

An elementary proof of Ramanujan's conjectures <sup>one of</sup> <sup>simple</sup> <sup>in number theory</sup>  $2^{n+2} - 7 = \square$  has just five solutions given by  $n = 1, 2, 3, 5, 13$ , has been given here based on the expressions for the convergents of the continued fraction for  $\sqrt{2}$ . <sup>Putting  $\square = (2y+1)^2$</sup>  <sup>Since  $x$  is an integer</sup> ~~and~~ the conjecture is reduced to the form  $1 + \Delta y = 2^{n-1} (*)$ . <sup>by putting  $x = 2y+1$</sup>  ~~is made~~ The proof is based on a result due to A. Boutin (Dickson's History, vol 2, p. 31) that the relation  $1 + \Delta y = x^2$  ~~(\*)~~ can be reduced to the Pellian equation  $p^2 - 2q^2 = -1$  by setting  $2x = 3q \mp p$ ,  $y = \frac{1}{2}(K-1)$ ;  $K = 3p \mp 2q$ . The solution of the Pellian equation is then carried out, by using the classical method, obtaining  $p$  and  $q$  in terms of the odd convergents of the continued fraction. ~~for  $n$~~ . ~~The~~ ~~A~~ ~~detailed~~ ~~and~~ ~~A~~ detailed examination of the two sets of values of  $x$  given <sup>in</sup> by  $(*)$  thus <sup>obtained</sup> <sup>derived</sup> shows that there are ~~only~~ ~~four~~ ~~values~~ ~~of~~  $x$  <sup>which are powers of 2 viz  $x = 1, 2, 2, 2$</sup>  ~~of~~  $x$ , thereby giving the four odd values <sup>1, 3, 5, 13</sup> of  $n$  for which  $(*)$  holds. A further elementary reasoning is given to show that the only even value of  $n$  for which  $(*)$  holds is given by  $n = 2$ , thus completing the proof of the conjecture.

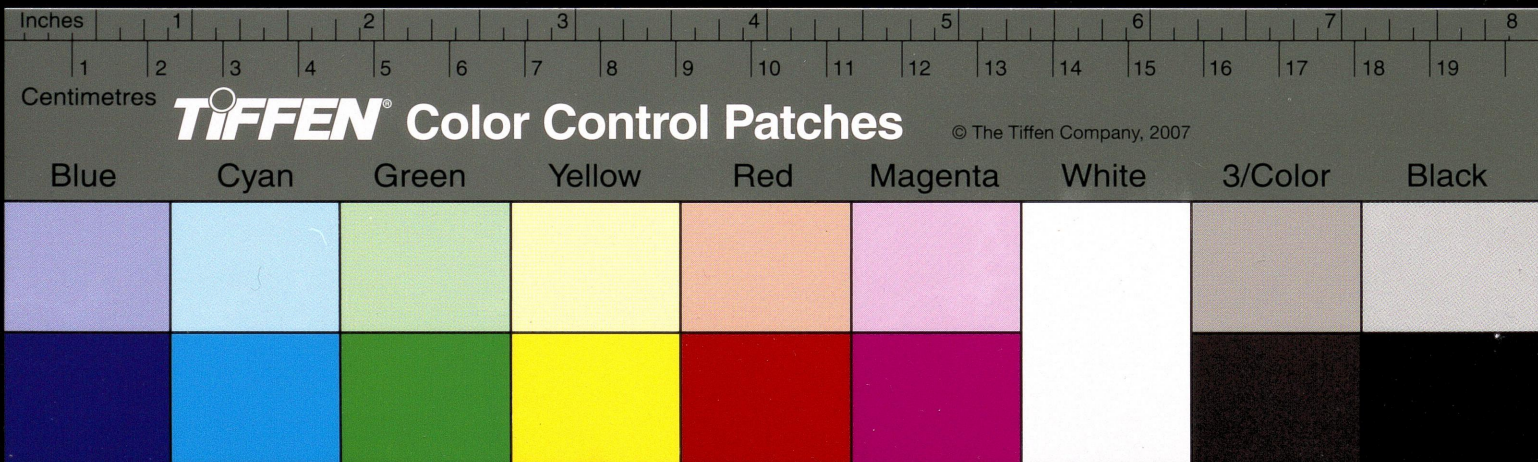
In order to show how apparently different results in elementary number theory are inter-related, three known results relating to the expressions of Fermat and Mersenne numbers as  $\Delta$ -numbers are derived from Ramanujan's conjecture. ~~Prop. Prop. of (2) on p. 448 of the paper might perhaps have been made more explicit.~~

Unfortunately, a number of misprints have occurred in the paper. Thus on p. 447 the last value of  $n$  given in the third line should be 13 instead of 12. The expression  $2^{m+1}$  in line 7, p. 448 should be  $2^m + 1$ . The page numbers 355 and 553 in the Reference at bottom of p. 449, and in the first Reference on p. 450 should both be 535. Finally ~~the~~ Reference 2 at the end of the paper ~~should be~~ <sup>is to</sup> Dickson instead of Dickens!

- For eg. (i) p. 447, <sup>third line 3,</sup> ~~for~~ last value of  $n$  ~~is~~ <sup>read 13 instead of 12;</sup> ~~12~~ <sup>for</sup> ~~12~~ <sup>B.S. Madhava Rao.</sup>
- (ii) p. 448, line 7, read  $2^m + 1$  for  $2^{m+1}$ ;
- (iii) p. 449, <sup>Ref.</sup> ~~at~~ reference at bottom, & p. 450, Ref. ~~(1)~~ should both read p. 535
- (iv) p. 450, Ref. 2, read Dickson for Dickens.

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$$\phi_n(x) = \frac{1}{\{2^n \cdot n! \cdot \pi^{1/2}\}^{1/2}} e^{-x^2/2} H_n(x)$$

Plotting  $\phi_0(x)$

In Russell's tables  $\phi_n(x)$  is mostly  $e^{-x^2/2} H_n(x)$  without any normalising factor. This makes the use of Russell's tables for higher values of  $n$  rather doubtful. Also in view of errata pointed out on p. 71 of Math. Tables for Computers Vol. 13 (1950) Nos. 65-68, p. 71 given by the three Senators viz.  $\phi_n(2.50)$  and  $\phi_n(6.60)$  are suspect for  $n > 3$  and result for  $3 < n < 7$  are suspect, I wonder how Russell's tables can be relied upon. Anyway let us try

(1)  $\phi_0(x) = \frac{1}{\pi^{1/4}} e^{-x^2/2} H_0(x) = \frac{1}{\pi^{1/4}} e^{-x^2/2}$   ~~$\phi_0^2 = \frac{1}{\sqrt{\pi}}$~~   $\phi_0^2 = \frac{1}{\sqrt{\pi}} e^{-x^2} H_0^2(x) = \frac{e^{-x^2}}{\sqrt{\pi}}$

|      |                |
|------|----------------|
| $x$  | $\phi_0^2$     |
| 0.00 | $1/\sqrt{\pi}$ |
| 0.04 |                |

But fig. in Durkman shows  $\phi_0^2(0) > 1$  (?)

Take classical case  $P = \frac{1}{\pi \sqrt{2n+1-x^2}}$

which for  $x=0$ ,  $P = \frac{1}{\pi \sqrt{1-x^2}} = \frac{1}{\pi}$  for  $x=0$

i.e.  $P(0) < \phi_0^2(0)$  all right

but the above  $\phi_0^2(0) > 1$  is still a riddle

For  $x=0.04$ ,  $\phi_0^2 = \frac{(0.99920)^2}{\sqrt{\pi}}$

Pauling & Wilson gives  $N_n^2 = \left(\frac{\alpha}{\pi}\right)^{1/2} \cdot \frac{1}{2^n \cdot n!}$  which is  $\frac{1}{N_n^2}$  of Durkman

$$\phi_0^2(0) = \frac{\phi_0^2 \left(\frac{\alpha}{\pi}\right)^{1/2}}{\pi^{1/2}} = \frac{\sqrt{\alpha}}{\sqrt{\pi}}$$

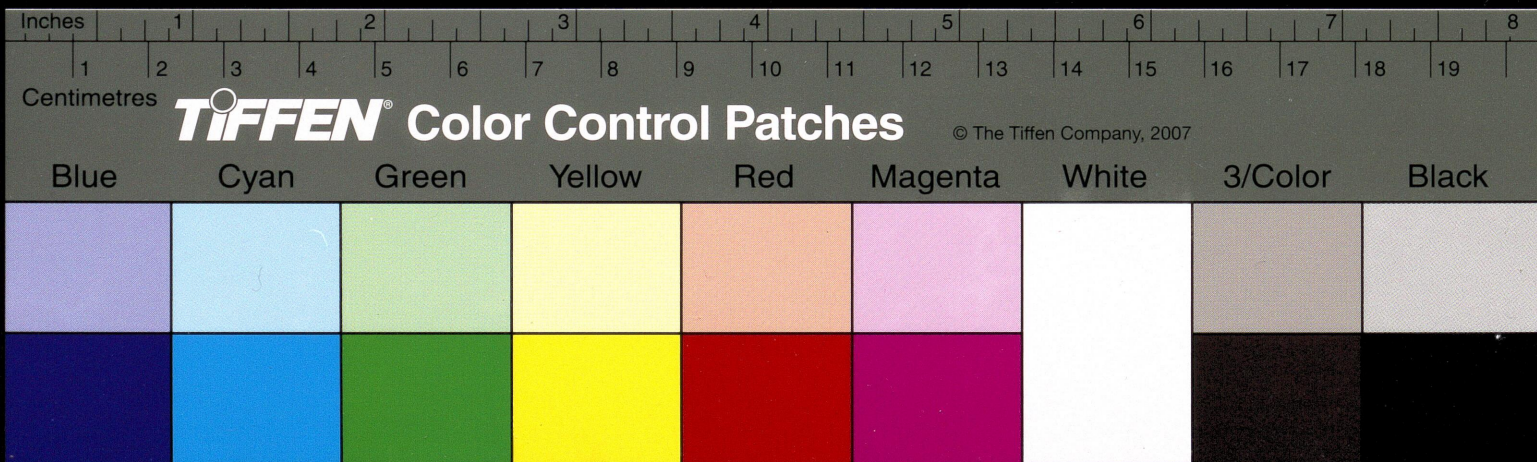
$$2^{n+2} - 7 = (2y+1)^2$$

$$2^{n+2} - 7 = 4y^2 + 4y + 1$$

$$2^{n+2} = 8 + 4y(y+1)$$

$$2^{n-1} = 1 + \frac{1}{2} y(y+1)$$

$$1 + \Delta_y = 2^{n-1}$$





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*soln of the*  
The Pellian equation is ~~solved~~ *solved* by the classical method which gives using the  
continued fraction  $\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \dots}}$  which gives  $\frac{p}{q}$  gives

$$2^{n+2} - 7 = \square$$

- $n = 1 \rightarrow 8 - 7 = 1 = 1^2$
- $n = 2 \rightarrow 16 - 7 = 9 = 3^2$
- $n = 3 \rightarrow 32 - 7 = 25 = 5^2$
- $n = 5 \rightarrow 128 - 7 = 121 = 11^2$
- $n = 12 \rightarrow 16384 - 7 = 16377$

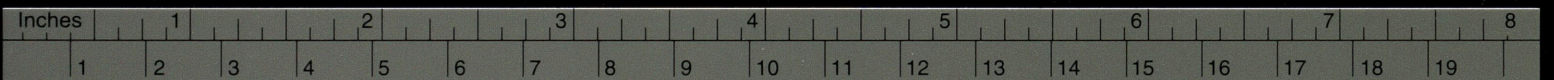
$$\begin{array}{r}
 181 \times 181 \\
 \hline
 181 \\
 1448 \\
 \hline
 181 \\
 \hline
 32768 + 7 = 32768
 \end{array}$$

$$32768 = 4 \times 8192$$

16384

$$\begin{aligned}
 2^{14} &= 2^5 \times 2^5 \times 2^4 \\
 &= 32 \times 32 \times 16 \\
 &= 1024 \times 16 \\
 &= 16384
 \end{aligned}$$

b 2<sup>15</sup>



Centimetres

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