4	1	·	·	2	4	3	2	1	0	12	12	0

16	3	2	13	3	1	4	2
9	6	7	12	2	4	1	3
5	10	11	8	2	4	1	3
4	15	14	1	3	1	4	2

Take X with 1, 2, 3, 4 in any order such that 1st & last add to 5 & 2nd & 3rd add to 5 } (*)

& then rows ⟷ cols ⟶ root square ⟶ add ⟶ associated square.

Same thing happens with (16). In ~~(15)~~ (17) there is a slight modification with the diagonals being 1 1 4 4 & 2 2 3 3 cond (*) holds

1	4	4	1
4	4	4	1
4	4	4	1
1	·	·	4

Does not work, must have different diagonals adding to 10 & satisfying (*)

1	2	4	3
3	4	2	1
4	3	1	2
2	1	3	4
1	·	4	3
·	4	2	·
·	3	1	·
2	·	3	4

~~Does not work although diagonals sum up to 10 & satisfy (*)~~

⟶

0	8	12	4	1	10	16	7
4	12	8	0	7	16	10	1
12	4	0	8				
8	0	4	12				

Does not work by rows ⟷ cols.

(26)

(27)

1	7	10	16
12	14	3	5
8	2	15	9
13	11	6	4

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4

0	4	8	12
8	12	0	4
4	0	12	8
12	8	4	0

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

(1, −2), (1,1), (1, −2) (0, −1)
(−1, −2), (−1, −1), (−1, −2) (−1, 0)
(−1, 2), (−1, 1), (−1, 2), (0, −2)
(1, 2), (1, −1), (1, 2)
(1, 2),
not obvious

of these 27, let us analyse some which are amenable to 2 x 2 squares & ascribe them to Narayana.

$nx + y + 1$. 6/4·1 7/14/15

2 x 2 square types are

1	2
3	4

1	4
3	2

4	1
2	3

4	3
1	2

3	4
2	1

1 3
4 2

1	1
4	4

2	2
3	3

(0,0), (1,2), (2,1), (3,3)
(2,3), (3,1), (0,2), (1,0)
(1,3), (0,1) (3,2), (2,0)
(3,0), (2,2), (1,1), (0,3)

1	·	·	4
·	3	2	·
·	3	2	·
4	3	·	4

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33

1	4	3	2
4	3	1	2
3	4	2	1
3	2	1	4

1	2	3	4
4	3	2	1
4	3	2	1
1	2	3	4

1	4	4	1
2	3	3	2
3	2	2	3
4	1	1	4

(9) is ascribed to

1	14	15	4
8	11	10	5
12	7	6	9
13	2	3	16

0	12	12	0
4	8	8	4
8	4	4	8
12	0	0	12

Only map (†') to l.d & (†) to u.d. yes

4	3	1	2	12	8	0	4	13	12	3	6
2	1	3	4	4	0	8	12	8	1	10	15
1	2	4	3	0	4	12	8	2	7	16	9
3	4	2	1	8	12	4	0	11	14	5	4

✓

Apply (†') ~~to both~~ diagonals.

4	2	1	3	12	4	0	8	13	8	3	10
3	1	2	4	8	0	4	12	12	1	6	15
1	3	4	2	0	8	12	4	2	11	16	5
2	4	3	1	4	12	8	0	7	14	9	4

✓ yes

So for cases where diagonals are 1 1 4 4 & 2 2 3 3 apply (†) & (†') in full or in parts & complete square B & add to A — All these lead to associated squares only viz (9), (16), (17) & (18).

4 1 1 4   4 3 2 1   1 3 2 4   0 8 4 12   4 11 6 13
. 2 1 .   1 2 3 4   1 2 3 4   0 4 8 12   1
. 4 3 .   1 2 3 4   1 2 3 4   0 4 8 12
1 3 . 1   4 3 2 1   1 3 2 4   0 8 4 12

1 2 3 4   4 7 10 13   12 0 0 12   16 3 2 13
4 3 2 1   13   8 4 4 8   9 6 7 12
4 3 2 1   4 1 1 4   4 8 8 4   5 10 11 8
1 2 3 4   3 2 2 3   0 12 12 0   4 15 14 1
2 3 3 2
1 4 4 1

4 4
2
1 3
1

4 1 1 4
3 2 2 3
2 3 3 2
1 4 4 1

(27)

1	2	4	3	1	4	2	3	0	12	4	8	1	14	8	11
3	4	2	1	4	1	3	2	12	0	8	4	15	4	10	5
4	3	1	2	3	2	4	1	8	4	12	0	12	7	13	2
2	1	3	4	2	3	1	4	4	8	0	12	6	9	3	16
	a				(A)				B						

To get square B from A interchange elements in central subsquare diagonally (†) & fill up square

~~In (13) there is a modification of this rule i.e. changing one diagonal in subsquare & interchanging extremes of the main diagonals. Does it work with rule (†)~~

1	4	3	2
4	1	2	3
2	3	4	1
3	2	1	4

1 4 3 2

4

3

1	3	4	2
2	4	3	1
4	2	1	3
3	1	2	4

0	8	12	4	1	12	15	6
4	12	8	0	8	13	10	3
12	4	0	8	14	7	4	9
8	0	4	12	11	2	5	16

This correct in case (18) also with rule (†)

But in (18) as it is (†) is modified by interchanging central elements of lower diagonal & extreme elements of u.d and finishing the square (†′)

1	4	3	2	1	2	4	3
4	1	2	3	3	4	2	1
2	3	4	1	4	3	1	2
3	2	1	4	2	1	3	4

This is exactly what we have in (18)

So rule (†′) is also valid

3	1	5	4	2
2	3	1	5	4
4	2	3	1	5
5	4	2	3	1
1	5	4	2	3

+

10	20	0	5	15
15	10	20	0	5
5	15	10	20	0
0	5	15	10	20
20	0	5	15	10

13	21	5	9	17
17	13	21	5	10
9				

X

+

(1, 4)

10	0	20	15	5
5	10	0	20	15
15	5	10	0	20
20	15	5	10	0
0	20	15	5	10

(1, 4)

13	1	25	19	7
7	13	1	25	19
19	7	13	1	25
25	19	7	13	1
1	25	19	7	13

X

Repetitions

(1) both rows central (any two steps).
(2) only one row central (and a middle to centre the diags)
(3) both rows non-central
(4) both rows asym → associative any two whatever steps)

$$\begin{vmatrix} 3 & 1 & 4 & 2 & 5 \\ 5 & 3 & 1 & 4 & 2 \\ 2 & 5 & 3 & 1 & 4 \\ 4 & 2 & 5 & 3 & 1 \\ 1 & 4 & 2 & 5 & 3 \end{vmatrix} + \begin{vmatrix} 5 & \cancel{15}\ 0 & \cancel{0}\ 15 & 10 & 20 \\ 10 & 20 & 5 & \cancel{15}\ 0 & \cancel{0}\ 15 \\ \cancel{15}\ 0 & \cancel{0}\ 15 & 10 & 20 & 5 \\ 20 & 5 & \cancel{15}\ 0 & \cancel{0}\ 15 & 10 \\ \cancel{0}\ 15 & 10 & 20 & 5 & \cancel{15}\ 0 \end{vmatrix}$$

$$\begin{vmatrix} 8 & 16 & 4 & 12 & 25 \\ 15 & 23 & 6 & 19 & 2 \\ 17 & 5 & 13 & 21 & 9 \\ 24 & 7 & 20 & 3 & 11 \\ 1 & 14 & 22 & 10 & 18 \end{vmatrix} \qquad \begin{matrix} 8 & 1 & 19 & 12 & 25 \\ 15 & 23 & 6 & 4 & 17 \\ 2 & 20 & 13 & 21 & 9 \\ 24 & 7 & 5 & 18 & 11 \\ 16 & 14 & 22 & 10 & 3 \end{matrix}$$

<u>a</u> not <u>n</u>   neither <u>a</u> nor <u>n</u>.

Tws

(29)

$4 \bigcirc^{3}_{1} {}^{2}$

Let us examine the several squares re. subsquare configuration (going backwards from (27)

(α) (β)

(27)

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4

(γ) (δ)

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

1	2	1	3	4	2	4	3
3	4	2	4	3	1	2	1
2	1	2	4	3	1	3	4
4	3	1	3	4	2	1	2

(27)

1	3	2	4	1	4	4	1	·	3	2	·
4	2	3	1	3	2	2	3	·	2	3	·
4	2	3	1	2	3	3	2	·	2	3	·
1	3	2	4	4	1	1	4	·	3	2	·

~~148877~~. $= 53^3$

1, 2, 3, 4, 5, 6, 7

0, 7, 14, 21, 28, 35, 42

148877 × 148877

1042139
1042139
1191016
1191016
595508
148877

22,154,361,129.

(2,5)

6	3	7	4	5	2	1
2	1	6	3	7	4	5
4	5	2	1	6	3	7
3	7	4	5	2	1	6
1	6	3	7	4	5	2
5	2	1	6	3	7	4
7	4	5	2	1	6	3

(2,5).

n = 7, (1,6), 6,1
(2,5), (5,4
(3,4), (4,3)

14	28	42	7	0	35	21
0	35	21	14	28	42	7
28	42	7	0	35	21	14
35	21	14	28	42	7	0
42	7	0	35	21	14	28
21	14	28	42	7	0	35
7	0	35	21	14	28	42

(3,4)

20	31	49	11	5	37	22
2	36	27	17	35	46	12
32	47	9	1	41	24	21
38	28	18	33	44	8	6
43	13	3	42	25	19	30
26	16	29	48	10	7	39
14	4	40	23	15	34	45

magic although ~~[illegible]~~ n

and also n.

324
45

$\frac{1}{2}$ 9(52
9441 (36)

(1,4) + (4,1) + (4,1) + ~~(4,4)~~ (4,1) + (1,4)

~~(2,3)~~

b c d e a
c d e a b
a b c d e
e a b c d
a b c d e

least repetitions

(3,2) + (1,4) + (2,3) + (1,4) + (3,2)

e a b c d
d e a b c
a b c d e
c d e a b
b c d e a

For (2)

c d e a b
e a b c d
a b c d e
e a b c d
b c d e a

(2) does not work but
(1,4) + (3,2) + (4,1) + (3,2) + (4,1) works but gives repetitions

(2,3) + (3,2) + (3,2) + (3,2) + (2,3)

repetitions

a b c d e
d e a b c
a b c d e

So we have to consider only the 8 cases in (F).

(9) & (6) of Frenson correspond to (1) & (6) of (F)

Should we start only with a symmetric series in middle row? Take (1) & try (2)

(4,1) + (2,3) + (1,4) + (2,3) + (1,4)

$$\begin{vmatrix} 3 & 2 & 5 & 1 & 4 \\ 5 & 1 & 4 & 3 & 2 \\ 1 & 4 & 3 & 2 & 5 \\ 5 & 1 & 4 & 3 & 2 \\ 4 & 3 & 2 & 5 & 1 \end{vmatrix} + \begin{vmatrix} 15 & 0 & 20 & 5 & 10 \\ 5 & 10 & 15 & 0 & 20 \\ 20 & 5 & 10 & 15 & 0 \\ 5 & 10 & 15 & 0 & 20 \\ 0 & 20 & 5 & 10 & 15 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 \end{vmatrix}$$

repetition

repetition

repetition

$$+ \begin{vmatrix} 10 & 5 & 20 & 0 & 15 \\ 20 & 0 & 15 & 10 & 5 \\ 0 & 15 & 10 & 5 & 20 \\ 20 & 0 & 15 & 10 & 5 \\ 15 & 10 & 5 & 20 & 0 \end{vmatrix} \rightarrow \begin{matrix} 13 & 7 & 25 & 1 & 19 \\ 25 & & & & \end{matrix}$$

Contd in ~~Bk 19~~ Bk. 16, p. 191

Generalisation of Reichmann's rule – why not call it Ramanujan's rule?

Bk. 17, p. 53 – Frierson's (9) & (10) generalisation with $(1,4)+(2,3)+(3,2)+(2,3)+(2,3)$

& $(4,1)+(3,2)+(2,3)+(3,2)+(3,2)$

(9)

$$\begin{vmatrix} 5 & 1 & 4 & 3 & 2 \\ 3 & 2 & 5 & 1 & 4 \\ 1 & 4 & 3 & 2 & 5 \\ 5 & 1 & 4 & 3 & 2 \\ 3 & 2 & 5 & 1 & 4 \end{vmatrix} + \begin{vmatrix} 5 & 10 & 15 & 0 & 20 \\ 15 & 0 & 20 & 5 & 10 \\ 20 & 5 & 10 & 15 & 0 \\ 5 & 10 & 15 & 0 & 20 \\ 15 & 0 & 20 & 5 & 10 \end{vmatrix} = \begin{vmatrix} 10 & 11 & 19 & 3 & 22 \\ 18 & 2 & 25 & 6 & 14 \\ 21 & 9 & 13 & 17 & 5 \\ 10 & 11 & 9 & 3 & 22 \\ 18 & 2 & 25 & 6 & 14 \end{vmatrix}$$

For former & 2nd gp.

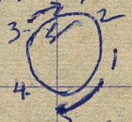
repetition

Does not work:

$$\begin{vmatrix} b & c & d & e & a \\ c & d & e & a & b \\ a & b & c & d & e \\ d & e & a & b & c \\ e & a & b & c & d \end{vmatrix}$$

$(2,3)+(4,1)+(3,2)+(4,1)+(2,3)$ works whatever a, b, c, d, e //

What about $(3,2)+(1,4)+(2,3)+(1,4)+(3,2)$



$$\begin{vmatrix} e & a & b & c & d \\ d & e & a & b & c \\ a & b & c & d & e \\ c & d & e & a & b \\ b & c & d & e & a \end{vmatrix}$$

also works whatever a, b, c, d, e //

(10) of Frierson.

$$\begin{vmatrix} d & e & a & b & c \\ b & c & d & e & a \\ a & b & c & d & e \\ e & a & b & c & d \\ c & d & e & a & b \end{vmatrix}$$

$(1,4)+(2,3)+(4,1)+(2,3)+(1,4)$

also works

whatever a, b, c, d, e.

So should its complement

√(1) $(1,4)+(2,3)+(4,1)+(2,3)+(1,4)$

(2) $(1,4)+(3,2)+(4,1)+(3,2)+(1,4)$

(3) $(4,1)+(2,3)+(1,4)+(2,3)+(4,1)$

√(4) $(4,1)+(3,2)+(1,4)+(3,2)+(4,1)$

(5) $(2,3)+(1,4)+(3,4)+$ –

√(6) $(2,3)+(4,1)+(3,4)+$

√(7) $(3,2)+(1,4)+(2,3)+$ –

(8) $(3,2)+(4,1)+(2,3)+$ –

A

F

15	10	20	5	0
20	5	0	15	10
0	15	10	20	5
10	20	5	0	15
5	0	15	10	20

~~3 2 4 1 5~~
~~5 3 2 4 4~~
~~1 5 3 2 4~~
~~4~~

+

4	1	5	3	2
3	2	4	1	5
1	5	3	2	4
2	4	1	5	3
5	3	2	4	1

(2,3)

+

5	0	15	10	20
10	20	5	0	15
0	15	10	20	5
20	5	0	15	10
15	10	20	5	0

(2,3)

~~[illegible]~~ → repetitions

(3,2)

4	5	2	1	3
3	4	5	2	1
1	3	4	5	2
2	1	3	4	5
5	2	1	3	4

(1,4) (A'')

4	3	2	1	5
3	2	1	5	4
2	1	5	4	3
1	5	4	3	2
5	4	3	2	1

5	0	20	15	10
0	20	15	10	5
20	15	10	5	0
15	10	5	0	20
10	5	0	20	15

(B'') (4,1)

p. (34)

9	5	22	16	13
3	24	20	12	6
21	18	14	10	2
17	11	8	4	25
15	7	1	23	19

(C'') semi-magic
not magic only

19 11

3	2	4	1	5
5	3	2	4	1
1	5	3	2	4
4	1	5	3	2
2	4	1	5	3

(1,4)

+

10	5	15	0	20
20	10	5	15	0
0	20	10	5	15
15	0	20	10	5
5	15	0	20	10

(1,4)

= X repetitions

+

5	15	0	20	10
15	0	20	10	5
0	20	10	5	15
20	10	5	15	0
10	5	15	0	20

(4,1)

+

15	0	20	10	5
10	5	15	0	20
0	20	10	5	15
5	15	5	20	10
20	10	5	15	0

(2,3)

✓

(33)

1	2	3	4	5
0	5	10	15	20

$$\begin{vmatrix} 3 & 2 & 4 & 1 & 5 \\ 5 & 3 & 2 & 4 & 1 \\ 1 & 5 & 3 & 2 & 4 \\ 4 & 1 & 5 & 3 & 2 \\ 2 & 4 & 1 & 5 & 3 \end{vmatrix} + \begin{vmatrix} 10 & 0 & 20 & 5 & 15 \\ 15 & 10 & 0 & 20 & 5 \\ 5 & 15 & 10 & 0 & 20 \\ 20 & 5 & 15 & 10 & 0 \\ 0 & 20 & 5 & 15 & 10 \end{vmatrix} \quad \begin{vmatrix} 0 & 20 & 5 & 15 & 10 \\ 20 & 5 & 15 & 10 & 0 \\ 5 & 15 & 10 & 0 & 20 \\ 15 & 10 & 0 & 20 & 5 \\ 10 & 0 & 20 & 5 & 15 \end{vmatrix}$$

(1,4) (1,4) (4,1)

[leads to relations 12 with 1-2]

$\alpha + \alpha' = -2$

$\beta + \beta' = -1$

8	1	24	17	15
5	23	16	14	7
22	20	13	6	4
19	12	10	3	21
11	9	2	25	18

Andrews fig. 17, p. 11.

$$\begin{matrix} 3 & 1 & 4 & 2 & 5 \\ 5 & 3 & 1 & 4 & 2 \\ 2 & 5 & 3 & 1 & 4 \\ 4 & 2 & 5 & 3 & 1 \\ 1 & 4 & 2 & 5 & 3 \end{matrix} \quad + \quad \begin{matrix} 5 & 0 & 20 & 15 & 10 \\ 0 & 20 & 15 & 10 & 5 \\ 20 & 15 & 10 & 5 & 0 \\ 15 & 10 & 5 & 0 & 20 \\ 10 & 5 & 0 & 20 & 15 \end{matrix}$$

(1,4) + (4,1)

(1,1)

(-2,-1)

$\begin{pmatrix} 1, & 1 \\ 2, & -2 \end{pmatrix}$

$\begin{pmatrix} 1, & 1 \\ 2, & 3 \end{pmatrix}$

-1,-1.

Fig. 23 of Andrews, p. 12.

$\begin{pmatrix} 2, & 3 \\ 2, & 1 \end{pmatrix}$

$$\begin{matrix} 2 & 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 & 4 \end{matrix} \quad \begin{matrix} 10 & 20 & 5 & 15 & 0 \\ 0 & 10 & 20 & 5 & 15 \\ 15 & 0 & 10 & 20 & 5 \\ 5 & 15 & 0 & 10 & 20 \\ 20 & 5 & 15 & 0 & 10 \end{matrix}$$

(3,2). (1,4)

Central symm. Central symm.

So it looks as if every square derivable from the generalised D.D.M can be obtained by the Narayana method

1	4	5	3	2
3	2	1	4	5
4	5	3	2	1
2	1	4	5	3
5	3	2	1	4

(2,3)

0	15	20	10	5
10	5	0	15	20
15	20	10	5	0
5	0	15	20	10
20	10	5	0	15

(2,3)

3	17	21	14	10
15	8	2	16	24
19	25	13	7	1
6	4	20	23	12
22	11	9	5	18

1451.59
466.00
2417.59
400 300
2817.59

p. 35

3	1	2	4	5
5	3	1	2	4
4	5	3	1	2
2	4	5	3	1
1	2	4	5	3

a (1,4) (A)

20	0	15	10	5
10	5	20	0	15
0	15	10	5	20
5	20	0	15	10
15	10	5	20	0

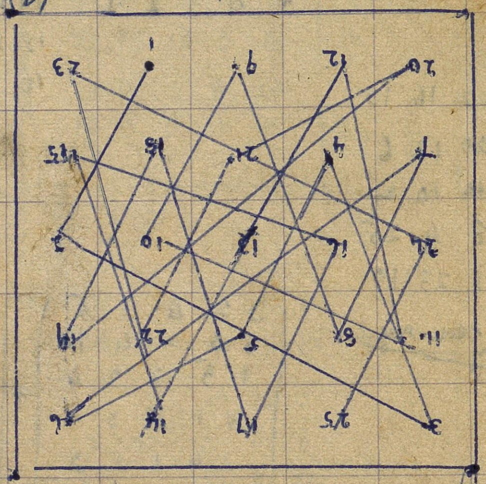
a (2,3) (B)

23, 1, 17, 14, 10
15, 8, 21, 2, 19
4, 20, 13, 6, 22
7, 24, 5, 18, 11
16, 12, 9, 25, 3

23	1	17	14	10
15	8	21	2	19
4	20	13	6	22
7	24	5	18	11
16	12	9	25	3

magic & semi-n (C)

4 5 3



160/9
17°C
-15
0°C = 32°F
47° x 5 / 9
25°C

3	2	1	4	5
5	3	2	1	4
4	5	3	2	1
1	4	5	3	2
2	1	4	5	3

(1,4) (A')

2	5	4	1	3
5	4	1	3	2
4	1	3	2	5
1	3	2	5	4
3	2	5	4	1

(4,1)

5	20	15	0	10
20	15	0	10	5
15	0	10	5	20
0	10	5	20	15
10	5	20	15	0

(B')

8	22	16	4	15
25	18	2	11	9
19	5	13	~~7~~	21
1	14	10	23	17
12	6	24	20	3

(C')

magical alright
but non-n
nor semi-n

VH 24(A) - No. 1

PART

CALCUTTA UNIVERSITY

..................Examination, 19

For Head Examiner

A+c

A+e B+d C+e

A+a B+c C+c D+d, E+e

Subject............

..........Paper ....Half

Registration No. and Year	Roll No.	Marks

Examiner..........

Date..........

PART

CALCUTTA UNIVERSITY

..................Examination, 19

First Tabulator

Subject............

..........Paper ....Half

Registration No. and Year.	Roll No.	Marks

Examiner..........

Head Examiner

Date..........

PART

CALCUTTA UNIVERSITY

..................Examination, 19

Second Tabulator

Subject............

..........Paper ....Half

Registration No. and Year.	Roll No.	Marks

Examiner..........

Head Examiner

Date..........

3330

PART

CALCUTTA UNIVERSITY

..................Examination, 19

For Examiner

Subject............

..........Paper ....Half

Registration No. and Year.	Roll No.	Marks

A+a	D+d	C+b	B+c
C+c	B+b	A+d	D+a
B+d	C+a	D+c	A+b
D+b	A+c	B+a	C+d

Examiner..........

Date..........

11, 16, 18, 19 → 111, 116, 118, 119, 161, 181, 191, 611, 616, 618, 619, 116, 166, 186, 196, 811, 816, 818, 819, 168, 188, 198, 911, 916, 918, 919, 169, 189, 199 (28)

61, 66, 68, 69 → ~~166~~, 661, 666, 668, 669, 686, 696, 861, 866, 868, 869, 688, 698, 961, 966, 968, 969, 669, 689, 699 (20)

681, 691

81, 86, 88, 89 → 681, 881, 886, 888, 889, 898, 981, 986, 988, 989, 899 (12) (12)

891, 898

91, 96, 98, 99 → ~~991, 99~~ 996, 998, 999, (4)

991, ~~996~~

(3) (4) (64)

33 / 28 / 8 / 69

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

mere reversed diagonals

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

16	13	3	2
5	8	10	11
9	12	6	7
4	1	15	14

X

16	13	3	2
4	1	15	14
9	12	6	7
5	8	10	11

semi-Nanth

	a_{11}	a_{21}	a_{31}	a_{41}
A_{11}	α_{11}	α_{12}	α_{13}	α_{14}
A_{12}	α_{21}	α_{22}	α_{23}	α_{24}
A_{13}	α_{31}	α_{32}	α_{33}	α_{34}
A_{14}	α_{41}	α_{42}	α_{43}	α_{44}

	1	2	5	6
2	3	4	7	8
8	9	10	13	14
10	11	12	15	16

1	15	8	10
14	4	11	5
12	6	13	3
7	9	2	16

✓

α_{11}	α_{43}	α_{42}	α_{14}
α_{34}	α_{22}	α_{23}	α_{31}
α_{24}	α_{32}	α_{33}	α_{21}
α_{41}	α_{13}	α_{12}	α_{44}

α_{11}	α_{43}	α_{24}	α_{32}
α_{34}	α_{22}	α_{41}	α_{13}
α_{42}	α_{14}	α_{33}	α_{21}
α_{23}	α_{31}	α_{12}	α_{44}

0	1	3	5	7
1	2	4	6	8
8	9	11	13	15
9	10	12	14	16

reversed diagonals

	1	2	3	4
	A_1+a_1	A_1+a_2	A_1+a_3	A_1+a_4
4	5	6	7	
8	9	10	11	12
12	13	14	15	16

12 → 34 | 13 → 42
23 → 14 | 32 → 41
31 → 24 | 21 → 43

1	15	8	10
12	6	3	5
14	4	11	
		2	16

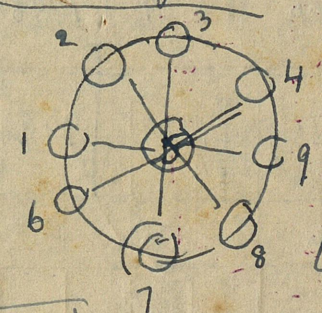
1	14	8	11
15	4	10	5
12	7	13	2
6	9	3	16

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	5	8	[illegible]
2	6		3
14		11	15
	9	12	16

8	1	6
3	5	7
4	9	2

Magic square



11
14
16
41

48976
4.3.43
2.21
197.63
73303

M M

1 + 9

2 + 9

6 + 4 = 4 + 6

Chris reversed diagonals

16	2	5	11
3	13	10	8
9	7	4	14
6	12	15	1

✓

81689

181 G u m 3 47 9 m

2 a h D A O

16	3	5	10
2	13	11	8
9	6	4	15
7	12	14	1

Λ γ z x A 7

XOHOXI ∩ ?

ΛV 8

hyhy H OHOHO

IHIHI 7

Z S nu hy S hy ⊥ !