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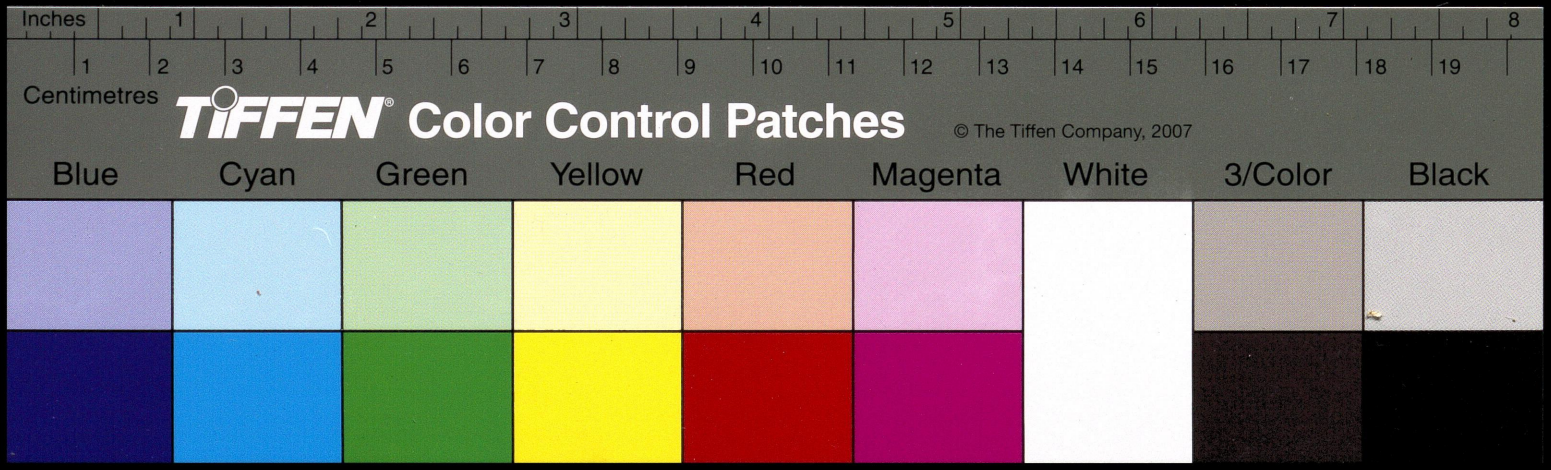

Confidential

Setter/Examiner

No. Misc./ 723

(In replying, please quote the number and date of this letter)

SENATE HOUSE:
CALCUTTA

The 19 JUN 1981, 197...

FROM

THE CONTROLLER OF EXAMINATIONS,
UNIVERSITY OF CALCUTTA.

To Prof. B. S. Madhava Rao

.....

DEAR SIR/MADAM,

I am desired to request you kindly to set/examine the papers noted overleaf in connection with your appointment as a member of the Board of Examiners in Appl. Math for the M.A/M.Sc. Part II Examination, 1980 to be held in ..., 197...

The question-papers written very legibly on one side only of the foolscap paper (herewith supplied) should be sent in sealed double covers, marked 'Confidential' and addressed to me by name so as to reach me on or before 30 JUN 1981 197...

Unless the contrary is indicated, you will be called upon to examine the answer-scripts of the candidates in the paper or papers set by you, and the relevant scripts will be forwarded to you for valuation immediately after the examination. The marks awarded together with the examined scripts have to be submitted to this office in accordance with the instructions to be issued separately.

It is not desirable that a person who has a near relation, or who has privately undertaken the coaching of a student, to appear in a subject, should be associated with paper-setting or examining in that subject. I am to request you to return without delay the following Declaration Form duly filled in and signed.

Enclosures:—

(a) *List of Text-books and/or Syllabus of study.*
(b) *Specimen question-paper.*
(c) *Foolscap paper for setting questions.*
(d) *Bill form.*
(e) *Two envelopes.*

Yours faithfully,

Controller of Examinations.

Rec'd BSM 24/8/81

Please detach the portion below and return it, duly filled in and signed, to the Controller of Examinations.

Sent acceptance form and 2-taper today BSM 25/6/81

Also wrote to Controller Banerjee re. return for 1975 to 1979. BSM 25/6/81

[P. T. O.]

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Instructions regarding paper-setting and examination :—

No copy of the question-papers should be retained by the setter.

Usually each paper is set by an External Examiner and one or two Internal Examiners, as the case may be, who have actually taught the subject. If the paper is divided into two halves, one half may be set by the Internal Examiner and the other half by the External Examiner. Where joint paper-setting is indicated, it is not required that the setters should of necessity get into touch and/or exchange papers with each other. The setting may be done by them independently. It will be the task of the Moderators to compile a full paper from the quesions set by the Examiners separately.

No questions should be set on a subject which do not come within the scope of the prescribed list of text-books and/or syllabus of study. The Internal Examiner for each paper or half paper is requested to furnish his Co-Examiner (External) with all the necessary information in this regard.

In this connection, I beg to draw your special attention to the following rules :—

(1) "Examiners are required to keep the results of the examinations and the marks assigned to candidates strictly secret."

(2) "If any Examiner at any University Examination fails to send his marks within the limits of time prescribed for the purpose, the amount of his remuneration shall, as a rule, be reduced by Rs. 5 for each day by which he exceeds that period. The Syndicate may, however, in very special circumstances, exempt an Examiner from the operation of this rule."

The scales of remuneration sanctioned by the Syndicate in this regard will be found enumerated in the enclosed bill form.

IV) Quantum Mechanics. Subject.....

3rd Paper { ~~First half~~ / ~~Second half~~ → Setter and Examiner

Paper { First-half / Second half

Paper { First-half / Second-half

Paper { First-half / Second-half

Paper { First-half / Second-half

SCUP 102C—30-7-79—5,000.

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BOARD OF EXAMINERS

SENATE HOUSE:
CALCUTTA,

The....., 198.....

28 MAY 1981

No. Misc./ 620

FROM : SRI G. C. BANERJEE, M.A.,
CONTROLLER OF EXAMINATIONS,
UNIVERSITY OF CALCUTTA.

To Prof. B. S. Madhava Rao.

DEAR SIR/MADAM,

I have the honour, by direction of the Vice-Chancellor and Council, to invite you to act as a member of the Board of Examiners in Applied Math for the M.A/M.Sc. Part II Examination 19 80 to be held in Aug, 19 81.

The Board of Examiners in any subject consists of Internal and External Examiners.

It is not intended that every member of the Board of Examiners thus constituted shall actually frame questions or examine answer-papers ; this work shall be shared by the members of the Board in such manner as they may determine. But the results of the Examination in any subject shall be submitted to, and reported upon, by the entire Board of Examiners in the subject.

Necessary intimation will be sent to you if the Board of Examiners desires you to set and/or examine any paper or portion of any paper or to participate in the examination work in any other way.

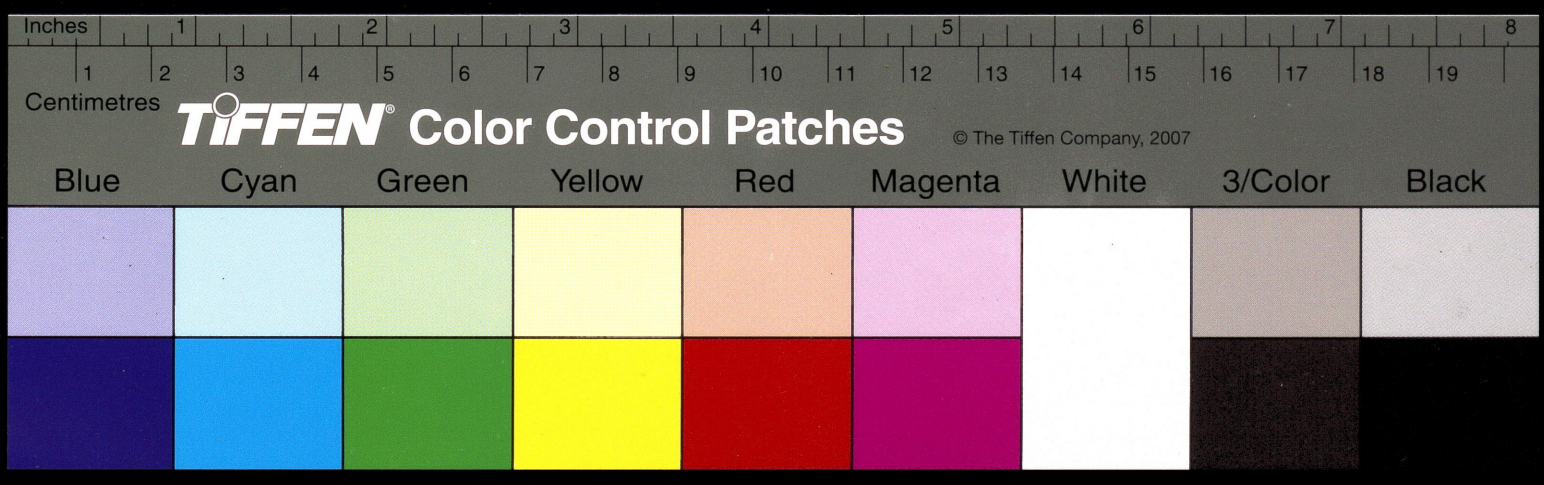
I should be obliged if you would kindly inform me at your earliest convenience whether you accept or decline the appointment.

Yours faithfully,

Controller of Examinations.

Replied today saying willing
4/6/81

N.B.—Please detach the following portion and return it, duly filled in and signed immediately.



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UNIVERSITY OF CALCUTTA

NOTICE

A meeting of the Board of Examiners in Applied Mathematics for the M.A./M.Sc. Part II Examination, 1980 will be held on Friday, the 5th June, 1981 at 3-30 P.M. in Room No.26 of the Department of Applied Mathematics, Calcutta University, 92, Acharyya P.C. Road, Calcutta-700009.

Business :

To recommend the names of persons for appointment as Paper-Setters, Examiners and Moderators from amongst the members of the Board constituted for the purpose for the abovementioned examination.

R.N. Chaudhuri

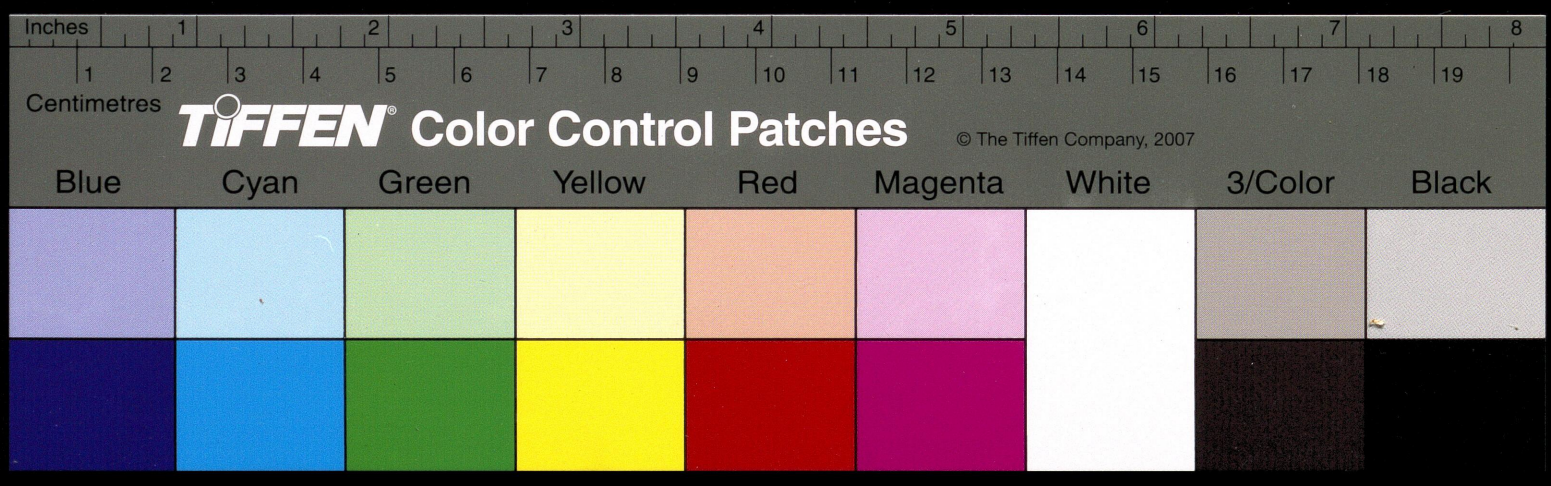
Senate House,
Calcutta.
The 28th May, 1 9 8 1.

(R. N. Chaudhuri)
Asstt.Controller of Examinations.

N.B. :- (1) Members are earnestly requested to attend the meeting.

(2) The University does not pay T.A. or any other allowance for the purpose of attending the meeting.

Sri. G.C. Banerjee M.A.,
The Controller of Examinations
Calcutta University
48, Hazra Road,
Calcutta-19



(1980 Exam) Marks awarded

(1) No. 4 –
Q.2 – 16
Q.3 – 14
Q.4 – 12
Q.7 – 12
Q.9 – 12
Q.10 – 16
84

(2) No. 23
Q.2 – 6
Q.3 – 2
Q.4 – 2
Q.6 – 16
Q.7 – 9
Q.10 – 14
49

(3) No. 24
Q.3 – 6
Q.4 – 12
Q.5 – 8
Q.7 – 16
Q.9 – 10
Q.10 – 16
68

(4) No. 25
Q.3 – 12
Q.5 – 8
Q.7 – 16
Q.9 – 14
Q.10 – 16
66

(5) No. 26
Q.2 – 6
Q.3 – 10
Q.4 – 12
Q.7 – 16
Q.9 – 12
Q.10 – 16
70

(6) No. 27
Q.2 – 12
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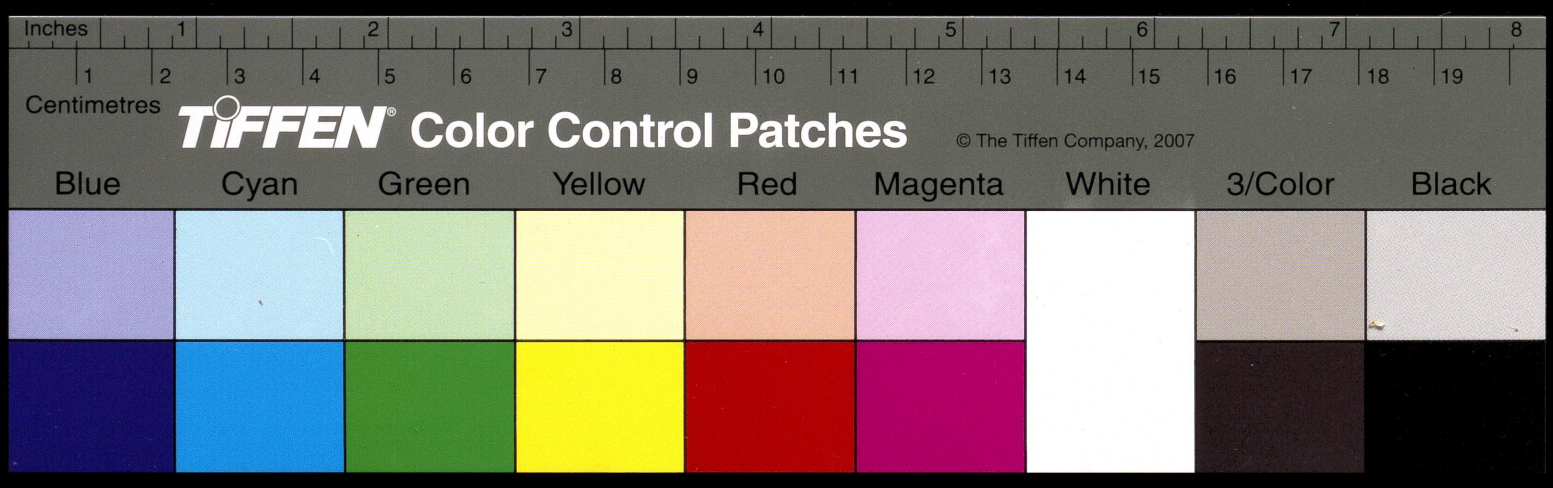
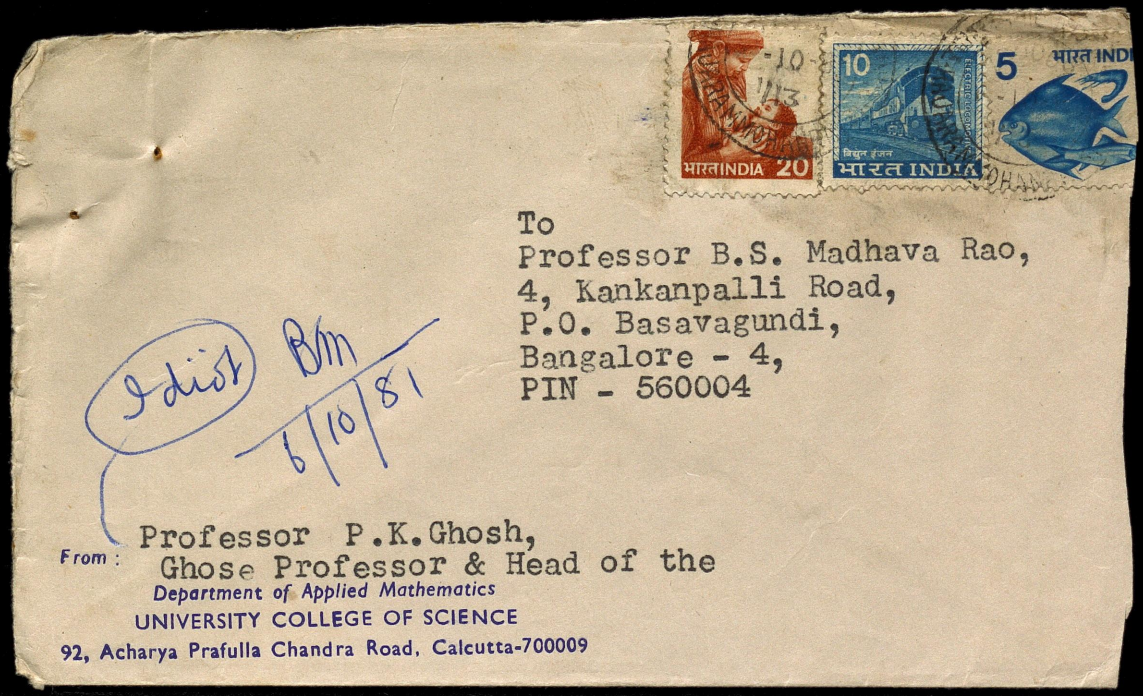
Q.5 – 12
Q.6 – 16
Q.7 – 12
Q.10 – 16
78





To
Professor B.S. Madhava Rao,
4, Kankanpalli Road,
P.O. Basavagundi,
Bangalore - 4,
PIN - 560004

1 dist BM
6/10/81

From : Professor P.K.Ghosh,
Ghose Professor & Head of the
Department of Applied Mathematics
UNIVERSITY COLLEGE OF SCIENCE
92, Acharya Prafulla Chandra Road, Calcutta-700009

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1980

APPLIED MATHEMATICS

THIRD PAPER

(Quantum Mechanics)

Full Marks—100

Answer SIX *questions, taking* THREE *from* EACH HALF

The questions are of equal value.

Candidates are required to give their answers in their own words as far as practicable

FIRST HALF

1. (*a*) Explain the basic ideas of the Bohr-Sommerfeld old quantum mechanics, and illustrate the same by an example.

(*b*) Explain Heisenberg's *uncertainty principle*, and use it to estimate the ground state energy of the simple linear harmonic oscillator.

2. (*a*) Starting from the Schrödinger equation for a particle, deduce the conservation equation,

$$\frac{\partial \rho}{\partial t} + \text{div } \vec{j} = 0$$

and interpret ρ and $\vec{j}$ physically.

(*b*) The matrices A, B, C satisfy the following equations :

$$A^2 = B^2 = C^2 = 1, \text{ the unit matrix}$$

$$\{A, B\} = \{B, C\} = \{C, A\} = 0,$$

where the curly brackets { } denote anticommutation. Find explicit representation of all the matrices when A is diagonal, assuming it to be nondegenerate.

3. (*a*) If p and q be the momentum and position operators of a particle, show that

$$qp^n - p^n q = ni\hbar p^{n-1}.$$

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(*b*) If U be an $n \times n$ unitary matrix, show that $U = \exp(iH)$, where H is hermitian, and that

$$\det U = \exp(i \text{ trace } H).$$

Further, if U be also unimodular, find the number of real parameters in U.

4. Assuming that the time evolution of the state vector is determined by the Schrödinger equation derive Heisenberg's form of the equation of motion for a dynamical variable A

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{i}{\hbar}\left[H, A\right].$$

Hence derive Ehrenfest's theorem

$$\left\langle \frac{d\vec{p}}{dt} \right\rangle = \left\langle -\text{grad } V \right\rangle$$

when $H = \vec{p}^2/2m + V(\vec{r})$ and $\langle\ \rangle$ denotes expectation value.

Also show that for a charged particle in an electromagnetic field with potentials $\vec{A}$, ϕ

$$\frac{d\vec{r}}{dt} = \frac{1}{m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)$$

5. (*a*) Obtain the bound state energy eigenvalues of a particle of mass μ placed in an attractive potential

$$V(x) = -\frac{l(l+1)\hbar^2}{2\mu}\,\text{sech}^2 x,$$

where l is a positive integer (consider the problem in one dimension).

(*b*) Apply the Born approximation to calculate the scattering amplitude (in the first approximation) for a particle scattered by the potential

$$V = ge^{-\mu r}/r,$$

and discuss the case where $\mu \to 0$.

Second Half

6. The Schrödinger equation for the radial part of the l th partial wave in a central field $V(r)$ is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_l}{dr}\right) + \left[k^2 - \frac{l(l+1)}{r^2} - \frac{2m}{\hbar^2}V(r)\right]R_l = 0.$$

Using the asymptotic expansion

$$j_l(\rho) + in_l(\rho) \xrightarrow[\rho \to \infty]{} \rho^{-1}\exp i\left[\rho - (l+1)\frac{\pi}{2}\right]$$

show that the effect of the scattering potential is to produce an energy dependent phase shift $\delta_l(k)$ in the l th partial wave.

Given the expansion

$$e^{ikr\cos\theta} = \sum_{l=0}^{\infty}(2l+1)\,e^{il\pi/2}\,j_l(kr)\,P_l(\cos\theta).$$

obtain an expression for the scattering cross-section in terms of the phase shifts $\delta_l(k)$. Hence show that the total cross-section is given by

$$\sigma = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l(k).$$

(All notations have their usual significance).

7. (*a*) Determine the spin eigenvalues and symmetrised eigenfunctions of

$$(\vec{S_1} + \vec{S_2})^2 \text{ and } (S_{1z} + S_{2z})$$

where $\vec{S_1}$ and $\vec{S_2}$ denote the spin angular momentum of two electrons.

(*b*) Define the density matrix operator ρ. Show that the expectation value of an observable A in an ensemble described by ρ can be expressed as $\langle A \rangle = Tr(\rho A)$.

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8. (*a*) Explain the notion of *isotopic spin*, and its connection with the group $SU(2)$.

(*b*) Show that the proper space rotation group $O_p(3)$ is isomorphic with $SU(2)$.

(*c*) Write down the 2×2 matrix representation of this group corresponding to the isospin for nucleons.

9. Determine the energy levels of an atom in a magnetic field, and explain the normal Zeeman effect.

Derive an expression for the corrections to the energy levels due to electron spin for the case of a weak homogeneous magnetic field, and the significance of the Landé factor.

10. Derive Fermi's golden rule for the transition probability per unit time from the initial state $| m >$ to the final state $| k >$

$$\omega = \frac{2\pi}{\hbar} \rho(k) \, | <k | H' | m> |^2$$

where $\rho(k)$ denotes the density of final states and H' is the perturbation.

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Calcutta University

M. A, M. Sc: Part II Exam. of 1980 – Appl. Math.

Third Paper

(Quant. Mech).

2 (a) Derive the Schrödinger wave equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar}{2m} \nabla^2 \psi + V(r,t)\psi$$

explaining the assumptions made.

(b) Starting from the Schrödinger equation for a particle derive the conservation equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{f} = 0$$

and interpret ρ and $\vec{f}$ physically

3 (a) If p and q be the position and momentum operators ~~of~~ of a particle, show that

$$qp^n - p^n q = ni\hbar p^{n-1}$$

(b) If U be an $n \times n$ unitary matrix, show that

$$U = \exp(iH),$$

where H is hermitian, and

$$\det U = \exp(i \operatorname{trace} H).$$

Further if U be also unimodular find the number of independent real parameters in U.

~~3 (a)~~
1 (a) Explain the basic ideas of the Bohr–Sommerfeld old quantum mechanics, and illustrate the same by an example.

(b) Explain Heisenberg's uncertainty principle and use it to estimate the ground state energy of the simple linear harmonic oscillator.

~~3 (a)~~
4 (a) Show that in a rigid rotator, the energy, the total angular momentum, and its z-component can all have the same constant eigen-values simultaneously

~~(b) Define dynamical symmetry and give an example of the same~~

(b) Evaluate $\int_{-\infty}^{\infty} u_n(x)\, x^2\, u_n(x)\, dx$, where the u's are normalised harmonic oscillator wave functions

~~4 (a) Define the group SU(2), and indicate the physical significance of the symmetry underlying it~~

~~(b) Show that the density matrix of a quantum has unit trace.~~

~~(c) Define dynamical symmetry and give an example of the same.~~

6 ~~(a)~~ ~~Define~~ Explain the terms: differential scattering cross-section, and total scattering cross section

Apply the Born approximation to calculate the scattering amplitude (in the first approximation) for a particle scattered by the potential

$$V = g e^{-\mu r}/r$$

Discuss the case when $\mu \to 0$

5 (a) ~~Write down the Pauli~~ State Pauli's exclusion principle, and ~~derive it~~ ~~illustrate it by an example~~ on the assumption that two electrons only anti-symmetrical states occur.

(b) Explain the connection between spin and statistics in elementary particle physics.

~~(c)~~

Second Half

~~7~~ 8 (a) Define unitary & hermitian matrices and show that two hermitian matrices can be diagonalised simultaneously ~~if and only if~~ by the same unitary matrix if & only if they commute with each other

(b) Show how a classical system can be taken over into quantum mechanics by the replacement of Poisson brackets by commutator brackets. Show, further, that the canonical variables then obey the quantum conditions:

$$[q_i, p_j] = i\hbar\delta_{ij}, \quad [q_i, q_j] = 0, \quad [p_i, p_j] = 0$$

9 8 (a) Deduce the commutation relations for the components (M_x, M_y, M_z) of the angular momentum in a central field

(b) Obtain these components in polar coordinates, and show that one can define ~~obtain~~ simultaneously the eigen-values of H, M^2 and M.

11 ~~10~~ 8 (a) State and prove the standard formula for the transition probability in the time-dependent perturbation theory.

(b) Use the variational method of perturbation theory to find an upper limit for the energy of the ground state of the helium atom.

9 8 (a) ~~Explain the normal Zeeman effect~~ Determine the energy levels of an atom in a magnetic field, and explain the normal Zeeman effect.

(b) Derive an expression for the correction to the energy levels for the case of

a
~~strong~~ weak homogeneous magnetic field and the significance of the Landé factor.

10 (a) Explain the notion of isotopic spin, and its connection with the group SU(2).

(b) Show that the proper space rotation group $O_p(3)$ is isomorphic with SU(2)

(c) Write down the 2×2 matrix representation of this group corresponding to the isospin for nucleons.

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