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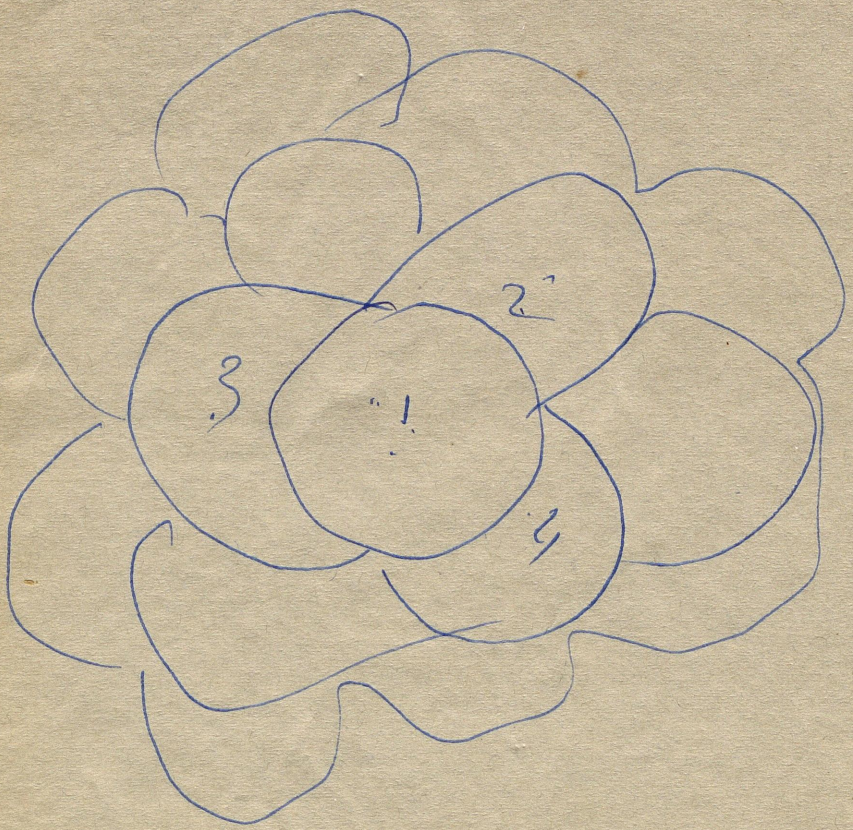
3469

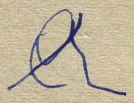


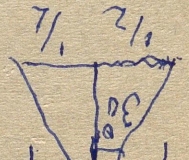
Note Book No. 9.

Verify for (2) the soln

$y = Ae^{-t} + B\cos t + C\sin t$







1(a) $x^2 D^2y + 3xDy + 2y = x^2 + 2x$ — (1)

$x = e^t$, $D^2y = \frac{1}{x^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$ $\left[t = \log x\right]$

$Dy = \frac{1}{x}\frac{dy}{dt}$.

$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 3\frac{dy}{dt} + 2y = e^{2t} + 2e^t$.

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = e^{2t} + 2e^t$ — (2)

$\frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

C.F of (2) is $y = c_1 e^{(-1+i)t} + c_2 e^{(-1-i)t}$.

$= c_1 e^t e^{-t} e^{it} + c_2 e^{-it}$

$= e^{-t}(c_1 e^{it} + c_2 e^{-it})$

$= e^{-t}(A\cos t + B\sin t)$

P.I is $\frac{1}{D^2+2D+2} \cdot e^{2t} + 2e^t$

$= \frac{1}{2^2+2\cdot 2+2} e^{2t} + \frac{2}{1^2+2\cdot 1+2} \cdot e^t$

$= \frac{1}{10} e^{2t} + \frac{2}{5} e^t$

$y = G.S = \frac{A}{x} + B\cos(\log x) + C\sin(\log x) + \frac{x^2}{10} + \frac{2x}{5}$.

Verify: $\frac{dy}{dx} = -\frac{A}{x^2} - B\sin(\log x)\frac{1}{x} + C\cos\log x \cdot \frac{1}{x} + \frac{x}{5} + \frac{2}{5}$

$\frac{d^2y}{dx^2} = \frac{2A}{x^3} - \frac{1}{x}B\cos(\log x)\cdot\frac{1}{x} - B\sin(\log x)\left(-\frac{1}{x^2}\right)$

$= -\frac{A}{x^2}\cos(\log x)\cdot\frac{1}{x} - \sin(\log x)\cdot\frac{1}{x^2} + \frac{1}{5}$

$\frac{d^2y}{dx^2} = \frac{2A}{x^3} + \cos(\log x)\left\{\frac{B}{x^2} + \frac{C}{x^2}\right\} + \sin(\log x)\left\{\frac{B}{x^2} - \frac{C}{x^2}\right\} + \frac{1}{5}$

$= \frac{2A}{x^3} + \frac{(B+C)}{x^2}\cos(\log x) + \frac{(B-C)}{x^2}\sin(\log x) + \frac{1}{5}$

$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 2y$

$= \frac{2A}{x} + (B+C)\cos(\log x) + (B-C)\sin(\log x) + \frac{x^2}{5}$

$-\frac{3A}{x} - 3(B+C)\cos(\log x) - 3B\sin(\log x) + 3C\cos(\log x)$

$+ \frac{3x^2}{5} + \frac{6x}{5}$

$+ \frac{2A}{x} + 2B\cos(\log x) + 2C\sin(\log x) + \frac{x^2}{5} + \frac{4x}{5}$

$y = \frac{A}{x}\cos(\log x) + \frac{B}{x}\sin(\log x) + \frac{x^2}{10} + \frac{2x}{5}$ Correct soln ..

Verification: $\frac{dy}{dx} = -\frac{A}{x^2}\cos(\log x) + \frac{A}{x^2}\sin(\log x)\cdot 1$

$- \frac{B}{x^2}\sin(\log x) + \frac{B}{x^2}\cos(\log x)\cdot 1 + \frac{x}{5} + \frac{2}{5}$

$\frac{d^2y}{dx^2} = \frac{2A}{x^3}\cos(\log x) + \frac{A}{x^3}\sin(\log x)\cdot + \frac{2A}{x^3}\sin(\log x) - \frac{A}{x^3}\cos(\log x)$

$+ \frac{2B}{x^3}\sin(\log x) - \frac{B}{x^3}\cos(\log x) - \frac{2B}{x^3}\cos(\log x) - \frac{B}{x^3}\sin(\log x)$

$= \frac{A}{x^3}\cos(\log x) + \frac{3A}{x^3}\sin(\log x) + \frac{B}{x^3}\sin(\log x) - \frac{3B}{x^3}\cos(\log x) + \frac{1}{5}$

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 2y$$

$$= \frac{A}{x}\cos(\log x) + \frac{3A}{x}\sin(\log x) + \frac{B}{x}\sin(\log x) - \frac{3B}{x}\cos(\log x) + x^2/5$$

$$- \frac{3B}{x}\sin(\log x)$$

$$- \frac{3A}{x}\cos(\log x) - \frac{3A}{x}\sin(\log x) - \frac{3B}{x}\sin(\log x) + \frac{3B}{x}\cos(\log x) + \frac{3x^2}{5} + \frac{6x}{5}$$

$$+ \frac{2A}{x}\cos(\log x) + \frac{2B}{x}\sin(\log x) + \frac{x^2}{5} + \frac{4x}{5}.$$

$$= x^2 + 2x$$

1(b). $(D^2+4)y = \sin 2x$

$$C.F = A\cos 2x + B\sin 2x$$

$$P.I = \frac{1}{D^2+4}\sin 2x = A'\sin 2x - \frac{x}{4}\cos 2x.$$

[Verify $(D^2+4)\left\{A'\sin 2x - \frac{x}{4}\cos 2x\right\}$

$$D\{\ \} = 2A'\cos 2x - \frac{1}{4}\cos 2x + \frac{x}{2}\sin 2x$$

$$D^2\{\ \} = -4A'\sin 2x + \frac{1}{2}\sin 2x + \frac{1}{2}\sin 2x + x\cos 2x$$

$$(D^2+4)\{\ \} = -4A'\sin 2x + \sin 2x + x\cos 2x$$

$$+4A'\sin 2x \quad - x\cos 2x = \sin 2x]$$

Complete soln is $y = A\cos 2x + B\sin 2x - \frac{1}{4}x\cos 2x$

(c) $(D^2+3D+2)y = 2\sin x + 4\cos x$

$(D+1)(D+2)y = -2\sin x + 4\cos x$

$C.F = c_1 e^{-x} + c_2 e^{-2x}$

$$\frac{1}{D^2+3D+2} \cdot 2\sin x = \frac{(D^2+2)-3D}{(D^2+2)^2 - 9D^2} \cdot 2\sin x$$

$$= 2 \cdot \frac{\{(-1)^2+2-3D\}\sin x}{\{(-1)^2+2\}^2 - 9(-1)^2} = 2\,\frac{3-3\sin x}{9}$$

$$= 2\,\frac{\{-1+2-3D\}\sin x}{(-1+2)^2 - 9(-1)} = 2\,\frac{\sin x - 3\cos x}{10}$$

$$\frac{1}{D^2+3D+2} 4\cos x = 4 \cdot \frac{\cos x + 3\sin x}{10} = \frac{2(\cos x + 3\sin x)}{5}$$

$\therefore P.I = \frac{1}{5}\cos x + \frac{3}{5}\sin x = \frac{7}{5}\sin x - \frac{1}{5}\cos x$

$$y = c_1 e^{-x} + c_2 e^{-2x} + \frac{7}{5}\sin x - \frac{1}{5}\cos x$$ ✓

Verification, $Dy = -c_1 e^{-x} - 2c_2 e^{-2x} + \frac{7}{5}\cos x + \frac{1}{5}\sin x$

$D^2y = c_1 e^{-x} + 4c_2 e^{-2x} - \frac{7}{5}\sin x + \frac{1}{5}\cos x$

$D^2y + 3Dy + 2y = c_1 e^{-x} + 4c_2 e^{-2x} + \frac{7}{5}\sin x + \frac{1}{5}\cos x$
$- 3c_1 e^{-x} - 6c_2 e^{-2x} + \frac{21}{5}\cos x + \frac{3}{5}\sin x$
$+ 2c_1 e^{-x} + 2c_2 e^{-2x} + \frac{14}{5}\sin x - \frac{2}{5}\cos x$

$= \sin x\left(-\frac{7}{5} + \frac{3}{5} + \frac{14}{5}\right) + \cos x\left(\frac{1}{5} + \frac{21}{5} - \frac{2}{5}\right)$

$= 2\sin x + 4\cos x$

2(a) Ramanathan, p. 673 (5)

Calling $e^{\int f(x)\,dx} = \mu$

condition for an exact equation in general is

$$\frac{\partial(\mu P)}{\partial y} = \frac{\partial(\mu Q)}{\partial x} \quad \text{where } \mu = f(x, y)$$

or $$P\frac{\partial \mu}{\partial y} - Q\frac{\partial \mu}{\partial x} + \mu\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$$

(i) If $\mu(x)$ be a fn of x only

$$\frac{d\mu}{dx} = \frac{\mu}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0 \quad \text{— } 1^{st} \text{ part}$$

Second part is converse of this

(b) Here $P = x^3 + xy^4$, $Q = 2y^3$

$$\frac{\partial P}{\partial y} = 4xy^3, \quad \frac{\partial Q}{\partial x} = 0$$

$$\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{4xy^3}{2y^3} = 2x = \text{function of } x \text{ only}$$

∴ integrating factor is $\mu(x) = e^{\int 2x\,dx} = e^{x^2}$.

$$e^{x^2}(x^3 + xy^4)\,dx + 2y^3 e^{x^2}\,dy = 0.$$

$$\int x^2 e^{2x}\,dx = \frac{x^2 e^{2x}}{2} - \frac{2}{2}\int e^{2x}\, x\cdot dx$$

$$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\cdot dx = \frac{1}{2}x^2 e^{2x} - \left[\frac{1}{2}\int e^{2x}\,dx\right]$$

$$= \frac{1}{2}x^2 e^{2x} - \frac{xe^{2x}}{2} + \frac{1}{2}\int e^{2x}\,dx$$

$$= \frac{1}{2}x^2 e^{2x} - \frac{xe^{2x}}{2} + \frac{1}{4}e^{2x}$$

$$\int x e^{2x} = \frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}.$$

$$\frac{1}{2}x^2 e^{2x} - \frac{xe^{2x}}{2} + \frac{1}{4}e^{2x} + y^4\left(\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}\right) + \frac{y^4}{2}e^{2x} = \text{const.}$$

$$xe^{2x}y^4\,dx + 2y^3 e^{2x}\,dy + x^2 e^{2x}\,dx = 0$$

$$e^{x^2}(x^3 + xy^4)\,dx + 2y^3 e^{x^2}\,dy = 0.$$

$$xe^{x^2}y^4\,dx + 2y^3 e^{x^2}\,dy + x^3 e^{x^2}\,dx = 0$$

$$\frac{1}{2}d(e^{x^2}y^4) + x^3 e^{x^2}\,dx = 0$$

$$+\frac{1}{2}d\left\{e^{x^2}(x^2-1)\right\} = 0$$

$$\text{or } e^{x^2}y^4 + e^{x^2}(x^2-1) = \text{const.}$$

$$e^{x^2}\{x^2 + y^4 - 1\} = \text{const.}$$ ✓

$e^{x^2}\cdot 4y^3 + 2xe^{x^2}y^4 \quad 2(2y^3 e^{x^2} + xe^{x^2}y^4)$

$x^2 = u$

$2x\,dx = du$

$\int x^3 e^{x^2}\cdot dx$

$= \int \frac{1}{2}du \cdot ue^u$

$= \frac{1}{2}\left[ue^u - \int e^u du\right]$

$= \frac{1}{2}e^u(u-1)$

$\cdot e^{x^2}(2x) + (x^2-1)e^{x^2}\cdot 2x$

Verification: Differentiate

$$e^{x^2}\cdot 2x\cdot y^4\,dx + 4e^{x^2}y^3\,dy + \cancel{2xe^{x^2}\,dx} + x^2\cdot e^{x^2}\cdot 2x\,dx - \cancel{e^{x^2}\cdot 2x\,dx} = 0$$

$$2xy^4\,dx + 4y^3\,dy + 2x^3\,dx = 0$$

$$xy^4\,dx + 2y^3\,dy + x^3\,dx$$

$$(x^3 + xy^4)\,dx + 2y^3\,dy = 0$$

3 (a) & (b) — postponed — Look into Forsyth.

4(a) $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0.$

Soln of $x^2y'' + xy' + (x^2 - \nu^2)y = 0$ (Bessel eqn of order ν)

by Frobenius method two linearly ind. solns, one of these

b(i) $$J_\nu(x) = \sum_{K=0}^{\infty} \frac{(-1)^K}{K!\,\Gamma(\nu+K+1)}\left(\frac{x}{2}\right)^{\nu+2K}$$ or with m for K

~~This soln for (a) is $J_{1/2}(x)$~~

~~as in Hilbert-Courant:~~

$$\cancel{J_n(x) = \frac{x^n}{2^n\cdot n!}\left\{1 - \frac{x^2}{2(2n+2)} + \frac{x^4}{2\cdot 4\cdot(2n+2)(2n+4)} - \cdots + \cdots\right\}}$$

(b) $$J_\nu(x) = \sum_{m=0}^{\infty}(-1)^m\left(\frac{x}{2}\right)^{2m+\nu} \Big/ m!\,\Gamma(m+\nu+1) \quad \text{— (A)}$$

$$\frac{d}{dx}\left[x^\nu J_\nu(x)\right] = x^\nu \sum_{m=0}^{\infty}(-1)^m\left(\frac{x}{2}\right)^{2m+\nu-1} \Big/ m!\,\Gamma(m+\nu) = x^\nu J_{\nu-1}(x) \quad \text{— (1)}$$

[using $\Gamma(m+\nu+1) = (m+\nu)\Gamma(m+\nu)$

(8) $\therefore \frac{d}{dx}\left[x^{-\nu} J_\nu(x)\right] = x^{-\nu} \sum_{m=1}^{\infty} (-1)^m \left(\frac{x}{2}\right)^{2m+\nu-1} \Big/ [(m-1)!\,\Gamma(m+\nu+1)$

$= -x^{-\nu} J_{\nu+1}(x)$ [In $J_{\nu+1}(x)$ put $m=k-1$]

— (2)

From (1) & (2) $\nu x^{\nu-1} J_\nu(x) + x^\nu J_\nu'(x) = x^\nu J_{\nu-1}(x)$

and $-\nu x^{-\nu-1} J_\nu(x) + x^{-\nu} J_\nu'(x) = -x^{-\nu} J_{\nu+1}(x)$

$x^{-\nu-1} \cdot x$

$$\nu J_\nu(x) + x J_\nu'(x) = x J_{\nu-1}(x) \quad -- (1')$$
$$-\nu J_\nu(x) + x J_\nu'(x) = -x J_{\nu+1}(x) \quad -- (2')$$

i.e.
$$\frac{\nu}{x} J_\nu(x) + J_\nu'(x) = J_{\nu-1}(x) \quad -- (1'')$$
$$-\frac{\nu}{x} J_\nu(x) + J_\nu'(x) = J_{\nu+1}(x) \quad -- (2'')$$

Subtracting $J_{\nu-1}(x) - J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x)$

Adding $J_{\nu-1}(x) + J_{\nu+1}(x) = 2 J_\nu'(x)$

(b) (ii). using the series (A) for $J_\nu(x)$ with $\nu = 1/2$, we have, use

$$J_{\frac{1}{2}}(x) = \frac{(x/2)^{1/2}}{\Gamma(3/2)} - \frac{(x/2)^{5/2}}{\Gamma(5/2)} + \frac{(x/2)^{9/2}}{\Gamma(7/2)} - \cdots$$

& using $\Gamma\left(n+\tfrac{1}{2}\right) = \frac{(2n)!}{n!\,2^{2n}}\sqrt{\pi}$ & simplifying

$$J_{1/2}(x) = \frac{(2x)^{1/2}}{\sqrt{\pi}}\left\{1 - \frac{x^2}{2\cdot 3} + \frac{x^4}{2\cdot 3\cdot 4\cdot 5} - \cdots\right\}$$

$$= \sqrt{\frac{2}{\pi x}} \left\{ x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \right\} = \sqrt{\frac{2}{\pi x}} \sin x.$$

Q.5 Put $u = (x^2-1)^n$; $u' = 2nx(x^2-1)^{n-1}$

1st part: $(x^2-1)u' = 2nxu$

Differentiating $(n+1)$ times

$$(x^2-1)u^{(n+2)} + \binom{n+1}{1} 2x u^{(n+1)} + \binom{n+1}{2} 2u^{(n)}$$

$$= 2n \left\{ x u^{(n+1)} + \binom{n+1}{1} u^{(n)} \right\}$$

ie $(x^2-1)u^{(n+2)} + 2x u^{(n+1)} + \left\{ \frac{n(n+1)}{2} \cdot 2 - 2n(n+1) \right\} u^{(n)} = 0$

$$(x^2-1)u^{(n+2)} + 2x u^{(n+1)} - n(n+1)u^{(n)} = 0$$

Putting $u^{(n)} = 2^n \cdot n!\, y$, this becomes

$(x^2-1)y'' + 2xy' - n(n+1)y = 0$ ie the eqn satisfied by $y = \frac{1}{2^n \cdot n!} u^{(n)} = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n}(x^2-1)^n$ ie the Legendre polynomial.

2nd part.

Q.8. 2nd part

$$\frac{d^2y}{dx^2} + (2x+1)\frac{dy}{dx} + (x^2+x+\tfrac{1}{4})y = 0$$

$y'' + P_1 y' + P_2 y = 0$ reduces to

$u'' + \left\{ P_2 - \frac{1}{2} P_1' - \frac{1}{4} P_1^2 \right\} u = 0$ by putting $y = ue^{-\frac{1}{2}\int P_1 dx}$.

So in this case putting $y = ue^{-\frac{1}{2}\int (2x+1)\,dx} = ue^{-\frac{1}{2}(x^2+x)}$

$4x^2+4x+1$

the eqn reduces

$$u'' + \left\{ (x^2+x+\tfrac{1}{4}) - \tfrac{1}{2}\cdot 2 - \tfrac{1}{4}(2x+1)^2 \right\} u = 0$$

$$u'' + \left\{ x^2 + x + \tfrac{1}{4} - 1 - \tfrac{1}{4}(4x^2+4x+1) \right\} u = 0$$

$$u'' - u = 0$$

$u = Ae^x + Be^{-x}$.

$u' = Ae^x - Be^{-x}$

$u'' = Ae^x + Be^{-x}$

$= u$

$y = (Ae^x + Be^{-x})\, e^{-\frac{1}{2}(x^2+x)}$

$= Ae^{-\frac{1}{2}x(x-1)} + Be^{-\frac{1}{2}x(x+3/2)}$

$-\frac{1}{2}x^2 + \frac{1}{2}x$.

$-\frac{1}{2}x^2 - \frac{3}{2}x$

Q.1(c) Is the soln $y = c_1 x^{i-1} + c_2 x^{-(i+1)} + \frac{x^2}{10} + \frac{2x}{5}$ correct?

$$\frac{dy}{dx} = c_1(i-1)x^{i-2} + c_2 x^{-i-2}(-i-1) + \frac{2x}{5} + \frac{2}{5}$$

$$\frac{d^2y}{dx^2} = c_1(i-1)(i-2)x^{i-3} + c_2(i+1)(i+2)x^{-i-3} + \frac{1}{5}$$

$$x^2\frac{d^2y}{dx^2} = c_1(i-1)(i-2)x^{i-1} + c_2(i+1)(i+2)x^{-(i+1)} + \frac{x^2}{5}$$

$$3x\frac{dy}{dx} = 3c_1(i-1)x^{(i-1)} + 3c_2(-i-1)x^{-i-1} + \frac{3x^2}{5} + \frac{6x}{5}$$

$$2y = 2c_1x^{i-1} + 2c_2x^{-i-1} + \frac{x^2}{5} + \frac{4x}{5}$$

Adding $c_1x^{i-1}\{(i-1)(i-2)+3(i-1)+2\}$

$+c_2x^{-(i+1)}\{(i+1)(i+2)-3(i+1)+2\} + x^2 + 2x$

$c_1x^{i-1}\{-\cancel{i}-\cancel{3i}+2+\overset{0}{\cancel{3i}}-\cancel{3}+2\}$

$c_2x^{-(i+1)}\{0-1+\cancel{3i}+\overset{0}{\cancel{2}}-\cancel{3i}-\cancel{3}+2\}$ correct

Q.1(b), Is the P.I $y = \frac{1}{16}\sin 2x - \frac{1}{4}x\cos 2x$ correct?

$$\frac{dy}{dx} = \frac{1}{8}\cos 2x - \frac{1}{4}\cos 2x + \frac{1}{2}x\sin 2x$$

$$= -\frac{1}{8}\cos 2x + \frac{1}{2}x\sin 2x$$

$$\frac{d^2y}{dx^2} = -\frac{\cancel{\sin 2x}}{4} = \frac{1}{4}\sin 2x + \frac{1}{2}\sin 2x + x\cos 2x.$$

$$(D^2+4)y = \frac{3}{4}\sin 2x + x\cancel{\cos 2x} + \frac{16}{4}\sin 2x - x\cancel{\cos 2x} = \sin 2x$$ ✓

So P.I $y = -\frac{1}{4}x\cos 2x$ correct

$$Dy = -\frac{1}{4}\cos 2x + \frac{x}{2}\sin 2x$$

$$D^2y = \frac{1}{2}\sin 2x + \frac{1}{2}\sin 2x + x\cos 2x$$

$(D^2+4)y = \sin 2x + x\cancel{\cos 2x} - x\cancel{\cos 2x} = \sin 2x$ ✓

also correct

Q 1(c) is P.I $y = -\sin x - \cos x$ correct.

$Dy = -\cos x + \sin x$

$D^2y = \sin x + \cos x$

$(D^2+3D+2)y = \sin x + \cos x - 3\cos x + 3\sin x - 2\sin x - 2\cos x$

$= 2\sin x - 4\cos x$ (not correct)

Q. 1(b) $y = \frac{1}{4} x \sin 2x$ Correct as P.I.

$Dy = \frac{1}{4}\sin 2x + \frac{1}{2} x \cos 2x$

$D^2y = \frac{1}{2}\cos 2x + \frac{1}{2}\cos 2x - x\sin 2x = \cos 2x - x\sin 2x$

$(D^2+4)y = \cos 2x - x\sin 2x + x\sin 2x$

not correct

1(c) is $y = \frac{1}{4}\cos x + \frac{1}{2}\sin x$ correct as P.I

$Dy = -\frac{1}{4}\sin x + \frac{1}{2}\cos x$

$D^2y = -\frac{1}{4}\cos x - \frac{1}{2}\sin x$

$(D^2+3D+2)y = -\frac{1}{4}\cos x - \frac{1}{2}\sin x - \frac{3}{4}\sin x + \frac{3}{2}\cos x$

$+\frac{1}{2}\cos x + \sin x$ not correct

Q. 1 (b) is P.I $y = -\frac{x}{4}\sin 2x$ correct

$Dy = -\frac{1}{4}\cos 2x - \frac{1}{4}\sin 2x - \frac{x}{2}\cos 2x$

$D^2y = \frac{1}{2}\sin 2x - \frac{1}{2}\cos 2x - \frac{1}{2}\cos 2x + x\sin 2x$

$(D^2+4)y = \sin 2x - \sin 2x$ correct!

$= -\frac{1}{2}\cos 2x - \frac{1}{2}\cdot - \cos 2x + x\sin 2x$

$+ -\sin 2x - x\sin 2x$

not correct

Q.1(a) ~~Does~~ $y = c_1 e^{-x(1-i)} + c_2 e^{-x(1+i)}$ a correct C.F

$$Dy = c_1 e^{-x(1-i)} \cdot (i-1) + c_2 e^{-x(1+i)} \cdot (-i-1)$$

$$D^2y = c_1 e^{-x(1-i)} \cdot (i-1)^2 + c_2 e^{-x(1+i)} \cdot (i+1)^2$$

$$x^2D^2y = c_1 x^2 e^{-x(1-i)} (i-1)^2 + c_2 x^2 e^{-x(1+i)} (i+1)^2$$

$$3xDy = 3c_1 x e^{-x(1-i)} (i-1) + 3c_2 x -$$

obviously incorrect

Q.1(b) Does $y = +\frac{1}{4}\cos 2x$ a correct P.I

$$Dy = -\frac{1}{2}\sin 2x$$

$$D^2y = -\cos 2x$$

$$D^2y + 4y = -\cos 2x$$

Does $+\frac{1}{4}x\cos 2x$ do?

$$y = \tfrac{1}{4}x\cos 2x$$

$$Dy = \tfrac{1}{4}\cos 2x - \tfrac{1}{2}x\sin 2x$$

$$D^2y = -\frac{1}{2}\sin 2x - \frac{1}{2}\sin 2x - x\cos 2x = -\sin 2x - x\cos 2x$$

$$D^2y + 4y = -\sin 2x - x\cos 2x + x\cos 2x$$ — no!

Q.1(b) $\frac{1}{8i} xe^{2ix} - \frac{1}{8i} xe^{-2ix} = \frac{x}{8i}\{\cos 2x + i\sin 2x - \cos 2x + i\sin 2x\}$

$$= \frac{x}{4i}\cos 2x = -\frac{ix}{4}\cos 2x$$

$$y = -\frac{ix}{4}\cos 2x$$

$$Dy = -\frac{i}{4}\cos 2x + \frac{ix}{2}\sin 2x$$

$$D^2y = \frac{i}{2}\sin 2x + \frac{i}{2}\sin 2x + ix\cos 2x$$

$$D^2y + 4y = i\sin 2x + ix\cos 2x - ix\cos 2x$$

Is $y = -\frac{1}{4i}xe^{-2ix}$ a P.I?

$$Dy = -\frac{1}{4i}e^{-2ix} - \frac{1}{4i}x\cdot e^{-2ix}(-2i)$$

$$= -\frac{1}{4i}e^{-2ix} + \frac{1}{2}xe^{-2ix}$$

$$D^2y = \frac{1}{2}e^{-2ix} + \frac{1}{2}e^{-2ix} - ixe^{-2ix} = e^{-2ix} - ixe^{-2ix}$$

$$D^2y + 4y = e^{-2ix} - ixe^{-2ix} + ixe^{-2ix} \neq \sin 2x$$

no

Meteorology & Oceanography

Paper I – Mathematics

1(a) Area of $\triangle$ formed by $P(1,3,2)$, $Q(2,-1,\frac{1}{2})$, $R(-1,2,3)$

Vol of tetrahedron $= OPQR = \frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & -1 & \frac{1}{2} & 1 \\ -1 & 2 & 3 & 1 \end{vmatrix}$

$$= -\frac{1}{6}\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & +1 \\ -1 & 2 & 3 \end{vmatrix} = -\frac{1}{6}\left[1(-3-2) - 3(6+1) + 2(4-1)\right]$$

$$= -\frac{1}{6}\left[-5 - 21 + 6\right] = -\frac{1}{6}(-20) = \frac{10}{3}.$$

Eqn to plane PQR is

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 3 & 2 & 1 \\ 2 & -1 & -1 & 1 \\ -1 & 2 & 3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 3 & 2 & 1 \\ 1 & -4 & -3 & 0 \\ -2 & -1 & 1 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-3 & z-2 & 0 \\ 1 & 3 & 2 & 1 \\ 1 & -4 & -3 & 0 \\ -2 & -1 & 1 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-3 & z-2 \\ 1 & -4 & -3 \\ -2 & -1 & 1 \end{vmatrix} = 0$$

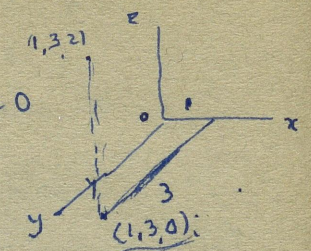
$$(x-1)(-7) - (y-3)(-5) + (z-2)(-9)$$

$$7(x-1) - 5(y-3) + 9(z-2)$$

$$7x - 5y + 9z = 10$$

$$p = \frac{10}{\sqrt{5^2+7^2+9^2}} = \frac{10}{\sqrt{25+49+81}} = \frac{10}{\sqrt{155}}$$

Plane PQR is $\begin{vmatrix} x & y & z & 1 \\ 1 & 3 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ -1 & 2 & 3 & 1 \end{vmatrix} = 0$



$$x\begin{vmatrix} 3 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} - y\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ -1 & 3 & 1 \end{vmatrix} + z\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ -1 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ -1 & 2 & 3 \end{vmatrix} = 0.$$

$$x\{3(1-3) - 2(-1-2) + 1(-3-2)\} - y\{1(1-3) - 2(2+1) + 1(6+1)\}$$
$$-6 \quad +6 \quad -5 \qquad -2 \quad -6 \quad +7$$
$$+z\{1(-1-2) - 3(2+1) + 1(4-1)\} - \{1(-3-2) - 3(6+1) + 2\{4-1\}\} = 0$$
$$-3 \quad -9 \quad +3 \qquad -5 \quad -21 \quad +6$$

$$-5x + y - 9z + 20 = 0$$
$$5x - y + 9z = 20.$$

$$p = \perp^r \text{ from } 0 \text{ on the plane} = \frac{20}{\sqrt{5^2+1^2+9^2}} = \frac{20}{\sqrt{107}}$$

25
81
118

$\frac{1}{\beta}$ p. area $= \frac{10}{\beta}$ area $= \frac{10}{20/\sqrt{107}}$

$= \frac{1}{2}\sqrt{107}$

Betts perhaps as candidate No. 987 has done to consider projections

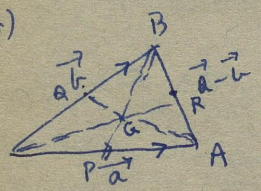
$\Delta_x, \Delta_y, \Delta_z$ of ΔPQR on planes yz, zx, xy i.e.

i.e. coords of Δ_x are $(0, 3, 2), (0, -1, 1), (0, 2, 3)$

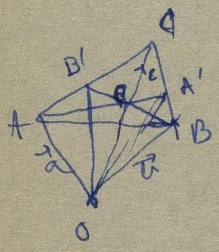
$$\therefore \Delta_x = \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} \quad \text{similarly } \Delta_y \text{ & } \Delta_z$$

$\Delta = \sqrt{\{\Delta_x^2 + \Delta_y^2 + \Delta_z^2\}} = \frac{1}{2}\sqrt{107}$ as obtained by tetrahedron method

(b)



$\frac{2}{3}\vec{a'} + \frac{1}{3}\vec{a} = \frac{2}{3}\vec{b'} + \frac{1}{3}\vec{b} = \frac{2}{3}\vec{c'} + \frac{1}{3}\vec{c} = \vec{p}$



$\vec{OA'} = \vec{a'}$, $\vec{OB'} = \vec{b'}$

$\vec{a'} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c}$ $\qquad \frac{1}{2}\vec{c} = \vec{a'} - \frac{1}{2}\vec{b}$ R

$\vec{b'} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{c}$ $\qquad = \vec{b'} - \frac{1}{2}\vec{a}$

i.e. $\vec{a'} + \frac{1}{2}\vec{a} = \vec{b'} + \frac{1}{2}\vec{b} = \vec{p}$.

Similarly $\vec{b'} + \frac{1}{2}\vec{b} = \vec{c'} + \frac{1}{2}\vec{c}$

i.e. $2\vec{b'} + \vec{b} = 2\vec{a'} + \vec{a}$

$\frac{2}{3}\vec{a'} + \frac{1}{3}\vec{a} = \frac{2}{3}\vec{b'} + \frac{1}{3}\vec{b} = \frac{2}{3}\vec{c'} + \frac{1}{3}\vec{c}$

$= \vec{p}$

$\vec{p} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$

or take P as dividing AA' or BB' in ratio 2:1 & similarly CC'

2(a) $f(x,y,z) = 2xz^2 - 3xy - 4x - 7 = 0$

Tangent plane at $(1,-1,2)$, [the point lies for $f=0$ since $2.1.2^2 - 3.1(-1) - 4.1 - 7 = 8 + 3 - 4 - 7 = 0$]

is $(x-1)\left(\frac{\partial f}{\partial x}\right)_{1,-1,2} + (y+1)\left(\frac{\partial f}{\partial y}\right)_{1,-1,2} + (z-2)\left(\frac{\partial f}{\partial z}\right)_{1,-1,2} = 0$

$(x-1)7 + (y+1)3 + (z-2)8 = 0$

$7x + 3y + 8z = 26$ $\quad 7+3+16$

$\frac{\partial f}{\partial x} = 2z^2 - 3y - 4$

$\frac{\partial f}{\partial y} = -3x$

$\frac{\partial f}{\partial z} = 4xz.$

(b) (i) $\text{curl curl } \vec{A} = \text{grad div } \vec{A} - \nabla^2 \vec{A}$

$\nabla \times \nabla \times \vec{A}$ being a vector having two ∇'s the only possibilities are $\nabla(\nabla \cdot \vec{A})$ and $(\nabla \cdot \nabla)\vec{A}$. A combination of the terms in

$\nabla \times \nabla \times A = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ as can be verified using rect. coords.

Alternatively

$$\text{curl}_x(\text{curl } A) = \frac{\partial}{\partial y}(\text{curl}_z A) - \frac{\partial}{\partial z}(\text{curl}_y A)$$

$$= \frac{\partial}{\partial y}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$$

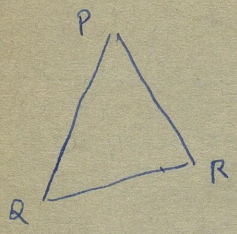
$$= \frac{\partial}{\partial x}(\text{div } \vec{A}) - \nabla^2 A_x$$

$\therefore \text{curl curl } \vec{A} = \text{grad div } \vec{A} - \nabla^2 \vec{A}$

$\left(\frac{\partial^2 A_y}{\partial x \partial y} + \frac{\partial^2 A_z}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial x^2}\right)$

$\left(-\frac{\partial^2 A_x}{\partial x^2} - \frac{\partial^2 A_x}{\partial y^2} - \frac{\partial^2 A_x}{\partial z^2}\right)$

Q. 1(a) 3rd method $P(1,3,2)$, $Q(2,-1,1)$, $R(-1,2,3)$



$$PQ = \sqrt{\{(1-2)^2+(3+1)^2+(2-1)^2\}} = \sqrt{18} = 3\sqrt{2}$$

$$PR = \sqrt{\{(1+1)^2+(3-2)^2+(2-3)^2\}} = \sqrt{6}.$$

D.C's of PQ ~~[illegible]~~ $= \frac{-1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}$

" $PR = \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$

$$\cos QPR = -\frac{-2}{3\sqrt{12}} + \frac{4}{3\sqrt{12}} - \frac{1}{3\sqrt{12}} = \frac{1}{3\sqrt{12}} = \frac{1}{6\sqrt{3}}$$

$$\sin QPR = \sqrt{1-\frac{1}{108}} = \sqrt{\frac{107}{108}} = \frac{\sqrt{107}}{6\sqrt{3}}$$

$9\times4\times3$

$$\text{area} = \frac{1}{2}\cdot 3\sqrt{2}\cdot\sqrt{6}\cdot\frac{\sqrt{107}}{6\sqrt{3}} = \frac{1}{2}\cdot\frac{6\sqrt{3}}{6\sqrt{3}}\cdot\sqrt{107} = \frac{1}{2}\sqrt{107}$$

i.e. same answer as in other two methods

(ii) If $\vec{v} = \vec{\omega}\times\vec{r}$, to prove that $\vec{\omega} = \frac{1}{2}\nabla\times\vec{v}$

$$\nabla\times\vec{\omega}\times\vec{r} = \vec{\omega}\,\text{div}\,\vec{r} - \vec{r}\,\cancel{\text{div}\,\vec{\omega}} + (\vec{r}\,\cancel{\text{grad}})\vec{\omega} - (\vec{\omega}\,\text{grad})\vec{r}$$

$$= \vec{\omega}\,\text{div}\,\vec{r} - (\vec{\omega}\,\text{grad})\vec{r}$$

$$(\vec{\omega}\,\text{grad})\vec{r} = \vec{i}\left\{\frac{\partial x}{\partial x}\omega_x + \frac{\partial y}{\partial x}\omega_y + \frac{\partial z}{\partial x}\omega_z\right\} + \vec{j}\{\quad\} + \vec{k}$$

$$= (i\omega_x + j\omega_y + k\omega_z) = \vec{\omega}$$

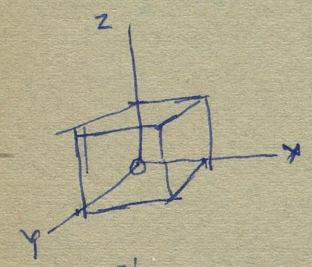
$\text{div}\,\vec{r} = 3 \quad \therefore \quad \nabla\times\vec{\omega}\times\vec{r} = 3\vec{\omega} - \vec{\omega} = 2\vec{\omega}$

or $\frac{1}{2}\nabla\times\vec{v} = \vec{\omega}$. $\because \vec{\omega} = \text{const}$, 2nd & 3rd terms vanish.//

3(a) Bookwork

(b) Integral is, by Gauss's theorem equal to

$$\iiint_V \operatorname{div} \vec{F}\, d\tau$$ where V is the volume bounded by $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$



$$= \int_0^1\int_0^1\int_0^1 (4z - 2y + y)\, dx\,dy\,dz.$$

$$= \int_0^1\int_0^1 (4z-y)\, dy\,dz = \int_0^1 \left(4z - \tfrac{1}{2}\right) dz$$

$$= 2z^2 - z/2\Big]_0^1$$

$$= 2 - \tfrac{1}{2} = 3/2$$

$4yz - \frac{1}{2}y^2\Big]_0^1$

$4z - \frac{1}{2}$

4(a) Dirichet's conditions:

If $f(x)$ is defined in the range $(-\pi, \pi)$ and $\int_{-\pi}^{\pi} |f(t)|\, dt$ exists & further if $f(x)$ is defined by the equation

$$f(x+2\pi) = f(x)$$

outside the range $(-\pi, \pi)$, then provided that

$$\pi a_n = \int_{-\pi}^{\pi} f(t)\cos nt\, dt \quad \& \quad \pi b_n = \int_{-\pi}^{\pi} f(t)\sin nt\, dt$$

the series $\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ converges to the sum $\frac{1}{2}\{f(x+0) + f(x-0)\}$ provided that these limits exist and that $f(x)$ has limited total fluctuation in an interval of finite

length of which x is an internal point.

(d) or more simply

A function continuous except at a finite number of ordinary discontinuities and has only a finite no. of maxima & minima.

(b) To expand $f(x) = \cos x$ in $0 \le x \le \pi$ in a Fourier sine series (Half range Fourier series)

Assuming $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$.

$$\text{with } b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \cos x \sin nx\,dx$$

$\cos x \sin nx$

$$= \frac{1}{2}\{\sin(nx+x) + \sin(nx-x) = \frac{1}{2}\{\sin(n+1)x + \sin(n-1)x\}$$

$$= \frac{1}{2}\{\sin nx\cos x + \cos nx \sin x + \sin nx \cos x - \cos nx \sin x\}$$

$$= \frac{2}{\pi}\left\{-\frac{\cos(n+1)x}{(n+1)} - \frac{\cos(n-1)x}{n-1}\right\}$$

$$= -\frac{2}{\pi}\left\{\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right\}_0^{\pi}$$

$$\text{When } n=1, \quad = -\frac{2}{\pi}\left\{\frac{\cos 3x}{3} + \frac{\cos x}{1}\right\}_0^{\pi} = -\frac{2}{\pi}\left[\right.$$

$$= -\frac{2}{\pi}\left[\frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} - \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= -\frac{1}{\pi}\left[\frac{\cos n\pi\cos\pi + \sin n\pi\sin\pi}{n+1} + \frac{\cos n\pi\cos\pi + \sin n\pi\sin\pi}{n-1} - \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= -\frac{1}{\pi}\left[-\frac{(-1)^n}{n+1} - \frac{(-1)^n}{n-1} - \frac{2n}{n^2-1}\right]$$

$$= \frac{1}{\pi}\left[(-1)^n\cdot\frac{2n}{n^2-1} + \frac{2n}{n^2-1}\right] = \frac{1}{\pi}\left[\frac{(-1)^{n+1}}{n^2-1} + \frac{n}{n^2-1}\right]$$

If $n=1$, this $= \frac{4}{\pi}\left[\frac{n+(-1)^{n+1}}{n^2-1}\right]_{n=1} = \frac{4}{\pi}\cdot\frac{2n}{\pi}$

$$= \frac{2n}{\pi(n^2-1)}\left\{1+(-1)^n\right\}$$

$$b_1 = \frac{2}{\pi}\int_0^{\pi}\cos x\sin x\,dx = \frac{1}{\pi}\int_0^{\pi}\sin 2x\,dx \qquad (b)$$

$$= -\frac{1}{2\pi}\cos 2x\Big]_0^{\pi} = 0.$$

Hence for $n>1$, $b_n = \frac{2n}{\pi(n^2-1)}\left\{1+(-1)^n\right\}$ i.e. $b_n = 0$ if n is odd

$$= \frac{4n}{\pi(n^2-1)} \text{ if } n \text{ is even}$$

5(a) $V = \iiint dx\,dy\,dz.$ $= 2a\iint\sqrt{1-y^2/b^2-z^2/c^2}\cdot dy\,dz$

$$= 2a\int_{-b}^{b}dz\int\sqrt{(1-z^2/c^2)-y^2/b^2}\,dy$$

Substitute $x = aX$, $y = bY$, $z = cZ$

$$dx\,dy\,dz = \frac{\partial(x,y,z)}{\partial(X,Y,Z)}\,dX\,dY\,dZ$$

$= \frac{128}{3}$

with this substitution ellipsoid $\rightarrow X^2+Y^2+Z^2=1$.

$$\iiint dx\,dy\,dz = \iiint \frac{\partial(x,y,z)}{\partial(X,Y,Z)}\,dX\,dY\,dZ.$$

& the Jacobian $= \begin{vmatrix} \partial x/\partial X & \partial x/\partial Y & \partial x/\partial Z \\ \partial y/\partial X & \partial y/\partial Y & \partial y/\partial Z \\ \partial z/\partial X & \partial z/\partial Y & \partial z/\partial Z \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} \equiv abc.$

Hence volume of ellipsoid $= abc \cdot$ Vol of unit sphere.

$= \cancel{\tfrac{4}{3}}\, abc \cdot \frac{4}{3}\pi = \frac{4}{3}\pi abc$

(b) ~~Area required~~ = Vol required

$$= \iint (4-x^2-y^2)\,dx\,dy \longrightarrow$$

taken over the circle $x^2+y^2=4$

$$= \int dx \int_{-\sqrt{4-x^2}}^{\sqrt{(4-x^2)}} (4-x^2-y^2)\,dy$$

$$= \int dx \left[(4-x^2)y - \frac{y^3}{3}\right]_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}$$

$$= \int dx \left[2(4-x^2)^{3/2} - \frac{1}{3}(4-x^2)^{3/2}\right]$$

$$= 2\int_0^2 dx \cdot \frac{4}{3}(4-x^2)^{3/2}$$

$$= 2\int_0^{\pi/2} 2\cos\theta \cdot d\theta \cdot \frac{4}{3} \cdot 8\cos^3\theta = \frac{128}{3}\int_0^{\pi/2} \cos^4\theta \cdot d\theta$$

$$= \frac{128}{\cancel{3}} \cdot \frac{\cancel{3}}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = 8\pi$$

$$\iint_0^{2\pi} (4-r^2)\,r\,d\theta\,dr$$

$$2\pi \int_0^2 (4r-r^3)\,dr$$

$$= 2\pi\left[2r^2 - \frac{r^4}{4}\right]_0^2$$

$$= 2\pi[8-4]$$

$$= 8\pi$$

$y^2 = 4-x^2$

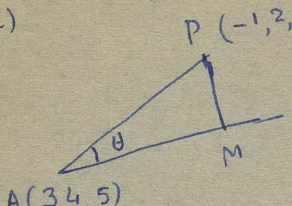
$y = \pm\sqrt{4-x^2}$

This polar method is far simpler than Wartikars

$x = 2\sin\theta \cdot (4-4\sin^2\theta)^{3/2}$ $\quad (4\cos^2\theta)^{3/2}$ $\quad 2\cos\theta$

Section B. 4, 2, 0 (24)

6(a)



1st method: D.c'ns of AP are $\left(\frac{4}{\sqrt{20}}, \frac{2}{\sqrt{20}}, 0\right)$

i.e. $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$

D.c'ns of AM are $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

$\therefore \cos\theta = \frac{4}{7\sqrt{5}} - \frac{3}{7\sqrt{5}} + 0 = \frac{1}{7\sqrt{5}}$.

$AP = \sqrt{\{4^2 + 2^2\}} = \sqrt{20}$.

$\sin\theta = \sqrt{1 - \frac{1}{245}}$

$= \sqrt{\frac{244}{245}} = \frac{2\sqrt{61}}{7\sqrt{5}}$.

$PM = \frac{\sqrt{20} \cdot 2\sqrt{61}}{7\sqrt{5}} = \frac{4\sqrt{61}}{7}$. ✓

2nd method: Coords of any point M on $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-5}{6} = k$

are $2k+3, -3k+4, 6k+5$

The D.c'ns of P.M are ∥l to $(2k+3+1), (-3k+4-2), (6k+5-5)$

i.e. $2k+4, -3k+2, 6k$

Since PM is ⊥r to AM, $2(2k+4) - 3(-3k+2) + 6(6k) = 0$

$4k + 8 + 9k - 6 + 36k = 0$

$49k = -2$ or $k = -\frac{2}{49}$.

$\therefore$ P is $\left(\frac{116}{49}, \frac{22}{49}, \frac{142}{49}\right)$

$PM^2 = \left(\frac{116}{49} + 1\right)^2 + \left(\frac{22}{49} - 2\right)^2 + \left(\frac{142}{49} - 5\right)^2$

$$= \left(\frac{165}{49}\right)^2 + \left(\frac{76}{49}\right)^2 + \left(\frac{103}{49}\right)^2$$

165×165

$$\text{ie } PM = \frac{\sqrt{(165)^2 + (76)^2 + (103)^2}}{49}.$$

$\therefore$ M is $\left(\frac{116}{49} + 3,\ -\frac{174}{49} + 2,\ \frac{348}{49} - 5\right)$

$$\text{ie } \left(\frac{263}{49},\ -\frac{76}{49},\ \frac{103}{49}\right)$$

$$PM^2 = \left(\frac{263}{49} + 1\right)^2 + \left(-\frac{76}{49} - 2\right)^2 + \left(\frac{103}{49} - 5\right)^2$$

$$= \left(\frac{312}{49}\right)^2 + \left(\frac{174}{49}\right)^2 + \left(\frac{142}{49}\right)^2$$

$$\therefore PM = \frac{2}{49}\sqrt{\{(156)^2 + (87)^2 + (71)^2\}}$$

$$\begin{array}{r} 116 \\ 147 \\ \hline 263 \end{array} \qquad \begin{array}{r} 174 \\ 98 \\ \hline 76 \end{array} \qquad \begin{array}{r} 348 \\ 245 \\ \hline 103 \end{array} \qquad 4$$

$$\begin{array}{r} 76 \\ 98 \\ \hline 174 \end{array} \qquad \begin{array}{r} 245 \\ 103 \\ \hline 142 \end{array}$$

$$\begin{array}{r} 263 \\ 249 \\ \hline 312 \end{array}$$

$$\begin{array}{r} 87 \times 87 \\ \hline 609 \\ 696 \\ \hline 7569 \end{array} \qquad \begin{array}{r} 71 \times 71 \\ \hline 71 \\ 497 \\ \hline 5041 \end{array} \qquad \begin{array}{r} 156 \times 156 \\ \hline 936 \\ 780 \\ 156 \\ \hline 24336 \\ 7569 \\ 5041 \\ \hline 36946 \end{array}$$

$$\begin{array}{r} 254 \times 254 \\ \hline 1016 \\ 1270 \\ 508 \\ \hline 64516 \\ 7569 \\ 5041 \\ \hline 77126 \end{array}$$

$\frac{8}{7}$

$\frac{2}{7} \cdot 4 - \frac{3}{7} \cdot 2$

$\frac{2}{7}$

$20 - \frac{4}{49}$

$4\left(5 - \frac{1}{49}\right)$

$20 - \frac{4}{49}$

$4\left(5 - \frac{1}{49}\right)$

$4\sqrt{\frac{215}{49}}$

looks ridiculously troublesome

$K = -2/49$

M is $(2K+3, -3K+4, 6K+5)$ with $K = -2/49$

$$\left(-\frac{4}{49}+3, \frac{6}{49}+4, \frac{-12}{49}+5\right)$$

$$\begin{array}{r} 147 \\ 4 \\ \hline 196 \\ 6 \end{array}$$

ie $\left(\frac{143}{49}, \frac{202}{49}, \frac{233}{49}\right)$

$$\begin{array}{r} 245 \\ 12 \\ \hline 233 \end{array}$$

$$PM^2 = \left(\frac{143}{49}+1\right)^2 + \left(\frac{202}{49}-2\right)^2 + \left(\frac{233}{49}-5\right)^2$$

$$\begin{array}{r} 143 \\ 49 \\ \hline 192 \end{array}$$

$$= \left(\frac{192}{49}\right)^2 + \left(\frac{104}{49}\right)^2 + \left(\frac{12}{49}\right)^2$$

$$\begin{array}{r} 202 \\ 98 \\ \hline 104 \end{array} \qquad \begin{array}{r} 233 \\ 245 \\ \hline 12 \\ 6 \end{array}$$

$$\therefore PM = \frac{4}{49}\sqrt{48^2+26^2+3^2}$$

$$\begin{array}{r} 26 \times 26 \\ \hline 156 \\ 52 \\ \hline 676 \end{array}$$

$$= \frac{4}{49}\sqrt{49 \times 61}$$

$$\begin{array}{r} 48 \times 48 \\ \hline 384 \\ 192 \\ \hline 2304 \\ 676 \\ 9 \\ \hline 2989 \end{array}$$

$$= \frac{4}{7}\sqrt{61}$$ same as before

obviously my method is preferable to the Wartikar's method!

427
61

6(b) Eqn of a plane through the line $\frac{x-\alpha}{l} = \cdot =$ is given by

$$a(x-\alpha) + b(y-\beta) + c(z-\gamma) = 0 \quad \text{—(1)}$$

with the condition $al + bm + cn = 0$ —(2)

If the plane (1) is $\parallel$ to the line $\frac{x}{l'} = \frac{y}{m'} = \frac{z}{n'}$

$$al' + bm' + cn' = 0 \qquad \text{— (3)}$$

Eliminating $a : b : c$ between (1), (2) & (3) the eqn of plane is

$$\begin{vmatrix} x-\alpha & y-\beta & z-\gamma \\ l & m & n \\ l' & m' & n' \end{vmatrix} = 0.$$

[Warrikar, p.246]

7(a) $S.D = \begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ l & m & n \\ l' & m' & n' \end{vmatrix} \div D = d$ say

where $D = \sqrt{\{(mn'-m'n)^2 + \dots + \dots\}}$.

$D = \sin\theta$ where θ is angle between the two straight lines

ie $d \sin\theta = \begin{vmatrix} & \\ & \end{vmatrix}$ [Warrikar, 27

(b) If P be (x, y, z), Vol of tetrahedron PABC

$$V = \frac{1}{6} \begin{vmatrix} x & y & z & 1 \\ 3 & 2 & 1 & 1 \\ -2 & 0 & -3 & 1 \\ 0 & 0 & -2 & 1 \end{vmatrix}$$

Locus is given by $V = 5$

~~Expanding the determinant in terms of~~

Subtracting the 4th row from the 3rd

$$V = \frac{1}{6}\begin{vmatrix} x & y & z & 1 \\ 3 & 2 & 1 & 1 \\ -2 & 0 & -3 & 1 \end{vmatrix}$$

Expanding in terms of elements of last row

$$V = \frac{1}{6}\left\{-2\begin{vmatrix} x & y & 1 \\ 3 & 2 & 1 \\ -2 & 0 & 1 \end{vmatrix} - 1\begin{vmatrix} x & y & z \\ 3 & 2 & 1 \\ -2 & 0 & -3 \end{vmatrix}\right\} = 5$$

4.2

$$= \frac{1}{6}\left\{2(2x - 5y + 4) + 1(-6x + 7y - 4z)\right\} = 5$$

$$4x - 10y + 8 - 6x + 7y - 4z = -30$$

$$-2x - 3y - 4z = -38, \quad 2x + 3y + 4z = 38$$ ✓

8 (a) A sphere passing through the given circle has the eqn

$$(x^2 + y^2 + z^2 - 5) + \lambda(x + 2y + 3z - 3) = 0$$

The condition that the plane $4x + 3y = 15$ touches the sphere is $x^2 + y^2 + z^2 + \lambda x + 2\lambda y + 3\lambda z - (3\lambda + 5) = 0$

is $(2\lambda+3\lambda)^2 = (4^2+3^2)\left(\frac{\lambda^2}{4}+\lambda^2+3\lambda+5\right)$

$49\lambda^2 = 25\left(\frac{5\lambda^2}{4}+3\lambda+5\right)$

$196\lambda^2 = 25(5\lambda^2+12\lambda+20)$

$71\lambda^2 - 300\lambda - 500 = 0.$

196
125
71

$(2\lambda+3\lambda+15)^2 = (4^2+3^2)\left(\frac{\lambda^2}{4}+\lambda^2+3\lambda+5\right)$
$+9\lambda^2/4$. [Wardlaw, p.291]

$(5\lambda+15)^2 = 5^2\left(\frac{15\lambda^2}{4}+3\lambda+5\right)$

$4(\lambda+3)^2 = 15\lambda^2+12\lambda+20$

$4\lambda^2+24\lambda+36 = 15\lambda^2+12\lambda+20$

$10\lambda^2 - 12\lambda - 16 = 0$

$5\lambda^2 - 6\lambda - 8$

$5\lambda^2 - 10\lambda + 4\lambda - 8 = 0$

$5\lambda(\lambda-2)+4(\lambda-2) = 0$

$\lambda = 2, -4/5$

$\lambda = \frac{12 \pm \sqrt{144+64}}{2}$

$= \frac{12 \pm \sqrt{208}}{2} = \frac{12 \pm 4\sqrt{13}}{2}$

$= 6 \pm 2\sqrt{13}.$

∴ The spheres are $x^2+y^2+z^2-5+2(x+2y+3z-3)=0$

$5(x^2+y^2+z^2-5)-4(x+2y+3z-3)=0$

$x^2+y^2+z^2+2x+4y+6z-11=0$, $5x^2+5y^2+5z^2-4x-8y-12z-13=0$

8(b) Wartikar Ex. 9, p. 300.

~~Let us~~ Taking constant radius as r instead of b

If the intercepts on the axes be a, b, c, eqn of sphere $OABC$

$$x^2+y^2+z^2-ax-by-cz=0 \quad \text{——(1)}$$

& eqn of plane ABC is $x/a + y/b + z/c = 1$ ——(2)

Since radius of sphere = const r,

$$a^2+b^2+c^2 = 4r^2 \quad \text{——(3)}$$

Eqns of the $\perp^r$ from O to (2) is

$$ax = by = cz = (k) \text{ say} \quad \text{——(4)}$$

If (x', y', z') be the foot of this $\perp^r$

$$\frac{x'}{a} + \frac{y'}{b} + \frac{z'}{c} = 1 \quad \text{——(5)}$$

$$\text{and } ax' = by' = cz' = k. \quad \text{——(6)}$$

Eliminating a, b, c ~~between (3), (5) & (6)~~ given by (6) from (3) & (5)

$$\frac{1}{k}(x'^2+y'^2+z'^2) = 1$$

$$\text{and } k^2(x'^{-2}+y'^{-2}+z'^{-2}) = 4r^2$$

Eliminating between the last two eqns gives

$$(x'^2+y'^2+z'^2)^2(x'^{-2}+y'^{-2}+z'^{-2}) = 4r^2$$

giving the reqd locus.

[Too complicated Ex for a part of a qn.

9(a) The cone $xy + yz + zx = 0$ passes through the coordinate axes. If (l', m', n') & (l'', m'', n'') be two generators of the cone intersected by the plane $ax + by + cz = 0$

$$\left.\begin{aligned} al + bm + cn &= 0 \\ al' + bm' + cn' &= 0. \end{aligned}\right\}$$

and $\frac{1}{l'} + \frac{1}{m'} + \frac{1}{n'} = \frac{1}{l''} + \frac{1}{m''} + \frac{1}{n''} = 0.$

$a : b : c = (mn' - m'n) : \dots : \dots$

If generators be $\perp^r$ $ll' + mm' + nn' = 0.$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{mn' - m'n} + \dots + \dots$$

$ax + by + cz = 0$

$yz + zx + xy = 0.$ or $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0.$

$$\left(\frac{1}{l} + \frac{1}{m} + \frac{1}{n}\right)\left(\frac{1}{l'} + \dots\right) = 0.$$

$$\frac{1}{ll'} + \frac{1}{lm'} + \frac{1}{ln'} + \frac{1}{ml'} + \dots = 0$$

?

$$\left.\begin{aligned} al + bm + cn &= 0 \\ \frac{1}{l} + \frac{1}{m} + \frac{1}{n} &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} \frac{al}{m} + b + \frac{cn}{m} &= 0 \\ 1 + \frac{l}{m} + \frac{l}{n} &= 0 \end{aligned}\right\} \frac{l}{m} = -1 - \frac{l}{n}$$

$$-a\left(1 + \frac{l}{n}\right) + b + \frac{cn}{m} = 0$$

~~$am - \frac{alm}{n} + bm + ln = 0$~~

9(b) Wartikar, p. 334

Let the plane touch the ellipsoid at (x_1, y_1, z_1)

Tangent plane at (x_1, y_1, z_1) is $a^2 x x_1 + b^2 y y_1 + c^2 z z_1 = 1$

Comparing with $lx + my + nz = p$

$$\frac{a^2 x_1}{l} = \frac{b^2 y_1}{m} = \frac{c^2 z_1}{n} = \frac{1}{p}.$$

$$x_1 = \frac{l}{a^2 p}, \; y_1 = \frac{m}{b^2 p}, \; z_1 = \frac{n}{c^2 p}.$$

Since (x_1, y_1, z_1) lies on ellipsoid

$$a^2 x_1^2 + b^2 y_1^2 + c^2 z_1^2 = 1$$

i.e. $\dfrac{a^2 l^2}{a^4 p^2} + \cdots = 1$ i.e. $\dfrac{l^2}{a^2} + \dfrac{m^2}{b^2} + \dfrac{n^2}{c^2} = p^2$

is the req'd cond^n

10(a) $\cos a = \cos b \cos c + \sin b \sin c \cos A$ [from G.V.R Appendix]

$\therefore \cos A = (\cos a - \cos b \cos c) / \sin b \sin c$

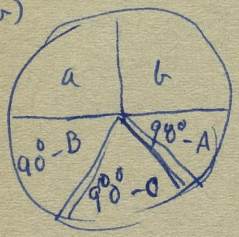
$$\sin^2 A = 1 - \cos^2 A = 1 - \frac{(\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c} = \frac{\sin^2 b \sin^2 c - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c}$$

$$= \frac{(1 - \cos^2 b)(1 - \cos^2 c) - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c} = \frac{1 - \cos^2 a + 2 \cos a \cos b \cos c}{\sin^2 b \sin^2 c}$$

i.e. $\frac{\sin^2 A}{\sin^2 a} = \frac{1 - \Sigma \cos^2 a + 2 \cos a \cos b \cos c}{\sin^2 a \sin^2 b \sin^2 c}$?.

R.H.S is symmetric in a, b, c & hence ch.

(b)



Napier's analogies

Sine of middle = product of tangents of adjacents

Sine of middle = product of cosines of others

$C = \pi/2$

$\cos C = \cos a \cos b$

$\cos C = \cos a \cos b$

$\tan^2 \frac{1}{2} C = \frac{1 - \cos a \cos b}{1 + \cos a \cos b}$

$\therefore \cos a = \frac{\cos C}{\cos b}$.

$\frac{1 - \cos a}{1 + \cos a} = \frac{1 - \cos C / \cos b}{1 + \cos C / \cos b}$

$\tan^2 \frac{a}{2} = \frac{\cos b - \cos B}{\cos b + \cos B} = \frac{2 \sin \frac{1}{2}(b + c) \sin \frac{1}{2}(c - b)}{2 \cos \frac{1}{2}(b + c) \cos \frac{1}{2}(b - B)}$

$= \tan \frac{1}{2}(b + c) \tan \frac{1}{2}(c - b)$ [$\because \cos \frac{1}{2}(b - c) = \cos \frac{1}{2}(c - b)$

$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

Only 8(a) left

8(a) The D.C's of the two generators lying on $ax + by + cz = 0$

Satisfy $al + bm + cn = 0$ —— (1)

$mn + nl + lm = 0$ —— (2)

Eliminate $l = -(bm+cn)/a$ from (1)

$$mn + \frac{n(bm+cn)}{a} - \frac{m(bm+cn)}{a} = 0$$

$$amn - bmn - cmn - cn^2 - bm^2 = 0$$

or $$(a-b-c)\frac{m}{n} - c - b\left(\frac{m^2}{n^2}\right) = 0$$

or $$b\left(\frac{m}{n}\right)^2 - (a-b-c)\frac{m}{n} + c = 0.$$

If roots of this be $\frac{m_1}{n_1}$ & $\frac{m_2}{n_2}$, then

$$\frac{m_1 m_2}{n_1 n_2} = \frac{c}{b}, \quad \text{Simily } \frac{l_1 l_2}{n_1 n_2} = \frac{c}{a}$$

Hence $$\frac{m_1 m_2}{n_1 n_2} + \frac{l_1 l_2}{n_1 n_2} + 1 = \frac{c}{b} + \frac{c}{a} + 1$$

ie the two functions are $\perp^r$ $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$ ie

$$\frac{c}{b} + \frac{c}{a} + 1 = 0$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \quad \checkmark$$

Andhra App. Physics

Paper I – Mathematics I

1(a) characteristic eqn of

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

is $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$

$(2-\lambda)^2 = 4 - 4\lambda + \lambda^2$

$(4-4\lambda+\lambda^2)(2-\lambda)$

$= 8 - 8\lambda + 2\lambda^2 + 4\lambda^2 - \lambda^3 - 4\lambda$

$8 - 12\lambda + 6\lambda^2 - \lambda^3$

$-6 + 3\lambda + 2 = 0$

$(2-\lambda)\{(2-\lambda)^2 - 1\} + 1\{-(2-\lambda)+1\} + 1\{1-(2-\lambda)\} = 0$

$(2-\lambda)^3 - (2-\lambda) + (2-\lambda) + 1 + 1 - (2-\lambda) = 0$

$(2-\lambda)^3 - 3(2-\lambda) + 2 = 0$; $8 - 12\lambda + 6\lambda^2 - \lambda^3 - 6 + 3\lambda + 2 = 0$

$4 - 9\lambda + 6\lambda^2 - \lambda^3 = 0$

~~$(2-\lambda)\{(2-\lambda)^2 - 3\} = 0$~~ ~~$(\lambda-2)\{\lambda^2 - 4\lambda + 3\} = 0$~~

~~$(\lambda-2)(\lambda-1)(\lambda-3) = 0$~~ // ~~$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$~~

$A-1 = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$; $A-2 = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

$A-3 = \begin{pmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$

$(A-1)(A-2) = \begin{pmatrix} 2 & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & 2 \end{pmatrix}$

$(A-1)(A-2)(A-3) =$

$\lambda - 1 \;)\; \lambda^3 - 6\lambda^2 + 9\lambda - 4 \;(\; \lambda^2 - 5\lambda + 4$

$\lambda^3 - \lambda^2$

$-5\lambda^2 + 9\lambda - 4$

$-5\lambda^2 + 5\lambda$

$4\lambda - 4$

$(\lambda-1)(\lambda-1)(\lambda-4)$

$(\lambda-1)^2(\lambda-4) =$ is the ch. eqn

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix}$$

$$A^3 - 6A^2 + 9A - 4$$

$$= \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - \begin{pmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{pmatrix}$$

$$+ \begin{pmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= 0.$$

$$A^3 - 6A^2 + 9A = 4$$

$$A^2 - 6A + 9 = 4A^{-1}.$$

$$4A^{-1} = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix} \quad \text{or} \quad A^{-1} = \begin{pmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{pmatrix}$$

(b) $x + 2y - z = 3$

$3x - y + 2z = 1$

$2x - 2y + 3z = 2$

$x - y + z = -1$

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

~~$3x = x + 2y$~~

$$\Delta = \begin{vmatrix} 1 & 2 & -1 & -3 \\ 3 & -1 & 2 & -1 \\ 2 & -2 & 3 & -2 \\ 1 & -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -1 & -3 \\ 1 & 1 & -1 & 1 \\ 2 & -2 & 3 & -2 \\ 1 & -1 & 1 & 1 \end{vmatrix} \quad (r_2 - r_3)$$

$$= \begin{vmatrix} 1 & 2 & -1 & -3 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -3 \\ 1 & -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & -1 & -3 \\ 0 & -1 & 0 & 4 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 2 & 4 \end{vmatrix} \quad \text{each row} - r_1$$

$$(r_3 - r_4) = \begin{vmatrix} -1 & 0 & 4 \\ -3 & 3 & 0 \\ -3 & 2 & 4 \end{vmatrix} = (-1)(12) + 4(3) = 0.$$

$\Delta = 0$ is cond^n for consistency

ie last column is a linear combination of 1st three columns.

To solve the eqns add 1st & 4th $\rightarrow 2x + y = 2$

" add $2(1^{st})$ & $2^{nd} \rightarrow 5x + 3y = 7$

$$\left.\begin{array}{ll} 6x + 3y = 6 & x = -1 \\ 5x + 3y = 7 & y = 4 \\ & \& \quad z = 4 \end{array}\right\}$$

far simpler than using determinant method.

[Eq no. 886 is right in setting $\Delta = 0$ & 887 is wrong in using $\Delta \neq 0$, infact his a_{44} should be $+1$ & not -1 — anyway both set correct solns]

2(a) $$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3.$$

If $a = 0$, $$\Delta = \begin{vmatrix} (b+c)^2 & 0 & 0 \\ b^2 & c^2 & b^2 \\ c^2 & c^2 & b^2 \end{vmatrix} = (b+c)^2 (b^2c^2 - c^2b^2) = 0$$

ie a is a factor. Simily b & c are factors. The determinat has a factor abc. Also putting $b+c = -a$, $c+a = -b$, $a+b = -c$ $\Delta = 0$ ie $(a+b+c)$ is a factor. But what about $(a+b+c)^2$ as the remaining factor?

or other $\begin{vmatrix} (b+c)^2-a^2 & a^2 & a^2 \\ 0 & (c+a)^2 & b^2 \\ c^2-(a+b)^2 & c^2 & (a+b)^2 \end{vmatrix}$ $(C_1 - C_3)$

$= \begin{vmatrix} (b+c)^2-a^2 & 0 & a^2 \\ 0 & (c+a)^2-b^2 & b^2 \\ c^2-(a+b)^2 & c^2-(a+b)^2 & (a+b)^2 \end{vmatrix}$ $C_2 - C_3$

$= (a+b+c) \begin{vmatrix} b+c-a & a^2 & a^2 \\ 0 & (c+a)^2 & b^2 \\ c-a-b & c^2 & (a+b)^2 \end{vmatrix}$

$= (a+b+c) \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & (c+a)^2-b^2 & b^2 \\ c-a-b & c^2-(a+b)^2 & (a+b)^2 \end{vmatrix}$

$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$

$= (a+b+c)^2 \{ a^2 \cdot \{(b-c)^2-a^2\} - b^2\{(c-a)^2-b^2\} + (a+b)^2 \}$

$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & -c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$

$= (a+b+c)^2 \{ a^2\{2b(c+a-b)\} + b^2 \cdot 2a(b+c-a) + 2ab\{(b+c-a)(c-a+b)\} \}$

$$= 2ab(a+b+c)^2\left[a(c+a-b)+b(b+c-a)+(b+c-a)(c-a+b)\right.$$

$$= 2ab(a+b+c)^2\left[a^2+b^2+ca-2ab+bc+(b+c)^2-a^2\right.$$
$$\left.-a^2+b^2+c^2+2bc\right]$$

~~$(a-b)^2+c(a+b)$~~ This is also Ex(1), Wantikar, p.9

as follows:- C_2-C_1, C_3-C_1 & remove $(a+b+c)^2$ & " 2 as common factor

$r_1-r_2-r_3$

Worked Ex. 4, p. 127 } 2 Barnard & Child }

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix} \quad \left| \begin{array}{l} b\cdot C_2 + c\cdot C_3 \text{ & divide } \Delta \text{ by } bc. \\ C_2+C_1,\ C_3+C_1 \end{array} \right.$$

$$= (a+b+c)^2 \begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ c^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix} \quad \left| \begin{array}{l} \text{Expanding } \Delta \text{ in terms of elements of } r_1. \\ C_2+C_1 \quad \text{& remove } (a+b+c) \text{ as fact} \end{array} \right.$$

$$\left.\begin{array}{l} r_1-(r_2+r_3) \end{array}\right\} = 2(a+b+c)^2 \times \begin{vmatrix} bc & -c & -b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix} \quad \left| \begin{array}{l} \text{& } \to 2bc(a+b+c)^3 \end{array} \right. \begin{vmatrix} 1 & b \\ 1 & a+b \end{vmatrix}$$

$$= 2abc(a+b+c)^3$$

~~Quite complicated~~ alternative proof in B & C is what I attempted.

(b) A is orthogonal matrix if $AA' = I$ (A' = transpose i.e. interchange rows & cols).

A is Hermitian if $A^{*\prime} = A$.

If H be hermitian and $HX = \lambda X$ (λ an eigenvalue & X an eigen vector) vector comp)



$$XH = \lambda X$$

Then $H^{*'} X^{*'} = \lambda^* X^{*'}$

Since H is hermitian $H^{*'} = H$

Then $(XHX^{*'}) = (XH)X^{*'} = (\lambda X)X^{*'} = \lambda(XX^{*'})$

and $XHX^{*'} = XH^{*'}X^{*'} = X(H^{*'}X^{*'}) = X(\lambda^* X^{*'})$

$$= \lambda^*(XX^{*'})$$

Hence $\lambda(XX^{*'}) = \lambda^*(XX^{*})$ so that $\lambda = \lambda^*$ since $XX^{*'} \neq 0$

Hence eigen value is real.

3(a) $$\int_S \vec{q} \cdot d\vec{S} = \int_S \vec{n} \cdot \vec{q}\, dS = \int_V \nabla \cdot \vec{q}\, dv$$

(b) Surface integral = $\iiint_V (2ax + 2by + 2cz)\, dV$

$$= \iiint_V 2(ax + by + cz)$$

$$= 2\iiint_V (ax + by + cz)\, dV.$$

$x^2 + y^2 + z^2 = a^2$

$$= 2\int_{-a}^{+a} \int_{-\sqrt{a^2 - x^2}}^{+\sqrt{a^2 - x^2}} \int_{-\sqrt{a^2 - x^2 - y^2}}^{+\sqrt{a^2 - x^2 - y^2}} (ax + by + cz)\, dx\, dy\, dz \quad ?$$

$$\iiint_V \int_0^{\sqrt{a^2-x^2-y^2}} z\,dz = \iiint_V z\,dx\,dy\,dz \quad \text{where } V \text{ is } x^2+y^2+z^2 = a^2$$

$$\int_0^R dx \int_0^{\sqrt{a^2-x^2}} dy \int_0^{\sqrt{a^2-x^2-y^2}} z\,dz = \pi a^5/16 \quad [\text{Goursat, p. 300}$$

Similarly $\iiint_V x\,dx\,dy\,dz = \iiint_V y\,dx\,dy\,dz = \pi a^5/16$

Hence integral $= \frac{\pi a^5}{16}(a+b+c) = \frac{\pi}{8}(a+b+c)$

4(a) ——

(b) $\operatorname{curl}\left(c \times \operatorname{grad}\frac{1}{r}\right) + \operatorname{grad}\left(c \cdot \operatorname{grad}\frac{1}{r}\right) = 0$

$\operatorname{curl}\left(\vec{c} \times \operatorname{grad}\frac{1}{r}\right) = \vec{c}\,\operatorname{div}\operatorname{grad}\frac{1}{r} - \operatorname{grad}\frac{1}{r}\operatorname{div}\vec{c}$

$+ (\vec{B}\cdot\operatorname{grad})$

$+ \left(\operatorname{grad}\frac{1}{r}\cdot\operatorname{grad}\right)\vec{c} - (\vec{c}\cdot\operatorname{grad})\operatorname{grad}\frac{1}{r}$ 2·5

$(\vec{B}\cdot\operatorname{grad})\vec{A}$

$= i\left(\frac{\partial A_x}{\partial x}B_x + \frac{\partial A_x}{\partial y}B_y + \frac{\partial A_x}{\partial z}B_z\right)$

$+ j\left(\frac{\partial A_y}{\partial x}B_x + \frac{\partial A_y}{\partial y}B_y + \frac{\partial A_y}{\partial z}B_z\right)$

$+ k\left(\frac{\partial A_z}{\partial x}B_x + \frac{\partial A_z}{\partial y}B_y + \frac{\partial A_z}{\partial z}B_z\right)$

$= \vec{c}\,\operatorname{div}\operatorname{grad}\left(\frac{1}{r}\right) - (\vec{c}\cdot\operatorname{grad})\operatorname{grad}\frac{1}{r}$

thus $+ \operatorname{grad}\left(\vec{c}\cdot\operatorname{grad}\frac{1}{r}\right)$

i.e. $(\vec{c}\cdot\operatorname{grad})\operatorname{grad}\frac{1}{r}$

$= \vec{c}\,\operatorname{div}\operatorname{grad}(1/r)$

$$\text{grad}_x\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{\partial r}{\partial x} \qquad r^2 = x^2+y^2+z^2$$

$$= -\frac{1}{r^3}\cdot\frac{x}{r} \qquad 2r\frac{\partial r}{\partial x} = 2x$$

$$\text{div grad}\left(\frac{1}{r}\right) = -\left\{\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right)\right\}$$

$$= -\left\{\frac{r^3 - x\cdot 3r^2\,\partial r/\partial x}{r^6} + - \right\}$$

$$= -\left\{\frac{r^3 - x\cdot 3r^2\cdot x/r}{r^6} + - \right\}$$

$$= -\left\{\frac{r^4 - 3x^2r^2}{r^7} + - \right\}$$

$$= -\left\{\frac{1}{r^5}\left\{\Sigma\left(r^2-3x^2\right)\right\}\right\} = -\frac{1}{r^5}\left\{3r^2 - 3\left(x^2+y^2+z^2\right)\right\}$$

$$= \underline{\underline{0}}.$$

2.5(a). Bookwork $-\int_C f(z)\,dz = 2\pi i\,(k_1+k_2+\cdots+k_n)$

(b) $$\int_C \frac{e^{-z}\sin z}{z^2}\,dz = \int_C \frac{\left(1-z+\frac{z^2}{2!}-\frac{z^3}{3!}\cdots\right)\left(z-\frac{z^3}{3!}+\frac{z^5}{5!}\cdots\right)}{z}\,dz$$

$$\int_C \left(1-z+\frac{z^2}{2!}+\cdots\right)\left(1-\frac{z^2}{3!}+\frac{z^4}{5!}\cdots\right)dz$$

C pole is at $z=0$. Residue at the pole

$$= \lim_{z\to 0} z f(z) = \lim_{z\to 0} z\,e^{-z}\frac{\sin z}{z}$$

Hence integral $= 0$.

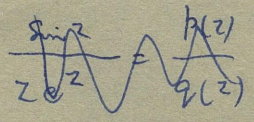
$\lim_{z\to 0} e^{-z}/z$

$\lim_{z\to 0} \frac{1}{z e^{z}}$

$\lim_{z\to 0} \frac{e^{-z}\sin z}{z} = \lim_{z\to 0} \frac{\bar{e}^{z}\left(z - z^3/3! + \cdots\right)}{z}$ (44)

$= \lim_{z\to 0} \bar{e}^{z}\left(1 - z^2/3! + \cdots\right) = 1.$

Hence integral $= 2\pi i$



6(a) This transformation is

$$\frac{(w+1)(i-1)}{(w-1)(i+1)} = \frac{(z+i)(-i)}{(z-i)(i)} = -\frac{z+i}{(z-i)}$$

$$(w+1)(z-i)(i-1) + (w-1)(i+1)(z+i) = 0$$

$$(w+1)(iz - z + 1 + i) + (w-1)(iz + z - 1 + i) = 0$$

$$iz w - zw + w + iw + iz - z + 1 + i$$

$$= iwz + zw - w + iw - iz - z + 1 - i$$

$$2zw - 2w - 2iz - 2i = 0$$

$$2w(z-1) = 2i(z+1)$$

$$w = i\,\frac{z+1}{z-1}.$$

If $z = iy$ (y-axis being $x = 0$)

$$w = i\,\frac{iy+1}{iy-1} = \frac{-y+i}{iy-1} = \frac{(i-y)(iy+1)}{-y^2-1}$$

$$= \frac{-y + i - iy - y}{-(1+y^2)}$$

$$w = u + iv = \frac{2y}{1+y^2} + i\,\frac{y-1}{1+y^2}$$

$$u = \frac{2y}{1+y^2}, \quad v = \frac{y-1}{1+y^2}$$

$$u - v = \frac{y+1}{1+y^2}$$

$2y = u(1+y^2)$

$y - 1 = v(1+y^2)$.

$$\frac{v}{u} = \frac{y-1}{2y}.$$

$2yv = uy - u$

$y(2v-u) = -u$

$$y = -\frac{u}{2v-u} = \frac{u}{u-2v}$$

$$u = \frac{2u/(2v-u)}{1 + u^2/(2v-u)^2} = u \cdot \frac{2u(2v-u)}{(2v-u)^2 + u^2}$$

$$u = u \cdot \frac{2(2v-u)}{4v^2 - 4uv + 2u^2}$$

$$1 = \frac{2(2v-u)}{4v^2 - 4uv + 2u^2}$$

$2v^2 - 2uv + u^2 = 2v - u = 0$ // (1)

$$v = \frac{u/2v-u - 1}{1 + u^2/(2v-u)^2}$$

$$v = \frac{u(2v-u) - (2v-u)^2}{(2v-u)^2 + u^2}$$

$$v = \frac{(2v-u)(u-2v+u)}{(2v-u)^2 + u^2}.$$

$$= \frac{2(2v-u)(u-v)}{4v^2 - 4uv + 2u^2} = \frac{(u-v)(2v-u)}{u^2 - 2uv + 2v^2}.$$

$u^2v - 2uv^2 + 2v^3 = 2uv - u^2 - 2v^2 + uv$. (2) (1) ≠ (2).

$$u = \frac{2u/(u-2v)}{1 + u^2/(u-2v)^2} = \frac{2u(u-2v)}{(u-2v)^2 + u^2}$$

$(u-2v)^2 + u^2 = 2(u-2v)$

$4v^2 - 4uv + 2u^2 = 2(u-2v)$

$u^2 - 2uv + 2v^2 - u + 2v = 0$



$$u = \frac{2y}{1+y^2}, \quad v = \frac{y-1}{1+y^2} \quad . \quad u - v = \frac{y+1}{1+y^2}.$$

$$\therefore \frac{v}{u-v} = \frac{y-1}{y+1} \;\&\; \frac{u}{u-2v} = \frac{2y}{2} \;;\; y = \frac{u}{u-2v}$$

$$\frac{u}{v} = \frac{2y}{y-1} = \frac{2u/(u-2v)}{2v/(u-2v)}$$

$$= \frac{u}{v} \quad (!)$$

$$y - 1 = \frac{u}{u-2v} - 1$$

$$= \frac{u - u + 2v}{u-2v}$$

$$= \frac{2v}{u-2v}.$$

$$u = \frac{2u/(u-2v)}{1 + u^2/(u-2v)^2} \;;\; v = \frac{2v/(u-2v)}{1 + u^2/(u-2v)^2}$$

$$u = \frac{2u(u-2v)}{(u-2v)^2 + u^2} \;;\; v = \frac{2v(u-2v)}{(u-2v)^2 + u^2}$$

we both give the locus.

$$(u-2v)^2 + u^2 = 2(u-2v)$$

$$2u^2 - 4uv + 4v^2 = 2(u-2v)$$

$$u^2 - 2uv + 2v^2 - u + 2v = 0. \quad (*)$$

a conic through O.

(b) $f(z)$ is analytic at z_0 if its derivative $f'(z)$ exists not only at z_0 but at every point z in some neighbourhood of z_0. ~~It~~ is analytic in a domain if it is analytic at every point in that domain

7(a) ⊛ Centre of conic is given by

$$\left.\begin{aligned} u - v - \tfrac{1}{2} &= 0 \\ -u + 2v + 1 &= 0 \end{aligned}\right\} \quad v + \tfrac{1}{2} = 0, \quad v = -\tfrac{1}{2}$$
$$u = 0.$$

centre $(0, -\tfrac{1}{2})$

transferring origin to this point the conic becomes

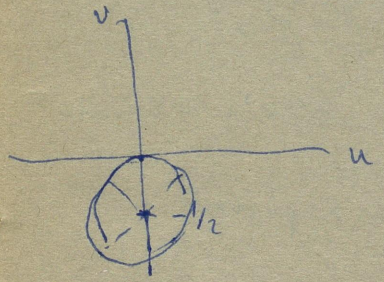
$$u^2 - 2uv + 2v^2 + \frac{\Delta}{ab - h^2} \qquad \Delta = \begin{vmatrix} 1 & -1 & -1/2 \\ -1 & 1 & 1 \\ -1/2 & 1 & 0 \end{vmatrix}$$

$u^2 - 2uv + 2v^2 - \tfrac{1}{4} = 0$ $\qquad ab - h^2 = 2 - 1 = 1.$

$\Delta = -\tfrac{1}{2}\left(-1 + \tfrac{1}{2}\right) - 1\left(1 - \tfrac{1}{2}\right)$

$ab - h^2 = 1 \ (+ve)$ $\qquad = \tfrac{1}{4} - \tfrac{1}{2} = -\tfrac{1}{4}$

ie an ellipse of centre $(0, -\tfrac{1}{2})$ passing through the origin



~~Tangent at (0,0)~~

$uu' - (uv' + u'v) + 2vv' - \tfrac{1}{4} = 0$

7(a) ~~Hartleian § 6.21, p. 206~~

Eqn of any plane through 1st line is

$$a(x - \alpha) + b(y - \beta) + c(z - \gamma) = 0 \qquad \text{——(1)}$$

with the cond^n $al + bm + cn = 0$ ——(2)

(1) contains the other line if $(\alpha', \beta', \gamma')$ lies on it ⇒ is $\perp^r$ to

to the normal to the plane is

$$a(\alpha'-\alpha) + b(\beta'-\beta) + c(\gamma'-\gamma) = 0 \quad \text{—(3)}$$

and $$al' + bm' + cn' = 0 \quad \text{—(4)}$$

Eliminating a, b, c from (3), (2) & (4) gives the required condn

$$\begin{vmatrix} \alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ l & m & n \\ l' & m' & n' \end{vmatrix} = 0$$

Let plane through the first two lines be

$$ax + by + cz = 0 \quad \text{—(1)}$$

with $$al_1 + bm_1 + cn_1 = 0 \quad \text{—(2)}$$

& $$al_2 + bm_2 + cn_2 = 0$$

If the third line also lies in the plane

$$al_3 + bm_3 + cn_3 = 0 \quad \text{—(3)}$$

Eliminating $a : b : c$ from (1), (2), (3) gives required result

(b) direct cosines of given line are $2/7, -3/7, -6/7$.

Any point on the line through $(1, -2, 3)$ ∥l to this is given

$$x = 1 + \frac{2r}{7},\ y = -2 + \frac{3r}{7},\ z = 3 - \frac{6r}{7}.$$

If this lies on the given plane

$$1 + \frac{2r}{7} + 2 - \frac{3r}{7} + 3 - \frac{6r}{7} = 5$$

i.e. $-r = -1$ or $r = 1$

ie the req'd point on the plane is $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

& distance of this from $(1, -2, 3)$

$$\sqrt{\left\{\left(\frac{9}{7}-1\right)^2 + \left(2-\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2\right\}} \qquad -\frac{11}{7} + 2$$

$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \underline{\underline{1}}$$

2.8(a) Let (x_1, y_1, z_1) be the point of contact

$$xx_1 + yy_1 + zz_1 + u(x+x_1) + v(y+y_1) + w(z+z_1) = 0.$$

is identical with $lx + my + nz = p$.

$$\therefore \quad \frac{x_1+u}{l} = \frac{y_1+v}{m} = \frac{z_1+w}{n} = \frac{ux_1 + vy_1 + wz_1}{-p}$$

Wentworth p. 298, Cor.

Centre & radius of sphere are

$(-u, -v, -w)$ and $r = \sqrt{u^2+v^2+w^2}$

$\perp^r$ from centre on $lx+my+nz = p$ is equal to radius

$$\sqrt{u^2+v^2+w^2} = r = \frac{-lu - mv - nw - p}{\sqrt{l^2+m^2+n^2}}$$

ie $(lu + mv + nw + p)^2 = (l^2+m^2+n^2)(u^2+v^2+w^2)$

(b) Same as Ex. worked on p. 30 (Wentzel Ex. 9, p. 300)

Applied Physics — Paper II - Mathematics III

Q. 2

Q. 3

1(a) Same q^n as in Paper I of Met & Ocean.

(b) " Q. 4(b) " " except that cosine series is asked for sin x, instead of sine series for cos x.

Assuming $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx, \quad a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} \sin x\,dx = \frac{2}{\pi}$$

$-\cos x\Big]_0^{\pi}$

$1-(-1)$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x \cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \{\sin(n+1)x - \sin(n-1)x\}\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right]_0^{\pi} \quad (n \neq 1$$

$\sin nx \cos x + \cos nx \sin x - \sin nx \cos x + \cos nx \sin x.$

$$a_1 = \frac{2}{\pi}\int_0^{\pi} \sin x \cos x\,dx = \frac{1}{\pi}\int_0^{\pi} \sin 2x\,dx$$

$$= -\frac{1}{2\pi}\left[\cos 2x\right]_0^{\pi} = 0.$$

Q. 4

for $n > 1$, $a_n =$ if n is odd, $a_n = 0$

if n is even, $a_n = \frac{4n}{\pi(n^2-1)}$ as on p. 22.

Q. 2. ———

Q. 3. Derivation of wave eqn (Carslaw, pp. 21–23)

Do. p. 31. Displacement is given by

$$u = \sum_r \sin\frac{r\pi x}{l}\left\{a_r \cos\frac{r\pi ct}{l} + b_r \sin\frac{r\pi ct}{l}\right\} \quad -(1)$$

when $t=0$, $u_{t=0} = \sum_r a_r \sin\frac{r\pi x}{l}$

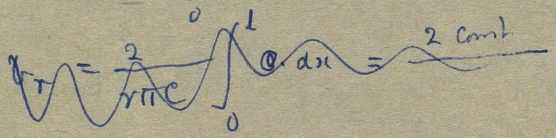
$$\dot{u}_{t=0} = \sum_r b_r \frac{r\pi c}{l} \sin\frac{r\pi x}{l}$$

in this case $u_{t=0} = x(l-x)$

$\dot{u}_{t=0} = 0$ as string is released from rest since $u(0,t) = u(l,t) = 0$

$$\therefore a_r = \frac{2}{l}\int_0^l x(l-x)\sin\frac{r\pi x}{l}\,dx \quad -(2)$$

& $b_r = 0$



Hence (1) becomes $u = \sum_r \sin\frac{r\pi x}{l} \cdot a_r \cos\frac{r\pi ct}{l}$.

with a_r given by (2). (to be multiplied)

Q. 4. (a) $q\ddot{y} - 6\dot{y} + y = 0$, with $y(0) = 3$, $\dot{y}(0) = 5$

Let $Y(s)$ be the L.T of $y(t)$

$$\mathcal{L}[\dot{y}(t)] = sY(s) - y(0) = sY(s) - 3.$$

$$\mathcal{L}[\ddot{y}(t)] = s^2Y(s) - sy(0) - \dot{y}(0)$$
$$= s^2Y(s) - 3s - 5.$$

Transformation given eqn leads to

$$9\{s^2Y(s) - 3s - 5\} - 6\{sY(s) - 3\} + Y(s) = 0.$$

$$9s^2Y(s) - 27s - 45 - 6sY(s) + 18 + Y(s) = 0$$

$$Y(s)(9s^2 - 6s + 1) = 27(s+1)$$

$$Y(s) = \frac{27(s+1)}{9s^2 - 6s + 1} = \frac{27(s+1)}{(3s-1)^2}$$

Let this $\equiv \frac{A}{3s-1} + \frac{B}{(3s-1)^2}$

$$27(s+1) \equiv A(3s-1) + B$$

giving $3A = 27$ or $A = 9$

$-A + B = 27$ i.e. $B = 36$.

i.e. $Y(s) = \frac{9}{(3s-1)} + \frac{36}{(3s-1)^2}$

$$= \frac{3}{s - 1/3} + \frac{4}{(s - 1/3)^2}$$

Taking inverse L.T's, we have

$$y(t) = 3e^{\frac{1}{3}t} + 4te^{\frac{1}{3}t} = e^{\frac{1}{3}t}(4t+3).$$

Verify: $y(0) = 3$ ✓,

$$\dot{y}(t) = \tfrac{1}{3}e^{\frac{1}{3}t}(t+1) + 3e^{\frac{1}{3}t}$$

$$\therefore \dot{y}(0) = \tfrac{1}{3}e^{\frac{1}{3}t}(4t+3) + e^{\frac{1}{3}t}\cdot 4$$

$$\dot{y}(0) = 1 + 4 = 5 \checkmark$$

$$y(t) = e^{\frac{1}{3}t}(4t+3)$$

$$\dot{y}(t) = \tfrac{1}{3}e^{\frac{1}{3}t}(4t+3) + 4e^{\frac{1}{3}t}$$

$$\ddot{y}(t) = \tfrac{1}{9}e^{\frac{1}{3}t}(4t+3) + \tfrac{4}{3}e^{\frac{1}{3}t} + \tfrac{4}{3}e^{\frac{1}{3}t}$$

$$9\ddot{y}(t) = e^{\frac{1}{3}t}(4t+3) + 24e^{\frac{1}{3}t}$$

$$6\dot{y}(t) = 2e^{\frac{1}{3}t}(4t+3) + 24e^{\frac{1}{3}t}$$

$$y(t) = e^{\frac{1}{3}t}(4t+3)$$

$$9\ddot{y}(t) - 6\dot{y}(t) + yt = 0. \checkmark$$

(b) (i) Miller Ex 3, p. 221, with $h = 1$, $a = 0$, $b \to a$

Answer here $\quad U(s) = \frac{1}{s}(1 - e^{-as})$

(ii) Ex. on p. 220 – the half-wave sine wave with $\omega = \frac{2\pi}{T}$.

L.T is given by $F(s) = \frac{2\pi T}{s^2T^2 + 4\pi^2} \cdot \frac{1}{1 - e^{-sT/2}}$

$T = \frac{2\pi}{\omega}$

$$= \frac{\omega}{s^2 + \omega^2} \cdot \frac{1}{1 - e^{-\pi s/\omega}} = \frac{4\pi^2/\omega^2}{s^2 \cdot 4\pi^2/\omega^2 + 4\pi^2} \cdot \frac{1}{1 - e^{-\pi s/\omega}}$$

5(a) ~~Qn in~~ (i) Wellknown Bkwork – Q. 4(b)(i) of ord. Diff Eqns of M. Sc. Applied

(ii) As on p. 8 of this notebook the relation (1'') is

$$\frac{\nu}{x} J_\nu(x) + J_\nu'(x) = J_{\nu-1}(x)$$

with $\nu = 1$, this becomes $\frac{J_1(x)}{x} + J_1'(x) = J_0(x)$

ie $J_1'(x) = J_0(x) - \frac{1}{x} J_1(x)$.

(b) Qn 3(b) of ord. Diff. Eqns paper – Soln to be obtained by using the method of Frobenius

6(a) First part of Q. 5 of ord. Diff. Eqns paper.

(b) Whittaker, pp. 299 – 300 or Courant – Hilbert, p. 68 } Worker.

7(a) The integral formula for $J_n(x)$ is [Whittaker, p. 355]

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ni\theta + ix\sin\theta} \cdot d\theta$$

Bisecting the range of integration and writing $-\theta$ for θ in the former part; $J_n(x) = \frac{1}{2\pi} \int_0^{\pi} e^{ni\theta - ix\sin\theta} + \frac{1}{2\pi} \int_0^{\pi} e^{-ni\theta + ix\sin\theta}$

ie $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\,d\theta$ (W, p. 357)

$= \frac{1}{\pi}\int_0^{\pi} \{\cos n\theta \cos(x\sin\theta) + \sin n\theta \sin(x\sin\theta)\}\,d\theta.$

when n is even $= 2m$

$\int_0^{\pi} \sin 2m\theta \sin(x\sin\theta)\,d\theta$ ——— (1)

$= -\frac{1}{2m}\cos 2m\theta \sin(x\sin\theta)\Big]_0^{\pi}$

$+\frac{1}{2m}\int_0^{\pi} \cos 2m\theta \cos(x\sin\theta)\cdot x\cos\theta\cdot d\theta.$

Courant-Hilbert p. 392.

$$e^{ix\sin x} = \sum_{n=-\infty}^{\infty} J_n(x) e^{in\theta}$$

from which we have

$$\cos(x\sin\theta) = \sum_{n=-\infty}^{\infty} J_n(x)\cos n\theta \quad ——— (3)$$

$$\sin(x\sin\theta) = \sum_{n=-\infty}^{\infty} J_n(x)\sin n\theta \quad ——— (4)$$

using $J_{-n}(x) = (-1)^n J_n(x)$, these can be written as

$$\cos(x\sin\theta) = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos 2n\theta$$

$$\sin(x\sin\theta) = 2\sum_{n=0}^{\infty} J_{2n+1}(x)\sin(2n+1)\theta \quad ——— (2)$$

(2) is Q. 7(4)

from (3) & (4)

$$\frac{1}{\pi}\int \cos n\theta \cos(x\sin\theta) = \begin{cases} J_n(x) & \text{for } n \equiv 0 \pmod 2 \\ 0 & \text{for } n \equiv 1 \pmod 2 \end{cases} \quad (5)$$

& $$\frac{1}{\pi}\int \sin n\theta \sin(x\sin\theta) = \begin{cases} 0 & \text{for } n \equiv 0 \pmod 2 \\ J_n(x) & \text{for } n \equiv 1 \pmod 2 \end{cases} \quad (6)$$

[When $n \equiv 0 \pmod 2$ i.e. when n is even, (6) shows that (2) reduces to Q. 7(a)]

By adding (5) & (6), $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\, d\theta$.

Q. 8(a) Convolution theorem is proved in Miller, Sec. 7, Ch. 5, p. 229.

(b) $$\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+\omega^2)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2}\cdot\frac{s}{s^2+\omega^2}\right\}$$

$\mathcal{L}^{-1}\left(\frac{s}{s^2+\omega^2}\right) = \cos\omega t$ | s by the convolution theorem

req'd answer $= \int_0^t \cos\omega(t-\xi)\cos\omega\xi \cdot d\xi$

$$= \int_0^t (\cos \omega t \cos \omega\xi + \sin \omega t \sin \omega\xi) \cos \omega\xi \cdot d\xi$$

$$= \int_0^t (\cos \omega t \cos^2 \omega\xi + \sin \omega t \sin \omega\xi \cos \omega\xi)\, d\xi$$

$$\int_0^t \cos^2 \omega\xi = \int_0^t \tfrac{1}{2}(1 + \cos 2\omega\xi) \cdot d\xi$$

$$= \frac{1}{2}\left\{ t + \left[\frac{1}{2\omega} \sin 2\omega\xi\right]_0^t \right\}$$

$$= \frac{1}{2}\left\{ t + \frac{1}{2\omega} \sin 2\omega t \right\}$$

$$\int_0^t \sin \omega\xi \cos \omega\xi \, d\xi = \frac{1}{2}\int_0^t \sin 2\omega\xi \cdot d\xi$$

$$= -\frac{1}{4\omega} \cos 2\omega\xi \Big]_0^t$$

$$= -\frac{1}{4\omega}\left\{ \cos 2\omega t - 1 \right\}$$

$\therefore$ req^{n} eqn $= \frac{1}{2}\cos \omega t \left\{ t + \frac{1}{2\omega} \sin 2\omega t \right\}$

$$- \frac{1}{4\omega} \sin \omega t \left(\cos 2\omega t - 1 \right)$$

$$= \frac{1}{2} t \cos\omega t + \frac{1}{4\omega} \sin\omega t + \frac{1}{4\omega} (\cos\omega t \sin 2\omega t - \sin\omega t \cos 2\omega t)$$

$$= \frac{1}{2} t \cos\omega t + \frac{1}{4\omega} \sin\omega t + \frac{1}{4\omega} \sin(2\omega t - \omega t)$$

$$= \frac{1}{2} t \cos\omega t + \frac{1}{2\omega} \sin\omega t.$$

Verification: $\mathcal{L}\{t\cos\omega t\} = -F'(s)$ with $f(t) = \cos\omega t$

ie $F(s) = \frac{s}{s^2+\omega^2}$

ie $-F'(s) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$. ie $F'(s) = \frac{(s^2+\omega^2) - s\cdot 2s}{(s^2+\omega^2)^2}$

$$= \frac{-s^2+\omega^2}{(s^2+\omega^2)^2}$$

$\mathcal{L}(\sin\omega t) = \frac{\omega}{s^2+\omega^2}$

$\therefore \mathcal{L}\left\{\frac{1}{2} t\cos\omega t + \frac{1}{2\omega}\sin\omega t\right\}$

$$= \frac{(s^2-\omega^2)}{2(s^2+\omega^2)^2} + \frac{1}{2(s^2+\omega^2)} = \frac{(s^2-\omega^2)+(s^2+\omega^2)}{2(s^2+\omega^2)^2}$$

$$= \frac{s^2}{(s^2+\omega^2)^2} \checkmark$$

papers!!!

Still 4 more questions!! (25/6/72).

Q. papers relating to Meteorology & Oceanography

(A) (Modified regulations)

(A) Hydrodynamics & Mathematical methods

1(a) Streak line ≠ path line i.e line of material particles [See p. 65]
= fluid motion in a way that an observer can see easily

Stream line determined by velocity vector

Stream line given by $\frac{dx}{q_1} = \frac{dy}{q_2} = \frac{dz}{q_3}$

Streak line by $\frac{d\vec{r}}{dt} = \vec{q}(r,t)$ with $\vec{r}(t_0) = \vec{r}_0$

(b) $\iiint_R \frac{\partial \rho}{\partial t} dV + \iint (\vec{n} \cdot \vec{q}) \rho dS$ follows from law of mass conservation → (1)

which leads to $\frac{\partial \rho}{\partial t} + \text{div}(\rho q) = 0$. from (1) as $R \to 0$

i.e $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho q_1)}{\partial x} + \frac{\partial(\rho q_2)}{\partial y} + \frac{\partial(\rho q_3)}{\partial z} = 0$.

2(a) Euler's eqns of motion are

$$\frac{D\vec{q}}{Dt} = \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = \vec{G} - \frac{\nabla p}{\rho}$$

$\vec{G}$ = body force = force acting on the body $\perp^l$ to its mass

or in cartesian

$$\frac{\partial q_k}{\partial t} + q_j \frac{\partial q_k}{\partial x_j} = G_k - \frac{1}{\rho}\frac{\partial p}{\partial x_k} \qquad (k = 1, 2, 3 \quad (x_1, x_2, x_3 = x, y, z)$$

Integration along a stream line – Look up Milne-Thomson

For steady state $\frac{\partial p}{\partial t} = 0$. Euler's eqn becomes

$$(\vec{q}\cdot\nabla)\vec{q} = \vec{G} - \frac{\nabla p}{\rho}$$

For barotropic fluid (ie ρ a fn of p only), the result can be written as

$$\vec{q} \times \vec{w} = \nabla\left\{H + \Omega + \tfrac{1}{2}\vec{q}^{\,2}\right\}$$

$\nabla \times \vec{v} = \vec{w}$ = instantaneous angular velocity

$G = -\nabla H$

$\nabla \Omega = \frac{\nabla p}{\rho}$

Integration of R.H.S. depends only on the end points so that we get $\int_C (\vec{q} \times \vec{w})\cdot\vec{r}\cdot d\vec{s}$

$$= \left[\tfrac{1}{2}q^2 + \Omega(p) + H\right]_{p_1} - \left[\quad\right]_{p_2}$$

If integration path C is a segment of stream line, tangent vector to C is $\parallel$ to $\vec{q} \times \vec{w}$ so that

$$\left[\quad\right]_{p_1} = \left[\quad\right]_{p_2} \text{ ie } \left[\;\right] : \text{const along a stream line}$$

(ii) If $\vec{q}$ is derivable from from a potential θ i.e. $\vec{q}$ analogous to a conservative force, motion is potential

Source – Defined by potential $\theta(Q) = m/4\pi r_{PQ}$

fine source at P i.e. steady state condition reached by creation of a fluid at the origin (P) at the constant rate 4π. Stream lines emanate radially from origin & equi-potential surfaces are concentric spheres. θ = potential due to a source of strength m at P. If m is -ve the terminology source is replaced by sink

Doublet – Case of source & sink at $x=a$ & $x=-a$

Complex potential is $W = K[\log(z-a) - \log(z-a)]$

Let $Ka = \frac{\mu}{a}$ & write $W = \frac{\mu}{2a} \log \frac{z-a}{z+a}$

Now let $a \to 0$, varying K so that μ remains constant

so that $W = \frac{\mu}{z} \cdot \frac{\partial}{\partial a}\left[\log(z-a) - \log(z+a)\right]_{a\to 0} = -\mu/z$

Giving complex potential of a doublet of moment μ oriented along the +ve z-axis. More generally for doublet

$W = \mu e^{i\gamma}/z-a$ for doublet at $z=0$ along $e^{i\gamma}$.

3(a) ——

(b) — Vide Truesdell – Kinematics of Vorticity for Bjerknes' general theorem

4(a) Complex Potential for a point vortex is (considering it as a limit of a $\odot^r$ vortex)

$$W = ik \log z.$$

(b) ?

5(a) $\nabla f(x) = f(x) - f(x-h)$

$$E f(x) = f(x+h)$$

$$\nabla E f(x) = \nabla f(x+h) = f(x+h) - f(x)$$

$$(E-1) f(x) = f(x+h) - f(x)$$

$$E \nabla f(x) = E\{f(x) - f(x-h)\}$$

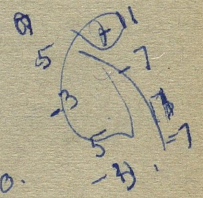
$$= f(x+h) - f(x). = \Delta$$

(b) – – –

$$\Delta f(x) = f(x+h) - f(x)$$

6(a) Bookwork

(b) – – .



7(a) $A = \begin{pmatrix} 7 & 3 \\ 11 & 5 \end{pmatrix}$ $\quad |A| = \begin{vmatrix} 7 & 3 \\ 11 & 5 \end{vmatrix} = 2 \neq 0.$

$$\cancel{A^{-1} = \begin{pmatrix} 5/2 & 11/2 \\ 3/2 & 7/2 \end{pmatrix}, \quad AA^{-1} = \begin{pmatrix} 7 & 3 \\ 11 & 5 \end{pmatrix}\begin{pmatrix} 5/2 & 11/2 \\ 3/2 & 7/2 \end{pmatrix}}$$

$A' = \begin{pmatrix} 7 & 11 \\ 3 & 5 \end{pmatrix}$ $\quad A^{-1} = \frac{1}{|A|} A'$

$A' = \begin{pmatrix} 7/2 & 11/2 \\ 3/2 & 5/2 \end{pmatrix}$

$AA' = \begin{pmatrix} 7 & 3 \\ 11 & 5 \end{pmatrix} \begin{pmatrix} 7/2 & 11/2 \\ 3/2 & 5/2 \end{pmatrix}$

$\begin{matrix} 7 & 11 \\ 3 & 5 \end{matrix}$

$\begin{matrix} 5 & -3 \\ -11 & -7 \end{matrix}$

adjoint of A is the transposed matrix of the cofactors of elements of A

ie adj $A = \begin{pmatrix} 5, -11 \\ -3, -7 \end{pmatrix}' = \begin{pmatrix} 5 & -3 \\ -11 & -7 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 5/2 & -3/2 \\ -11/2 & -7/2 \end{pmatrix}$

$AA^{-1} = \begin{pmatrix} 7 & 3 \\ 11 & 5 \end{pmatrix} \begin{pmatrix} 5/2 & -3/2 \\ -11/2 & -7/2 \end{pmatrix} = \begin{pmatrix} 1 & \\ 0 & \end{pmatrix}$

$\begin{pmatrix} 5, -11 \\ -3, 7 \end{pmatrix}$

$\begin{pmatrix} 5 & -3 \\ -11 & 7 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 5/2 & -3/2 \\ -11/2 & 7/2 \end{pmatrix}$; $AA^{-1} = \begin{pmatrix} 7 & 3 \\ 11 & 5 \end{pmatrix} \begin{pmatrix} 5/2 & -3/2 \\ -11/2 & 7/2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^{-1}A = \begin{pmatrix} 5/2 & -3/2 \\ -11/2 & 7/2 \end{pmatrix} \begin{pmatrix} 7 & 3 \\ 11 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Note cofactor $= (-1)^{i+j}$ minor

$\therefore$ cofactor of $7 = 5\,(-1)^{1+1} = 5$

" " $11 = 3\,(-1)^{2+1} = -3$

" " $3 = 11\,(-1)^{1+2} = -11$

" " $5 = 7\,(-1)^{2+2} = 7$

(b) Bookwork.

8 (a) Hartree pp. 168.–69.

(b) ?

~~(B) Modified Regulations~~

~~Paper III – Statistics~~

Streak line (from p. 60) is a curve traced out by all particles passing through some fixed point r_0

Q.5(b) Divided difference table [unworked Ex. 3 m

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$
0	−18			
2	6			
4	55			
8	104			

$\frac{684}{9} = 76$

$\frac{560}{5}$

11 ~~65~~

~~640~~

$$\begin{array}{r} 687 \\ 47 \\ \hline 640 \end{array}$$

ordinary table

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$
0	−18			
		24		
2	6		25	
		49		−25
4	55		0	
		49		
8	104			

$\frac{49-24}{8}$

25

$\Delta u_x = \frac{6-18}{2-0}$

Divided difference table

x	~~−18~~ u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_3$
0	~~−18~~	12		
2	6	49/2	$\frac{49/2-12}{4-0} = \frac{25}{8}$	
4	55	49/2	0	$\frac{-25/8}{6-0} = -\frac{25}{48}$
~~6~~ 8	104			

Q. 8(a) — Worked Ex 8.4 in Stanton's numerical methods for Science & Engineers, p. 189

Operations table

$\sum_{j=1}^{n} a_{ij} x_j - c_i = 0 \quad (i = 1, \ldots n)$

$R_i = \sum a_{ij} x_j - c_i$

make R_i smaller & smaller

	Δx	Δy	Δz	ΔR_1	ΔR_2	ΔR_3
	1	0	0	3	4	11
	0	1	0	9	2	−4
	0	0	1	−2	13	3

Take trial soln $x=0, y=0, z=0$

$x=0$	$y=0$	$z=0$	$R_1=-11$	$R_2=-24$	$R_3=8$
0	0	2	−15	2	14
−1	0	0	−18	−2	3
0	2	0	0	2	−5
−1	2	2	0	2	−5

Second trial soln is $x=-1, y=2, z=2$, making $R_1=0$, $R_2=2$ & $R_3=-5$. Multiplying by 5 continuing the procedure

−10	20	20	0	20	−30
5	0	0	15	40	5
0	0	−3	21	1	−4
0	−2	0	3	−3	4
−5	18	17	3	−3	4

This is up to one decimal place

ie $10x=-5$ $\quad x=-0.5$

$10y=18$ $\quad y=1.8$

$10z=17$ $\quad z=1.7$

5(b) is solved example 8.10, p. 199 of Stanton.

Write down the matrix

3	−5	47	20
56	23	11	−19
12	16	17	18
17	65	−13	7

Starting with the maximal entry 65 and deleting to find the corresponding minor, obtain

3	47	20
56	11	−19
12	17	18

We then choose 56, 47 & 18 in that order; so our pivotal coefficients become 65, 56, 47, 18. With this choice the eqns can be written as

$$\underline{65}\,x_2 + 17x_1 - 13x_3 + 7x_4 = 84$$
$$23\,x_2 + \underline{56}\,x_1 + 11x_3 - 10x_4 = 36$$
$$-5\,x_2 + 3x_1 + \underline{47}\,x_3 + 20x_4 = 28$$
$$16\,x_2 + 12x_1 + 17x_3 + \underline{18}\,x_4 = 25$$

The iteration scheme then reds

$$x_2 = \frac{1}{65}(84 - 17x_1 + 13x_3 - 7x_4)$$

$$x_1 = \frac{1}{56}(36 - 23x_2 - 11x_3 + 19x_4)$$

$$x_3 = \frac{1}{47}(18 + 5x_2 - 3x_1 - 20x_4)$$

$$x_4 = \frac{1}{18}(25 - 16x_2 - 12x_1 - 17x_3).$$

This is all that is reqd in 8(b), but solve by the Jacobi iteration technique.

7(b) Stanton §8, p. 181

Let A be a given matrix whose eigen values are required

Let X_1 be an arbitrary vector, which we use as an approximation to an eigen vector; suppose we "normalise" X_1 by requiring that one component, say the last, be unity

Compute the sequence

$$AX_1 = \lambda_1 X_2,\ AX_2 = \lambda_2 X_3,\ AX_3 = \lambda_3 X_4 \cdots$$

In this sequence all the vectors X_i are to be normalised in whatever manner we chose originally. Then if the X_i approach a limit X, the recursion relation will approach the limiting relation

$$AX = \lambda_i X$$

& we see that the constant λ_i will also approach the limit λ. The relation approached by the successive iterates will thus be

$$AX = \lambda X;$$

thus $\lambda = \lim \lambda_i$ is an eigen-value of A & $X = \lim X_i$ is the corresponding eigen-vector of A

In general the process gives the largest eigen value.

6(b) is unworked Ex. 3(a), p. 51 of Stanton. — Answer not given

use $f^{(n)}(x) = h^{-n} \Delta^n f(x)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1·00	e			
1·01	$e^{1.01}$			
1·02	$e^{1.02}$			
1·03	$e^{1.03}$			
1·04	$e^{1.04}$			

At Institute: 10/7/72

New Books Section.

(1) Group Theory & its applications edited by Ernest M. Loebl
Vol II

AP (1971)

(a) The representation & tensor operators of the unitary group $U(n)$
by Holman & Biedenharn

– Gives the Group $SU(2)$ & its representations, p-21

– Also deals with $SU(n)$

(b) Symmetry & degeneracy by H.V. McIntosh

(c) Dynamical groups in atomic & molecular physics by
C.E. Wulfman

(d) Symmetry adaptation of physical states
by means of computers by Flodmark & Blokker

(e) $\S$ VI on Group Theory & Computers

(e) Galilei groups & Galilean invariance by
Jean-Marc Lévy-Leblond

Vol I – contents seen on the flap of Vol II at the end

Articles by Wigner, Behrends & O'Raifeartaigh

(2) Relativistic Astrophysics by Zeldovich & Novikov – Vol I
(Stars & Relativity)

Vol II (not available yet) – to be devoted to The Universe and Relativity [Univ. of Chicago Press, 1971]

§ 2.5 on the possibility of calculating the gravitation constant from elementary particle theory

§ 2.6. Quantisation of gravity

are both very interesting & appear entirely new to me.

Chapter 14 on quasi-stellar objects ie quasars contains all recent results relating to them.

11/7/74 (1) Astronomy & Cosmology by A. P. Rowe

(A sixth-form course!)

Radio galaxies – transition from ordinary galaxy to radio galaxy involves some catastrophic event in the nucleus which

Quasars:

releases energy equivalent to the annihilation of half a million suns.

Quasars. – Nearest known (3 C 273) consists of two radio components about 20" apart. The nuclei of them coincided with a particular blue star and the other with a jet of bluish nebulosity. Schmidt found a redshift of 16% & if associated with the expansion of the Universe it if it is cosmological in origin, then distance of the quasar exceeds 1000 million light years, the length of the jet about 130,000 light years and the diameter of the quasar about 2 or 3 light years. Hoyle suggests that its mass lies between the prodigious figures of one million and a 100 million times the mass of the sun. – Longair discovers that out to a certain distance (or, what is the same thing, back to a certain time) there are more & more sources per unit volume. Before this time there appears to be a cut-off with fewer sources per unit volume. – Prodigious energies of the quasars; suggestions (i) chain reaction of a large number of supernovae (ie those caused by the collapse of a star into a white dwarf & temporarily having luminosity of 200 million suns) in the central regions of the objects (ii) consist of antimatter & contact of this with ordinary matter

and the resultant annihilation, (iii) left overs from the "big bang" and failing to share the general expansion; (iv) some form of gravitational collapse.

(2) Dynamics of gravitational collapse by S. V. Szekl (Inst. for Th. Physics, Univ. of Vienna, Austria) — Int. Conf. on development in high energy Physics — edited by P. Urban pp. 308 – 354.

(a) Brief summary of general relativity — Schwarzschild soln of the vacuum field eqns $R_{ik} = 0$ (R_{ik} = Ricci tensor $= R_{hikl}\, g^{hi}$) with the line element etc

$$ds^2 = \left(1 - \frac{2GM}{\ldots}\right)$$

(b) Experimental tests of general relativity

(c) Gravitational waves

(d) use of exterior forms in Riemannian Geometry

(e) Appln: Spherically symmetric soln of the Einstein field equations.

(f) Gravitational Collapse

(f) The Tolman-Oppenheimer-Volkoff equation

(g) Incompressible matter.

(h) The geometry of the Schwarzschild soln

(i) Eqn of state at high pressures

(j) Kruskal space

(k) Dynamics of gravitational collapse.

12/7/72 Reprint of a paper on the "Electron-muon Mass

(1) doublet from general relativity". (Il Nuovo Cimento, Vol 7 B, N.2, 11/2/72)

by M. Sachs (Dept of Physics, State Univ. of N.Y, Buffalo N.Y)

Summary :- ---. It is shown that the inertial mass field for spinor particles being a 2-dimensional matrix field, has two distinct mass eigen-values. From a study of the space-time curvature as due to the background gas of particle-antiparticle pairs, whenever the mass of an observed particle might be measured, semi-quantitative arguments are presented which imply that the ratio of these

two eigen-values for the electron mass doublet is of the order of $3/2\alpha \cong 205$ (α = fine structure constant) — a number that is quite close to the μ-e mass ratio — specific volume of the background gas of pairs assumed to be of order $(10^{-15}\,\text{cm})^3$.

. . . . possible explanation for some of the observed characteristics of quasars.

— makes linear approximation

$$\frac{e}{hc}\sigma^{\mu}A_{\mu} = \frac{e}{hc}\begin{pmatrix} A_0 - A_2 & 0 \\ 0 & A_0 + A_2 \end{pmatrix} \text{cm}^{-1}$$

so that mass eigen-values themselves are

$$m_1 = (h/c)\lambda_1 = (e/c^2)\,|A_0 + A_2|\,g,$$

$$m_2 = (h/c)\lambda_2 = (e/c^2)\,|A_0 - A_2|\,g$$

etc . . etc.

giving $m_1 = 2\left(\frac{4\pi\rho}{3N}\right)^{1/3}\alpha\frac{h}{c}$, $m_2 = \left(\frac{4\pi\rho}{3N}\right)^{1/3}\frac{\alpha\delta h}{c}$

and then argues intuitively that α should be $\simeq$ fine structure constant — Not very convincing.

(2) 14/7/72

(1) Annual review of Fluid mechanics for 1970

(Reference Section)

p. 95 - The turbulence boundary layer by Leslie S.G. Kovasznay

(e)

(a) Structure of turbulent boundary layers; exptl work —

(b) finding an understanding of turbulent shear flows of which the boundary layer is probably the most interesting example.

(c) Symposium in 1966 & two in 1968 (one under NASA on compressible boundary layers)

(d) In Exptl turbulence research, the general availability of hot-wire anemometry on a routine basis is now taken for granted. — novel visualisation methods that even permit quantitative analysis

(e) The turbulent boundary layer is worth studying because it is especially rich in details and complications, although it is a simple, well-defined flow

14/6/69

(f) Stanford Conference of 1968 was a remarkable event, not only because of the subject matter, but also because of the

"Confrontation of the predictors". There were 28 methods presented for predicting certain important mean flow properties in incompressible boundary layers. Exptl. data available in the technical literature were first scrutinised and finally approved by a small committee then the same flows were calculated by each participating predictor. In addition to the calculated numerical results, each predictor was required to submit the outline of his method as well as an actual calculating scheme, including all the necessary arbitrary functions, and arbitrary constants used, so that the organisers of the Conference themselves were able to repeat the computations

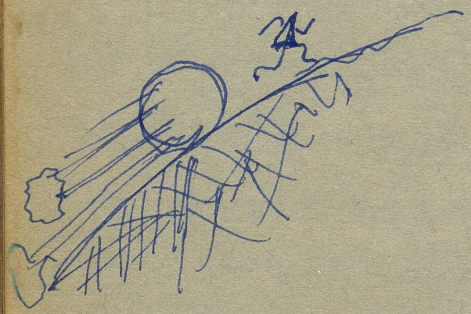
The Conference revealed many interesting aspects of the technical problem, but also some sociological aspects of the field (e.g. the proponents of different methods could not ignore each other completely). One of the puzzling outcomes of the Conference was the lack of a clear relationship between the sophistication of a method and its relative success in predicting the gross parameters; so ~~some~~ many ~~crude~~ crude methods appeared to yield nearly as good results as those with more elaborate physical assumptions. — Study of flow by taking movies.

(2) Principles of Statistical Mechanics by R. C. Tolman

– Analogue of 2nd law in Stat. Mech

(3) The Second Law by Henry A. Bent.

An interesting picture on the frontispiece



Norbert Weiner

(1) Ex-prodigy – my childhood & youth

(2) Cybernetics of the nervous system.

(3) Papers in Biocybernetics Vols 1 & 2. & III

(4) Cybernetics (Control & Communication in the animal & the machine)

(5) Human use of human beings – Cybernetics & Society

(6) Extrapolation, Interpolation and smoothing of

stationary time series

(7) Nerve, brain & memory models.

(8) Ed. by Rose – Survey of Cybernetics (B.87, 4, N.69)

(9) Differential space, quantum systems & prediction.

How the
1) Initial presentation of the stored fine-grained data looks upon representation

2) Examples of the accuracy achievable in fine-grain bearing measure.

3) Sonar search systems

4) Information storage & display – Concepts for an active detection system

5) A rapid-scan core memory and display concept

---

6) Monograph by Forney about concatenated codes

7) Schalkwijk & Kailath have exhibited systems for Channels

which allow noiseless feedback.

(8) Etc. --. have designed systems for non-erasure channels without feedback.

(9) Srivastava Codes

algebraic Coding theory

Ex. $q = 2$, $R = 1/2$, $K = 1$, $n = 2$

$$H^{(D)}(y, z) = y(1 + z^2 + z^5 + z^6)$$

An encoder for this code is shown in fig

memory is 6 blocks, constraint is 14 digits

2 diagrams:

Let it be, said Dhawan,

O.K; said Sudarshan,

~~And~~ Solid Sinha saw through all the fun,

and with hesitation to ~~join~~ be Chairmen in there was none..

Then the great scientists came from far & near

To unravel the unsolved without any fear

And did their endeavours reach a stalemate?

Let us leave it to Kailath to estimate.

But don't forget that the bounty of Nature

is'nt strained

It droppeth ~~from~~ like the gentle rain from Heaven

It blesseth him that seeks and him that fails.

Prayer to the Almighty.

Questions on weak interactions (S. B. Treiman, Sc. Amer, March 1959, (200), pp. 72-84

(1) Why not β-decay of all nuclei? Feynman's answer)

(2) Unity underlying the whole set of weak interaction (merely based on strength & conservation of energy?) (Weakness & strength relative)

(3) Confirmation of ~~parity~~ violation of parity by a decay process not involving neutrinos?

(4) Neutrinos trapped?

(5) Exptl. confirmation of overthrow of C (matter - antimatter)

(6) CP - validity in 1959

(~~7) e-μ puzzle~~

~~Questions on Leptons.~~

The Muon (Sc. Am. July, 1961 (205), 46-55

(1) Life of muon $= 2.2 \cdot 10^{-16}$ sec – quite long) (except instability same as muon – only $m_\mu \cong 200 m_e$)

(2) Why is S not conserved in weak interactions?

(3) Δ^μ of weak interactions & rates of β-decay.

(4) ~~Similarities~~ Exptl. evidence for $J^\mu = 1/2$.

(5) When μ decays $\underline{e}$ is emitted preferentially in a dirn opposite to that in which the spin axis of μ points – failure of a conservation law (no interest by nature about conservation of P)

(6) magnetic moment of the μ – shortest distance of space in which e.m. forces are relevant.

(7) g-factor of $e = 2$ (according to Dirac's theory) & slight anomaly, work of Schwinger – g factor of $\mu = 2$ with uncertainty of 10% (tantalizing)

(8) methods of measuring mass of (mystery) μ combined with magnetic moment – asymmetry eqn

(h) μ is a "riddle wrapped in a mystery inside an enigma"

25/7/72 In the Time of 24/7/72 there is a joke about Fred Hoyle on p. 521

31/7/72 (1) Universitas – Vol. 14, Aug. 1, No. 2, 1972

p. 97 – Willy Braadt's Nobel Peace Prize Lecture

(2) Commentationes Physico-Mathematicae

Vol 14, No. 4, Dec. 1971

p. 333 – Die Klassische Grenzfall der Quantentheorie by Manfred Andrié

Abstract :– The classical limit of quantum mechanics characterised by $h \to 0$ is based on a rigorous mathematical proof. It is shown that under certain conditions the soln of the Schrödinger equation of a quantum mechanical problem defines a density which in the limit $h \to 0$ is correlated with a δ-function, the carrier space of which exactly being a definite classical orbit. The expectation values of a self-adjoint operator tends to the corresponding classical observable. The proposed construction essentially uses the

theory of complete solutions of an equation of Hamilton-Jacobi.

(3) Scientific American – April, 1972, p. 93

Environmental Control in the Beehive – Honeybees effectively regulate temperature and humidity, eliminate polluted air, remove foreign objects, wastes and dead bodies and control parasites and pathogens that attack them & their food.

p. 100 – Martin Gardiner's "Mathematical Games" column consists of a mention of Coxeter's remarkable class of frieze patterns [Ref. Coxeter – Acta Arithmetica, Vol. 18 (1971) pp. 297-30] – Also ref. to his modern classic – Introduction to Geometry (Wiley, 1961), pp. 47-49)

New Books Section

(1) I am a Mathematician by Norbert Weiner, 1956
M. I. T. Press.

X | The scoundrel has mentioned his trip to Bangalore, but not mentioned my name at all

(2) Proc. Int. School of Physics "Enrico Fermi"

Course 47 — General Relativity & Cosmology

✓✓ Edited by R. K. Sachs

Comprehensive series of articles on all modern topics

(3) Introductory Group Theory for Chemists by George Davidson

Elsevier Pub. Co. Lond, 1971.

Good type of book to write

Preface — ~~Symmetry~~ Chaps — Symmetry elements & symmetry operations — Groups & their basic properties — Matrices — Repns of groups — Reducible & irreducible repns — Some important reducible representations — Group theory & vibrational spectroscopy — Further aspects of vibrational spectroscopy — Some applns to Bonding theory —

Bibliography

Answers to Exercises.

Appendix: Character tables for some chemically important symmetry groups —

Index.

(~~Pot~~ deals with Raman Spectroscopy also)

2|8|72 Book (2) of New Books Section

(1) on "General Relativity & Cosmology" (1969 Varenna summer school on rel. & cosm.

In the article by G.F.R. Ellis on Relativistic Cosmology, there appears on p. 127 a mention of "Raychaudhuri's equation" - on the photograph of participants, one finds name of K.B. Marathé, but I could not figure him out.

Contents (a) General Relativity & Kinetic Theory by J. Ehlers.

(a) §2. Consists of summary of Einsteins theory of Gravitation and its mathematical apparatus

§3. Differential forms & integration on manifolds

(b) Space-time structure from a global view point by R. Geroch.

(c) Relativistic cosmology by G.F.R. Ellis.

(d) Astrophysical Cosmology by D.W. Sciama.

(e) Relativistic stars, black holes and gravitational waves by K.S. Thorne

(f) Quasi-stellar objects - their importance for cosmology and general relativity by G.R & E.M. Burbridges.

(g) Optical observations relevant to cosmology by E.M. Burbidge

(h) Some current ideas on galaxy formation by M.J. Rees.

(i) Detection of gravitational waves by B. Bertotti

(j) The stability of relativistic star clusters by J.R. Ipser.

(K) General relativistic neutron and hyperon star models by H. Heintzmann.

(l) Elementary particles in De Sitter space by G. Börner

(m) Cosmological density fluctuations during hadron stage by W. Kundt

(n) Antimatter & Cosmology by G. Steigmann.

(o) Creation of particles by gravitational fields by H. Urbantke.

(2) Reference Section.

(a) Physical Review D – Particles & Fields (1/1/72) – nil

" (15/1/72)

" p. 329 – Tachyons & quantum statistics by Raymond Fox – No relativity reference to E.C.G.S's work.

— Do — 1/2/72.

p. 686 — Tests for the internal quantum numbers of partons
(used by Feynman for constituents of nuclei which scatter incoherently)
by O. Nachtmann

Talks of the charm-conserving $SU(2) \times SU(2) \times Y$ algebra
(Ref. to H. J. Lipkin, Phys. Rev. Letters 28, 63 (1972)

"charm" is another internal quantum number, not defined but mention is made of "charmed" particles not having been observed so far.

p. 760 — Time reversal & β-decay. A generalisation of the Kim-Primakoff theorem by S. P. Rosen

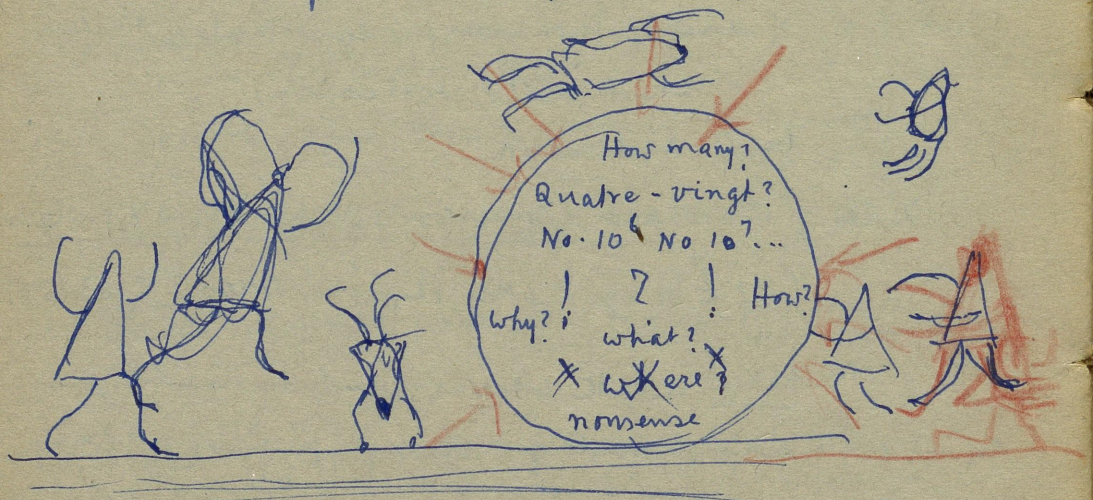
— Do — 15/2/72

p. 799 — Momentum & A.M. in relativistic classical particle mechanics by F. F. Jordan & C. S. Shukre
— Looks elementary

p. 950 — An article on magnetic quarks and electric quarks

p. 981 — CPT invariance & weak interactions by J. P. Hsu. [See p. 94]

Physicists solving unsolved problems





Entropy after Endarshan

Let it be, said Shawan

O.K. said Sudarshan

Solid Sinha managed all the fun

Hesitatant to be Chairmen there were none

—

Men the great Scientists came from far and near

To unravel the unsolved without any fear

But did their endeavours reach a stalemate?

Let us leave it to Kailath to estimate

—

But Remember ~~But don't forget~~ that Nature's bounty is'nt strained

It droppeth ~~too~~ like the gentle rain from Heaven

It blesseth him that ~~seeks~~ seeks ~~finds~~ and him that ~~fails~~ seeks fails.

—

Prayer to the Almughty

—

बद्धोनांजलिना नतेन शिरसा

गात्रैस्सर्व मोद्गमैः

कंठेन स्वार गद्गनेन नयने नोद्गीर्ण बाष्पंबना

नित्यं त्वच्चरणारविंद युगळ ध्यनामुतास्वादिनां

अस्माकं सरसीरुहाक्ष सतत संपध्यातं जीवित.

Since the discovery of CP non-invariance, it has become clear that the exptl. foundations for CPT and T invariances in weak interactions should be re-examined. If CPT be violated Lorentz-invariance is also violated, for these two invariance principles are intimately related. CPT invariance is a ~~necessary~~ sufficient ~~but for~~ but not a necessary condition for the equality of total lifetime and mass of particle & antiparticle. Ref:- So far exptl. tests of the possible CPT non invariance by measuring the mass, the magnetic moment or the total lifetime difference between particle & antiparticle have failed; and there is no direct evidence for T non invariance although CP violation has been established --- Constructs some kind of an exptl model & uses notion of aoraton (does not carry electric charge, energy, momentum or mass, however, it does carry one unit of spin A.M. According to uncertainty principle, the space-time posn of the aoraton is completely unknown, so that the aoraton itself is not directly observable) and spurions (?) ---- from consistency with experiment: it follows that both CPT and L-invariances are violated. It is apparently not possible to have CPT non invariance and Lorentz invariance at the same time

without losing at the same time some form of unitarity or contradicting the known expts --- Suggests some other expts set up to detect CPT non invariance using techniques with $m_s \approx 200$ GeV.

p. 1033 – Some remarks on symmetry groups by T.K. Kuo.

Intn – meaning of bootstrap principle – Hidden symmetries – An example – Ambiguities of the parity operator – Ambiguities in broken chiral symmetries – Further examples

– Appears a very interesting paper

4/8/72 Staff room

Full many a gem of the purest ray serene,
The unfathomed caves of ocean bear
Full many a flower is born to blush unseen
And waste its sweetness on the desert air

(1) J. Franklin Institute, Vol. 288, No. 4, Oct. 1969, p. 275

for about one hr

The Laplace transform of the derivative of a function with finite jumps by Leon. V. Bahar (with good bibliography)

(2) Ibid – ~~N~~ Jan. 1969, p. 19 – New rectangles for ~~recta~~
(a)
dynamic symmetry by Parry Moon – Mentions logarithmic
spiral being obtained not only by the golden-section rectangle
but also by any Ω-rectangle (Ω any number usually
irrational) – mentions the ratio 1.272 which is the square root
of the golden ratio $\Phi = 1.618$ and says the former is the shape of
many paintings & photographs. – Gives a bibliography of 16 authors
like "The curves of life", "on art & unity" & "Geometry of Art &
Life" etc^ns – Gives 2 tables of dynamic rectangles & a selected
list of 7 such rectangles particularly promising.

Fibonacci

(3) p. 159 – on the design of quadratic weirs by K. Kesha Murthy,
Feb, 1969 I.I.Sc – Thanks to N.S. Govinda Rao & Prof. K. Seetharamiah

(4) Ibid – June, 1968, p. 436.

(a) Applns of L.T's of matrix functions

(b) p. 400 – Fundamental classical & quantum dynamical
theory by G. Rosen.

14/8/72 – Took round C.S.P's son to several Departments in the Institute – Introduced him to Sinha, Narayanan, Narasimha etc – Borrowed 2 books on number theory for Lecture on 25th – Town E.C.G

lecture on 19th at 5.30 P.M.

Notes for Lecture on 25th

(1) The fascination of numbers by W. J. Reichmann –

The same title as my lecture

p.37 – Prof. Yahya's proof of Fermat's last theorem in 1957 – joke about Yahya Khan !

cube = sum of three other cubes two eg

$$3^3+4^3+5^3 = 6^3$$
$$1^3+6^3+8^3 = 9^3$$

p.50 – Primes of numbers with digits all = 1.

p.66 – Divisibility by 7 & 13.

p.90 – Perfect, amicable, triangular, automorphic, palindromic, numbers, [(factorial numbers), (Power numbers), (Mersenne's numbers), (Fermat numbers), (oblong numbers = $2\Delta^r$ nos.), (hexagonal nos. = $6\Delta^r$ no +1) (digital roots of nos)]. [Ex. of palindromic nos : $11111\cdots111^2$.

pp. 91 – 96 – Other peculiar numbers.

p. 111 – Wilson's theorem – Congruence notation

p. 115 $\sqrt{2} = 1 + \frac{1}{2+} \frac{1}{2+} \cdots$ – Continued fraction

Chap 13: one example of pseudo-telepathy

pp. 133-34 – Remarks re. "Calculating prodigies"

Chap. 15 – Magic squares – Rx of making slides for the lecture

Chap. 16 – Number mystic – Find corresponding Indian superstitions.

(2) Excursions in number theory – Ogilvy & Anderson.

Chap 1 – p. 4: Remark about enhancing fascination

p. 7 Importance of 0 as a place holder – Binary system – Roman numeral system.

Chap 2 – one whole book devoted to perfect nos – No odd perfect nos. known; if they exist they must be $> 10^{25}$ – Perfect nos & Mersenne nos ($2^n - 1$ is composite) – Deficit & abundant numbers, problem relating to this & remarks re. computers (p. 24) – Story about Gauss.

Chap. 3. Prime number theorem – Probability of two random numbers being relatively prime – No. of primes as – No

polynomial expn for primes – Fermat numbers – Remark

Chap 4. ie. 79 explt. verifications (ie. $n^2-79n+1601$)

Chap 4 – Congruence arithmetic – Proof of 999999 being divisible by 7 – Fifth Fermat number being divisible by 641 – Fermat's theorem $a^{p-1} \equiv 1 \pmod p$ [a not divisible by prime p] – Binomial coefficients of Pascal's triangle – Converse of Fermat's theorem is "if $a^p - a$ is divisible by p, then p is prime" not always being true proved by Congruence methods (using properties of Pascal's Δ^{le}, $n = 341$) – "Monkey & Coconuts problem" using Congruence.

Chap 5. Irrationals and iteration.

$\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$ irrational – period lengths of reciprocals of primes & the table – Problems regarding primality of strings of 1's – Iterated convergent radical – $2^{\sqrt{2}}$ transcendental.

Chap 6 – Diophantine equations

Length of radius of in-circle of a Pythagorean Δ^{le} is an integer [for 3-4-5 this radius = 1 from $r = n(m-n)$] – Fermat's Last Theorem – All points on $cle\ x^2+y^2 = \pi^2$ are transcendental – $y^2-7 = x^3$ has no solutions – 8 solns of $y^2-17 = x^3$ ($x = -2, -1, 2, 4, 8$ & $42, 53$, & lastly 5234) [p 107]

(16/8/72) Number the Language of Science by Dantzig (100)

Chap 2. Quotation. (p. 12)

" It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit. But its very simplicity and the great ease which it has lent to all computations put our arithmetic in the front rank of useful inventions, and we shall appreciate the grandeur of this achievement the more when we remember that it escaped the genius of Archimedes & Apollonius, two of the greatest men produced by antiquity"

— Laplace

[figure on p. 20 may be used for a slide]

p. 21 - All numerals of the ancient Babylonians, the hieroglyphics of the Egyptian papyri, the queer figures of the early Chinese records, the Hebrew, Chaldean Greek & Roman numerals are all based a "Cardinal" principle

p. 25 - The average man identifies mathematical ability with quickness in figures even at the present day — Income tax! irony about mathematicians having no excessive income.

p. 29. One who reckons upon the history of reckoning up to the invention of the principle of position is struck by the paucity of achievement. This long period of nearly 5000 years saw the fall and rise of many a civilisation, each leaving behind it a heritage of literature, art, philosophy & religion. But what was the net achievement in the field of reckoning, the earliest art practised by man? An inflexible numeration so crude as to make progress wellnigh impossible, and a calculating device so limited in scope that even elementary calculations called for the services of an expert. And what is more, man used these devices for thousands of years without making a single worthwhile improvement in the instrument, without contributing a single ~~idea~~ important idea to the system!

When viewed in this light, the achievement of the unknown Hindu who sometime in the first century of our era "or earlier" discovered the principle of position assumes the proportions of a world-event. ----

Particularly puzzling is the fact that the Greek nation which gave us ~~geometry the rotating~~ geometry and carried this science so far, did not create even a rudimentary algebra! Is it not equally strange that

algebra, that corner stone of modern mathematics also originated in India and at about the same time when positional numeration did?

p. 31 – The discovery of (Sunya) Zero by the Hindus for purpose of unambiguous ~~record~~ of permanent record of the positional notion.

How the Indian Sunya became the Zero of today constitutes one of the most interesting chapters in the history of culture. Arabs of 10th century adopted sunya by their own sifr (empty in Arabic) → In Italy afri latinised into zephirum (beginning of 13th century). In ~~Europe~~ Germany ~~sifra~~ sifr → Cifra (Even the great Gauss who wrote in Latin used Cifra in this sense). In English this → Cipher – deciphering (secrecy of the sign of Cifra) → Zifferum → Zero.

—

p. 35 – The Indian sunya was destined to become the turning point in a development without which the progress of modern science, industry or commerce is inconceivable. And the influence of this great discovery was by no means confined to arithmetic. By having the

way to a generalised number concept, it played just as fundamental role in practically every branch of mathematics. In the history of culture the discovery of zero will always stand out as one of the greatest single achievements of the human race.

[Unknown Hindu – modesty, humility & nobility of Hindu nature & thought]

Pp. 82–83 – Arabs good at translation – Richest Hellenistic library at Alexandria was twice pillaged & destroyed; first by the Christian vandals in the 4th century, then by Moslem fanatics in the 7th. But many of these ancient manuscripts have survived thanks to Arabic translations

mention of Omar Khayyam (Rubaiyat) who used Greek geometry & Hindu algebra to solve cubic & quartic equations – anticipates Newton in discovery of the binomial formula

Yet for all this, the Arabs have not advanced one iota in symbolic notation. It is one of the strangest phenomena in the history of mathematics that the Arabs, in adopting Hindu algebra, did not retain their quaint syncopated symbolism. Quite the contrary; they dropped back to the rhetorical algebra of the Greeks, and for a time even

went so far as to eliminate numeral symbols from their treatises on algebra, preferring to write numerals out in full. Was it that the Arabs pushed their claim of being the intellectual heirs of the Hellenes to the point of refusing to acknowledge the debt they owed to the Brahmins?

p. 84 – Revival of European learning due to Arabs of Spain & of Levant – first began in Italy for eg. the notable work of Fibonacci (Leonardo de Pisa ?)

p. 118 – Kronecker wanting to banish the introduction of the irrationals from mathematics. His remark "God made the integers; all the rest is the work of man"

(2) Hardy & Wright.

p. 39. $\sqrt{2}$ is irrational – $\sqrt[m]{N}$ is irrational, – nos. defined by algebraic eqns with integral coefficients are irrational unless "obviously" rational – e is irrational easily proved – π is irrational but of this there is no really simple proof – All powers of e or π & polynomials in e or π with rational coefficients are irrational – $e^{\sqrt{2}}$, $e^{\sqrt{5}}$, $\sqrt{7}e^{5\sqrt{2}}$, $\log 2$ are irrational – e^{π}, $2^{\sqrt{2}}$, $e+\pi$, $e\pi$ are irrational –

irrationality of such numbers as 2^e, π^e, $\pi^{\sqrt{2}}$ or Euler's constant γ $\left[= \lim_{n\to\infty}\left(1+\frac{1}{2}+\cdots\frac{1}{n}-\log n\right)\right]$ is still unproved

p. 94 – Th. 121 – If $m_1, m_2, \cdots m_k$ are coprime [ie $(m_i, m_j)=1$]

then the system

$$x \equiv c_1 \pmod{m_1},\ x \equiv c_2 \pmod{m_2},\ \cdots\ x \equiv c_k \pmod{m_k}$$

has a unique solution (mod m) given by

$$x = n_1 M_1 c_1 + n_2 M_2 c_2 + \cdots + n_k M_k c_k$$

[$m = m_1 m_2 \cdots m_k = m_1 M_1 = m_2 M_2 = \cdots = m_k M_k$ and since $(m_i, M_i)=1$, there is an n_i (unique to modulus m_i) such that $n_i M_i \equiv 1 \pmod{m_i}$]

[This theorem known to the Chinese mathematician Sun-Tsu in the first century A.D]

Problem of 6 Professors

Six professors begin courses of lectures on Monday, Tuesday, Wednesday, Thursday, Friday and Saturday, and announce their intentions of lecturing at intervals of two, three, four, one, six and five days respectively. The regulations of the University forbid Sunday lectures (so

that a Sunday lecture must be omitted). When first will all six professors find themselves compelled to omit a lecture?

The proof of this is an illustration of the case where the moduli are not coprime.

Proof :- If the day in question is the xth (counting from and including the first Monday), then

$$x = 1 + 2k_1 = 2 + 3k_2 = 3 + 4k_3 = 4 + k_4 = 5 + 6k_5 = 7k_7$$

where the k's are integers

i.e. (1) $x \equiv 1 \pmod 2$; (2) $x \equiv 2 \pmod 3$, (3) $x \equiv 3 \pmod 4$, (4) $x \equiv 4 \pmod 1$, (5) $x \equiv 5 \pmod 6$, (6) $x \equiv 6 \pmod 5$ & (7) $x \equiv 0 \pmod 7$

Of these congruences (4) is no restriction & (1) & (2) are included in (3) & (5). Of the two latter, (3) shows that x is congruent to 3, 7 or 11 (mod 12) and (5) that x is congruent to 5 or 11, so that (3) & (5) are together equivalent to $x \equiv 11 \pmod{12}$. Hence the problem is that of solving $x \equiv 11 \pmod{12}$, $x \equiv 6 \pmod 5$, $x \equiv 0 \pmod 7$

or $x \equiv -1 \pmod{12}$, $x \equiv 1 \pmod{5}$, $x \equiv 0 \pmod{7}$ (107)

This is a case of the problem solved by Th. 121 (p. 94). Here

$m_1 = 12$, $m_2 = 5$, $m_3 = 7$, $m = 420$

$M_1 = 35$, $M_2 = 84$, $M_3 = 60$

The n are given by

$35 n_1 \equiv 1 \pmod{12}$, $84 n_2 \equiv 1 \pmod{5}$

$\&$ $60 n_3 \equiv 1 \pmod{7}$

or $-n_1 \equiv 1 \pmod{12}$, $-n_2 \equiv 1 \pmod{5}$, $4 n_3 \equiv 1 \pmod{7}$

$\&$ we can take $n_1 = -1$, $n_2 = -1$, $n_3 = 2$. Hence

$x = (-1)(-1)35 + (-1)1 \cdot 84 + 2 \cdot 0 \cdot 60$

$= -49 \equiv 371 \pmod{420}$

The first x satisfying the condition is 371

[Cattle from p. 99] $a^b - c^d = 1$ (a, b, c, d all different integers, b & $d \neq 1$. one soln is $3^2 - 2^3 = 1$. Are there any others? How many? what are they? No one knows — Sierpinski's problems ($n^n + 1$ theories) & the problems of Goldbach & twin primes — $a^2 + 2ab + b^2$ & $a^2 + 3ab + b^2$.

Chap. 7 — Number curios

(108) 17/8/72 – Hardy & Wright [found at F.R.L that slides required should be 36 mm □]

p. 45. $x = \sqrt{2} + \sqrt{3}$ is irrational since it is not an integer

& satisfies $x^4 - 10x^2 + 1 = 0$

$\log_{10} 2$ is irrational

for if $\log_{10} 2 = \frac{a}{b}$, $2^b = 10^a$ which is impossible

Irrationality of e & π proved by Lambert in 1761 & that

of e^{π} by Gelfond in 1929

π & π^2 are ~~[illegible]~~ irrational

If $\pi^2 = a/b$, we write

$$G(x) = b^n \left\{ \pi^{2n} f(x) - \pi^{2n-2} f''(x) + \pi^{2n-4} f^{(4)}(x) + \cdots + (-1)^n f^{(2n)}(x) \right\}$$

so that $G(0)$ & $G(1)$ are integers

$$\left[\text{where } f(x) = \frac{x^n (1-x)^n}{n!} = \frac{1}{n!} \sum_{m=n}^{2n} c_m x^m \right.$$

where c_m are integers]

we have $\frac{d}{dx} \left\{ G'(x) \sin \pi x - \pi G(x) \cos \pi x \right\}$

$$= \left\{ G''(x) + \pi^2 G(x) \right\} \sin \pi x = b^n n^{2n+2} f(x) \sin \pi x$$

Hence $\pi \int_0^1 a^n \sin \pi x f(x)\,dx = \left[\frac{G'(x)\sin \pi x}{\pi} - G(x)\cos \pi x\right]_0^1$

$= G(0) + G(1)$ — — (A)

an integer [But from the properties of $f(x)$ viz

$0 < f(x) < \frac{1}{n!}$

and $f(0) = 0$ & $f^{(m)}(0) = 0$ if $m < n$ or $m > 2n$. But

if $n \leq m \leq 2n$, $f^{(m)}(0) = \frac{m!}{n!} c_m$. an integer

Hence $f(x)$ & all its derivatives take integral values at $x = 0$ & since $f(1-x) = f(x)$, the same is true at $x = 1$]

Thus (A) gives $0 < \pi \int_0^1 a^n \sin \pi x f(x)\,dx < \frac{\pi a^n}{n!} < 1$

for large enough n, a contradiction.

Chap XX — Repn of nos. by decimals

p. 111 — A decimal which neither terminates nor recurs represents an irrational number.

Exx. (i) $x = .0100100010000001\ldots$

(ii) $x = .0110101000010\ldots$

where the digit a_n is 1 if n is prime and 0 otherwise

(iii) $x = .2357111317192329\ldots$

where the sequence of digits is formed by the primes in ascending order.

---

p. 121 – Sets of measure Zero – Any enumerable set is of measure zero.

p. 122. Rationals (R) can be arranged as

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \ldots$$

ce in the form $x_1, x_2, \ldots x_n, \ldots$

Hence R form a set of measure zero.

Chapter X – Continued fractions

p. 148 – The series of Fibonacci and Lucas

Fibonacci (u_n) is 1, 1, 2, 3, 5, 8, 13, 21, –

Lucas (v_n) is 1, 3, 4, 7, 11, 18, 29, 47, –

Properties (i) $(u_n, u_{n+1}) = 1, (v_n, v_{n+1}) = 1$

(ii) u_n & v_n are both odd or both even.

& $(u_n, v_n) = 1, (u_n, v_n) = 2$ in these two cases

(iii) $u_n \mid u_{rn}$ for every r

(iv) If $(m, n) = d$, then $(u_m, u_n) = u_d$

& in particular u_m, u_n are coprime if m & n are coprime.

(v) If $(m,n) = 1$, $u_m u_n \mid u_{mn}$

Also we can define u_n & v_n for all integral n with $u_0 = 0$, $v_0 = 2$

then $u_{-n} = -(xy)^{-n} u_n = (-1)^{n-1} u_n$ & $v_{-n} = (-1)^n v_n$

(vi) $u_n^2 - u_{n+1} u_{n+1} = (-1)^{n-1}$

(vii) Every prime p divides some Fibonacci number (and therefore an infinity of the numbers)

Chapter XI

p. 159 – A number x which satisfies an algebraic eqn ie

$$a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$$

($a_0, a_1 \dots a_n$ integers) is algebraic. A number which is not algebraic is transcendental.

—

$\mathfrak{A}$: The aggregate of algebraic numbers is enumerable

$\mathfrak{N}$: Almost all real numbers are transcendental

Few classes of transcendental numbers are known even now. These classes include

$$e, \pi, \sin 1, J_0(1), \log 2, \frac{\log 3}{\log 2}, e^{\pi}, 2^{\sqrt{2}}$$

but not 2^e, 2^{π}, π^e or γ. It has never been proved even that any of these last numbers are irrational.

p. 172 – Proof of e being transcendental

p. 173 – " π "

Ch XVI – The arithmetical fns $\phi(n)$, $\mu(n)$, $d(n)$, $\sigma(n)$, $\vartheta(n)$

p. 233 $\phi(n)$ = no. of $+^{ve}$ integers less than & prime to n

$$\phi(n) = n \prod_{p|n} (1 - 1/p)$$

p. 237 – Evaluation of Ramanujan's sum

Defn of Ramanujan's sum (p. 55)

$$c_q(m) = \sum_{h^*(q)} e\left(\frac{hm}{q}\right)$$

$h^*(q)$ implying that h runs only through residues prime to q

$[e(\tau) = e^{2\pi i \tau}$ where τ is rational.
we have $e(m/n) = e(m'/n)$ when $m \equiv m' \pmod n]$

ρ is a primitive q^{th} root of unity if $\rho^q = 1$ but $\rho^r \neq 1$ for any $+^{ve}$ $r < q$.

Theorem: Any q^{th} root of unity is a primitive r^{th} root for some divisor r of q

Theorem: the qth roots of unity are the numbers

$e\left(\frac{h}{q}\right)$, $(h = 0, 1, \ldots q-1)$

and a necessary & sufficient condition that the root should be primitive is that h should be prime to q.

Now Ramanujan's sum can be written as

$$c_q(m) = \Sigma \rho^m$$

where ρ runs through the primitive q^{th} roots of unity

Theorem: If $(q, q') = 1$, $c_{qq'}(m) = c_q(m)\, c_{q'}(m)$

~~Ramanujan's~~

Mobius fn $\mu(n)$ is defined as follows:

(i) $\mu(1) = 1$

(ii) $\mu(n) = 0$ if n has a squared factr

(iii) $\mu(p_1, p_2, \ldots p_k) = (-1)^k$ if all the primes $p_1, p_2, \ldots p_k$ are different.

Thus $\mu(2) = -1$, $\mu(4) = 0$, $\mu(6) = 1$

Ramanujan's sum is given by the theorem

$$c_n(m) = \sum_{d|m,\, d|n}{}' \mu\left(\frac{n}{d}\right) d.$$

p. 241. fn $r(n)$ is defined as the number of representations of n in the form $n = A^2 + B^2$ (A & B rational integers)

p. 239. $d(n)$ = no. of divisors of n including 1 & n

& $\sigma_k(n)$ = sum of the kth powers of divisors of n

$\sigma_1(n) = \sigma(n)$ = sum of divisors of n

Perfect number n has $\sigma(n) = 2n$

ie n = no. of divisors of n other than itself

Th. If $2^{n+1} - 1$ is prime, then $2^n(2^{n+1} - 1)$ is perfect

Th. Any even perfect number is a Euclid number, that is to say of the form $2^n(2^{n+1} - 1)$ where $2^{n+1} - 1$ is prime

p. 141 – Equivalent numbers

If $\xi = \dfrac{a\eta + b}{c\eta + d}$

where a, b, c, d are integers such that $ad - bc = \pm 1$, then ξ is said to be equivalent to η. In particular ξ is equivalent to itself. This relation is symmetrical

Th. If ξ & η are equivalent, & η and ζ are equivalent then ξ & ζ are equivalent

Th. Any two rational nos are equivalent

---

p. 62 – 'Ramanujan's sum' can be seen in earlier writings (for eg. Jensen) but Ramanujan was the first

mathematician to see its full importance and use it systematically. It is particularly important in the theory of the repⁿ of numbers by sums of squares.

p. 201. The diophantine eqns

$$x^3 + y^3 + z^3 = t^3 \quad \text{— (a)}$$

and $$x^3 + y^3 = u^3 + v^3 \quad \text{— (b)}$$

Ramanujan gave

$$x = 3a^2 + 5ab - b^2, \quad y = 4a^2 - 4ab + 6b^2$$
$$z = 5a^2 - 5ab - 3b^2, \quad t = 6a^2 - 4ab + 4b^2$$

as a soln of (a)

[p. 245. $f(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ is convergent for $s > 1$ and its sum $f(s)$ is called the Riemann Zeta function. In particular $f(2) = \sum_{n=}^{\infty} \frac{1}{n^2} = \pi^2/6$.]

p. 256. Ramanujan proved

$$\text{Th}: \quad \frac{f^4(s)}{f(2s)} = \sum_{n=1}^{\infty} \frac{\{d(n)\}^2}{n^s} \qquad (s > 1)$$

Similarly $$\text{Th}: \quad \frac{f(s) f(s-a) f(s-a-b)}{f(2s-a-b)} = \sum_{n=1}^{\infty} \frac{\sigma_a(n)\,\sigma_b(n)}{n^s}$$

These two theorems were first published stated by Ramanujan.

p. 265 – Ramanujan proved that

the sum $d^2(1), + \cdots + d^2(n)$

is of order $n(\log n)^{2^2-1} = n(\log n)^3$

& $d^3(1) + \cdots + d^3(n)$

is of order $n(\log n)^{2^3-1} = n(\log n)^7$, ... & so on.

Ch XIX – Partitions – Additive arithmetic

Partition of number n is a rep^n of n as the sum of any number of $+^{ve}$ integral parts. Thus

$$5 = 4+1 = 3+2 = 3+1+1 = 2+2+1 = 2+1+1+1$$
$$= 1+1+1+1+1$$

has 7 partitions. Thus $p(5) = 7$.

p. 287 – Congruence properties of $p(n)$

Ramanujan, examining Macmahon's table of $p(n)$ was led first to conjecture & then to prove, three striking arithmetic properties associated with the moduli 5, 7 & 11 viz

Th: $p(5m+4) \equiv 0 \pmod 5$
$p(7m+5) \equiv 0 \pmod 7$
$p(11m+6) \equiv 0 \pmod{11}$

Proof given on p. 288 for first theorem.

Ramanujan made the general conjecture that if

$$\delta = 5^a 7^b 11^c$$

and $24n \equiv 1 \pmod{\delta}$, then $p(n) \equiv 0 \pmod{\delta}$

Ramanujan proved the congruences for 5^2, 7^2, 11^2, Kronecker for 5^3 & Watson for 5^4. But Gupta, in extending Macmahon's table up to 300 found

$$p(243) = 133\,978\,259\,344\,888$$

is not divisible by $7^3 = 343$ & since $24 \cdot 243 \equiv 1 \pmod{343}$ this contradicts Ramanujan's conjecture for 7^3. The conjecture for 7^b is therefore to be modified & Watson showed that $p(n) \equiv 0 \pmod{7^b}$ if $b > 1$, and $24n \equiv 1 \pmod{7^{2b-2}}$

D. H. Lehmer used quite a different method based upon the analytic theory of Hardy & Ramanujan and of Rademacher to calculate $p(n)$ for particular n. By this means he verified the truth of the conjecture for the first values of n associated with the modulii 11^3 & 11^6. For eg. he finds $p(721)$ & $p(2052)$ are both divisible by $11^3 = 1331$ and that $p(14031)$ a number of 127 digits is divisible by $11^4 = 14641$

pp. 290-91 — Rogers-Ramanujan identities (a) & (b) too long to write

Simple interpretation in terms of partitions as combinatorial theorems

(a) The no. of partitions of n with minimal difference 2 (parts without repetitions or sequences) is equal to the number of partitions into parts of the forms $5m+1$ & $5m+4$

(b) The no. of partitions of n into parts not less than 2, and with minimal difference 2, is equal to the number of partitions of n into parts of the form $5m+2$ and $5m+3$.

p. 295 — Ramanujan's continued fractions viz

$$1+\frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+}\cdots \quad \text{or general} \quad 1+\frac{ax}{1+}\,\frac{ax^2}{1+}\cdots$$

[Cf this with usual simple continued fraction

$$a_0+\frac{1}{a_1+}\,\frac{1}{a_2+}\,\frac{1}{a_3+}\cdots\Big]$$

Ramanujan showed that the former is equal to $F(1)/F(x)$

$$\text{ie} = \frac{1-x^2-x^3+x^9+\cdots}{1-x-x^4+x^7+\cdots} = \frac{(1-x^2)(1-x^7)\cdots-(1-x^3)(1-x^8)\cdots}{(1-x)(1-x^6)\cdots-(1-x^4)(1-x^9)\cdots}$$

It is known from the theory of elliptic functions that these products and series can be calculated for certain special values of x, and in particular when $x = 2\pi\sqrt{h}$ and h is rational. In this way Ramanujan proved that, for example,

$$1 + \frac{e^{-2\pi}}{1+}\,\frac{e^{-4\pi}}{1+}\,\frac{e^{-6\pi}}{1+\cdots} = \left\{\sqrt{\frac{5+\sqrt{5}}{2}} - \frac{\sqrt{5}+1}{2}\right\} e^{2\pi/5}.$$

p. 302. The four square theorem (Ch. 369)

Every positive integer is the sum of four squares (Lagrange's theorem).

Ch. 370. Any prime p is the sum of four squares

Two lemmas leading to proof of the 4-square theorem were given by Ramanujan. (using elliptic fns)

p. 316. A more analytic method, applicable also to representations by an odd number of squares, has been developed by Hardy, Mordell & Ramanujan

Ch. XXII: The series of primes (3)

19/8/72 New Books Section

The Born - Einstein Letters with Foreword by (1916-1955)

(120) Bertrand Russel
(Macmillan & Co, 1971)

X ~~I~~ was rather disappointed to find no mention of my name anywhere – References to Raman & his quarrels with Born in number of letters to Einstein

(2) Collected papers of G. H. Hardy – Vol IV

~~p.50~~ p. 517 – Gives Ramanujan's formula

$$1 - 5\left(\frac{1}{2}\right)^3 + 9\left(\frac{1.3}{2.4}\right)^3 - 13\left(\frac{1.3.5}{2.4.6}\right)^3 + \cdots = \frac{2}{\pi}$$

p. 537 – Another formula

$$\sum \frac{n^{4m+1}}{e^{2n\pi}-1} = \frac{B_{2m+1}}{8m+4}.$$

Vol II, p. 358 – In his remarkable memoir "on certain arithmetical functions" Ramanujan considers, among other fns of much interest the fn $\tau(n)$ defined by

$$f(x) = x\{(1-x)(1-x^2)(1-x^3)\cdots\}^{24} = \sum \tau(n)\, x^n$$

This fn is important in the theory of the representation of a number as the sum of 24 squares.

Vol I . p. 276 (in a joint paper with Ramanujan)

$p(61) = 1121505,\ p(62) = 1300156,\ p(63) = 1505499$

p. 277 - Number $H(x)$ of highly composite numbers not exceeding x appears to be one of extreme difficulty. Mr. Ramanujan has proved, by elementary methods, that the order of $H(x)$ is at any rate greater than that of $\log x$, as great as that of

$$\frac{\log x \sqrt{(\log\log x)}}{(\log\log\log x)^{3/2}}.$$

{Defn of a highly composite number (Dickson, Vol I, p. 323)

A highly composite number N is one for which $\tau(N) > \tau(n)$ [$\tau(n)$ = number $\sigma_0(n)$ of the divisors of n] when $N > n$. If $N = 2^{a_1} 3^{a_2} \dots p^{a_p}$, then $a_1 \geq a_2 \geq a_3 \dots \geq a_p$ while $a_p = 1$ except for $N = 4$ or 36. — The ratio of two consecutive highly composite numbers tends to unity.}

{Dantzig p. 22 — Table of ancient numeration (cut this out)

	1	2	3	4	5	9	10	12	23	60
Sumerian (3400 B.C.)			etc							
Hieroglyphics ("										
Greek	α′	β′	γ′			etc				

(28/8/72) New Books Section

(1) Prospects in Mathematics – Annals of Mathematics Studies by Princeton Univ. Press (1971)

(a) The signature theorem: Reminiscences & Recreation by F. Hirzebruch, pp. 3–33.

Gives an account of the Atiyah–Bott–Singer index and fixed point theorems.

Highly technical & difficult to understand

p. 28 – Gives a diophantine eqn

$$45 b_0 b_1 b_2 b_3 b_4 = 7\left(\sum_{i<j} b_i^2 b_j^2\right) - \sum_i (b_i^2)^2$$

& asks find all solns

Added in proof: using the I.B.M. 7090 Computer at Bonn, Don Zagier found the soln

$$(2, 7, 19, 47, 59).$$

It is the only (non-trivial) relatively prime soln in integers $\leq 10^4$

– Gives 27 references (& big work of Atiyah & others)

(4) Cohomologie des groupes discrets – by Jean-Pierre Serre

- References in two places of M. S. Raghunathan's (?) work

(C) Future extensions of index theory and elliptic operators by O. M. Singer
— A sort of highly technical "popular" paper giving future trends — Anyway appears quite interesting.

Also saw on the cover of the book 38 books of the Annals series one by Abhyankar

(2) Advances in Nuclear Physics Vol 4

(3) Book of wave-systems — Lorden elementary

(4) Ring Theory by Ernst-August Behrens - Acad. Press (Vol 44 in Pure & Appl. Maths

12 Chapters - Basic concepts - Primitive rings - Rings with a faithful family of irreducible modules - Completely reducible modules - Tensor products, fields and matrix representations - Separable algebras - Rings with identity - Frobenius algebras - distributively representable rings - Noetherian ideal theory in nonassociative rings - orders in semi-simple Artinian rings - Rings of continuous fns

may be useful for Calcutta Higher Algebra paper.

—

Saw a lot of books on Space research (obviously because of D being made Chairman of Space Commission)

—

(5) Modern Optics by E. B. Brown – Reinhold Publishing Corporation, 1965

p. 113: dispersion – when a beam of light traverses a ~~to~~ transparent liquid or solid, it is observed that the velocity is a function of the wave length or frequency; usually the velocity is reduced for the higher frequencies. This means that the index of refraction varies with wave length, usually becoming greater as the wave length becomes shorter. This phenomenon is called dispersion

[λ a property of the medium in which light is propagated & $\lambda' = c/\nu\mu$ (μ = index of refraction) – {Also velocity $V = \lambda\nu$} – the velocity changes so that $V = c/\mu$ and frequency remains constant; consequently the wave length changes to $\lambda' = \lambda/\mu$]

– Absorption bands – Figs of typical index of refraction curves – Cauchy's formula for dispersion

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

Behaviour of curves in the vicinity of absorption points is referred to anomalous dispersion – Theoretical Dispersion formulas

$$\mu^2 = 1 - \sum_j^m A_j \lambda^2 / (\lambda^2 - \lambda_j^2)$$

(fails at absorption point $\lambda = \lambda_j$)

– Dispersion as the result of scattering using models of S.H.M

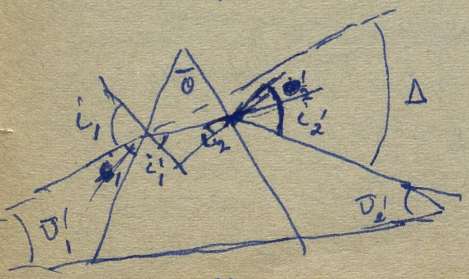
– Dispersion in quantum theory

p. 385–87 Dispersion of a prism – The dispersing action of a prism ~~The change in the deviation~~ is due to the fact of the dependence of index of refraction on wave length



(i)

the angle of refraction for an ~~oblique ray~~ obliquely incident ray increases with wave length. A prism therefore separates a beam of light into its component wave lengths when the light falls obliquely on one or more of the prism surfaces



(ii)

Δ = deviation of prism (Fig ii)

which obviously $= i_1 + i_2' - \theta$ —— (1)

It would be possible to derive a formula for Δ as a fn of i_1, μ, & θ but it would be complicated

Minimum deviation occurs (Fig (i)) when ray inside the prism is $\parallel^l$ to the base & the incident & refracted rays are symmetrical w.r.t prism apex angle. This may be shown by differentiating eqn (1) w.r.t i_1, setting result equal to 0 & substituting the necessary values from the law of refraction

For this case (1) reduces to

$$\Delta = 2i_1 - \theta \quad \text{--- (2)}$$

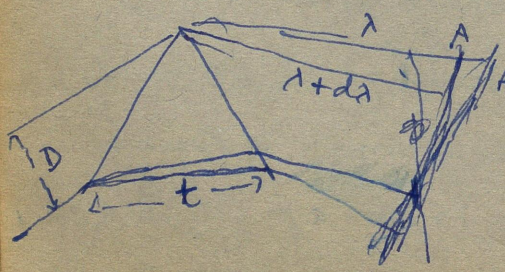
& it is found that $\sin i_1 = \mu \sin \frac{1}{2}\theta$ (3)

Eqns (2) & (3) provide a basis for measuring μ, for they may be combined & written in the form

$$\mu = \frac{\sin\left(\frac{\theta+\Delta}{2}\right)}{\sin \theta/2} \quad \text{—— (4)}$$

<u>Dispersion of a prism</u> – The change in deviation as a fn of the wave length is the <u>dispersion</u> of the prism ie defined as

$$\frac{d\Delta}{d\lambda} = \frac{d\Delta}{d\mu} \cdot \frac{d\mu}{d\lambda} \quad \text{—— (5)}$$



It can be shown in case of minimum deviation that

$$\frac{d\Delta}{d\mu} = \frac{1}{D} \quad \text{—— (6)}$$

& (5) become $\mathcal{D} = \frac{d\Delta}{d\lambda} = \frac{t}{D} \cdot \frac{d\mu}{d\Delta}$ (7)

p. 400-1. Dispersion of a grating

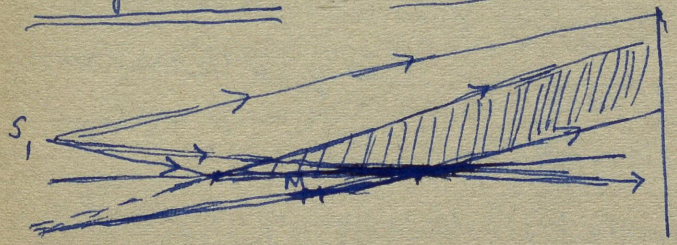
$$D = d\theta/d\lambda = m/d\cos\theta$$

From $d(\sin i - \sin\theta) = m\lambda$, we find with $i = 0$ for normal incidence & $\sin\theta \simeq \theta$ for small θ, we have $\theta \propto \lambda$, this is known as normal dispersion. It is also to be noted that the dispersion of a grating is the reverse of that of a prism – the longer wave lengths have the greater dispersion.

(31|8|72) – Read some portions of Born's Optics & also Modern Optics by E. B. Brown. – re.

Interference

Lloyd's mirror – From Born: p. 262.



The coherent sources are the primary source S_1 & its virtual image S_2 in the mirror. The $\perp^r$ bisector of S_1S_2 then lies in the plane of the mirror surface

(7)

p. 265: with Lloyd's mirror, the fringe which lies in the plane of the mirror surface is an intensity maximum, and with a white light source it appears black. This is because (§ 1.5.2) the phase of the wave reflected from the mirror is changed by π on reflection irrespective of its wave length.

§1.52 (p. 40)

From Fresnel's formulae

$$R_{\parallel} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta + n_1 \cos\theta_t} A'_{\parallel}$$

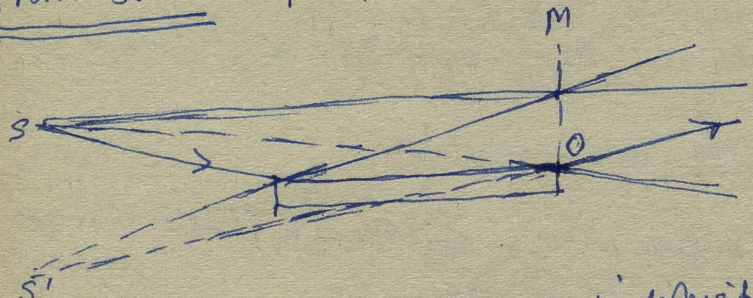
$$R_{\perp} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} A'_{\perp} \qquad \text{——(1)}$$

In case of reflected wave, the phase will depend on the relative magnitudes of θ_i & θ_t. For, if the second medium is optically denser than the first ($\epsilon_2 > \epsilon_1$) then $\theta_t < \theta_i$. According to (1) the signs of $R_\perp$ and $A_\perp$ are different and the phases therefore differ by π

p. 278: for grazing incidence ($\cos\theta_i \to 0$) for which

the reflectivity approaches unity even when n_2/n_1 is large, we get $R_{\parallel} = -A_{\parallel}$, $R_{\perp} = -A_{\perp}$. This corresponds to the case of the Lloyd's mirror

From Brown, p. 79



The viewing plane is taken at M, in which case the point O at the surface of the mirror, corresponds to the 0th order maximum. Observations will show that this is at the centre of a dark fringe, which is explicable only on the assumption that the reflected beam has undergone a phase change of π. It will be remembered that theory indicated this to be the case.

For Lloyd's mirror, the condns of maxima & minima expressed by eqns

Young's expt. $\left\{ \begin{array}{l} \text{for a max. } s \sin\theta = m\lambda \\ \text{" min } s \sin\theta = (2m+1)\lambda/2 \end{array} \right\}$ $\delta = ks$, s = diff of paths

are reversed. In addition this arrangement produces only one-half of a full set of fringes.

1|9|72 — At the Physics Dept. Leiden

Handbook of Physics (McGraw Hill), p. (Chapter on Diffraction & Interference). §6.91.

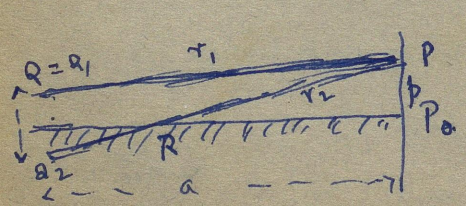
Two-beam Interference: Consider 5 cases (last is Lloyd's mirror)

Path difference = $D = \frac{pd}{a}$.

providing a phase difference $\delta = \frac{D}{\lambda} = \frac{pd}{\lambda a}$

There are bright (Constructive) fringes considered at points

$$\left.\begin{aligned} p_{bright} &= \frac{m\lambda a}{d} \quad m = 0, \pm 1, \pm 2, \\ p_{dark} &= \frac{(m+\frac{1}{2})\lambda a}{d}, \quad m = 0, \pm, \pm 2 \end{aligned}\right\} \text{(A)}$$



(Lloyd's mirror)

one mirror RP_0 in a plane close to Q causing light to P directly and by reflection at almost grazing incidence (This

is different from the remaining 4 types and less symmetric).
ie Q_1 is Q itself; the path Q_1P is always the shorter
and only in the longer path $Q_2P = QRP$ is there a reflection
Regardless of the nature of the reflecting substance, there
is always a ~~change of~~ phase change of π in light
reflected at grazing ~~in~~ incidence; so all the fringes (A)
are shifted by $\lambda a/2d$ and the centre of the system P_0
is dark instead of bright.

~~unlike~~ While in all the other 4 cases, one
part of the original beam is laterally displaced by
its whole width w.r.t. the other, in Lloyd's mirror
the beam is folded back on itself.

4/9/72 ~ Neutrino Astrophysics

(1) High-energy Astrophysics by Trevor C. Weeks
(Chapman & Hall, London ~~1969~~ 1969)

Chap. 10 – Neutrino Astrophysics

Reaction used to detect neutrinos by Cowan & Reines in 1956
(1930, Pauli, 1933, Fermi)

(132) [neutrino interaction weak for the nuclear physicist, but this very weakness makes it important for the Astrophysicist neutrinos are the only form of radiation that escapes from the stellar core, thus providing the sole means of direct investigations of nuclear mechanisms responsible for stellar energy. — As central T^0 increases, rate of neutrino emission increases leading to a catastrophic gravitational collapse.] was

$$\bar{\nu} + p \rightarrow n + e^+.$$

[cross section of this $= 10^{-44}\ cm^2$ ⇒ intense neutr 10^{18} MeV antineutrino flux obtained]

weak interactions

β-decay : $n \rightarrow p + \bar{e} + \bar{\nu}_e$

K-Capture : $e^- + p \rightarrow n + \nu_e$

νchange exchange : $\bar{\nu}_e + p \rightarrow e^+ + n$

Coupling constant (g) giving rise of matrix element for the above reactions is given by

$$g = 1.4 \times 10^{-49}\ ergs - cm^{-2}.$$

cross-section for typical neutrino interactions

is given by $\delta \sim 10^{-44} E_\nu (c.m)/mc^2\ cm^2$

Uncertainty about possible existence of W-Boson; a heavy particle postulated to explain weak interactions. It is produced & decays as follows:-

$$\nu + e^+ \rightarrow W^+ \begin{cases} \rightarrow \text{mesons} \\ \rightarrow \nu_\mu + \mu^+ \\ \rightarrow \nu_e + e^+ \end{cases}$$

Still not detected. Hope that high energy cosmic neutrino exps may lead to it

Solar neutrinos – detection – confirming thermonuclear reactions

Neutrino – electron detectors.

$$\nu_e + e \rightarrow \nu_e + e$$

is predicted by most weak interaction theories, but not yet ~~detected~~ observed in the laboratory.

URCA-process – converting K.E. of initial electron to a neutrino & antineutrino.

Weak interaction theory of Gellmann & Feynman:- predicting many interactions between e & μ which may be impt. in astrophysics.

eg: Photo-neutrino: $\gamma + e \rightarrow e + \nu_e + \bar{\nu}_e$ – Equivalent to compton effect with neutrino pair replacing scattered photon

The Threshold of neutrinos in cosmology (ie background Flux)

Muon neutrinos:

Interactions: $\nu_\mu + N \rightarrow \mu + N$ (N = nucleon) – Elastic collision

$\nu_\mu + N \rightarrow \mu + N + \pi$ mesons – Inelastic collision

Interaction in which W is produced.

– Three ways in which technique relating to ν_e & ν_μ differ.

– Ref. to Kolar Gold field mines (7600 ft. deep)

neutrino emits leading to probability of

$$\nu_\mu + p \rightarrow p + \mu + W$$

$$W \rightarrow \mu + \nu_\mu$$

Astrophysics & neutrinos: – possibility of detection of the W-Boson

Neutrino properties from Astrophysics

	ν_e	ν_μ
Charge (e)	$\leq 10^{-18} e$	$\leq 10^{-12}$
Mag. moment (Bohr m-	$\leq 10^{-10}$	$\leq 10^{-11}$
Charge radius (cm)	$< 4 \times 10^{-15}$	$\leq 4 \times 10^{-15}$

(2) Lectures on High energy Astrophysics

by G. R. Burbidge (T.I.F.R. 1966 Bangalore Symposium)

Mentions an important process

$$e^+ + e^- \rightarrow \nu + \bar{\nu}$$

High energy neutrinos ($E_\nu \geq 100$ MeV) – mentions number of possible interactions eg

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Suggests the reaction

$$\bar{\nu}_e + e^+ \rightarrow \bar{\nu}_\mu + \mu^+$$

may have a resonance & remarks about use of a neutrino telescope having good angular resolution, and that above eqn may enable the devising of such a telescope.

Background flux of thermal neutrinos. – The total mass energy density in the Universe, probably, amounts to 10^{-8} ergs (~ 10 KeV) per cc, and only a few percent appears not to be visible matter condensed into galaxies. If the remainder were in the form of neutrinos, it must have a cosmological origin ie it must have been created

(136) in the early evolution of the Universe, or form the bulk of energy appearing in the steady-state Universe.

From laboratory expts of β-decay spectrum, the energy density of neutrinos is ≤ about 10^{10} erg/cm^3 so that we are very far away (a factor 10^{18}) from setting a meaningful result

Method of Cowsik, Pal & Tandon to set a limit of this energy density: They point out that the high-energy cosmic ray protons will collide with low-energy neutrons & this will give rise to a noticeable change in the form of the cosmic-ray spectrum at energies between 10^{13}–10^{20} eV. They have concluded from the form of this spectrum that the energy density of the neutrino sea is less than ~ 10^4 erg/cm^3

Thus it is not out of question that we are living in a Universe whose main constituent is neutrinos, but, at this point, observation & expt are far apart.

(3) Nuclear Astrophysics by W. A. Fowler – Jayne Lecture for 1965.

p. 39 – About neutron having a magnetic moment –
Perhaps due to neutron itself being made up of still more fundamental particles (quarks?) having bits + & – charge. The charges balance to give a net charge of Zero, but the motions of the charges are such that their magnetic contributions do not cancel & hence the neutron is magnetic

p. 50. Neutrino interaction cross-sections increase roughly as the square of the neutrino energy.

p. 52 – Davis' expt. estimates central temp of Sun by neutrino counting upto 10% – this observation cost about one-half million dollars – justification for this in terms of cost of ordinary thermometers

p. 53: what happens to neutrinos crossing the wide expanse of the Universe! Scientific guesses there are, but none so richly poetical as that of T. S. Eliot in East Coker III (1940)

"O dark dark dark. They all go into the dark
The vacant interstellar space, the vacant into the vacant"

Eliot must be thinking of neutrinos – Steven Weinberg's answer is that $\nu \bar{\nu}$ will be redshifted to low energies by the

Universal redshift & will then constitute a "sea" of low energy particles throughout the Universe which would be very difficult to detect. Eventually, even this will be accomplished

(4) Astrophysics — Progress in Physics Series, Benjamin, 1969
(contributors include J. V. Narlikar) of Cambridge)

pp. 151 - 202 — Astrophysical neutrinos
by M. A. Ruderman (N. Y. Univ)

p. 165 — We assume that the neutrino mass is not larger than a few KeV (the mass of ν_e is known to be < 200 eV, & that of $\nu_\mu < 3.5$ mev); otherwise there is generally not enough energy to create them in stellar interiors where temperatures are typically $10^7 - 10^{80}$ K. In our estimates we take the neutrino mass to be zero

p. 197 — Prospects — Four general remarks.

(5) Enrico Fermi Series Course 35, 1966.

p. 367 – Article by W. A. Fowler on neutrino astrophysics

Is this the same as (3)? No.

p. 397 – The mass density equivalent to the neutrino energy density, is related to the Fermi energy, E_ν by the well-known expression

$$\rho_\nu = \frac{1}{8\pi^2}\left(\frac{E_\nu}{\hbar c}\right)^2\left(\frac{E_\nu}{c^2}\right) \sim 3\cdot 10^{-21} E_\nu^3 \text{ g cm}^{-3}$$

for E_ν in eV. E_ν can be measured in beta-decay & present results set an upper limit $E_\nu < 200$ ev. This corresponds to an upper limit $\rho_\nu < 5\cdot 10^{-18}$ g cm^{-3} which is $\sim 10^{10}$ times the nucleon rest mass density of 10^{-28} g cm^{-3} required in steady state cosmology which in turn may be one or two orders of magnitude greater than that observed in luminous stars. Alternatively it will be noted that $E_\nu \sim 0.01$ eV if the "missing mass" energy is indeed in the form of neutrinos. This is too low for detection in terrestrial labs

However it is possible to set observational limits on the possible neutrino degeneracy in the Universe from

a consideration of the shape of the energy spectrum of the cosmic rays between 10^{10} & 10^{20} eV. Cowsik et al (1964) have pointed out that the low energy neutrinos in a Fermi sea appear to have quite large energies to a high-energy cosmic-ray proton in its rest mass system. Proton–neutrino interactions elastic & inelastic, increase in probability quite rapidly with energy but eventually flatten off. This will have the result that the cosmic-ray energy spectrum should steepen at high energies, and then eventually regain the original shape. Such an effect has been observed at 10^{15} to 10^{16} eV. If this is attributed entirely to proton–neutron interactions then Cowsik et al's show that the height of the Fermi sea is at most a few eV.

6/9/72 Sat down to set the two Calcutta Papers — It is impossible to find what exactly are the papers they want. Is it Pure I ~~or Paper VII~~ of which copy of 1970 set (or Pure I A & B i.e. Abstract Algebra & Metric, affine & projective geometries reply of which syllabi have been sent).

Paper I — Group ~~Part~~ B.

2 Qns from B.C.C's Linear Algebra

3 Qns on Projective Geometry

~~[illegible]~~

1(a) from Wylie Jr (to be borrowed) & also use

2.3(a) S. Suryanarayana Iyer & Annamalai papers

Group A

5 Questions from B.C.C.

Paper VIII — Higher Algebra

[P. T. O

1(a) ~~Define~~ Explain the notions of minimum & maximum conditions satisfied by a poset, and show that such a set can satisfy both the above conditions if and only if every one of its subchains is finite.

[Gabor Szasz – Intn to Lattice Theory pp. 24–25]

(b) Define a lattice, and verify that there are only five non-isomorphic lattices with exactly five elements. Also draw their Hasse diagrams

[Intn to Lattice Theory by D. E. Rutherford, p. 4]

2(a) Find the lattices all subsets of which are sublattices [Szasz, Ex. 7, p. 56 – Answer on p. 210 viz. they are precisely the Chains]

(b) Show that the lattice of invariant subgroups of any group is modular. [Jacobson, p. 184]

(c) State and prove the Jordan-Hölder theorem relating to modular lattices. [Rutherford or McCoy & Burkhardt]

3(a) Define a noetherian ring, and state and prove Krull's theorem relating to an associative ~~and~~ commutative noetherian ring with identity.

[Ring Theory by Behrens, Th. 18, p. 258

(b) If R be a semi-simple ring, and $M \neq 0$ a finitely generated R-module, show that M is a finite direct sum of simple submodules.

[D. G. Northcott's Lessons on ring modules etc., Th. 19, p. 54.

4 (a) Explain the notions of primitive, prime and primary ideals in a commutative ring, and prove that

(i) a primitive ideal is prime [Behrens, p. 67

(ii) every ideal in a noetherian ring is a finite intersection of primary ideals [Jacobson, p. 176]

(b) (iii) the radical of a primary ideal is prime

[Northcott, p. 97]

12/9/72 – Returned Born's Optics & C.R. Wylie's Projective Geometry

(1) ✓✓ || – Looked at some small books in the Reference Section about Mathematics – may be used in connection with books for Children's Book Trust

(2) Rev. Mod. Phys – April 1972, p. 250

Current status of lepton g-factors (See p. 132 of this note book)

by A. Rich & J. C. Wesley.

(3) Progress of Theoretical Physics – Feb, 1972, p. 671

A class of linear relativistic wave equations describing particles with spin 1/2 by P.S. Chandrasekharan, Nalini B. Menon & T.S. Santhanam (All of Madras)

In the bibliography the authors give references to all kinds of papers including Bhatia, Harish-Chandra & Alladi (Grammar of Divine Matrices), Duffin, Kemmer, Fierz & Pauli – all except myself as if it were a sort of conspiracy against me! – most deplorable

X

6/10/72 New Books Section

(1) Lectures on rings and modules ~ Lecture Notes in Maths No. 246 - Springer Verlag - Inlaw Univ Ring & Operator Theory Year 1970-71. - Vol I. (Nearly 700 pages) - There are two more Vols to come!

From Preface: Vol I - represents the purely algebraic side of the program with lectures on in the structure theory of rings & modules.

Vol II - Collected contributions to the analysis ~~program~~ section of the program with - lectures on operator algebras and applns to the theory of operators in Hilbert space and to harmonic analysis.

Vol III - contains applications of topology to ring theory in the form of selected lectures in the theory of representation of rings by sections in sheaves (?)

Vol I appears to be readable & understandable with a lecture on "Structure Noetherian rings"

(2) Seminars in Maths - Vol 12 (Russian) - Investigations in the theory of stochastic processes by edited by Sudakov

(3) Applied magneto-hydrodynamics (Russian translation)
(AEC-tr-3602) - U.S. Atomic Engy Commission -
A series of 9 articles
Not a single reference to any Indian work.

(3) Advances in Applied Mechanics by Edited by Chia-Shun-Yih
Vol II (College of Engg, Univ of Michigan, A.P. 1971)
(meant for non-experts to be used even for enjoyment)

Contributions: (i) Hydromechanics of swimming fishes and cetaceans by H. Yao-Tsu Wu

(ii) Biomechanics: A survey of the blood flow problem by Y. C. Fung

(iii) Two dimensional shock structure in transonic & hypersonic flow by Martin Sichel

(iv) Non-linear theory of random vibrations by T. K. Caughey.
- Two references to Ariaratnam (?)

(v) Physical theory of plasticity by T. H. Lin.
(A reference to Sinnotsenko's Mech of Elasticity)

(vi) Thermodynamic theory of viscoplasticity by Piotr Perzyna
[Ref. to Bridgman: Rev. Mod. Phys 22, 56-63.
[Ref. to two papers of S. K. Mitra (Trans ASME 224 & 227

(4) Compressible fluid dynamics by philip. a. thompson (all small letters !)

– McGraw Hill, 1972 (Reference only)

– Book introducing fundamentals of compressible fluid mechanics or gas dynamics

(5) Matrices in Control Theory by Stephen Barnett (van Nostrand Reinhold Co, 1971.

Quotation : He who does not increase his knowledge, decreases it. Say not, when I have leisure I will study; perchance thou will have no leisure.

Hillel.

– Very interesting book with no. of examples with answers & dealing with all kinds of topics for applications. Also talks of Smith's form – Very interesting book

Chap I – Polynomial matrices

" II – Polynomials

" III – Rational matrices

" IV – Stability & Inertia (Liapounov Theory)

" V – Matrix Riccati eqn

" VI – Generalised inverses (applns to linear programming)

Chap. VII – Unimodular matrices

~~Chapter VIII~~ – with 5 appendices

(6) Book referred to in my notes of 12/9/72 viz

Putting mathematics to work – Marshall Cavendish Books, London W1 (Learning system)

Men must measure – two and two make why – Man-made brains – damned lies & statistics – Mathematics in industry – Mathematics in art – video

– Beautiful illustrations

12/10/72 (1) Brown: Modern optics

p. 111: interaction of light & matter – 3 categories viz dispersion, absorption, dispersion and scattering.

(1) Photon retains its original energy, as does the particle and the photon continues on, possibly in a new direction.

(2) Photon gives up parts of its energy to the particle, and proceeds onwards, usually in a new direction, with less energy and hence with lower frequency & longer wave length.

(3) The photon is completely absorbed – it gives up all of its energy to the material particle.

In (1) photon merely bounces off the particle and continues on, undisturbed as to energy, but perhaps changed in direction. We have scattering, if scattering particles are small w.r.t a wave length we have Rayleigh scattering & if they are large we have Mie scattering. This phenomenon also accounts for dispersion & for index of refraction

In (2) photon becomes a new photon of lower energy. If the energy is given up to produce an increase in the K.E. of the particle, the particle does not change its quantum state, and we have Compton scattering. If the particle absorbs its part of the energy by undergoing a change of state, we have Raman scattering.

In (3) the particle absorbs all the energy of the incident photon and undergoes a change of state. This takes place only when the energy of the photon corresponds to the energy required for specific changes to take place; in other words, this phenomenon occurs only in specific frequency bands of the incident light.

After absorption of the photon, it may be reemitted at the same frequency almost immediately, in which case we have

resonance radiation; it may be emitted at a different frequency, as the particle spontaneously reverts to a lower energy level in which case we have flourescence; it may be emitted with a ~~lower~~ different frequency after a long interval of time (say 10^{-8} secs ~ 10 secs) in which case we have phosphorescence, or the energy absorbed may be sufficient to ionise the atom, and eject an electron, which then takes up the excess energy in the form of K.E., in which case we have photoelectric emission one ~~or~~ other rare ~~occurrence~~ event may take place; the incident photon may be annihilated & converted to a positron-electron pair. This is referred to as pair-production

p. 122: Flourescence in short interval of order of 10^{-8} Secs; in Phosphorescence time interval may be long, perhaps hours (observing what is written above is not in defining flo & phos is wrong! Author a big fool)

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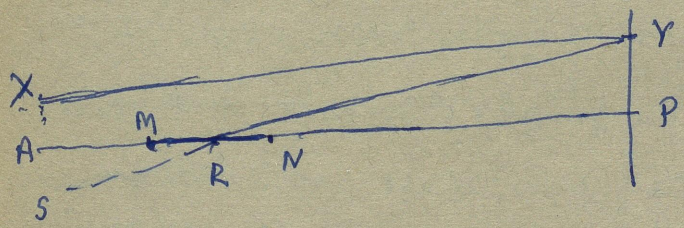
(2) Theory & interpretation of flourescence & phosphoresence by R.S. Becker (Wiley Interscience, 1969)

[with 525 References at end of the Book]

(3) A text book of light by Middleton., 1939 (Bell & Sons)

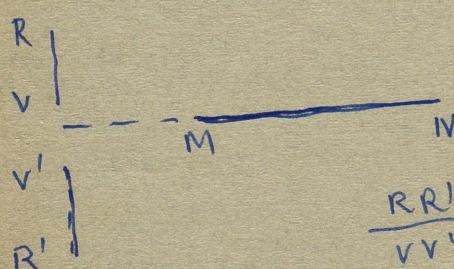
p. 225. Lloyd's single mirror experiment



By arranging that light from a slit X should fall on a plane mirror MN at practically grazing incidence, Lloyd obtained interference bands. The interference occurs between the direct rays from X and the reflected rays which appear to come from S, the image of X formed by the mirror. The slit is arranged to be parallel to the mirror and $\perp^r$ to the line MP. Since X is slightly above the level of the mirror no reflected rays reach the screen at P; which lies along MN produced. This means that no central band can be observed. To produce such a band a thin sheet of mica is placed in the path of the direct rays to the screen so that the lengths of the paths XY and XRY may be optically equal. Then a point such as Y is the centre of the system of interference bands. The central band is found by experiment to be dark with bright bands on each side, so that for no

path difference the two interfering rays are out of phase. This indicates that the reflected ray must have its phase changed by 180° when reflection occurs.

In Young's slits, Fresnel's mirrors and Biprism the distance between the fringes depends on λ, so that if white light is used the fringes, apart from the very central ones, are coloured & overlap. By Lloyd's mirror, it is possible to produce achromatic fringes ie fringes which are successively white & black. This is due to the lateral inversion of the image. If a spectrum is formed at RV by use of a diffraction grating, the reflected image is R'V'. Then provided we make



$\frac{RR'}{VV'} = \frac{\lambda_{red}}{\lambda_{violet}}$, we shall have $\frac{\lambda}{d}$ constant for all wavelengths, since in the spectrum formed by the diffraction grating the dispersion is proportional to the wavelength. Then the distance between the fringes will be the same whatever wavelength we use, for this distance is $\frac{\lambda D}{2d}$ and λ/d is const.

So if a piece of red glass is placed in the path of the light, red and black fringes result, while using blue glass, blue & black fringes ~~result~~ are caused, the distance between successive bands always being the same. Hence taking white light, black & white bands are obtained

(Width $\underline{d} = VV'$ and $D = PA$

---

p. 167 — Flourescent substances ~~Quinine~~ sulphate,

~~Flourspar~~ ~~Actual~~ Substance	Actual colour	Flourescent light
Quinine Sulphate	Colorless	Blue
Flourspar	"	Violet
Vaseline	Yellow	Green
Chlorophyll	Green	Red

Common phosphorescent substances are Calcium sulphide, barium sulphide & strontium sulphide - Becquerel's expt.

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(154) (13/10/72) Maths for Children's Book Trust

(A) Putting Mathematics to work (Gen. Sechn. Ref. only)

(a) Introduction:- Maths. plays a significant part in the life of every child, both in the home and at school, more time is devoted to it than any other subject except, perhaps, the natural language, and many educationists are convinced that mathematics ought to be a compulsory subject for all children until they leave school.

Importance of maths in everyday life, in industry, in the university, in the world of art & craft, and the immensely important part which is now being played by the large digital computers.

(b) Man must measure - p. 14 - Remarks on universal system.

(c) Two & two make why - Mention of decimal system - Pictures on pp. 22 & 23 - Counting by weight (Last paragraph)

(d) Man-made brains - Too complicated.

(e) Damned lies & statistics - Do -

(f) Mathematics in industry:- Use in engineering - From end of p. 61 to end of p. 62; specially the last two paragraphs.

(g) Mathematics in art:- The whole thing is interesting, specially

mention of golden section & its use by Leonardo da Vinci - only Indian counterparts have to be found, perhaps as in the book on which "Smarahara" is mentioned.

(2) 19/10/72 (1) Returned E.V.K's article (with Sen) on Knots with written comments saying it is suitable for publication, etc

(2) Borrowed to books on Artificial Intelligence

20/10/72 - Attended lecture at Raman Inst. by one V. Chandrasekharan of Ohio Univ. on Artificial Intelligence & Agnosticism = Stupid lecture - made only one comment.

21/10/72 → Some wild idiotic word-hunting → (at home

ab

Cab
Dab (Expert person)
Gab (Chatter)
Jab (Stab)
Lab
Mab (any fairy)
Nab (Hill top)
Sab (Scot. for Sob)

Tab (small strap)

only 9.

ac Lac
Sac (~~bag~~ bag to contain liquid)

only 2

(156) ad

Bad
Cad
Dad
Fad
Gad (Ramble in speech)
Had
Lad
Mad
Pad
Rad (Short for radical)
Sad
Tad (Street boy U.S)
Wad (matter for packing)

13

ag

Bag
Dag (Dagger)
Fag
Gag
Hag
Jag (Saddle bag)
Lag
Mag (Chatter)
Nag
Rag
Sag
Tag
Vag (Turf for fuel)
Wag (a wit)

14

am

Bam (slang for hoax)
Cam
Dam
Gam (Herd of whales)
Ham
Jam
Kam (Crooked)
Lam (To beat)
Nam (Distraint) & seizure of goods)
Pam (Knave of Clubs)
Ram
Sam (together)
Yam (large root)

an

Ban
Can
Dan (A title = "Sir")
Fan
Han (Pl. of have)
Man
Pan
Ran
Tan
Van
Wan (Languid; Sloomy)

11

ap

Cap
Dap (drop bait gently into water)
Fap (Drunk)
Gap
Hap (A cloak or covering)
Jap
Lap
Map
Nap
Pap (woman's breast or nipple)
Rap
Sap
Tap
Wap (Strike)

13

Yap (Yelp, Bark contents)

15

ar

Bar
Car
Ear
Far
Gar (Kind of fish)
Jar
Lar (House deity)
Mar
Oar
Par
Tar
War

12

at

Bat
Cat
Eat
Fat
Gat (A strait)
Hat
Kat (Egy. unit of wt)
Lat (pillar in Ind. archi)
Mat
Oat
Pat
Rat
Sat
Tat
Vat (Vessel for liquor)
Wat (Drunken)

16

ay

Bay
Cay (low islet)
Day
Fay (fairy)
Gay
Hay
Jay (Bird of Crow family)
Kay (Same as Cay)
Lay
May
Nay (no)
Pay
Ray
Say
Way

15

[illegible]
Sed

aw

Caw (Cry as a crow)
Daw (Jackdaw)
Faw (Gypsy)
Haw (Hedge)
Jaw
Kaw (Same as Caw)
Law
Maw (Stomach in lower animal)
Paw
Raw
Saw
Taw (Chosen marble in marble game)
Yaw (To move unsteadily)

13

ed

Bed
Fed
Ged (Pike or luce)
Led
Ped (a basket)
Red
Ted (Spreading grass)
Wed
Zed (Bar of Metal of form Z)

9

[illegible]

en

Den
Hen
Men
Pen
Ten
Yen
Ben (mountain peak)
Een (contraction of even)
Fen (Morass or Bog)
Ken
Sen (Jap. copper coin)
Wen (Cyst in scalp)

12

et

Bet
Fet (Fetch)
Get
Jet
Ket (matted wool)
Let
Met
Net
Pet
Ret (Expose flax etc to moisture)
Set
Vet (abbr. Veterinary)
Wet

13

id

Bid
Cid (Chief-)
Did
Fid (conical part of hard wood)
Gid (Staggers or sturdy in sheep)
Hid
Kid
Lid
Mid (Middle)
Rid
Tid (Fit time)

11

ig

Big
Dig
Fig
Gig (Kind of carriage)
Jig (quick dance)
Lig (To lie)
Pig
Rig
Tig (Game of tag)
Wig

10

in

Bin
Din
Fin
Gin
Kin
Pin
Sin
Tin
Win
Hin (Hebrew liquid measure)
Lin (to quiver)

11

ip

Dip
Hip
Lip
Nip
Sip
Tip
Zip (Bullet screw)
Rip
Pip (Seed of fruit)
Kip (Play truant)
Gip (Same as Gyp)

11

it

Bit
Fit
Hit
Kit
Lit
Pit
Sit
Wit
Dit (a little prem)
Nit (Egg of a louse)
Rit (to strike)
Tit (a small bird)

12

on

Bon
Don
Son
Ton
Won
Eon
Con (to study carefully)
Fon (A fool)
Jon
Non
Yon (yonder)

11

ot

Cot
Dot
Got
Hot
Lot
Not
Pot
Rot
Sot (habitual drunkard)
Bot (larvae of botfly)
Jot (an iota)
Mot (pithy saying)
Tot (to add)
Wot (to know)

14

(160)

ow

Bow
Cow
How
Low
Now
Row
Sow
Dow (to be good or able)
Jow (Stroke of a bell)
Mow (cut with a scythe)
Tow (Pull with rope)
Vow

12

un

Bun
Fun
Gun
Hun
Nun
Pun
Run
Sun
Dun (of a dark colour)
Mun (Man)
Tun (large Cask)

11

ug

Bug
Dug
Hug
Jug
Mug
Rug
Tug
Lug (To pull along)
Pug (a monkey)

9

ut

But
Cut
Gut
Hut
Jut
Nut
Out
Put
Rut
Tut (exclamation of rebuke)

10

End of the nonsense

26/10/72

(1) Concepts in Hadron Physics — Edited by Paul Urban

Springer Verlag — International Conference held in Austria 1971

(2) Advances in Astronomy & Astrophysics — Edited by Zdeněk Kopal (A.P. 1972) — Vol. 9 — 4 Contributions

(3) Appl. Math. for Engrs & Physicists by L. A. Pipes

p. 398. In sph. polar coords

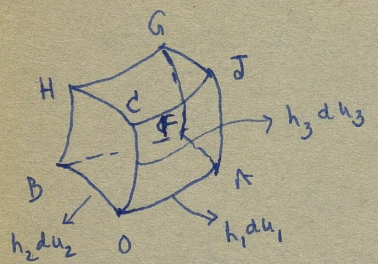
$$\left.\begin{aligned}
(\nabla \times \vec{A})_r &= \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] \\
(\nabla \times \vec{A})_\theta &= \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial (r A_\phi)}{\partial r}\right] \\
(\nabla \times \vec{A})_\phi &= \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial\theta}\right]
\end{aligned}\right\} \quad (1)$$

To be deduced from (general coords)

$$\left.(\nabla \times \vec{A})_1 = \frac{1}{h_2 h_3}\left[\frac{\partial}{\partial u_2}(A_2 h_3) \frac{\partial}{\partial u_3}(A_3 h_2)\right]\right\} \quad (2)$$

— — —

where $ds^2 = h_1^2 du_1^2 + h_2^2 du_2^2 + h_3^2 du_3^2$



The component 1 of curl $\vec{A}$ is obtained by applying Stokes' theorem to the surface OBHC. We calculate

$$\oint_{OBHC} A \cdot dl = \int_O^B \vec{A} \cdot dl + \int_B^H \vec{A}\, dl + \int_H^C \vec{A}\, dl + \int_C^O \vec{A}\, dl$$

$$= A_2 h_2 du_2 + A_3 h_3 du_3 + \frac{\partial}{\partial u_2}(A_3 h_3)\, du_3 du_2$$

$$- \left[A_2 h_2 du_2 + \frac{\partial}{\partial u_3}(A_2 h_2)\, du_2 du_3 \right] - A_3 h_3 du_3$$

$$= \left[\frac{\partial}{\partial u_2}(A_3 h_3) - \frac{\partial}{\partial u_3}(A_2 h_2) \right] du_2 du_3 \quad \text{--(2')}$$

By Stokes' theorem this equals the 1 – component of the curl of $\vec{A}$, $(\nabla \times \vec{A})_1$, multiplied by the area of the face OBHC. That is

$$(\nabla \times A)_1 h_2 h_3 du_2 du_3 = (2')$$

thus getting the 1st eqn of (2)

For cylindrical coords: $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$

i.e. $u_1 = r$, $u_2 = \theta$, $u_3 = z$, $h_1 = 1$, $h_2 = r$, $h_3 = 1$

∴ hence using (2) we get

$$\left.\begin{aligned}(\nabla \times \vec{A})_r &= \frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\\ (\nabla \times \vec{A})_\theta &= \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\\ (\nabla \times \vec{A})_z &= \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\end{aligned}\right\} \quad (3)$$

In Sph. polar coors; $ds^2 = dr^2 + r^2 \sin^2\theta\, d\phi^2 + r^2 d\theta^2$

$u_1 = r$, $h_1 = 1$

$u_2 = \theta$, $h_2 = r$

$u_3 = \phi$, $h_3 = r\sin\theta$

and (2) gives (1)

Put this as Q. 3 of Andhra B.E. IV as follows (for May 73) :–

State and prove Stokes' theorem, and use it to find the cond expressions for the components of $(\nabla \times \vec{A})$ in orthogonal curvilinear coordinates $(u_1 u_2 u_3)$ for which

$$ds^2 = h_1^2 du_1^2 + h_2^2 u_2^2 + h_3^2 u_3^2.$$

for September – Ask the other part deducible from Gauss' theorem (without asking ~~proof of~~ statement of) mentioning Gauss' theorem – Add deduction of expn for $\nabla^2\phi$ in sph. polar coordinates.

(1164) May

Q.1(a) Show that the vector triple product

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}.$$ [Stein pp. 24–26]

[or Ex. 15, p. 31]

~~(b) Show that~~

~~(b)~~ and deduce that

$$\vec{d} \times (\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}](\vec{a} \cdot \vec{d})$$

where the square bracket represents the scalar triple product.

{ [Ex. 2, p. 26 of Stein

$$\vec{d} \times (\vec{a} \times \vec{b}) = (\vec{d} \cdot \vec{b})\vec{a} - (\vec{d} \cdot \vec{a})\vec{b}$$

Hence L.H.S $= (\vec{d} \cdot \vec{b})\{\vec{a} \cdot (\vec{a} \times \vec{c})\} - (\vec{d} \cdot \vec{a})\{\vec{b} \cdot (\vec{a} \times \vec{c})\}$

$= 0 + (\vec{a} \cdot \vec{d}) \cdot [\vec{a}\,\vec{b}\,\vec{c}]$ }

(b) Show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is a vector lying along the line of intersection of the plane of $\vec{a}$ and $\vec{b}$, and the plane of $\vec{c}$ and $\vec{d}$ [Stein Ex 1.8.1, solved p. 26]

[

~~Q.1~~ September

Q.1(a) Define the scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}]$ of three vectors

$\vec{a}$, $\vec{b}$ and $\vec{c}$, and show that

(i) ~~$[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{b}\,\vec{a}\,\vec{c}]$~~

(ii) $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$

(Stein p. 24)

(iii) $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$

(Stein unworked Ex. 4, p. 26)

$$\text{L.H.S} = (\vec{a}\times\vec{b})\cdot\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\}$$
$$= (\vec{a}\times\vec{b})\cdot\{[\vec{b}\,\vec{c}\,\vec{a}]\,\vec{c} - [\vec{b}\,\vec{c}\,\vec{c}]\,\vec{a}\}$$
$$= \{(\vec{a}\times\vec{b})\cdot\vec{c}\}\{[\vec{b}\,\vec{c}\,\vec{a}]\} \qquad [\vec{b}\,\vec{c}\,\vec{c}] \to 0$$
$$= [\vec{a}\,\vec{b}\,\vec{c}]^2$$

(b) Show that ~~for four vectors~~

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c})$$

[Stein Ex. 1.8.1 first part]

Q.2 (a) Prove the following relations:

[May] (i) $\nabla\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot(\nabla\times\vec{a}) - \vec{a}\cdot(\nabla\times\vec{b})$,

(ii) $\nabla(\vec{a}\cdot\vec{b}) = \vec{a}\times(\nabla\times\vec{b}) + \vec{b}\times(\nabla\times\vec{a}) + (\vec{a}\cdot\nabla)\vec{b} + (\vec{b}\cdot\nabla)\vec{a}$

} Stein pp. 50-51

(166) (b) If $\vec{v}$ be a constant vector, show that $\nabla \times (\vec{v} \times \vec{r}) = 2\vec{v}$

From $\nabla \times (A \times B) = \vec{A}(\nabla \cdot \vec{B}) + (\vec{B} \cdot \nabla)\vec{A} - \vec{B}(\nabla \cdot \vec{A}) - \vec{A}(\nabla \cdot \vec{B}) - (\vec{A} \cdot \nabla)\vec{B}$

putting $A = \vec{v}$, $B = \vec{r}$, we have $\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \dots = 0$

$$(\vec{B} \cdot \nabla)\vec{A} = x\frac{\partial v_1}{\partial x} + y\frac{\partial v_2}{\partial y} + z\frac{\partial v_3}{\partial z} = 0.$$

Hence we req. expn $= \vec{v}(\nabla \cdot \vec{r}) - (\vec{v} \cdot \nabla)\vec{r}$

$$= 3\vec{v} - \left(v_x \frac{\partial}{\partial x} + \dots\right)(\vec{i}x + \vec{j}y + \vec{k}z)$$

$$= 3\vec{v} - (\vec{i}v_x + \dots) = 3\vec{v} - \vec{v} = 2\vec{v}$$

[Stein, unworked Ex. 9, p. 51]

[Change $\vec{v}$ to $\vec{u}$

Q. 2 (a) Prove the relations:

Sept] (i) curl grad $\phi = 0$,

(ii) div curl $\vec{a} = 0$

(iii) $\nabla \times (\nabla \times \vec{a}) = \nabla(\nabla \cdot \vec{a}) - \nabla^2 \vec{a}$.

(b) If ϕ_1 and ϕ_2 be two arbitrary functions, prove that

$$\text{div}(\text{grad}\,\phi_1 \times \text{grad}\,\phi_2) = 0$$ [Stein, Ex 18, p. 59

[using $\nabla \cdot (A \times B)$

$$= B \cdot \nabla \times A - A \cdot \nabla \times B$$

the given expn grad $\phi_2 \cdot$ curl grad ϕ_1 − grad $\phi_1 \cdot$ curl grad ϕ_2

$= 0$].

Q.4) (A) 4 questions on complex variable (Qns 5, 6, 7, 8)

(1) Cauchy-Riemann eqns & operations with complex numbers.

(2) Cauchy's integral formula

(3) Residues & contour integration

(4) Conformal mapping.

one question on second order tensors (Q.4)

two questions on Laplace transformation: ie 10 altogether (Qns. 9 & 10).

Q.9 (a) Find the Laplace transform of the function t^n $(n > -1)$

May

(b) Find the inverse Laplace transform of $\dfrac{s+a}{(s+a)^2+b^2}$.

(c) Solve by the L.T. method the linear differential equation

$$\ddot{x} + 4\dot{x} + 4x = 4e^{2t}$$

with the boundary conditions. $x(0) = 1$, $\dot{x}(0) = -4$

[Miller pp. 205, 207]

(168)

10(a) Find the Laplace transform of the function $e^{at}\sinh at$.

Sept (b) If $\mathcal{L}\{f(t)\} = F(s)$, find the transforms of $f'(t)$, $f''(t)$ & $f'''(t)$.

~~Find the inverse Laplace transform of~~ $b/(s^2+b^2)$

(c) Solve by the L.T. method the linear differential equation

$$y''' + 2y'' - y' - 2y = 0$$

with the boundary conditions, $y(0) = y'(0) = 0$ & $y''(0) = 6$, and the dashes denoting differentiation with respect to t.

[(a) $\sinh at = \frac{1}{2}(e^{at} - e^{-at})$ & hence $\mathcal{L}(\sinh at)$

$$= \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right) = \frac{1}{2}\,\frac{2a}{s^2-a^2} = \frac{a}{s^2-a^2}$$

(b). Miller p. 205

(c) $\mathcal{L}(y') = sY(s) - y(0) = sY(s)$

$\mathcal{L}(y'') = s^2Y(s) - sy(0) - \dot{y}(0) = s^2Y(s)$

$\mathcal{L}(y''') = s^3Y(s) - s\mathcal{L}(y'') - y''(0)$

$= s^3Y(s) - 6$

Hence diff. eqn $\rightarrow s^3Y(s) - 6 + 2s^2Y(s) - sY(s) - 2Y(s) = 0$

ie $Y(s)\{s^3 + 2s^2 - s - 2\} = 6$

$$Y(s) = \frac{6}{s^3+2s^2-s-2} = \frac{6}{(s-1)(s+1)(s+2)}$$

$-8+8+2-2$

Resolving into partial fractions putting

$$\frac{6}{(s-1)(s+1)(s+2)} \equiv \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$6 \equiv A(s+1)(s+2) + B(s-1)(s+2) + C(s^2-1)$$

ie $A + B + C = 0$.

$3A + B = 0$ $\quad$ or $\quad$ $3A + B = 0$

$2A - 2B - C = 6$ $\quad$ $3A - B = 6$

$6A = 6$

$A = 1$

$B = -3$

$C = 2$

Hence $Y(s) = \dfrac{1}{s-1} - \dfrac{3}{s+1} + \dfrac{2}{s+2}$

Hence $y(t) = e^{t} - 3e^{-t} + 2e^{-2t}$. Miller Ex. 8, p. 210 unworked]

10 (a) Show ~~Find the Laplace transform~~ that if $\mathcal{L}\{f(t)\} = F(s)$ then

May

(i) $\mathcal{L}\left[\int_a^t f(t)\,dt\right] = \frac{1}{s}\int_a^0 f(x)\,dx + \frac{1}{s}F(s)$

See p. 17 of new note book

(ii) $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)\,du$ [Miller, p. 202] [p. 204]

(b) ~~Find~~ the Laplace transform of the square wave of period T defined

Show that by

$x(t) = +h, \; 0 < t < T/2$

$x(t) = -h, \; T/2 < t < T$

$x(t) = 0$, otherwise

Replace by Q. at bottom of p. 3 of new note book giving answer

given by $X(s) = \frac{h}{s}\tanh\frac{sT}{4}$ [Miller pp. 219-20]

5(b) using polars in the xy-plane.

$$V = \int_0^a \int_0^{2\pi} z\, r\, dr\, d\theta = \int_0^a (a^2 - r^2)\, r\, dr \int_0^{2\pi} d\theta$$

$$= 2\pi \int_0^2 (4 - r^2)\, r\, dr$$

$$= 2\pi \left[\frac{a^2 r^2}{2} - \frac{1}{4} r^4 \right]_0^a = 2\pi \left[8 - 4 \right] = 8\pi$$ ✓

$$= 2\pi \int_0^a \sqrt{a^2 - r^2} \cdot r\, dr \qquad r = a \sin\theta$$

$$= 2\pi \int_0^{\pi/2} a\cos\theta \cdot a\sin\theta \cdot a\cos\theta \cdot d\theta = 2\pi a^3 \int_0^{\pi/2} \sin\theta \cdot \cos^2\theta \cdot d\theta$$

$$= 2\pi a^3 \cdot \left[-\frac{\cos^3\theta}{3} \right]_0^{\pi/2} = \frac{2\pi a^3}{3}$$

~~(a) If $\mathcal{L}\{f(t)\} = F(s)$, show that~~

[Sept]

~~$$\int_s^\infty F(z)\,dz = \mathcal{L}\left[\frac{f(t)}{t}\right]$$~~ [Murray p 206]

~~(b) One question on a pair of simultaneous linear equations of first order~~ [Q. at bottom of p. 3 of new note book]

31/10/72 At Dept.

Q. 3 (a) Curvilinear coordinates from L. A. Pipes, p. 354

[Sept] To calculate $\operatorname{div}\vec{A}$, we use Gauss's Theorem

$$\iiint (\nabla\cdot\vec{A})\,dV = \int_S\int \vec{A}\cdot d\vec{S} \qquad \text{---(1)}$$

The contribution to $\iint_S$ through the area OBHC taken in terms of outward normal is $-A_1 h_2 h_3\, du_2 du_3$ while that through the area AFGJ is

$$\left[A_1 h_2 h_3\, du_2 du_3 + \frac{\partial}{\partial u_1}(A_1 h_2 h_3)\, du_1 du_2 du_3\right]$$

From this & corresp. expns for the other two pairs of surfaces we have by (1)

$$\lim_{V\to 0} \iiint \nabla\cdot\vec{A}\cdot dV = (\nabla\cdot\vec{A})\, h_1 h_2 h_3\, du_1 du_2 du_3$$
$$= \iint \vec{A}\cdot d\vec{S}$$

we thus obtain

$$\nabla\cdot\vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 A_1) + \text{--} + \text{--}\right] \qquad \text{---(2)}$$

I think this very unfair criticism for the author has not obviously understood the method of proof.

15|6|72 – After nearly 2 two months

(1) Sir some worked examples etc. for Andhra Geophysics Math. Paper which I had set

(2) Andhra. Mech & Oce – Paper I, Math.

2.5(a) Finding vol of ellipsoid $x^2/a^2 + - = 1$

Bewry, worked Ex. 3, p. 595.

Vol of octant $= \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} \int_0^{c\sqrt{1-x^2/a^2-y^2/b^2}} dx\,dy\,dz$

$= \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} c\sqrt{1-x^2/a^2-y^2/b^2}\,dx\,dy$

$= \frac{c}{b}\int_0^a \left[\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} \sqrt{b^2(1-x^2/a^2)-y^2}\,dy\right] dx$

$= \frac{c}{b}\int_0^a \left[\frac{y}{2}\sqrt{b^2(1-x^2/a^2)-y^2} + \frac{1}{2}b^2(1-x^2/a^2)\sin^{-1}\frac{y}{b\sqrt{1-x^2/a^2}}\right]_0^{\frac{b}{a}\sqrt{a^2-x^2}} dx$

$= \frac{c}{b}\int_0^a \frac{1}{2}b^2(1-x^2/a^2)\frac{\pi}{2}\,dx = \frac{\pi bc}{4}\int_0^a (1-x^2/a^2)\,dx$

$= \frac{\pi bc}{4}\cdot\frac{2}{3}a = \pi abc/6 \quad \therefore \text{whole vol} = 8\times\frac{\pi abc}{6} = \frac{4}{3}\pi abc$

If $\vec{A} = \nabla\phi$, then

$$\nabla^2\phi = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\cdot\frac{\partial\phi}{\partial u_1}\right) + \cdots\right]$$

In sph. polar coords, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

$u_1 = r$, $u_2 = \theta$, $u_3 = \phi$ & $h_1 = 1$, $h_2 = r$, $h_3 = r\sin\theta$

$$(\text{grad } V)_r = \frac{\partial V}{\partial r},\quad (\text{grad } V)_\theta = \frac{1}{r}\frac{\partial V}{\partial\theta},\quad (\text{grad } V)_\phi = \frac{1}{r\sin\theta}\cdot\frac{\partial V}{\partial\phi} \qquad (3)$$

[From the figure for curvilinear coords in general case

$$(\text{grad }\phi)_1 = \lim_{du_1\to 0}\frac{\phi(A)-\phi(0)}{h_1\,du_1} = \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}$$

& similarly $(\text{grad }\phi)_2$ & $(\text{grad }\phi)_3$]

$$\nabla\cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

[From (2) $\nabla\cdot A = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 u_1) + \cdots + \cdots\right]$

$$= \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta\, u_1) + \frac{\partial}{\partial\theta}(r\sin\theta\cdot u_2) + \frac{\partial}{\partial\phi}(r u_3)\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_1) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\cdot u_2) + \frac{1}{r\sin\theta}\frac{\partial u_3}{\partial\phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\cdot A_\theta) + \frac{1}{r\sin\theta}\cdot\frac{\partial A_\phi}{\partial\phi}$$]

be 92 non-uniform r fcp.

(3) Ibid – Dec, 1971 (Vol. 292, No. 6) – Special issue on Modern Dimensional Analysis; similitude & similarity

(4) American Mathematical Monthly (Vol 79, No. 1, Jan, 1972) pp. 2-43 – Article by T. L. Saaty of Pennsylvania on "Thirteen Colorful variations on Guthrie's four-colour conjecture" (Dedicated to the memory of O. Ore)

[This is my co-examiner on Saxena's thesis]

(5) Math. Reviews – August 1971; p. 1759 – Review No. 1759. Review of my article on Ramanujan's conjecture

"It is known that $2^{n+2} - 7 = u^2$ has just five solutions $(n, u) = (1,1), (2,3), (3,5), (5,11), (13,181)$. The author proposes a new proof based on certain convergents of the continued fraction for $\sqrt{2}$, but at the final, crucial steps, the argument is poorly expressed and not convincing. Several misprints appear, and the notation $(2n+1)C_K$ for the binomial coefficient $\binom{2n+1}{K}$ is confusing"

[B. M. Stewart (E. Lansing, Mich]

If $A = \text{grad}\, V$, ~~$A_r =$~~ we have from (3), using $A_r = (\text{grad}\, V)_r$ etc.

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin\theta}\cdot\frac{\partial}{\partial \phi}\left(\frac{1}{r\sin\theta}\cdot\frac{\partial V}{\partial \phi}\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\cdot\frac{\partial^2 V}{\partial \phi^2}$$

~~using $A_r = (\text{grad}\, V)_r$~~

Questions on Complex Variables (Pipes, p. 447–78)

Ex. p. 469 on evaluation of integrals

$$\int_0^\infty \frac{dx}{x^4 + a^4} = \frac{\pi}{2\sqrt{2}\, a^2}$$

$\frac{1}{z^4 + a^4}$ has simple poles at $ae^{\pi i/4}$, $ae^{3\pi i/4}$, $ae^{5\pi i/4}$, $ae^{7\pi i/4}$.

only the first two are in upper half plane

[N.B. Evaluation $\int_{-\infty}^{\infty} Q(x)\, dx$, where $Q(z)$ satisfies conditions

(ii) (ⅱ) It has no poles on real axis

(i) (ⅰ) It is analytic in the upper half plane except at a finite no. of poles

(iii) $zQ(z) \to 0$ as $|z| \to \infty$ for $0 \le \arg z \le \pi$

(iv) when x is real $xQ(x) \to 0$ such that $\int_0^\infty Q(x)\, dx$ and $\int_{-\infty}^0 Q(x)\, dx$ both converge.

Institute again

(21/4/72) – New Books Section

(1) The Viceroy at Bay by John Glendevon (Linlithgow's vicenyally described by his son)

(2) J. Franklin Inst. May, 1971 (Vol. 291, No. 5) p. 329 – Article on regular faced convex polyhedra by Martin Berman – The article which I mentioned to K. V.

Abstract: A regular-faced convex polyhedron is one in which all the faces are regular polygons, not necessarily with the same number of sides. Counting the prisms and anti-prisms as one type of solid each, there seem to be 112 regular-faced convex polyhedra as conjectured by Johnson and later proved by Zalgaller. These polyhedra can all be built from combinations of 28 simple polyhedra plus prisms and antiprisms. The properties of these solids including the volume and sphericity are presented. Photographs of all the solids are shown. Nets for construction of the polyhedra are given, as well as methods of construction.

[Besides the Platonic and Archimedean solids and the two classes of prisms and antiprisms, there are known to

If these conds are satisfied then $\int_{-\infty}^{\infty} Q(x)\,dx = 2\pi i \sum R^+$

= sum of residues at poles in the upper half plane – Pdf p. 469]

The function $Q(z)$ satisfies conds (i) – (iv) above

$$\therefore \int_{-\infty}^{+\infty} \frac{dz}{z^4+a^4} = 2\pi i \sum \text{Residues at } ae^{\pi i/4} \text{ \& } ae^{3\pi i/4}$$

$$\text{\& Res}_{z=ae^{\pi i/4}} Q(z) = \frac{1}{4a^3} e^{-3\pi i/4} \quad \text{\&} \quad \text{Res}_{z=ae^{3\pi i/4}} Q(z) = \frac{1}{4a^3} e^{-9\pi i/4}$$

$$\therefore \int_{-\infty}^{\infty} \frac{dx}{x^4+a^4} = 2\pi i \left(\frac{1}{4a^3} e^{-3\pi i/4} + \frac{1}{4a^3} e^{-9\pi i/4} \right)$$

$$= \frac{\pi}{\sqrt{2}\, a^3}$$

Since $Q(x)$ is an even fn

$$\int_0^{\infty} \frac{dx}{x^4+a^4} = \frac{\pi}{2\sqrt{2}\, a^3}.$$

---

pp. 489 – 492

The transformation $z = K\cosh w = K\cosh(u+iv)$

Now $x + iy = K(\cosh u \cos v + i \sinh u \sin v)$

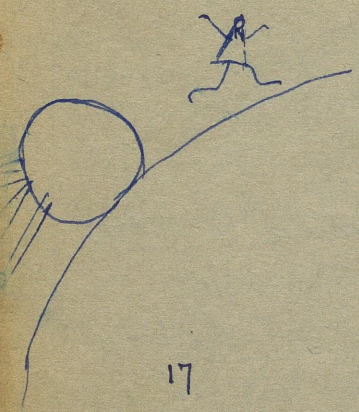
i.e. $x = K\cosh u \cos v$, $y = K \sinh u \sin v$

or $\cos v = x / K\cosh u$

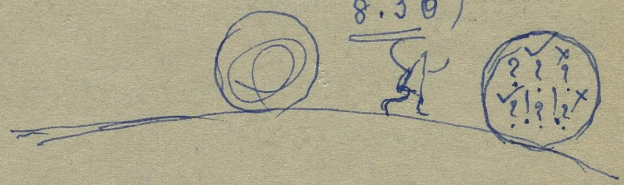
$\sin v = y / K \sinh u$

[See new note book

Q



4.50
2.50
0.70
0.60
8.30



17

30

~~16! + 1~~

~~1.2.3.4.5.6.7.8.9.10.11.12.13.14~~

64
16
8
4
2

31
63
94 $= 2^6 + 2^4 + 2^3 + 2^2 + 2^1$

111111
11111
1011110

1000
1000

~~1011110~~

1000000
10000
1000
100
10
1011110

$25 \times 60 \times 60$

$3600 \times \frac{360000}{4} = 90000$

9×10^4

$10,0000$

$1 \cdot 10^5$ seconds.

Just mention the works of Samuelson as mathematical economics

$\frac{4}{19} + 8$

$= \frac{-4 + 147}{49}$

$2z^2 - yz \Big]_0^1 \quad (2 - y)$

$2y - \frac{y^2}{2} \Big]_0^1$

$2 - \frac{1}{2} = \frac{3}{2}$

(1) Mathematics & social sciences. - Logic -

(2) ~~Groups of Scientists all due to~~ Attitude of Scientists to social sciences

(3) Point of view of scientists - Humility - Bridges of man - Concrete bridges.

div. curl $\vec{A}$

$11^2 \times 11^2$

11^0

121×121

121

242

121

not palindromic