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2+2=4 ; BUT EINSTEIN MAY NOT AGREE*

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"It is as clear as 2+2=4". This is a common saying used by lay persons who may not, in the least, be mathematically inclined. But, as is suggested by the title, if Einstein has some doubts about two and two adding to four, that certainly will make us wonder ! Will 2+2 be equal to 4 ? We shall try to understand the statement 2+2=4.

There are many different way of understanding a mathematical statement. One such way is that of negating the statement. This method is known as the method of *reductio ad absurdum*. We wish to understand the statement 2+2=4. So let us negate the statement *i.e.* assume that $2+2\neq 4$. But if 2+2 is not 4, then how much it ? So this is a new problem. 2+2=X find X, or is it that we do not want any addition ? We can't say that we do not want to add one number to another, because to collect things is a natural human instinct and to collect means to add. So we want to carry out operations like 2+2 or 3+7 or $\frac{5}{2}+\frac{4}{5}$. So we must attach some meaning to 2+2. Now we began with $2+2\neq 4$. So the problem is how much is 2+2 ? Let us find a way out. I do not want 2+2=4 so let me say that $2+2=\frac{4}{5}$ Now it is for you to prove me wrong.

One method of proving me wrong could be the following: When we write 2+2=4, we do not mean that such a result is true for 2 only. As a matter of fact we know the operation of addition and can add up any two numbers: 2+2=4, 3+3=6, 7+3=10 *etc.* So you can now ask me : 'When you say $2+2=\frac{4}{5}$, you might have used a new definition of addition or perhaps without any thought you just put down $2+2=\frac{4}{5}$ in an adhoc manner. So you can ask me a few questions. "If your 2+2 is 4/5 then What are 3+3 or 7+7 or 4+7 ?

Now it appears as if you have started taking me to task. But my mind is on the right track. I have with me a novel definition and I put down 2+2=4/5 on the basis of that definition. So I shall be able to reply to all your questions.

$$3+3=\frac{6}{10}=\frac{3}{5}, \quad 7+7=\frac{14}{50}=\frac{7}{25}, \quad 4+7=\frac{11}{29}.$$

* (The Golden-cum-Diamond Jubilee lecture of the Calcutta Mathematical Society is expected to be devoted to pedagogy. The lecture of Professor Vaidya apparently is not on padagogy but something more. In this lecture some basic results of special theory of relativity were presented in a very interesting and illustrative way, pin-pointing the basic mathematics involved. The lecture is a super illustration of how an able experienced intelligent teacher can present the topic to be taught in an ingenious intelligent manner.—*M. Dutta*)

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Can you guess my new definition of addition ?

2. *New rule of addition :* One thing becomes clear from the above discussion that 2+2=4 because of our definition of addition. If we change our definition the result of 2+2 will also change. So let us now write down in an organised manner the definition of the new process of addition suggested above.

With the new definition we must also introduce new notations. + is the symbol indicating usual addition. For the new process of addition, we shall introduce the symbol $\oplus$ and call it the 'new plus'. If a and b are any two numbers then our new definition will be :

$$a \oplus b = \frac{a+b}{1+ab}$$

One can now verify that $2 \oplus 2 = 4/5$ $3 \oplus 3 = 6/10$ *etc...*

But in a logic-oriented subject like mathematics one is not allowed to choose any definition as one likes. The definition chosen must possess certain specific properties. We know that our usual addition + has following properties :

i. $a + b = b + a$ (commutativity)

ii. $a + 0 = a$ (existence of zero)

iii. $a + (-a) = 0$ (existence of opposite)

iv. $(a + b) + c = a + (b + c)$ (associativity)

One can easily verify that our new plus $\oplus$ possesses all these properties. As an illustration we shall verify the last property.

$$(a \oplus b) \oplus c = \frac{a+b}{1+ab} \oplus c = \frac{\frac{a+b}{1+ab} + c}{1 + \frac{a+b}{1+ab}c} = \frac{a+b+c+abc}{1+ab+ac+bc}$$

This last result is symmetric in a,b and c and $a \oplus (b \oplus c)$ will also reduce to the same result *etc...*

Many conclusions can be deduced from this new addition. We shall note here only one of these conclusions because it is intimately related to Einstein's theory of relativity. Let us obtain $a \oplus 1$. From the definition

$$a \oplus 1 = \frac{a+1}{1+a.1} = \frac{a+1}{1+a} = 1 .$$

According to old $+$, only $0 + 1 = 1$. But according to new plus $2 \oplus 1 = 1$, $10{,}000 \oplus 1 = 1$. For any number a, $a \oplus 1 = 1$ *i.e.* anything $\oplus\, 1 = 1$.

This means that 1 plays the same role in new addition-rule as the role played by infinity in our usual summation. But all the numbers used in the usual summation are smaller than infinity. Since 1 takes the place of infinity in the new $\oplus$, the new summation rule will give logically consistent results only if numbers to be added are all less than 1, Enthusiastic readers may verify the above statement. However, we have reached a stage very near to Einstein's relativity and so we shall now proceed in that direction.

3. **Einstein's law of addition:** According to our new summation-rule $a \oplus 1 = 1$, *i.e.* the role of infinity in the old rule is taken up by 1 in the new rule. But we can amend our new rule in such a way that instead of 1 any other number (say 10) takes on the role of infinity. Take the new rule for sum of a and b as

$$a \oplus b = \frac{a+b}{1+\frac{ab}{100}}, \text{ Then for any } a$$

$$a \oplus 10 = \frac{a+10}{1+\frac{10a}{100}} = 10\,.$$

i.e. now in place of $a \oplus 1 = 1$ we have $a \oplus 10 = 10$.

Now it will be easy to understand Einstein's rule of addition. Imagine a river flowing with a velocity u. Suppose a swimmer who can swim with a velocity v drops into the river and swims in the direction of the flow of the river. Then we know that the flow helps the swimmer and his total velocity will be $u+v$. But in 1905, Einstein proved, with the help of his theory of Relativity that this total velocity is not $u+v$ but $u \oplus v$ where.

$$u \oplus v = \frac{u+v}{1+\frac{uv}{c^2}}$$

[c=the velocity of light : $c=3\times10^{10}$ cms/sec.]

If the swimmer can swim with the velocity c, then $v=c$ $\therefore$ the total velocity is

$$u \oplus c = \frac{u+c}{1+\frac{uc}{c^2}}$$

i.e. $u \oplus c = c$. If the swimmer's velocity is c, then however large the velocity u of the flow is, it would not add anything to the velocity of the swimmer. He will swim with his original velocity c only.

It may not be possible here to go into mathematical details of how Einstein was led to our new rule of addition. But the logical arguments used by Einstein are easy to follow. We shall now describe this logic.

4. Space, Time and Motion : We shall follow Einstein and as a first step, forget all about our common-sense notions of space, time and motion. We shall start with a clean slate. One point will be easily understood. Of the three concepts of space, time and motion, those of space and time are basic from which the concept of motion can be derived. So we must first try to clarify our notions about space and time measurements and then only formulate the concept of motion.

The concept of time in Newtonian dynamics is very simple. Newton had assumed a 'true evenflowing time, the same for all observers.' Due to this assumption the problem of measuring time-interval between any two events is similar to measuring distance between two points on a highway. On the highway a place A is near to 6.7 Km. –stone and another place B is at the 7.9 Km. –stone. Then the distance $AB = 7.9 - 6.7 = 1.2$ Km. Well, in the same way, as per Newton's assumption there is a highway of time and on this highway there are stones indicating time. An event X occures at 1946.82 and another event Y occures at 1967.93. Then the time interval between the two events will be $1967.93 - 1946.82 = 21.11$ years.

In order to follow Einstein, we shall forget the above Newtonian method of time-measurement and shall try to work out a logically consistent method of measuring this time-interval. The instrument which measures time is a clock. An event X occures at a place A at the clock-time t_1 and at the same place A another event Y occures at clock-time t_2. Then the time-interval between the two events will be $t_2 - t_1$; this seems to be logically consistent.

Now suppose that these two events X and Y occure at two different places A and B. Further suppose that the events occure at A at clock-time t_1 and at B at clock-time t_2. Will the time-interval between the two events be $t_2 - t_1$? The answer is yes provided the clocks at A and B agree *i.e.* they show the same time. But how to find out whether two clocks placed at different places show the same time ? Well, this is quite simple. Take one of these clocks say the clock at B. Carry this clock from B to A and then see for yourself whether the two clocks keep the same time ; But this involves 'carrying' a clock from B to A *i.e.* of moving a clock. But the concept of motion is to be derived from the concept of time measurement and for measuring time at two different places we need the concept of motion. This is like moving in a circle.

We can think of several methods of comparing times shown by two clocks placed at two different places. But in all these methods we shall have to move (*i e.* give 'motion') to atleast one clock or an 'observer' from one place to another. In order to define motion it is necessary to measure time-intervals between events occuring at different places and for measuring such time-intervals we need the concept of motion. This viscious circle pervades the basic concept of time measurement. Einstein suggested a simple way to break this viscious circle. In order to make the measurement of time-interval between two events at two different places logically consistent, we must predetermine a fundamental observer (*FO*). The velocity of this *FO* must also be predetermined. The motion of *FO* is predetermined. But all motions other than that of *FO* should be derived from the basic concepts of space and time measurements. If we follow this suggestion of Einstein we can use the fundamental observer for comparing clocks at two different places and so time measurement will become consistent.

Using this additional notion of *FO*, Einstein constructed a logically consistent system of Dynamics (Science of Motion). The result of the famous Michelson-Morley experiment of 1885 which could not be explained by Newton's laws of motion could be easily explained by Einstein's laws. Not only that but this experiment proved that Einstein's assumtion of the predetermined Fundamental Observer is correct and that the predetermined velocity of *FO* is the velocity of light c.

Again using this Einsteinian law if we find the resulting velocity of a swimmer swimming with a velocity V in the direction of river flowing with a velocity u, we do not get $u+v$, but

$$u \oplus v = \frac{u+v}{1+\frac{uv}{c^2}}$$

Now the velocity of the *FO* is predetermined so the above formula must also give its resultant velocity as c. Thus if our swimmer is the Fundamental Observer himself then $v=c$ and so his resultant velocity $u \oplus c$ must turn out to be c. And we know that

$$u \oplus c = \frac{u+c}{1+\frac{uc}{c^2}} = c$$

Well, 2+2=4 ; but we now know that Einstein may not agree :
