A. M. D. G.

ST. JOSEPH'S COLLEGE

TRICHINOPOLY

NOTE BOOK

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Form B.Sc. (Hons)

Subject Light

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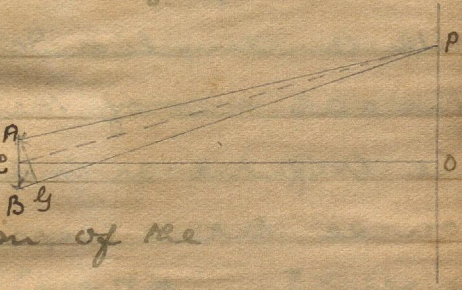
Interference

1-2-38 Conditions of a perfect pattern – As already seen (cf – B.A. Notes) a pattern is perfect if two sources of light continuously emitting light of the same intensity, frequency & phase are made to illuminate the same medium. The pattern is clear if the sources are very near each other & if the screen which catches the images is at a great distance.

Bandwidth



Let A & B be the sources, PO the screen, P the position of the n^{th} bright band.

It can be proved ~~the~~ analytically that BG should be $n\lambda$ where n is zero or an integer.

Let y_1, y_2 be displacements at P due to the two sources.

$y_1 = a \sin \omega t \quad = a \sin 2\pi \frac{v}{\lambda} t.$

$y_2 = a \sin(\omega t + \alpha) = a \sin \frac{2\pi}{\lambda}(vt + \delta)$

Where α is a phase difference due

to the retardation $\delta = Bq. \quad \alpha = \frac{2\pi\delta}{\lambda}$

$y_1 + y_2$ is the resultant displacement

$\therefore Y = y_1 + y_2 = a(\sin \omega t + \sin \omega t \cos\alpha + \cos\omega t \sin\alpha).$

$= a\left(1 + \cos \frac{2\pi\delta}{\lambda}\right) \sin \omega t + \sin \frac{2\pi\delta}{\lambda} . \cos \omega t.$

When $\delta = 0$ or $n\lambda$

$Y = 2a \sin \omega t$

ie when ~~posto ret~~ the particle vibrates

with an amplitude $2a$ & frequency the

same as that of the two sources.

The brightness is 4 times that of each

source taken separately.

) when $\delta = \frac{2n+1}{2} \lambda$

$Y = 0$

ie there is no vibration; destructive

interference results in complete darkness.

Hence if P is the position of the n^{th}

bright band, we have $Bq = n\lambda$

& from ||r Δs ABG & OPC,

$$\frac{n\lambda}{d} = \frac{OP}{D} \quad (\text{taking } PC = PO = D)$$

$$\therefore n\lambda = \frac{d}{D} OP$$

& the bandwidth $\beta = \frac{D}{d}\lambda$.

It now becomes evident that perfect definition of bright bands is obtained only if the source is perfectly monochromatic. If not, the band exten over a certain breadth & so ∴ the total no. of bands observed will be sma

If white light is used the centr band is seen white, & all the other bands are violet inside & red outsid After a small number of bands the light is confused.

The locus of the point P is not exactly a st. line. It moves in such a way that the difference of its distan from two points is a constant $n\lambda$ & ∴ the locus is a ~~para~~ hyperbola. If taken Space, the surface traced by P is a

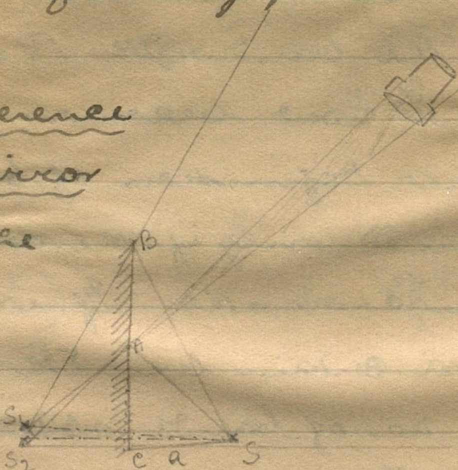
A.

e ~~par~~hyperboloid. The foci of the ~~par~~hyperbola re very near each other & ∴ ~~their~~ its section n the focal plane of the eyepiece is almost a st. line.

<u>Expts in interference</u>

<u>Fresnel's Bimirror</u>

Let AB, AC be the two mirrors inclined at an ∠ to each other.



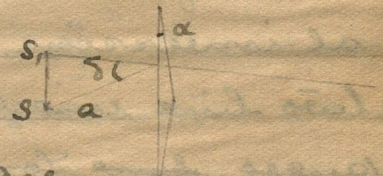
θ should be very small & is measured by a micrometer. First the mirror AB is brought in the same plane as AC (as can be seen if the two images of a source coincide) & ~~then~~ the screw head is made to touch AB. The screw head is advanced through a small distance & thus the reqd inclination θ produced.

If $S_1 S_2$ be the two images of S, $\angle S_1 S S_2 = \theta$

$\therefore S_1 S_2 = 2a\theta$. If b be the distance of the eyepiece from the mirror $\beta = \frac{b+a}{2a\theta}\lambda$.

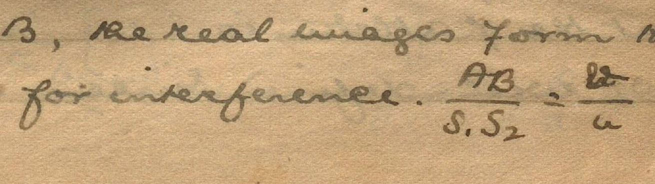
The mirrors should both be metallic, & ∴ having only the surface of reflection. There is a practical inconvenience due the hinge concealing a certain portion of the ~~image~~ light.

<u>Biprism</u> $\beta_2 - \beta_1 = \frac{D_2 - D_1}{\sqrt{d_1 d_2}}\lambda$ is the formula used as fully explained in the record book. If the ∠ of the prism is α, the distance between the sources can be calculated as follows. Since i & r are small, $\delta = (\mu - 1)\alpha$ & ∴ $SS_1 = a(\mu - 1)$



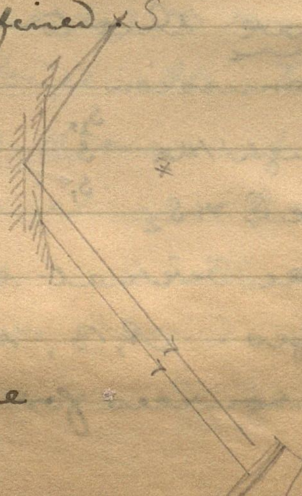
∴ $d = 2a(\mu - 1)\alpha$

<u>Biplate</u> Due to transmission through the plate S_1 & S_2 are the virtual sources & A B their images. A, B, the real images form the two sources for interference. $\frac{AB}{S_1 S_2} = \frac{v}{u}$



r $S_1S_2 = \frac{2e\sin(i-r)}{\cos r}$. Since i approxi-
nates to 90° & r to the critical ∠, the
xpression can be evaluated by the
igonometrical expansion.

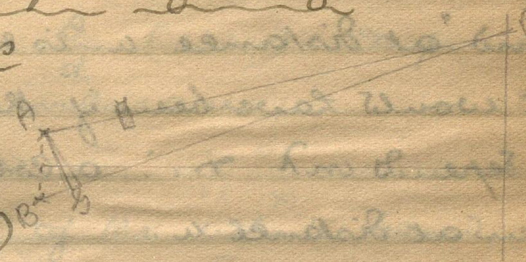
<u>Lloyd's Single Mirror</u> The defects of the
attern are many. First ~~the~~ only one
ide of the pattern is formed. Secondly,
ince one is reflected & the other
ot, one of the sources suffers a
ateral inversion. Hence ||| parts of
he two line images are at various
istances; ~~since~~ for the lines must neces-
sarily be of some thickness. The consequence
o that β ~~the~~ attains various values;
the bands are ill-defined



<u>resnels three mirror</u>
It have been devised
o avoid the inconve-
ience of the hinges,
ut the pattern is
ot well-defined since.

one of the images is only once reflec
whereas the other is twice reflected

Thin film in the path of one of the interfering beams



δ the path difference

$= BP - (AP - e + \mu e)$

$= BP - AP - (\mu - 1)e$

$= \frac{x}{D} d - (\mu - 1)e$ ∴ Since $\delta = n\lambda$

$x = \frac{D}{d}\{n\lambda + (\mu - 1)e\}$

This is the shift when monochromatic light is used.

10-2-'38 Shift under Composite light Stokes worked out mathematically the shift when composite light is used instead of a monochromatic light. If the light be of one wavelength λ, the distance x from the central pt O of the n^{th} ~~band~~ bright band is

$$x = \frac{D}{d}\{n\lambda + \overline{\mu - 1}\, e\}.$$

The central ~~pt~~ band is also shifted by $\frac{D}{d}(\mu-1$

Let $u = \frac{D}{d}(\mu - 1)e$.

Now when Composite light replaces his monochromatic source the "Central band" at distance u is no more white as it would have been if there had been no film. ... depends on λ & ∴ a Coloured band is seen at distance u.

If the film is removed this Coloured band will shift to O, & become white.

Now it is possible to have a particular band on which all the Colours overlap & the Colour of which is ~~bright~~ white. Let it be ... n the band at distance x from O

$$x = \frac{D}{d}\left(n\lambda + \overline{\mu - 1}\, e\right)$$

Since the rate of change of x with λ should ... be zero, ie $\frac{dx}{d\lambda} = 0$, $n + \frac{d}{d\lambda}\left\{\overline{\mu - 1}\, e\right\} = 0$.

Now $(\mu - 1)e \times \frac{D}{d} = u$

$$\therefore \frac{d}{d\lambda}(\mu - 1)e = \frac{d}{D}\frac{du}{d\lambda}.$$

∴

$$x = \frac{D}{d}\left\{-\frac{d}{D}\frac{du}{d\lambda}.\lambda + \overline{\mu - 1}\, e\right\}$$

$$= \frac{\theta}{d}\left\{-\frac{d}{\theta}\,\frac{du}{d\lambda}.\lambda + \frac{d}{\theta}.u\right\}$$

$$= u - \frac{du}{d\lambda}.\lambda$$

& Since $u = \frac{\theta}{d}(\mu-1)e$ & μ decreases as λ increases,

$\frac{du}{d\lambda}$ is $-$ive

$\therefore$ The "shift" of the white band, i.e. the distance of the position of the new white band from its original position when there had been no film is

$$u + \lambda\frac{du}{d\lambda}.$$

The same expn̤ can be express as $u + \beta.\frac{du}{d\beta}$. For $\beta = \frac{\theta}{d}.\lambda$; (1)

$u = \frac{\theta}{d}.(\mu-1)e$ (2)

$$\frac{du}{d\lambda} = \frac{du}{d\beta}.\frac{d\beta}{d\lambda} = \frac{du}{d\beta}.\frac{\theta}{d}$$

& $\lambda.\frac{\theta}{d} = \beta$

$$\therefore x = u + \lambda.\frac{du}{d\lambda}.$$

If ~~white~~ monochromatic light had been used the shift of the central band would have been u. Now when composite light is sed the corresponding shift is

$$u + \lambda \frac{du}{d\lambda}$$

Hence the relative error due to composite light is

$$\frac{\lambda \frac{du}{d\lambda}}{u}.$$

Since $u = \frac{\theta}{d}(\mu - 1) e$

$$\frac{du}{d\lambda} = \frac{\theta}{d} e \frac{d\mu}{d\lambda}$$

$$\therefore \frac{\lambda}{u} \cdot \frac{du}{d\lambda} = \frac{\lambda}{\frac{\theta}{d}(\mu-1) e} \times \frac{\theta}{d} \cdot e \frac{d\mu}{d\lambda}$$

ie Relative error $= \frac{\lambda}{\mu - 1} \cdot \frac{d\mu}{d\lambda}$.

Now by Cauchy's formula $\mu = A + \frac{B}{\lambda^2}$

$$\therefore \frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^3}$$

$$\therefore \text{Relative error} = \frac{\lambda}{\mu-1} \cdot -\frac{2B}{\lambda^3} = \frac{-2}{\mu - 1} \cdot \frac{B}{\lambda^2}$$

$$= \frac{-2(\mu - A)}{\mu - 1}$$

If A is known, the relative error can be calculated. A is known by expt. by determining the μ for two of the Fraunhofer lines ~~of the~~ of which the wave lengths are known. When substituted in the formula

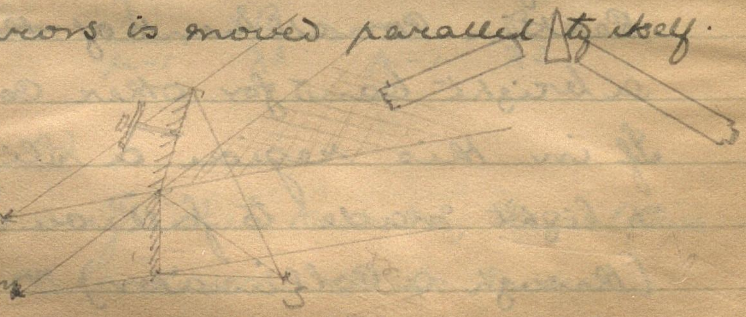
$\mu = A + \frac{B}{\lambda^2}$, two simultaneous eqns are obtained from which A can be calculated.

Interference under relatively high path retardation — Spectroscopic analysis.

Theoretically, under monochromatic light, the number of fringes visible should be without limit. But in practice no source is absolutely monochromatic & ∴ after the few first fringes have ~~been~~ passed there comes a region where it is a dark band for certain constituents of the source & a bright band for other constituents. If in this region a slit be opened, & light made to fall on a prism (through a collimator) & the spectrum be examined a ~~narr~~ strip of a com-

lete spectrum will be seen — the
trip say in the neighbourhood of the D_1, D_2
ines, if sodium source is ~~observed~~
sed. In this strip will be seen a
e dark bands showing that ~~the~~ certain components of light
ve been extinguished by interference.

Fizeau & Foucault made a study of this
henomenon by Fresnel's mirrors. The
ource used was of composite light. In-
tead of moving the Collimator slit from
ntre to side ~~to~~ in order to give path
ifference, they employed an easier device.
e Collimator slit was directed towards
Centre of the interference pattern. The
pectrum shows all the Colours. ~~The~~ One
the two mirrors is moved parallel to itself.
e effect is to
hift the whole
attern to one
ide away from
e moving mirror.



Fizeau — Foucault's expt

When the collimator slit reached the position of the first dark violet, ie retardation = $\frac{D}{d} \times \frac{4047}{2}$ A.U., a dark line is seen at the violet end. As the mirror is moved forward the line pas from the violet to the red end. When the line is in red δ (for the slit) is $\frac{D}{d} . \frac{7660}{2}$. Another line follows, which reaching the red gives ~~D~~ δ to be $\frac{D}{d}$ 3 $\frac{7660}{2}$ which is equal to $\frac{D}{d} \times 5 \times \frac{4596}{2}$. Hence there is another line which has appeared & has by this time reached the green. When this 3rd line is in the red a fourth will be seen in green (5471 A.U.) & a fifth in blue (4256 A.U.) Thus as the mirror is moved more and more the number of lines ~~Su~~ crowd together on the spectrum. In regions where the ordinary eyepiece shows uniform illumination, the spectroscope reveals a spectrum crow

y a series of dark bands at equal
istances. Thus we have a spectroscopic
vidence for interference in regions
ar away from the central band.

From the closeness of the lines we
an calculate the relative path retar-
ation as follows. Suppose the line in
e red ($\lambda_1 = 7660$) is the 20^{th}.

$= \frac{\theta}{d} \times \frac{39}{2} \times \lambda_1$. The 21^{st} line is of λ_2 7300
. e. in the red itself; the 22^{nd} is also
n the red of λ_3 6940.

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{1}{\delta} \cdot \frac{\theta}{d} \times 1$$

$$\frac{1}{\lambda_3} - \frac{1}{\lambda_1} = \frac{1}{\delta} \cdot \frac{\theta}{d} \times 2$$

If altogether $n+1$ lines are visible,

$$\frac{1}{\lambda_{n+1}} - \frac{1}{\lambda_1} = \frac{1}{\delta} \cdot \frac{\theta}{d} \times n.$$

Now by directing a beam of sunlight
the collimator slit, Fraunhofer lines
an be superposed on the interference

spectrum. ~~Suppose~~ If any two of them of known λ coincide with two of the interference lines, then,

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{1}{\delta} \cdot \frac{D}{d} n$$

where $(n-1)$ is the number of interference lines between the two ~~tines~~ two lines Fraunhofer lines & λ_2, λ_1, the wavelength of

$$\therefore \delta = \frac{nD}{d}\left\{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right\}.$$

Or if N be the number of fringes ~~we ha~~ between the two lines we have

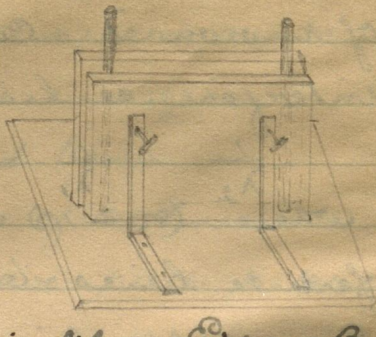
$$\delta = \frac{N+1}{\left\{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right\}}$$

Butler's figures — Arrangement of thin films due to Edser & Butler:

The above formula can be utilized to determine the the small thicknesses (say of wires) or to find the wavelength of an unknown Fraunhofer line.

Two glass plates having plane sides

semisilvered on the
inner face are placed
|| to each other with
two bits of a thin wire
between them. When
white light falls upon



it, part is transmitted, & Thin film – Edser & Butler.
part doubly reflected
then transmitted thus introducing a relative
retardation 2e (incidence is normal)
The silvering should be such that the
direct & reflected beams ~~of~~ are nearly of
the same intensity. Spectroscopic analysis
will show a pattern exactly as in the
Fizeau Foucault expt with Fresnel mirrors.

$$\delta = 2e = \frac{N+1}{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}}$$

where λ_2 & λ_1 are the wavelengths of
two Fraunhofer lines which coincide
with two of the interference lines &
enclose N lines between them. The

two sets of lines can be a easily distinguished as the former are very thin. If λ_1 & λ_2 are known, e can be calculated. Also λ of any unknown line can be found out.
Let there be n interference lines betw. λ & λ_2

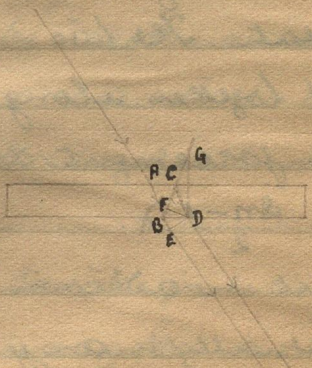
$$\therefore \frac{1}{\lambda_2} - \frac{1}{\lambda} = \frac{n+1}{2e} \; ; \quad (1)$$

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{N+1}{2e} \; ; \quad (2)$$

Dividing (1) by (2) λ can be calculated. e can be separately determined from (2)

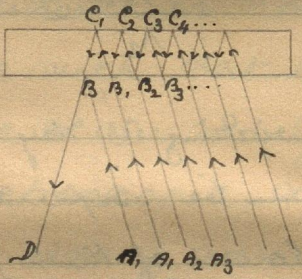
<u>Colours of thin films</u>

It has been shown (B. A. Notes) that path BE in air is equivalent to path BF in the film n. $\therefore$ path difference is FG $= 2\mu e \cos r$.



Haidinger's Fringes When a film of 12-2-'38
ransparent medium of with ∥l sides is
llumined by an extended source of
ight, & the reflected or refracted light is
iewed through a lens, a system of clear fringes is
en which is called Haidinger's fringes.

When investigating the colours of
hin films we considered two ∥l in-
ident ~~beams~~ rays
B & $A_1 B_1$ the first
which ~~emerging~~ re-
lected at B & the
ther reflected at the
arther surface C_1.
~~nd that~~ The two



avel together along BD and cause interference
reinforcement according as $2\mu e \cos r =$
λ or $\frac{2n-1}{2}\lambda$.

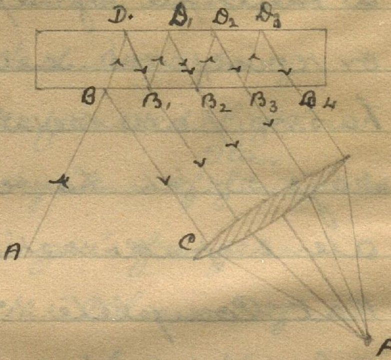
But in addition to the ray B, C, B, there
e actually many others refracted & reflected
number of times and emerging at BD, all

of which have a path retardation which is a multiple of $2\mu e \cos r$. Hence interference or reinforcement is between ABD & which has no path retardation & all the other rays which have $(n \times 2\mu e \cos r + \frac{\lambda}{2})$ equivalent path retardation. Hence it is that complete & not partial absence of colour is observed in the dark fringes.

The direction of the rays can be rever[sed]. A single ray DB incident on the film gives rise to one reflected beam BA & a series of multiple refracted beams $B_1 A_1$, $B_2 A_2$ .... If $2\mu e \cos r = n\lambda$ the effect of bringing all these rays together to a common focus will be destructive interference. If a point source is used & the film be viewed through a lens, it will appear dark.

If an extended source is used light falls at different angles on the plate & hence there are many convergen[t]

encils of light issuing
rom the lens C to pts
t and near the focus.



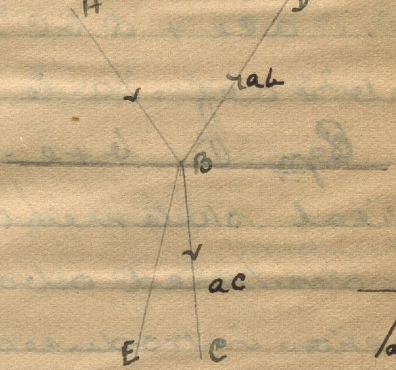
ll the ~~bear ray~~ inci-
ent rays which
ave the same value
r $2\mu t \cos r$ (i.e.
r r since $2\mu t$ is
nstant) will form one Convergent pencil.
s ✱ Those for which $2\mu t \cos r$ is greater by $\frac{\lambda}{2}$ will form
series of foci ✱ round the focus of the
ns i.e. will form the first bright ring.
round it is formed ~~and~~ a dark ring & thus
system of rings is formed.

Hardinger's fringes in Reflected light
n be observed if a mercury arc
ith a filter is made to illumine the
~~[illegible]~~ plate.

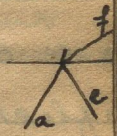
A similar effect is observed in
ansmitted light also, the one difference
ing that a ring is bright when $2\mu t \cos r = n\lambda$.

Directing a thin film towards a bright cloud the effect is seen distinctly. The lens of the ~~e~~ eye forms the fringes upon the retina.

Reflection & Refraction Coefft.



Let a beam AB of amplitude a be partially reflected along BD & partially refracted along BC.



The ratio of the amplitude of BD to a is the Reflection Coefft. Let it be b. The ratio of the ampl. of BC to a is the refraction Coefft. Let it be c. Amplitude of BD is ab & of BC is ac. Let refl. & refr. coeffts from the lower to the upper medium be e & f respectly.

Consider both the rays DB & CB travell back along their own paths. DB will split into one of ~~ampl~~ amplitude ab^2 along BA & another of amp. abc along BE. CB will split into one of amp. ace along BE & another of cf along BA. The net result should be the original ray, ie ampl. a along BA & zero along BE.

$\therefore ace + abc = 0$ ie $b + e = 0$ (1)

& $ab^2 + acf = a$ ie $b^2 + cf = 0$; $cf = 1 - b^2$ (2)

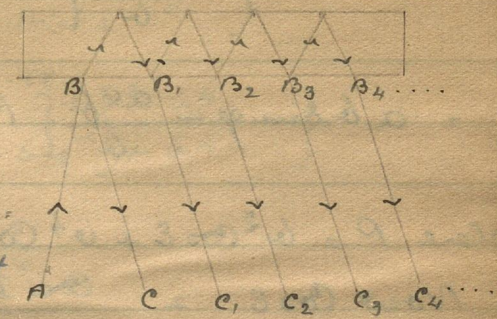
Eqn (1) $b + e = 0$ is the mathematical statement of the fact we have assumed above that dissimilar reflection introduced a phase change of π. Since cf is +ive there is no phase change under dissimilar refrn.

Calculation of intensity in Haidingers Fringes (I) Reflected Light

Let the displacement at B when AB is incident on it be $y = a \sin \varphi$.

The amplitude of BC is ab; of B_1C_1 $a \times c \times e \times f$, of B_2C_2 $a \times c \times e^3 \times f$, of B_3C_3 ace^5f etc.

In BC there is no phase change; in B_1C_1 phase change is due partly to the path retardation $2\mu e \cos r$ & partly due to dissimilar refl. That due to dissimilar refl. is ~~compensated~~ expressed in the displacement eqn by



the sign of e. & that due to $2\mu e \cos r$ is $\varepsilon = \frac{2\pi}{\lambda} 2\mu e \cos r$.

In $B, C,$ phase change is 2ε ...

$\therefore$ If y_1, y_2, y_3 .... be ~~a~~ displacements disturbances at B, B_1, B_2, B_2 ....

$$y_1 = ab \sin\varphi$$
$$y_2 = ace f \sin(\varphi + \varepsilon)$$
$$y_3 = ace^3 f \sin(\varphi + 2\varepsilon)$$
$$y_4 = ace^5 f \sin(\varphi + 3\varepsilon) \quad \dots \quad \dots \quad \dots$$

etc etc.

The sum total of the disturbances is

$$Y = y_1 + y_2 + y_3 + \dots\dots$$
$$= ab\sin\varphi - acbf\sin(\varphi+\varepsilon) - acb^3 f \sin(\varphi + 2$$
$$- \dots\dots \quad \text{ad inf.}$$

Note The series may be summed up & A^2+B^2 found out as on page 95 A^2+B^2 $(A+iB)(A-i$

$$= ab\sin\varphi - \frac{acf}{b}\left\{ b^2\sin(\varphi+\varepsilon) + b^4\sin(\varphi+2\varepsilon) \ldots \right\}$$

$$= ab\sin\varphi - \frac{acf}{b}\left\{ P\sin\varphi + Q\cos\varphi \right\}$$

where $P = b^2\cos\varepsilon + b^4\cos 2\varepsilon + \ldots\ldots$

Now $\cos\varepsilon = \frac{e^{i\varepsilon} + e^{-i\varepsilon}}{2}$

$$\therefore\ P = \frac{1}{2}\left\{ \begin{array}{l} b^2e^{i\varepsilon} + b^4e^{2i\varepsilon} + b^6e^{3i\varepsilon} + \ldots\ldots \\ + b^2e^{-i\varepsilon} + b^4e^{-2i\varepsilon} + b^6e^{-3i\varepsilon} + \ldots\ldots \end{array} \right\}$$

$$= \frac{1}{2}\left\{ \frac{b^2e^{i\varepsilon}}{1-b^2e^{i\varepsilon}} + \frac{b^2e^{-i\varepsilon}}{1-b^2e^{-i\varepsilon}} \right\}$$

$$= \frac{1}{2}\left\{ \frac{b^2e^{i\varepsilon} - 2b^4 + b^2e^{-i\varepsilon}}{1 - b^2e^{i\varepsilon} - b^2e^{-i\varepsilon} + b^4} \right\}$$

$$= \frac{b^2\cos\varepsilon - b^4}{1 - 2b^2\cos\varepsilon + b^4}$$

$$Q = b^2\sin\varepsilon + b^4\sin 2\varepsilon + \ldots\ldots$$

Now $\sin\varepsilon = \frac{e^{i\varepsilon} - e^{-i\varepsilon}}{2i}$

$$\therefore\ Q = \frac{1}{2i}\left\{ \begin{array}{l} b^2e^{i\varepsilon} + b^4e^{2i\varepsilon} + \ldots \\ - b^2e^{-i\varepsilon} - b^4e^{-2i\varepsilon} - \ldots \end{array} \right\}$$

$$= \frac{1}{2i}\left\{\frac{b^2e^{i\varepsilon}}{1-b^2e^{i\varepsilon}} - \frac{b^2e^{-i\varepsilon}}{1-b^2e^{-i\varepsilon}}\right\}$$

$$= \frac{1}{2i}\left\{\frac{b^2e^{i\varepsilon} - b^2e^{-i\varepsilon} + b^4 - b^4}{1-(b^2e^{i\varepsilon}+b^2e^{-i\varepsilon})+b^4}\right\}$$

$$= \frac{b^2\sin\varepsilon}{1-2b^2\cos\varepsilon+b^4}.$$

$$y = ab\sin\varphi - \frac{acf}{b}\left\{P\sin\varphi + Q\cos\varphi\right\}$$

$$= A\sin\varphi + B\cos\varphi$$

where $A = ab - \frac{acf}{b}P$ and $B = -\frac{acf\,Q}{b}$

$= 0$

$$A = ab\left\{1 - cf\cdot\frac{1}{b^2}\left(\frac{b^2\cos\varepsilon - b^4}{1-2b^2\cos\varepsilon+b^4}\right)\right\}$$

$$= ab\left\{1-(1-b^2)\frac{\cos\varepsilon - b^2}{1-2b^2\cos\varepsilon+b^4}\right\}$$

$$= ab\left\{1 - \frac{\cos\varepsilon - b^2 + b^4 - b^2\cos\varepsilon}{1-2b^2\cos\varepsilon+b^4}\right\}$$

$$= ab\left\{\frac{1-b^2\cos\varepsilon - \cos\varepsilon + b^2}{1-2b^2\cos\varepsilon+b^4}\right\}$$

$$= ab\left\{\frac{(1+b^2)(1-\cos\varepsilon)}{1-2b^2\cos\varepsilon+b^2}\right\}$$

ie $A = \dfrac{2ab(1+b^2)\sin^2\frac{\varepsilon}{2}}{1-2b^2\cos\varepsilon+b^4}$. (1) 17-2-'38

$$B = -\frac{acf}{b}Q$$

~~$= -a(1-b^2$~~

$$= -\frac{a(1-b^2)}{b}\cdot\frac{b^2\sin\varepsilon}{1-2b^2\cos\varepsilon+b^4}$$

$$= \frac{-2ab(1-b^2)\sin\frac{\varepsilon}{2}\cos\frac{\varepsilon}{2}}{1-2b^2\cos\varepsilon+b^4} \quad (2)$$

Thus we have $y = A\sin\varphi + B\cos\varphi$ A & B being given by (1) & (2). y is the resultant of two S. H. Ms in along directions ⊥ to each other &.: ~~the curve~~ is of the form $C\sin(\varphi+\varepsilon)$

Equating coeffts of $\sin\varphi$ & $\cos\varphi$,
$\sin\varepsilon = B$ & $C\cos\varepsilon = A$.

The ~~I~~ intensity I_R of the reflected beam is $C^2 = A^2 + B^2$

$$\therefore I_R = \left\{\frac{2ab\sin\frac{\varepsilon}{2}}{1-2b^2\cos\varepsilon+b^4}\right\}^2\left\{(1+b^2)^2\sin^2\frac{\varepsilon}{2} + (1-b^2)^2\cos^2\frac{\varepsilon}{2}\right\}$$

$$= \left\{\frac{2ab\sin\frac{\varepsilon}{2}}{1-2b^2\cos\varepsilon+b^4}\right\}^2\left\{1+b^4-2b^2\left(\cos^2\frac{\varepsilon}{2}-\sin^2\frac{\varepsilon}{2}\right)\right\}$$

$$= \frac{4a^2b^2\sin^2\frac{\varepsilon}{2}}{1-2b^2\cos\varepsilon+b^4}.$$

To discuss the conditions for max. & min. brightness a slightly different form is more convenient.

$$I_r = \frac{4a^2b^2\sin^2\frac{\varepsilon}{2}}{1+b^4-2b^2(1-2\sin^2\frac{\varepsilon}{2})}$$

$$= \frac{4a^2b^2}{\frac{(1-b^2)^2}{\sin^2\frac{\varepsilon}{2}}+4b^2}$$

I_r is a max. when den. is a minimum i.e. when $\sin^2\frac{\varepsilon}{2}$ is a max. viz. 1

$\therefore$ when $\sin\frac{\varepsilon}{2}$ is ± 1 ie $\frac{\varepsilon}{2}$ is $(2n+1)\frac{\pi}{2}$

But $\varepsilon = \frac{2\pi}{\lambda} 2\mu e \cos r$

$$\therefore\ 2\mu e \cos r = \frac{\varepsilon}{2}\cdot\frac{\lambda}{\pi} = (2n+1)\frac{\lambda}{2}$$

The film appears bright by reflected light when $2\mu e\cos r = (2n+1)\frac{\lambda}{2}$.

I_r is a min. when denominator is a max. i.e. when $\sin^2\frac{\varepsilon}{2}$ is zero; i.e. when $\frac{\varepsilon}{2} = n\pi$. & since $\frac{\varepsilon}{2} = \frac{\pi}{\lambda} 2\mu e\cos r$,

$$2\mu e \cos r = n\lambda.$$

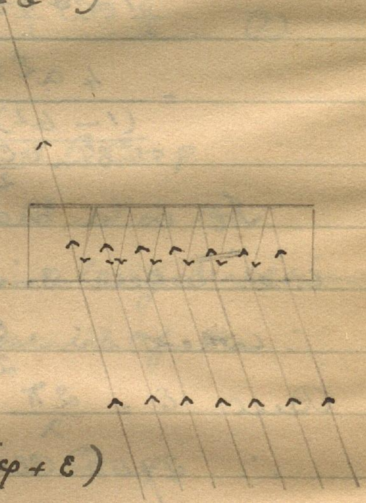
The film appears dark (or min. brigh

hen $2\mu e \cos r = n\lambda$.

$$I_r = \frac{4a^2b^2}{\left(\frac{1-b^2}{\sin \varepsilon/2}\right)^2 + 4b^2}$$

The min. value is zero since in this
ase the denominator is infinite

The max. value is $\frac{4a^2b^2}{(1+b^2)^2}$



I) Transmitted light.

Using the symbols of
art I, y the disturbance
the emitted ray is given
y

$$\begin{aligned} y &= y_1 + y_2 + y_3 + \ldots\ldots \\ &= acf \sin\varphi + ace^2 f \sin(\varphi+\varepsilon) \\ &\quad + ace^4 f \sin(\varphi + 2\varepsilon) + \ldots\ldots \quad \text{ad inf.} \\ &= a(1-b^2)\sin\varphi + acf\{b^2\sin(\varphi+\varepsilon)+\ldots\ldots\} \\ &= a(1-b^2)\sin\varphi + acf(P\sin\varphi + Q\cos\varphi) \\ &= D\sin\varphi + E\cos\varphi \end{aligned}$$

where $D = a(1-b^2) + acfP$

$E = acfQ$.

$$\mathcal{D} = a\,e\,f\,(1+\rho)$$

$$= a(1-b^2)\left\{1 + \frac{b^2\cos\varepsilon - b^4}{1-2b^2\cos\varepsilon + b^4}\right\}$$

$$= a(1-b^2)\left\{\frac{1-b^2\cos\varepsilon}{1-2b^2\cos\varepsilon+b^4}\right\}$$

$$\mathcal{E} = a(1-b^2)\left\{\frac{b^2\sin\varepsilon}{1-2b^2\cos\varepsilon+b^4}\right\}$$

If I_t is the intensity of the transmitted beam, $I_t = \mathcal{D}^2 + \mathcal{E}^2$

i.e. $I_t = \left\{\frac{a(1-b^2)}{1-2b^2\cos\varepsilon+b^4}\right\}^2\left\{(1-b^2\cos\varepsilon)^2 + (b^2\sin$

$$= \frac{a^2(1-b^2)^2\{1-2b^2\cos\varepsilon + b^4(\cos^2\varepsilon+\sin}{(1-2b^2\cos\varepsilon+b^4)^2}$$

$$= \frac{a^2(1-b^2)^2}{1-2b^2\cos\varepsilon+b^4}.$$

I_t is max. when den. is min. i.e when the -ive term $2b^2\cos\varepsilon$ is max.
The max. value of $\cos\varepsilon$ is $+1$ n. $\therefore$ $\varepsilon = 2n\pi$

$\varepsilon = \frac{2\pi}{\lambda}\, 2\mu e\cos r$

$\therefore 2\mu e\cos r = n\lambda$

∴ The film is seen brightest by transmitted light when $2\mu e \cos r = n\lambda$.

I_t is min. when $2b^2\cos\varepsilon$ is min. i.e. when $\varepsilon = (2n+1)\pi$ thus making $\cos\varepsilon = -1$

∴ $2\mu e \cos r = (2n+1)\frac{\lambda}{2}$

∴ The film appears least bright when $\mu e \cos r = (2n+1)\frac{\lambda}{2}$.

The max. value is $\frac{(1-b^2)^2 \cdot a^2}{1-2b^2+b^4} = a^2$

& the min. value is $a^2\left\{\frac{1-b^2}{1+b^2}\right\}^2$.

Thus for reflected light the intensity varies between zero & $a^2 \cdot \frac{4b^2}{(1+b^2)^2}$.

For transmitted light the intensity varies between $a^2 \cdot \frac{(1-b^2)^2}{(1+b^2)^2}$ & a^2.

The sum of the reflected & transmitted intensities at any instant

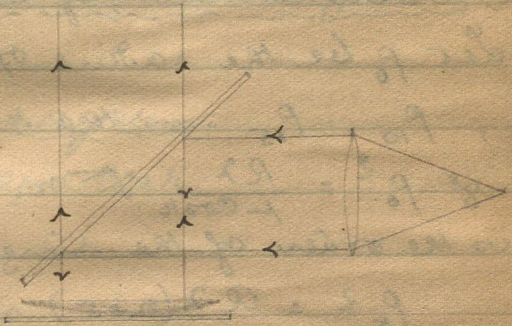
$$I = I_r + I_t = \frac{4a^2b^2\sin^2\frac{\varepsilon}{2} + a^2(1-b^2)^2}{1-2b^2\cos\varepsilon + b^4}$$

$$= a^2\,\frac{1-2b^2+b^4+2(1-\cos\varepsilon)b^2}{1-2b^2\cos\varepsilon+b^4}$$

$_2 a^2$.

Sum of the refl. & ~~refra~~ transmitted intensities is equal to the total incident intensity.

Since the reflected system varies between zero & a certain value, whereas in the transmitted system the variation is between maximum brightness & ano a less brightness, the reflected fringes appear clearer to perception than the ~~ref~~ transmitted.



Newton's Rings

1) Radius of a ring.

In the Centre where radius is zero we have a dark spot. For the first bright ring $2\mu e \cos r = \frac{\lambda}{2}$ & for the n^{th} bright ring, $2\mu e \cos r = (2n-1)\frac{\lambda}{2}$. If ρ_n be the radius of the ring, R the radius of curvature of the lens $\rho_n^2 = 2Re \;\therefore\; 2e = \rho_n^2/R$

$\therefore (2n-1)\frac{\lambda}{2} = \rho_b^2/R \times \mu \cos r.$

$$\therefore \rho_b = \sqrt{\frac{R \times (2n-1)\frac{\lambda}{2}}{\mu \cos r}} = \sqrt{\frac{R\lambda}{2\mu \cos r}} \times \sqrt{2n-1}$$

For the n^{th} dark ring $2\mu e \cos r = n\lambda$

if ρ_d is the radius

$$\rho_d^2 = 2e \times R.$$

$$= R \times \frac{n\lambda}{\mu \cos r}$$

$$\therefore \rho_d = \sqrt{\frac{Rn\lambda}{\mu \cos r}} = \sqrt{\frac{R\lambda}{2\mu \cos r}} \times \sqrt{2n}$$

Thus the radii of the bright and dark rings are $\propto$ to the sq. roots of the odd & even natural numbers resp. ly.

2) Determination of wavelength.

Let ρ_0 be the radius of the first clear dark ring, ρ_5, ρ_{10}, ρ_{15} ... the radii of the 5^{th}, 10^{th} ... rings $\rho_0^2 = \frac{R\lambda}{\mu \cos r} \times n = R\lambda n$ where n is the order of the ring, not known.

$$\rho_5^2 = R\lambda(n+5) \quad \ldots \quad \rho_{50}^2 = R\lambda(n+50)$$

$\therefore 50 R\lambda = \rho_{50}^2 - \rho_0^2 = \rho_{55}^2 - \rho_5^2 = \ldots$

$\rho_{95}^2 - \rho_{45}^2$ The mean of the 9 values is found out. If it is D then $\lambda = \frac{D}{50R}$

3) Refractive index of a liquid.

When the liquid is introduced all the

rings shrink to a smaller radius. If D' be the difference $\rho_{50}'^2 - \rho_0^2$, since $\rho_0 = \frac{R\lambda n}{\mu}$, we have $D' = \frac{50R\lambda}{\mu}$

$\therefore \mu = \frac{D}{D'}$, which is also the ratio of the sqq. of ~~any~~ the radii of any particular ring before & after water has been introduced.

Since in this case part of the light is absorbed by the liquid, the rings are feebler & this defect is compensated for by semi-silvering the surface of the lens.

4) <u>Differential lens system.</u>

The ring is dark if $2\mu e \cos r = n\lambda$ where $e = BC$.

Let $AC = e_1$ & $AB = e_2$, R_1, R_2 the radiis of the two surfaces.

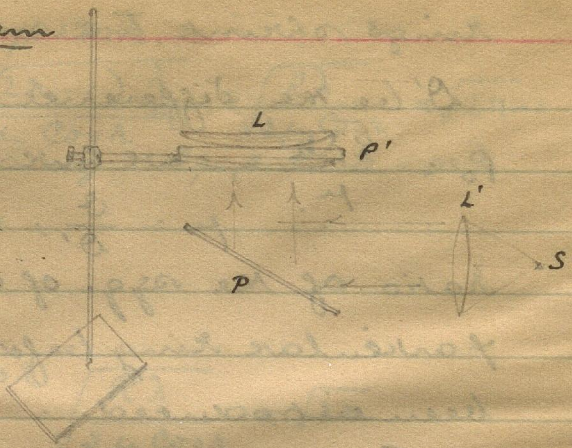
$$e = e_1 - e_2 = \frac{\rho^2}{2}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$\therefore$ Radius for the n^{th} dark ring $= \sqrt{\frac{2e}{\frac{1}{R_2} - \frac{1}{R_1}}}$

$$= \sqrt{\frac{n\lambda}{\mu \cos r\left\{\frac{1}{R_2} - \frac{1}{R_1}\right\}}}.$$

5) Transmitted system

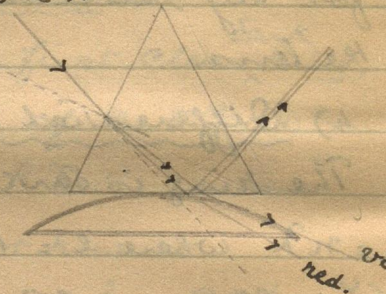
The arrangement is similar to that of the reflected system except regarding the inclination of the reflecting plate P for the comparatively darker ring of (there is no ring perfectly dark)



$\rho^2 = \frac{(2n-1)R\lambda}{2\mu \cos r}$ where n is the order of the ring.

Here again $\rho_{50}^2 - \rho_0^2 = 50 R\lambda$

Bright, broad, achromatic bands.



With the lens & plate shown above the radius of the ring $= \sqrt{Rn\lambda}$

If the $\angle$ of refr. is r $\rho = \sqrt{\frac{Rn\lambda}{\cos r}}$

Thus as r increases from 0 to $\frac{\pi}{2}$, ρ increases, the bands become brighter broader. But r cannot be increased without causing a large amount of partial reflection ~~of~~ at the

upper
Convex side of the lens, & thus diminishing the brightness.

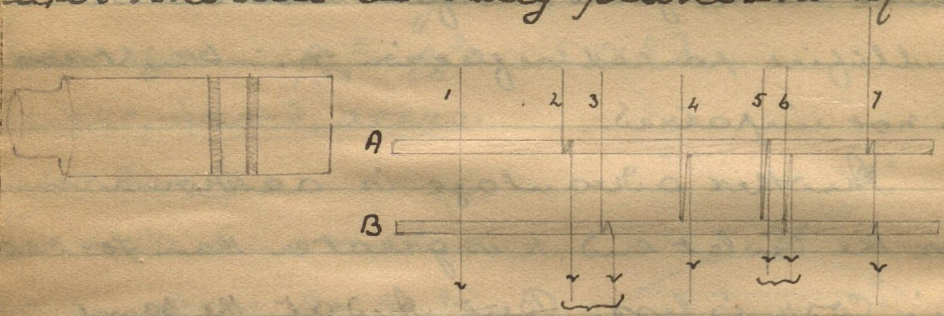
But with a prism this difficulty can be avoided. The light that is incident almost along the normal ~~is the~~ passes in the prism & falls on the lower surface at an ∠ just below the critical ∠. Thus r is nearly 90°, $\cos r$ almost zero & ρ is ∴ very great. The normal incidence on the first surface ~~is~~ almost nullifies partial reflection & ∴ brightness is not impaired.

Another advantage is achromatism. For the violet end r is greater than for red & ∴ $\cos r$ is less. ~~But~~ And at the same time λ too is less for violet than for red. ~~&~~ Hence $\rho^2 = Rn \cdot \frac{\lambda}{\cos r}$ is very nearly the same for violet as well as the red. The rings instead of being highly coloured are all white. Achromatism is especially noticeable when r approaches 90°.

Brewster's bands or fringes

19-2-38

Brewster observed a system of ringes due to interference in thick films air. Two glass plates are fixed vertically o very nearly parallel to each other, ithin a box which is illumined through small hole on one side & is provided ith an eyepiece ~~at~~ on the other. The inside the ~~of~~ box is blackened. The two lates should be truly plane and of the

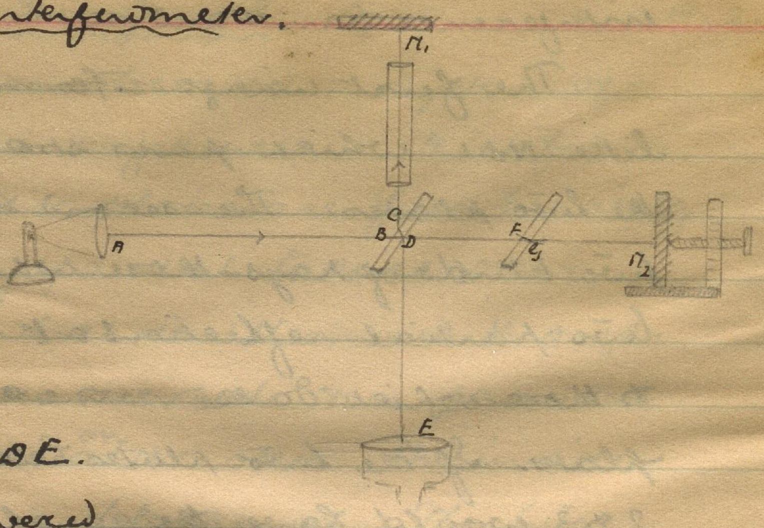


me ~~index~~ thickness and refractive index. everal images can be seen even with the aked eye, the first & the brightest being ure, the next less bright crossed ith interference bands, a third yet dimmer ure, a fourth still more dim crossed with interference bands and so on.

The first image is formed by rays like No 1 which pass straight through the two plates. The second ~~i~~ is formed by two kinds of rays – those which suffer two partial reflections at the first plate & those which do the same at the second plate. If the two plates had been ‖l, 2 & 3 would have the equal paths, but since the plates are inclined there is a path difference $2\mu e(\cos r - \cos r')$ which is of the order of λ & hence 2 & 3 interfere or reinforce according to various values of λ. Rays No 4 cause the third & pure image; while 5 & 6 can interfere & cause a fringed image.

The principle of interference has been utilized in a set of instruments known as interferometers or interference refractometers for measuring ~~λ for~~ μ of gases

Michelson's interferometer.



l B. A. Notes
Page 109 —
The two paths
followed by
the rays are
$BDCM_1CDE$
& $ABDFGM_2GFDE$.
M_1 & M_2 are silvered
on their front faces. The two plates are optically
plane & semisilvered so as to make the two
interfering ~~bands~~ beams of equal intensity. The micro-
meter ~~is~~ used to slide forward or backward
~~at it~~ is of such fine pitch that it can read
up to 100^{th} of the division on the ~~micro~~head scale,
i.e. up to $\frac{1}{10^4}$ of a m.m. The tube containing
the gas is to be introduced in front of M_1. The
shift of the interference bands is compensated
by moving M_2 back or forth. ~~If~~ It is moved
~~back~~ forward if the gas be of lower μ than air. The
fringes seen are as in Haidinger's fringes

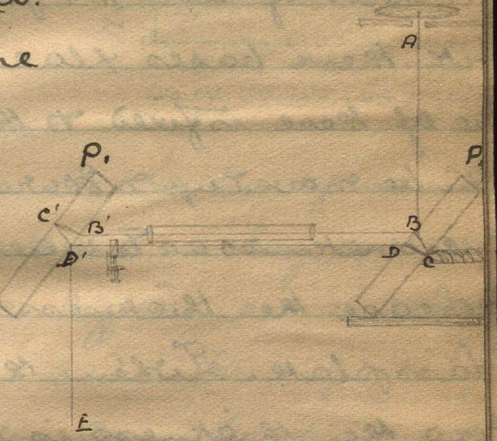
circular, ~~the~~ the source being an extended one. If d is the distance through which the mirror is moved backwards,

$$(\mu - 1) l = d \quad \therefore \mu = \frac{d}{l} + 1$$

Here the refr. index of air is taken to be uni[ty] & the zero position of M_2 is that when air is contained in the gas chamber.

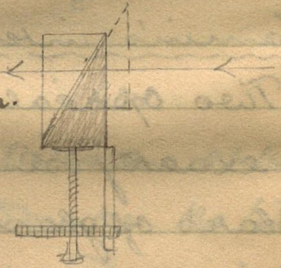
Jamin's interferometer.

Two optically plane & exactly similar plates of glass are fixed in a vertical plane ~~an~~ inclined to each other at a very small angle. A ray



of light incident on P from a narrow slit A takes two different paths, ABC D'E & ABB'C'D'E, which have a small path difference between them & ∴ interference fringes are seen in the eye piece a[t] E. In the path BB' is introduced the gas

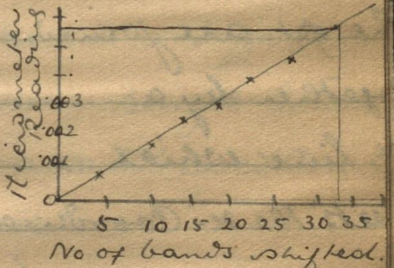
chamber. To make sure whether ~~the~~ one
the
bands passes through the chamber & the
ther outside, first the chamber is closed
the interference bands must disappear.
Next the sides of the chamber are closed.
Again the bands must disappear. In the
path of DD' is a compensator which
consists of two ||r prisms
with their bases placed together.
One of these is fixed & the other
can be moved up & down by a
micrometer so as to increase or
decrease the thickness of the equivalent
glass plate. When the micrometer reads
zero the thickness is equal to that of the two
glass sides of the gas chamber.



If a is the ~~i~~ increase in thickness necessary
to remove the shift due to the replacing of
air in the chamber by a gas, then

$(\mu_c - 1) a = (\mu - 1) l$ where μ_c is the
refractive index of the compensator.

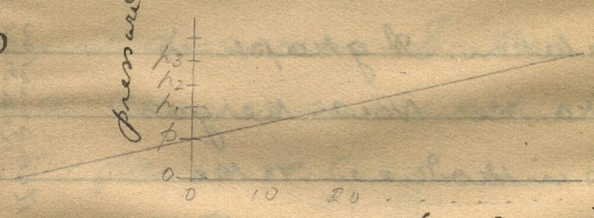
Since a is not easily determined, we may calibrate the compensator. The micrometer is turned till the shift of the interference is through 5, 10, ... band widths & each time the micrometer read is taken. A graph is drawn. Now the gas is introduced & the necessary movement of the micrometer is read out. On the graph we see the no. of bands ~~that this~~ equivalent to this — say 33.



Then 33λ is the path difference introduced by the gas $(\mu - 1)\,l = 33\lambda$ Hence $\mu = 1 + \frac{33\lambda}{l}$

The compensator can be dispensed with & the instrument can be used to find the variation of μ with pressure. Make the pressure as low as possible (1 cm of mercury) & ~~find~~ adjust the pointer of the eyepiece to coincide with a fixed band. Note the pressure p.

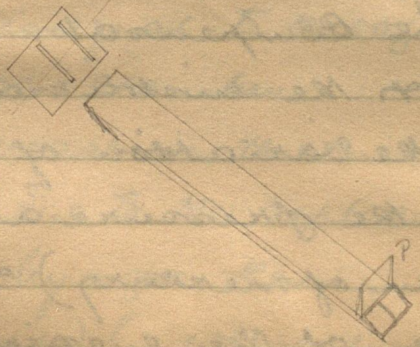
Plot (0, p) on a graph. Increase the
ressure & Count the bands as they pass.
When 10 bands have passed note the pressure
, . Plot (10, p_2). Continue the process
until ~~all the~~ atmos. press. is reached.



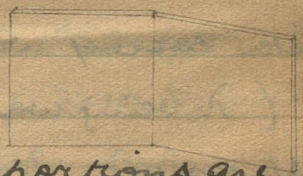
The pt. are joined
together by a
st. line which
meets the x-Coordinate
at some pt. say ($-n$)
Then $n\lambda$ is path difference due to gas at pressure
p & if ~~p ...~~ $l(\mu - 1) = n\lambda \therefore \mu = 1 + \frac{n\lambda}{l}$
At press. p_1, $\mu = 1 + \frac{(n+10)\lambda}{l}$ and so on.

Rayleigh's interferometer.

Two very fine
narrow vertical slits
illuminated by ||l
light are arranged
to cause interference.
The upper portion of
the light from these



pass through air, while the lower portion thro
two air tight ~~tubes~~ Chambers into which a gas at any
pressure can be introduced. The interference
pattern is seen ~~out~~ in two parts, the lines bei
continuous if the gas ~~the~~ in the two chambers
are at the same press. If there is a difference
in press, ~~they~~ the lower pattern shifts to one
side or other. By means of a calibrated Com-
pensator the equivalent path
difference $n\lambda$ can be found
out. To note ~~the f shift~~



whether the lines in the two portions are
exactly continuous, the upper pattern has
to be deflected slightly downwards which is
done by a prism P placed above the chamb
Rayleigh's compensator is shown in the diagra
Two plates of glass hinged together at a sligh
inclination, will cause no relative path
difference if the inclination of one plate to
the chamber behind it is equal inclination
of the other to its corresponding chamber.

y rotating the Compensator about the hinge any 24-2-38
...ge of path difference can be introduced. The Compen-
...ator is Calibrated ~~...~~ using monochroma-
...c light & keeping the same gas at the
...ame pressure in the two chambers. The
...o. of lines that cross the eye-piece, ~~as the~~
...d the rotation of the Compensator are read.
...ince every band shift corresponds to a path difference
... we can calculate the rotation of the micro-
...eter ~~...~~ which causes the same path diff.
... (λ being wave length of the monochromatic
...ght used). Next white light is used &
...e chambers are filled with the gases to be
...mpared. The rotation of micrometer which
...stores continuity to the central band is noted &
...e corresponding retardation $n\lambda$ in the terms of
...e light used for calibration is computed.

$$n\lambda = (\mu_1 - \mu_2)\, d.$$

Gladstone and Dale's Law, ~~Gladstone~~ is an
...mpirical one & states $\dfrac{\mu - 1}{\rho} = K$
...here ρ is the density of a gas whose ref. ind. is μ.

k is a constant ~~of~~ for the gas.

The law can be proved from molecular theory. Suppose a medium of the gas l cms thick. The path retardation it causes is μl.

Now, let all the molecules in it be brought together to a compact ~~to~~ whole without interspace, & let it occupy a thickness l', having the same area of section. The rest of the medium is empty space. Total path retardation is $l'\mu' + (l - l')$ μ' being ~~the~~ ref. ind. of the gas in the compact form. The path ~~tot~~ in both cases is equal

$$\therefore \quad \mu l = l'\mu' + l - l'$$

$$\therefore (\mu - 1) l = (\mu' - 1) l' \quad \text{ie} \quad (\mu - 1) = \frac{l'}{l}(\mu' -$$

Now μ' is a constant for the gas, $\frac{l'}{l}$ $= \frac{\text{amount of matter}}{\text{Space occupied}}$ = density

$$\therefore \quad \frac{\mu - 1}{\rho} = \text{Constant.}$$

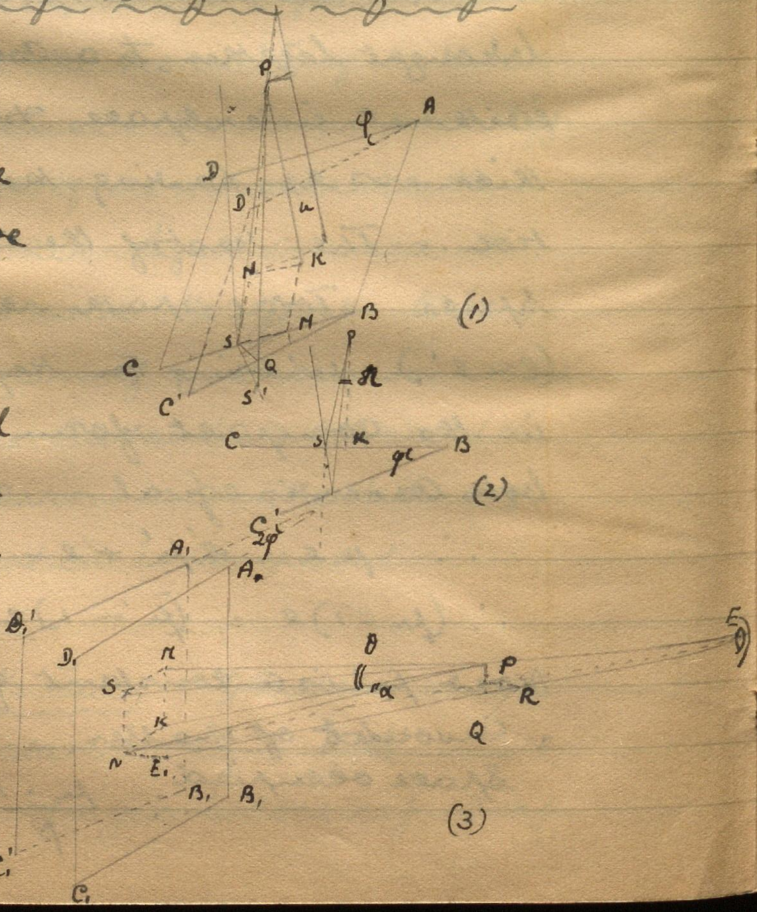
From the above can be derived the ref. ind. at any given press. p, & temp. , if the refr. ind. at p_0, T_0 is known be μ_0. $\frac{p_0}{\rho_0 T_0} = \frac{p}{\rho T}$ $\therefore \rho = \rho_0 . \frac{T_0}{T} . \frac{p}{p_0}$

$$\therefore \frac{\mu - 1}{\rho_0 \frac{T_0}{T} . \frac{p}{p_0}} = K = \frac{\mu_0 - 1}{\rho_0} \quad \therefore \mu - 1 = (\mu_0 - 1)\frac{T_0}{T}.\frac{p}{p_0}$$

Position and Shape of interference fringes.

Let BC, BC' be two mirrors (fig 2) & S bright point on the surface BC. Let S' be the image of S & P the point ~~of~~ at which the interference image is sought. P need not necessarily be in the plane of the paper. Through P draw PK normal to the first mirror

The path difference Δ



between the rays reflected at the two surfaces is $2e \cos r$ where e = thickness of the film at S & $r = \angle S'SQ = \angle SPK$

Let $A_1 B_1 C_1 D_1$, $A_1 B_1 C_1' D_1'$ be two plane images inclined at an $\angle 2\varphi$ (φ being inclination between the mirrors); and let their line of intersection be ||l to $A_1 B_1$. Let $PK =$

Describe the rectangle (fig 3) SMKN, KM SN being ||l to $A_1 B_1$. Let e_0 be the thickness of the film at KM. Thickness at S = that at N = e $\therefore e = e_0 + NK \tan\varphi$.

Let $\angle NPK = \alpha$ & $\angle KPM = \theta$

$\therefore e = e_0 + w \tan\alpha \tan\varphi$.

$$\cos r = \cos S\hat{P}K = \frac{PK}{SP} = \frac{w}{(SK^2 + w^2)^{1/2}}$$

$$= \frac{w}{(w^2 + NS^2 + NK^2)^{1/2}} = \frac{w}{(w^2 + w^2\tan^2\theta + w^2\tan^2\alpha)^{1/2}}$$

$$= \frac{1}{(1 + \tan^2\theta + \tan^2\alpha)^{1/2}}$$

$$\therefore \Delta = 2(e_0 + w \tan\alpha \tan\varphi)\frac{1}{(1 + \tan^2\theta + \tan^2\alpha)^{1/2}}$$

Thus Δ may have all possible values for [d]ifferent values of α & θ. But if the beam is [s]upposed to be limited, say by a pupil at the difference in Δ can be made small. [Th]e interference fringe is most definite when [f]or given maximum values of α & θ, the [v]ariation of Δ is a minimum i.e. $\frac{d\Delta}{d\theta}$ & $\frac{d\Delta}{d\alpha}$ are both zero.

$$\Delta = \frac{2(e_0 + w \tan\alpha \tan\varphi)}{(1+\tan^2\alpha + \tan^2\theta)^{1/2}}$$

$$\frac{d\Delta}{d\theta} = \frac{2(e_0 + w\tan\alpha\tan\varphi)2\tan\theta\sec^2\theta \times -\frac{1}{2}}{(1+\tan^2\alpha+\tan^2\theta)^{3/2}}$$

which is zero if

$$e_0 = -w\tan\alpha\tan\varphi \quad \text{or if}$$

$$\theta = 0$$

In the former case Δ becomes zero, $\therefore$ the condition for $\frac{d\Delta}{d\theta}$ being zero is

$$\theta = 0.$$

$$\frac{d\Delta}{d\alpha} = \frac{2\left[\left\{(1+\tan^2\alpha+\tan^2\theta)^{1/2}(w\tan\varphi\sec^2\theta)\right\} - \left\{(e_0+w\tan\alpha\tan\varphi)\frac{1}{2}(1+\tan^2\alpha+\tan^2\theta)^{-\frac{1}{2}}2\tan\alpha\sec^2\alpha\right\}\right]}{(1+\tan^2\alpha+\tan^2\theta)}$$

which is zero if

$$(1+\tan^2\alpha+\tan^2\theta)\,u\tan\varphi = (e_0+u\tan\alpha\tan\varphi)\tan\alpha$$

i.e. if $(1+\tan^2\theta)\,u\tan\varphi = e_0\tan\alpha$

Since by the former condition $\theta=0$

$$u\tan\varphi = e_0\tan\alpha$$

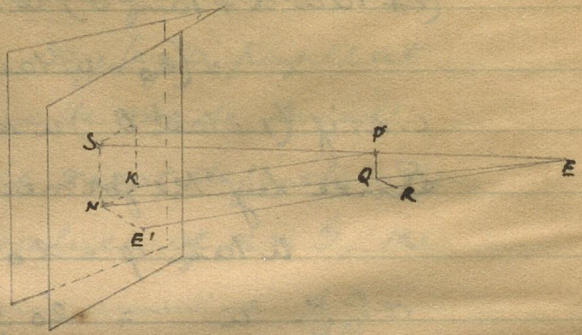
$$\therefore\ u = e_0\frac{\tan\alpha}{\tan\varphi}.$$

1) When e_0 is not ~~small~~ zero nor a small quantity which approaches zero, & φ also is not zero, i.e. if the film is wedge shaped and of large thickness, u can have any value. Hence, in Jam[in] the position of the image is indeterminate.

2) When $\varphi=0$ but $e_0\neq 0$, as in Michelson's, $u=\infty$, the fringes are formed at infinity.

3) When e approaches zero & $\varphi\neq 0$, as in Haidinger's fringes, ~~the~~ $u=0$, the fringes are formed on the surface of the mirror itself.

Let the fringe at P be viewed by an eye placed at Q. Draw PQ' normal to the surface. Let a plane through P ∥l to SNK cut NE & E'E at Q & R.



Referred to R let P be (x, y), $QR = x$ & $PQ = y$.

As in the former case Δ

$$= \frac{2\,(e_1 + w_1 \tan\varphi \tan N\hat{E}E')}{(1 + \tan^2 N\hat{E}E' + \tan^2 NES)^{1/2}}$$

where $w_1 = E'E$, e_1 = thickness at E'.

Let $PE = v$

$$\Delta = \frac{2(e_1 + w_1 \tan\varphi \cdot \frac{x}{v})}{\left(1 + \frac{x^2}{v^2} + \frac{y^2}{v^2}\right)^{1/2}}$$

$$= \frac{2\,(e_1 v + w_1 \tan\varphi\, x)}{(v^2 + x^2 + y^2)^{1/2}}$$

$$\therefore \Delta^2 (v^2 + x^2 + y^2) = 4\,(e_1 v + w_1 x \tan\varphi)^2$$

$$\Delta^2 y^2 = x^2\,(4 w_1^2 \tan^2\varphi - \Delta^2)$$
$$+ x\,(8 v e_1 w_1 \tan\varphi)$$
$$+ \quad (4 e_1 v^2 - \Delta^2 v^2)$$

The above eqn is a ~~par~~hyperabola, ~~hyper~~parabola or ellipse according as

$4u_1^2 \tan^2\varphi - \Delta^2$ is +ive, zero or −ive.

ie $4u_1^2 \tan^2\varphi \gtrless \Delta^2$.

In Jamin's all three conditions are possible, & the fringes can have any of the above shapes.

In Michelson's $\varphi = 0$ ∴ $4u^2 \tan^2\varphi < \Delta^2$ & further the term in x vanishes & the coeffts of x^2 & y^2 are equal. Hence the fringes are Olar with the ~~ft~~ projection of C viz C' as centre.

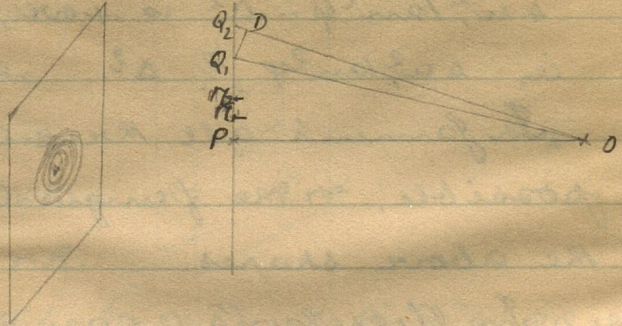
~~In~~ The fringes of Fabry & Perot, ~~Lummer & Gheroke~~, (& Haidinger?) ~~the fringes~~ are again Olar since $\varphi = 0$.

26-2-'38

~~Diff~~raction

The intensity of light at any pt is ∝ the sq. of the amplitude, and when ∝ the pt. is illuminated by ||l light, we may consider the amplitude or displacement

t the pt O to be that due to wavelets starting
rom various points on a plane wavefront
A B. For the



ary calculation
the resultant
isplacement
e divide the
ave front into
hat are called half period zones. Let OP be
ormal to AB & = b. Spheres with O as
entre & radii $b+\frac{\lambda}{2}$, $b+2\frac{\lambda}{2}$, + . . . are described
tting the AB in Ols which have P as centre.
reas enclosed between consecutive Ols are
e h. p. zones.
Resultant amplitude at O due to wavelets
rom a zone depend on

1) area of the zone
2) Distance of the zone from O
3) Phase of the wavelets
4) Obliquity of propagation.

1). <u>Area</u> : Let Q_1 Q_2 be the bounding ~~surface~~

lines of the n^{th} zone.

Area $= \pi PQ_2^2 - \pi PQ_1^2$
$= \pi (OQ_2^2 - OQ_1^2)$
$= \pi (OQ_1 + OQ_2) \frac{\lambda}{2}$
$= \pi \lambda r$ where r = mean distance of the zone from O

∴ Area varies directly as the distance r. Since displacement $\propto$ area, other factors being constant, we have displacement $\propto r$

2) Intensity varies inversely as r^2, ie displacement $\propto \frac{1}{r}$ other factors bein equal.

Hence due to (1) & (2) together, the displacement is not affected, the effect due one being cancelled by the other.

3) The average phase of wavelets from 2 zone is behind that of wavelets from first zone by $\frac{\lambda}{2}$ or π. Hence resultant amplitudes due to wavelets from alternate zones are of opposite sign.

4) The displacement must vary with

e obliquity of the wavelet, it must be a
unction of r. Hence if A = displacement due
any zone, & A' that due to the other zone

$A = a\,f(r)$ where a = a const.

$A' = a\,f\left(r + \frac{\lambda}{2}\right) = a\left\{f(r) + \frac{\lambda}{2}\,f'(r)\right\}$

$\therefore A - A' = a \times -\frac{\lambda}{2}\,f'(r)$

Stokes gives the effect of obliquity to
vary as $\cos\theta$ where θ is the $\angle$ made by
, O to PO & $\therefore$ $\cos\theta = \frac{b}{r}$

$\therefore f(r) = \frac{b}{r}$; $f'(r) = -\frac{b}{r^2}$

$\therefore A - A' = \frac{ab\lambda}{2} \cdot \frac{1}{r^2}$

Hence the diminution is $\propto \frac{1}{r^2}$

If d_1, d_2, d_3 ... be displacements due
different zones, & d_1 is +ive, due to (1) & (2)
$d_1 = d_2 = \ldots$; due to (3) d_2 is −ive, d_3 is +ive
& due to (4) $d_1 > d_2 > d_3 > \ldots$ though
differences $d_1 - d_2$, $d_2 - d_3$ ... are very small.

$d_2 = \frac{d_1 + d_3}{2}$, ...

Resultant displacement at O

$= d_1 - d_2 + d_3 - \ldots = d_1 - \frac{d_1 + d_3}{2} + d_3 - \ldots$

$= \frac{d_1}{2}$.

The intensity is $\frac{1}{4}$ that due to the 1st L. p zone alone.

If the first few zones are covered by an opaque body, the intensity is small, but not zero. If many zones are covered intensity is practically zero. Hence the truth of rectilinear propagation of light.

Graphical representation of Diffraction

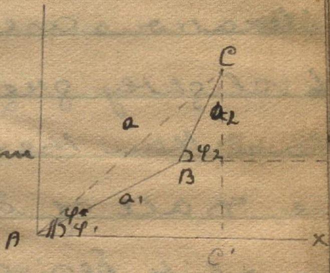
The ordinary polygon method for vibrat of the same period and along the same line but of different amplitudes and phases can be applied here.

The method for two vibrations

$$y_1 = a_1 \sin(\omega t + \varphi_1)$$

$$\& \; y_2 = a_2 \sin(\omega t + \varphi_2)$$

is illustrated in the diagram



The resultant S.H.M.

$y = a \sin(\omega t + \varphi)$ is the orthogonal projection of C, where ABC is a Δ, $AB = a_1$, $BC = a_2$, $\angle BAX = \varphi_1$, $\angle CBX$ ... $AC = a$ & $\angle CAX = \varphi$.

he result can be proved anglytically

$y = y_1 + y_2 = $

ie $a \sin(\omega t + \varphi) = a_1 \sin(\omega t + \varphi_1) + a_2 \sin(\omega t + \varphi_2)$

$a \cos\varphi = a_1 \cos\varphi_1 + a_2 \cos\varphi_2$ (1)

$a \sin\varphi = a_1 \sin\varphi_1 + a_2 \sin\varphi_2$ (2)

$\therefore a^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\varphi_2 - \varphi_1)$

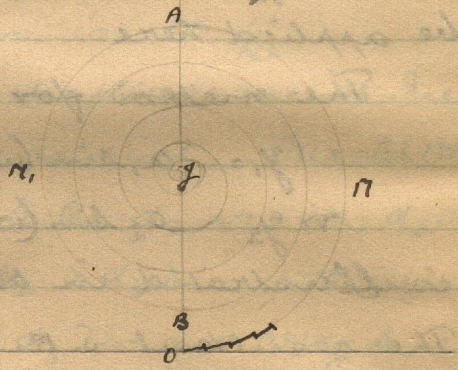
But $AC^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\varphi_2 - \varphi_1)$

$\therefore AC = a$

Besides eqns (1) & (2) are satisfied if $\varphi = \angle CAX$.

What has been proved of 2 S. H. M, can be proved about any number of S. H. M's.

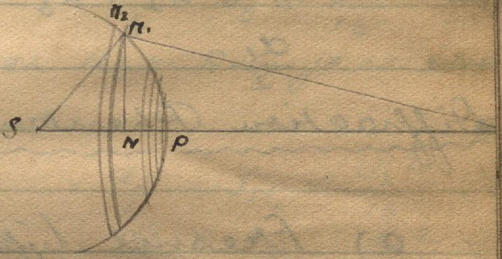
When the ~~corresp~~ Component vibrations are each of slightly greater phase than the preceding one, & all of small amplitudes, as happens in the plane wave front, the polygon takes the form of a curve.



At the boundary of the 1st h. p. zone, the phase is π & so the line is ‖ to the x-axis but in the $-$ive direction. ∴ The polygon for the 1st h. p. zone is a semi circular curve OM_1A & the resultant amplitude is OA & has phase $\frac{\pi}{2}$. Due to 2nd h. p. zone the polygon is AM_2B, B being slightly higher than O. Taking all the h. p. zones together we shall have a spiral starting at O & ending at J. OJ is $\frac{d_1}{2}$ & gives the resultant amplitude for all the h. p. zones together

3-3-'38 Spherical or cylindrical wave front.

H. p. zones are constructed as in the previous case.



Area of the n^{th} h. p. zone, bounded by M_1, M_2 is

$$2\pi NM_1 \times M_1M_2 = 2\pi a \sin\theta \times a\,d\theta$$

where a = radius SP, $\theta = \angle M_1SP$

$$A = 2\pi a^2 \sin\theta\, d\theta.$$

The distance OM, $= r$ of the zone from O
is given by $r^2 = a^2 + (a+b)^2 - 2a(a+b)\cos\theta$.

$\therefore \quad 2r\,dr = 2a(a+b)\sin\theta\,d\theta$

$\therefore \quad \sin\theta\,d\theta = \dfrac{r\,dr}{a(a+b)}$ & in this case

$dr = \dfrac{\lambda}{2}$

$\therefore \quad \sin\theta\,d\theta = \dfrac{r\lambda}{2a(a+b)}$

$\therefore \quad A = 2\pi a^2 \times \dfrac{r\lambda}{2a(a+b)}$

$= \dfrac{\pi a r \lambda}{(a+b)}$ $\therefore$ area $\propto r$.

The other three factors on which displacement at O depends viz phase, distance obliquity are as in a plane wave front.

$\therefore$ Total displacement at O

$= d_1 - d_2 + d_3 - \dots$

$= \dfrac{d_1}{2}$.

Diffraction Phenomena — are classified into

a) Fresnel type

b) Fraunhofer type

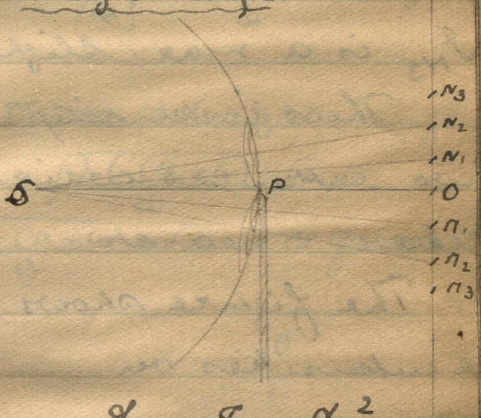
In the Fresnel type, both source & pattern are at finite distance; whereas in Fraunhofer type they are at infinite distance i.e.

rays are collimated on to the object causing the pattern & then are viewed through a telescope adjusted for ∥l beams, as in a spectrometer.

a) Fresnel type

1) Diffraction at a straight edge.

Cf B.A. Notes. Let S ~~being~~ the source, & P the st. edge, ~~At~~ & O the pt where SP meets the screen.



At O, $d = \frac{d_1}{2} - \frac{d_2}{2} + \dots = \frac{d_1}{4}$ $I_0 = \frac{d_1^2}{16}$

At n_1, which is such that P is just above the 1st h.p. zone,

$$d = -\frac{d_2}{2} + \frac{d_3}{2} - \dots = -\frac{d_2}{4} \qquad I_{n_1} = \frac{d_2^2}{16} <$$

At n_2 $d = \frac{d_3}{2} - \frac{d_4}{2} + \dots = \frac{d_3}{4}$ $I_{n_2} = \frac{d_3^2}{16} <$

The intensity goes on diminishing.

At N_1 $d = d_1 - \frac{d_2}{2} + \frac{d_3}{2} - \dots = \frac{d_1}{2} + \frac{d_1}{4} = \frac{3d_1}{4}$

$$I_{N_1} = \frac{9d_1^2}{16}$$ (a max. value

At N_2 $d = d_1 - d_2 + \frac{d_3}{2} - \dots = (d_1 - d_2) + \frac{d_3}{4}$

$$= \frac{\alpha_1}{4} + \frac{3\alpha_1}{4} - \frac{\alpha_1 + d_3}{2} + \frac{d_3}{4}$$

$$= \frac{\alpha_1}{4} + \frac{\alpha_1}{4} - \frac{d_3}{4}$$

$$= \frac{\alpha_1}{4} + \text{a small quantity}$$

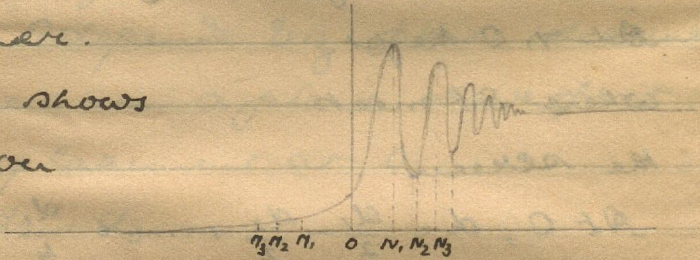
I_{N_2} is a min. slightly > than I_0.

$$I_{N_3} = \alpha_1 - \alpha_2 + d_3 - \frac{d_4}{2} + \dots$$

$$= \frac{\alpha_1}{2} + \frac{d_3}{4} = \frac{3\alpha_1}{4} - \text{a small quantity}$$

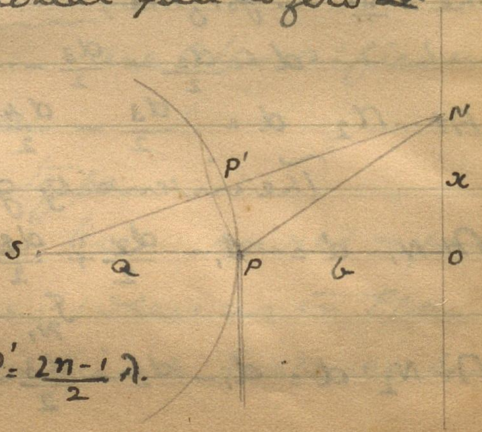
I_{N_3} is a max. slightly < than I_{N_1}

Thus intensity varies between two extremes which themselves come nearer & nearer.



The figure shows e intensities on oth sides of , the max. & mini. at , N_2 ... & the gradual fall to zero at long On_1, n_2 ...

Bandwidth



N is the position of e n^{th} dark band if $P - NP' = n\lambda$ & it is e n^{th} bright band if $NP - NP' = \frac{2n-1}{2}\lambda$.

$$\therefore n\lambda \text{ or } \frac{2n-1}{2}\lambda = (b^2+x^2)^{1/2} - \left\{(\overline{a+b}^2+x^2)^{1/2} - a\right\}$$

$$= b\left(1+\frac{x^2}{2b^2}\right) - \left\{(a+b)\left(1+\frac{x^2}{2\,\overline{a+b}^2}\right) - a\right\}$$

$$= \frac{x^2}{2b} - \frac{x^2}{2(a+b)}$$

$$= \frac{x^2}{2}\cdot\frac{a}{b(a+b)}$$

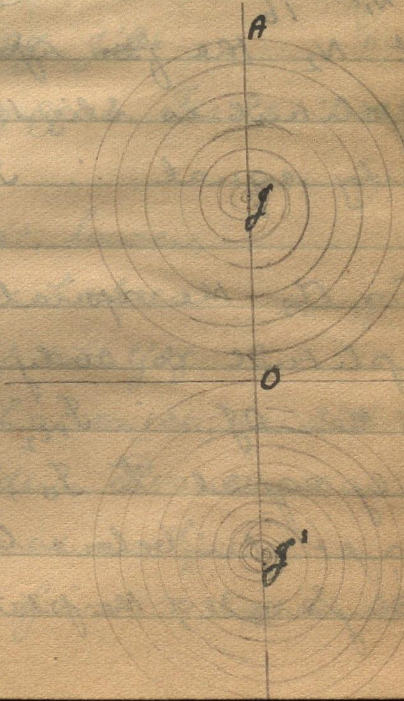
$$\therefore x = \sqrt{\frac{2\lambda b(a+b)}{a}}\cdot\sqrt{n} \quad \text{for dark bands}$$

$$= \left\{(2n-1)\lambda b(a+b)/a\right\}^{1/2}\sqrt{\cdots} \quad \text{bright}$$

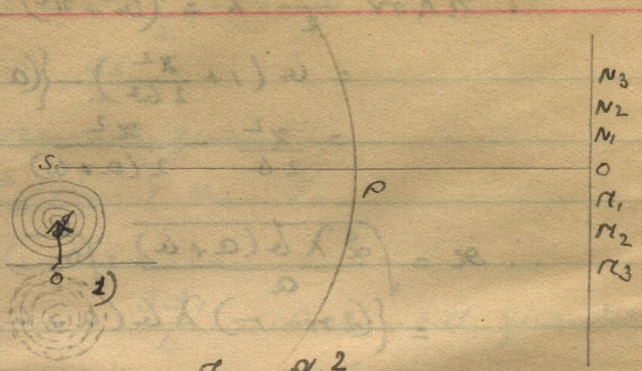
i.e. The alternate dark & bright rings are at distances zero, $\sqrt{\frac{\lambda b(a+b)}{a}}\sqrt{1}$, $\sqrt{\frac{\lambda b(a+b)}{a}}\times\sqrt{2}$, $\sqrt{\frac{\lambda b(a+b)}{a}}\times\sqrt{3}$ .... from central dark fringe.

<u>Amplitude spiral</u>

Here we take the upper spiral to represent displacements due to wavelets from points above P & the lower spiral for the same from below P. $\therefore OA = \frac{d_1}{2}$ & $OJ = \frac{d_1}{4}$.



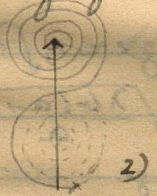
t O, displacements
e due to the upper
alf only ∴ the
mplitude spiral
ill have the form :



isplacement is $d_0 OJ = \frac{a_1}{4}$

Intensity $I_0 = \frac{I}{16}$ where $I = a_1^2$

t N_1, the lower half of the 1^{st} h.p.z all
lso is effective.

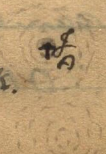


Hence we have
piral (2) $d_{N_1} = d_1 + \frac{a_1}{2} = \frac{3d_1}{4}$

$I_{N_1} = \frac{9I}{16}$

At N_2 the ~~foo~~ spiral is (3)
mplitude is slightly > OJ, but
early equal ∴ I_{N_2} is nearly
o (3)

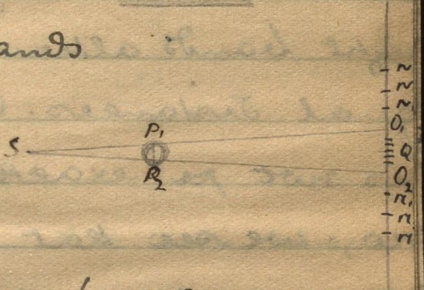
For n_2 the spiral is (4)
mplitude is only slightly
ss than OJ ∴ I_{n_2} is very (4)
early equal to I_0, but yet smaller.
At a pt far below O, the ampl.
JA
is very small; the pt is ∴ practically dark.



(5)

2) Narrow wire

The pattern consists of interference bands in the centre and diffraction bands on the two sides.

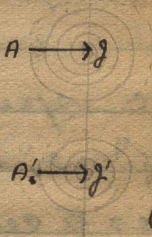


Within the shadow:—

Take a pt Q, such that the wire hides 1st h.p. zone both in upper & ~~of~~ lower halves, & 2nd & 3rd h.p. zones ~~of~~ in their lower halves only.

The spiral is No (1) & the amp. is $AJ + A'J' = AJ - J'A'$ which is very nearly zero. Hence we ha[ve] a dark band. (1)

Let Q_2 be another pt, lower, such that of the upper halves of the h.p. zones the 1st & half of the 2nd & of the lower halves 1st, ~~be~~ 2nd & half of the third are hidden. Resultant amp. $AJ + A'J'$ which is a max $= \frac{a_1}{2}$. (2)



At Q_3 for which the first & 2nd in both

[u]pper & lower halves are hidden

[th]e amp. is zero, hence a dark

[b]and. Thus we have dark &

[b]right bands alternating at

(3)

[e]qual distances. But since $AJ - A'J'$ in

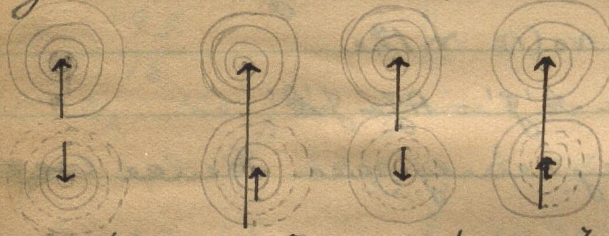
[(]? is not ~~pe~~ exactly zero, while in (3) it is

[ze]ro, we see that the dark bands on either

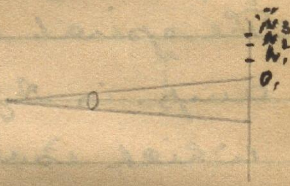
[s]ide are not ~~so~~ as perfectly defined as those

[i]n the centre.

Figures 4, 5, 6, 7, show the ampli-



4 5 6 7



[tu]de spirals at pts outside the geometrical

[s]hadow, O_1, N_1, N_2, N_3 respsly. The wire

[h]ides a certain fixed number (4 in the figure) of the lower

[h]alves of the h. p. zones. At O_1 the zones

[h]idden are 1 — 4, at N_1 2 — 5, at N_2 3 — 6 &

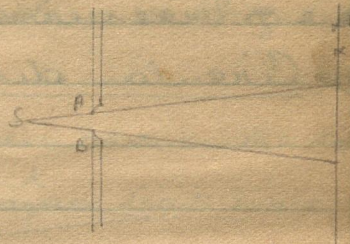
[s]o on. Thus when the spirals are drawn we

[s]ee that ~~the~~ pts of maximum & ~~&~~ minimum

brightness alternate with each other.

8-3-'38 3) Diffraction at a rectangular aperture.

As we proceed along the Screen outside the geometrical Shadow, pts $r_1, r_2, \ldots$ are reached

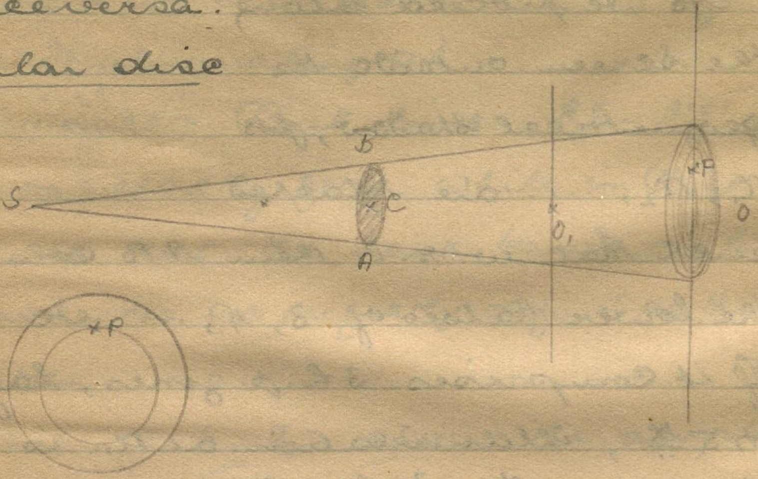


such that w.r.t. them AB comprises the lower halves of 3, 4, ... etc h.p. zon[es]. If it comprises 3 h.p. zones, say 3rd, 4th & 5th, illumination at r_1 is ~~nearly~~ $\left(\frac{d_3}{2} - \frac{d_4}{2} + \frac{d_5}{2}\right)^2 = \left(\frac{d_3}{2}\right)^2$ which is a max. If AB comprises 4 h.p. zones, say 4th to 7th, illumination at r_2 is practically zero. $Br_2 - Ar_2 = 2\lambda$ while $Br_1 - Ar_1 = 3\frac{\lambda}{2}$. The bands are bright or dark according as path difference $Br - Ar$ is $(2n+1)\frac{\lambda}{2}$ or $~~2~~ n\lambda$. The case is similar to sim[]ple interferce, the bands are equispace[d] & bandwidth $= \frac{b}{d}\lambda$.

The graphical method can be employe[d] here also as for the rectangular obsta[cle]

[th]e difference being that while lines are
[d]otted in the previous case are continuous
[h]ere & vice versa.

[(b)] Circular disc



[W]ithin the geometrical shadow and
[co]ncentric with it are seen a system of
[b]right and dark rings, caused by the
[in]terference of light bending round the
[ed]ges of the disc. At O the centre, BO − BO = 0
∴ pt is bright. Let P be such that $BA - AB = \frac{\lambda}{2}$.
[T]his the ∴ a dark point, ~~All~~ due to inter-
[f]erence of light from A & B, ~~1st~~ all pts
[th]e ends of that diameter of the disc which
[is] ∥ to OP. ∥ly all pts at distance OP from O are
[da]rk due to light bending round the ends of

their corresponding diameter. Beyond at P_1 where $AP_1 - BP_1 = \lambda$ there is a bright ring. Thus equispaced dark & bright rings alternate.

By bringing the screen nearer the di[sc] the rings gradually ~~collapse~~ shrink but none actually collapses. The centre always remains bright since it is always equi[-]distant from ~~A~~ A & B. The pt. P comes nearer & nearer to O, thus reducing its brightness.

The brightness of O decreases bec[ause] the no. of h.p. zones covered by AB is gradually ~~less & less~~ more & more. At O brightness is say $\left(-\frac{\cancel{d_4}}{\cancel{4}} + \frac{d_1}{5} - \dots\right)^2 = \left(-d_4 + d_5 - \dots\right)$
$= + \frac{d_4^2}{4}$ while at O, it will be $\frac{d_{10}^2}{4}$ due to the ∮ 7 no. of h.p. zo[nes] covered.

Disc covers n h.p. zones if

$SA + AO - SCO = n\frac{\lambda}{2}$

ie $(a^2+r^2)^{1/2} - (b^2+r^2)^{1/2} - (a+b) = n\frac{\lambda}{2}$

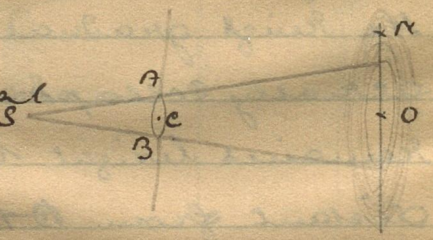
$$a + \frac{r^2}{2a} + b + \frac{r^2}{2b} - a - b = n\frac{\lambda}{2}$$

ie $r^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda.$

$\therefore\ b = 1/\left(\frac{n\lambda}{r^2} - \frac{1}{a}\right) = \frac{r^2 a}{an\lambda - r^2}.$

Thus as b decreases n increases, thus causing less brightness at O.

<u>Circular aperture</u>



Outside the geometrical image there are bright and dark rings alternating at equal distances. R of e.g. is bright if $RB - RA = (2n+1)\frac{\lambda}{2}$ (as in the case of the rectangular aperture) $\therefore$ pts at distance RO from O are all bright.

The illumination ~~of~~ at O ~~is~~ depends upon the no of h.p. zones w.r.t. O comprised by AB. If the no. of zones is odd the pt is bright, & if the no. is even the pt is dark. Now the disc covers n h.p. zones if $SA + AO = SC + CO + n\frac{\lambda}{2}$ $\therefore\ b = \frac{r^2 a}{an\lambda - r^2}$. Thus as b decreases n increases ~~from~~ alternating between odd & even numbers. $\therefore$ The pt O is successively dark and bright.

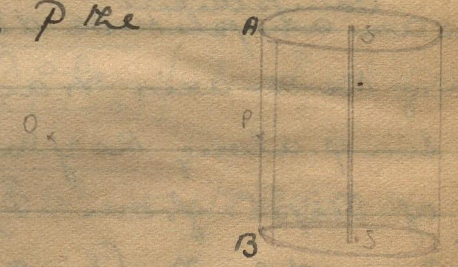
~~Zone~~ If white light is used instead of monochromatic, as the formula shows for the same value of n, as b ~~inc~~ dec λ increases. ∴ O will pass ~~f~~ through the spectrum colours from violet to re[d]

Familiar diffraction phenomena are the colours seen when looking throu[gh] a fine silk kerchief, or when looking with a ~~squinted~~ partially open eye. In the latter case the eyelashes act as obstacles or apertures.

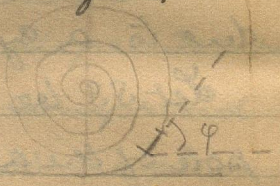
<u>Zone Plate.</u> cf B.A. Notes Page 135

9 - 3 - '38 Fresnel's investigations on Diffraction due to a cylindrical wave front.

Let SS be the slit, P the pole of the wave-front w.r.t. O. The wavefront can be divided into h.p. zones, or into strips ‖l to AB. The displace-ment at O due to any strip can be suppose[d]



to be that due to a small effective portion at the middle. These portions of the different strips form an equatorial band. As we go ~~to~~ along the band ~~to~~ from P to either side, distance from O increases. Fresnel assumed that at a short distance from P, the distance from O is so great that the consequent displacement at O can be neglected. He further neglected the effect of obliquity. Let S be the distance of a pt P' on the band from P. The amplitude due to ~~x~~ element ds at P' can be assumed $\propto$ to the length of the element since obliquity is neglected. The displacement

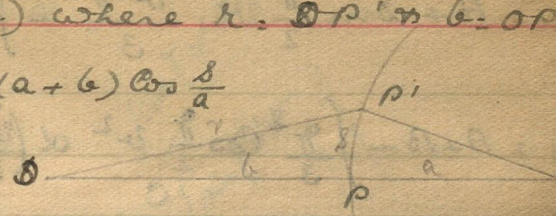


has an ~~amplitude~~ component $ds \cos\varphi$ along the x-axis & another $ds \sin\varphi$ along the y-axis (φ being the phase lag of displ. from P')

$\int \cos\varphi \, ds$ & $\int \sin\varphi \, ds$ give the sum of the components & $\therefore$ I (intensity at O)

$$= \left\{\int \cos\varphi \, ds\right\}^2 + \left\{\int \sin\varphi \, ds\right\}^2$$

$$\varphi = \frac{2\pi}{\lambda}(r - b) \text{ where } r = OP' \text{ \& } b = OP$$



$$r^2 = (a+b)^2 + a^2 - 2a(a+b)\cos\frac{s}{a}$$

$$= \cancel{b^2}$$

$$= (a+b)^2 + a^2$$

$$\quad - 2a(a+b)\left(1 - 2\sin^2\frac{s}{2a}\right) \qquad \left[\frac{s}{a}\right. \text{ being sm}$$

$$= (a+b)^2 + a^2 - 2a(a+b)\left(1 - \frac{s^2}{2a^2}\right)$$

$$= b^2 + \frac{a+b}{a}s^2$$

$$\therefore r^2 - b^2 = \frac{a+b}{a}s^2$$

$$r - b = \frac{a+b}{2ab}s^2 \quad \text{since } 2b = r+b \text{ nearly}$$

$$\therefore \varphi = \frac{2\pi}{\lambda}\cdot\frac{a+b}{2ab}s^2$$

$$= \frac{\pi}{2}v^2 \text{ where } v^2 = \frac{4(a+b)}{\lambda\, 2ab}s^2$$

$$s = \sqrt{\frac{\lambda ab}{2(a+b)}}\; v$$

$$\therefore \int \cos\varphi\, ds = \sqrt{\frac{\lambda ab}{2(a+b)}}\int \cos\frac{\pi}{2}v^2\, dv.$$

$$\text{\&} \int \sin\varphi\, ds = \sqrt{\frac{\lambda ab}{2(a+b)}}\int \sin\frac{\pi}{2}v^2\, dv$$

$$\int \cos\frac{\pi}{2}v^2\, dv = \overbrace{\cos\frac{\pi}{2}v^2 \times v}^{A} + \int v\cdot\pi v\, dv \sin\frac{\pi}{2}v^2$$

$$= A + \pi\int \sin\frac{\pi}{2}v^2\, d\,\frac{v^3}{3}$$

$$\overbrace{}^{B}$$
$$= A + \sin\frac{\pi}{2}v^2 \; \frac{v^3\pi}{3} - \int \frac{v^3\pi}{3}\cdot \pi v\,dv \cdot \cos\frac{\pi}{2}v^2$$

$$= A + B - \int \frac{\pi}{3}^2 \cos\frac{\pi}{2}v^2 \; d\left(\frac{v^5}{5}\right)$$

$$= A + B - \overbrace{\cos\frac{\pi}{2}v^2 \; \frac{v^5\pi^2}{3\cdot 5}}^{C} - \int \frac{v^5\pi^2}{3\cdot 5}\cdot \pi v\,dv \, \sin\frac{\pi}{2}v^2$$

$$= A + B - C - \int \frac{\pi^3}{3\cdot 5}\cdot \sin\frac{\pi}{2}v^2 \; d\left(\frac{v^7}{7}\right)$$

$$= A + B - C - \sin\frac{\pi}{2}v^2 \cdot \frac{\pi^3 v^7}{1\cdot 3\cdot 5\cdot 7} + \cdots\cdots$$

$$= \cos\frac{\pi}{2}v^2 \left\{ v - \frac{\pi^2 v^5}{1\cdot 3\cdot 5} + \frac{\pi^4 v^9}{1\cdot 3\cdot 5\cdot 7\cdot 9} - \cdots \right\} - M$$

$$+ \sin\frac{\pi}{2}v^2 \left\{ \frac{\pi v^3}{1\cdot 3} - \frac{\pi^3 v^7}{1\cdot 3\cdot 5\cdot 7} + \frac{\pi^5 v^{11}}{1\cdot 3\cdot 5\cdot 7\cdot 9\cdot 11} - \cdots \right\} - N$$

$$= M \cos\frac{\pi}{2}v^2 + N \sin\frac{\pi}{2}v^2.$$

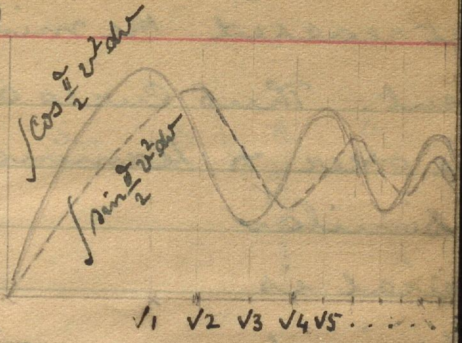
Sly it can be shown that $\int \sin\frac{\pi}{2}v^2 = M \sin\frac{\pi}{2}v^2 - N\cos\frac{\pi}{2}v^2.$

$$I = \left(\int_a^x \cos\varphi\, ds\right)^2 + \left(\int \sin\varphi\, ds\right)^2 = \frac{\lambda ab}{2(a+b)} \left\{ \left(\int_0^v \cos\frac{\pi}{2}v^2\,dv\right)^2 + \left(\int_0^v \sin\frac{\pi}{2}v^2\,dv\right)^2 \right\}$$

$$= \frac{\lambda ab}{2(a+b)} \left\{ \left(M\cos\frac{\pi}{2}v^2 + N\sin\frac{\pi}{2}v^2\right)^2 + \left(M\sin\frac{\pi}{2}v^2 - N\cos\frac{\pi}{2}v^2\right)^2 \right\}$$

$= \frac{\lambda ab}{2(a+b)} \cdot (r^2 + v^2)$

On examining the values for $\int_0^v \cos\frac{\pi}{2} v^2 dv$ & $\int_0^v \sin\frac{\pi}{2} v^2 dv$



for values of v ranging from zero to ∞, it is seen that the former has maxima & minima values alternating when $v = \sqrt{1}, \sqrt{3}, \sqrt{5} \ldots$ the values gradually approaching $\frac{1}{2}$.

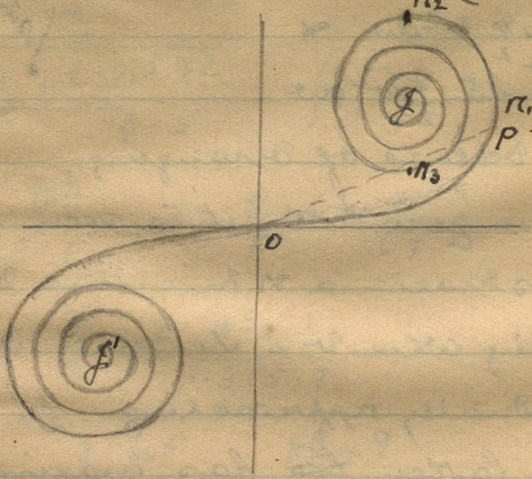
The latter ~~too~~ has maxima & minima values alternating when v is $\sqrt{2}, \sqrt{4}, \sqrt{6}$. the values here also gradually approach $\frac{1}{2}$ (See page 296 of Preston)

Taking $\int_0^v \cos\frac{\pi}{2} v^2 dv$ on the x-axis & $\int_0^v \sin\frac{\pi}{2} v^2 dv$ on the y-axis Cornu drew a spiral the shape of which is shown below.

When X reaches its max. Y is still increasing. Hence the curve bends back at r_1. At r_2 Y has its max, but x has

[no]t reached the minimum, hence the 2nd [cu]rve. Thus curls are formed one within [th]e other & terminate at the pt $\left(\frac{1}{2}, \frac{1}{2}\right)$.

[A] Similar [s]piral is [o]btained [i]n the 4th [qu]adrant [on] giving [ne]gative values [of] v.



[T]he radius vector $OP = r$ joining the origin [to] any pt on the curve is $\sqrt{x^2+y^2}$

r^2 represent the intensity due to ele-[m]ents comprised between O & P; & r is [th]e amplitude of vibration, $\angle POx$ is the [p]hase.

The spiral is ∴ the amplitude spiral.

$[O\pi]_2$ is the 1st h.p. zone, $\pi_2\pi_3$ the 2nd & so on.

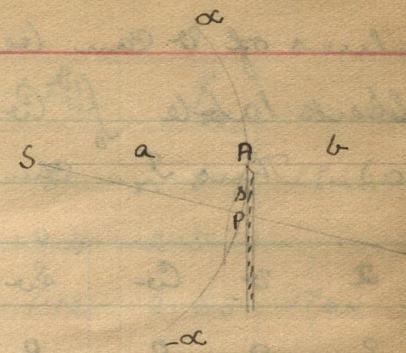
Diffraction pattern by a st. edge. 11-3-'38

Intensity at any pt is $\left(\int C\,dv\right)^2 + \left(\int S\,dv\right)^2$

[where $C = \cos \frac{\pi}{2} v^2$

& $S = \sin \frac{\pi}{2} v^2$.]

with proper limits for v.



Let P be at distance x from O. $\frac{x}{a+b} = \frac{s}{a}$

$$\therefore x = s \cdot \frac{a+b}{a} = \cancel{s} \cdot \frac{a+b}{a} \cdot \sqrt{\frac{\lambda ab}{2(a+b)}}\, v$$

$$= \sqrt{\frac{\lambda ab (a+b)^2}{a^2\, 2(a+b)}} \cdot v$$

$$= \sqrt{\frac{\lambda b (a+b)}{2a}}\, v$$

$\therefore v = Kx$ where

$$K = \sqrt{\frac{2a}{\lambda b (a+b)}}.$$

Illumination at P

$$= \left(\int_v^\infty C\,dv\right)^2 + \left(\int_v^\infty S\,dv\right)^2$$

$$= \left(\int_0^\infty C\,dv - \int_0^v C\,dv\right)^2 + \left(\int_0^\infty S\,dv - \int_0^v S\,dv\right)^2$$

$$= \left(\frac{1}{2} - C_v\right)^2 + \left(\frac{1}{2} - S_v\right)^2$$

where $C_v = \int_0^v C\,dv$ & $S_v = \int_0^v S\,dv$.

When x has diff. values, the corresponding

alues of v can be calculated & from
Gilberts table $\int_0^v C_v\,dv$ & $\int_0^v S_v\,dv$ can be
read. Thus I_r can be calculated.

x	v	C_v	S_v	$\frac{1}{2}-C_v$	$\frac{1}{2}-S_v$	I_r
0	0	0	0	·5	·5	·5
—	·5	·494	·065	·008	·435	·189
—	1·0	·780	·434	-·28	·066	·084
—	1·5	·444	·698	—·056	·198	·042
—	2·0	·488	·343	·012	·157	·026
—	2·5	·457	·619	·043	·119	·015

Thus we see that as r recedes from
, I_r goes on diminishing at first
apidly & then slowly.

At a pt N above O, the intensity

$$I_N \text{ is } \left(\int_{-v}^{\infty} C\,dv\right)^2 + \left(\int_{-v}^{\infty} S\,dv\right)^2$$

$$= \left(\int_0^{\infty} C\,dv + \int_0^{v} C\,dv\right)^2 + \left(\int_0^{\infty} S\,dv + \int_0^{v} S\,dv\right)^2$$

$$= \left(\frac{1}{2} + C_v\right)^2 + \left(\frac{1}{2} + S_v\right)^2$$

v	C_v	S_v	$\frac{1}{2}+C_v$	$\frac{1}{2}+S_v$	I_v	
0	0	0	·5	·5	·5	Min
1	·78	·44	1·28	·94	2·57	
1·2	·72	·62	1·22	1·12	2·65	Max.
1·5	·45	·70	·95	1·20	2·34	
1·9	·39	·37	·89	·87	1·54	Min.
2·2	·64	·46	1·14	·96	2·23	
2·4	·56	·62	1·06	1·12	2·31	Max.
2·8	·47	·39	·97	·89	1·73	Min.

The table shows how intensity varies between maxima & minima, giving a graph of the kind on page 62.

Merits and defects of Fresnel's theory.

The assumptions made about 3 of the four factors (Page 54) are untenable. Area of the zone is taken $\propto$ to the amplitude. Distance of the zone is not considered different for different zones since the strip is narrow & $r + \delta - 2b$ according to Fresnel. The factor of obli-

-uity is not mentioned at all. Nor is it possible that ~~phase~~ obliquity & distance thus discarded can cancel each other. The amp. due to a farther zone is less both due to obliquity and due to distance.

Yet in spite of these ~~transparent~~ outstanding errors, the final result agrees closely at least qualitatively with the exptal observations. The reason is that inadvertently the function $\int C\,dv$ & $\int S\,dv$ included ~~functions~~ a law of obliquity as can be seen from the following correct formula. This law is called the fortuitous law of obliquity. Amplitude varies as a function of obliquity, as the area ds of an element, ~~[illegible]~~ and inversely as the distance of the ~~be~~ element.

Let f be the function of obliquity.

Area of a h. p. zone $\propto r \times \frac{\lambda}{2}$

Area of a small element $\propto r \times d\varphi$

where $d\varphi$ is the phase change between the extremes of the element.

$\therefore$ ampl $\propto f \frac{ds}{r} \propto f \frac{r\,d\varphi}{r} \propto f\,d\varphi$

This has a component $\propto f\,d\varphi \cos\varphi$ along the x-axis &. $\therefore$ the sum of the x components $= \int f \cos\varphi\, d\varphi$.

If Fresnel's result $\int \cos\frac{\pi}{2} v^2\, dv$ is to be correct, it should be identical with $\int f \cos\varphi\, d\varphi$.

$\therefore$ Let $f \propto \frac{a}{\sqrt{\varphi}} \propto \frac{a}{z}$ where $z = \sqrt{\varphi}$

$d\varphi = 2z\,dz$

$$\int f \cos\varphi\, d\varphi = \int k \frac{a}{z} \cos z^2\, 2z\, dz$$

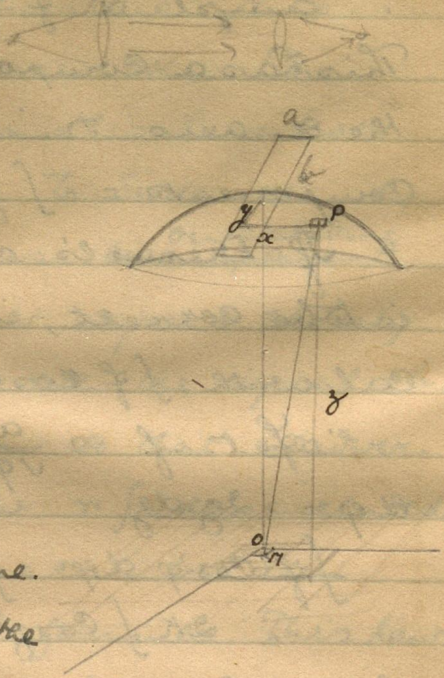
$$= 2k \int \cos z^2\, dz$$ which is of the same form as $\int \cos\frac{\pi}{2} v^2\, dv$.

$\therefore f = \frac{a}{\sqrt{\varphi}} \propto \frac{a}{\sqrt{\delta}} \propto \frac{a}{\sqrt{r-b}}$

Hence the fortuitous law of obliquity inadvertently introduced by Fresnel is that f varies inversely as the sq. root of the phase retardation, & $\therefore$ its decrease as we recede from the pole is very rapid. The assumption that only a narrow strip is effective becomes almost correct.

Fraunhofer Phenomena.

In Fraunhofer Phenomena two lenses are used one to collimate & the other to focus. The illumination at the pt O is due to a concave wave surface. Let O be the origin, the screen ∥l the lens in the x, y plane. Consider an element P on the wavefront (x, y, z) area $dx\,dy$. The vibration at P at time t



$= a \sin \omega t = a \sin \frac{2\pi}{T} t$

Vibration at some pt R $(\eta, \zeta, 0)$ very near O, $u_1 = a \sin\left(\frac{2\pi}{T} t + \frac{2\pi}{\lambda}\rho\right)$ [where $\rho = PR$]. Taking displacement ∝ area due to the element $= a \sin 2\pi \left(\frac{t}{T} + \frac{\rho}{\lambda}\right) dx\,dy$

Now $\rho^2 = (x-\eta)^2 + (y-\zeta)^2 + z^2$

$= x^2 + y^2 + z^2 + \eta^2 + \zeta^2 - 2x\eta - 2y\zeta$

$$\rho = \sqrt{R^2+\xi^2+\eta^2}\left(1 - \frac{x\eta + y\xi}{+R^2+\xi^2+\eta^2}\right)$$

Since ξ & η are small

$$= R\left(1 - \frac{x\eta+y\xi}{R^2}\right) = R - \frac{x\eta + y\xi}{R}$$

$$\therefore\ u_1 = k \sin 2\pi\left\{\left(\frac{t}{T} + \frac{R}{\lambda}\right) - \frac{x\eta+y\xi}{R\lambda}\right\} dx\,dy$$

$$= k \sin 2\pi\,(A - B)\,dx\,dy \quad \text{Where } A = \left(\frac{t}{T} + \frac{R}{\lambda}\right),\ B = \left(\frac{x\eta}{R ...}\right)$$

$$= \left\{(\sin 2\pi A)\cos 2\pi B - (\cos 2\pi A)\sin 2\pi B\right\} dx ...$$

These are two S. H. M of ~~period~~ frequ... A, amplitudes $\cos 2\pi B\,dx\,dy$ & $\sin 2\pi B\,dx ...$ in directions at rt. Ls to each other.

$\therefore$ Resultant vibration at P due to the element has intensity

$$(\cos 2\pi B\,dx\,dy)^2 + (\sin 2\pi B\,dx\,dy)^2$$

$\therefore$ Total intensity at P due to all the elements

$$= \left\{\iint \cos 2\pi B\,dx\,dy\right\}^2 + \left\{\iint \sin 2\pi B\,dx\,dy\right\}$$

15-3-'38 When a rectangular aperture is placed between the collimating lens & the objective of the telescope, the intensity I can be found by giving the proper limits to the intergral. Let a, b be the lengths of the aperture ‖l to x & y axis resply

$$I = \left\{\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \cos\frac{2\pi}{R\lambda}\left(x\eta + y\zeta\right) dx\,dy\right\}^2$$

$$+\left\{\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \sin\frac{2\pi}{R\lambda}\left(x\eta + y\zeta\right) dx\,dy\right\}^2$$

$$= \left\{\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \cos(px + qy)\, dx\,dy\right\}^2$$

$$+\left\{\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} \sin(px + qy)\, dx\,dy\right\}^2$$

where $p = \dfrac{2\pi\eta}{R\lambda}$ & $q = \dfrac{2\pi\zeta}{R\lambda}$

$$= \left\{\int_{a/2}^{a/2} \cos px\, dx \int_{-b/2}^{b/2} \cos qy\, dy\right\}^2$$

Since the four other terms which contain sine integrations reduce to zero $\displaystyle\int_{-a}^{a} \sin\theta\, d\theta = \left(-\cos\theta\right)_{-a}^{a}$

$$= -\cos a + \cos a = 0$$

$$\therefore I = \left\{ \frac{1}{p} \left(\sin px \right)_{-a/2}^{+a/2} \times \frac{1}{q} \left(\sin qy \right)_{-b/2}^{b/2} \right\}^2$$

$$= \left\{ \frac{2}{p} \sin \frac{pa}{2} \times \frac{2}{q} \sin \frac{qb}{2} \right\}^2$$

$$= \left\{ \frac{\sin \frac{pa}{2}}{\frac{pa}{2}} \right\}^2 \times \left\{ \frac{\sin \frac{qb}{2}}{\frac{qb}{2}} \right\}^2 \times a^2 b^2.$$

Since p is a function of η & q of ζ, the intensity varies according as the coordinates of N vary.

Let $F = \left(\frac{\sin \varphi}{\varphi} \right)^2$. The variation of F will show how I varies.

F is max. or min when $F' = 0$

F is max when $F' = 0$ & F'' is $-$ive

F is min. when $F' = 0$ & F'' is $+$ive

$$F' = 2 \frac{\sin \varphi}{\varphi} \times \frac{\varphi \cos \varphi - \sin \varphi}{\varphi^2}$$

F' is zero if

$$\frac{\sin \varphi}{\varphi} = 0 \qquad (1)$$

or if $\qquad \varphi = \tan \varphi. \qquad (2)$

$$F' = 2\,\frac{\sin\varphi}{\varphi} \times \frac{\varphi\cos\varphi - \sin\varphi}{\varphi^2}$$

$$F'' = 2\left[\frac{\frac{\sin\varphi}{\varphi}\left\{\begin{array}{l}\varphi^2\left(-\varphi\sin\varphi + \cos\varphi - \cancel{\cos\varphi}\right) \\ -\left(\varphi\cos\varphi - \sin\varphi\right)2\varphi\end{array}\right\}}{\varphi^4} + \left\{\frac{\varphi\cos\varphi - \sin\varphi}{\varphi^2}\right\}^2\right]$$

When ~~F' is +ive~~ $\varphi = \tan\varphi$,

$F'' = 2 \times \frac{\sin\varphi}{\varphi} \times -\frac{\sin\varphi}{\varphi}$ & is $-$ive

When $\sin\varphi/\varphi = 0$ $\quad F'' = 2\left(\frac{\varphi\cos\varphi - \sin\varphi}{\varphi^2}\right)^2$

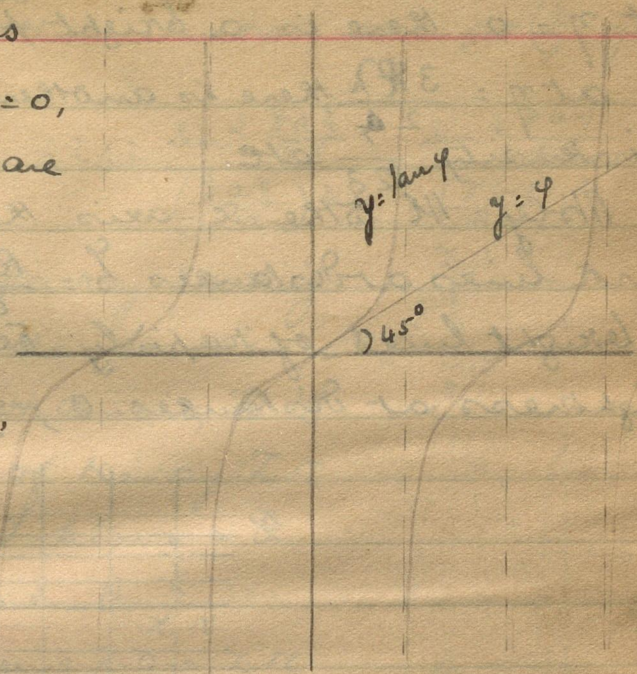
& is +ive

F is max. when $\varphi = \tan\varphi$

F is min. when $\frac{\sin\varphi}{\varphi} = 0$.

The max. roots are found by drawing the curves of $y = \varphi$ & $y = \tan\varphi$, φ being the abscissa. The values of φ for which eqn $\varphi = \tan\varphi$ is satisfied are first zero, ~~0,~~ (2) then ~~no~~ values less than $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\frac{7\pi}{2}$ ... by very small amounts.

The values of F when $\varphi = 0$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$ .... are 1, $\frac{1}{23}$, $\frac{1}{56}$, $\frac{1}{110}$ etc.



F is min. when $\frac{\sin\varphi}{\varphi} = 0$, i.e. $\sin\varphi = 0$ i.e. the values are π, 2π, 3π ....

∴ the minimum values of F are all ze

Now illumination at any pt (η, ζ) is

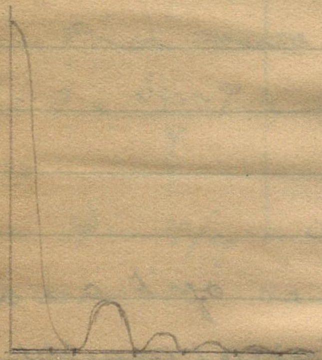
$$a^2 b^2 \cdot \left\{ \frac{\sin \frac{\pi a}{R\lambda} \cdot \eta}{\frac{\pi a}{R\lambda}\eta} \right\}^2 \times \left\{ \frac{\sin \frac{\pi b}{R\lambda} \cdot \zeta}{\frac{\pi b}{R\lambda} \cdot \zeta} \right\}^2$$

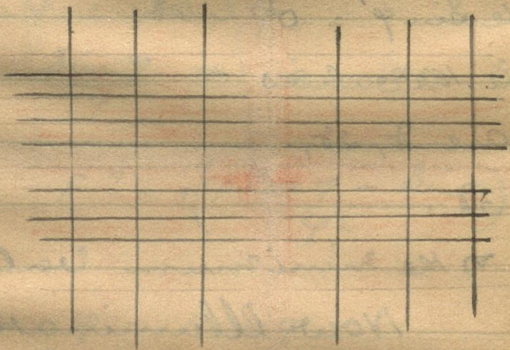
For any given value of ζ the illumination varies bet. max & min. according as $\frac{\pi a}{R\lambda}\eta$ has values 0, π, $\frac{3\pi}{2}$, 2π, $\frac{5\pi}{2}$, 3π etc.

Thus there are dark lines ||l to the y-axis at distances $\eta = \frac{R\lambda}{a}$, $\frac{2R\lambda}{a}$ ....

At $\eta = 0$ there is a bright ~~patch~~ band of intensity ; at $\eta = \frac{3 \cancel{R}\lambda}{2 \cancel{a}}$ there is another bright ~~pat~~ band of intensity $\frac{1}{23}$ etc.

Illy ‖l to the x-axis there are dark lines at distances $\zeta = \frac{R\lambda}{b}, \frac{2R\lambda}{b}, \ldots$ bright lines of rapidly decreasing brightness at distances $0, \frac{3}{2}\frac{R\lambda}{b}, \frac{5}{2}\frac{R\lambda}{b} \ldots$



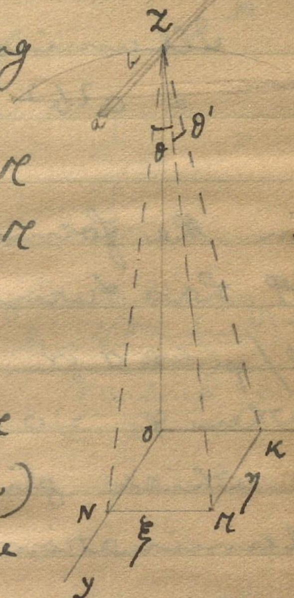


It is specially to be noticed that if the ~~bright~~ aperture is longer along the y-axis, the bright patch in the centre of the pattern is longer along the x-axis & vice versa.

Diffraction pattern due to a slit.

It was shown above that when wavefront is restricted by a rectangular aperture illumination at a pt (ξ, η) is given by

$$a^2b^2 \frac{\sin^2 \frac{\pi a \xi}{R\lambda}}{\left(\frac{\pi a \xi}{R\lambda}\right)^2} \times \frac{\sin^2 \frac{\pi b \eta}{R\lambda}}{\left(\frac{\pi b \eta}{R\lambda}\right)^2}$$



Deviation of a ray starting from Z (the position of the slit) & coming to M ~~from~~ i.e. of the ray ZM from the ~~Z~~Y plane = ~~θ~~

$NZM = \theta$ & $\sin\theta = \frac{\xi}{ZM}$

$= \frac{\xi}{R}$ (R being ZO = ZM to a first approximation)

Illy deviation θ' from the XZ plane is given by

$\sin\theta' = \frac{\eta}{R}$.

∴ Illumination at M (in polar Coordinates

$$= a^2b^2 \frac{\sin^2 \frac{\pi a \sin\theta}{\lambda}}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2} \times \frac{\sin^2 \frac{\pi b \sin\theta'}{\lambda}}{\left(\frac{\pi b \sin\theta'}{\lambda}\right)^2}$$

If the ~~slit~~ aperture is replaced by ~~an aperture~~ slit ~~&~~ b ~~comes~~ becomes very great ompared to a & the deviation θ' becomes zero $\therefore$ $\sin\theta' = 0$. Hence the second factor ~~x~~ = unity for $\frac{\sin\alpha}{\alpha}$ ($Lt\ \alpha \to 0$)

$$\frac{\alpha - \frac{\alpha^3}{\lfloor 3} + \cdots}{\alpha} = 1 - \frac{\alpha^2}{\lfloor 3} + \cdots = 1$$

$\therefore$ Illumination at R

$$= a^2b^2 \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}$$

As in the former case the function $\frac{\sin^2\varphi}{\varphi^2}$ has turning values when φ $\sin\varphi/\varphi = 0$ or when $\varphi = \tan\varphi$

when $\varphi = 0$, $= \frac{3\pi}{2}$ (nearly), $= \frac{5\pi}{2}$ (nearly) etc which gives the maximum values of illumination to a^2b^2, $a^2b^2/\left(\frac{3\pi}{2}\right)^2$, ... etc.

The min. values occur when $\varphi = \pi$, 2π, ... etc & the min. values are zero. Thus parallel to the slit a series of bands be seen at ~~distan~~ points such that

$$\frac{\pi a \sin\theta}{\lambda} = \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

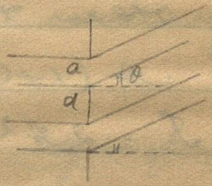
$$\text{ie } \sin\theta = \frac{3}{2}\frac{\lambda}{a}, \frac{5}{2}\frac{\lambda}{a}, \ldots$$

Since θ is a function of λ, when the

Source is monochromatic, a series of spectra are seen. Fraunhofer named this series the spectra of the 1st Class.

Two equal & parallel Slits.

Let a be the width of each slit & d the distance between them.



At inclination θ, illumination due to a single slit is

$$I = A^2 = a^2b^2 \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}.$$

Where A is the amplitude of vibration at the point in question.

The displacement at instant t is of the form $y_1 = A \sin \omega t$.

Due to the second slit there is a similar displacement, but with a phase lag $\frac{2\pi}{\lambda}(a+\delta)\sin\theta = \varepsilon$.

$\therefore$ Displacement due to the Second Slit is $y_2 = A\sin(\omega t + \varepsilon)$.

$\therefore$ Resultant displacement

$$y = y_1 + y_2 = A\{\sin\omega t(1+\cos\varepsilon) + \cos\omega t \sin\varepsilon\}$$

$\therefore$ Illumination ~~I~~ at ~~the p~~ an [a]ngular ~~d~~ inclination θ is

$$I = A^2\{(1+\cos\varepsilon)^2 + \sin^2\varepsilon\}$$ since y [i]s represented by 2 S.H.Ms at rt ∠s [o]f each other & of amplitudes

$$A(1+\cos\varepsilon) \text{ \& } A\sin\varepsilon$$

i.e. $I = 2A^2(1+\cos\varepsilon)$

$$= 4A^2\cos^2\frac{\varepsilon}{2}.$$

$$= 4a^2b^2\frac{\sin^2\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\left(\frac{\pi a\sin\theta}{\lambda}\right)^2}\cos^2\left\{\frac{\pi(a+d)\sin\theta}{\lambda}\right\}$$

$\cos^2\left\{\frac{\pi(a+d)\sin\theta}{\lambda}\right\}$ has max. values when $\left(\frac{\pi(a+d)\sin\theta}{\lambda}\right) = 0, \pi, 2\pi$ etc,

[i.]e $\sin\theta = 0, \ \frac{\lambda}{a+d}, \ \frac{2\lambda}{a+d} \ldots\ldots$

[T]he max. values of I at these [in]clinations are

$$4a^2b^2 \times \frac{\sin^2 0}{0} \times 1$$

$$= 4a^2b^2.$$

Thus a series of maxima each [o]f intensity $4a^2b^2$ is seen at [v]alues of θ given by $0, \frac{\pi\lambda}{a+d}, \frac{2\pi\lambda}{a+d} \ldots \frac{n\lambda}{a+d}$ etc

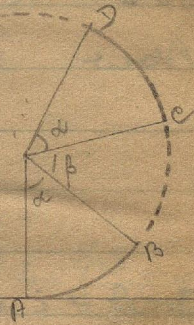
Here again the inclination $\propto \lambda$ ∴

Coloured bands are produced under monochromatic ligh[t].

These Fraunhofer termed the Spectra [of] the 2nd class. These are evidently muc[h] more brilliant.

<u>Any number of equal equispaced parallel slits or the diffraction gra[ting]</u>

The full theory of the diffraction grating is easily solved by reference to the amplitude spiral. The displacement curve due to the first slit is the arc AB of a ☉ which subtends an ∠ at the centre equal to $\alpha = \frac{2\pi}{\lambda} a \sin\theta$.



Due to the opaque portion d the displacement from the 1st poi[nt] in the 2nd slit is of phase not α but $\alpha + \beta$ where $\beta = \frac{2\pi}{\lambda} d \sin\theta$. The result

isplacements are represented by
ords AB, CD etc, & the phase
ag between each is $\gamma = \frac{2\pi}{\lambda}(a+d)\sin\theta$.
et η be the phase of the displacement
e to the first slit. The displacement
$B = A = \left\{a^2b^2 \sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \Big/ \left(\frac{\pi a \sin\theta}{\lambda}\right)^2\right\}^{1/2}$
Sum of the x components of the
t displacements is
$X = A\left\{\cos\eta + \cos(\eta+\gamma) + \dots \cos(\eta+\overline{n-1}\gamma)\right\}$
um of the y components is
$Y = A\left\{\sin\eta + \sin\overline{\eta+\gamma} + \dots \sin(\eta+\overline{n-1}\gamma)\right\}$
The intensity at $\angle$lar displacement
is $I = X^2 + Y^2$

Now $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

$\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$\cos\theta + i\sin\theta = e^{i\theta}$

$\cos\theta - i\sin\theta = e^{-i\theta}$

Using these formula

$X + iY = A\left\{(\cos\eta + i\sin\eta) + \dots\dots\right\}$

$$= A\left\{e^{i\eta} + e^{i(\eta+\gamma)} + \dots\dots e^{i(\eta+\overline{n-1}\gamma)}\right.$$

$$= A e^{i\eta}\left\{\frac{e^{in\gamma}-1}{e^{i\gamma}-1}\right\}$$

Illrly $X - iY = A\{(\cos\eta - i\sin\eta) + \dots\dots\}$

$$= A\left\{e^{-i\eta} + e^{-i(\eta+\gamma)} + \dots\ e^{-i(\eta+n-\dots}\right.$$

$$= A e^{-i\eta}\left\{\frac{e^{-in\gamma}-1}{e^{-i\gamma}-1}\right\}$$

$$I = X^2 + Y^2 = (X+iY)(X-iY)$$

$$= A^2\frac{(e^{in\gamma}-1)(e^{-in\gamma}-1)}{(e^{i\gamma}-1)(e^{-i\gamma}-1)}$$

$$= A^2\left\{\frac{2-2\cos n\gamma}{2-2\cos\gamma}\right\}$$

$$= A^2\frac{\sin^2\frac{n\gamma}{2}}{\sin^2\frac{\gamma}{2}}$$

$$= \cancel{A^2}\, a^2 b^2\,\frac{\sin^2\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\left(\frac{\pi a\sin\theta}{\lambda}\right)^2}\times\frac{\sin^2\left\{\frac{n\pi}{\lambda}(a+d)\dots\right.}{\sin^2\left\{\frac{\pi}{\lambda}(a+d)\dots\right.}$$

Now $\frac{\sin^2 nx}{\sin^2 x}$ is max. or min when $\frac{\sin nx}{\sin x} = 0$ or hen $\sin x \times n \cos nx = \sin nx \cos x$.

when $\sin nx = 0$ but $\sin x$ is not zero

r when $n \tan x = \tan nx$.

1) $\sin nx = 0$ & $\sin x \neq 0$ when

$x = \pi, 2\pi, 3\pi$ etc. For these values

x, $I = A^2 \dfrac{\sin^2 nx}{\sin^2 x}$ becomes zero

ence these are minimum values.

$= \dfrac{\pi}{\lambda}(a+d)\sin\theta = \dfrac{\pi}{n}; 2\dfrac{\pi}{n}, 3\dfrac{\pi}{n}$ etc.

ie $\sin\theta = \dfrac{1}{n}\dfrac{\lambda}{a+d}, \dfrac{2}{n}\dfrac{\lambda}{a+d}, \ldots\ldots$

Thus ~~min. illu~~ bands of zero illu-

ination are seen at displacements

iven by $\sin\theta = \cancel{\pi}\dfrac{1}{n}\dfrac{\lambda}{a+d}, \dfrac{2}{n}\dfrac{\lambda}{a+d}$ ... etc.

2) $n \tan x = \tan nx$ when $x = 0, \pi, 2\pi$,

, etc. These will give the maximum

alues of $\sin^2 nx / \sin^2 x$ to be n^2.

r $\dfrac{\sin^2 nx}{\sin^2 x} = \dfrac{\sin^2 nx - \sin^2 m\pi}{\sin^2 x - \sin^2\pi}$

hich when $x \rightarrow$

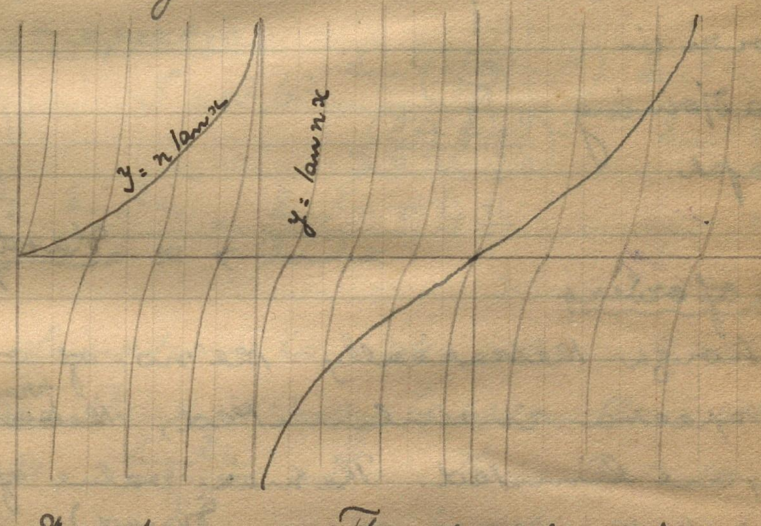
Hence the primary maxima values

ccur when $x = 0, \pi, 2\pi$ etc

. e. when $\sin\theta = \dfrac{\lambda}{a+d}\times 0, \dfrac{\lambda}{a+d}\pi, \dfrac{\lambda}{a+d}2\pi \ldots$

& the max. values are all $n^2 A^2$.

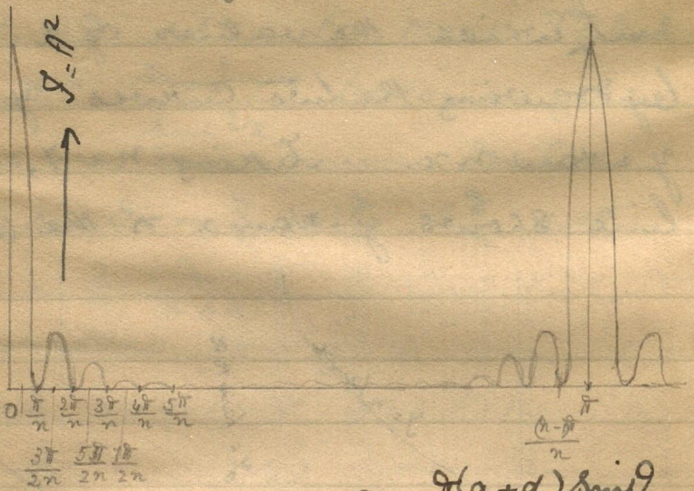
Now it is clear that between two adjac[ent] primary maxima there are $(n-1)$ minima ∴ there should be $(n-2)$ secondary max[i]ma for which the values of x can be foun[d] by drawing the two curves $y = n\tan x$ & $y = \tan nx$. Taking $n = 7$, the inked line shows $y = n\tan x$ & the pencil lines



show $y = \tan nx$. The graph shows that the roots of the eqn are nearly $\frac{3}{2}\frac{\pi}{n}, \frac{5}{2}\frac{\pi}{n}, \frac{7}{2}\frac{\pi}{n}$ etc & on calculation the secondary maxima will be seen to be much smaller than the primary maxi[ma]

a. The ~~1st~~ secondary nearest to the primary are about $\frac{1}{20}$th the primary & hence almost dark compared to the primary. Thus it is that the images are seen very narrow.

The relative intensities & positions are shown in the adjoining graph.



No. of orders.

Though theoretically the no. of orders of spectra is unlimitted, ~~theoretically~~ practically they are limited. The max. value of $\theta = 90°$, & ∴ the max. value of $x = \frac{\pi(a+d)}{\lambda}$. If the mth order is the greatest visible,

$m\pi = \frac{\pi(a+d)}{\lambda}$ ie $m = \frac{a+d}{\lambda \times \cancel{\pi}} = \frac{2\cdot 56}{5890 \times 10^{-8} \times \cancel{\pi} \times 14000}$

$= 3\cdot 10\delta$

Thus only up to the third order of sodium will

be visible on either side with a grati[ng]
of 14000 lines to the inch.

$\sin\theta = Nn\lambda$ & since $2\lambda_r = 15300 A^\circ U$
& $3\lambda_v = 12000 A^\circ U$, the 2nd order & the
third order overlap. The violet of the
third order begins immediately after the
Sodium $D_1\, D_2$ of the 2nd order. The over-
lapping is still greater in the higher order

<u>Absent spectra</u>

$$I = a^2 b^2 \frac{\sin^2\left(\frac{\cancel{\pi a}\sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2} \times \frac{\sin^2 nx}{\sin^2 x}$$

$$= a^2 b^2 \frac{\sin^2 u}{u^2} \times \frac{\sin^2 nx}{\sin^2 x}$$

The function is max. when $x = 0, \pi$
2π etc. But it may happen that whe[n]
$x = m\pi$, $\frac{\sin^2 u}{u^2} = 0$. Then that parti-
cular spectrum will be absent.

$\frac{\sin^2 u}{u^2} = 0$ when $u = \pi, 2\pi, 3\pi$ etc.

~~ie $\sin\theta = \frac{\lambda}{a}\pi, \frac{\lambda}{a}2\pi, \frac{\lambda}{a}3$~~

$\sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \ldots$

$\frac{\sin^2 nx}{x}$ = max when $\sin\theta = 0, \frac{\lambda}{a+d}, \frac{2\lambda}{a+d}, \ldots$

Hence if $\frac{n\lambda}{a+d} = \frac{m\lambda}{a}$ m & n being integers the n^{th} order spectrum will be completely absent. For the same reason the $2n^{th}$, $3n^{th}$... etc also are ~~all~~ absent. Thus if $\frac{a+d}{a} = 2$ the 2^{nd}, 4^{th} ... orders are absent. If $\frac{a+d}{a} = 3$, the 3^{rd}, 6^{th}, ... orders are absent.

Purity of the grating spectrum. If the different lines are seen separate the spectrum is said to be pure. Hence the image of any particular band should occupy a very small portion of the spectrum. This condition is satisfied in the grating spectrum. Between two principal maxima there are $(n-1)$ dark lines. Hence the adjacent dark line is very close to the maximum. The width of a band of wavelength λ is ~~only~~ given by $d\alpha = \frac{2\pi}{n}$ $\alpha = \frac{\pi(a+d)\sin\theta}{\lambda}$ $\frac{2\pi}{n}$ i.e. the width is ∴ very small. Thus there is no overlapping

Prism & Grating In a prism the dispersion depends on the kind of glass, & the mean deviation of the rays & the relative

~~the~~ spacings between different bands vary widely from prism to prism. This is called the irrationality of the prism spectrum. In a grating the deviation is connected to the wavelength by a constant law which is independent of the material on which the lines are drawn. Dispers[ion] too is constant for small values of the deviation. A grating spectra is ∴ sai[d] to be normal.

Dispersive power $= \frac{d\theta}{d\lambda}$ where

$\cos\theta \, d\theta = Nn\lambda \quad \therefore d\theta/d\lambda = Nn/\cos\theta.$

<u>Minimum Deviation</u>

It can easily be shown (cf B.A. Notes) that when the ∠ of incidence is not zero but i, the position of the n^{th} order spectrum is given by

$$\sin i + \sin\theta = Nn\lambda.$$

The deviation $\delta = i + \theta$ & is a minimum when $\frac{d\delta}{di} = 0$

$$\sin i + \sin(\delta - i) = Nn\lambda$$

$\cos i \, di + \cos(\delta - i)(d\delta - di) = 0$.

$$\frac{d\delta}{di} = -\frac{\cos i - \cos(\delta - i)}{\cos(\delta - i)}$$ & equal to zero, when $i = \delta - i$

i.e. $i = \theta = \frac{\delta}{2}$.

Hence if δ_m be the min. deviation,

$\sin \delta_m/2 = Nn\lambda \therefore \lambda = \frac{2 \sin \delta_m/2}{Nn}$

At the position of min. deviation the spectrum is clearest. Hence adjust the telescope to view the 1st spectrum when incidence is normal. Then gradually move the telescope towards the direction of the collimator, & correspondingly rotate the grating the keep the spectrum in view. When the spectrum begins to turn back upon its ~~position~~ path, we have the min. deviation position.

Resolving power. Two lines which are very near each other wavelengths ~~λλ'~~ λ & λ' are said to be just resolved if they are just seen separate which will be the case if the ~~1~~ 1st minimum after the mth principal maximum of λ ~~etc~~ coincides with the minimum immediately preced the m^{th} principal maximum of λ'. For in this case the two images of λ & λ' are on the limit of overlapping.

It was seen that intensity is of the form $I = a^2 b^2 \frac{\sin^2 u}{u^2} \cdot \frac{\sin^2 nx}{\sin^2 x}$. From this we can derive an expression for the resolving power which is defined a the ratio of any particular wavelength to the difference between λ and another λ which can just be resolved from it

Resolving power $= \frac{\lambda}{\lambda \sim \lambda'} = \frac{\lambda}{d\lambda}$

Now $\frac{\sin^2 nx}{\sin^2 x}$ is max. when $x = m\pi$. & ~~Sa~~ is min. when $nx = m'\pi$. Value of x for the first min. after the m^{th} max.
$= m\pi + \frac{\pi}{n}$ & value of x for the min just preceding the m^{th} max. $= m\pi - \frac{\pi}{n}$

Now $x = \frac{\pi(a+d)\sin\theta}{\lambda}$ $\therefore \sin\theta = \frac{\lambda N}{\pi} x$

N being no. of lines per cm. Let θ be the devia of the 1^{st} min. after m^{th} max. of λ & θ' the

...eviation of the min. just before the m^{th} max.

λ'. $\sin\theta = \frac{\lambda N}{\pi}\left(m\pi + \frac{\pi}{n}\right) = \lambda N\left(m + \frac{1}{n}\right)$

$\sin\theta' = \lambda' N\left(m - \frac{1}{n}\right)$

At the limit of resolvability $\theta = \theta'$

i. e. $\lambda\left(m + \frac{1}{n}\right) = \lambda'\left(m - \frac{1}{n}\right)$

$\therefore$ Resolving power $= \frac{\lambda}{\lambda' - \lambda}$

$$= \frac{\lambda}{\lambda\left\{\frac{m + \frac{1}{n}}{m - \frac{1}{n}} - 1\right\}} = \frac{m - \frac{1}{n}}{\frac{2}{n}}$$

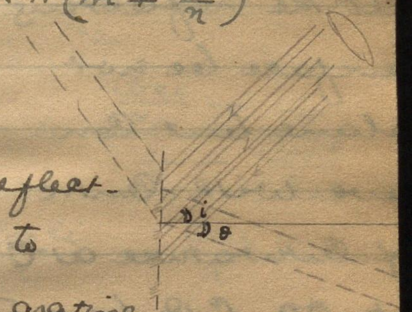
$$= \frac{mn - 1}{2} = \frac{mn}{2}$$ since 1 is very small compared to mn.

Thus for resolving the $d_1 d_2$ lines

R. P $= \frac{mn}{2} = \frac{5893}{3}$ $\therefore$ In the 1st order spectrum the lines are just resolved if $n = \frac{2 \times 5893}{3}$; if there are 3928 lines on the grating. Raleigh tried to verify the result by placing a slit in front of the grating and gradually narrowing the slit so as to reduce the total number of lines. He found that $d_1 d_2$ lines were seen ~~of the~~ even when n was less than 3900. The limit was reached when n = nearly $\frac{1}{2}$ of 3928. $\therefore$ He concluded that R.P. should be mn, not $\frac{mn}{2}$. To resolve it is not necessary to have a perfectly dark line between; it suffices if there is comparative darkness between the lines. This condition is satisfied if the m^{th} max. of λ' lies on the min. immediately after the m^{th} max. of λ. i.e. $\sin\theta' = mN \times \lambda'$

$= \lambda' N m \quad = \sin\theta = \lambda N\left(m + \frac{1}{n}\right)$

$\therefore$ R. P = ~~mn~~ $\underline{mn}$



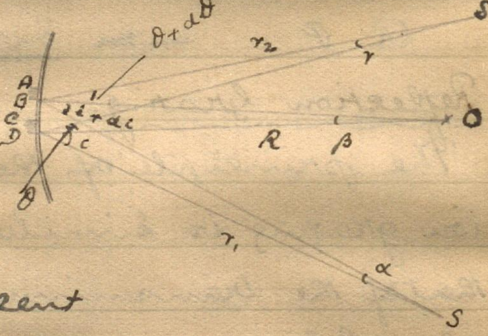
Reflection Grating.

The principle of the reflection grating is similar to that of the transmission grating for oblique incidence. When the image is on the same side of the grating normal (in transmission grating) the eq$\underline{n}$ for the n^{th} order spectrum is $(\sin i + \sin\theta) = Nn\lambda$. If the image is on the opposite side, $(\sin i \sim \sin\theta) = Nn\lambda$.

[Th]e same eqns hold good here. The streams [of] light regularly reflected from the sur[fa]ces between the rulings proceed from [a] virtual image of the source as if they [c]ame from behind the surface. ∴ $n\lambda = (\sin i \mp \sin\theta)$ The rulings [ar]e usually made on a polished metal [su]rface (aluminized glass).

[C]oncave or Rowland grating 13-7-38

[L]et the reflecting [s]urface be not [p]lane but Con[c]ave with Centre [o]f Curvature at O



[Le]t AB, CD be adjacent [r]eflecting strips & BC a ruling. [L]et the rays SB, SD from a source S [c]onverge to a point S'. ~~Let~~ We shall [d]enote $\angle ODS = i$, $ODS' = \theta$, $OBS = i + di$, $[O]BS' = \theta + d\theta$ $SD = r_1$, $DO = R$, $S'D = r_2$ $\angle BSD = \alpha$, $\angle BOD = \beta$, $\angle BS'D = \gamma$.

S' is a bright point if $(a+d)(\sin i - \sin \theta)$

$= n\lambda \quad \therefore \cos i \, di = \cos\theta \, d\theta$

$$\frac{di}{d\theta} = \frac{\cos\theta}{\cos i}.$$

$$i + \alpha = i + di + \beta$$

$$\therefore di = \alpha - \beta$$

$$\theta + d\theta + \gamma = \theta + \beta$$

$$\therefore d\theta = \beta - \gamma$$

$$\therefore \frac{di}{d\theta} = \frac{\alpha - \beta}{\beta - \gamma}.$$

Now $\alpha = (a+d)\dfrac{\cos i}{r_1}$

$$\beta = \frac{a+d}{R}, \qquad \gamma = (a+d)\frac{\cos\theta}{r_2}.$$

$$\therefore \frac{\cos\theta}{\cos i} = \frac{di}{d\theta} = \frac{\alpha-\beta}{\beta-\gamma} = \frac{\frac{\cos i}{r_1} - \frac{1}{R}}{\frac{1}{R} - \frac{\cos\theta}{r_2}}$$

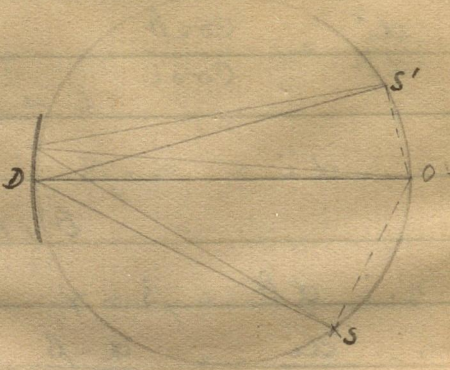
$$\cos\theta \, \frac{1}{R} - \frac{\cos^2\theta}{r_2} = \frac{\cos^2 i}{r_1} - \frac{\cos i}{R}$$

$$\text{i.e.} \quad \frac{\cos^2\theta}{r_2} = \frac{\cos\theta + \cos i}{R} - \frac{\cos^2 i}{r_1}$$

$$r_2 = \frac{R\, r_1 \cos^2\theta}{r_1(\cos\theta + \cos i) - R\cos^2 i}$$

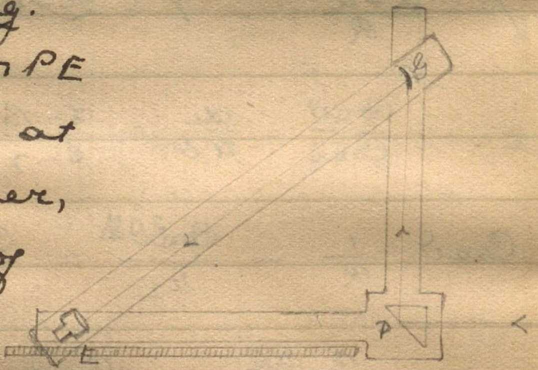
Suppose $r_1 = R\cos i$ which will be the case if S lies on a circle with OD a

diameter
radius. Then $R_2 = R\cos\theta$ i.e.
also will lie
on the ⊙ with
D as diameter.
This arrangement
was used by Rowland for mounting
his grating. Hence
the ⊙ is known as Rowland's circle.



Rowland's mounting.

Two beams PG & PE
are fixed rigidly at
rt. ∠s to each other,
& a third beam of
constant length
G can move on
~~the other~~ keeping its ends on the other two.
At G a grating of radius of curvature
GE is mounted with the grating normal
along GE & at E an eyepiece is placed
directed towards G. A rt. ∠d prism at P



reflects a beam of light to G & hence acts as source. Thus it is seen that the Source is always on a Semicircle having GE (the line joining the grating to its Centre of Curvature) as diameter. Further the arrangement is such that $\theta = 0$

$\therefore$ The eqn to the spectrum is $\sin i = Nn$

But $\sin i = \dfrac{x}{\rho}$ where x is distance EP

$\therefore \lambda = \dfrac{x}{\rho N n}$ Hence with the Rowland mounting the different orders are all seen on the line EP at distances from P proportional to the wavelength. The arrangement is especially suitable for photographic purposes.

Advantages of the Concave grating

The advantages of the concave grating are many. It combines the ~~dis~~ chromatic dispersive power of the prism or plane grating, and the focussing power of the lens. The collimating lens and eye piece are unnecessary for the ~~the~~ incident an

iffracted beams are pencils of rays, not
arallel beams. This renders the instrument
asy for photography. The sharpness of
e lines excels that of plane grating. Since
o Collimator or telescope is used the
ltra violet lines are not absorbed. Hence
~~e use of~~ quartz lenses needed for ultra violet photo-
raphy in other instruments can here be
ispensed with. The spectrum is truly nor-
al, much more so than the plane grating
pectrum for x is $\propto \lambda$, ~~not~~ s where-
s in plane grating $\sin\theta \propto \lambda$. All the
rders of spectra "stay in focus". Thus a
late which prints the d_1 d_2 lines of the
st order will also print the 2950 A°U.
ltraviolet of the 2nd order. No separate
ocussing is necessary

But Concave gratings have one
reat defect of astigmatism. Point
~~images~~ sources are extended into line
images, & thus the image suffers in

definiteness. Astigmatism is least when the slit is perfectly parallel to the grating rulings. This however is a condition which cannot be fulfilled easily. Hence what is generally done is to take several photographs with different positions of the slit all nearly ‖l to the rulings. That which shows best definiteness is chosen. Astigmatism can be minimized by shortening the rulings & ~~or~~ taking a larger width of the grating.

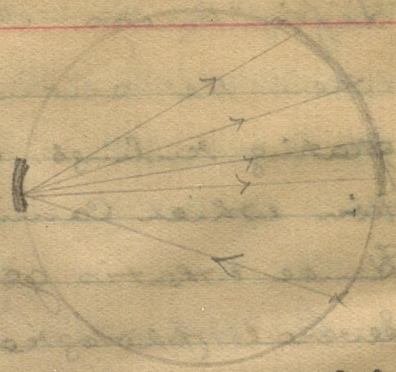
Dispersive power $\frac{dx}{d\lambda} = \rho m A$ & $\propto$ i.e. is $\propto$ to no. of rulings & the order of the spectrum.

R.P. is $\propto$ to the total no. of lines.

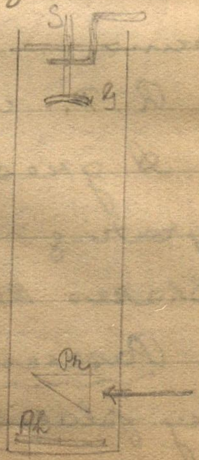
A great disadvantage is that the grating moves in a slot and necessarily shakes the camera.

<u>Paschen mounting</u> obviates this difficulty by placing the source at some point on

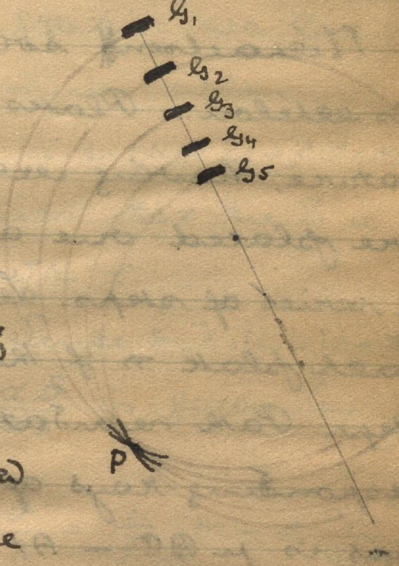
Rowlands circle. The
plate is a long flexible
one & is placed along
the Occ of the Rowlands
circle. Thus all the
lines of the spectrum
are simultaneously photographed. This
arrangement is found to give less astig-
matism. But yet the arrangement is
cumbrous.



Eagle Mounting is more compact, gives
spectra of increased brightness, has least
astigmatism, can reach higher orders,
& can focus on either side of the central
white. The Grating is mounted
on a screw track within a
light tight box & by means
of an external handle it can
be moved ||l to itself. The
source is an illuminated slit
reflected by a suitable prism.



Thus: the virtual source is some pt S on ~~along~~ the normal to the grating. The plate [is] placed at the bottom of the box & by suita[ble] screws its inclination can be adjusted from outside. Let P be the position of the ~~plat~~ plate & the virtual source. As the grating is moved forward, the Rowland's circle changes its position but ~~it~~ the grating is so adjusted that P is always on the Rowland's ⊙. The ~~are~~ plates have to be inclined suitably to lie along the ⊙

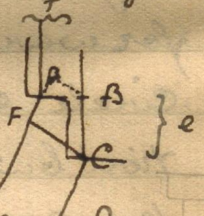


Thus the arrangement can be made to photograph any part of the spectr[um]

<u>Michaelson's Echelon Grating</u> The resolv[ing] power of a ruled grating which is the product of the number of rulings and the order of the spectrum, cannot be increas[ed]

yond a certain limit; for orders higher than the first are faint, & rulings cannot in practice be made more than 10000 to a ...n.

Michaelson solved the difficulty by his echelon. Plates of the same optical nature, having exactly the same thickness are placed one above the other forming a series of steps. Let e be the thickness of each plate & f the width of each step. Path retardation between corresponding rays of two adjacent



steps is $\mu BC - AF = \mu e - e\cos\theta + f\sin\theta$
$(\mu-1)e + f\theta$ when θ is small. For the principal maximum of the m^{th} order
$m\lambda = (\mu-1)e + f\theta$.

In the grating where path diff. between corresponding rays is $(a+d)\sin\theta$ we saw intensity to be $a^2b^2\dfrac{\sin^2\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\left(\frac{\pi a\sin\theta}{\lambda}\right)^2} \times \dfrac{\sin^2\left\{\frac{n\pi(a+d)\sin\theta}{\lambda}\right\}}{\sin^2\left\{\frac{\pi(a+d)\sin\theta}{\lambda}\right\}}$

Hence here where ~~n~~ $(\mu-1)e + f\theta$

stands in place of $(a+d)\sin\theta$, we have
Intensity given by

$$I = A^2 \frac{\sin^2 \frac{n\pi}{\lambda}\{(\mu-1)e + f\theta\}}{\sin^2 \frac{\pi}{\lambda}\{(\mu-1)e + f\theta\}}$$

As in the case of the grating we can see the expn a maximum when
$\frac{\pi}{\lambda}\{(\mu-1)e + f\theta\} = m\pi$, i.e. $(\mu-1)e + f\theta = m\lambda$
The intensity is n^2 times that due to a single opening of breadth f.

<u>Dispersive power</u>. Since $m\lambda = (\mu-1)e + f\theta$
the dispersive power $\frac{d\theta}{d\lambda}$ is given by

$$m = \frac{d\mu}{d\lambda}e + f\frac{d\theta}{d\lambda}$$

i.e. $$\frac{d\theta}{d\lambda} = \frac{1}{f}\left(m - e\frac{d\mu}{d\lambda}\right)$$

Since m is approximately $\frac{(\mu-1)e}{\lambda}$ when θ is small,

$$\frac{d\theta}{d\lambda} = \frac{1}{f}\left\{\frac{e}{\lambda}(\mu-1) - e\frac{d\mu}{d\lambda}\right\}$$

$$= \frac{e}{f\lambda}\left\{(\mu-1) - \lambda\frac{d\mu}{d\lambda}\right\}$$

$$= a\frac{e}{f\lambda}$$ where a is a constant depending on the nature of glass

Resolving power. According to Rayleigh's [th]eory two lines λ & $\lambda + d\lambda$ are just resolved [when] the m^{th} order max. of $\lambda + d\lambda$ coincides [w]ith the 1^{st} min. after the m^{th} max. of λ.

Let $\theta + d\theta$ be the position of this min.

$$(\mu - 1) e + f(\theta + \cancel{d\theta}) = \cancel{m(\lambda + d\lambda)} (m+1)\lambda$$

$$\text{but } (\mu - 1) e + f\theta = m\lambda$$

$$\therefore \cancel{f d\theta = m\, d\lambda}\ f\varepsilon = \cancel{m+1}\ \lambda$$

[ε] being the $\angle$lar separation of the m^{th} [&] $(m+1)^{th}$ orders of λ.

Since there are n minima within ε,

$$[d]\theta = \frac{e}{n} = \frac{\lambda}{fn}. \quad \text{ie } \lambda = fn\, d\theta$$

$$\text{But } \frac{d\theta}{d\lambda} = \frac{ae}{f\lambda}$$

$$\therefore d\lambda = \frac{f\lambda}{ae} d\theta \quad \therefore \frac{\lambda}{d\lambda} = \frac{aen}{\lambda}$$

$$[\therefore] \text{ R. P. } \frac{\lambda}{d\lambda} = \frac{en}{\lambda}\left\{(\mu - 1) - \lambda \frac{d\mu}{d\lambda}\right\}$$

If $\frac{d\mu}{d\lambda}$ is negligible, & $m\lambda = (\mu - 1)e$ for a first approximation $\frac{e}{\lambda}(\mu - 1) = m$

$$\therefore \text{ R. P. } \frac{\lambda}{d\lambda} = \underline{\underline{mn}}$$

The echelon grating like all other high resolving instruments cannot be used to form spectra. The order of the spectrum being high all the colours overlap if the source is ~~mono~~ white. Hence the necessity of using strictly monochromatic sources. A slit may be opened in the spectrum of an ordinary grating and this narrow band of wavelengths is made to fall on a the grating. A more usual procedure is to use a series of filters with a source like the copper arc or the mercury lamp.

Calculate the thickness of a flint glass echelon which will just avoid the overlapping of the resolved components of sodium light and confine them to a particular order. [Cauchy's Const. $A = 1.78$; $B = 1.50 \times 10^{-10}$]

Let λ & $\lambda + d\lambda$ be the components of sodium. ~~and~~ The condition for being just resolved & confined to a particular order so that $(m+1)^{th}$ order of λ just coincides with m^{th} order of $\lambda + d\lambda$.

$$m\lambda = (\mu - 1)e + f\theta$$

$$\lambda \times dm = \cancel{(\mu-1)e} + f\,d\theta.$$

When $dm = 1$, $f\,d\theta = \lambda - \cancel{(\mu-1)}e \quad (1)$

$$m\,d\lambda = \cancel{\mu-1}\,e\,d\mu + f\,d\theta \quad (2)$$

For resolution $d\theta$ in both cases should be equal $\therefore$ $m\,d\lambda = e\,d\mu + \lambda - \cancel{(\mu-1)}e. \quad (3)$

$\mu = A + \frac{B}{\lambda^2}$ $\quad d\mu = \cancel{A} - 2B/_{\lambda^3} \times d\lambda$

Taking θ to be zero,

$$m = \frac{(\mu-1)e}{\lambda}$$

Substituting the above values

$$d\lambda \times \frac{(\mu-1)e}{\lambda} = e\,d\mu + \lambda - \cancel{(\mu-1)}e$$

$$(\mu-1)e\left(\frac{d\lambda}{\lambda}\cancel{+1}\right) = e\,d\mu + \lambda$$

ie $\quad e = \dfrac{\lambda}{(\mu-1)\left(\frac{d\lambda}{\lambda}\cancel{+1}\right) - d\mu} = \dfrac{\lambda/d\lambda}{\frac{\mu-1}{\lambda} - \frac{d\mu}{d\lambda}}$

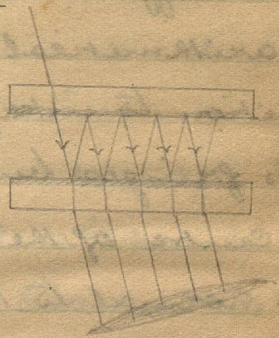
$$= \frac{\lambda^2}{(\mu-1)(\lambda+d\lambda) - d\mu} = \frac{\frac{\lambda}{d\lambda}}{\frac{\cancel{A}}{\lambda} + \frac{3B}{\lambda^3} - 1/\lambda}$$

$$= \frac{5890^2}{\cancel{A + \frac{B}{\lambda}}} = \frac{\lambda^2}{\cancel{\left(A + \frac{B}{\lambda^2} - 1\right)}(\cancel{\lambda} + d\lambda) - \cancel{A} + \frac{2B}{\lambda^3}}$$

Substituting for A, B etc we shall have

l = ·66 mm nearly.

Fabry Perot Interferometer.



In Michalson's interferometer the ~~two~~ rings seen under ~~mono~~ say sodium light were all broad instead of being split up into two narrow rings each. In other words the resolving power was small. This was due to the number of interfering sources being only two & R. P. = n m.

Fabry Perot devised a similar apparatus with high resolving power. Two truly plane plates of glass are semisilvered on one side each in the same bath and to the same extent. They are mounted silvered sides facing each other parallel to each other and one of them is provided with a screw track to move it perpendicular to its plane. The rays after multiple reflect

[b]etween the semisilvered faces are trans-
[m]itted and focussed by a lens. Since the
[p]ath difference of the various rays form
[a]n arithmetical progression they are in a
[p]osition to interfere. ~~Hf~~ Rings ||r to Haidin-
[ge]r's fringes but with more Contrast are seen,
[th]e centre of the rings corresponding to the
[r]ay normal to the plates. Since the relative
[p]ath retation is great the order of the ~~fringes~~
[s]pectrum is ~~g~~ high. Further the number of
[v]irtual sources which cause interference
[i]s large. Hence the resolving power of the
[i]nstrument is very high.

The influence of semisilvering may
[b]e easily deduced ~~from the~~ as follows.
~~I~~ The intensity of the ~~raf~~ transmitted
[b]eam $= a^2 \dfrac{(1-b^2)^2}{(1-b^2)^2 + 4b^2 \sin^2 \frac{\varepsilon}{2}}$ Since the
fringes are exactly those of Haidinger's
[e]xcept for the fact that ~~they are~~ film is thick.
For normal incidence $b^2 = \cdot 04$ for un-
silvered surfaces & $\cdot 8$ for semisilvered.

$\therefore I_t$ ~~for~~ when surfaces are not silver[ed]

$$= a^2 \frac{(1-\cdot 04)^2}{(1-\cdot 04)^2 + \sin^2\frac{\varepsilon}{2} \times \cdot 16}$$

$$= a^2 \frac{1}{1+\cdot 16 \times \sin^2\frac{\varepsilon}{2}} = a^2\left(1 - \cdot 16 \times \sin^2\frac{\varepsilon}{2}\right.$$

When $\varepsilon/2 = 0, \pi, \ldots$ $I_t = a^2$ nearly

When $\frac{\varepsilon}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$ $I_t = \cdot 84 a^2$ nearly.

Thus the intensity varies between $\cdot 84$ ~~a~~ & a^2. The dark & bright rings are hardly distinguishable.

When the surfaces are ~~s~~ silvered

$$I_t = a^2 \frac{(1-\cdot 8)^2}{(1-\cdot 8)^2 + \cdot 64 \times \cancel{4} \sin^2\frac{\varepsilon}{2}}$$

$$= a^2 \frac{\cdot 04}{\cdot 04 \times 3\cdot 2 \sin^2\frac{\varepsilon}{2}}$$ ~~nearly~~

$\therefore$

Which when $\frac{\varepsilon}{2} = 0, \cancel{\frac{\pi}{2}}\ \pi, 2\pi$ etc is a^2

& when $\frac{\varepsilon}{2} = \frac{\pi}{2}, \frac{3\pi}{2}$ etc is $a^2 \frac{\cdot 04}{3\cdot 24} = \cdot 01$

The intensity of the trans. beam varies between 100% & 1%. The rings are very clear. Further since the sq. of $\sin\frac{\varepsilon}{2}$ is involved in the function, the variation

[-] with $\frac{\varepsilon}{2}$ is very rapid. The lines of [m]ax. brightness are ~~very narrow~~ sharply [d]efined.

The above expn for I_t is only an [a]pproximate one since when deducing [i]t, the assumption was made that ~~the~~ the [n]umber of interfering beams is infinite. [A]ctually it is not so since the film is [t]hick. If n be the no. of beams

$$I_t = a^2 \frac{(1-b^2)^2}{(1-b^2)^2 + 4b^2 \sin^2\frac{\varepsilon}{2}} \left\{ (1-b^{2n})^2 + 4b^{2n} \sin^2 \frac{n\varepsilon}{2} \right\}$$

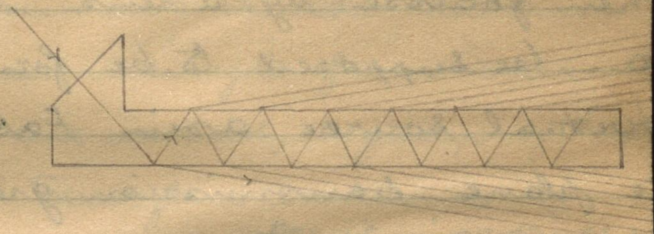
The above expn shows that for every one [f]luctuation of the denominator there are [n] fluctuations of the numerator. Since n is large the resolving power of the interfero-[m]eter is also large.

Lummer Gehrcke Plate.

Although the Chromatic R. P. is high [an]d the images are ~~sharp~~ clear (due to semisil-[v]ering), the Fabry Perot interferometer [h]as one great disadvantage viz the silvering

absorbs a good portion of the light &
hence only a fraction of the incident
intensity is available in the pattern.

The Lummer Gehrcke plate aims at an equally high ~~resolving~~ reflecting power without any absorption.



It is a plane sheet of glass with exactly parallel sides; & ~~a rt. ∠d prism at~~ one end is shaped like a rt. ∠d prism. The acute ∠s of the prism are such that light incident normally on the hypotenuse surface * reaches the lower face of the plate at an incident angle slightly less that the critical angle. Hence a small part is transmitted while the rest is reflected towards the upper surface. ~~A~~ Here it is again reflected after a small fraction has been transmitted. Thus several transmitted beams with path retardations in a regular arithmetical progressions emerge from either face, all of which can interfere or reinforce ~~each ot~~ when focussed by a lens. * Each beam can be supposed to be formed by a virtual source as in Fabry Perot or the plane transmission grating, and hence the chromatic R. P $= n m$

$$m\lambda = 2\mu e \cos r \qquad n \qquad n \times 2e \tan r = l$$

$$\therefore R.P = \frac{2\mu e \cos r}{\lambda} \times \frac{l}{2e \tan r}$$

$$= \frac{\mu l}{\lambda} \frac{\cos^2 r}{\sin r} = \frac{\mu l}{\lambda} \frac{1 - \sin^2 r}{\sin r}$$

Taking $\sin r$ as approximately $\frac{1}{\mu}$

$$C.R.P = \frac{\mu l}{\lambda} \frac{1 - \frac{1}{\mu^2}}{\frac{1}{\mu}}$$

$$= \frac{l}{\lambda}(\mu^2 - 1)$$

Thus for a plate of length 10 cm, $\mu = 1{\cdot}5$, $\frac{\lambda}{d\lambda} = \frac{10}{6 \times 10^{-5}} (1{\cdot}25)$

$$= 2{\cdot}1 \times 10^5 \quad \therefore d\lambda = \frac{6 \times 10^{-5}}{2{\cdot}1 \times 10^5} = 3 \times 10^{-10}$$

Å

Hence two lines of λ, 6000 & 6000·03
Can be resolved by the plate.

Uses of High Resolving Power

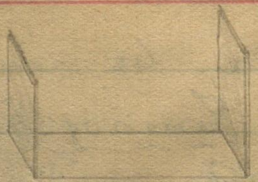
1) These instruments reveal that lines which appear single from other instruments are actually a group of ~~two~~ fine lines. This fine structure of spectral lines has a ~~deep bearing on~~ great importance in Mod. physics

2) The constituents of an apparently single star or of nebulae can be detected by high R. P.

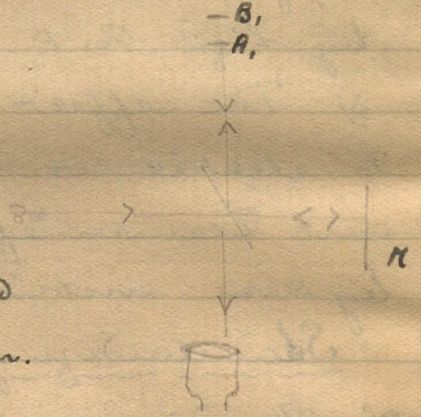
3) The effects of subjecting a source to electric or magnetic field — Stark and Zeeman effects can be studied only by these instruments.

Standardizing the metre. The expt of standardizing the metre in terms of three standard lines of Cadmium was conducted by Michaelson at the request of the bureau international des mesures

oids. The apparatus
was a Combination of
ichelson's interfero-
eter & Fabry Perot



lates. Two semisilvered plates one shorter than the
ther were mounted ~~at~~ parallel to each
ther at distances 10 Cms $\frac{10}{2}$, $\frac{10}{4}$, $\frac{10}{8}$,
$\frac{10}{16}$, $\frac{10}{32}$ $\frac{10}{64}$, $\frac{10}{128}$, $\frac{10}{256}$ Cms ~~etc~~ from each other there
eing on the whole 9 such ~~on~~ Fabry
erot interf. or etalons as they are called.
he Smallest etalon of length ·039 Cm
s placed in the position
f one of Michelson's
irrors, ~~with~~ the front
late A, ~~coinciding with~~
e being where the silvered
urface should have been.
hus ~~no int~~ rings are



roduced between M & A. Adjust the position
f M till a crosswire touches the centre of
e system. Now lower the etalon. Interference

is between M + the upper portion of B, +

so ~~rings~~ the system is displaced. Rings

will still be seen if the source is strictly

monochromatic. Move M backwards slo

counting the rings as they pass the crossw

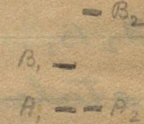
till the centre of the system again comes

the crosswire. The no. of rings was 780.

∴ The length of the etalon is 780 wavelength

of Cd red

Next place the 2nd etalon



(length nearly twice that of the

first) near the first. See that

when interference is between M + A_1 or

bet. M + A_2 the system is not disturbed.

Hence A_1 + A_2 are at the same ~~d~~ distan

Then ~~move~~ lower A_1B_1 till int. is between B_1 + M, +

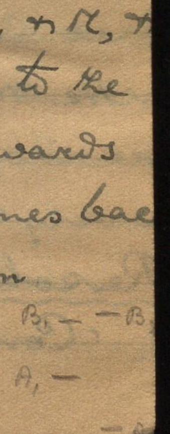
move M to bring back the system to the

crosswire. Next, raise A_1B_1, ~~move A_1B_1~~ backwards

till the displaced system again comes bac

to centre. Thus A_1 is in the position

originally occupied by B_1.



hen get the interference between B_1 & R

~~So~~ Shift the etalon stage to the left so

at int. may be between B_2 & R. See whether

e system is displaced. If so B_2 is not

t the same distance as B_1. Move R

ack till system is replaced. ~~If~~ Note

e rings that pass the cross wire which will

e only 30 or 40. Hence length $A_2 B_2 = 2 \times 780$

30. In this way $A_3 B_3$ can be compared

ith $A_2 B_2$, $A_4 B_4$ with $A_3 B_3$ and so on. The

st etalon can be compared with the metre

y means of a microscope. Thus ~~he~~

ichelson computed the number of

wavelengths in one metre to be 1,555,163·5.

le calculated the wavelengths of the

hree Cd lines in terms of the standard

etre & termed them the standard lines. They

re unalterable standards ~~of~~ depending on

e very nature of light.

Resolving power of a telescope.

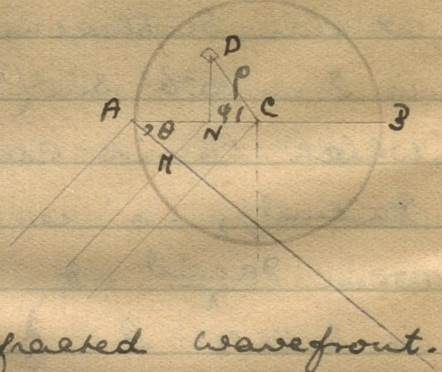
Consider a telescope objective with

a rectangular ~~aperture~~ slit in front of it the length b of the aperture being large compared to breadth a. The illumination at $\angle$lar displacement θ $\perp$r to the slit is $I = a^2b^2 \sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \Big/ \left(\frac{\pi a \sin\theta}{\lambda}\right)^2$. which has its central max. when $\pi a \sin\theta / \lambda = 0$ i.e. $\theta = 0$ & its next min when $\frac{\pi a \sin\theta}{\lambda} = \pi$, i.e. $\sin\theta = \frac{\lambda}{a}$ & since θ is small $\theta = \frac{\lambda}{a}$.

Now if two sources at distances d from each other (d being measured $\perp$r to the telescope axis) ~~&~~ are to be ~~not~~ resolved the image of one should be where the first minimum of the other lies i.e. $\angle$ subtended by the images at the field lens should be $\frac{\lambda}{a}$ But the $\angle$ subtended by the images = that subtended by the objects $\therefore \frac{\lambda}{a} = \frac{d}{u}$ u being distance of the objects. This gives the condition for the objects being just resolved.

The larger the value of u, the larger should a b

When the ~~slit is~~ aperture is ⊙lar and
not rectangular a different procedure is
necessary. Let R be the radius
of the aperture. Consider
the ~~wavelets~~ diffracted
rays inclined at $\angle \theta$ to
the aperture normal. AM



inclined at θ to the plane
of the aperture is the diffracted wavefront.
The path difference ~~of the~~ between the
rays starting from A & from any other
point P = path difference between rays
from A & from N (N being the projection
of P on the diameter ACB) $= MN = AN \sin\theta$
$= (R - \rho \cos\varphi) \sin\theta$ where $\rho = PC$ & $\varphi = P\hat{C}N$.
∴ ~~Am~~ Displacement due to a small
area $\rho\, d\varphi \times d\rho$ about P

$$= \rho\, d\varphi \times d\rho \times \left[\sin\left\{ \omega t + \frac{2\pi}{\lambda} \sin\theta \,(R - \rho\cos\varphi) \right\} \right]$$

$$= \rho\, d\varphi\, d\rho \times \sin(\omega t + kR - k\rho\cos\varphi)$$

where $k = \frac{2\pi}{\lambda} \sin\theta$

$$= \rho\, d\varphi\, d\rho \sin \cancel{\omega t} (\alpha - k\rho\cos\varphi)$$

$$= \sin\alpha \left\{ \rho\, d\varphi\, d\rho \cos(k\rho\cos\varphi) \right\}$$

$$- \cos\alpha \left\{ \rho\, d\varphi\, d\rho \sin(k\rho\cos\varphi) \right\}$$

α being a function of time, the expn denotes two simple harmonic motions at rt $\angle$s.

The intensity due to the small area

$$= \left\{\cos(k\rho\cos\varphi)\,\rho\, d\varphi\, d\rho\right\}^2 + \left\{\sin(k\rho\cos\varphi)\,\rho\, d\varphi\, d\rho\right\}^2.$$

Intensity due to the whole area of the aperture

$$= \left\{\int_0^{2\pi}\!\!\int_0^{R} \cos(k\rho\cos\varphi)\,\rho\, d\varphi\, d\rho\right\}^2$$

$$+ \left\{\int_0^{2\pi}\!\!\int_0^{R} \sin(k\rho\cos\varphi)\,\rho\, d\varphi\, d\rho\right\}^2$$

The sine integration reduces to zero

$\therefore$ Intensity I at inclination θ

$$I = \left\{\int_0^{R}\int_0^{2\pi} \cos(k\rho\cos\varphi)\,\rho\, d\varphi\, d\rho\right\}^2.$$

Integrating we shall get

$$\sqrt{I} = \pi R^2 \left\{ 1 - \frac{1}{2}\left(\frac{m}{\lfloor 1}\right)^2 + \frac{1}{3}\left(\frac{m^2}{\lfloor 2}\right)^2 - \frac{1}{4}\left(\frac{m^3}{\lfloor 3}\right)^2 + \dots\right.$$

ad inf

$= \pi R^2 S$

m being $\frac{kR}{2}$ & since $k = \frac{2\pi}{\lambda} \sin\theta$

$m = \frac{\pi R \sin\theta}{\lambda}$.

The tabulation below gives values of $\frac{m}{\pi}$ i.e. of $\frac{R \sin\theta}{\lambda}$ for which I has turning values, and the corresponding illumination

$\frac{m}{\pi} = \frac{R\sin\theta}{\lambda}$	$I = (\pi R^2 S)^2$	
0	1	1st maximum
·061	0·0000	1st minimum
0·81	0·0175	2nd max.
1·16	0·0000	2nd min.
1·33	0·0041	3rd max
1·62	0·0000	3rd min.

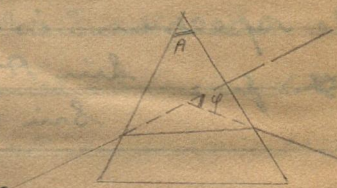
Thus dark & bright rings are seen the ~~diameters~~ radii of which subtend at the centre of the aperture $\angle$s given by the first column. In the centre is a bright patch of $\angle$lar radius $\sin\theta = \theta = \cdot 61 \times \frac{\lambda}{R}$. The ~~other~~ bright rings are of so feeble an illumination that their presence is undetected except by ultra photograph

precautions. This is the wave theory expla-
nation for the point image due to ~~a~~
parallel ~~beam~~ rays of light. Two
images are resolved if the
∠ subtended by ~~that~~ ~~cent~~ line
joining their centres subtends an ∠ .61
at the centre of the aperture. But the
∠ ~~x~~ is $\frac{d}{u}$ d being the distance between the
objects & u their distance from the lens.
Thus at the limit of resolution

$$\frac{\delta}{u} = .61\frac{\lambda}{R} = 1.22\frac{\lambda}{a}$$ a being diame

ie $\frac{d}{u} = \frac{\lambda}{a}$ nearly which is the
same as ~~that~~ eqn. of a rectangular aperture.

To find the relation between



<u>∠ of deviation in a prism</u>
<u>& μ</u>

We have 4 eqns $r + r' = A$, (1)

$\varphi = i + i' - A$ (2) $\sin i = \mu \sin r$ (3)

& $\sin i' = \mu \sin r'$ (4)

Differentiating,

$$dr + dr' = 0 \qquad (5)$$

$$d\varphi = di' \qquad (6)$$

$$0 = \mu \cos r\, dr + \sin r\, d\mu \qquad (7)$$

$$\cos i'\, di' = \mu \cos r'\, dr' + \sin r'\, d\mu \qquad (8)$$

$$d\mu = -\mu \frac{\cos r}{\sin r} \times dr$$

$$dr = -\frac{\sin r}{\mu \cos r} \cdot d\mu = -dr'$$

$$\cos r' \frac{\sin r}{\cos r}\, d\mu + \sin r'\, d\mu = \cos i'\, di'$$

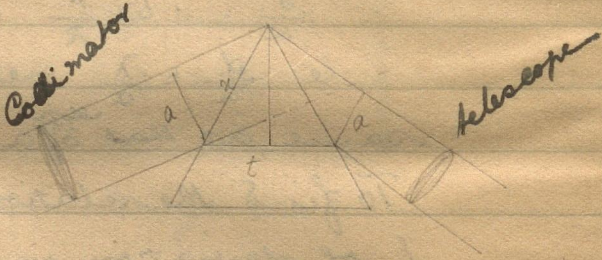
$$d\mu\, (\cos r' \sin r + \sin r' \cos r) = \cos i' \cos r\, di'$$

$$\text{i.e. } d\mu \sin(r + r') = \cos i' \cos r\, d\varphi$$

$$\text{i.e. } d\mu = \frac{\cos i' \cos r}{\sin A}\, d\varphi.$$

Resolving power of a prism spectroscope.

The prism is placed for minimum deviation where ~~sin i/sin~~ the spectrum is clearest.



$$\mu = \frac{\sin \frac{A+\varphi}{2}}{\sin A/2}$$

$$\therefore d\mu = \frac{1}{\sin A/2} \cdot \cos \frac{A+\varphi}{2} \times \frac{d\varphi}{2}$$

$$d\mu = \frac{\cos \frac{A+\varphi}{2}}{2 \sin \frac{A}{2}}\, d\varphi.$$

If t be the effective base of the prism, a the aperture,

$$d\mu = \frac{a/x}{2 \frac{t}{2}/x} d\varphi = \frac{a}{t} d\varphi$$

$$\text{ie} \quad d\varphi = \frac{t}{a} d\mu.$$

That the telescope may resolve the 2 lines Separation $d\varphi$, $d\varphi$ should be $\frac{\lambda}{a}$

$\therefore \lambda = t\, d\mu$ for the limit of resolution

Resolving power $\lambda / d\lambda = \frac{t\, d\mu}{d\lambda}$

& by Cauchy's formula

$$\mu = A + B/\lambda^2$$

$$\frac{d\mu}{d\lambda} = -2B/\lambda^3$$

$$\therefore R.P = -\frac{2Bt}{\lambda^3}.$$

The above eqn can be used to find the difference of in wavelength say of the two sodium lines

$\frac{10^{-8} \times 5893}{d\lambda} = -\frac{2Bt}{\lambda^3}$ B being known & t exptly determined by gradually lessen the effective base, $d\lambda$ can be calculated.

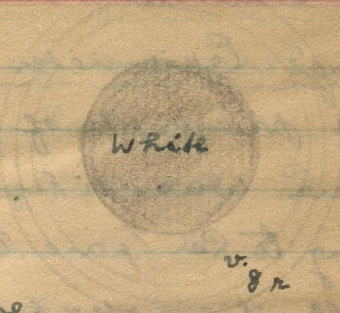
Babinet's Principle* first enunciated in 1837 at holds good for points in a diffraction pattern where the intensity is zero. If the aperture be covered by a screen certain portions of which are

ransparent and other portions opaque,
en the points of zero illumination will
ave a certain displacement & intensity
iven by $y_1 = a \sin \varphi$; $I_1 = a^2$. Now
uppose the screen is replaced by another
xactly its complement i.e. one having
e opaque & transparent portions just
~~eplacing one anot~~ reversed in shape
nd contour. The displacement due to
e ~~opaque~~ transparent portions in this
creen should be $y_2 = -y_1$ since due
 transparent portions of both the screens
ogether i.e. due to the whole aperture, the
 displacement is zero.

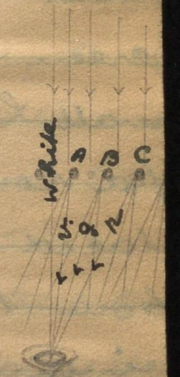
$\therefore$ $y_2 = -a \sin \varphi$. And $\therefore$ illumi-
nation at the point $I_2 = (-a)^2 = a^2 = I_1$
i.e. Illumination due to complementary
diffraction patterns are equal.

Halo or Corona is the coloured ring
seen ~~as~~ round the sun's or moon's disc
when viewed through a light fleecy

cloud. The rings are
violet inside and red
outside. The water
particles in the cloud
are so many opaque
obstacles each producing



a diffraction pattern. Particles like A at
small angular deviation from the
observers eye ~~Cause~~ send the violet
portion of their patterns into the
eye. Particles like B at a greater
angular deviation have their green
portions incident ~~of~~ on the eye; while part
like C still farther away ~~&~~ have their red
portions incident on the eye. Thus rings
violet within & red without are seen
surrounding the ~~so~~ disc.

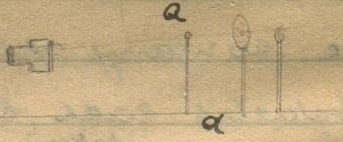


M. Verdet produced a similar
pattern by placing in front of the telescope a copper disc pierced
by a very large number of equal holes.
The screen is complementary and the pattern

s. : Similar

Young's Eriometer.

The principle of the corona was utilized by Young to compare very small diameters. A

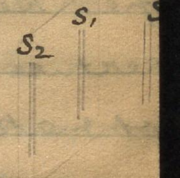


screen with a central hole and a ring of small holes around was mounted in front of a source of light & a small object was placed in front. The distance of the object from the screen adjusted till the corona coincided with the ring of the ~~x~~ holes.
Theory shows that when distance d from the source is constant, the radius of the corona varies inversely as the diameter of the particle causing it & that when the diameter is constant, radius of the corona is $\propto \frac{1}{d}$ ∴ when ~~two~~ two particles of a diameters a_1, a_2 at distances d_1, d_2 from the screen give equal coronas

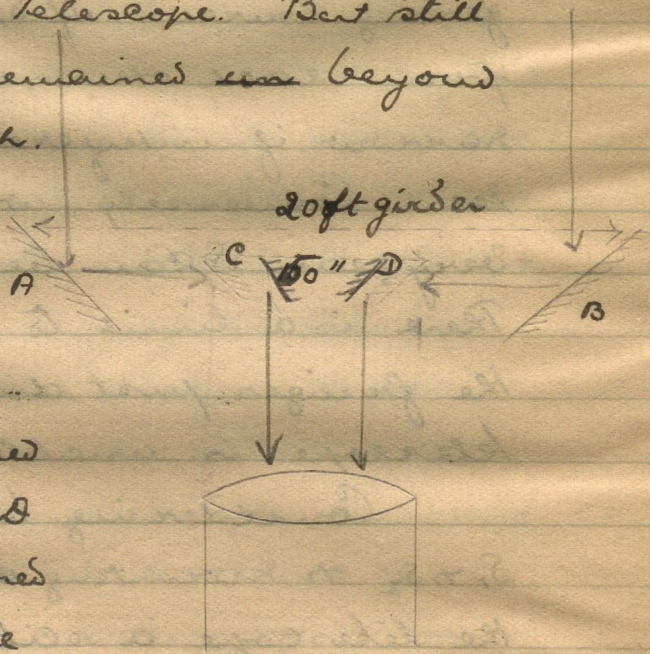
$$\therefore \frac{a_1}{a_2} = \frac{d_1}{d_2}.$$

Michelson's measurement of the diameters

Stars. The principle of the expt. is ~~of~~ well illustrated by Raleigh's interferometer. Two slits S_1 & S_2 ||l to each other & illumined by a third slit S & are the interfering sources. If S_1 & S_2 are very far from each other, S has to be extremel narrow if interference fringes are to be seen. Conversely, when S_1 & S_2 are very near S can be broader; ~~but the~~ There is a limit to the width of S when the fringes just disappear, i.e. the telescope is unable to resolve the frin



Considering two distant stars as S_1 & S_2 & mounting ~~a slit~~ in front of the telescope a slit just ~~wide~~ narrow enou to resolve the fringes, we have here a method of determining the distance betw the components ~~the~~ any ~~two~~ double ~~near~~ stars or of measuring the diameter of a star. The method was fir proposed in 1864 by Fizeau. It was first tried by Stephen in 1874 with the Marseilles telescope. But he found that even with the widest aperture that telescope allowed he could not make the fringes disappear. In 1890 measured the diameter of the moons of Jupiter with a 36" telescope. But still stellar diameters remained beyond experimental reach.



The problem was solved by Michelson in 1920. In front of the 100" telescope of Mt. Wilson he placed two plane mirrors C, D at 100" apart, inclined at 45° to the telescope axis facing the telescope. In line with CD at a distance of 20 ft he placed two other mirrors A, B also inclined at 45° to the axis, ∥ to C & D but facing the stars. A & B could be moved

on a frame work near to C & D.
The arrangement was directed towards α Orionis. When AB was 20 ft the fring[e]
could not be seen. AB was made gradually smaller. The fringes just appea[red]
when AB was 121". From this he calcu-
lated the ∠ subtended at the ~~tele~~ telescop[e]
by the diameter of α Orionis.

At the limit of resolution $\frac{1.22\lambda}{a} = \frac{d}{u}$.
a being the aperture which in this
case is $121'' = 121 \times 2.56$ cm.

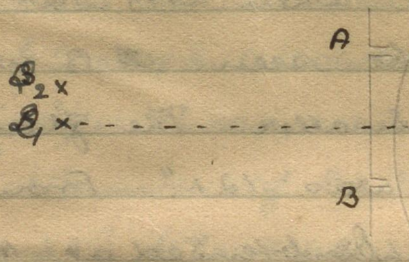
$$\therefore \quad \theta = \frac{1.22 \times 5.890 \times 10^{-5}}{121 \times 2.56} \text{ radians}$$
$$= .0479 \text{ seconds of arc.}$$

The diameter of the star, ~~when~~ ~~c~~ from the above expt, was calculated to be 250×10^6 miles. Further the section of the star normal to the line of vision is elliptical & not ⊙lar, as was shown by the variation in the distance between the periscope mirrors at different times, the maximum and minimum being

21" & 148".

Components of a double Star

Two slits



A & B are
mounted in
front of the
objective, and
the distance between
them is adjusted till ~~they are~~ the ~~two com-~~
~~ponents are just seen separate~~ fringes just disappear.

Let b be the distance AB, a the ~~distance~~
breadth of a slit.

Illumination due to the two slits from
one of the stars S_1

$$I_1 = a^2 \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}} \times \frac{\sin^2 2\left(\frac{\pi(a+b)\sin\theta}{\lambda}\right)}{\sin^2\left(\frac{\pi(a+b)\sin\theta}{\lambda}\right)}$$

$$= a^2 \sin^2 u / u^2 \times \frac{\sin^2 2v}{\sin^2 v}$$

$$= a^2 \frac{\sin^2 u}{u^2} \cancel{\sin}\ 4 \cos^2 v$$

$$= 4a^2 \frac{\sin^2 u}{u^2} \cos^2 v.$$

Rays from the star S_2 do not fall normally, but at an inclination $i = \frac{d}{D}$
d being distance between the stars,
& D distance of the stars from the objec[t]
Thus for u we have u'

$$= \frac{\pi a (\sin\theta + \sin i)}{\lambda}$$

which is nearly equal to u since
a sin i is a very small quantity.
For v we have v' $= \frac{\pi(a+b)(\sin\theta + \sin i)}{\lambda}$

$$= \frac{\pi b (\sin\theta + \sin i)}{\lambda}$$

which $\neq$ v since b sin i is not negligi[ble]
$\therefore$ Illumination due to star S_2

$$= I_2 = 4a^2 \frac{\sin^2 u}{u^2} \times \cos^2 v'$$

The resultant illumination $= I_1 + I_2$
& is $\therefore \propto \cos^2 v + \cos^2 v'$

$$\propto 2 + \cos 2v + \cos 2v'$$

The fringes just disapper when the values of v & v' are such that the expression has no maxima & minima
which is satisfied when $v' - v = (2n+1)\frac{\pi}{2}$

$$2v' - 2v = (2n+1)\pi$$

$$\cos 2v' = \cos\{2v + (2n+1)\pi\}$$
$$= -\cos 2v$$

∴ $\cos 2v' + \cos 2v = 0$ i.e. the expression is a constant

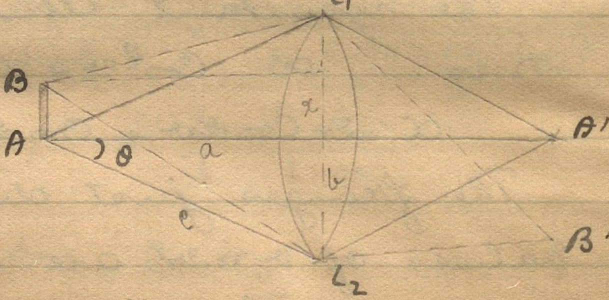
For the first position when fringes disappear $n = 0$, $v' - v = \frac{\pi}{2}$

$$v' = \pi b(\sin\theta + i)/\lambda$$
$$v = \pi b \sin\theta/\lambda \quad \therefore \frac{\pi}{2} = \frac{\pi b i}{\lambda}$$
$$\therefore i = \frac{\lambda}{2b}.$$

b being exptly determined the ∠lar separation i can be readily calculated.

Resolving power of a microscope

Let AB be an object in front of the objective A being on the principal axis. The image of A



is a patch of light with A' as centre, & of B is another patch with B' as centre. A & B are just resolved if the first dark ring round the patch B' passes through A, i.e. if the path difference of the extreme rays BL_1A' & BL_2A' is λ, i.e. if $BL_2 - BL_1 =$

$$\lambda = \sqrt{a^2 + (b+x)^2} - \sqrt{a^2 + (b-x)^2}$$

$$= \sqrt{a^2 + b^2 + 2bx} - \sqrt{a^2 + b^2 - 2bx}$$

$$= c\left\{1 + \frac{2bx}{c^2}\right\}^{1/2} - c\left\{1 - \frac{2bx}{c^2}\right\}^{1/2}$$

$$= c\left\{\left(1 + \frac{bx}{c^2}\right) - \left(1 - \frac{bx}{c^2}\right)\right\}$$

$$= \frac{2bx}{c} = 2x \sin\theta.$$

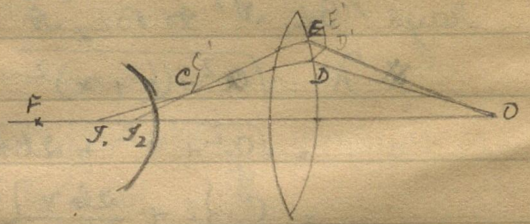
∴ The smallest object that can be resolved has the dimension $\frac{\lambda}{2\sin\theta}$

If the object is immersed in oil of refr. index μ, then $\lambda = \mu \times 2x \sin$

$$x = \frac{\lambda_0}{2\mu \sin\theta}.$$ (Num. ap. = μ Sin (Ex sex 1/22)

Spherical aberration The laws &c $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ &c were derived on the supposition that the ∠ subtended by the object at the lens is small; ~~wh~~ but when the aperture of the lens is large

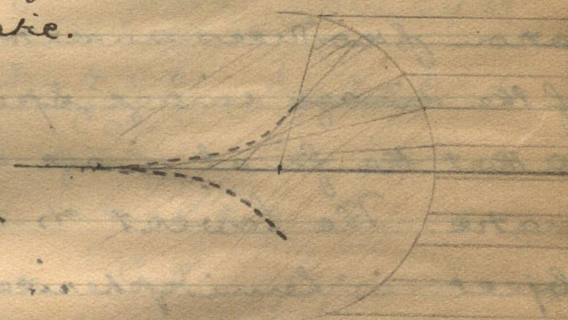
is condition is not satisfied. Rays starting
from a point object do not form a
point image. Two rays OE, OD ~~Con~~ near
each other meet at C



after ~~too~~ refraction,
and only after passing
through C so they meet
the axis of the lens. If the element ED be
rotated about the axis OI_1, OED forms
a small oblique centric pencil (oblique
since OD is oblique to ~~OI~~ the axis of the
lens, & centric because all the rays of
the pencil are not far from the centre of EDD'E')
the ~~line~~ pt C describes a line CC' ⊥r to
the plane of the paper. Thus the pencil
gives rise to two focal lines CC' ~~⊥r~~ to the
plane of the paper, & $I_1 I_2$ in the plane of
the paper. As we pass from C to I_1 the
width ~~⊥ b~~ of the pencil decreases in the
plane ⊥r to the paper, & increases in the
plane of the paper. At some point between C & I_1 both

widths are equal & the pencil is ○lar. T
is called the <u>circle of least confusion</u>. If
A sphere about the focus on the surface of
which this ○ lies will also have on its sur-
face all the ○les of least confusion refrac
from the various elements of the lens. This
surface will give the ~~lead~~ ~~least distorted~~ best definition
image of ~~an extended~~ point source at O.

The envelope of all the refracted rays
is called a caustic.



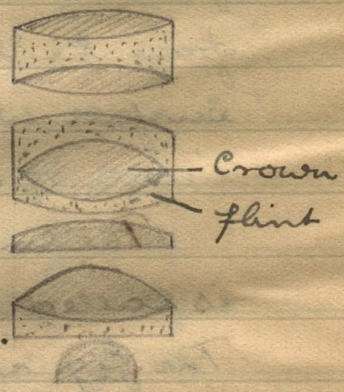
The figure shows the caustic for refraction of a ∥l beam into a spherical surface.

It is impossible to design a lens such that rays from <u>any</u> point on its axis are brought to focus at ~~any~~ some other point. But a lens ~~be~~ may be constructed which can bring accurat to a point focus rays from some <u>particu</u>

point on its axis. Such a lens is called aplanatic & the particular positions of source & image are called aplanatic foci.

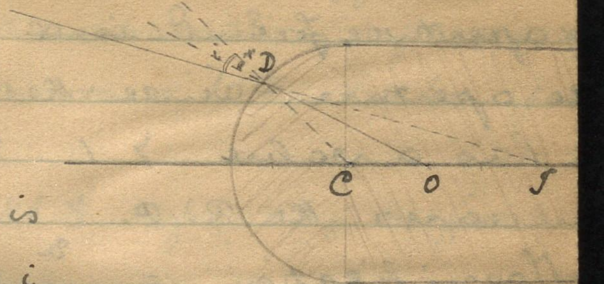
Abbe's oil immersion microscope objective (Ed. sen 98)

This high power objective consists of different combinations of divergent flint glass lenses & convergent crown glass lenses. Each combination produces an extra fold of the ~~image~~ image spectrum so that the final image is perfectly achromatic. The lowest & the nearest to the object is hemispherical & aplanatic as is shown below. The eyepiece ~~image~~ & the object to be examined are immersed in Cedar wood oil which has refr. index 1·5 nearly the same as that of crown glass.



Let the object be at a pt O on the axis & let C be the centre of curvature, $CO = \frac{R}{\mu}$

ρ being radius of curvature & μ refr. index of the lens & the oil.



The virtual image is at I. Let $\angle COI = i$,

$\angle CDO = r$. $\dfrac{\sin i}{\sin r} = \mu$ But $\dfrac{\sin i}{\sin C\hat{I}D} = \dfrac{CI}{\rho}$

& $\dfrac{\sin r}{\sin C\hat{O}D} = \dfrac{CO}{\rho}$

$\therefore \dfrac{CI}{CO} = \dfrac{\sin i}{\sin r} \times \dfrac{\sin C\hat{O}D}{\sin C\hat{I}D}$

But $C\hat{O}D = C\hat{I}D + i - r$

Also $\sin C\hat{O}D = \dfrac{\sin r \times \rho}{CO} = \dfrac{\sin r \times \rho \times \mu}{\rho}$

$= \mu \sin r = \sin i$

$\therefore C\hat{I}D = r$ & $C\hat{D}O = i$

$\dfrac{CI}{CO} = \mu^2$ i.e. $CI = \mu\rho$

Thus whatever be the ∠s of incidence the refracted rays when produced pass through a pt I at distance $\mu\rho$ from the Centre ~~of~~ C.

Thus there is no spherical aberrat[ion] however large be the aperture. Since resolving power is $\propto \dfrac{2\mu \sin\theta}{\lambda}$, ~~sin~~ the

R. P. is largely increased by increasing the aperture, for Sin θ is ~~∝~~ increases with the aperture. Since the object is in oil μ has a value > 1 ∴ this factor also increases the R. P.

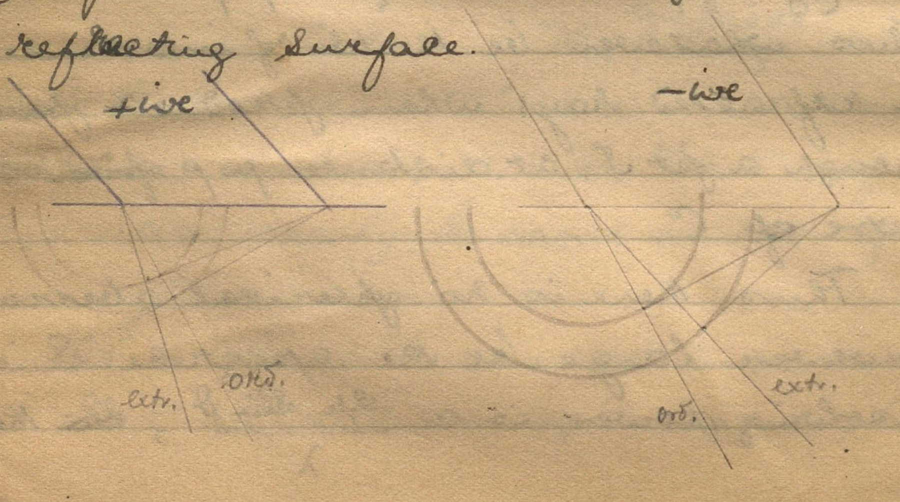
Magnification $= \dfrac{v}{1/\mu \times u} = \mu \dfrac{PI}{PO} = \mu^2$

Thus the ~~intr~~ presence of the hemispherical lens increases the magnification of the eyepiece μ^2 times.

Polarization

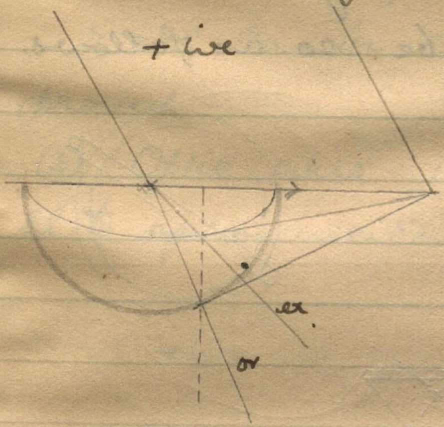
The three principal sections to verify Huyghen's theory of Double wave Surface in a uniaxial Crystal.

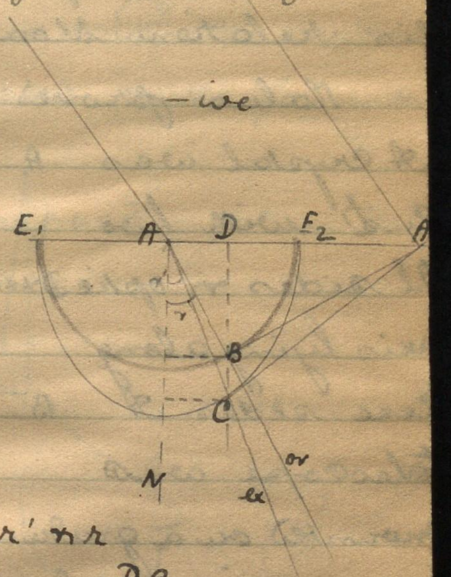
1) Optic axis ⊥r to incident plane & ∥l to reflecting Surface.



If the theory is correct, for a crystal cut in this ~~structo~~ way the extraordinary refractive index should be constant. This has been pro[ved] to be the case

2) Optic axis ‖l to refracting surface, ‖l to incident plane.





Let ∠s NAC & NAB be r' & r

$\tan r' / \tan r = \frac{AD}{DC} \times \frac{DB}{AD} = \frac{DB}{DC}$ for BC is ⊥r to AA' since the ⊙ is within the ellipse & tangents are drawn from a pt on the comm[on] diameter. Let AD be y, & DC be x

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\therefore \frac{x^2}{a^2} = \frac{b^2 - y^2}{b^2} = \frac{(b+y)(b-y)}{b^2}$$

$$= \frac{E_1D \times DE_2}{b^2} = \frac{DB^2}{b^2}$$

ie $\frac{DB}{DC} = \frac{b}{a}$.

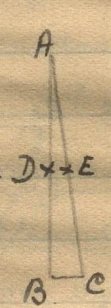
Now $\frac{b}{a}$ in a -ve crystal $= \frac{\mu_e}{\mu_o}$

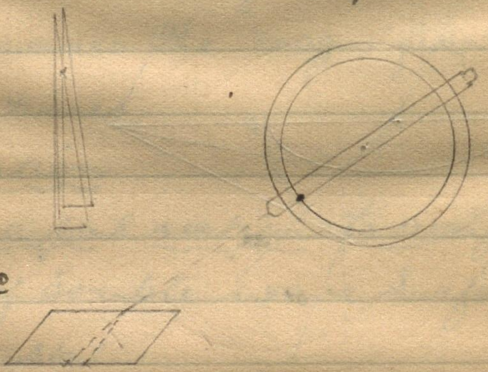
$\therefore \tan r' / \tan r = \mu_e / \mu_o$

If the section of the wavesurface in the plane of the paper is an ellipse this relation should hold good.

Malus proved it to be so as follows.

A crystal was cut with two ‖l sides & optic axis lying along these sides. A telescope was

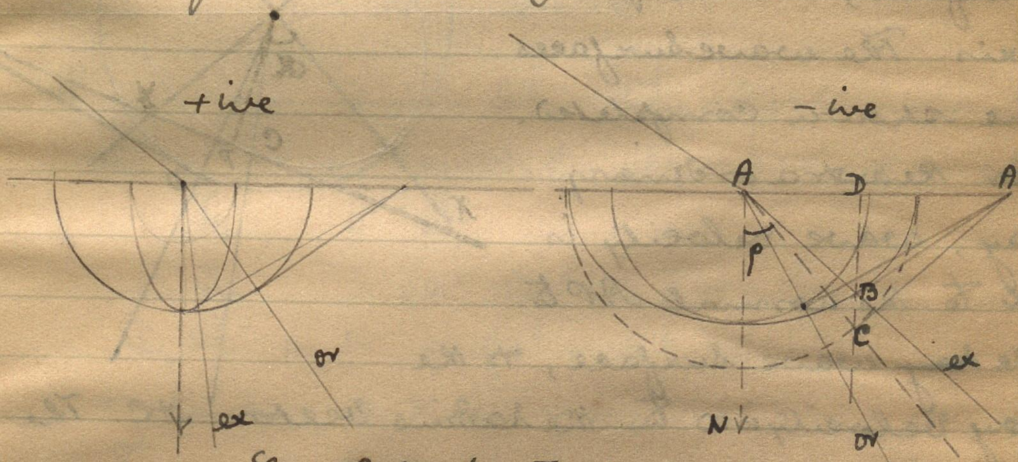




mounted on a graduated vertical circle & the optic axis adjusted to lie in the same vertical plane as the ⊙. A ~~bx~~ steel strip shaped like a narrow set square having its long sides graduated is placed below the crystal. Two images of the set square are seen in the telescope & some point D on AB of one image coincides with

some other point E on AC. DE can be calculated from the graduations on the set square. If e be the thickness of the plate $e(\tan r - \tan r') = DE$.
But ~~tan r =~~ ~~Sin~~ $\sin r = \frac{\sin i}{\mu_o}$ & $\sin i$ is the $\angle$ the telescope makes with the vertical
$\therefore$ r & r' can be calculated & the formula proved.

③ Optic axis $\perp$r to refracting plane & ‖l to ~~optic~~ incident plane.

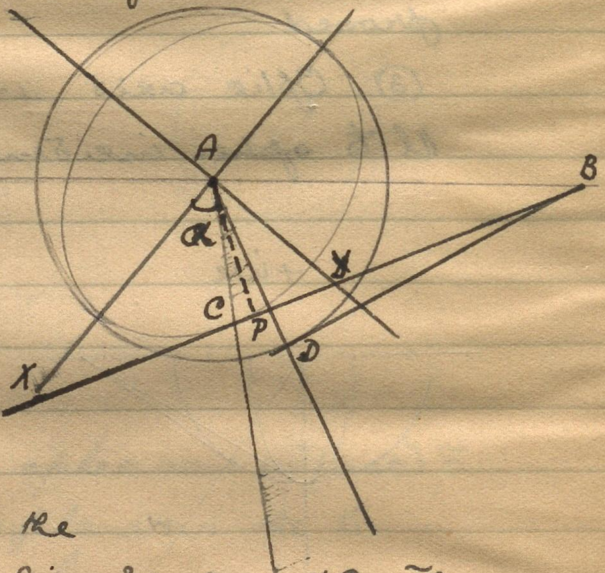


Let ~~A~~ ⊙ be drawn with A as centre & radius = a. Let A'C be tangent to the ⊙ Let $\angle NAC$ be ρ, $\angle NAB$ be r'
$\frac{\tan \rho}{\tan r'} = \frac{DB}{DC} = \frac{b}{a} = \frac{\mu_e}{\mu_o}$ But ρ is the $\angle$ of

refraction in the first case, i.e.

$\frac{\sin i}{\sin \rho} = \mu_e$. ~~Thus ρ is~~ By conducting an expt $\parallel$r to the former but with a crystal having its optic axis $\perp$r to the refracting surface, ρ & r' can be found out & the relation $\frac{\tan \rho}{\tan r'} = \frac{\mu_e}{\mu_o}$ can be verified.

Wave Velocity & Ray Velocity



In the figure AB is the refracting surface, Ax the optic axis. The wave surfaces are shown completed. For the extraordinary ray, wave velocity is $\propto$ to the normal AP to the ~~ray~~ wave surface, & the ray velocity is $\propto$ to the radius vector AC. The two are not equal.

Let $\angle APx$ be α & $AP = p$

Equation xPY is $x\cos\alpha + y\sin\alpha = p$.

& eqn to the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since the line is a tangent to the ellipse, there is only one point of intersection

$$\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}$$

$$\text{ie} \quad x^2 b^2 = a^2 b^2 - a^2 y^2.$$

but $y = (p - a\cos\alpha)/\sin\alpha \times x$

$$\therefore x^2 b^2 = a^2 b^2 - a^2\{(p - \cancel{x}\cos\alpha)/\sin$$

$$\text{ie} \quad x^2\left\{b^2 + \frac{a^2\cos^2\alpha}{\sin^2\alpha}\right\} - 2\,\frac{a^2 p\cos\alpha}{\sin^2\alpha}\,x$$

$$+ a^2 p^2\frac{\cos^2\alpha}{\sin^2\alpha} - a^2 b^2 = 0$$

Since here $4ac = b^2$, $p^2 = a^2\cos^2\alpha + b^2\sin^2$

In a +ive crystal wave vel. of ordina

$$w_o = a$$

$$w_e = (a^2\cos^2\alpha + b^2\sin^2\alpha)^{1/2}$$

$$\therefore w_o^2 - w_e^2 = (a^2 - b^2)\sin^2\alpha.$$

& since $a > b$, $w_o > w_e$.

In an ~~*~~ negative crystal, α is the inclination of the wave normal to the normal to the optic axis. If α ~~β~~ is the inclination to the wave normal, then,

$p^2 = a^2\sin^2\alpha + b^2\cos^2\alpha$

$\therefore W_o^2 - W_e^2 = (a^2 - b^2)\cos^2\alpha.$

Here $b > a$ $\therefore W_e > W_o$.

The ray velocity is $\propto$ to AC.

Let $\angle xAC = \theta$

If $\overset{AC}{\cancel{AB}} = r$, & C is (xy)

$x = r\cos\theta$, $y = r\sin\theta$

& since $x^2/a^2 + y^2/b^2 = 1$

$$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{r^2}.$$

In a +ive crystal ~~the~~ ray velocity of ordinary $R_o = a$ $\quad \frac{1}{R_o^2} = \frac{1}{a^2}$

$$\frac{1}{R_e^2} = \cos^2\theta/a^2 + \sin^2\theta/b^2$$

$$\therefore \frac{1}{R_o^2} - \frac{1}{R_e^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)\sin^2\theta$$

& since $a > b$

$R_o > R_e$

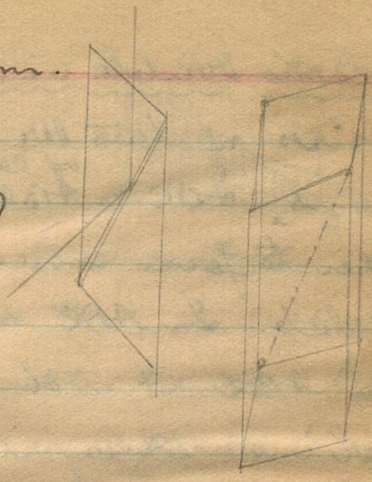
In a -ive crystal

$$\frac{1}{R_o^2} - \frac{1}{R_e^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)\cos^2\varphi$$

& since $a < b$

$R_o < R_e$.

Polarizers Nicol prism.

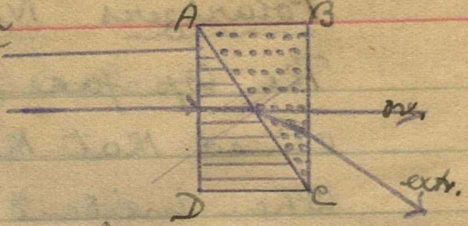


The top face is so inclined that the ordinary when incident upon the balsam is totally reflected internally & is absorbed by black casing on the sides. The $\angle$ of incidence should be greater than $i \left(= \sin^{-1} \frac{1.53}{1.658} = 67^\circ\text{-}1\right.$
The extraordinary is ~~not at all~~ reflected on very slightly.

Foucault prism has the advantage of being shorter since the film of separation is air. If the ordinary should be reflected, but not the extraordinary
$i > \sin^{-1} \frac{1}{1.658}$ but $< \sin^{-1} \frac{1}{1.486}$.
ie $i > 34^\circ < 42^\circ$. The disadvantage is that the extraordinary is also copiously reflected & $\therefore$ the image is faint.

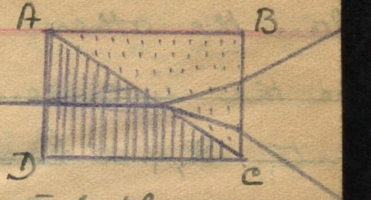
Tourmaline is a uniaxial crystal with a high absorptive power for the ordinary ray

Rochon's double image prism



onsists of two ||r rt. ∠d prisms, of calcite, ADC & ABC, joined together to form a rectangu-lar slab. In ADC the optic axis is || to DC, in ABC it is || to the refracting edge of e prism. A ray incident on face AD normally splits up into or. & extraord., & since ey travel along the optic axis, have the ame speed of propagation until they each AC. Beyond AC the ordinary ravels along the same St. line, but the xtraordinary emerging from a denser o a lighter medium is deflected away rom the normal. On reaching BC a urther deflection takes place. Thus ord. extraord. are seen quite separate.

Wollaston's prism is the same as Rochon's, ut light is incident on DC. In the first rism both ordinary & extraord. travel long the same paths, but with different velocities. On reaching ABC ~~both~~ ordinary of 1st becomes



ext. & vice versa; one travels from denser to rarer medium & ∴ is deflected away from the normal, the other travels from rarer to denser & is deflected towards the normal. On reaching BC further deflections occur & ∴ the two rays are seen more separate than in the former case.

Elliptic & Cr polarization – Theory, production & characteristic analysis
Cf B. A. Notes Page 187 - 193.

Fresnel's Theory of rotatory polarization

Fresnel assumed that when a plane polarized beam is incident on an optically active medium, it is split into two Circularly polarized beams revolving in opposite directions, one advancing with a velocity greater

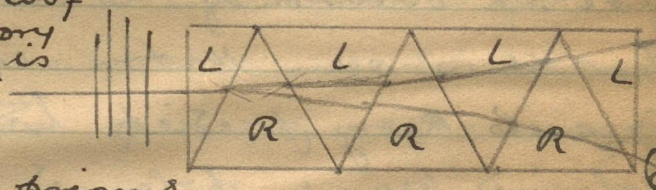
than the other. Thus if $y = 2a \sin \omega t$
be the incident beam, it is split up
into $x_1 = a \cos \omega t$, $y_1 = +a \sin \omega t$ (1)
$x_2 = -a \cos \omega t$, $y_2 = +a \sin \omega t$ (2)
If μ_1, μ_2 be refractive indices,
$\frac{v_0}{v_1}$, & $\frac{v_0}{v_2}$ for the two beams,
path retardation in traversing a distance
e in the medium $= (\mu_1 - \mu_2) e$ & phase
retardation $\varepsilon = \frac{2\pi}{\lambda} (\mu_1 - \mu_2) e$.
$\therefore$ On emergence the two ⊙ls become
$x_1 = a \cos \omega t \quad y_1 = a \sin \omega t$
$x_2 = -a \cos(\omega t + \varepsilon) \; y_2 = a \sin(\omega t + \varepsilon)$.
The resultant vibration is
$x = (x_1 + x_2) = 2a \sin \frac{\varepsilon}{2} \sin\left(\omega t + \frac{\varepsilon}{2}\right)$
$y = (y_1 + y_2) = 2a \cos \frac{\varepsilon}{2} \sin\left(\omega t + \frac{\varepsilon}{2}\right)$
& which is evidently a St. line
inclined at $\varphi = \frac{\varepsilon}{2}$ to the former direction.
Thus the rotation $= \frac{\varepsilon}{2}$.
$e = n_1 \lambda_1 = n_2 \lambda_2$ where n_1, n_2
are ~~frequencies~~ no. of waves in distance e, λ_1, λ_2 wavelengths
of the two rays. The phase retardation

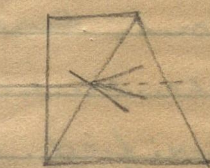
$$\mathcal{E} = 2\pi(n_1 - n_2) = 2\pi e\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$
$$= (2\pi\, d\lambda)\ \frac{e}{\lambda^2}$$

∴ Thus Biot's laws $\varphi \propto e$ & $\varphi \propto \frac{1}{\lambda^2}$ get established.

Fresnel's pile of prisms

A direct proof of the above theory is afforded by a pile of quartz prisms arranged alternately with the ~~face~~ base in opposite directions. They are all cut with optic axis ||l to the base & ||l to the incident beam & alternately they are laevogyrate & dextrogyrate.





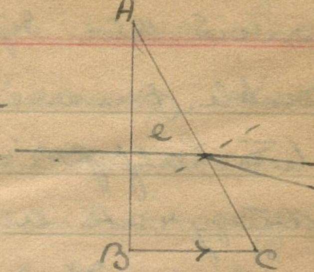
Thus ~~with to~~ the ⊙lar motion which is faster in one prism is slower in the other. Hence when ~~now~~ passing from the first to the second prism, one of the ⊙larly

polarized rays is deflected towards the normal i.e. downwards, & the other is deflected upwards. When passing the third prism, the first is deflected away from the normal, that is, downwards since the direction of the normal is different. Thus it is seen that on emergence, Fresnel's theory predicts two ⊙larly polarized beams, polarized in opposite directions, emerging at different angles. This phenomenon has been experimentally verified.

Cornu's Prism

Cornu observed that a monochromatic plane polarized beam passed through a quartz 60° prism cut with optic axis $\parallel$l to the base & $\perp$r to the refracting edge, & adjusted for minimum deviation, gave rise to two ⊙larly polarized beams with an angular separation of 23''.

Consider a half prism



The ~~phase~~ retardation between the ~~two~~ ⊙larly polarized beams $= \frac{2\pi}{\lambda}(\mu_1 - \mu_2)\,e$

where μ_1 & μ_2 are the corresponding indices, e is the thickness of the medium traversed, & this is ~~2 × rotatn~~ twice the rotation of the plane of polariza

If $\theta =$ rotation per cm,

$$\theta e = \frac{\pi}{\lambda}(\mu_1 - \mu_2)\,e$$

ie $$d\mu = \frac{\theta\lambda}{\pi}.$$

If φ be the $\angle$ of emergence,

$$\mu = \sin\varphi / \sin\theta_1 = 2\sin\varphi$$

$$d\mu = 2\cos\varphi\, d\varphi$$

$$\therefore d\varphi = \frac{\theta\lambda}{\pi \times 2\cos\varphi} = \frac{\theta\lambda}{\pi}\,\frac{1}{\sqrt{4-\mu^2}}$$

Since $\mu = 1.544$, θ per mm is 21.7°,

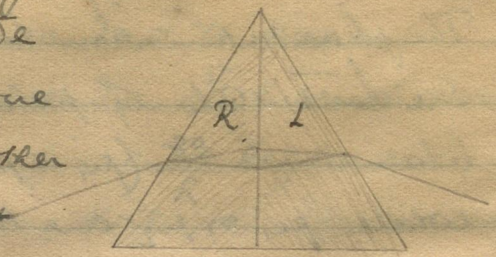
ie $\theta = 217^\circ$,

$$d\varphi = \frac{217 \times 5.893 \times 10^{-3}}{\pi \times \sqrt{4 - 1.544^2}}$$

$$= 11.5''.$$

Hence for the full prism ~~rotatn~~ $\angle$lar deviation is $23''$.

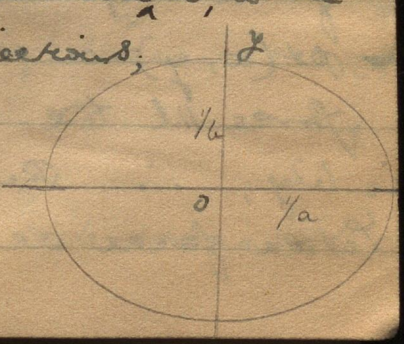
To avoid this difficulty, Cornu constructed a double prism made of two half prisms one dextrogyrate and another laevogyrate, ~~The result~~ for both having ~~same~~ equal rotatory power, Cut with optic axis ||l to the base. The resultant effect of the two is zero.



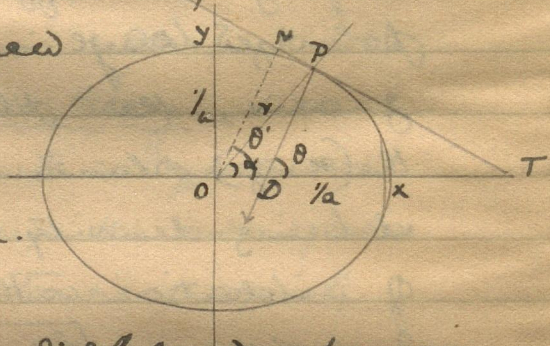
Fresnel's elastic solid theory

Fresnel assumed a hypothetical medium ether filling up the interspaces of matter, & having its elastic property modified by the ~~ma~~ properties of the matter with which it is entangled. Light waves are propagated by transverse & vibrations in ether. In isotropic media the entangled ether has identical properties in all directions, but the properties of the entangled ether are different in different directions when the matter is aelotropic — that is, one which has sp. conductivity for heat, or electricity, elastic coeff. of expansion ~~etc~~ & other such physical properties different in different directions. Thus Fresnel assumes the the elasticity e_1, e_2, e_3 of the entangled ether along the three principal axes x, y, z are different. Suppose a plane polarized wave is propagated along the z-axis, the plane of vibration being the $(x-z)$ plane. Its velocity is $\cancel{\propto}\ \sqrt{\frac{e_1}{d}}$ d being density of ether. If the plane of vibration is the $(y-z)$ plane, the velocity is $\sqrt{\frac{e_2}{d}}$. Let $a^2 \propto e_1$, $b^2 \propto e_2$.

Then the ellipse $a^2x^2 + b^2y^2 = 1$ is called the ellipse of elasticity. Its property is that the axes are inversely $\propto$, to the root of elasticities in the $x-y$ directions; & that the velocity of a wave propagated $\perp$r to the plane of the ellipse,



and having vibrations ‖l to the major (or minor) axis has a velocity $\propto$ to the reciprocal of the semi major (or minor) axis. Similar ellipses can be drawn for all the infinite possible directions of propagations, the envelope of all of which will form an ellipsoid $a^2x^2 + b^2y^2 + c^2z^2 = 1$, called the ellipsoid of elasticity.



If a particle is displaced along the x-axis force of restitution is along XO, & is a^2 per unit displacement, since $a^2 \propto e$,

Ill'ly if the particle is ~~dis~~ displaced along OY, the force of restitution is along YO, & is b^2 per unit displacement.

If the displacement is along OP, P being a pt (x, y) on the ellipse, force ‖l to x-axis is a^2x, ‖l to y-axis is b^2y, $\therefore$ Resultant force $R = \sqrt{a^4x^2 + b^4y^2}$

Let the force act along PD, & $\angle PDx = \theta$

$\tan\theta = \frac{b^2y}{a^2x}$. But $\tan\alpha = \frac{y}{x}$

α being $P\hat{O}X$. Thus the force of restitution is not necessarily along PO.

Now ~~tan~~ Cot $P\hat{T}D = -\frac{dx}{dy} = \frac{b^2y}{a^2x} = \tan$

ie $\theta + P\hat{T}D = \frac{\pi}{2}$,

ie the force acts along the normal to the tangent of the ellipse at P.

It can be shown that the velocity of propagation of a wave having vibrations ∥ to a given radius vector (say PO) is inversely $\propto$ to that radius vector.

Velocity $\propto \sqrt{\text{Elasticity}}$

& elasticity $\propto$ the force

Now force of restitution $\propto$ to PO

$= R\cos(\theta - \alpha) = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$= R.\left(\frac{a^2x^2}{Rr} + \frac{b^2y^2}{Rr}\right) = \frac{a^2x^2 + b^2y^2}{r} = \frac{1}{r}$

$\therefore$ force per unit displacement $= \frac{1}{r^2}$

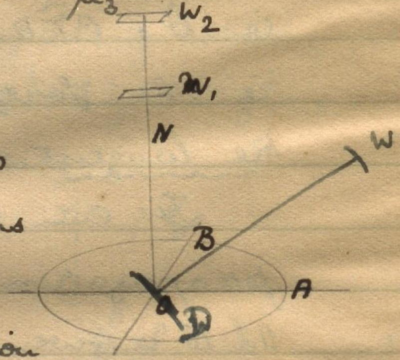
ie Elasticity $\propto \frac{1}{r^2}$

& hence velocity $\propto \frac{1}{r}$.

Since velocity v, of vibrations ∥ to

..l axis $\propto 1/\frac{1}{a}$ ~~i.e.~~ $\propto a$ ~~&~~ $\mu_1 = \frac{v_0}{v_1}$ μ_1 being
..e corresponding refr. index $\therefore \mu_1 \propto \frac{1}{a}$
$\mu_2 \propto \frac{1}{b}$, $\mu_3 \propto \frac{1}{c}$.

Hence eqn to the ellipsoid of elasticity ..can be written as $\frac{x^2}{\mu_1^2} + \frac{y^2}{\mu_2^2} + \frac{z^2}{\mu_3^2} = 1$



Eqn. to a wave Surface

Suspose at any instant ..a large number of plane waves ..are passing in different directions ..through a pt O in a crystal. ..Corresponding to each direction ..of transmission there will in general ..be two elements of plane wavefronts W_1 & W_2 travelling with different velocities. ..The envelope of all wavefronts corresponding ..to W_1 produces one wave surface & the ..envelope of wavefronts corresponding to W_2 produces another ..wave surface & they together form ..the wave surface in a crystal.

Consider an element W of the wave ..surface. ~~Let~~ $WO = v$ the velocity & let l, m, n

be direction cosines of WO w.r.t. the x-y-z axes. Let W be the pt (x, y, z)

$\therefore \quad v = lx + my + nz$ (1)

This eqn when the variables are eliminated ~~y~~ gives the eqn to the wave surface.

Let displacement be r in direction OD ~~inclin~~ whose direction cosines are ξ, η, ζ.

Components of r along the 3 axes are $r\xi$, $r\eta$, $r\zeta$.

Compounding these along OW

displacement along OW $= r\xi l + r\eta m + r\zeta n$

which should be zero

$\therefore \quad l\xi + m\eta + n\zeta = 0$ (2)

Let R be the force of restitution & F & f its components along OD & OW.

Components of F along the 3 axes are

$F\xi$, $F\eta$, $F\zeta$

Components of f along the 3 axes are

fl, fm, fn.

Now $F\xi + fl = a^2 x = a^2 r\xi$.

If $r = 1$, $v^2 = F$ $\therefore v^2\xi + fl = a^2\xi$

ie $\xi = \dfrac{fl}{a^2 - v^2}$

ubstituting in ② $\quad \dfrac{l^2}{v^2-a^2} + \dfrac{m^2}{v^2-b^2} + \dfrac{n^2}{v^2-c^2} = 0 \quad$ ③

Again $l^2 + m^2 + n^2 = 1$ ④ being sum of the sqq. of direction Cosines.

Thus $\quad lx + my + nz = v \quad$ ①

$$\frac{l^2}{v^2-a^2} + \frac{m^2}{v^2-b^2} + \frac{n^2}{v^2-c^2} = 0 \quad ③$$

$$l^2 + m^2 + n^2 = 1 \quad ④$$

Differentiating $\cancel{l\,x}\ x\,dl + y\,dm + z\,dn = dv$ ⑤

$$\frac{l\,dl}{v^2-a^2} + \frac{m\,dm}{v^2-b^2} + \frac{n\,dn}{v^2-c^2} - v\,dv\left\{\frac{l^2}{(v^2-a^2)^2} + \frac{m^2}{(v^2-b^2)^2} + \frac{n^2}{(v^2-c^2)^2}\right\} = 0 \quad ⑥$$

$$\text{\&}\quad l\,dl + m\,dm + n\,dn = 0. \quad ⑦$$

Equating like coeffts in $\{⑤\}$ & $\{⑥ \times B + ⑦ \times A\}$

$$x = \frac{Bl}{v^2-a^2} + Al;\quad y = \frac{Bm}{v^2-b^2} + Am;\quad z = \frac{Bn}{v^2-c^2} + An \quad ⑧$$

$$Bv\left\{\frac{l^2}{(v^2-a^2)^2} + \frac{m^2}{(v^2-b^2)^2} + \frac{n^2}{(v^2-c^2)^2}\right\} = 1 \quad ⑨$$

Substituting (8) in (1)

$$l^2\left\{\frac{B}{v^2-a^2}+A\right\}+m^2\left\{\frac{B}{v^2-b^2}+A\right\}+n^2\left\{\frac{B}{v^2-c^2}+A\right\}=v$$

$$\text{ie } B\sum\frac{l^2}{v^2-a^2}+A\sum l^2=v$$

By (3) $\sum\frac{l^2}{v^2-a^2}=0$; & by (4) $\sum l^2=1$

$\therefore\quad A=v.$ (10)

From (8) $(x-Al)^2=\left\{\frac{Bl}{v^2-a^2}\right\}^2$

$$\text{ie } \sum\left\{x^2-2Alx+A^2l^2\right\}=B^2\sum\left\{\frac{l^2}{(v^2-a^2)^2}\right\}$$

Now $\sum x^2=r^2$; $\sum lx=v$ by (1)

$\sum l^2=1$ by (4)

& $\sum\frac{l^2}{v^2-a^2}=\frac{1}{Bv}$ by (

$\therefore\quad r^2-2Av+A^2=\frac{B^2}{Bv}$ & Since $A=$

ie $r^2-v^2=\frac{B}{v}$ ie $B=v(r^2-v^2)$ (11

$\therefore\quad x=lv+\frac{l}{v^2-a^2}\cdot v(r^2-v^2)$ from (8) ie $x = lv\left\{1 + \frac{r^2 - v^2}{v^2 - a^2}\right\} = vl \cdot \frac{r^2 - a^2}{v^2 - a^2}$.

ie $l = \frac{x(v^2 - a^2)}{v(r^2 - a^2)}$ (12)

Substituting in (1) $\sum x^2 \cdot \frac{v^2 - a^2}{v(r^2 - a^2)} = v$

ie $\sum x^2 \cdot \frac{v^2 - a^2}{r^2 - a^2} = v^2 = v^2 \times \sum \frac{x^2}{r^2}$.

ie $\sum x^2 \cdot \left\{\frac{v^2 - a^2}{r^2 - a^2} - \frac{v^2}{r^2}\right\} = 0$

ie $\sum x^2 \cdot \left\{\frac{a^2 v^2 - a^2 r^2}{r^2(r^2 - a^2)}\right\} = 0$

ie $\sum x^2 \cdot \frac{a^2(v^2 - r^2)}{r^2(r^2 - a^2)} = 0$

ie $x^2 \frac{a^2}{r^2 - a^2} + y^2 \frac{b^2}{r^2 - b^2} + z^2 \frac{c^2}{r^2 - c^2} = 0$. (13)

The above is the reqd eqn to the wave Surface

The eqn to the normal velocity Surface is obtained from this by putting $\frac{1}{a}$ for a, $\frac{1}{r}$ for r etc. Thus $\frac{a^2}{r^2 - a^2}$ becomes

$$\frac{(1/a)^2}{(1/r)^2-(1/a)^2} = \frac{r^2}{a^2-r^2}$$ Hence eqn to the normal velocity surface is

$$\Sigma \frac{x^2 r^2}{a^2-r^2} = 0$$

ie $x^2 \cdot \frac{1}{r^2-a^2} + y^2 \frac{1}{r^2-b^2} + z^2 \cdot \frac{1}{r^2-c^2} = 0$

To obtain the plane figures got by the intersection of any of the three planes xy, yz, or zx with the figures of eqn (13) the following transformation is necessary.

Multiplying through out by $(r^2-a^2)(r^2-b^2)(r^2-c^2)$

$$\Sigma x^2 a^2 (r^2-b^2)(r^2-c^2) = 0 \qquad (15)$$

~~In this putting~~ $z = 0$

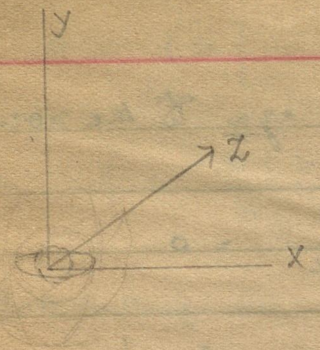
$x^2 \cancel{a^2} (r^2-b^2)(\cancel{r^2}-c^2) + \cancel{y^2 b^2 (r^2-c^2)(r^2-a^2)} =$

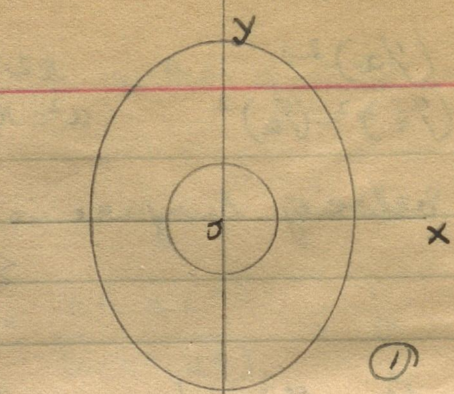
ie $\Sigma x^2 a^2 \{r^4 - r^2(b^2+c^2) + b^2c^2\} = 0$

$+ \cancel{y^2 b^2 (r^4 - r^2(}$

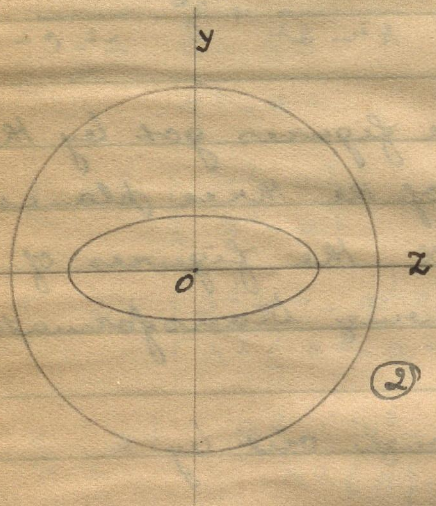
ie $r^4 \Sigma a^2x^2 - r^2 \Sigma x^2 a^2 (b^2+c^2)$

$+ a^2b^2c^2 \Sigma x^2 = 0$

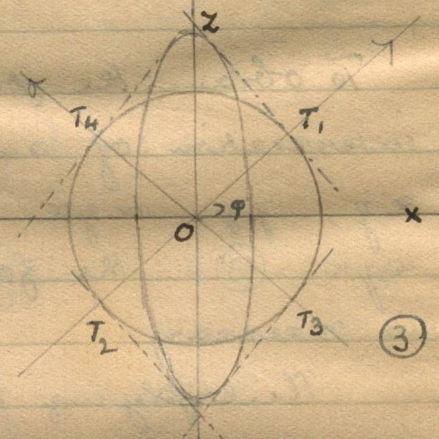




(1)



(2)



(3)

Since $\Sigma x^2 = r^2$.

$$r^2 \Sigma a^2x^2 + a^2b^2c^2 - \Sigma x^2a^2(b^2+c^2) = 0.$$

In this putting $z = 0$

$$r^2(a^2x^2 + b^2y^2) + a^2b^2c^2 - x^2a^2(b^2+c^2)$$
$$- y^2b^2(c^2+a^2) = 0$$

i.e. $$(x^2+y^2)(a^2x^2+b^2y^2) - c^2(a^2x^2+b^2y^2)$$
$$\div \cancel{*}\, b^2a^2(x^2+y^2) + c^2a^2b^2$$

ie $\left\{(x^2+y^2)-c^2\right\}\left\{\left(\frac{x^2}{b^2}+\frac{y^2}{a^2}\right)-1\right\}=0$

This comprises a circle of radius c & an ellipse of semi axes b & a, thus giving the section in the (x, y) plane.

By Symmetry, we have in the (y, z) plane a ⊙ of radius a, & an ellipse of semi axes c & b (axis ∥ to y axis is c)

In the $(z-x)$ plane it is a ⊙ of radius b & an ellipse of semi-axes a & c, (the axis ∥ to the Z axis is a.)

The four planes represented by the tangents in fig ③ are common to both the wave surfaces. ~~An~~ Hence along the normal to these tangents both the waves are propagated with equal velocity. The four planes form two pairs & each pair has a common normal. Thus there are two ~~ax~~ and only two directions in a crystal along which the two waves are propagated with the same velocity.

Crystals are in general ~~uni~~ biaxial.

Eqn to the optic axis ($O r_1$)

Eqn to a tangent normal to an ellipse is $p^2 = a^2 \sin^2\varphi + c^2 \cos^2\varphi$

φ being $\angle r_1 O x$

But $p = b$ ie $b^2 = a^2 \sin^2\varphi + c^2\cos^2\varphi$

$\therefore\ b^2 - c^2 = (a^2 - c^2) \sin^2\varphi$.

$\therefore\ \tan^2\varphi = \left\{(b^2 - c^2)/(a^2 - b^2)\right\}^{1/2}$.

$\therefore$ Eqn to $O r_1$ or $O r_2$ is

$$y = 0 \quad ; \quad z = \pm x \times \sqrt{\frac{b^2 - c^2}{a^2 - b^2}}$$

Since $\mu_1 \propto \frac{1}{a}$, $\mu_2 \propto \frac{1}{b}$, $\mu_3 \propto \frac{1}{c}$,

The eqn for the optic axes in terms of the refr. indices is

$$y = 0 \quad ; \quad z = \pm x \left\{\frac{(\mu_3^2 - \mu_2^2)\mu_1^2}{(\mu_2^2 - \mu_1^2)\mu_3^2}\right\}^{1/2}$$

In a uniaxial -ive crystal, $b = c$

~~Subs~~ Putting b for c in eqn (13)

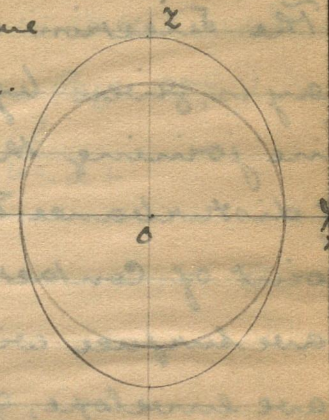
& simplifying,

$$(x^2 + y^2 + z^2 - b^2)\left(\frac{x^2}{b^2} + \frac{y^2}{a^2} + \frac{z^2}{a^2} - 1\right) = 0$$

Which is the eqn for a sphere of radius b & for a spheroid ~~eg~~ generated by an ...

ellipse rotating about its major axis

The section in the $(x-z)$ plane gives a circle & an ellipse.



That the two ~~axes~~ optic axes coincide is shown by $\tan\varphi = \left(\frac{b^2-c^2}{a^2-b^2}\right)^2 = 0$ since $b=0$. Hence both the axes lie along OX

In a +ive uniaxial crystal, $a = b \neq c$ & the wave surface is

$$(x^2+y^2+z^2-a^2)\left(\frac{x^2}{c^2}+\frac{y^2}{c^2}+\frac{z^2}{a^2}-1\right)=0.$$

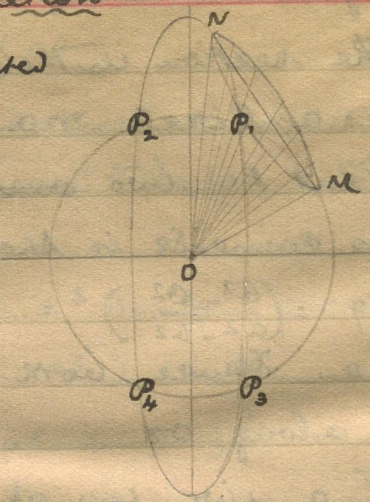
Here the spheroid is generated by the ellipse revolving about the major axis. Section in the $(x-z)$ plane gives the ⊙ outside the ellipse.

∴ The optic axes coincide & lie along the z-axis.

In isotropic media, where $a = b = c$, the eqn becomes $(x^2+y^2+z^2-a^2)^2 = 0$

The two wave surface are two coincident spheres.

Internal Conical Refraction



The direction of a refracted ray is given by the line joining the centre of disturbance to the point of contact of the wave surface with the wave envelope, as determined by Huyghen's construction. Now if the wave envelope be normal to the optic axis, i.e. lies along MN, it can be shown to touch the wave surface not at two points but along a circle. Hence any line joining the point of disturbance O (a pt on the face of the crystal on which the plane wave is incident) to any pt on this O is a possible direction of the refracted ray. Thus we should expect that any wave incident in such a direction that the refracted wavefront travels along the optic axis will be divided not into two rays but into a cone of rays. This phenomenon predicted by Sir W. Hamilton is called internal conical refraction. (Houstoun Page 209)

From eqn (1) $\dfrac{x}{r^2-a^2} = v.\dfrac{l}{v^2-a^2}$.

$$\therefore \sum \frac{xl}{r^2-a^2} = v\sum \frac{l^2}{v^2-a^2} = 0 \quad \text{by (3)}$$

$$\text{ie } \frac{xl}{r^2-a^2} + \frac{nz}{r^2-c^2} = 0 \quad \text{since } y = 0$$

$$\text{ie } xl(r^2-c^2) + nz(r^2-a^2) =$$

But $lx + nz = \cancel{v}$ (2)

$$\text{ie } xl(r^2-c^2) + (v-lx)(r^2-a^2) = 0$$

~~ie~~ ∴ Since $\cancel{v} = b$,

$$x = \frac{b(r^2-a^2)}{l(c^2-a^2)}$$

$$nz = b - lx = b\left\{1 - \frac{r^2-a^2}{c^2-a^2}\right\} = \frac{b.(c^2-r^2)}{c^2-a^2}$$

$$\text{ie } z = \frac{b(c^2-r^2)}{n(c^2-a^2)}$$

$$\therefore c^2lx + a^2nz = b.\frac{a^2(c^2-r^2)+c^2(r^2-a^2)}{c^2-a^2}$$

$$= br^2 = b(x^2+y^2+z^2)$$

In Eqn (1) (x, y, z) determine the point of contact of the ray with the wave surface in the direction (l, m, n)

The above is a sphere passing through the origin & having its centre in the (x, z) plane.

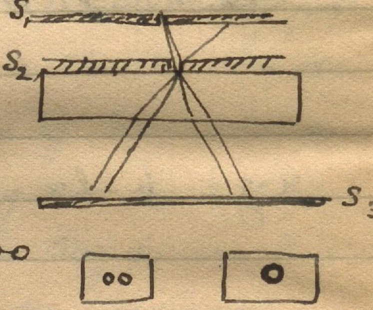
The intersection of this sphere ~~with~~ by the plane MN is the line along which the wave surface is touched by the wave envelope ~~base of the cone of refraction~~

Since any plane intersects a sphere in a circle, the line ~~base of the cone of refraction~~ is a ⊙.

An exptal verification was carried out by Lloyd.

A plate of arragonite was cut with its surface ⊥r to the bisector of the optic axes

A screen S_2 with a small hole is placed on it, & another screen S_1 also with a hole is supported at some height. S_1 is illumined ~~a~~ by an extended source. Two dots are seen on a screen S_3



placed below the plate. As S_1 is moved (keeping the line joining the two holes in the plane containing the optic axis) in one particular position the two dots disappear & in their place a circle appears. If S_3 is moved nearer to the plate the ○ remains of the same size, thereby showing that the cone is internal. The ~~radius of the~~ ∠ of the cone ~~circle~~ as well as the ∠ of incidence in this position show very good agreement with the calculated values. The ∠ of the cone is 1°-50' & the ∠ of incidence 15°-40' (1°-55' & 15°-11' from theory).

When the ring is examined by an analyzing nicol, it is bright at one end of a diameter & dark at the opposite end, the brightness gradually fading away. This also has been explained by theory, for it can be shown that two rays OA & OB which meet opposite ends of a diameter of the base of the cone, are polarized in planes perpendicular to each other.

External conical refraction As internal

[C]onical refraction is produced when a ray passes along the direction of single wave velocity, [E]xternal Conical refraction is produced when [th]e ray passes along the direction of single ~~wave~~ ray velocity i.e. in the directions OP_1 [o]r OP_2. In general, the ray has two ray [v]elocities since the radius vector cuts the [t]wo sheaths of the wave surface at different [p]oints, & velocity is $\propto$ to the radius vector. [B]ut at pts P_1, P_2, P_3, P_4 there is only one [r]adius ~~vector~~ $= b$, & $\therefore$ ~~the~~ hence only [o]ne ray velocity.

The eqn to OP_1, OP_2 can be derived as follows.

P_1 is the intersection of $\frac{x^2}{c^2} + \frac{z^2}{a^2} = 1$

& $x^2 + z^2 = b^2$

$$\therefore \frac{x^2}{c^2} + \frac{b^2 - x^2}{a^2} = 1$$

i.e. $x^2(a^2 - c^2) = a^2c^2 - c^2b^2$ i.e. $x^2 = \frac{c^2(a^2 - b^2)}{a^2 - c^2}$

[S]imilarly $z^2 = \cancel{\frac{a^2(c^2 - a^2)}{c^2 - b^2}}$ $b^2 - x^2 = \frac{a^2(b^2 - c^2)}{a^2 - c^2}$

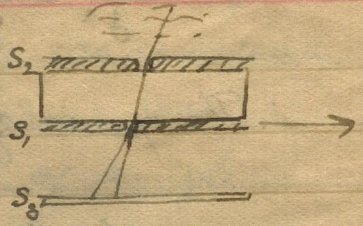
$$\therefore \frac{x^2}{z^2} = \frac{c^2(a^2-b^2)}{a^2(b^2-c^2)}$$

$\therefore$ The eqn is $z = \pm \frac{a}{c}\sqrt{\frac{b^2-c^2}{a^2-b^2}}\, x$, $y = 0$.

At these pts P_1, P_2, P_3, P_4 which are the ~~apex~~ of conical pts, an infinite ~~to~~ number of tangent planes can be drawn to the wave surface, & these tangent planes constitute a tangent cone. Now the direction of an emergent beam from a crystal is determined by the tangent drawn to the wave surface at the point where the beam meets the surface of the crystal. Hence if a ray travelling through the crystal along the direction of OP_1 or OP_2 should on emergence divide into a cone of rays. This phenomenon was kno as external conical refraction was also ~~veri-fied~~ predicted by Hamilton & verified by Lloyd.

The same plate of aragonite was used,

The movable screen S_1 being placed below the plate. S_2 is illumined by an extended source, & as S is moved, in one particular position the two dots on S_2 become a circle. As the S_3 is moved towards the plate, the radius of the ⊙ becomes less & less. The ∠ of the cone was $3^\circ - 1'$ & ∠ of incidence $15^\circ - 58'$. The values predicted from theory were $2^\circ - 59'$ & $16^\circ - 25'$.



Fresnel's Theory of Reflection & Refraction

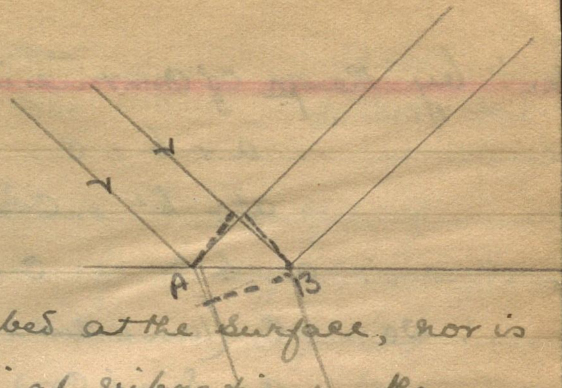
Fresnel assumed that no phase change is introduced by reflection or refraction. Thus if $y = a \sin \omega t$ is incident, the reflected & refracted rays are given by $y_1 = b \sin \omega t$ & $y_2 = c \sin \omega t$.

The principle of Conservation of energy was supposed to hold true.

Energy incident on the surface p. s.

$$= k\, a^2 \rho \times AB \cos i \times v$$

ρ being ether density above the surface, v vel. in that surface. If



no energy is absorbed at the surface, nor is spent as longitudinal vibrations, the sum of the energies emitted p.s. in reflect & refracted beams ie $b^2 \rho$ AB cos i v + $c^2 \rho'$ AB cos r v' should be equal to incident energy (ρ' & v' refer to the medium below the surface

$$\text{ie } a^2 - b^2 = c^2 \cdot \frac{\rho' v'}{\rho v} \cdot \frac{\cos r}{\cos i} = c^2 \cdot \frac{v}{v'} \frac{\cos r}{\cos i}$$

Since $\rho \propto \frac{1}{v^2}$ $\therefore a^2 - b^2 = c^2 \tan i \cot r$.

Fresnel also assumed that the displacements ‖ to the surface are the same both above & below the refracting surface; that there is no slipping of ether.

1) <u>Light polarized</u> in <u>the incident</u> plane.

Displacement is ⊥r to incident plane, & ∴ ‖ to the refracting surface

∴ by the eqn of Continuity

$$a + b = c$$

$$a^2 - b^2 = c^2 \tan i \cot r.$$

$$\therefore \quad a - b = c \tan i \cot r.$$

$$\therefore \quad 2a = c \left(\tan i \cot r + 1\right)$$

$$= c \, \frac{\sin i \cos r + \cos i \sin r}{\cos i \sin r}$$

$$= c . \frac{\sin(i + r)}{\cos i \sin r}.$$

Illy $2b = - c . \dfrac{\sin(i - r)}{\cos i \sin r}$

$$\therefore \quad b = -a \frac{\sin(i - r)}{\sin(i + r)} \quad \& \quad c = + a \frac{2 \cos i \sin r}{\sin(i + r)} \quad (1)$$

Aliter in terms of μ,

$$a + b = c \ ; \quad a^2 - b^2 = c^2 \mu \frac{\cos r}{\cos i}$$

$$\therefore \quad a - b = c \mu \frac{\cos r}{\cos i}$$

$$\therefore \quad 2a = c . \frac{\mu \cos r + \cos i}{\cos i}$$

$$2b = - c . \frac{\mu \cos r - \cos i}{\cos i}$$

$\therefore\ b = -a \dfrac{\mu \cos r - \cos i}{\mu \cos r + \cos i}$ & $c = a \dfrac{2\cos i}{\mu \cos r + \cos i}$ — (6)

1) Sign of b is opposite to sign of a if $i - r$ is +ive i.e. if the ray passes from a rarer to a denser medium. (eqn ①)

2) Value of b for $i = 0$ $\quad b = -a \dfrac{\mu - 1}{\mu + 1}$

& for $i = 90°$ $\quad b = -a$

3) Value of c for $i = 0$ $\quad c = \dfrac{2a}{\mu + 1}$

& for $i = 90°$ $\quad c = 0$.

4) The intensities at any incidence ~~are the~~ in the incident, refl. & refracted rays are

$\propto \quad a^2,\ b^2,\ c^2 \tan^2 i \cot^2 r$

ie $\propto (\mu \cos r + \cos i)^2, (\mu \cos r - \cos i)^2$ & $4\cos^2 i \tan^2 i \cot$

At normal incidence, they are $\propto$

$$a^2,\ a^2\left(\frac{\mu - 1}{\mu + 1}\right)^2,\ a^2 \cdot \frac{4\mu}{(\mu + 1)^2}$$

This latter result has been verified for light by Arago, while other experimenters using a thermopile have verified it for heat rays.

The reflected intensity increases with the value of i

2) Light polarized $\perp$r to the plane of incidence.

The Continuity eqn for components $\parallel$l to the refracting surface,

$$(a+b) = c\,\frac{\cos r}{\cos i}$$

$$a^2 - b^2 = c^2\,\frac{\sin i}{\sin r}\cdot\frac{\cos r}{\cos i}$$

$$\therefore\ a - b = c\cdot\frac{\sin i}{\sin r}.$$

$$\therefore\ 2a = c\cdot\left(\frac{\sin i}{\sin r} + \frac{\cos r}{\cos i}\right)$$

$$= c\cdot\frac{\sin i \cos i + \cos r \sin r}{\sin r \cos i}$$

$$= c\cdot\frac{(\sin i \cos r + \cos i \sin r)(\cos i \cos r + \sin i \sin r)}{\cos i \sin r}$$

$$= c\cdot\frac{\sin(i+r)\cos(i-r)}{\cos i \sin r}$$

Illy $$2b = -c\,\frac{\sin(i-r)\cos(i+r)}{\cos i \sin r}$$

$$\therefore\ b = -a\,\frac{\tan(i-r)}{\tan(i+r)} \quad \&\quad c = a\,\frac{2\cos i \sin r}{\sin(i+r)\cos(i-r)}$$

Aliter in terms of μ $a + b = c \frac{\cos r}{\cos i}$

$a - b = c\mu$

$$\therefore 2a = c \frac{\mu \cos i + \cos r}{\cos i}$$

$$2b = c. \frac{\mu \cos i - \cos r}{\cos i}$$

$$\therefore b = -a. \frac{\mu \cos i - \cos r}{\mu \cos i + \cos r} \quad \& \quad c = a \frac{2\cos i}{\mu \cos i + \cos r}$$

1) Sign of b is opposite to sign of a if $\tan(i+r)$ is +ive i.e. $i + r < 90°$ & is the same as that of a if $i + r > 90°$.

2) Value of b for $i = 0$, $b = -a \frac{\mu - 1}{\mu + 1}$

$i + r = 90°$, $b = 0$

$i = 90°$ $b = +a$

Thus as i increases, b changes from a -ive to a +ive value, passing through zero when $i + r = 90°$, which is the $\angle$ of max. polarization for an ordinary ray.

3) Value of c for $i = 0$ $c = \frac{2a}{\mu + 1}$

$i + r = 90°$, $c = a. \frac{2 \cos i \sin r}{\cos(i - r)}$

& for $i = 90°$ $c = 0$.

C & b for $i = 0$ ~~are~~ the ~~same~~ same for polarization
⊥r to the incident plane as ~~well as~~ for polariz-
tion ‖l to the incident plane, which is as it
should be since in both cases ~~light is~~
the vibrations are ‖l to the surface.

(4) The intensities of the rays are proportional to

$a^2 \quad , \quad b^2 \quad , \quad c^2 \tan i \cot r.$

i.e. $a^2 \quad , \quad a^2 \dfrac{\tan^2(i-r)}{\tan^2(i+r)} \quad , \quad a^2 \quad \dfrac{4 \cos i \sin r \sin i \cos r}{\sin^2(i+r)\cos^2(i-r)}$

At $i = 0$, $\quad 1 \quad : \quad \dfrac{(\mu-1)^2}{(\mu+1)^2} \quad : \quad \dfrac{4\mu}{(\mu+1)^2}$

At $i + r = 90°$ $\quad 1 \quad : \quad 0 \quad : \quad 1$

At $i = 90$ $\quad 1 \quad : \quad 1 \quad : \quad 0$

Light polarized at any azimuth say
at α to the incident plane.

If a be the vibration,
its component polarized ‖l to the plane
of incidence is $a \cos \alpha$
& component polarized ⊥r to the plane
of incidence is $a \sin \alpha$.
The reflected components due to the two are

[illegible] for small dispersion. $\mu = A + \frac{B}{\lambda^2} \ldots$ (Cauchy's)

$$I_r = \frac{4a^2 b^2 \sin^2\frac{\varepsilon}{2}}{1 - 2b^2\cos\varepsilon + b^4} \qquad I_t = \frac{a^2(1-b^2)^2}{1 - 2b^2\cos\varepsilon + b^4}$$

$$I_r = a^2 \frac{4b^2 \sin^2 \varepsilon/2}{(1-b^2)^2 + 4b^2\sin^2\frac{\varepsilon}{2}} \qquad I_t = a^2 \frac{(1-b^2)^2}{(1-b^2)^2 + 4b^2\sin^2\frac{\varepsilon}{2}}$$

I. Fraunhofer $= a^2 b^2 \frac{\sin^2 \frac{\pi a \xi}{R\lambda}}{\left(\frac{\pi a \xi}{R\lambda}\right)^2} \cdot \frac{\sin^2 \frac{\pi b \eta}{R\lambda}}{\left(\frac{\pi b \eta}{R\lambda}\right)^2}$

Grating $\alpha^2 s^2 \frac{\sin^2 \frac{\pi a \sin\theta}{\lambda}}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2} \cdot \frac{\sin^2 \frac{n\pi(a+\delta)\sin\theta}{\lambda}}{\sin^2 \frac{\pi(a+\delta)\sin\theta}{\lambda}}$

Pol[illegible] in inc. plane $b = -a\frac{\sin(i-r)}{\sin(i+r)}$; [illegible]

$\perp$r to inc plane. $b = -a\frac{\tan(i-r)}{\tan(i+r)}$ [illegible]