A. M. D. G.

ST. JOSEPH'S COLLEGE
TRICHINOPOLY

NOTE BOOK

Name: Mathew Thekkekara S.J.

Form: B.Sc. (Hons)

Subject: Supplement.

ST. JOSEPH'S I. S. PRESS, TRICHY.—1937

Electricity

at the end of the uniform field ~~tubular~~ Cathode.

In the case where the screen is at a certain distance, the eqn is more complicated. We can here derive an expn for the case where the field between the polepieces is uniform & that at any pt outside is zero.

The path due to magnetic field is a circular arc ~~where~~ ~~r~~ of radius r where $\frac{m v_x^2}{r} = He v_x$

i.e. $r = \frac{m v_x}{He}$

Let the uniform field extend for a ~~ba~~ distance l & let k be the distance of the screen from the pt where the field ends. The ∠lar deflection θ in the (x, z) plane is $\frac{z}{k + \frac{l}{2}}$ considering l to be small compared to k.

θ is also $\frac{l}{r}$ $\therefore r = \frac{l\left(k + \frac{l}{2}\right)}{z}$

$\therefore z = He\, l\left(k + \frac{l}{2}\right) / m v_x$.

Due to the electric field, component

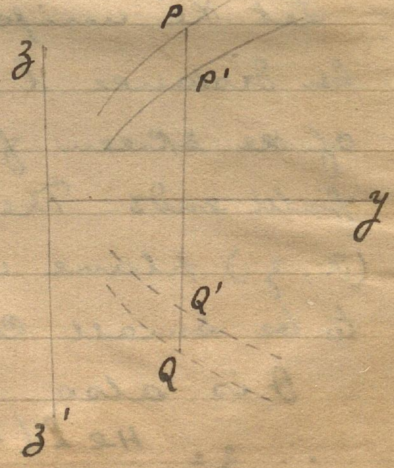
of the velocity along the y-axis at the instant the particle emerges from the field is $v_y = at = \frac{Xe}{m}\cdot\left(\frac{l}{v_x}\right)$

ie $v_y v_x = \frac{Xel}{m}$

Now $y/_{K+\frac{l}{2}} = v_y/v_x$ $\therefore y = \left(K+\frac{l}{2}\right)\frac{v_y}{v_x}$

$= \frac{Xel\left(K+\frac{l}{2}\right)}{m v_x^2}$. & $z^2 = \frac{H^2e^2l^2\left(K+\frac{l}{2}\right)^2}{m^2 v_x^2}$

$\therefore \frac{z^2}{y} = \frac{H^2 l\left(\frac{l}{2}+K\right)}{X}\cdot\frac{e}{m}$ Here again we see that path traced is a parabola (Experimental atomic physics by Harnwell & Livingwood - Page 135).



When hydrogen was used in the bulb, the two continuous curves in the upper half were obtained. Half way through the experiment, the direction of the magnetic field also was changed & the two dotted curves were obtained. $\frac{\rho Q^2}{\rho' Q'^2}$ was seen to be of value 2, i.e. the ratio's of $\frac{e}{m}$ for the two curves was as 1:2. These were explained to be due to the fact that one curve is formed by molecules (i.e. ~~mas~~ two atoms) with one electronic +ive charge, while the other is formed by atoms with unit charge. $\frac{e}{m}$ in the latter case was nearly 10^4 which is the charge carried by one gm of hydrogen in electrolysis.

When helium was used a line showing $\frac{e}{m} = \frac{10^4}{4}$ was obtained as is to be expected from the atomic wt. 4 of helium.

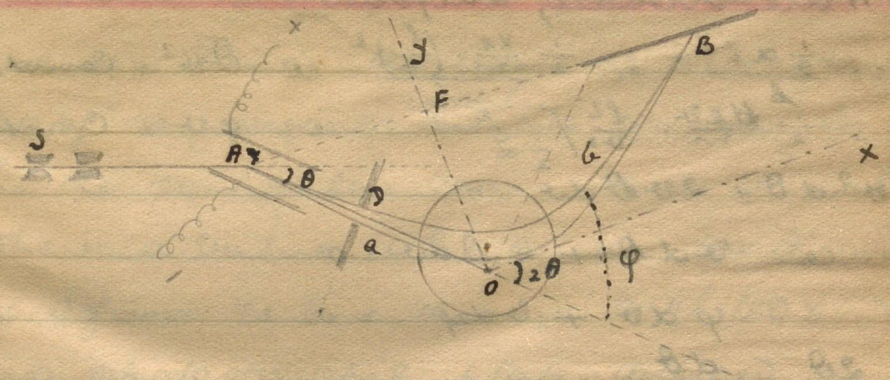
With this apparatus J. J. Thomson first discovered isotopes in non-radio-active elements. Neon gas (at. wt 20.18) gave two parabolas which when meas showed the masses of the ~~atom~~ impinging particles to be in the ratio 20 : 22; i.e.

Hence we are to conclude that in the neon gas there are two kinds of atoms of masses 20 & 22, having equal free charges, i.e. same physical & chemical properties, & ∴ inseparable by any ordinary means. In other other words in Neon there are two isotopes, two chemically identical substances having the same resultant +ive charge on the nucleus but different ~~atomic~~ wts.

Aston's Mass Spectrograph. For definiteness & clearness of image, the parabola method is very disadvantageous.

Aston's method is entirely new, & gave very definite information about isotopes of most known elements.

The rays after arriving at the Cathode pass through two very narrow slits, and then between two ∥l plates maintained at a high p.d. The plates are slightly inclined to the path of the rays. After emerging from the plate, the thin ribbon is spread



into an electric spectrum, & may be taken (to a first order of approximation) to emerge from a virtual source half way between the plates. A group of these rays ~~are~~ is then selected by a diaphragm D & made to pass through the poles of a magnet. For simplicity we shall take the poles to be ⊙lar &, : the field acting as if concentrated at some pt O. A further deviation (in the opposite direction & a concentration of the dispersed rays takes place say at some point B.

Let θ & φ be the deviations at A & O, l, L the distances through which the fields a

X & H are virtually uniform.

$\theta = \dfrac{\frac{1}{2}\alpha t^2}{s} = \dfrac{\frac{1}{2}\frac{Xe}{m}\cdot\left(\frac{l}{v}\right)^2}{\frac{1}{2}l}$ i.e. $\theta v^2 = \text{Const.}$

$\varphi = \frac{1}{2}\frac{Hev}{m}\cdot\frac{l^2}{v^2}\Big/\frac{1}{2}l$ i.e. $\varphi v = \text{Const.}$

$\therefore\ v^2 d\theta + 2v\theta\, dv = 0$

i.e. $v\,d\theta + 2\theta\, dv = 0$

& $\varphi\, dv + v\, d\varphi = 0$

$\therefore\ \dfrac{2\theta}{\varphi} = \dfrac{d\theta}{d\varphi}$.

The group selected will spread out to a breadth $a\,d\theta$ at O & at B since it is zero $(a+b)\,d\theta = b\,d\varphi$

$\therefore\ \dfrac{d\theta}{d\varphi} = \dfrac{b}{a+b}$ or $\dfrac{d\varphi}{d\theta} = \dfrac{a}{b}+1$

$\therefore\ \dfrac{\varphi}{2\theta} = \dfrac{a}{b}+1$.

i.e. $\dfrac{\varphi - 2\theta}{2\theta} = \dfrac{a}{b}$

i.e. $b(\varphi - 2\theta) = 2a\theta$.

Now taking OX, OY as axes of coordinates, OX being at $\angle\ \theta$ to SA & OY $\perp$r to OX, the position of B in Cartesian coordinates is $[b\cos(\varphi - 2\theta),\ b\sin(\varphi - 2\theta)]$

i.e. $[b,\ b(\varphi - 2\theta)]$

i.e. $[b,\ 2a\theta]$

Now $2a\theta$ is a constant whatever be the value of $\frac{e}{m}$ for the particles that arrive. Hence the locus of the different pts of convergence is a line ~~are all~~ parallel to OX. In the particular case where ϕ is adjus to have a value 4θ, $a = b$, $2a\theta$ = dista of O from the line through A ‖l to OX &

∴ a photographic plate placed on this line will be in focus for all the mass-spectra. The distance FB is very nearly ∝ to $\frac{e}{m}$ for the particles which for the image at B.

The method actually employed was to calibrate the photographic plate for differe masses, by using CO_2 in the discharge bulb. CO_2 gave lines at pts corresponds to $\left(\frac{m}{e}\right)$ masses 6 (C^{++}), 8 (O^{++}), 12 (C^{+}), ~~O~~ 16 (O^{+}), 28 (CO^{+}), & 32 (O_2^{+}) & 44 (CO_2^{+}). The plate could thus be calibrated & then used to examine the isotopes of any element. Cf Aston's Mass Spectra & isotopes - (Page 39 sqq)

Metallic Conduction (Grimsehl 361)

The theory of metallic conduction is
till confronted with ~~may~~ many obstacles
ince our present knowledge of the inner
tructure of molecules and atoms is
very limited, & in the ~~for~~ case of
solids the mutual interactions of molecules
resent a complicated phenomenon. Hence
e mechanism of conduction in metals is
uch less clear than in gases & liquids.
ut one thing is certain; conduction
n metals is not accompanied by trans-
ort of matter from one side to another;
& so we conclude charged molecules
r parts of molecules (ions) are not
e carriers in metals. Then, are the
arriers those particles of pure electricity
f mass $\frac{1}{1840}$ of a ~~pure~~ hydrogen atom
— those practically weightless particles
alled electrons?

Two experiments among others

seem to confirm this view, & a theory has been propounded ~~to~~ known as the electron theory of electric conduction.

The chief postulates of the theory are as follows. In the interior of the conductor there are a certain number of electrons actually separated from the atoms. When ~~no potential~~ is appl[ied] the conductor is free of any electric potential, these electrons travel equally in all direction, with all velocities, through the interstices of the lattice of atoms. When they impinge upon an atom they are brought to a stand still & their energy is imparted either to ~~a~~ the atom or to ~~the~~ another electron. The mean free path depends on the space lattice, & hence on the nature of the ~~ele~~ metal. The mean kinetic energy ~~of~~ is proportional to the absolute

emperature. The distribution of velocities
s analogous to that of molecules in gases;
ut Sommerfeld has shown that Max-
well's law of distribution does not exactly
hold good here.

When a potential is applied to the
onductor, the component of the velocities
ll to the direction of the current is decreased
y a certain amount $\propto$ to the electric
ield; & each electron has a tendency
 travel from -ive to +ive end of the wire.
This causes a net transmigration of
lectrons in that direction, i.e. a current
n the direction of the field. ~~D~~ The current
value depends on the no. of free electrons
& on the mean free path (both factors
depending on the nature of the conductor.)
Due to the added velocity, impacts of
electrons with atoms cause greater
transference of energy & greater
velocity of the atoms themselves. This

explains heating due to a current.

The expts which seem to confirm the theory ~~so~~ show that the specific charge $\left(\frac{e}{m}\right)$ of freely moving particles ~~have been~~ is the same as that of electron. If a ~~conductor~~ metal is rapidly moved in one direction, say along the +ive direction of the x-axis, the lattices of atoms in the Y–Z– plane form a sort of barrier to the freely moving particles. A number of these particles will thus be heaped up in front of the lattices, while there is a scarcity of them behind the lattices. If the particles are charged (say, –ively) the end of the wire $A \xleftarrow{\qquad} B$ which is forward will have a ~~+ive~~ negative potential & the other end will have a +ive potential; & the value of the p.d. can be expressed in terms of the specific charge $\frac{e}{m}$, the velocity of the conductor & the length AB. The expt was conducted, the p.d. found out between the ends of spiral which was revolved rapidly about its axis & suddenly braked. The $\frac{e}{m}$ was calculated. The value agreed closely with that of electrons obtained by J. J. Thomson and others.

Hence the freely moving particles in conductors are electrons.

Though there are electrons moving along a metal wire, ~~at rest~~, the wire does not exhibit any magnetic properties since the number of electrons moving in one direction is equal to those moving in the other direction. ~~The~~ But when the conductor is rapidly moved to & fro the electrons (or the particles, whatever they are) are shaken like peas in a box; the electrons also move ~~a p~~ to & fro along with the conductor, & this causes a variable magnetic field. From the magnitude of the field the specific charge of the ~~el~~ particles has been calculated & the particles have been proved to be electrons.

Qualitatively the electron theory ~~proves~~ explains various phenomena; but quantitatively the theory has been found wanting.

Let v be the mean velocity of an electron (when free of a electric field) & λ the mean free path. ∴ The ~~If~~ time for which the electron travels between two successive collisions is $\left(\frac{\lambda}{v}\right)$ sec. If an electric field E is applied, the electron is subject to a force Ee & to an acceleration $\frac{Ee}{m}$, in the -ve direction of the field; ∴ displacement of the electron in that direction $= \frac{1}{2} \times$ acceleration $\times (\text{time})^2$.

$= \frac{1}{2} \times \frac{Ee}{m} \times \left(\frac{\lambda}{v}\right)^2$. [The initial drift velocity is zero]

Hence ~~the~~ mean velocity of the electrons in this direction $= \frac{\text{Displacement}}{\text{time}}$

i.e. $V = \frac{1}{2} \frac{\mathcal{E}e}{m} \times \frac{\lambda}{v}$.

If there are N free electrons per c.c.
∴ the ~~current~~ quantity of electricity flowing across unit area of the conductor in unit time $= Nev$

i.e. Current i per unit sectional area $= NeV$

$$= Ne \frac{1}{2} \frac{\mathcal{E}e}{m} \cdot \frac{\lambda}{v}.$$

Since the mean kinetic energy of the electrons $\frac{1}{2} mv^2$ is $\propto$ to the absolute temperature T

$$\frac{1}{2} mv^2 = \alpha T$$

α being a constant

$$\therefore \quad \frac{1}{v} = \sqrt{\frac{m}{2\alpha T}}$$

∴ Current $i = Ne \cdot \frac{1}{2} \frac{\mathcal{E}e}{m} \cdot \lambda \times \sqrt{\frac{m}{2\alpha T}}$

$$= \frac{1}{2} \frac{N\mathcal{E}e^2\lambda}{\sqrt{2m\alpha}} \times \frac{1}{\sqrt{T}} = k\mathcal{E}$$

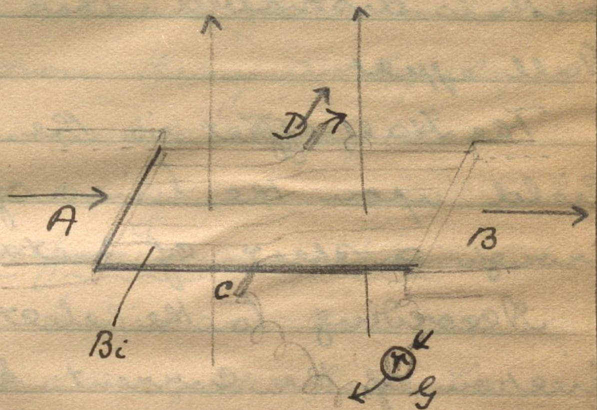
But current $= \frac{\mathcal{E}}{R}$ (where R is the resistance) by Ohm's law

∴ The eqn also shows that the current is ~~independent of~~ proportional to the e.m.f. thereby proving

Ohm's laws.

According to the eqn R resistance is $\propto \sqrt{T}$ the root of the absolute tempera
But expts shows that in all pure metal resistance is $\propto$ to T, the absolute temperature.

Hall effect



Two metal pieces A & B which are connected to the poles A of a high tension battery are bridged across by a thin sheet of metal say Bismuth and a strong current thus passed through the sheet. The current distributes itself uniformly in the sheet, & hence if two pts C & D on opposite sides of the sheet & symmetrically placed are joined together through a galvanometer.

he galvanometer will show no deflection since
 & D are at the same potential. But if a
~~current~~ strong magnetic field is applied
+r to the sheet (upwards as shown in
he figure) the balance in the galvano-
meter is disturbed. This is known as the
Hall effect

The Hall effect is the effect of a magnetic
field upon the lines of current flow
hrough a sheet of ~~metal~~ conducting material.

According to the electron theory the
direction of the current should be from
 to D within the plate & D to C through the
galvanometer. For a stream of electrons
s equivalent to a conductor. In a
magnetic field the motion of a conductor
is governed by the left hand thumb
rule is from D to C. Thus we
electricity flows from D to C through
the plate. In other words, the current
flow within the plate is from C to D.

This is found to be the case if the plate is made of bismuth, Copper, & most other metals. But if the plate of ~~iron~~, Cobalt or some substance which has the slightest magnetic properties, the current flow is the reve Hence the electron theory as developed at present is unable to explain Hall effect completely.

Dia – para – & ferro – magnetism

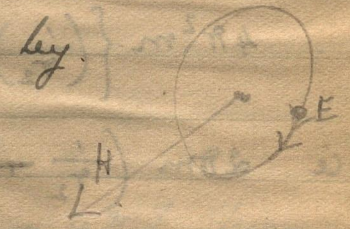
Paramagnetic bodies are those which when placed in a magnetic field have a tendency to move from a ~~stronger~~ weaker to a ~~stronger~~ stronger field. Diamagnetic Substances move from a ~~wea~~ stronger to a weaker field. When a field is applied the lines of force through the material increase if it is paramagnetic, ~~bu~~ but decrease if it is ~~ferro~~ dia-magnetic.

An explanation for this behavior has been given by Langevin on the basis of the electron theory. The electrons have various orbits in various planes. Now if there are two electrons in the same plane, having equal orbits but in opposite directions, then their effects anull each other. The resultant magnetic moment is zero. Thus all substances can be divided into two classes, ~~one~~ 1) those in which every electronic orbit has ~~another~~ one or more other orbits such that the total resultant magnetic moment is zero & 2) those in which there are one or more unbalanced electronic orbits.

When a magnetic field is applied two effects take place. The unbalanced electronic orbits are brought into a plane at rt ∠s to the field & thus the induction is increased. Substances of this class exhibit paramagnetic properties. It may also happen that the orbits have sufficient mutual interaction as to form larger groups, in which case the induction is increased very greatly. Such substances are ferromagnetic. A second effect is that the moving elect under the action of the field changes its frequency in such a way as to oppo the field. The induction is thus decreas Such a substance is diamagnetic.

The change in frequency can be deduced as follows.

Any motion can be resolved into a longitudinal vibration ∥l to the field & two ⊙lar motions ⊥r to the field in opposite directions.



The first is unaffected by the field; but the other two are affected. Consider an electron moving in the direction of the arrow. Due to the field H a force He v, acts on it which by the left hand

[...]ule is towards the centre of the orbit.

$$\therefore \quad \cancel{\frac{1}{}}\ m\frac{v_1^2}{r} = pr - Hev_1$$

$$\text{ie} \quad \cancel{\parallel}\ m\omega_1^2 = p - He\omega_1.$$

This causes a field in the direction ~~opposite~~ of the magnetizing field.

Electron which moves along the same orbit but in anticlockwise direction yields the eqn

$$\cancel{\parallel}\ m\omega_2^2 = p + He\omega_2.$$

If T_1 & T_2 are the periods of the two ~~roto~~ rotations

$$m\left(\frac{2\pi}{T_1}\right)^2 = p - He\left(\frac{2\pi}{T_1}\right)$$

$$m\left(\frac{2\pi}{T_2}\right)^2 = p + He\left(\frac{2\pi}{T_2}\right)$$

$$\therefore \quad 4\pi^2 m\left\{\left(\frac{1}{T_2}\right)^2 - \left(\frac{1}{T_1}\right)^2\right\} = He \times 2\pi\left(\frac{1}{T_2} + \frac{1}{T_1}\right)$$

$$\text{ie} \quad 2\pi m\left(\frac{1}{T_2} - \frac{1}{T_1}\right) = He$$

$$\text{ie} \quad \frac{1}{T_2} - \frac{1}{T_1} = \frac{He}{2\pi m}$$

Now an electron moving in orbit with

period T is equivalent to a current ~~&~~ $\frac{e}{T}$ & if the area of the orbit is a, the magnetic field $= \frac{ea}{T}$. $\left[\frac{ea}{CT}\right.$ if e is in C.S.U.

Magnetic field due to the ~~rota~~ clockwise direction of rotation is $\frac{ea}{T_1}$ ~~&~~ is in the direction of the field. Magnetic field due to anticlockwise direction of rotation is $\frac{ea}{T_2}$ & is opposed to the field.

Resultant magnetic field $=$ ~~Ea~~ $ea\left(\frac{1}{T_1} -\right.$

If there are N_1 such electrons in unit vol, magnetic ~~induction~~ intensity due to all the electrons in unit vol

$$= N_1 ea\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$= -\frac{N_1 e^2 H a}{2\pi m}$$

Thus the eqn $B = H + 4\pi I$ become in the case of a diamagnetic substance

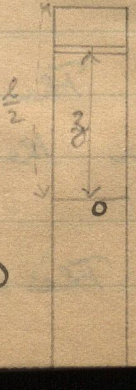
$$B = H - \frac{N_1 e^2 H a}{2\pi m}$$

The magnetic susceptibility $= -\frac{N_1 e^2 a}{2\pi m}$

Now N_1 is the no. of electrons whose
rbits are ⊥r to the field H ∴ if N is the
otal number of electrons per unit vol.
$N_1 = \frac{N}{3}$
∴ Diamagnetic Susceptibility $k_1 = -\frac{Ne^2a}{6\pi m}$.
Suppose that there are n unbalanced
lectronic orbits per unit vol. & that
ach of these is brought normal to the
ield, which will be case when the
agnetism has reached Saturation value.
he magnetic intensity $= \frac{nea}{T}$
s ∴ paramagnetic susceptibility is $k_2 = \frac{nea}{TH}$.
A substance is para- or dia-magnetic
ccording as the sum $k_2 + k_1$, $\frac{nea}{TH} - \frac{Ne^2a}{6\pi m}$
s +ive or negative.

Capacity of a cylinder (Pidduck page 67)

Let the length l be large com-
pared to the radius a.



The distribution of electricity
n the cylinder is nearly uni-
orm in the middle & is disturbed

appreciably only at distances from the ends comparable to the radius. Thus at O (the midpt. of the axis) the potential is that due to a uniform layer of surface density σ over the curved surface (the electricity on the ends being neglected.) The charge on a strip of height dz at distance z from the axis is $2\pi a\, dz \times \sigma$ ∴ the potential at O due to this charge is $\dfrac{2\pi a \sigma\, dz}{\sqrt{z^2+a^2}}$.

∴ Potential at O i.e. potential of the cylinder is given by

$$V = \int_{-l/2}^{l/2} 2\pi a \sigma \frac{dz}{\sqrt{z^2+a^2}}$$

$$= 4\pi a \sigma \int_0^{l/2} \frac{dz}{z^2+a^2}$$

$$= 4\pi a \sigma \Big[\log_e \left(z + \sqrt{a^2+z^2}\right)\Big]_0^{l/2}$$

$$= 4\pi a \sigma \log_e \left\{\frac{\frac{l}{2} + \sqrt{a^2+\frac{l^2}{4}}}{a}\right\}$$

$$= 4\pi a \sigma \log_e \frac{l}{a} \quad \text{approximately}$$

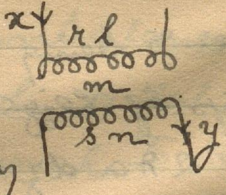
The charge on the cylinder $= 2\pi a l \sigma$

$\therefore$ Capacity $= \dfrac{l}{2\log_e(l/a)}$

Transformer. Complete Solution (Starling 358)

~~[illegible] is the e.m.f.~~

Let e be the e.m.f. impressed in the primary at any instant & x, y the currents in primary & secondary, x being $a \sin pt$ & y being $b \sin(pt + \varphi)$



$$l\frac{dx}{dt} + m\frac{dy}{dt} + rx = e \qquad 1)$$

$$\& \quad n\frac{dy}{dt} + m\frac{dx}{dt} + sy = 0 \qquad 2)$$

Now

$$\frac{dx}{dt} = ap\cos pt \quad \& \quad \frac{dy}{dt} = bp\cos(pt+\varphi)$$

Substituting for x & y in 2)

$$nbp\cos(pt+\varphi) + map\cos pt + sb\sin(pt+\varphi) = 0$$

Equating Coeff$\underset{..}{ts}$ of $\cos pt$ & $\sin pt$,

$$nbp\cos\varphi + map + sb\sin\varphi = 0 \qquad 3)$$

$$\& \; -nbp\sin\varphi + sb\cos\varphi = 0 \qquad 4)$$

From 4) $\tan\varphi = \dfrac{s}{np}$

$$\therefore \sin\varphi = \frac{s}{\sqrt{n^2p^2+s^2}} \qquad \cos\varphi = \frac{np}{\sqrt{n^2p^2+s^2}}$$

Putting $\dfrac{mp}{\sqrt{n^2p^2+s^2}} = \beta$

$$\sin\varphi = \frac{s\beta}{mp} \quad \cos\varphi = \frac{np\beta}{mp}.$$

Substituting in 3)

$$b \times \frac{n^2p^2\beta}{mp} + a \cdot mp + b \cdot \frac{s^2\beta}{mp} = 0$$

i.e. $b = -\dfrac{m^2p^2}{\beta(n^2p^2+s^2)} \times a = -a\beta.$

$\therefore\ y = -a\beta \sin(pt+\varphi)$ where $\tan\varphi = \dfrac{s}{np}$

Substituting for x & y in 1)

$$\begin{aligned} e &= lap.\cos pt + mbp\cos(pt+\varphi) + ra\sin \\ &= \cos pt\,(lap + mbp\cos\varphi) \\ &\qquad + \sin pt\,(ra - mbp\sin\varphi) \\ &= \cos pt \times pa.(l - n\beta^2) + \sin pt\, a(r+\beta^2 \\ &= a\,(\cos pt\sin\theta + \sin pt\cos\theta)\sqrt{L^2p^2+R} \end{aligned}$$

Where $\tan\theta = \dfrac{Lp}{R}$ $\quad L = l - \beta^2 n$

$$R = r + \beta^2 s$$

$$= a\sqrt{L^2p^2+R^2}\ \sin(pt+\theta)$$

Now if we take the phase of e as zero

i.e. $e = E\sin pt$,

$$x = \frac{E}{\sqrt{L^2p^2+R^2}}.\sin(pt-\theta) \qquad \text{I}$$

$$y = \frac{-\mathcal{E}}{\sqrt{\mathcal{L}^2 p^2 + \mathcal{R}^2}} \times \beta \sin(pt - \theta + \varphi)$$

$$= \frac{\mathcal{E}}{\sqrt{\mathcal{L}^2 p^2 + \mathcal{R}^2}} \cdot \beta \cdot \sin\{pt - \theta - (180 - \varphi)\}$$

$$= \beta \frac{\mathcal{E}}{\sqrt{\mathcal{L}^2 p^2 + \mathcal{R}^2}} \sin(pt - \theta - \varphi' - \frac{\pi}{2}) \qquad \text{II}$$

where $\varphi' = 90 - \varphi$

Eqns I & II give the values of x & y

If $\mathcal{E} \cos pt$ is the impressed e.m.f

$$x = \frac{\mathcal{E}}{\sqrt{\mathcal{L}^2 p^2 + \mathcal{R}^2}} \cos(pt - \theta) \qquad \text{I}$$

$$y = \frac{\mathcal{E}}{\sqrt{\mathcal{L}^2 p^2 + \mathcal{R}^2}} \beta \cdot \cos(pt - \theta - \varphi' - \frac{\pi}{2}) \qquad \text{II}$$

where $\mathcal{L} = l - \beta^2 n$; $\mathcal{R} = r + \beta^2 s$

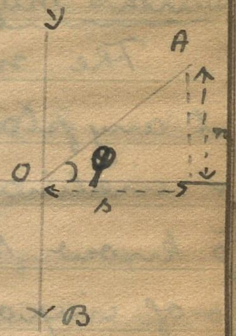
$\beta = \dfrac{mp}{\sqrt{n^2 p^2 + s^2}}$ $\tan\theta = \dfrac{\mathcal{L}p}{\mathcal{R}}$; $\tan\varphi' = \dfrac{np}{s}$.

The relation between the amplitudes & phases of x & y can also be derived from the eqn of complex quantities on page 37

$$x(lip + r) + y(mip) + \mathcal{E}e^{ipt}(-1) = 0$$

$$x(mip) + y(nip + s) + \mathcal{E}e^{ipt}(0) = 0$$

$$\frac{x}{+(nip+s)} = \frac{y}{-mip} \quad \therefore \frac{x}{y} = \frac{nip+s}{mip}$$



A line OA making $\angle \varphi$ $\left(= \tan^{-1}\frac{np}{s}\right)$ with the x-axis represents the complex quantity $nip+s$ & another line OB making $\angle -\frac{\pi}{2}$ $\left(= \tan^{-1}\frac{-mp}{0}\right)$ with the x-axis represents $-mip$.

The diagram shows that the phase of y lags behind that of x by $\left(\frac{\pi}{2}+\varphi\right)$

Since s is small compared to np, $x/y = n/m$.

If N_1 & N_2 are the no. of turns in primary & secondary & F the magnetic flux due to unit current in the primary, $n = FN_1$; $m = FN_2$

$\therefore \; x/y = N_1/N_2$.

$\therefore$ The currents are in the ratio of the turns in the two coils.

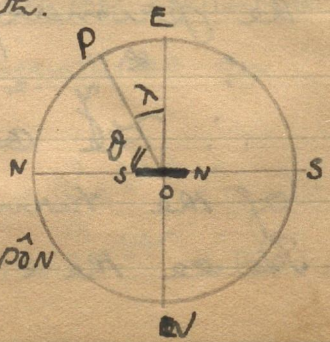
Hence the e.m.f.'s are in the inverse

ratio of the turns.

Earth's magnetic field (Starling 36)

The magnetic elements of the earth at any place are 1) Declination 2) Dip or inclination 3) Horizontal intensity. To know the nature of their variations is of importance to the scientist as well as to the navigator. Hence maps are drawn showing the isogonal lines (i.e. lines of same declination) or the isoclinal lines (lines of the same dip) on the surface of the earth.

To a first approximation the magnetic field of the earth may be considered as that due to a small magnet placed at the centre of the earth, with its S-pole pointing north.



Potential at a pt P

$V = \frac{M\cos\theta}{r^2}$ where M is the moment of the magnet & r the radius of the earth, & $\theta = \angle P\hat{O}N$

The vertical intensity $v = \frac{dV}{dr} = \frac{2M\cos\theta}{r^3}$

The horizontal int $h = \frac{1}{r}\frac{dV}{d\theta} = \frac{M\sin\theta}{r^3}$

$\therefore$ Dip $\delta = \frac{v}{h}$ is given by

$$\tan\delta = \frac{v}{h} = 2\cot\theta = 2\tan$$

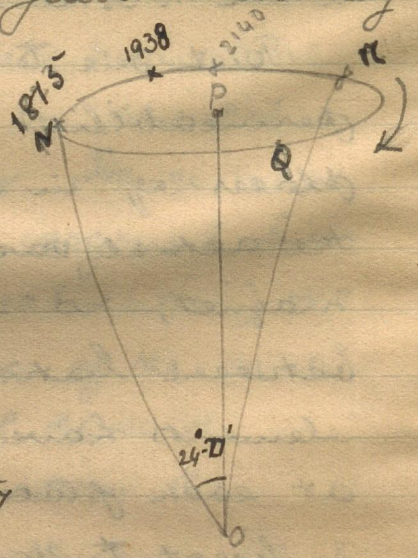
Where λ is the latitude of the place.

But due to the greater magnetic permeability of the earth in certain places eg in iron mines, & because the actual case is not that of a smal magnet, no simple relation exists between latitude & dip. The magnet elements have actually to be determine at each place, & since they are subject to variations, the determin-ations should be revised from time to time.

The magnetic elements are subject to three different variations secular, ~~ye~~ annual, & diurnal.

1) The secular variation comple one cycle in about 960 years. The

hus the declination ~~rises~~ changes from a maximum on the west to zero & reaches a maximum on the east in 480 years, then comes back to the maximum on he west in another 480 years. This may e explained as due to rotation of the magnetic axis from ast to west. If OP epresents the axis of he earth & OM ~~&~~ he magnetic axis, OM revolves with its extremity n the ⊙ MNQ. Thus in 960 years the magnetic axis makes ne revolution less than the earth.

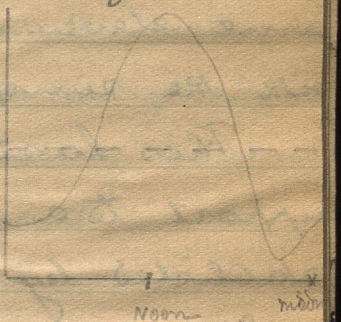


2) Annual variation. A variation f very small amplitude occurs nnually.

3) Daily Variation - A daily variation lso has been noticed. dθ the difference

between the actual declination at any hour of the day & the average declination for the whole day is for a part of the day +ive & for another part of the day negative

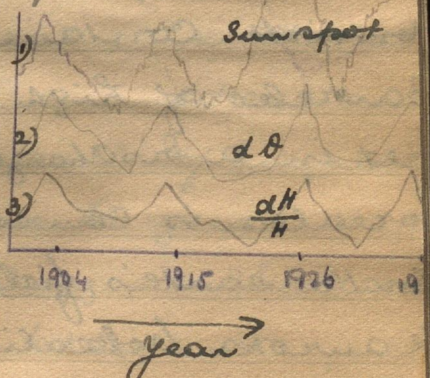
The figure shows a typical curve; but the nature & amplitude of the curve varies from place to place & from day to day.



The horizontal intensity also has been seen to vary.

It has also been noticed that the variations occur simultaneously with the ~~the~~ frequency of sunspots.

The curve (1) shows the frequency of sunspots every year; (2) shows the variation of $d\theta$ with year, $d\theta$ being the amplitude of the diurnal variations of declination;



and ③ shows the variation of $\frac{dH}{H}$ with year, dH being the amplitude of the diurnal variation of the horizontal intensity H. It is evident that $d\theta$ & $\frac{dH}{H}$ go through the same eleven-year period cycle along with the sunspots.

This last fact lends further support to a view held for long & established by the particular nature of the diurnal variations, that the cause of these variations is not the earth itself, but something external to it, perhaps the effects of the sun on the earth's atmosphere. The sun's radiations ~~consists~~ contain certain minute particles of high velocity, like those met in a discharge tube. The aurora borealis is caused by their deflection in the earth's field. The spectrum of the aurora borealis is a series of lines of nitrogen, CO_2 etc & hence we

Conclude that the aurora is caused by ~~the~~ ~~part~~ charged high velocity particles which can excite gases & not by reflected light. Now high velocity particles can ionize the atmosphere & make it conduct electricity. Hence the potential gradient between the different layers of the atmosphere must send a current in the vertical direction, & the current should produce a horizontal magnetic field. It is this magnetic field which affects the recording instruments on the surface of the earth. If that is so, it is natural that when the sun-spot frequency is great; i.e. the sun is more active, the emanations & the consequent variations of magnetic elements should also be great.

Field within a Olar ring – endless solenoid. Work done in taking unit pole once through the ring = $2\pi r H$.

Work done in taking unit pole through one Coil $= 4\pi C$

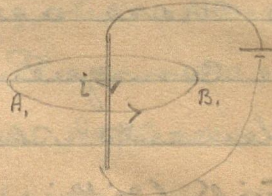
No. of Coils $= n \times 2\pi r$

$\therefore n \times 2\pi r \times 4\pi C = 2\pi r H$

$\therefore H = 4\pi n C.$

Magnetomotive force, Reluctance etc.

Magnetomotive force is ~~the~~ the term applied to in a magnetic circuit in an analogy with e.m.f. in an electric circuit. Every magnetic line forms a closed curve, & in the field of a magnet there are many such curves. So too round a wire carrying a current there many magnetic ~~curve~~ closed curves of magnetic lines.



Each one of these curves can be considered to be a magnetic circuit. Now, when unit pole is taken along the curve through a distance

dl, work done is Hdl H being the field at the element which is supposed to be uniform since dl is small. When the pole is taken round the whole magnetic circuit, work done is the line integral of Hdl for the circuit which is called the curl of Hdl & is written $\int_0 Hdl$. The term magneto-motive force is applied to $\int_0 Hdl$; for in the analogous of case of an elect. circuit in which there is an e.m.f. E, work done in taking unit charge round the circuit is $E \times 1$, $= E$ = e.m.f.

Let N be the flux through a small area s presented normal to the length dl. If s is so small that the induction over the whole area can be supposed to be uniform, $N = Bs = \mu Hs$, B being induction & μ the permeability of the medium. $\therefore H = \dfrac{N}{\mu s}$.

$\therefore$ m.m.f $= \int_0 H\,dl = \int_0 \frac{N}{\mu s}\,dl$

$\therefore N = \dfrac{m.m.f.}{\cancel{\mu s}\int_0 \frac{dl}{\mu s}}$ The quantity

$\int_0 \frac{dl}{\mu s}$ is called reluctance or magnetic resistance, for taking flux analogous to current as

current $= \dfrac{e.m.f}{\text{resistance}}$ we have flux $= \dfrac{m.m.f.}{\text{reluctance}}$

In an electric circuit, if R be specific resistance at a pt, resistance of an element of area s & length dl is $\frac{R\,dl}{s}$ $\therefore$ resistance of a whole circuit ~~which~~ through which the current is continuous $= \int_0 \frac{R\,dl}{s} = \int_0 \frac{dl}{\frac{1}{R}\times s}$ which is an expn similar to $\int_0 \frac{dl}{\mu s}$.

The above expression

flux $N = \dfrac{\int_0 H\,dl}{\int_0 \frac{dl}{\mu s}}$ enables us to calculate the flux at any point in a magnetic field provided the shape of the tubes of induction are such as to allow the two integrations $\int_0 H\,dl$ & $\int_0 \frac{dl}{\mu s}$

to be made. The following are two simple Cases.

1) A ringshaped or endless solenoid.

Work done in taking unit pole through the ring $= 4\pi i \times$ no. of turns

$$= 4\pi i \times 2\pi r n = \text{m.m.f.}$$

Since the sectional area of the tube of induction is the same throughout, magnetic reluctance $\int_0 \frac{dl}{\mu s} = \frac{2\pi r}{\mu s}$.

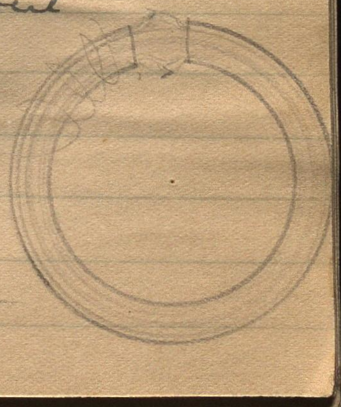
$$\therefore \text{flux} = 4\pi i \times 2\pi r n \times \mu s / 2\pi r$$

$$= 4\pi n i \times \mu s$$

$$\therefore \text{Field } H = 4\pi n i.$$

Here it was tacitly assumed that the area S is small compared to the ~~flux~~ radius r or else the flux will not be uniform throughout

2) Endless solenoid with an air gap in the iron core. Strictly speaking the tube of induction is not ⊙lar in section throughout.



At the air gap due to lateral pressure of the lines they bulge outwards. But if the gap is small this effect is negligible &

the reluctance $= (2\pi r - d)/\mu s + \frac{d}{s}$

$= 2\pi r + (\mu - 1) d / \mu s$

m.m.f $= (2\pi r - d) n \times 4\pi i$

$= 2\pi r n \times 4\pi i$ very nearly.

$\therefore$ Flux N in the circuit

$= \mu H s = \frac{2\pi r n \times 4\pi i}{2\pi r + (\mu - 1) d} \times \mu s$

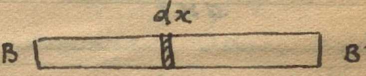
$\therefore H = \frac{4\pi i n}{1 + \frac{(\mu - 1) d}{2\pi r}}$

Heat.

Conductivity of Heat

Fourier's eqn for unidirectional flow of heat through an uncovered bar before the steady state is reached.

Let BB' be a bar along which heat flows from B to B'.



At distance x from B, let θ be the diff. of temp. from the outside air. If K is the thermal conductivity, A the sectional area, rate at which heat is received by an element of length dx at distance x. $-KA\frac{d\theta}{dx}$. [The $-$ive sign shows that $\frac{d\theta}{dx}$ is $-$ive]. Heat leaving the element p.s. $= -KA\frac{d}{dx}\left(\frac{d\theta}{dx}.d\theta+\theta\right)$

$\therefore$ Excess of inflow over outflow

$$= KA\frac{d}{dx}\left(\frac{d\theta}{dx}.dx\right)$$

$$= KA\frac{d^2\theta}{dx^2}\times dx.$$

Heat used up to raise the temp. of the element $= Adx \times \rho \times s \times \frac{d\theta}{dt}$

ρ being density & s sp. heat.

Heat radiated from the surface $= E\,p\,\theta\,dx$

$= E \times \theta \times p\,dx$

E being emissivity or heat radiated p.s.

from unit area for unit diff. of temp.

Thus we have the differential eqn for the flow of heat

$$k.A.\frac{d^2\theta}{dx^2}.dx = A\rho s.\frac{d\theta}{dt}.dx + E\theta p\,dx.$$

i.e. $$\frac{k}{\rho s}\frac{d^2\theta}{dx^2} = \frac{d\theta}{dt} + \frac{Ep}{A\rho s}\theta \quad 1)$$

The quantity $\frac{k}{\rho s}$ which is a constant for the material has been called thermal diffusivity by Kelvin, & thermometric conductivity by Maxwell, & is usually denoted by k.

Putting $\frac{Ep}{A\rho s}$ as m,

$$k\frac{d^2\theta}{dx^2} - m\theta = \frac{d\theta}{dt}. \quad 2)$$

Special cases

1) Bar covered, before steady state,

$E = 0 \therefore m = 0$

$$\text{i.e. } k\frac{d^2\theta}{dx^2} = \frac{d\theta}{dt}$$

or from eqn 1) $$\frac{k}{\rho s}\frac{d^2\theta}{dx^2} = \frac{d\theta}{dt}$$

2) Bar uncovered, after steady state

$\frac{d\theta}{dt} = 0$

$$\therefore \frac{k}{\rho s}\frac{d^2\theta}{dx^2} = \frac{Ep}{A\rho s}\theta \text{ or } kA\frac{d^2\theta}{dx^2} = Ep\theta$$

3) Bar covered, after steady state is reached

$kA \frac{d^2\theta}{dx^2} = 0$ ie $q = kA \frac{d\theta}{dx}$

where q is a constant.

Temp. of an element, distant x from the heated end, on an uncovered bar, after steady state has been reached.

$$kA \frac{d^2\theta}{dx^2} = A\rho s \frac{d\theta}{dt} + Ep\,\theta$$

becomes after steady state is reached,

$$kA \frac{d^2\theta}{dx^2} = Ep\,\theta$$

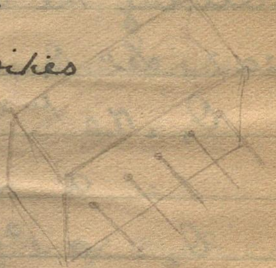
$$\text{ie } \frac{d^2\theta}{dx^2} = \mu^2\theta \qquad \mu \text{ being } \sqrt{\frac{Ep}{kA}}$$

∴ Indicial eqn $m^2 - \mu^2 = 0$ gives $m = \pm\mu$

∴ The solution is

$$\theta = Me^{\mu x} + Ne^{-\mu x}.$$

Expts to compare conductivities



1) Ingen Hauss' method

Let θ be the temp. at distance x on any bar.

$\theta = Me^{\mu x} + Ne^{-\mu x}$

When $x = \infty$ $\theta = 0$; ∴ $0 = M \times \infty + 0$

∴ $M = 0$

When $x = 0$, $\theta = \theta_0$ ∴ $\theta_0 = Ne^0 = N$.

$\therefore \; \theta = \theta_0 e^{-\mu x}$

At lengths $l_1, l_2, \ldots$ on the different bars temp θ is the same viz the melting pt. of wax.

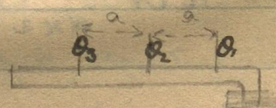
$$\text{ie} \quad e^{-\mu_1 l_1} = e^{-\mu_2 l_2} = \ldots$$

$$\text{ie} \quad l_1^2 \times \frac{Ep}{K_1 A} = l_2^2 \times \frac{Ep}{K_2 A} = \ldots$$

$$\text{ie} \quad K_1 / l_1^2 = K_2 / l_2^2 = \ldots$$

Despretz method.

A bar maintained at constant temperature at one end has three thermometers ~~on its~~ reading the temps at distances x, $x+a$, $+ 2a$. Let $\theta_1, \theta_2, \theta_3$ be the temps.



$$\theta_1 = M e^{\mu x} + N e^{-\mu x}$$

$$= \alpha + \beta \quad \text{say}$$

$$\theta_2 = \alpha e^{a\mu} + \beta e^{-\mu a}$$

$$\theta_3 = \alpha e^{2\mu a} + \beta e^{-2\mu a}.$$

The eqns can be solved by the method of determinants.

$$\begin{vmatrix} \theta & \theta_1 & \theta_2 & \theta_3 \\ \text{Coefft of } \alpha & 1 & e^{\mu a} & e^{2\mu a} \\ \text{Coefft of } \beta. & 1 & e^{-\mu a} & e^{-2\mu a} \end{vmatrix} = 0$$

ie $\theta_1 (e^{-\mu a} - e^{\mu a}) + \theta_2 (e^{2\mu a} - e^{-2\mu a})$

$+ \theta_3 (e^{-\mu a} - e^{\mu a}) = 0.$

ie $(e^{\mu a} - e^{-\mu a}) \{ \theta_2 (e^{\mu a} + e^{-\mu a}) - (\theta_1 + \theta_3) \}$

Putting $\dfrac{\theta_1 + \theta_3}{\theta_2} = 2n$

$e^{\mu a} + e^{-\mu a} - 2n = 0$

& putting $e^{\mu a} = x$,

$x^2 - 2nx + 1 = 0$

ie $x = n \pm \sqrt{n^2 - 1}$

Since $e^{\mu a} = x$ is > 1,

the $-$ive value of the root is inadmissib

$\therefore e^{\mu a} = n + \sqrt{n^2 - 1}$

ie $\mu a = \log(n + \sqrt{n^2 - 1})$

If another bar of Coefft of Conductivi k' is taken, & $\dfrac{\theta_1 + \theta_3}{\theta_2} = 2n'$ for this bar

$\mu' a = \log(n' + \sqrt{n'^2 - 1})$

$\therefore \sqrt{\dfrac{k'}{k}} = \dfrac{\mu}{\mu'} = \dfrac{\log.\{ n + \sqrt{n^2 - 1} \}}{\log\{ n' + \sqrt{n'^2 - 1} \}}.$

(3) Wiedman & Franz. ~~found that elect~~
used Ingen Hauss expt, but in order to make
exactly the same they electroplated the
different rods to the same extent. The temp-
eratures were measured by a sliding thermo-
couple.

[If the bar is covered

$kA\frac{d^2\theta}{dx^2} = 0$; $kA\frac{d\theta}{dx} = C_1$; $kA\,\theta = C_1 x + C_2$.

At $x = 0$; $\theta = \theta_0$ ie $C_2 = kA\,\theta_0$

At $x = l$ $\theta = \theta_l$ ie $C_1 = kA.\left(\frac{\theta_l - \theta_0}{l}\right)$

$\therefore \theta = \theta_0 + x\left\{\frac{\theta_l - \theta_0}{l}\right\}$.]

(4) <u>Forbe's expt.</u>

$kA\,\frac{d^2\theta}{dx^2} = E p\,\theta$

but by the static expt $E p\,\theta\,dx = A dx\,\rho s\,\frac{d\theta}{dt}$.

ie $E p\,\theta = A\rho s R$ R being the rate of
fall of temp. when temp. diff. is θ

$\therefore \frac{k}{\rho s}\,\frac{d^2\theta}{dx^2} = R$

ie $\frac{k}{\rho s}\left[\frac{d\theta}{dx}\right]_{x_1}^{\infty} = \int_{x}^{\infty} R\,dx$

At x_1 $\frac{d\theta}{dx}$ can be read from the graph (1)

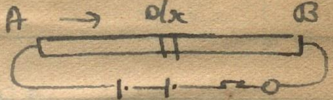
At ∞ $\frac{d\theta}{dx} = 0$ & $\int_{x_1}^{\infty} R\,dx$ is the area of the

$y\left(\frac{d\theta}{dt}, x\right)$ graph.

Electrical method for K.

$\frac{K}{\sigma}$ is found exptally, & knowing σ, K is calculated. The method was suggested first in 1900 by Kohlrausch, & was successfully carried out by Jaegar & Diesselhurst. Later improvements were made by Simleder.

Along a bar AB, a current flows from A to B.



After the steady state is reached, & if the radial flow of heat is negligible, then, the sum of the heat generated electrically, & of the excess of heat flowing into an element due to conduction should be zero.

$$K A \frac{d^2\theta}{dx^2} \cdot dx + \left\{\frac{dE}{\frac{dx}{A\sigma}}\right\}^2 \times \frac{dx}{A\sigma} = 0$$

where A is the sectional area, σ the sp. conductivity, $\therefore$ $\frac{dx}{A\sigma}$ the resistance of the element of length dx, dE the difference in e.m.f. between the ends of the element, θ the

Temp. at the element.

ie $kA\left(\frac{d^2\theta}{dx^2}\right)dx + A\sigma\left(\frac{dE}{dx}\right)^2 \times dx = 0$

ie $\frac{k}{\sigma}\frac{d^2\theta}{dx^2} + \left(\frac{dE}{dx}\right)^2 = 0$

Now $\frac{d^2\theta}{dx^2} = \frac{d}{dx}\left(\frac{d\theta}{dE}\cdot\frac{dE}{dx}\right)$

$$= \frac{d\theta}{dE}\cdot\frac{d^2E}{dx^2} + \frac{dE}{dx}\cdot\frac{d^2\theta}{dE\,dx}$$

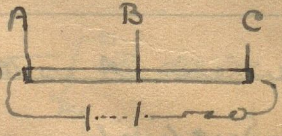
$$= \frac{dE}{dx}\cdot\frac{dE}{dx}\cdot\frac{d^2\theta}{dE^2} = \left(\frac{dE}{dx}\right)^2\times\frac{d^2\theta}{dE^2}.$$

for the current i being constant,
$A\sigma\frac{dE}{dx}$ is a const, ie $\frac{d^2E}{dx^2} = 0$.

$\therefore \frac{k}{\sigma}\cdot\frac{d^2\theta}{dE^2} = -1$ ie $\frac{k}{\sigma}\frac{d\theta}{dE} = -E + C_1$

$\frac{k}{\sigma}\theta = -\frac{E^2}{2} + C_1E + C_2$. (1)

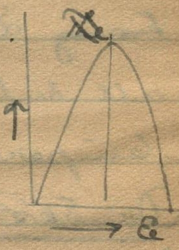
From the above eqn



C_1, C_2 & $\frac{k}{\sigma}$ can be determined
if the temps $\theta_1, \theta_2, \theta_3$ & the
potentials E_1, E_2, E_3 at three points A B C
on a bar are known.

$$\frac{k}{\sigma} = \frac{1}{2}\,\frac{(E_1 - E_2)(E_2 - E_3)(E_3 - E_1)}{\Sigma\, \theta_1 (E_2 - E_3)} \qquad (2)$$

Jaegar & Diesselhurst in their expt simplified the above expn by attaching two water chambers at A & C, making $\theta_1 = \theta_3$. E_2 at the midpt is $\frac{E_1 + E_3}{2}$ for the curve of eqn ① is parabolic



Thus ② reduces to

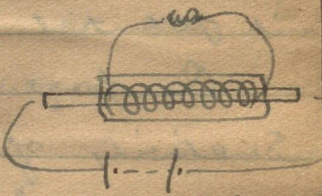
$$\frac{k}{\sigma} = \frac{1}{8}\,(E_1 - E_3)^2 / \theta_2 - \theta_1$$

If $dE = E_1 - E_2$ & $d\theta = \theta_2 - \theta_1$

$$\frac{k}{\sigma} = \frac{dE^2}{d\theta}.$$

The exptal rod was surrounded with a constant temp. bath, & correction was made for the small radial flow.

Simindu's improvement was to eliminate radial flow completely by surrounding the rod with a jacket containing electric heating coils. The current in the heater was adjusted to have its temp. the same as



the average temp. of the rod.

Wiedman - Franz & Lorentz Law.

Wiedman & Franz stated the law that for all substances at the same temp. $\frac{k}{\sigma}$ is constant.

~~Franz~~ Lorentz modified it by saying that for all substances ~~at~~ $\frac{k}{\sigma}$ is inversely $\propto$ to the temperature or $\frac{k}{\sigma T}$ is a constant.

Drude's proof on the free electron theory.

On page 203 (Electricity) the current per unit area is shown to be Saha 354

$\frac{Ne}{2} \cdot \frac{\mathcal{E}e}{m} \frac{\lambda}{\bar{c}}$ where N is the number of free electrons per unit area, $\mathcal{E}$ is the electric field, λ the mean free path.

$\therefore$ Conductivity, σ i.e. current per unit potential gradient $\sigma = \frac{Ne^2\lambda}{2m\bar{c}}$.

On page 40 (K. Theory) the expn for conductivity of heat in a gas is shown to be $\frac{1}{3} n m \lambda \bar{c} C_v$

Assuming that in ~~metals~~ solids heat is conducted by free electrons only

k for a solid $= \frac{1}{3} N m \lambda \bar{c}\, C_v$.

Now $J C_v (\tau_1 - \tau_2) = \frac{1}{2}(C_1^2 - C_2^2) \times m . N$,

ie $m C_v \, d\tau = \cancel{m} \; \cancel{d\mathcal{E}} \; \frac{1}{2} m (C_1^2 - C_2^2)$

$= d\mathcal{E}/J$

ie $m C_v = \frac{1}{J} . \frac{d\mathcal{E}}{d\tau}$.

$\therefore \quad k = \frac{1}{3} N \lambda \bar{C} . \frac{1}{J} . \frac{d\mathcal{E}}{d\tau}$.

Now $\mathcal{E} = \frac{3}{2} k_0 \tau$

$\therefore \quad \frac{d\mathcal{E}}{d\tau} = \frac{3}{2} k_0$

$\therefore \quad k = \frac{1}{2} N \lambda \bar{C} k_0 / J$.

$$\therefore \frac{k}{\sigma} = \frac{\frac{1}{2} \frac{\cancel{N}\lambda \bar{c} k_0}{J}}{\frac{N e^2 \lambda}{2 m \bar{c}}} = \frac{m \bar{c}^2 k_0}{J e^2}$$

~~&~~ Since $\frac{1}{2} m \bar{c}^2 = \frac{3}{2} k_0 \tau$,

$$\frac{k}{\sigma} = \frac{3\tau}{J}\left(\frac{k_0}{e}\right)^2$$

ie $\frac{k}{\tau\sigma}$ is a constant.

$k_0 = 1.372 \times 10^{-16}$ ergs

$= 1.372 \times 10^{-23}$ Joules

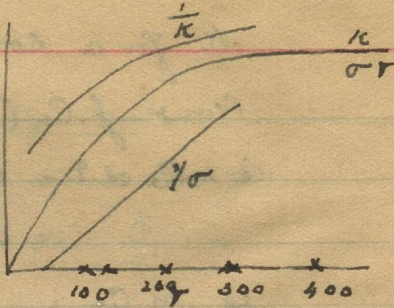
$e = 1.591 \times 10^{-19}$ Coul.

$$\therefore \frac{k}{\sigma\tau} = \left(\frac{1.372}{1.591}\right)^2 \times 10^{-8} \times 3$$

$$= 2.230 \times 10^{-8}$$

This is very nearly the value obtained

xptly.



Exact observations show
at the ratio $\frac{k}{\sigma T}$ is not constant
xcept at ordinary temps.
At very low temps $\frac{k}{\sigma T}$ falls
apidly being $1 \cdot 5 \times 10^{-8}$ at about 100° A.
–, & $\frac{1}{\sigma}$ also decrease rapidly as the
ubstance is cooled.

That stage where the conductivity increases
ery rapidly with temp. is called super-
onductivity.

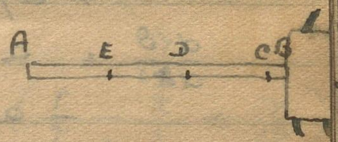
The objections to the electron gas theory
re many. According to the law of equi-
artition of energy, each of the free electrons
ossesses as much energy as the atoms.
nd since the number of free electrons is larger in
onductors than in non-conductors, the mole-
ular sp. heat of conductors should be
reater. But this is ~~ag~~ contrary to expt.
olecular sp. heats are the same for
onductors & non conductors (Dulong & Petit's law.)

The rapid increase of σ at very low temps $\left(\sigma = \frac{\lambda n e^2 \bar{c}}{6 k_0 T}\right)$ requires λ to be very large. But expts do not warrant this assumption.

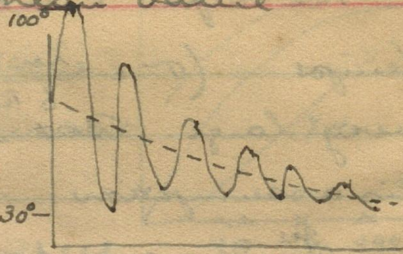
Lindemann suggested that the atoms form lattices through which the free electrons can shoot under special conditions. But this is too much of an ad hoc assumption.

K by Periodic flow of heat

One end of a bar AB is heated with steam for 10 minutes, then cooled with water at for 10 minutes, again heated etc.



When 6 or 7 cycles are finished the temps at diff. points settle down to a uniform rise and fall. At C the amplitude is ~~max~~ greater than at D, & at D it is greater than at E & so on. If at any instant the temp.-distance curve be drawn it will show a ~~less~~ wave form with lessening mplitude & lessening mean value



1) <u>Covered bar.</u>

$$k A \frac{d^2\theta}{dx^2} = A \rho s \frac{d\theta}{dt}$$

$$k \frac{d^2\theta}{dx^2} = \frac{d\theta}{dt} \quad \text{where } k = \frac{K}{\rho s}.$$

Let $\theta = a e^{-\alpha x} \sin(\omega t + \beta x + \gamma) = a e^{-\alpha x} S.$

Temp. at $x = 0$ is max. a & at $x = \infty$ is zero & kept by expt.

$$\frac{d\theta}{dx} = -a \alpha e^{-\alpha s} \sin(\omega t + \beta x + \gamma)$$
$$+ a \beta e^{-\alpha s} C.$$

$$\frac{d^2\theta}{dx^2} = a \alpha^2 e^{-\alpha s} S - a \alpha \beta e^{-\alpha s} C$$
$$- a \alpha \beta e^{-\alpha s} C - a \beta^2 e^{-\alpha s} S$$
$$= a e^{-\alpha s} S (\alpha^2 - \beta^2)$$
$$+ a e^{-\alpha s} C (-2\alpha\beta)$$

$$\frac{d\theta}{dt} = a \omega e^{-\alpha s} C$$

Equating coeffts

$$k a e^{-\alpha s} (\alpha^2 - \beta^2) = 0$$

$$\alpha^2 = \beta^2 \qquad \alpha = \pm \beta.$$

$$-2\alpha\beta a e^{-\alpha s} \times k = a \omega e^{-\alpha s}$$

$$\text{ie} \quad 2\alpha\beta = -\frac{\omega}{k}.$$

If the n^{th} maximum at distance x be θ & is reached at time t, & the n^{th} max. at distance x' be θ' & is reached at time t',

$$\omega t + \beta x + \gamma = 2n\pi + \frac{\pi}{2}$$
$$= \omega t' + \beta x' + \gamma$$

$$\text{ie } -\frac{\omega}{\beta} = \frac{x'-x}{t'-t} = v = \lambda n = \frac{\lambda}{T}$$

$$\text{ie } \beta = -\frac{\omega T}{\lambda} = -\frac{2\pi}{\lambda} \quad \therefore \alpha = \pm \frac{2\pi}{\lambda}$$

But the -ive value is inadmissible since $e^{-\alpha x}$ $(x \to \infty) = 0$

$$\therefore \theta = a e^{-\frac{2\pi x}{\lambda}} \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} + \frac{\gamma}{2\pi} \right)$$

1) Amplitude diminishes with time.

2) The mean temp ie θ for sine func being zero is $ae^{-2\pi x/\lambda}$ which also diminishes as distance increases.

To calculate k.

$$2\alpha\beta = -\frac{\omega}{k}. \text{ ie } k = -\frac{\omega}{2\alpha\beta}.$$

$$\text{ie } k = -\frac{2\pi}{2T} \times -\frac{\lambda^2}{4\pi^2} = \frac{\lambda^2}{4\pi T}$$

Hence if λ the distance between successive crests be known k can be calculated.

2) Bar exposed.

$$k \frac{d^2\theta}{dx^2} = m\theta + \frac{d\theta}{dt} \quad \text{where } m = \frac{Ep}{Ap s}.$$

Supposing the solution to be simple harmonic

$$\theta = a e^{-\alpha x} \sin(\omega t + \beta x + \gamma)$$

$$\therefore k\frac{d^2\theta}{dx^2} = S.\left\{k a e^{-\alpha x}(\alpha^2 - \beta^2)\right\} + C.\left\{k a e^{-\alpha x} - 2\alpha\beta\right\}$$

$$m\theta = S\left\{m a e^{-\alpha x}\right\}$$

$$\frac{d\theta}{dt} = C\left\{a \omega e^{-\alpha x}\right\}$$

Equating Coeffts $\alpha^2 - \beta^2 = \frac{m}{k}$ & $2\alpha\beta = -\frac{\omega}{k}$.

$$(\alpha^2 + \beta^2)^2 = (\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2 = \frac{m^2}{k^2} + \frac{\omega^2}{k^2}$$

$$\therefore \alpha^2 + \beta^2 = \frac{1}{k}\sqrt{m^2 + \omega^2}$$

$$\alpha = +\sqrt{\frac{1}{2k}\left\{(m^2 + \omega^2)^{1/2} + m\right\}} \quad \& \quad \beta = \mp\sqrt{\frac{1}{2k}\left\{(m^2+\omega^2)^{1/2} - m\right\}}$$

But the fn value of θ is not a pure Sine function. It contains a full retinue of upper partials

Let $\theta = a_0 e^{-\alpha_0 x} + a_1 e^{-\alpha_1 x} \sin(\omega t + \beta_1 x + \gamma_1)$

$$+ \dots + a_n e^{-\alpha_n x} \sin(n\omega t + \beta_n x + \gamma_n)$$

$$k \frac{d^2\theta}{dx^2} = \cancel{\alpha} k \alpha_0^2 a_0 e^{-\alpha_0 x}$$
$$+ S_1 \{k a_1 e^{-\alpha_1 x} (\alpha_1^2 - \beta_1^2)\} + C_1 \{k a_1 e^{-\alpha_1 x} x - 2\alpha_1$$

$$+ S_2 \{k a_2 e^{-\alpha_2 x} (\alpha_2^2 - \beta_2^2)\} + C_2 \{k a_2 e^{-\alpha_2 x} x - 2\alpha_2$$

$$+ \dots \quad + \dots$$

$$m\theta = m a_0 e^{-\alpha_0 x}$$
$$+ S_1 \{m a_1 e^{-\alpha_1 x}\} + \dots S_n \{m a_n e^{-\alpha_n x}\}$$

$$\frac{d\theta}{dt} = C_1 \{a_1 \omega e^{-\alpha_1 x}\} + C_2 \{a_2 2\omega e^{-\alpha_2 x}\} \dots$$

Equating like coeff^{ts} $\alpha_n^2 - \beta_n^2 = \frac{m}{k}$

$$2\alpha_n \beta_n = -\frac{n\omega}{k}.$$

$$\therefore \alpha_n = +\sqrt{\frac{1}{2k}\{(m^2 + n^2\omega^2)^{1/2} + m\}} \; ; \; \beta_n = \mp\sqrt{\frac{1}{2k}\{(m^2 + n^2\omega^2)^{1/2} - m\}}$$

Angstrom's Expt. The time for one [c]omplete period was 24 min. Temps. [a]t two pts x & x' were noted at phases [d]iffering from each other by 30°, i.e. [a]t times 0, 2^{min}, 4^{min}, 6^{min} ..... [2]4^{min}. He found that terms above the ~~2nd~~ 1st harmonic could be neglected. Thus ~~θ at dis~~ temp. at distance x is

$$\theta = A_0 + A_1 \sin(\omega t + \delta_1) + A_2 \sin(2\omega t + \delta_2) \quad (1)$$

$$A_0 = a_0 e^{-\alpha_0 x}$$

$$A_1 = a_1 e^{-\alpha_1 x} \qquad A_2 = a_2 e^{-\alpha_2 x}$$

$$\delta_1 = \beta_1 x + \gamma_1 \qquad \delta_2 = \beta_2 x + \gamma_2.$$

The four terms A_1, A_2, δ_1, δ_2 which [a]re dependent on distance can be found [f]rom eqn (1) by substituting the values [f]or θ at four different values of time.

Similarly A_1', A_2', δ_1', δ_2' the quantities [w]hich refer to distance x' can be found.

$$\frac{A_1}{A_1'} = \frac{a_1 e^{-\alpha_1 x}}{a_1 e^{-\alpha_1 x'}} = e^{\alpha_1 (x'-x)}$$

$$= e^{\alpha_1 l}$$

$$\therefore \alpha_1 = \frac{1}{l} \log\left(\frac{A_1}{A_1'}\right)$$

Knowing A_1 & A_1', α_1 can be calculated.

~~But α_1~~ $\delta_1' - \delta_1 = \beta_1 l$

$\therefore \beta_1 = \dfrac{\delta_1' - \delta_1}{l}$. β_1 also can be calculated

Now $2\alpha_1\beta_1 = -\dfrac{\omega}{k} = \dfrac{2\pi}{T k}$

$\therefore k = \dfrac{\pi}{T\alpha_1\beta_1}$.

From the values of A_2 A_2', δ_2, δ_2', α_2 & β_2 can be ||ly calculated.

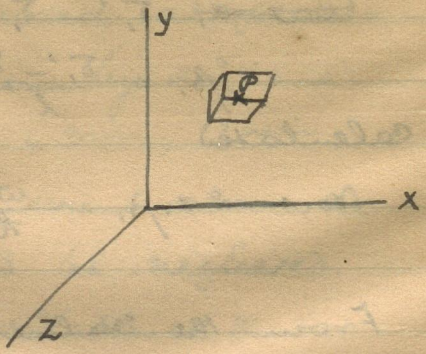
$2\alpha_2\beta_2 = -\dfrac{2\omega}{k}$

$\therefore k = \dfrac{2\pi}{T\alpha_2\beta_2}$.

The above result has been utilize to find the conductivity of the earth's Crust & hence also to compute the age of the earth. The surface of the earth is subjected to a period heating during day time & cooling during night. Sensitive thermometers can detect this heating & cooling effect to a depth of 4 feet or even more. There is also an annual heatin during summer & cooling during winter. The amplitude of this heating & cooling reduc zero only at a depth of nearly 40 ft.

Three dimensional flow of heat.

Let a rectangular
parallelopiped having
sides ∥l to the axes of
coordinates & be con-
sidered about a point
P whose coordinates are
x, y, z. Let the sides of the rect. p-d be
dx, dy, dz.



Influx (p.s.) into the element ∥l to the x-axis

$$= -kA\,\cancel{d\theta}\,\frac{d}{dx}\left(\theta - \frac{1}{2}\frac{d\theta}{dx}.dx\right)$$

where k is conductivity A is area
$dy\,dz$ & θ is temp. at the pt P.

Efflux out of the element ∥l to the x-axis

$$= -kA\,\frac{d}{dx}\left(\theta + \frac{1}{2}\frac{d\theta}{dx}\,dx\right)$$

Net excess of heat lodged in the element
due to heat conducted ∥l to the x-axis

$$= k\,dy\,dz.\frac{d}{dx}.\frac{d\theta}{dx}.dx$$

$$= k\frac{d\theta}{dx^2}\,dx\,dy\,dz.$$

Illrly for y & z axis.

∴ Total heat lodged p.s. in the element

$= k \nabla^2 \theta \, dx dy dz.$

$\nabla^2\theta$ being Laplacian $\theta = \frac{d^2\theta}{dx^2} + \frac{d^2\theta}{dy^2} + \frac{d^2\theta}{dz^2}$.

Heat taken up to raise the temperature

$= (dx dy dz) \rho s \frac{d\theta}{dt}$

Since the two quantities are equal

$\nabla^2\theta = \frac{1}{k} \cdot \frac{d\theta}{dt}$ k being diffusivity.

If the steady state has been reached

$\nabla^2\theta = 0.$

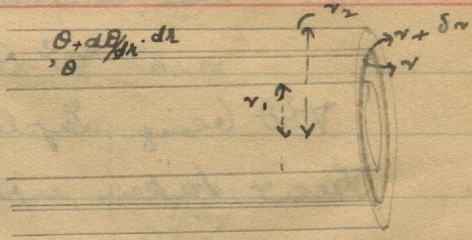
Thermal Conductivity for poor Conductors

The guard ring method is unsuitable since we cannot assume the radial flow negligible when compared to the axial flow. Hence the material is taken as a thin slab or a tube, or a spherical shell. In the case of a shell, the heat is supplied electrically at the ~~tem~~ centre of the shell.

Equation for radial flow of heat.

Let the tube be of internal radius r_1, external radius r_2. Consider a tubular elemen

f the tube wall
l internal & exter-
al diameters r & r
+dr & of length
l.



Influx of heat into the element p.s.

$$= -k \times 2\pi r l . \frac{d\theta}{dr}$$

Efflux of heat from the element

$$= -k \, 2\pi(r+dr) l . \frac{d}{dr}\left(\theta + \frac{d\theta}{dr} dr\right)$$

$$= -k \, 2\pi r l \frac{d\theta}{dr} - k \times 2\pi l . dr . \frac{d}{dr}\left(\theta + \frac{d\theta}{dr} dr\right)$$

$$\cancel{+} \cancel{2} - k . 2\pi l \, r \frac{d}{dr}\left(\frac{d\theta}{dr} dr\right)$$

The excess of influx over efflux

$$= 2\pi l k \left(d\theta + r \frac{d^2\theta}{dr^2} dr\right)$$

neglecting $d^2\theta$ of the term $\left(dr . \frac{d}{dr} . d\theta\right)$

If $d\theta/dt$ be the rate of rise in temp.

$$2\pi l k \left(d\theta + r \frac{d^2\theta}{dr^2} dr\right)$$

$$= 2\pi r \, dr \, l \rho \sigma \frac{d\theta}{dt}$$

ie $\frac{k}{\rho s}\left(\frac{d\theta}{dr} + r \frac{d^2\theta}{dr^2}\right) = r \frac{d\theta}{dt}$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} = \frac{\rho s}{k} \frac{d\theta}{dt} .$$

After the Steady State has been reached $\frac{d\theta}{dt}$ being zero

$$\frac{d^2\theta}{dr^2} = -\frac{1}{r}\frac{d\theta}{dr}$$

ie $\frac{d}{dr}\left(\frac{d\theta}{dr}\right) = -\frac{1}{r}\frac{d\theta}{dr}$ ie $\int \frac{d\left(\frac{d\theta}{dr}\right)}{\frac{d\theta}{dr}} = -\int\frac{dr}{r}$

$\therefore \log \frac{d\theta}{dr} = \log a - \log r$ a being a Constant

ie $\frac{d\theta}{dr} = -\frac{a}{r}$ the negative sign being used since $\frac{d\theta}{dr}$ is $-$ive

$\therefore \int d\theta = -a\int\frac{dr}{r}$ ie $\theta = -a\log r + b.$

If θ_1, θ_2 be the temps at r_1 & r_2, θ at distance r can be evaluated

$$\theta_1 = b - a\log r_1$$

$$\theta_2 = b - a\log r_2$$

$$\therefore \theta_1 - \theta_2 = a\log\frac{r_2}{r_1}$$

$\therefore a = \frac{\theta_1 - \theta_2}{\log r_2/r_1}$ $\therefore b = \theta_1 + a\log r_1$

$$= \left\{\theta_1 \log r_2/r_1 + (\theta_1 - \theta_2)\log r_1\right\}\frac{1}{\log r_2/r_1}$$

$$= \frac{+\theta_1 \log r_2 - \theta_2 \log r_1}{\log r_2/r_1}$$

$\therefore \theta = b - a\log r = \dfrac{\theta_2 \log \frac{r}{r_1} - \theta_1 \log \frac{r_2}{r}}{\log \frac{r_2}{r_1}}$

Again heat flowing p.s. through an annular element of radius r

$$Q = KA\frac{d\theta}{dr} = KA\frac{a}{r}$$

$$= K \times 2\pi r l . \frac{1}{r} . \frac{\theta_2 - \theta_1}{\log r_1/r_2}$$

$$= 2\pi l K \frac{\theta_1 - \theta_2}{\log r_2/r_1} .$$

If d be the thickness $r_2 - r_1$ which is small compared to r_1 or r_2,

$$\log \frac{r_2}{r_1} = \log\left(1 + \frac{d}{r_1}\right) = \frac{d}{r_1}$$

$$\therefore Q = 2\pi r_1 l K \frac{\theta_1 - \theta_2}{d} .$$

~~Radial fl~~ The error introduced by taking the simplified form $Q = 2\pi \frac{r_1 + r_2}{2} l K . \frac{\theta_1 - \theta_2}{r_2 - r_1}$ can be shown to be small.

If $r_1 = 2$, $r_3 = 3$ cms, the accurate value of

$$Q = 2\pi l K \frac{\theta_1 - \theta_2}{\log r_2/r_1} \propto \frac{1}{\quad}$$

ie $K \propto 2.3026 \log 1.5$

$\propto .4055$

Taking the approximate formula

$$k \propto \frac{2}{5} = \cdot 4000$$

$\therefore$ The % error $= -1\cdot 375$ %.

If $r_1 = 10$, $r_2 = 11$,

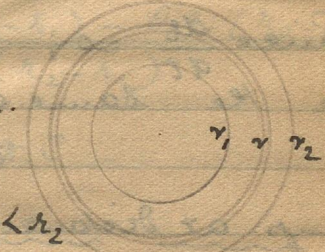
the accurate value of $k \propto \cdot 09528$

& the approximate " $\propto \cdot 09523$

$\therefore$ % error $= -\cdot 05$ % which is far smaller than expt.al errors.

Radial flow of heat in a hollow sphere.

Let the inside be at temp θ_1 & outside at temp θ_2 & the let $\theta_1 > \theta_2$.



Consider an element of thin shell radius $r > r_1 \,\&< r_2$ & thickness dr. Let θ be temp at distance r.

Influx p.s. of heat into the shell

$$= -k.\ 4\pi r^2.\ \frac{d\theta}{dr}$$

Efflux p.s. from the shell

$$= -k\ 4\pi(r+dr)^2.\ \frac{d}{dr}\left(\theta + \frac{d\theta}{dr}dr\right)$$

Excess lodged in the element

$$= k \cdot 4\pi \frac{d}{dr}\left(r^2 \frac{d\theta}{dr}\right) dr$$

$$= k \cdot 4\pi r^2\left(\frac{d^2\theta}{dr^2} + \frac{2}{r}\frac{d\theta}{dr}\right) dr.$$

This quantity is the heat req^d to raise the temp. of the element by $\frac{d\theta}{dt}$

ie $= 4\pi r^2 dr \times \rho \times s \times \frac{d\theta}{dt}$.

ie $k \cdot \frac{d^2\theta}{dr^2} + \frac{2}{r}\frac{d\theta}{dr} = \rho s \frac{d\theta}{dt}$

ie $\frac{1}{k}\frac{d\theta}{dt} = \frac{d^2\theta}{dr^2} + \frac{2}{r}\frac{d\theta}{dr}$.

Since $\frac{dr}{dt} = 0$, the above can be shown to be the same as

$$k \cdot \frac{d^2(\theta r)}{dr^2} = \frac{d(\theta r)}{dt}$$

Temp. at distance r after steady state is reached.

Can be determined as in the ~~ab~~ former case by integration.

But a simpler method is possible since the ~~temp. at an~~ heat passing through any layer is a constant, no heat being absorbed anywhere.

ie $k \, 4\pi r^2 \frac{d\theta}{dr}$ is a constant ie $r^2 \frac{d\theta}{dr}$ is a constant

ie $\int d\theta = -c\int \frac{dr}{r^2}$.

The negative sign is ~~__~~ used since $\frac{d\theta}{dr}$ is $-$ve.

ie $\theta = c/r + b$.

Applying end conditions

$\theta_1 = \frac{c}{r_1} + b \qquad \theta_2 = \frac{c}{r_2} + b.$

Whence we have

$c = \frac{r_1 r_2(\theta_1 - \theta_2)}{r_2 - r_1}$ & $b = \frac{r_2\theta_2 - r_1\theta_1}{r_2 - r_1}$

$\therefore \theta = \frac{c}{r} + b = \frac{\theta_2 r_2(r - r_1) - \theta_1 r_1(r_2 - r)}{r(r_2 - r_1)}$

$Q = k.\ 4\pi r^2 \frac{d\theta}{dr} = k.\ 4\pi c$

$= 4\pi k.\ \frac{r_1 r_2(\theta_1 - \theta_2)}{(r_2 - r_1)}$

The simplified formula assuming the thickness of the shell to be small compared to r_1 or r_2 is

$Q = k.\ 4\pi\left(\frac{r_1 + r_2}{2}\right)^2 \frac{\theta_1 - \theta_2}{r_2 - r_1}$.

Proceeding as before it can be shown that the error introduced by this simplifica

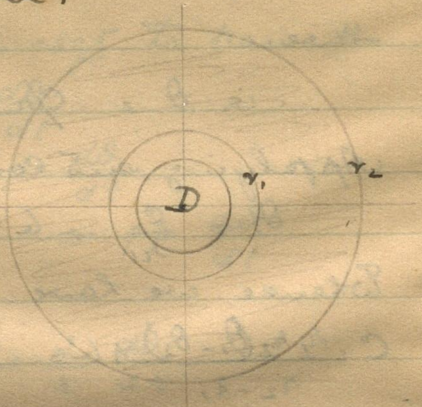
$- 4\%$ for $r_1 = 2$, $r_2 = 3$ cms

$- \cdot 2\%$ for $r_1 = 10$; $r_2 = 11$ cms.

Expts for Coefft. of Conductivity

Bad Conductors - Solids.

[N]usselt 1908



The principle of [r]adial flow in a hollow [s]phere was employed. [T]wo metallic shells [cu]t into halves contain [be]tween them the substance, powdered [co]rk, carbon or some other material. [Th]e shells can be split into two & reunited [thu]s to help the filling in. D is an electrically [h]eated body at the centre

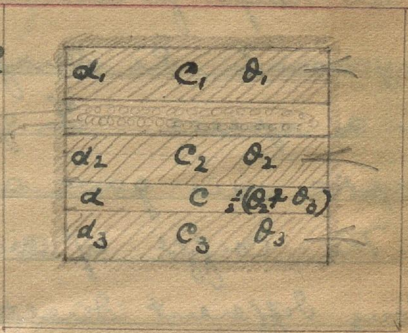
$$\frac{C^2R}{J} = Q = k\,4\pi\, r_1 r_2 \frac{\theta_1 - \theta_2}{r_2 - r_1}.$$

Temps. are measured by thermocouples.

Lees' Film Method (1898)

C_1, C_2, C_3 are copper discs & C the crystal [s]ubstance. A heater coil is placed between C_1 & C_2. [T]he outside of all the plates is given the same

paint so as to have the
Same emissivity power



If S be the emitting
Surface area

$$\frac{C^2R}{J} = E.\left\{ S_1\theta_1 + S_2\theta_2 + S_3\theta_3 + S\,\frac{\theta_2+\theta_3}{2}\right\}$$

Now $S_1 = \pi r^2 + 2\pi r d_1$, $S_3 = \pi r^2 + 2\pi r d_3$

$S_2 = 2\pi r d_2$ $S = 2\pi r d$.

All quantities except E are known ∴
E is calculated.

Heat passing through the middle layer of

$$C = k.\pi r^2.\frac{\theta_2-\theta_3}{d}$$
$$= E.\left\{ S_3\theta_3 + \frac{1}{2} S.\frac{\theta_2+\theta_3}{2}\right\}$$

k is calculated.

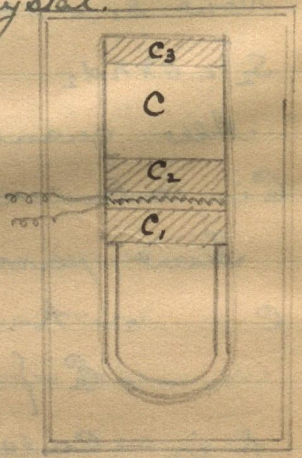
Conductivity of Crystals.

Senarmont (1847)

The principle is similar
to that of Ingen Hausz expt
A crystal was coated
uniformly with
wax as ~~sunlight~~

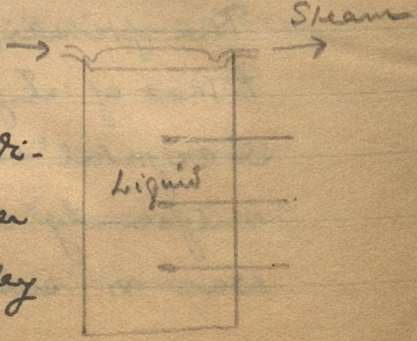
olar rays focussed on to a pt. on the surface.
In general the surface on which wax melted
was a sphere. In quartz χ cut surface normal
optic axis gave a ○lar isothermal surface.

The coefft of conductivity is different
along different directions & these directions
coincide with the axes of the crystal.



Eucken (1911) conducted
expts on crystals at different
temps. The method was a
slight modification of Lees'.
The pile of plates was
supported on a hollow pine
wood base & enclosed within
a liquid air or water bath.

He formulated the empirical law $k \propto \frac{1}{\theta}$
at low temperatures.

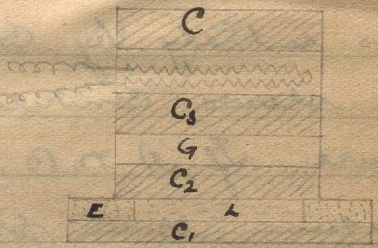


Liquids

Bottomley The different modifications are all to avoid transfer
of heat by convection. Bottomley

heated the top of the liquid ~~to~~ with a chamber ~~contain~~ of steam & noted temps at various points.

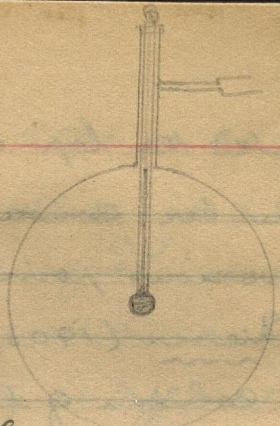
Weber (1903) Surrounded the column of liquid with a guard ring. Heat was supplied from an electrically heated oil bath. The amount of ice melted gave the heat conducted.



Lees (1899) Film method. (Saha 366). Between the copper plates C_3 & C_2 is a plate of glass. The liquid is surrounded by ring of ebonite. Heat passing through the central layer of glass — heat ~~passing~~ radiated from C_2 & half of G $= \left\{ K_l S_l \frac{\theta_2 - \theta_1}{d} + B.(\theta_2 - \theta_1) \right\}$ heat conducted through the liquid & through ebonite. B is determined by doing the expt with a liquid whose conductivity is kno

Gases.

Cooling thermometer method of Kundt & Warburg.



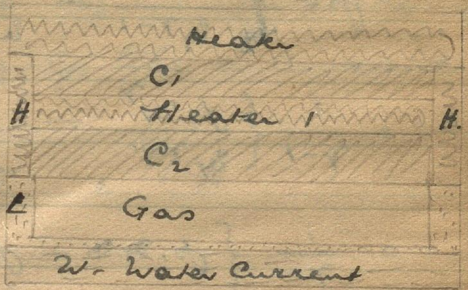
It was found that if the flask is evacuated cooling is independent of size i.e. loss of heat is entirely through radiation

Between pressures of 1 mm & 150 mm. cooling is dependent on size but independent of pressure; ∴ at this press. ~~the~~ loss of heat is entirely by conduction, for as kinetic theory shows conduction is independent of pressure. If A & B be rates of cooling in vacuum & in pressure > 1 mm & < 150 mm then B-A is rate of cooling due to conduction alone.

Other experimenters used the hot wire method. Along the axis of the a cylindrical vessel is placed a heated Pt wire or a silver tube through which steam is passing. The pressure is <150 & >1 mm.

Film method was employed by Laby & Stevens

The gas is placed in an ebonite dish, heat supplied from a heater coil H_1 & received by a current of water flowing through W.



Expts have shown that k varies with temp.

$$k_t = k_0 \frac{b+273}{b+T} \cdot \left(\frac{T}{273}\right)^{3/2}.$$

Thermodynamics

Work done during a large isothermal expansions

$$W = \int_0^W dW = \int_{v_1}^{v_2} p\,dv = RT \int_{v_1}^{v_2} \frac{dv}{v}$$

$$= RT \log \frac{v_2}{v_1} = p_1 v_1 \log \frac{v_2}{v_1} = p_2 v_2 \log \frac{v_2}{v_1}$$

Work done during a large adiabatic expansion

$$w = \int_0^w dW = \int_{v_1}^{v_2} p\,dv = pv^\gamma \cdot \int_{v_1}^{v_2} \frac{dv}{v^\gamma}$$

$$= pv^\gamma \left\{ \frac{1}{v_1^{\gamma-1}} - \frac{1}{v_2^{\gamma-1}} \right\} \cdot \frac{1}{\gamma-1}$$

$$= \frac{1}{\gamma-1} \left\{ \frac{p_1 v_1^\gamma}{v_1^{\gamma-1}} - \frac{p_2 v_2^\gamma}{v_2^{\gamma-1}} \right\}$$

$$= \frac{1}{\gamma-1} \left\{ p_1 v_1 - p_2 v_2 \right\} = \frac{R}{\gamma-1} (\tau_1 - \tau_2).$$

Work done in the adiabatic expansion to infinity $= \dfrac{R}{\gamma-1} \tau_1$

$$= \frac{p_1 v_1}{\gamma-1}.$$

Equations to the Curves.

1) Iso thermal Curve connecting p & v at constant temp.

$$pv = R\tau$$

2) Isentropic Curve Connecting p & v at Constant entropy

$$pv^\gamma = k.$$

3) Isobar curve Connecting τ & φ at Constant pressure.

$dQ = \tau\, d\varphi$ from defn of entropy

$= C_p\, d\tau$

$\therefore \int \frac{d\tau}{\tau} = \int \frac{d\varphi}{C_p}$ ie $\log \tau = \frac{\varphi}{C_p} + \log \alpha$

taking C_p to be a constant.

$\therefore \log \frac{\tau}{\alpha} = \frac{\varphi}{C_p}$

ie $\frac{\tau}{\alpha} = e^{\varphi / C_p}$

ie $\tau = \alpha e^{\varphi / C_p}$

4) Isometric curve connecting τ & φ at constant volume.

$dQ = \tau\, d\varphi = C_v\, d\tau$

$\therefore \tau = \beta e^{\varphi / C_v}$.

Heat Engines

are based on the three kno facts of nature:—

1) The lifting power of steam on wh Steam engines are based

2) Impulsive ~~&~~ energy of gunpowder on which are based the internal combustion engines &

3) The driving force of wind, from

which turbines have their origin.

Efficiency of in all engines is defined as the ratio of the available energy to the energy supplied. ~~Heat~~ Steam engines did not at first have an efficiency more than 5%; & today's most perfect ~~heat~~ steam engines do not exceed 20%; while an electric engine may easily have an efficiency of 95%.

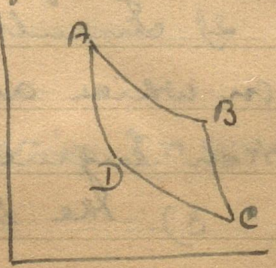
Carnot's theorem states that of all engines working between the same temps. the reversible has the max. efficiency & that all reversible engines have the same efficiency. Even in reversible engines effy cannot exceed a certain limit & this defect ~~which~~ is inherent in heat engines

The efficiency of a Carnot Cycle.

(1) In the expansion AB

heat absorbed = Q_1

Work done by the engine



$= \frac{R t_1}{J} \log \frac{v_b}{v_a}$

2) During the adiabatic expansion BC, heat absorbed $= 0$

Work done $= \frac{R}{J(\gamma-1)} \cdot (T_1 - T_2)$

3) During the isothermal contraction CD, heat absorbed $= -Q_2$.

Work done $= -\frac{RT_2}{J} \log \frac{v_c}{v_d}$.

4) During the adiabatic contraction DA, heat absorbed $= 0$

Work done $= -\frac{R}{J(\gamma-1)} (T_1 - T_2)$

Efficiency $\eta = \frac{\text{Total work done}}{\text{heat absorbed}}$

$$= \frac{T_1 \log \frac{v_b}{v_a} - T_2 \log \frac{v_c}{v_d}}{T_1 \log \frac{v_b}{v_a}}$$

But $\left(\frac{v_c}{v_b}\right)^{\gamma-1} = \frac{T_1}{T_2} = \left(\frac{v_d}{v_a}\right)^{\gamma-1}$

$$\therefore \frac{v_b}{v_a} = \frac{v_c}{v_d}$$

$$\therefore \eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

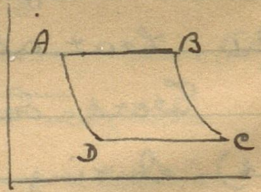
Now $\frac{T_2}{T_1} = \left(\frac{v_b}{v_c}\right)^{\gamma-1} = \left(\frac{1}{\rho}\right)^{\gamma-1}$

where $\rho = \frac{v_c}{v_b}$ = adiabatic expansion ratio

$$\therefore \eta = 1 - \left(\frac{1}{\rho}\right)^{\gamma - 1}$$

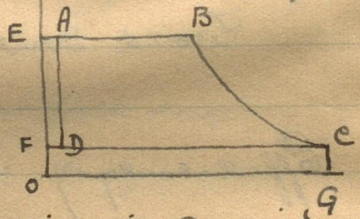
If the Carnot cycle is worked with team as the working substance, the wo isothermals reduce to st. lines

The Carnot cycle is owever never used even e steam engines.

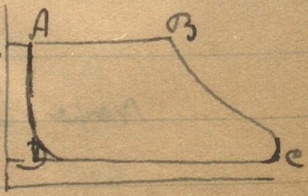


Its place is taken by the Rankine Cycle

Rankine Cycle is in ssentials similar to the rnot cycle, but the 3rd tep of isothermal compression is carried till all the steam is reduced to liquid. e liquid is then forced into the boiler & thence o the cylinder by a separate pressure ump. In the above diagram EFDA epresents work done in forcing the iquid into the boiler. Efficiency



$$\eta = \frac{EBCF - ADFE}{EBCGO} = \frac{ABCD}{EBCGO}.$$



In actual engines this efficiency

is not however achieved since the 2nd step is not purely adiabatic; nor is CD continued till all steam is converted into liquid. This modification makes the cylinder more compact, & the time of one complete cycle smaller. Efficiency as given in the above eqn is 70% for the ideal engine & 60% for ~~the~~ actual engines. This ofcourse is computed without taking into consideration heat loss in the furna boiler etc.

Internal Combustion engines

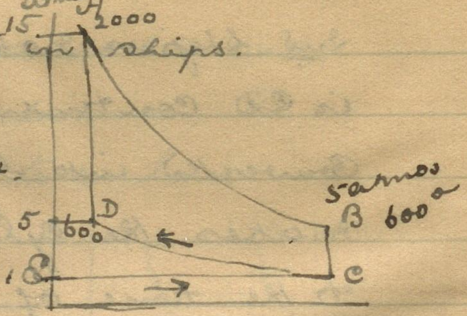
About 90% of modern engines belong to this type where the fuel is made to burn in the cylinder itself & its maximum heat value is made use of. The large losses in furnace & boiler are thus avoided.

They are of two ~~typ~~ kinds

The Otto engine, used in cars & aero

planes, &

The Diesel engine used in ships.



Otto Engine

absorbs heat at const.
nt volume. The fuel
s supplied as vapour
petrol.

The cylinder has
ree valves at its
losed end, A for petrol gas, B for air,
for exhaust.

The working is in the following stages.

) Charging stroke.

A & B being open & C closed, a suitable
mixture of air & petrol gas is intro-
duced into the cylinder at constant
pressure. The process is indicated by
e line E C.

2) Compression stroke. When the piston
has gone farthest back, & the two inlet
valves are closed, & the piston is moved

forward. The gas is compressed adiabatically to $\frac{1}{5}$ its original volume thereby raising its temperature to 600 At this stage a series of sparks are produced by the magneto & thereby raising the temp. to 2000°C & the press to 15 atmospheres. The two stages are shown by CD & DA.

3) Working stroke. The piston is now violently pushed forward by the high pressure gas until the pressure falls to 5 atmosp. nearly the same as at D. The process is indicated by AB. The exhaust valve is now opened, the pressure comes down to one atmosp. the release being represented by BC.

4) Scavenger stroke. Lastly the piston is pushed forward to remove all the inert gas from the cylinder. At the end of this stroke the exhaust

valve is closed & the other two valves
pened, thus making the cylinder ready
or a fresh cycle.

Heat supplied $Q_1 = C_v(T_a - T_d)$ per gm of the gas

Heat given out $Q_2 = C_v(T_b - T_c)$

$$\therefore \eta = 1 - \frac{T_b - T_c}{T_a - T_d}$$

Now $pv^\gamma = \text{const}$ & since $p = R\frac{T}{v}$

$T v^{\gamma-1}$ const.

$\therefore T_b v_b^{\gamma-1} = T_a v_a^{\gamma-1}$

$T_c v_b^{\gamma-1} = T_d v_a^{\gamma-1}$ $\therefore \frac{T_b - T_c}{T_a - T_d} = \frac{1}{\rho^{\gamma-1}}$

where $\rho = \frac{v_b}{v_a}$

$$\therefore \eta = 1 - \frac{1}{\rho^{\gamma-1}}.$$

The chief advantages of the Otto engine compared to the Carnot engine are that it is quick in its strokes, compact in design, & more economical. In a Carnot engine where heat is supplied at constant ~~pressure~~ temp, the supply should be made slowly. So also the giving out of heat to the sink should be slow. The Otto cycle can be much quicker.

It can be shown ~~of~~ that a Carnot ~~engine~~ cylinder having the same output of work as an Otto cyl. should be made of much thicker walls & should have a far greater length. ~~than a~~

The total expansion ratio $\frac{v_c}{v_a}$ in the Carnot cyl. should be nearly 100 times that in the Otto cylinder. In the Otto cylinder the ratio of the max. pressure to the mean pressure is 3·5 whereas in the Carnot cylinder it is more than 1000. These two factors make the ~~cyl.~~ Carnot cyl. excessively bulky & heavy which means that a Carnot engine is much more costly.

The difficulty has been partly overcome in locomotives where the whole expansion is not performed in one & the same cylinder. The steam after ex-

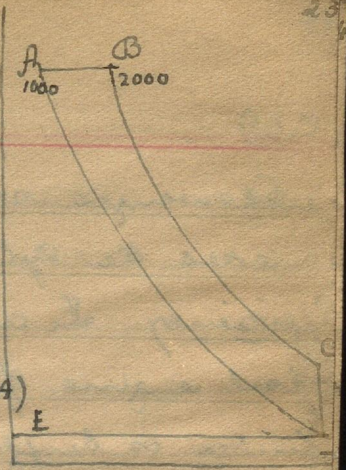
anding to some extent in one cylinder
s transferred to another with thicker
walls for the next stage in expansion
from this again to a third.

~~But~~ The efficiency of the otto engine
only 45% whereas a Carnot
ngine can have an efficiency of 85%
more. (neglecting losses in the furnace etc.)

Diesel engine

In the Otto engine efficiency is
low because the adiabatic expansion
atio ρ is only 5; nor can ρ be increased
much farther since if too highly com-
ressed the gas would ignite spontaneously.
iesel argued that ρ can be increased
f the fuel is introduced after the com-
pression stroke.

The Diesel engine absorbs heat at
constant pressure. The fuel can be
supplied as crude oil & need not be
vapour.



1) Charging stroke

(ED) The air valve alone is open, & air is taken in at atmos. pressure

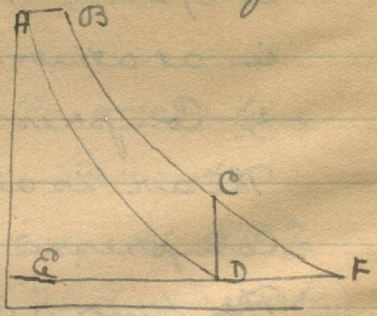
2) Compression Stroke (DA)

The air is adiabatically compressed to $1/17$ th the original vol. The temp rises to nearly 1000°. ~~At~~

3) Working Stroke. At this stage the fuel is introduced at constant press by means of a force pump. As soon as the fuel enters the cylinder it is ignited & the working stroke begins. When a part (AB) of the working stroke is finished the supply of fue is cut off & the stroke allowed to continue thus expanding the gas adiabatically along BC. When the piston reaches ~~it~~ the end of its stroke the exhaust valve is opened & the pressure brought down to that of the atmosphere

(D)

) Scavenger stroke D E follows next.
makes the cyl. ready for another cycle.



Efficiency In an ideal
Diesel engine where the
cylinder is sufficiently long,
that the working stroke
may be allowed to continue
till the initial pressure is reached.
The cycle will then be ABFD

Heat absorbed $= C_p\,(T_b - T_a)$

$\quad$ ... given out $= C_p\,(T_f - T_d)$

$$\therefore \eta = 1 - \frac{T_f - T_d}{T_b - T_a}.$$

$p v^{\gamma} = \text{Const} \qquad \& \quad v = R\frac{T}{p}$

$\therefore \dfrac{T^{\gamma}}{p^{\gamma-1}} = \text{Const.} \quad T / p^{1-1/\gamma}\ \text{Const.}$

$$\therefore T_b\, p_f^{1-1/\gamma} = T_f\, p_b^{1-1/\gamma}$$

$$T_a\, p_f^{1-1/\gamma} = T_d\, p_b^{1-1/\gamma}$$

$$\therefore \frac{T_f - T_d}{T_b - T_a} = \left(\frac{p_f}{p_b}\right)^{1-1/\gamma}$$

$$p v^{\gamma} = \text{Const.}$$

$$\therefore p^{\frac{\gamma-1}{\gamma}} . v^{\gamma-1} = \text{Const}$$

$$\therefore \frac{T_f - T_d}{T_b - T_a} = \left(\frac{v_b}{v_f}\right)^{\gamma-1} = \frac{1}{\rho^{\gamma-1}}$$

$$\therefore \eta = 1 - \frac{1}{\rho^{\gamma-1}}$$

The efficiency of the ideal engine ~~is~~ is 65%; but in actual engines where cylinder is not so very long efficiency does not exceed 55%.

Thus the superior merits of the Diesel engine are that its efficiency is higher, it can taken in cheaper fuel such as crude oil, & that it does not need any sparking device.

But the Diesel engine easily gets out of order, & is not easily manipulated except by skilled workmen. Its ~~a~~ total expansion ratio is 4.9 [which is slightly greater than that of the Otto engine given in the previous problem].

Entropy

Change of entropy over a large change of temp -

Say in heating a solid at temp T_1

to saturated vapour at temp θ_2.

$$dQ = \theta \, d\phi = s_1 \, d\theta$$

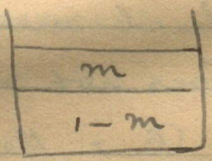
$$\therefore d\phi = s_1 \frac{d\theta}{\theta}$$

$$\therefore \phi_1 - \phi_2 = \int_{\theta_1}^{\theta_m} s_s \frac{d\theta}{\theta} + \frac{L_m}{\theta_m} + \int_{\theta_m}^{\theta_b} s_l \frac{d\theta}{\theta} + \frac{L_b}{\theta_b} + \int_{\theta_b}^{\theta_2} s_v \frac{d\theta}{\theta}.$$

Latent Heat equations

Clausius proofs

Let unit mass of a substance be contained in a vessel of which



m is in vapour state & 1 − m in liquid state. Let vapour & liquid be in dynamic equil. at temp θ.

If heat dQ ($= \theta \, d\phi$) is imparted which changes dm gm of liquid into vapour & raises the temp. by $d\theta$

$$dQ = \theta \, d\phi$$

$$= \left\{(1-m) s_1 + m s_2\right\} d\theta + L \, dm$$

$$\therefore \left(\frac{d\phi}{d\theta}\right)_m = \frac{1}{\theta}\left(\overline{1-m}\, s_1 + m s_2\right)$$

$$\left(\frac{d\varphi}{dm}\right)_\tau = \frac{L}{\tau}.$$

$$\left(\frac{d}{dm}\right)_\tau \cdot \left(\frac{d\varphi}{d\tau}\right)_m = \left(\frac{d}{d\tau}\right)_m \cdot \left(\frac{d\varphi}{dm}\right)_\tau$$

ie $s_2 - s_1 = \tau \frac{d}{d\tau}\left(\frac{L}{\tau}\right)$. (1)

If v_2, v_1 be the sp. volumes of q vapour and liquid,

$$(v_2 - v_1)\, dm = dv$$

$$\left(\frac{d\varphi}{dm}\right)_\tau = \frac{L}{\tau} \text{ ie } \left(\frac{d\varphi}{dv}\right)_\tau (v_2 - v_1) = \frac{L}{\tau}$$

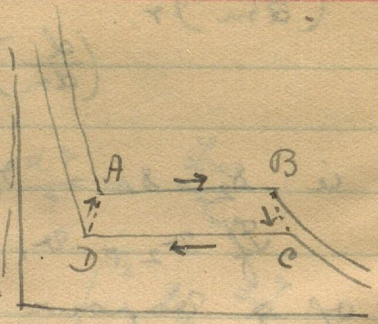
But $\left(\frac{d\varphi}{dv}\right)_\tau = \left(\frac{dp}{d\tau}\right)_v$

$$\therefore d\tau = \frac{\tau(v_2 - v_1)\, dp}{LJ} \quad (2)$$

Equation (1) gives the change of sp. heat when a substance changes from solid to liquid state or from liquid to vapour state.

Eqn. (2) gives the elevation of boiling point or the change in freezing point due to the increase of ~~temp~~ pressure.

Clayperon's proof.



When a substance is taken through a complete cycle the ~~st~~ change in entropy is zero.

Let the cycle be ABCD AB being at $\tau + d\tau$, D at τ.

$$\frac{s_1 d\tau}{\tau + d\tau} + \frac{L + dL}{\tau + d\tau} - \frac{s_2 d\tau}{\tau + d\tau} - \frac{L}{\tau} = 0$$

$$(s_1 - s_2) d\tau + L + dL - L\left(1 + \frac{d\tau}{\tau}\right) = 0$$

$$(s_2 - s_1) = \frac{dL}{d\tau} - \frac{L}{\tau}$$

$$\text{But } \tau \frac{d}{d\tau}\left(\frac{L}{\tau}\right) = \tau\left(\frac{1}{\tau}\frac{dL}{d\tau} - \frac{L}{\tau^2}\right) = \frac{dL}{d\tau} - \frac{L}{\tau}$$

$$\therefore s_2 - s_1 = \tau \frac{d}{d\tau}\left(\frac{L}{\tau}\right). \quad (1)$$

Net gain of heat $s_1 d\tau + L + dL - s_2 d\tau - L$

$$= (s_1 - s_2) d\tau + dL = (v_2 - v_1) dp.$$

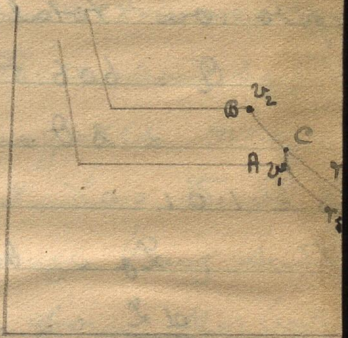
$$\therefore (v_2 - v_1)\frac{dp}{d\tau} = (s_1 - s_2) + \frac{dL}{d\tau}$$

$$= \frac{L}{\tau}$$

$$\therefore d\tau = \frac{\tau(v_2 - v_1)\,dp}{LJ}.$$

Negative sp. heat of saturated vapour

If saturated vapour at temp τ_1 is ~~heated~~ raised in temp. to τ_2, the heat given it ~~nee~~ may be −ive or +ive; for the change from



A to B may be considered as the sum of changes from A to C & from C to B

Heat absorbed $s(\tau_2 - \tau_1) = C_v(\tau_2 - \tau_1)$
$- R\tau_2 \log\frac{v_1}{v_2}$

If $R\tau_2 \log\frac{v_1}{v_2} > C_v(\tau_2 - \tau_1)$ then the sp. heat is −ive.

$$s_2 = s_1 + \frac{dL}{d\tau} - \frac{L}{\tau}$$

This eq$\underset{..}{\underline{n}}$ again shows that s_2 can be −ive.

The sp. heat is negative since on raisin e temp, the volume diminishes which liberates more energy than is reqd to increase K. E. of translation of the molecules.

$\frac{dL}{dT}$ for water may be evaluated from Regnault's expts on total heat.

$$Q = 606 \cdot 5 + \cdot 305\,\theta$$

$$= s\theta + L_\theta \quad s \text{ being sp. heat}$$

$s = 1 \cdot 01$

$$\therefore L_\theta = 606 \cdot 5 - \cdot 705\,\theta$$

$$\frac{dL}{dT} = - \cdot 705$$

$$\therefore s_2 = 1 \cdot 01 - \cdot 705 - \frac{536}{373}$$

Callender's more accurate expts give $\frac{dL}{dT}$ to be $- \cdot 64$

$$\therefore s_2 = 1 \cdot 01 - \cdot 64 - \frac{536}{373}$$

$$= \underline{\underline{-1 \cdot 03}}$$

That temp at which $s_2 = 0$ is called i.e. changes from $-$ive to $+$ive is called the temp. of sp. heat inversion.

If T be that temp,

$$\frac{L}{T} = s_1 + \frac{dL}{dT}.$$

Hirn demonstrated +ive & -ive values of sp. heat by a simple expt.

A chamber with transparent walls is filled with ~~sa~~ dry saturated

steam. The steam is quite transpare[nt]. The chamber is made to expand sudd[enly]. The temp. of the steam falls, & since sp. heat is -ive, heat has to be supplied to it. The change being sudd[en] the reqd heat is taken from the body of the steam, thereby condensing a part of it. The chamber should appear foggy. This was actually observed.

If however ether vapour which has +ive sp. heat was used, compressi[on] caused fogging.

Variation of sp. heat with temp.

$$s_2 = \frac{dQ}{dT} = T\left(\frac{d\phi_2}{dT}\right)_{sat.}$$

$$T\left(\frac{d\varphi_2}{dT}\right)_{sat} = T\left(\frac{d\varphi_2}{dT}\right)_p + T\left(\frac{d\varphi_2}{dp}\right)_T\left(\frac{dp}{dT}\right)_{sat}$$

$$= T\left(\frac{d\varphi_2}{dT}\right)_p - T\left(\frac{dv_2}{dT}\right)_p \cdot \left(\frac{dp}{dT}\right)_{sat}$$

$$s_2 = C_{p_2} - T\left(\frac{dv_2}{dT}\right)_p\left(\frac{dp}{dT}\right)_{sat}$$

$$s_1 = C_{p_1} - T\left(\frac{dv_1}{dT}\right)_p \cdot \left(\frac{dp}{dT}\right)_{sat}$$

$$s_2 - s_1 = C_{p_2} - C_{p_1} - T\left(\frac{dp}{dT}\right)_{sat}\left\{\left(\frac{dv_2}{dT}\right)_p - \left(\frac{dv_1}{dT}\right)_p\right\}$$

But $s_2 - s_1 = \dfrac{dL}{dT} - \dfrac{L}{T}$

$$T\left(\frac{dp}{dT}\right)_{sat} = \frac{L}{v_2 - v_1}$$

$$\therefore \frac{dL}{dT} = \frac{L}{T} + C_{p_2} - C_{p_1} - \frac{L}{v_2 - v_1}\left\{\left(\frac{dv_2}{dT}\right)_p - \left(\frac{dv_1}{dT}\right)_p\right\}$$

Contd in ~~Kinetic theory~~.
Modern Physics.

$$-a\cos\alpha\,\frac{\sin(k-l)}{\sin(i+k)}$$

Light

$$-a\cos\alpha.\frac{\sin(i-r)}{\sin i+r} \text{ \& } -a\sin\alpha\frac{\tan(i-r)}{\tan(i+r)}$$

polaryzn the planes ~~\|\|~~ & \|\| to incident plane

If these be denoted by

$b\cos\beta$ & $b\sin\beta$,

the inclination of the plane of polariza of the reflected beam to the incident plane β is given by

$$\tan\beta = \tan\alpha.\frac{\cos(i+r)}{\cos(i-r)}.$$

The refracted vibrations are

$$a\cos\alpha.\frac{2\cos i\sin r}{\sin(i+r)} \text{ \& } a\sin\alpha\frac{2\cos i\sin}{\sin(i+r)\cos}$$

If these be denoted by $c\cos\gamma$ & $c\sin$ the inclination of the plane of pol. of the refracted beam to the incident plane is given by

$$\tan\gamma = \tan\alpha.\frac{1}{\cos(i-r)}$$

$$\therefore \tan\beta = \tan\alpha.\frac{\cos(i+r)}{\cos(i-r)}\;;\; \tan\gamma = \tan\alpha\frac{1}{\cos(i-r)}$$

Thus in general the planes of polari zation of refracted & reflected rays

~~be~~ rotated. ~~by β~~

For the reflected ray, when $i = 0$, $\beta = \alpha$,

when $i + r = 90°$ $\quad\beta = 0$

when $i \quad = 90°$ $\quad\beta = -\alpha$

Thus as i increases the inclination β changes from $+\alpha$ to $-\alpha$, being zero at the $\angle$ of maximum polarization.

For the refracted ray, when $i = 0$ $\quad\gamma = \alpha$

when $i = 90°$, $\quad\tan\gamma = \dfrac{\tan\alpha}{\sin r} = \tan\alpha \times \mu$

Circularly polarized light

It can be resolved into two components at rt. $\angle$s to each other, $a\cos\omega t$ vibrating in the plane of incidence, & $a\sin\omega t$ vibrating $\perp$r to the plane of incidence.

Reflected components are

$$a\cos\omega t\ \frac{\tan(i-r)}{\tan(i+r)} \quad \& \ -a\sin\omega t\ \frac{\sin(i-r)}{\sin(i+r)}$$

Refracted components are

$$a\cos\omega t\ \frac{2\cos i\sin r}{\cos(i-r)\sin(i+r)} \quad \& \ a\sin\omega t\ \frac{2\cos i\sin r}{\sin(i+r)}$$

If these be denoted by $A\cos\omega t$, $B\sin\omega t$,

$C \cos \omega t$, $D \sin \omega t$ resply,

Reflected & refracted beams are the ellipses $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ & $\frac{x^2}{C^2} + \frac{y^2}{D^2} =$

When $i = 0$

$$A = B = a \frac{\mu - 1}{\mu + 1}$$

$$C = D = a \frac{2}{\mu + 1}$$

Both reflected & refracted beams are circularly polarized.

When $i + r = 90°$

$$A = 0 \quad ; \quad C \neq D$$

Reflected ray is plane polarized, & refrac elliptically polarized.

When $i = 90°$

$$C = D = 0 \quad , \quad A = B = a.$$

The reflected is elliptically polarized, & refracted intensity is zero.

Unpolarized light

If common light falls upo a doubly refracting medium, it is split into two components one ∥ to the principal plane & the other ⊥r to it.

~~B~~ Since the in unpolarized light the ibrations change many times p.s., e probability is that both components are equal intensity. If a is the amplitude of the incident beam, the intensity the reflected & refracted beams are iven by

$$I_1 = \frac{1}{2}a^2 \left\{ \frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)} \right\}$$

& $$I_2 = \frac{1}{2}a^2 \left\{ \frac{\sin 2i \sin 2r}{\sin^2(i+r)} + \frac{\sin 2i \sin 2r}{\sin^2(i+r)\cos^2(i-r)} \right\}$$

The 2nd term in I_1 is always less than e first term & is zero when $i+r = 90°$ ~~So this~~ When $i+r = 90°$, ∴, the reflected ght is polarized in the plane of incidence.

In the expn for I_2 the 1st term is lways ~~greater~~ less than the second.

But when $i = 0$, both the terms in I_1 are equal, & the terms in I_2 are also qual. ie the reflected & refracted beams emain unpolarized. When $i = 90°$, $I_2 = 0$;

& both the terms in I_1 are equal. ∴ The reflected beam is unpolarized. For all other values of i, both reflected & refracted beams are ~~plain~~ partially polarized

Total internal reflection

Component of light reflected & polarized ∥l to the incident plane

$$b = -\frac{a \sin(i-r)}{\sin(i+r)} = a \cdot \frac{\cos i \sin r - \sin i \cos r}{\cos i \sin r + \sin i \cos r}$$

& since $\sin r = \mu \sin i$ for reflection at the surface of a rarer medium,

$$b = a\,\frac{\mu \sin i \cos i - \sin i \sqrt{1-\mu^2 \sin^2 i}}{\mu \sin i \cos i + \sin i \sqrt{1-\mu^2 \sin^2 i}}$$

If the quantity within the radical is −ve ~~the~~ partial reflection is imaginary, i.e. total reflection takes place.

The condition for total reflection is $\mu \sin i = 1$ i.e. $\sin r = 1$

i.e. $r = 90°$.

Aliter, the refracted intensity,

$$I_2 = a^2 \frac{\sin 2i \sin 2r}{\sin^2(i+r)}$$

If $r = 90°$, I_2 becomes zero. Hence there is no refracted beam at the critical angle.

The same term $\sqrt{1-\mu^2\sin^2 i}$ is contained in the expression for light polarized ⊥ to the plane of incidence, & hence the same reasoning can be applied.

A totally reflected ray is elliptically polarized.

For component polarized ∥l to the incident plane

$$b = a \frac{\mu \cos i - \sqrt{1-\mu^2\sin^2 i}}{\mu \cos i + \sqrt{1-\mu^2\sin^2 i}}$$

$$= a \frac{(\mu c - \sqrt{1-\mu^2 s^2})^2}{\mu^2 c^2 - (1-\mu^2 s^2)}$$

$$= a . \frac{\mu^2 c^2 + 1 - \mu^2 s^2 - 2\mu c\sqrt{1-\mu^2 s^2}}{\mu^2 - 1}$$

$$= a\ (p - iq)$$

$$\text{where } p = \frac{\mu^2 + 1 - 2\mu^2 s^2}{\mu^2 - 1}$$

$$\therefore q = \frac{2\mu c\sqrt{\mu^2 s^2 - 1}}{\mu^2 - 1}.$$

$$\text{Now } p^2 + q^2 = \frac{1}{(\mu^2-1)^2}\left\{(\mu^2+1)^2 + 4\mu^4 s^4 - 4\mu^2 s^2(\mu \right.$$
$$\left. + 4\mu^2 c^2(\mu^2 s^2 - 1)\right.$$

$$= \frac{1}{(\mu^2-1)^2}\left\{(\mu^2+1)^2 + 4\mu^4 s^4 - 4\mu^2 - 4\mu^4 s^2(1-s\right.$$

$$= \frac{(\mu^2-1)^2}{(\mu^2-1)^2} = 1$$

Hence if $p = \cos\delta$, $q = \sin\delta$

$$b = a(\cos\delta - i\sin\delta)$$

$\therefore$ the vibration of the reflected beam ~~polo~~ is given by the eqn

$$y_1 = a(\cos\delta - i\sin\delta)\sin\omega t.$$

An eqn to a st. line multiplied by i is equivalent to rotating it through $\frac{\pi}{2}$

$$\therefore y_1 = a\cos\delta\sin\omega t - a\sin\delta\cos\omega t$$
$$= A\sin(\omega t - \delta)$$

In a similar manner the component polarized $\perp$r to the plane of incidence can be shown to be $b_2 = (p_2' - q_2' i q_2')$

$y_2 = B\sin(\omega t - \delta')$

where $\tan\delta' = \dfrac{q'}{p'}$.

Hence common light is on total reflection split into two components

$y_1 = A\sin(\omega t - \delta)$

$y_2 = B\sin(\omega t - \delta')$

in planes $\perp^r$ to each other, & differing in phase by $\delta - \delta'$.

The two together form an elliptic vibration.

It can be shown that

$$\cos(\delta - \delta') = pp' + qq'$$
$$= \frac{1-(\mu^2+1)s^2+2\mu^2 s^4}{(\mu^2+1)s^2-1}$$

When $(\delta - \delta') = 0$ the ~~one~~ reflected light is plane polarized.

The condition is $\cos(\delta - \delta') = 1$.

$$1-(\mu^2+1)s^2+2\mu^2 s^4 = (\mu^2+1)s^2-1$$

ie $\mu^2 s^4 - (\mu^2+1)s^2 + 1 = 0$

ie $s^2 = \dfrac{(\mu^2+1) \pm \sqrt{(\mu^2+1)^2 - 4\mu^2}}{2\mu^2} = \dfrac{(\mu^2+1) \pm (\mu^2-1)}{2\mu^2}$

ie $s^2 = 1$ or $\frac{1}{\mu^2}$; $s = 1$ or $\frac{1}{\mu}$

ie $i = 90°$ or C C being the critical angle.

For all other positions the reflected light is elliptically polarized.

For the ellipse to have one of its axes in the incident plane, $\delta - \delta'$ sho[uld] be $\frac{\pi}{2}$ ie $\cos(\delta - \delta') = 0$

$$2\mu^2 s^4 - (\mu^2+1) s^2 + 1 = 0$$

~~ie~~ s

$$s^2 = \frac{(\mu^2+1) \pm \sqrt{(\mu^2+1)^2 - 8\mu^2}}{4\mu^2}$$

For the value of s to be real,

$(\mu^2+1)^2 > 8\mu^2$

ie $\mu^4 - 6\mu^2 + 1 > 0$

ie $\mu^2 > \frac{6 \pm \sqrt{36-4}}{2}$

$> 3 \pm \sqrt{8}$ ie > 5.82

ie $\mu > 2.4$

Since no known material has ~~$\mu > 2$~~ μ so high except diamond, Fresnel attempted to give two total refle[ctions]

ions each time producing a phase
g of $\frac{\pi}{4}$

i.e. $\cos(\delta - \delta') = \frac{1}{\sqrt{2}}$

$\therefore \sqrt{2}\left(1 - \overline{\mu^2+1}\, s^2 + 2\mu^2 s^4\right) = \overline{\mu^2+1}\, s^2 - 1$

i.e. $2\cdot 8\mu^2 s^4 - 2\cdot 4(\mu^2+1)s^2 + 2\cdot 4 = 0$

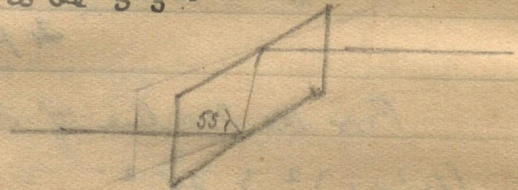
For the value of s to be real

$2\cdot 4^2(\mu^2+1)^2 > 4 \times 2\cdot 8 \times 2\cdot 4\,\mu^2$

i.e. $\mu > 1\cdot 45$

Substituting the value $1\cdot 55$ which is μ for the glass which Fresnel sed, he found i to be 55°

By using plane polarized light it s also possible to btain Olar polarization from Fresnel's Rhomb.



Theory of Neumann & Mac Cullah.

The principle of continuity of ether dopted by Fresnel requires that displacements ∥ to the surface of separation should be the same in both media. But

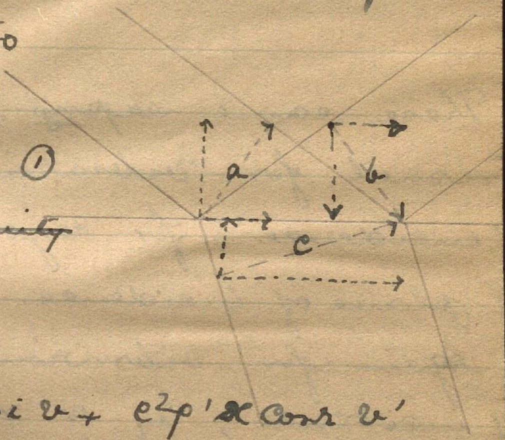
if it also requires that vibrations $\perp^r$ parall
to the plane of incidence are also the same
∴ If we have the eqn

$$(a+b)\cos i = c\cos r \quad \text{(no slipping)}$$

we should also have

$$(a-b)\sin i = c\sin r \quad \text{(no separation}$$

Combining the two



$$(a^2-b^2)\sin i\cos i = c^2\sin r\cos r. \quad (1)$$

Fresnel's ~~continuity~~ energy eqn is

$$a^2\rho\, x\cos i\, v = b^2\rho\, x\cos i\, v + c^2\rho' x\cos r\, v'$$

ie $(a^2-b^2)\rho\cos i\sin i = c^2\rho'\cos r\sin r$

(1) & (2) are the same if we assume ρ & ρ' equal.

Hence Fresnel's assumption that ether density is different is inconsistent with the continuity of ether $\perp^r$ to the surface.

Thus for vibrations in the plane of

incidence we have

$$\left.\begin{aligned}(a+b)\cos i &= c\cos r\\ (a-b)\sin i &= c\sin r.\end{aligned}\right\}$$

Whence $b = a.\dfrac{\sin(i-r)}{\sin(i+r)}$

$$c = a.\frac{2\sin i\cos i}{\sin(i+r)}$$

These same expns were derived by Fresnel for vibrations $\perp$r to the plane of incidence, for rays polarized ||l to the plane of incidence.

Again for vibration $\perp$r to the plane of incidence, by N.'s theory,

$$a+b=c$$

$$(a^2-b^2) = c^2\frac{\sin 2r}{\sin 2i}$$

$$\therefore a-b = c\times\frac{\sin 2r}{\sin 2i}$$

$$\therefore 2a = c.\left\{\frac{\sin 2i+\sin 2r}{\sin 2i}\right\} = c.\frac{2\sin(i+r)\cos(i-r)}{\sin 2i}$$

$$2b = c.\frac{2\sin(i-r)\cos(i+r)}{\sin 2i}$$

$$\therefore \; b = a\frac{\tan(i-r)}{\tan(i+r)} \; ; \quad c = a \cdot \frac{2\sin i \cos i}{\sin(i+r)\cos(i-r)}$$

These expns were derived by Fresnel for vibrations ∥l to plane of incidence.

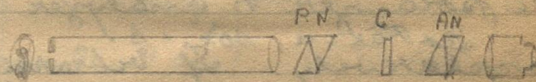
Hence according to N. & R. direction of vibration lies in the plane of polariza[tion] & not ⊥r to it as in Fresnel's theory.

Colours of thin crystalline films

Interference of polarized light.

Parallel light

If a parallel beam of light polarized by a nicol

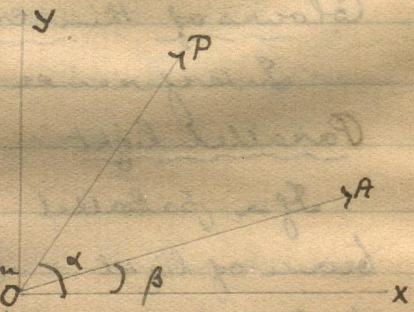


is passed through a thin crystalline plate & examined by another nicol, brilliant Colours are exhibited.

Malus ~~who~~ called the phenomen[on] 'depolarization'; but actually, the plate produces elliptic polarization, & when the two components which have a certain phase difference are resolved by the analy[ser]

-N along its principal plane, they are
in a position to interfere; & thus the
colours are produced. The plate should
be thin; otherwise phase difference
will be several times 2π, & hence
colours of different orders will be super-
posed, thus resulting in confusion.

Let Ox, Oy be
principal planes of
the crystal, OP, ON of
the polarizer & analyzer
Let $P\hat{O}x = \alpha$, $A\hat{O}x = \beta$
respectively. If vibration



$y = a \sin \omega t$
|| to OP is incident on the crystal, it
is split into $y_1 = a \cos\alpha \sin\omega t$ ||l to
Ox & $y_2 = a \sin\alpha \sin\omega t$ parallel to Oy.
On emergence a phase difference δ is
introduced, & the resolved components
of the two vibrations along the principal
plane of the analyzer are

$(a \cos\alpha \cos\beta) \sin\omega t + a \sin\alpha \sin\beta \sin(\omega t + \delta)$

$$= a\left[\sin\omega t\left\{\cos\alpha\cos\beta + \sin\alpha\sin\beta\cos\delta\right\}\right.$$

$$\left. + \cos\omega t\left\{\sin\alpha\sin\beta\sin\delta\right\}\right]$$

Hence intensity of the resultant vibrat

$$= a^2\{(\cos\alpha\cos\beta + \sin\alpha\sin\beta\cos\delta)^2 + (\sin\alpha\sin\beta\sin$$

$$= a^2\{(\cos\alpha\cos\beta + \sin\alpha\sin\beta)^2$$

$$- \cancel{4}\cos\alpha\cos\beta\sin\alpha\sin\beta\sin^2\frac{\delta}{2}\}$$

$$= a^2\cos^2(\alpha-\beta) - a^2\sin 2\alpha\sin 2\beta\sin^2\frac{\delta}{2}.$$

The first term is independent of δ & gives the intensity uniform ov ~~the~~ of white light in the field of view. It is called the intensity term.

The 2nd dependent on δ is called the Colour term.

1) Colour term is subtractive if α & β are both $> \frac{\pi}{2}$ or $< \frac{\pi}{2}$.

2) a) When Nicols are parallel, $\alpha = \beta$ Intensity term is a maximum.

Colour term $-a^2\sin^2 2\alpha\sin^2\frac{\delta}{2}$ is subtractive

b) $\alpha = \frac{\pi}{4}$

Colour term also is maximum.

(3) a — Nicols crossed $\alpha - \beta = \frac{\pi}{2}$.

Intensity term is zero.

Colour term $- a^2 \sin 2\alpha \sin(180 - 2\alpha) \sin^2 \frac{\delta}{2}$

$= + a^2 \sin^2 2\alpha \sin^2 \frac{\delta}{2}$

is additive.

b — $\alpha = 45^\circ$

Maximum colour on a dark background.

) Crystal rotated

$(\alpha - \beta)$ is constant, but α & β both vary. Hence the colour term varies.

When OX is ‖l to OB, $\alpha = 0$

$I = a^2 \cos^2 \beta$

When OX is ⊥r to OP, $2\alpha = 180^\circ$

$I = a^2 \sin^2 \beta$.

When OX is ‖l to OA $\beta = 0$

$I = a^2 \cos^2 \alpha$

When OX is ⊥r to OA $2\beta = \pi$

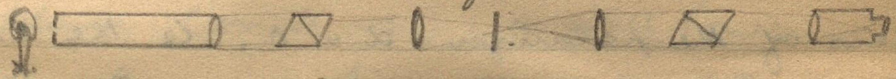
$I = a^2 \sin^2 \beta$.

If nicols are ‖l, there are only two positions of achromatism, & white light is seen on the screen

If nicols are crossed, again there are only two positions of achromatism where the screen is dark.

The effect of thin plates can be achieved by placing together two plates of nearly the same thickness with their principal planes at rt. $\angle$s to each other.

2 Rings & Brushes. A thin plate of uniaxial crystal cut $\perp$r to the optic axis & viewed under convergent or divergent light produces the phenomenon known as rings and brushes.



(Cf. Preston 421.)

The colour effect which depends on relative retardation varies according to the inclination at which the ray passes through the plate. Rays at equal distance from the centre of the plate have the sam inclination & ∴ a series of coloured rings are seen upon the screen. The rings are not equally coloured all round. The intensity at any pt is given by

$$I = a^2\left\{\cos^2(\alpha-\beta) - \sin 2\alpha \sin 2\beta \sin^2\frac{\delta}{2}\right\}$$

~~α and~~ α & β are the ∠s ~~of~~ which the principal planes of ~~analyzer~~ polarizer & analyzer make with the principal ~~axis~~ plane of the crystal. Now in the crystal cut with optic axis ⊥r to the refracting surface any plane ~~through~~ normal to the ~~centre~~ surface of the crystal can be the principal plane. Thus along a line ∥l to the principal axis of the polarizer $\alpha = 0$, ie the rings are of uniform colour. The β) same is true along a line ⊥r to the principal axis of the polarizer. Thus a cross is seen on the screen. Another cross is seen coinciding with the principal axis & ~~the~~ its normal in the analyzer, making ~~β~~ $\beta = 0$, $\beta = \frac{\pi}{2}$.

When $\alpha - \beta = 0$ or $\alpha - \beta = \frac{\pi}{2}$, the cross superpose being white in the former case & dark in the latter.

When the two crosses are not superposed but are separate, ~~a different~~ the ring in alternate sectors are seen to be displaced relative to those in the others. The intensity which is the sum of two terms one of uniform illumination & the other $(-a^2 \sin 2\alpha \sin 2\beta \sin^2 \frac{\delta}{2})$ the colour term varies from one sector to the other. On one side of a line showing the direction of ~~optic~~ principal plane (or its +r) α will have say the +ive sign & on the other side α will have a -ive sign. Hence on one side the colour term is additive & on the other it is subtractive. The colour of the ring on the two sides are complementary. The same happens for each of the four brushes

Hence the rings in alternate sectors seem to be displaced.

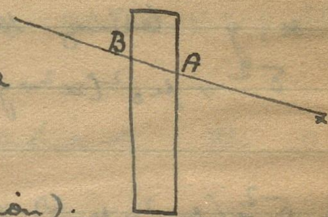
If nicols are ‖l or crossed one set of alternate sectors are suppressed. Hence rings are of same colour. This can also be seen from the term $a^2 \sin 2\alpha \sin 2\beta \sin^2\frac{\delta}{2}$ which when $\alpha = \beta$, becomes $a^2 \sin^2 2\alpha \sin^2\frac{\delta}{2}$ which is a perfect square & ~~has no~~ is +ive whatever be the value of α.

When monochromatic light is used only dark & bright bands are seen. The phenomena are the same.

Sign test :- This pattern can be used to find whether a given crystal is +ive or −ive. Form a pattern with a crystal whose sign is known (say calcite). Then ~~test~~ interpose a slice of the given crystal cut with optic axis ⊥r to surface; ~~&~~ If the rings dilate the given crystal is +ive; if they contract it is negative.

Isochromatic surfaces.

Let a ray pass in the direction AB through a ~~any~~ crystal (Cut with optic axis in any direction).



Let $AB = \rho$.

~~Path~~ Equivalent path of ordinary ray $= \mu_0 \times \rho$ & of the extraordinary ray $= \mu \times \rho$ ($\mu \neq \mu_e$ Since the crystal is not cut with optic axis normal to incident plane)

Relative path difference

$$\delta = \rho\,(\mu_0 - \mu).$$

$$\therefore \mu^2 = \left(\frac{\delta}{\rho} - \mu_0\right)^2.$$

Now $\mu_0^2 x^2 + \mu_e^2 y^2 = 1$ ~~be~~ is the section of the ~~ellipsoid of elasticity~~ wave surface

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

& $\frac{1}{r} = \mu$,

$$\mu_0^2 \cos^2\theta + \mu_e^2 \sin^2\theta = \frac{1}{r^2} = \mu^2.$$

$\therefore$ ~~$\left(\frac{\delta}{\rho} - \mu_0\right.$~~

$$\left(\frac{\delta}{\rho} - \mu_0\right)^2 = \mu_0^2 \cos^2\theta + \mu_e^2 \sin^2\theta$$

ie $(\delta - \rho\mu_0)^2 = \mu_0^2 x^2 + \mu_e^2 y^2$,

x, y being the coordinates of the pattern.

$$\delta^2 + \mu_0^2(x^2+y^2) - 2\mu_0\delta\sqrt{x^2+y^2} = \mu_0^2x^2 + \mu_e^2y^2.$$

ie $\delta^2 + (\mu_0^2 - \mu_e^2)y^2 = 2\mu_0\delta\sqrt{x^2+y^2}$.

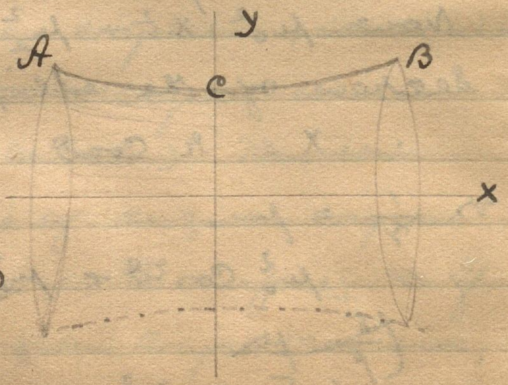
ie $\{\delta^2 + (\mu_0^2 - \mu_e^2)y^2\}^2 = 4\mu_0^2\delta^2(x^2+y^2)$. (1)

This gives the ~~isochromatic section~~ of x-y section of the isochromatic surface for which δ is a constant.

When this section is rotated about the x-axis, the isochromatic surface is obtained & its equation is

$$\{\delta^2 + (y^2+z^2)(\mu_0^2 - \mu_e^2)\}^2 = 4\mu_0^2\delta^2(x^2+y^2+z^2). \quad (2)$$

Equation (1) gives the line AB & (2) gives the anticlastic surface represented in the figure.



The section of the surface in the $(y\text{-}z)$

plane is a circle & hence the rings are formed by a plate having optic axis ||l to x-axis.

When x has a fixed value e, (1) becomes

$$A(y^2+z^2)^2 + B(y^2+z^2) + C = 0$$

The value of y^2+z^2 can be found, & the radius of a particular ring calculated for any given value of e.

When the crystal is cut with surface parallel to the optic axis, the section of the surface is to be taken in the x-z or x-y plane, & hence ~~two~~ a hyperbola is obtained. The pattern is a series of hyperbolae.

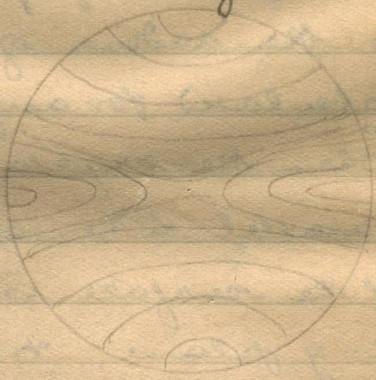
uniaxial, optic ax
||l to refracting surf

In biaxial crystals there are two optic axis, & hence the isochromatic surface is

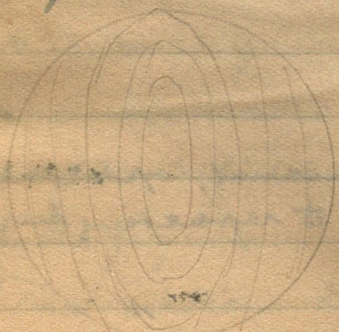
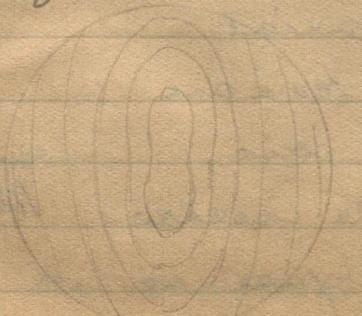
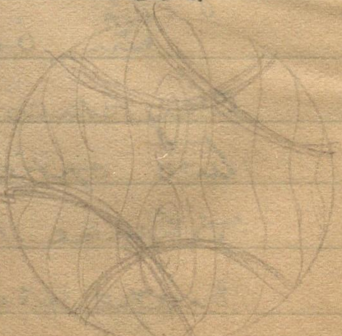
combination of two anticlastic drum
haped figures.

For a crystal
ut with optic axis
oth ||l to the refracting
urface,

the pattern ~~is a~~ consists of two
ets of hyperbolae.

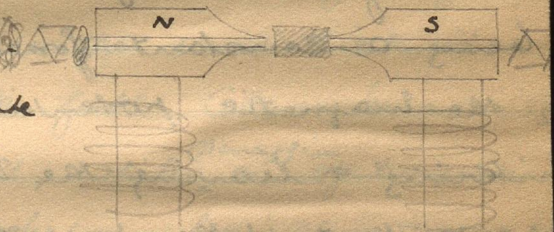


If the crystal is
ut with the refracting
urface normal to the
bisector of the ∠ between
he optic axis, the
attern is elliptic   biaxial, optic axes ~~not~~ along the refracting surface.
or small thicknesses, & consists of two
lliptic whirls for larger thicknesses.

The brushes ~~are~~ sections of achromatic surfaces are hyperbolic.

Faraday Effect. Preston 463 Faraday discovered in 1845 that ~~tr~~ substances not ordinarily optically active become so when under the influence of a strong magnetic field. The figure shows an exptal arrangement to demonstrate the effect

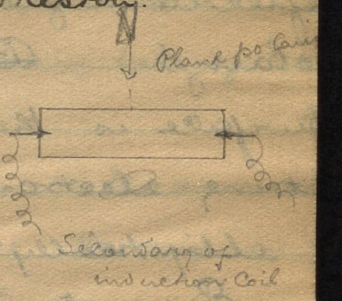


The direction of rotation in ~~ordinary~~ most substances is governed by cork screw rule. If a solenoid ~~Coil~~ is supposed to cause the magnetic field, the direction of rotation is the same as that of the current. Faraday gives the watch rule — the direction is that of the hands of a watch placed facing the N. pole. This rotation is called +ive & is caused by glass, gases like oxygen, thin plates of iron

But solutions of ferric salts produce —ve rotation. The stronger the field, the greater s the rotation; when the field is removed r reversed, the rotation ceases or is reversed, instantaneously if the ~~current~~ field is due to a solenoid, ~~too~~ not so very suddenly in the field of an electromagnet. erdet's formula gives $\theta = c(V_1 - V_2)$ being a constant for the medium, V_1, $_2$ the magnetic potentials ~~traversed~~ on ntering & leaving the medium. This hows that if the ray travels normal to e field there is no rotation. The power f rotation is possessed mostly by substances aving a high refr. index. One peculiar eature is that by retracing the path, e rotation is doubled, while for media which are ~~naturally~~ by nature optically ctive, retracing causes ~~a~~ neutralization. This enables the effect to be studied ~~even~~ in ubstances with a very weak susceptibility for

~~this~~ Faraday effect.

Kerr effects. Cf Page 428 Preston.

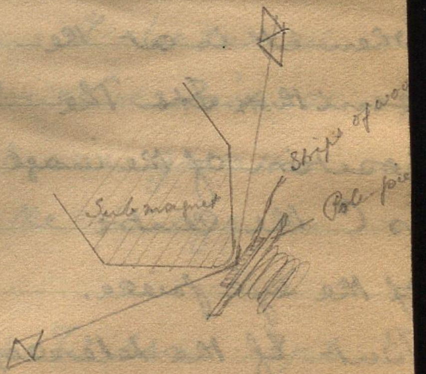


Electrostatic effect.

Under a strong electro-static field certain substances chiefly ~~quartz~~ glass become doubly refracting with optic axis in the direction of the field. The phase difference per unit thickness is ∝ to the sq. of the field. Glass behaves like a -ive crystal, while paraffin oil ~~of turpentine~~ & other substances behave like +ive crystals. The effect is well illustrated by connecting two terminals of an induction coil to the ends of the glass strip.

Magnetic effect.

Light polarized at any azimuth becomes elliptically polarized on reflection. But if the plane of polarization is



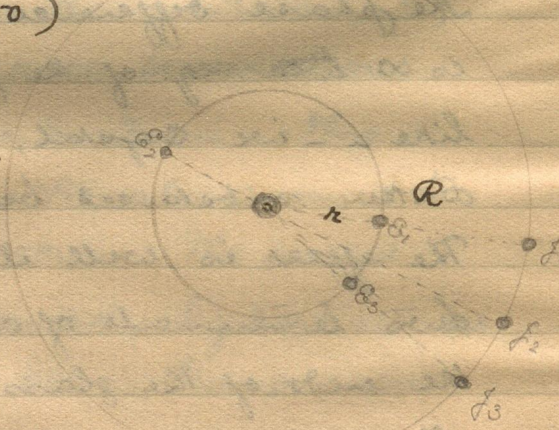
‖ or ⊥r to the plane of incidence the reflected light ~~become~~ remains plane polarized. ~~But~~ If however the reflecting surface is the polished polepiece of a strong electromagnet, the reflected light is elliptically polarized.

Expts for Velocity of Light.

Romer's method 1627

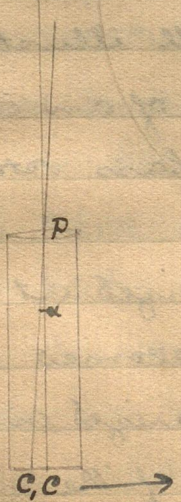
(Cf B.A. Notes Page 50)

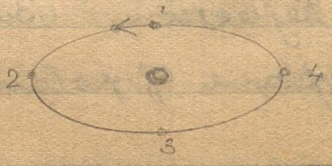
The result obtained was 187500 miles/sec.



Bradley's astronomical aberration method

A Star is viewed when it is at the zenith. ~~Its~~ The position of the image is C the centre of the eye piece. But If the telescope





telescope is moved in the direction of the arrow, the position is C′ such that

$CC'/PC = u/c$ u being vel. of the telescope.

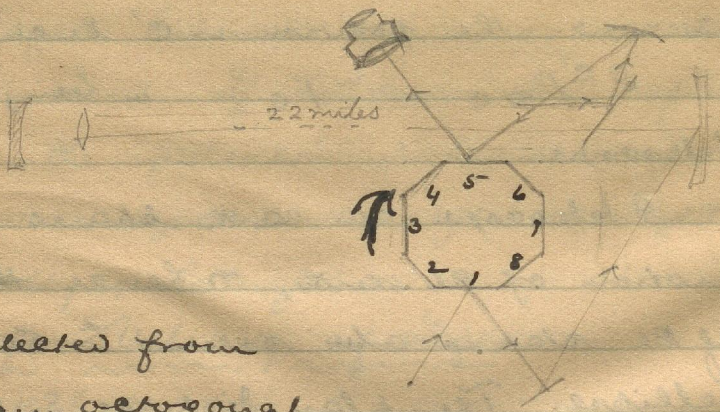
A telescope on earth shares the annual motion of the earth, & hence ~~the apparent~~ if the star ~~is ab~~ seems to describe an ellipse. The ∠lar distance S_2 to S_4 is $40{\cdot}89''$ ∴ $\theta = \frac{u}{c} = 20{\cdot}445°$, u being vel. of earth in 1st or 3rd positions. Knowing u, c can be calculated.

~~Fresnel's~~ Fizeau's Toothed wheel & ~~Fresnel's~~ Foucault's rotating mirror cf B.A. Notes Page 52.

Michaelson's ~~&~~ improved Foucault's expt by placing the lens L between the mirror M & the concave mirror S. Thus he obtained a much greater ~~reflection~~ deflection of the image in the eyepiece.

Michaelson's Mount Wilson expt

s the latest & most reliable.



Light reflected from
ace (1) of an octagonal
evolving mirror is sent by a series of
eflectors to a distant station 22 miles away & reflected
ack to the mirror ~~face~~ (5) & received in an eye piece.
f the mirror is stationary, the 2nd
eflection takes place at face (5). The
irror is rotated at such a rate that
e returning light meets face (4) at nearly
e same angle of incidence. The rate of
otation is adjusted till the image
n the eyepiece is perfectly stationary.
he no. of revol. p.s. is determined by
stroboscopic method and the velocity
of light computed. (Preston Page 551)

Group velocity or Ray velocity

If a disturbance be excited on a sheet of water, ~~the disturbance~~ eg. by throwing a stone into a still pond, the disturbance is propagated on the surface in various regions of waves, or groups of waves. These groups are themselves separated by comparatively motionless surfa[ces] of water. If any particular group be observed, it will be seen that individual waves in the group trave[l] faster than the group itself, that wave[s] die out as they reach the outer border of a group and new waves arise ~~be~~ on the inner border.

This difference between group velocity and wave velocity is due to the fact that waves of different wave lengths travel with different velocities. Consider the simple case of two waveengths superposed upon a medium. et the wavelengths be λ & λ'. If d be the east common multiple of λ & λ', at

oints in the medium separated by d, ~~are~~ two waves will be in the same phase. The disturbance is a max. at ese pts. At points midway between ese, the two waves are in opposite hase, & there is comparatively no isturbance. If ~~the~~ λ & λ' have the ame velocity, the ~~point~~ two waves greeing in phase will continue to be n phase, & the group will have the ame velocity as the individual wave rains. If the longer wavelength (say λ) as a greater velocity, the waves behind hose agreeing in phase will come to e more nearly in phase; the group will ravel slower than the waves.

Let A be the pt. where two waves are in phase, C & B the crests of λ & λ' just behind A. Let C overtake B in time t. If v & v' be velocities corresp to λ & λ', $t = \dfrac{\lambda - \lambda'}{v - v'} = \dfrac{d\lambda}{dv}$.

Distance travelled by the waves in time t = vt

Distance travelled by the group in the same interval = $vt - \lambda$.

$\therefore$ Velocity of group $u = \dfrac{vt - \lambda}{t}$

$$= v - \frac{\lambda}{t} = v - \lambda . \frac{dv}{d\lambda}.$$

1) Non dispersive media

$$\frac{dv}{d\lambda} = 0 \quad \text{since } \mu = \frac{v_0}{v} \;\&\; \frac{d\mu}{d\lambda} = 0$$

2) Ordinary media

$$\mu = A + \frac{B}{\lambda^2} \qquad \frac{d\mu}{d\lambda} =$$

$$\mu = \frac{v_0}{v} \quad \therefore v = \frac{v_0}{\mu} \therefore \frac{dv}{d\lambda} = -\frac{v_0}{\mu^2}\frac{d\mu}{d\lambda} = \frac{-v}{\mu}\frac{d\mu}{d\lambda}$$

$$\therefore u = v - \lambda\frac{dv}{d\lambda} = v + \frac{\lambda v}{\mu}\frac{d\mu}{d\lambda}$$

$$= v\left(1 + \frac{\lambda}{\mu}\frac{d\mu}{d\lambda}\right)$$

3) In strongly dispersive media
$d\mu/d\lambda$ has diff. values & hence the above formula does not hold good.
When refr. index is calculated from the formula $\mu = \frac{v_0}{v}$ the value obtained should be greater that that found in spectroscopy since v is the group velocity is less than the ray velocity. If the value of ray vel. is calculated from $v = u - \frac{\lambda}{\mu}\frac{d\mu}{d\lambda}$ substituted, the value will agree with the exptal value of μ.

In air the % error in the value of μ due to not substituting the ray vel. is only ·0015%; in water it is 1·5% & in carbon disulphide 7·5%.

This explains a discrepancy discovered by Michaelson which was at first inexplicable. He found μ for CS_2 1·64, while the value $\frac{v_0}{v}$ was 1·75. The difference is $\frac{\cdot 11}{1\cdot 75} \times 100\% = \frac{11}{1\cdot 75}\% = 6\cdot 3\%$
The error is very near that predicted by theory.

A controversey raged for long as to whe ~~the~~ the two velocities are different in inter stellar media. All expts on velocity of light give the group velocity; but Raleigh argued on certain theoretical grounds tha Bradley's aberration method should give the wave velocity. ~~But~~ Accurate expts showed no difference in the value obtain by Bradley's method & those from other methods. Hence it was concluded that in inter stellar space no dispersion takes place.

But Ehrenhaft & others showed later that Raleigh's argument was unsound. Bradley's method also gives ~~the~~ group velo Hence the question ~~is~~ was again unsolved.

Another discovery soon gave a defin answer. Algol is a double star in the constellation of ~~Perseus~~ Perseus. ~~To~~ Periodi cally one of its components is hidden ~~fro~~ behind the other, & the star changes from second to 4th magnitude & vice versa. If ~~there~~ the red wave travels even ·001 % faster than the violet, the red light should reach the earth atleast an hour before the violet when the star changes from ~~one~~ 4th to 2nd magnitude; for the star is thousands of light years away. But no such procession of colours is observed. Hence we conclude that ~~no disp~~ interstellar space is non-dispersive.

(Preston 513)

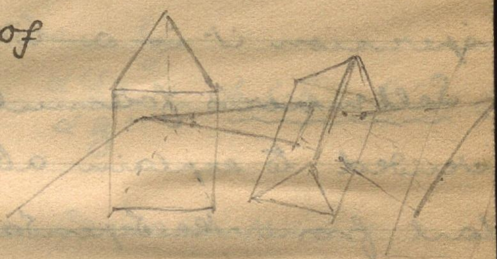
Anomalous Dispersion is a phenomenon exhibited by certain substances which produce a selective absorption, or as Kundt found out, by all substances which exhibit surface colour i.e. whose colour by reflection is different from the colour by transmission.

Le Roux observed in 1860 that when light is passed through a prism of iodine vapour

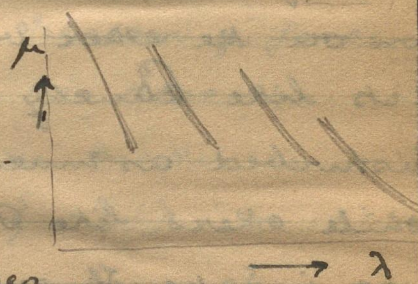
~~red~~ Violet & red alone are transmitted, & the red is refracted ~~m~~ more than the violet contrary to ordinary dispersion law

Christianson in 1870 continued the investigation for several substances.

Kundt employed the method of crossed prisms & found that the image instead of being continuous was made up of several patches.



Kundt formulated the law that starting from the longer wave-length side ~~the~~ ~~re~~ μ abnormally increases just ~~abo~~ before an absorption band & abnormally decreases just after it.



If there are several absorption bands then there will be several wavelengths having the same refractive index.

Theories

Cauchy gave the first satisfactory formula $\mu = A + B/\lambda^2 + C/\lambda^4 + \dots$ which is based for the most part on empirical results. It is ~~not~~ found not to hold good in the infra red & ultra violet & in the case of abnormalous dispersion it is a total failure.

Sellmeier's formula. Any formula intended to explain absorption should depart from the dependance of matter vibration on the ether vibration. Some ~~Sc~~ scientists like Cauchy assumed matter to be disturbed as much as the entangled ether while others like Briot postulated a perfect independence between matter and ether. Boussinesq argued that matter is disturbed but only to a very small extent. Sellmeier assumed this, & further stated that the displacement of matter is so small to call into play any elastic restoring force; & that the force of reaction between matter & ether is directly ∝ to the relative displacement between them.

Let ξ, ξ_1 be displacements of ether & matter, taking place in the z-direction. Let ρ, ρ_1 be their densities.

The force of restitution on unit mass of ether is given by.

$$\rho \frac{d^2\xi}{dt^2} = \alpha \frac{d^2\xi}{dz^2} - k(\xi - \xi') \quad 1)$$

Where α & k are constants.

$\alpha \dfrac{d^2\xi}{dz^2}$ is the elastic restoring force & $k(\xi - \xi_1)$ the reaction.

Force of restitution on matter is

$$\rho_1 \frac{d^2\xi_1}{dt^2} = k(\xi - \xi'). \quad 2)$$

From 2) we have $\dfrac{d^2\xi_1}{dt^2} = \dfrac{k}{\rho_1}(\xi - \xi_1)$

Which shows that the motion of matter is S. H of period

$$\tau_1 = 2\pi\sqrt{\frac{\rho_1}{k}} \quad \therefore k = \frac{4\pi^2\rho_1}{\tau_1^2}$$

($1/\tau_1$ is the natural frequency of matter)

The Solution of (1) is that of a progressive wave $\xi = b \cos \omega \left(t - \frac{x}{v}\right)$

But $\mu = \frac{n\lambda}{v}$ $\therefore$ $\frac{x}{v} = \frac{\mu x}{n\lambda}$ & $\omega = 2\pi n$

[v = vel. in med.; λ wavelength in air of frq. n]

$$\therefore \xi = b \cos 2\pi \left(tn - \frac{n\mu z}{n\lambda}\right)$$

$$= b \cos 2\pi \left(\frac{t}{\tau} - \frac{\mu z}{\lambda}\right)$$

Sly $\xi_1 = b_1 \cos 2\pi \left(\frac{t}{\tau} - \frac{\mu z}{\lambda}\right)$

$$\rho_1 \frac{d^2\xi_1}{dt^2} = k\left(\xi - \xi_1\right)$$

ie $-\rho_1 b_1 \cdot \left(\frac{2\pi}{\tau}\right)^2 \cos\theta = k\left(b - b_1\right)\cos\theta$

where $\theta = 2\pi\left(t/\tau - \mu z/\lambda\right)$

ie $-\rho_1 b_1 \left(\frac{2\pi}{\tau}\right)^2 = \left(b - b_1\right)\frac{4\pi^2\rho_1}{\tau_1{}^2}$

$$\therefore \frac{b}{b_1} - 1 = -\left(\frac{\tau_1}{\tau}\right)^2 = -\left(\frac{\lambda_1}{\lambda}\right)^2$$

$$\therefore \frac{b_1}{b} = 1\Big/1 - \left(\frac{\lambda_1}{\lambda}\right)^2 = \frac{\lambda^2}{\lambda^2 - \lambda_1{}^2}.$$

λ_1 is the wavelength in ether for a frequency equal to the natural frequency $\frac{1}{\tau_1}$ of the atom.

$$\rho \cdot \frac{d^2\xi}{dt^2} = \alpha \frac{d^2\xi}{dz^2} - k(\xi - \xi_1)$$

$$\rho \cdot b \cdot \left(\frac{2\pi}{\tau}\right)^2 \cos\theta = \alpha \cdot b \cdot \left(\frac{2\pi\mu}{\lambda}\right)^2 \cos\theta + \left(\frac{2\pi}{\tau_1}\right)^2 \rho_1 (b - b_1) \cos\theta$$

$$\frac{\rho b}{\tau^2} = \frac{\alpha b \mu^2}{\lambda^2} + \frac{\rho_1 (b - b_1)}{\tau_1^2}$$

$$\frac{\rho}{\tau^2} = \frac{\alpha \mu^2}{\lambda^2} + \frac{\rho_1}{\tau_1^2} \cdot \frac{-\lambda_1^2}{\lambda^2 - \lambda_1^2}$$

$$\text{ie } \mu^2 = \frac{\lambda^2}{\alpha} \left\{ \frac{\rho}{\tau^2} + \frac{\lambda_1^2}{\lambda^2 - \lambda_1^2} \cdot \frac{\rho_1}{\tau_1^2} \right\}$$

$$= \frac{\lambda^2}{\tau^2} \frac{\rho}{\alpha} - \frac{\lambda_1^2}{\tau_1^2} \cdot \frac{\rho_1 \lambda^2}{\alpha(\lambda^2 - \lambda_1^2)}$$

$$= \cancel{c^2} \; c^2 \frac{\rho}{\alpha} - c^2 \frac{\rho_1}{\alpha} \frac{\lambda^2}{\lambda^2 - \lambda_1^2}$$

$$= 1 + A_1 \cdot \frac{\lambda^2}{\lambda^2 - \lambda_1^2}.$$

If there are several natural frequen

$$\mu^2 = 1 + A_1 \frac{\lambda^2}{\lambda^2 - \lambda_1^2} + A_2 \frac{\lambda^2}{\lambda^2 - \lambda_2^2} + \dots$$

or in general $\mu^2 = 1 + \sum A_m \dfrac{\lambda^2}{\lambda^2 - \lambda_m^2}$.

If λ is small compared to λ_m,

$$\frac{\lambda^2}{\lambda^2 - \lambda_m^2} = 0$$

$$\therefore \mu = 1$$

This explains how X-rays are not refracted.

If λ is very great compared to λ_m,

$$\mu^2 = 1 + \sum A_m \left\{ \frac{\lambda_m^2}{\lambda^2 - \lambda_m^2} + 1 \right\}$$

$$= 1 + \sum A_m \qquad \text{since } \frac{\lambda_m^2}{\lambda^2 - \lambda_m^2} = 0$$

ie μ is a constant.

Hertzian waves are refracted but not dispersed.

The eqn explains the behaviour observed by Kundt

Suppose a substance has absorption bands corresponding to $\lambda = 4$ & $\lambda = 10$.

Then $\mu^2 = 1 + A_1 + A_2 + A_1 \dfrac{\lambda_1^2}{\lambda^2 - \lambda_1^2} + A_2 \dfrac{\lambda_2^2}{\lambda^2 - \lambda_2^2}$

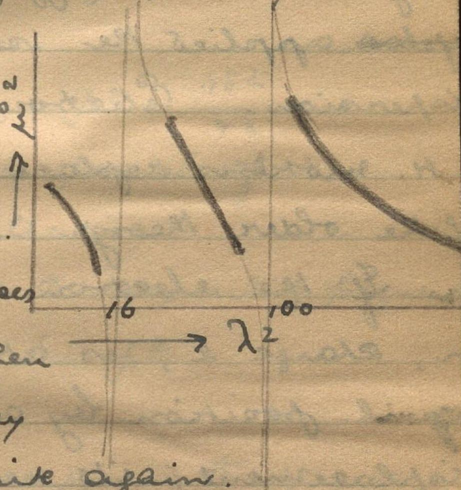
$$= 1 + A_1 + A_2 + A_1 \frac{16}{\lambda^2 - 16} + A_2 \frac{100}{\lambda^2 - 100}$$

$= (1 + A_1 + A_2)$ + a quantity k depending upon λ.

When $\lambda = \infty$ $k = 0$

As λ approaches 10, k becomes greater & greater, until at $\lambda^2 = 100$ $k = \infty$.

When λ^2 is slightly less than 100 k is $-$ive ~~&~~ of very high magnitude. ~~As~~ Thus for a short range after λ 10, ~~k~~ μ^2 is $-$ive ie μ is imaginary.



Kelvin explains this as showing the ~~ban~~ absorbed Colour.

As λ further decreases μ^2 becomes zero & then +ive & when λ^2 is very near 16 μ^2 is infinite again.

Just after 16 μ^2 is $-\infty$ & for a certain region is $-$ive. Again μ^2 increases. The full curve is shown in the above diagram. The ~~thick~~ lines corresponds to the results obtained by Kundt

A further modification was introduced by Helmholtz to explain the transformation of light energy which occurs in absorption. (cf Preston 526).

Helmholtz's application of electromagnetic theory.

After the successful application of e.m. theory to Zeeman effect by Lorentz, Helmholtz ~~also~~ applied the same to anomalous dispersion. Electrons executing their natural S.H. motions replace the atomic oscillators of the older theory.

If the electron in the atom of mass m, charge e, is attracted towards an equil. position by a force f per unit displacement, its natural period of vibration is $\tau_1 = 2\pi\sqrt{m/f} \therefore f/m = 4\pi^2/\tau_1^2$.

If at any instant displacement be y, electric field E, total force

$$m\frac{d^2y}{dt^2} = -fy + Ee.$$

$$\therefore \text{ie} \quad m\frac{d^2y}{dt^2} + fy = Ee = eE_0 \sin pt$$

E_0 being the max. value of E & $\frac{p}{2\pi}$ the frequency of the electric oscillation.

$$\therefore \frac{d^2y}{dt^2} + \frac{f}{m} y = \frac{e}{m} E_0 \sin pt.$$

which is the eqn to an undamped forced oscillation &

$$\therefore y = \frac{\frac{e}{m} E_0}{\left(\frac{f}{m}\right) - p^2} \sin pt.$$

Now $\frac{f}{m} = \frac{4\pi^2}{T_1^2}$; $p^2 = \frac{4\pi^2}{T^2}$

T being the ~~free~~ period of the impressed oscillation.

If ν_1 & ν be corresponding frequenc

$$y = \frac{e}{m} \frac{E_0}{4\pi^2(\nu_1^2 - \nu^2)} \sin pt$$

If ν is great compared to ν_1, as as happens in X-rays or $\frac{\nu}{r}$ is very small as happens in Hertzian waves, in both cases y is small.

Resonance takes place when $\nu = \nu_1$.

If there are N electrons per c.c.

~~total~~ electric displacement due to
ll the electrons = Ney.
~~If~~ Displacement ~~X~~ due to
the ether (unentangled with matter)

$$\frac{\mathcal{E}}{4\pi} = \frac{\mathcal{E}_0 \sin pt}{4\pi}$$

$\therefore$ Total electric displacement

$$D_0 \sin pt = \left\{ Ney + \frac{\mathcal{E}}{4\pi} \right\}$$

$$D_0 = \left\{ \frac{Ne^2 \mathcal{E}_0}{4\pi^2 m(v_1^2 - v^2)} + \frac{\mathcal{E}_0}{4\pi} \right\}$$

But dielectric constant k (of entangled
ether) $= \frac{4\pi D_0}{\mathcal{E}_0}$

$$\therefore k = 1 + \frac{Ne^2}{\pi m (v_1^2 - v^2)}$$

Refractive index $n = \frac{v_0}{v} = \sqrt{\frac{\mu k}{\mu_0 k_0}}$

$k_0 = 1$; $\mu_0 = \mu$ for all transparent
substances $\therefore n = \sqrt{k}$ ie $n^2 = k$
If further there are diff. natural frequencies

$$\therefore n^2 = 1 + \sum \frac{Ne^2}{\pi m (v_1^2 - v^2)}.$$

Since $\nu = c/\lambda$,

$$n^2 = 1 + \Sigma \frac{Ne^2}{\pi m c^2 \left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda^2}\right)}$$

$$= 1 + \Sigma \frac{Ne^2 \lambda^2 \lambda_1^2}{\pi m c^2 (\lambda^2 - \lambda_1^2)}$$

$$= 1 + \Sigma \left(\frac{Ne^2 \lambda_1^2}{\pi m c^2}\right) \frac{\lambda^2}{\lambda^2 - \lambda_1^2}$$

which is the same as Sellmeir's form

Restrahlen ~~Of Bst~~

The word means residual rays. Rubens and Nichols observed that polished crystalline surfaces like those of Rock salt, silvine, quartz exhibit selective reflection, ie certain rays are reflected 80 % or more while all others are not reflected more than 10 or 5 %. If a beam of composite light is caused to be reflected success at different quartz surfaces, the final light will consist almost entirely of ~~cont~~ a particular wavelength about 8μ. This ray after repeated reflections is called Restrahlen.

It can be shown that the reflectivity $R = \dfrac{(n^2-1)^2 + n^2k^2}{(n^2+1)^2 + n^2k^2}$ where n is the refr. index; k is defined as follows.

If a ray on falling on the material has intensity I_0 & ~~after~~ on passing through a plate of thickness dx, its intensity is ~~μdx, then let~~ diminished by a fraction μdx, of incident intensity, $-dI = \mu I dx$

$$\int_{I_0}^{I} \frac{dI}{I} = -\int_0^x \mu\, dx$$

$$\text{ie } \log(I_0/I) = \mu x$$

$$\text{ie } I = I_0 e^{-\mu x}$$

μ is called the absorption coefft.

The extinction coefft $k = \dfrac{\lambda}{4\pi}\mu$.

$$\therefore k = \frac{\lambda}{4\pi x} \log_e\left(\frac{I_0}{I}\right)$$

For ordinary thickness of transparent material, I_0 is nearly equal to I & $\dfrac{\lambda}{x}$ is very small $\therefore$ k is practically zero.

$\therefore R = \left(\dfrac{n^2-1}{n^2+1}\right)^2$ (very nearly) which is very small $\left(\dfrac{1.25}{3.25}\right)^2$

But if the substance has a high absorption coefft for a particular wavelength, I_0/I is very great for that wavelength, & ∴ k also becomes great. & In this case, $(n^2-1)^2$ & $(n^2+1)^2$ become negligib[le] compared n^2k^2. ∴ R is very nearly unity.

Thus selectively absorbing media ~~to~~ exhibit selective reflection.

Restrahlen have been obtained from various crystals by Rubens and his collaborators, & by this means the characteristic frequency of the particular substance has been found out. The method is ~~not~~ unhappily not applicable to most elemen[ts] since they do not form crystals.

Reflection and refraction on the electromagnetic theory. ~~Fresnel~~ (Preston 60

According to the electromagnetic theory a light wave consists of electric & magnetic vectors, oscillatin[g]

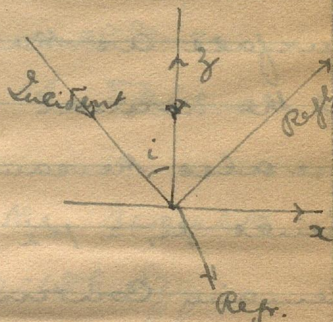
simple harmonically in transverse
to the direction of propagation, and
the magnetic & electric vectors are
mutually ⊥r to each other. They
are propagated in the medium with
a velocity $= \frac{1}{\sqrt{\mu k}}$ times the velocity
in free ether, μ being permeability,
& k susceptibility. The
differential eqns to the vectors are the
Maxwell eqns $\frac{d^2 \alpha}{dt^2} = \frac{1}{\mu k} \nabla^2 \alpha$ etc,
& the solution is of the type
of a progressive wave $\alpha = \alpha_0 \cos \frac{2\pi}{\lambda}(lx + my + nz - vt)$
l, m, n being dir. Cos. of direction
of propagation ... etc. It can be
shown that $\Sigma l \alpha = 0$; $\Sigma l X = 0$; $\Sigma X \alpha = 0$.
Hence the three directions are
mutually ⊥r.

~~One~~ Refractive Index $n = \frac{v_0}{v} = \sqrt{\mu k} = \sqrt{k}$
as has been shown on page 240.
This has been ~~accurat~~ verified to be
true accurately in the case of gases;

& approximately for solids (if k is taken in the region where k can be exactly found out). This gives one of the first confirmations of the theory.

The phenomenon of Hertzian waves is a most striking confirmation.

Knowing the general field eqns and their solutions we may proceed to find the change in amplitude on reflection & refraction.



Let a wave travel in the x-z plane inclined at i to the z-axis; & let z-axis be the normal to the surface of separation. If k_1, μ_1 be diel. & per. in the upper medium, k_2 μ_2 in the lower medium, by the boundary conditions, the tangential components (ie components ∥l to x & y axis) at the surface of separat

of the el. field is the same above
the surface as that below. The same holds
true ~~of~~ for the magnetic field.
The product of the normal component of el. field
(i.e. of the comp. ‖l to the z-axis) and
of the ~~sude~~. is the same above the
surface as that below ~~for the~~ ~~s~~ The ~~product~~ product
of the normal comp. and the permeability
are also the same above & below. But
since $\mu_1 = \mu_2$, the ~~p~~ normal components of mag. field
remain continuous. Further $\sqrt{\frac{k_2}{k_1}} = \frac{\mu_1}{\mu_2}$
From these data, ampl. of
refl. & refr. waves can be calculated.
~~If~~ Let a, a' be the amps. of the incident
electric wave ⊥r to & ‖l to the
plane of incidence, b b' corresponding
ampl. for refl. wave & c c' for refracted
wave.
It can be shown that

$$b = -a\frac{\sin(i-r)}{\sin(i+r)} \quad ; \quad c = \frac{2a\cos i \sin r}{\sin(i+r)} \qquad (1)$$

$$b' = -a\,\frac{\tan(i-r)}{\tan(i+r)}\;;\quad c = 2a\,\frac{\cos i \sin r}{\sin(i+r)\cos(}$$

The above eqns have been deduced by Fresnel on the elastic solid theory eqn ① for light polarized $\perp$r to plane of incidence i.e. for light whose vibrat are ~~at~~ $\perp$r to the plane of incidence & 2) for light whose vib. are ‖l to plane of incidence.

The discussion of the results is the same as on pages 186 sqq.

Since $(i+r) = 90$ for the $\angle$ of max polarization, & since the other conseq of the eqn have been verified, we ~~conc~~ conclude that the eqns are correct.

Accepting Fresnel's definition of the plane of polarization, the electric wave vibrates ~~in~~ $\perp$r the plane of polarization & the magnetic wave in ~~a~~ the plane ~~at rt $\angle$s to it~~ of incidence.

E. M. theory does not settle the

rival claims of the defns of Fresnel
& of N. & MacCullah.

Dichromatism

Certain transparent substances e.g. Cobalt glass
appear greenish yellow in when in
thin layers & red or some other
colour in thick layers. This is due to
the absorption coefft being diff. for
different colours $I_x = I_0 e^{-\mu x}$

$\therefore I_{xr} / I_{xg} = \frac{I_{0r}}{I_{0g}} \cdot \frac{e^{\mu_g x}}{e^{\mu_r x}}$.

In sunlight
$I_{0g} > I_{0r}$. Though $\mu_g > \mu_r$, when
x is small $e^{\mu_g x}$ is very nearly
equal to $e^{\mu_r x}$ & $\therefore I_{xr} < I_{xg}$
$\therefore$ the glass appears green.

But if x is large the exponential
term predominates, & $\therefore I_{xr} > I_{xg}$.

That thickness for which both I_{xg} & I_{xr}
are equal is called the critical thickness x_c

$e^{(\mu_g - \mu_r)x_c} = \frac{I_{0g}}{I_{0r}}$

$\therefore x_c = \frac{1}{\mu_g - \mu_r} \log_e\left(\frac{I_{0g}}{I_{0r}}\right)$

Spectrum & its teachings Cf B. A. Notes

{Transmission { Series solids
{Absorption { Fluted liquids
{ Line gases.

Method of excitation
Burning a volatile salt
Geisler tube
Mercury arc, sodium arc ~~lamps~~ &c
Weak sparks (say from pt knobs)

Doeppler effect Explanation & application to Saturn's rings, velocity of stars, broadening of spectral lines.

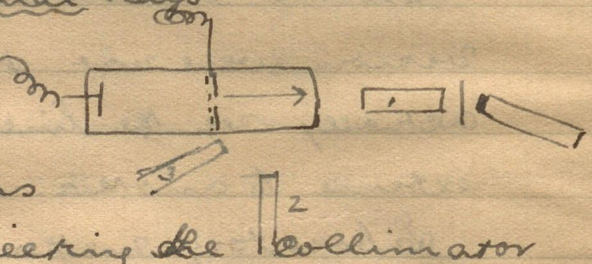
In the broadening & of lines, if our vision were not persistent we should actually see the line moving from one extreme to another.

Laboratory methods for illustrating Doppler effect. $\lambda_1 = \lambda(1 + \frac{u}{c})$.
Since c is very great, no appreciable shift is produced by merely moving a source. Belopotsky was the first

to overcome the difficulty by
a system of 6 mirrors which
were moved rapidly. The image moved
2^6 times as the vel. of any mirror
& thus an appreciable shift was
produced. Poincaré Galitz & later
Fabry & Buisson repeated the expts
with high resolving power instruments
like the echelon. In 1920 F. & B.
rotated rapidly a Olen disc with
polished edge. The disc appeared
elliptic.

Stark's work on canal rays

The strong F
line in a hydrogen
discharge tube was



examined by directing the collimator
against the canal rays produced by
a suitably perforated cathode.
Up to 1mm of pressure the
line is quite sharp at the edges; but

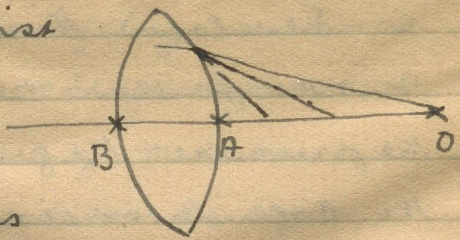
When pressure is still lowered, the edge on the shorter wavelength first becomes diffuse, then a wing is developed there, which gradually moves farther & farther. The lower the pressure the greater is the shift. If the Collimator is rotated to make an $\angle\theta$ with the direction of the rays, the shift becomes less, obeying the Cosine law. In position 2 there is no shift & in position 3 the shift is towa the longer wavelength side.

~~Series Relations in Line Spectra~~

Thick lenses.

If an object is situated at distance u from the pole A of the 1st surface of a thick lens, the image is formed at distance v' due to refraction at the first surface.

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{R_1} = \frac{1}{f_1} \quad (1)$$



where f_1 is the first focal length of the first surface & is defined as that distance at which an object has to be placed, in order that the image may be at infinity.

Due to refraction at the 2nd surface image is formed at distance v from the pole of the 2nd surface, so that

$$\frac{1/\mu}{v} - \frac{1}{v'+t} = \frac{1/\mu - 1}{R_2} = \cancel{\frac{1}{f_2}}$$

$$\text{ie} \quad \frac{1}{v} - \frac{\mu}{v'+t} = \frac{1-\mu}{R_2} = \frac{1}{f_2} \quad (2)$$

where f_2 is the ~~be~~ 2nd focal length of the 2nd surface, the distance of the point from the pole B to which all rays converge after refraction.

Eliminating v',

between ① & ②

$$uv + u\,\frac{\mu f_1 f_2 - f_2 t}{\mu(f_2 - f_1) + t} + v\,\frac{-\mu f_1 f_2 - f_1 t}{\mu(f_2 - f_1) + t}$$

$$+ \frac{t f_1 f_2}{\mu(f_2 - f_1) + t} = 0 \qquad 3)$$

Let us suppose the above eqn identical with

$$\frac{1}{v-\beta} - \frac{1}{u-\alpha} = \frac{1}{F}$$

ie $uv + u(-\beta - F) + v(-\alpha + F)$

$$+ \alpha\beta + (\alpha - \beta)F = 0 \qquad (4)$$

Equating like Coeff's in 3) & 4)

$$-\beta - F = \frac{\mu f_1 f_2 - f_2 t}{\mu(f_2 - f_1) + t} \qquad 5$$

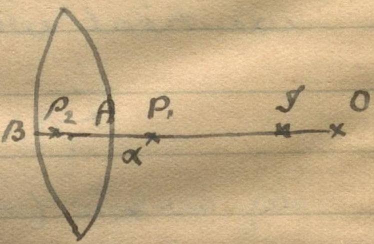
$$-\alpha + F = \frac{-\mu f_1 f_2 - f_1 t}{\mu(f_2 - f_1) + t} \qquad 6$$

$$\alpha\beta + (\alpha - \beta)F = \frac{t f_1 f_2}{\mu(f_2 - f_1) + t} \qquad 7$$

Multiply ⑤ & ⑥ & add ⑦ to ⑤ x ⑥

$$F = \pm \frac{\mu f_1 f_2}{\mu(f_2 - f_1) + t}. \quad (8)$$

Thus instead of measuring distances from the poles A & B, we measure the distance of the ~~p.~~ object from a point P_1 which is to the +ive side of A & call $P_1 O = u$ & measure the distance of the image from a point ~~β~~ P_2 to the left of ~~B~~ & call $P_2 I = v$, then the focal length of the thick lens can be expressed by an eqn similar to that of ~~the~~ a thin lens



viz $\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$.

Eqn (8) gives F in terms of f_1, f_2 & t

α & β are given by

$$\alpha = \frac{f_1 t}{\mu(f_2 - f_1)t} \qquad \beta = \frac{f_2 t}{\mu(f_2 - f_1)t} \quad (9)$$

All measurements of the object are taken from P_1 which is called the first principal point. All measurements of the image are taken from P_2 which is called the 2nd principal point

The first principal focus is ∴ at F from P_1 & the 2nd principal focus at distance F from P_2.

To know the sign of F in the two cases, we compare with a thin lens

The focal length which ⨯ to the second principal focal length is given by $\frac{1}{f_2} - \frac{1}{f_1} = \frac{1}{F}$ ie $\cancel{f =} \cancel{A}$ $F = -\frac{f_1 f_2}{f_2 - f_1}$ The above eqn is identical with 8) if $t = 0$ & sign of F is $-$ive.

∴ The sign of F the focal length is $-$ive.

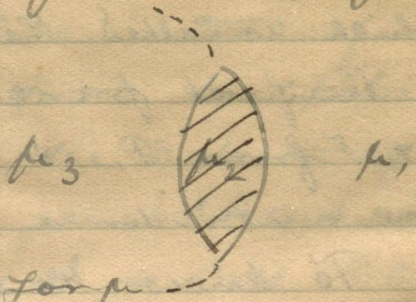
From 9 we see $f_1 \beta = f_2 \alpha$.

Thick lens with media on two sides other than air

The vitreous humour of the eye is such a lens (The lens is abnormally thick in short sighted ~~pe~~

persons, or abnormally thin in long-sighted persons)

The method of procedure is the same except that



in (1) we put $\frac{\mu_2}{\mu_1}$ for μ or in (2) $\frac{\mu_3}{\mu_2}$ for $1/\mu$.

Solving we get

$$\Delta F = -\frac{\mu_2 f_1 f_2}{\mu_2(\mu_1 f_2 - \mu_3 f_1) + \mu_1 \mu_3 t}$$

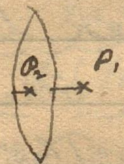
$$\alpha = \frac{\mu_1 \mu_3 f_1 t}{\mu_2 \text{ --- etc}}$$

$$\beta = \frac{\mu_1 \mu_3 f_2 t}{\mu_2 (\text{--- etc}} \text{ --}$$

To show that principal points (or principal planes) are conjugate.

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{F}$$

If measured from the poles



$$\frac{1}{v-\beta} - \frac{1}{u-\alpha} = \frac{1}{F}.$$

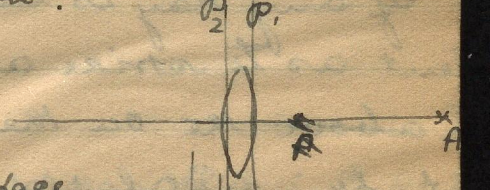
Let the object is at the 1st principal point $V = 0$; $u = \alpha$

$$\therefore \frac{1}{u-\beta} = \frac{1}{0} + \frac{1}{F}$$

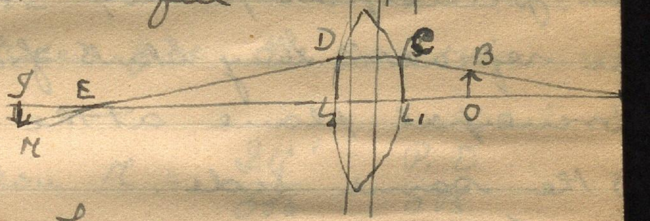
$$\therefore v-\beta = 0 \quad \text{i.e.} \quad v = \beta.$$

The ~~objects are~~ image is at P_2

The principal planes are planes of unit magnification.



If an object be at A the 1st focal point of the 1st surface



The image is formed at IM.

$$\frac{CL_1}{OB} = \frac{AL_1}{OA} = \frac{f_1}{f_1 - u}$$

$$\frac{DL_2}{IM} = \frac{EL_2}{IE} = \frac{f_2}{v - f_2}$$

$$\therefore \frac{IM}{OB} = \frac{f_1(v-f_2)}{f_2(f_1-u)} = -\frac{(f_2-u)f_1}{(f_1-u)f_2}$$

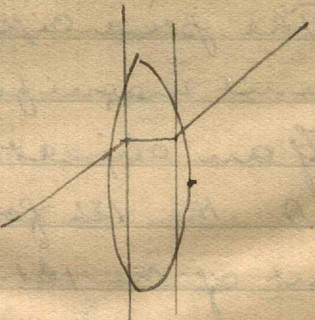
The -ive sign shows that the image is inverted.

$$m = \frac{f_1(f_2-v)}{f_2(f_1-u)}$$ gives the gen. eqn for magnification.

For the principal planes $u = \alpha$; $v = \beta$.

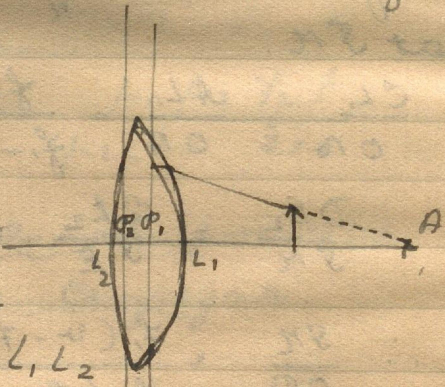
$$m = \frac{f_1 f_2 - f_1\beta}{f_1 f_2 - f_2\alpha} \quad \text{But } f_1\beta = f_2\alpha$$

$\therefore m = 1$



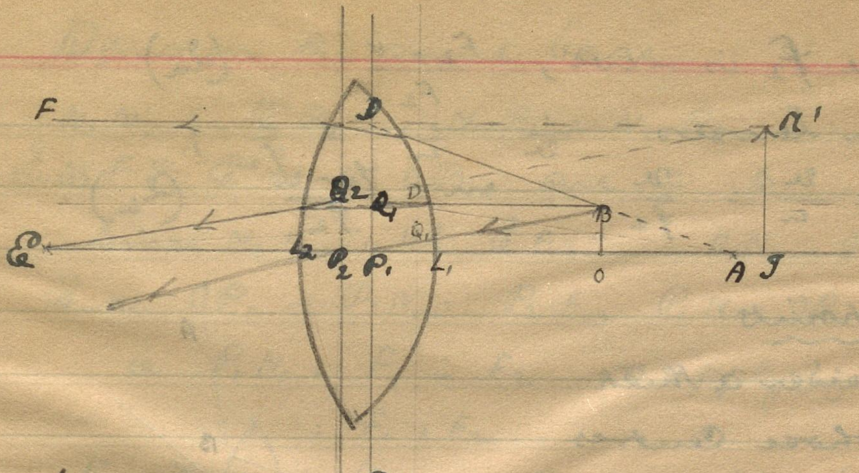
If any ray is incident as to strike at ~~above the~~ on the 1st principal plane, the refracted ray starts from the 2nd principal plane at the same height & the same side.

To ~~draw~~ show the image graphically



Let OB be an object. Ray AB should emerge from the lens ||l to ~~A~~ L_1L_2

~~Let~~ A be the 1st principal focus of the ~~1st surface~~ lens. ~~A~~ & E is the 2nd principal focus. A ray BQ_1 which is ||l to the principal axis & when produced meets P_1Q_1 at Q_1 should on refraction



travel along EQ_2 such that $P_1Q_1 = P_2Q_2$.
EQ_2 & FD produced give the position of the image.

Magnification $n = \frac{IN'}{OB} = \frac{P_1D}{OB}$

$= \frac{P_1A}{OA} = \frac{F_1}{F_1 - u_1}$ (1a) In thin lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Since f is f_2, & $f_1 = -f_2$, $\frac{1}{v} - \frac{1}{u} = -\frac{1}{f_1}$

i.e. $\frac{u}{v} - 1 = -\frac{u}{f_1}$ ∴ $\frac{1}{m} = 1 - \frac{1}{f_1} = \frac{f_1 -}{f_1}$

∴ $m = \frac{f_1}{f_1 - u}$. (1b)

$m = \frac{IN'}{OB} = \frac{IN'}{P_2Q_2} = \frac{IN}{P_2N} = \frac{V+F_2}{F_2}$.

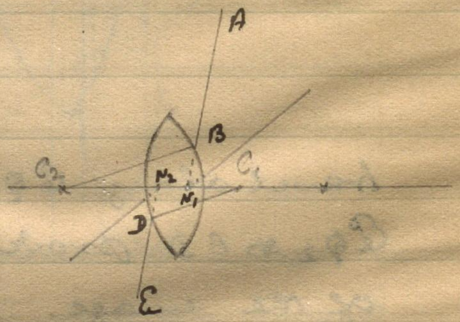
& Since f_2 is −ive, $\frac{F_2 - v}{F_2}$. (2a)

For thin lenses $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}$;

$\frac{v}{v}$ $1 - \frac{v}{u} = \frac{v}{f_2}$ $\therefore m = \frac{f_2 - v}{f_2}$ (2b).

Nodal points

Consider a thick lens whose centres of curvature be C_1, C_2. Let a ray be incident in direction AN, meeting 1st surface at B. Draw $C_1 D \parallel C_2 B$



Elements at B & D are ||l. Draw DE || to AB, to meet $C_1 C_2$ at N_2. The points such that a ray incident towards one of them, it emerges undeviated from the other, the two points are called nodal points. The two nodal points are conjugate since a beam converging to the 1st appears to diverge from the second.

Since $\angle K P_1 P_2$ & $\angle K N_2 N_1$ are both

~~||~~gms on the same base,

$N_1 N_2 = P_1 P_2$; $N_1 P_1 = N_2 P_2$

In Δs $F P_2 E$ & $N_1 O B$ are $\equiv$ Δs

Since $N_2 H$ is ||l EF, $\therefore ON_1 = P_2 F$

Since $OP_1 - P_1 N_1 = P_2 F$, $P_1 N_1 = OP_1 - P_2 F$.

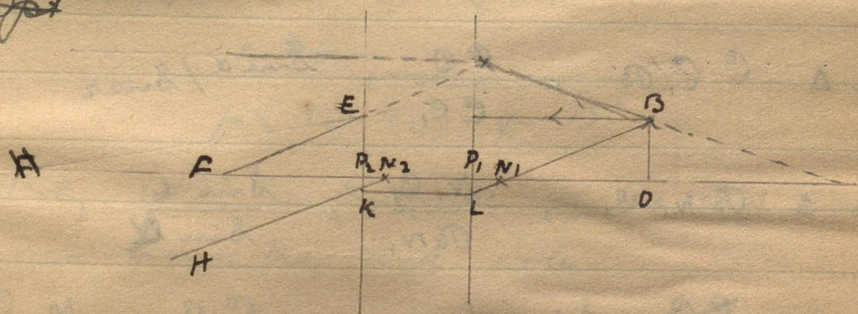
i.e. $P_1 N_1 = F_1 - F_2$ & applying signs

$= F_1 + F_2$.

For lenses bounded on ~~obo~~ both sides by air $F_1 = F_2$.

$\therefore$ the nodal points & principal points are coincident.

~~Opt~~

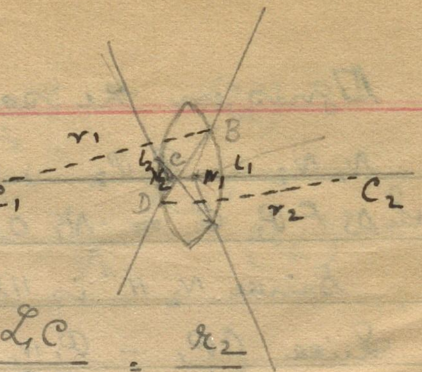


Optic Centre In the following diagram C is the optic Centre — the point through which the undeviated rays pass.

Δs $C C_1 B$ & $C C_2 B$ are |||r.

$\frac{CB}{CD} = \frac{r_1}{r_2}$

This relation holds good



for all undeviated rays.

Hence C is a universal point for all rays. $\therefore \frac{L_1 C}{C L_2} = \frac{r_2}{r_1}$

$\therefore \frac{L_1 C}{t} = \frac{r_1}{r_1 + r_2} \quad \therefore L_1 C = t \frac{r_1}{r_1 + r_2}$

Since r_1 is –ive,

$$L_1 C = t \frac{r_1}{r_1 - r_2}.$$

From the 2nd pole, distance $= t \cdot \frac{r_2}{r_2 - r_1}$

In Δ $C C_1 B$, $\frac{CB}{CC_1} = \sin \alpha / \sin r$

In Δ $B N_1 C_1$, $\frac{N_1 C_1}{B N_1} = \frac{\sin i}{\sin \cancel{\alpha}}$

$\therefore$ ~~CB~~ $\sin i / \sin r = \frac{CB}{CC_1} \times \frac{N_1 C_1}{B N_1} = \mu$

i.e. ~~t~~ t_1. In the limiting case of the ray coinciding with the principal axis,

$$\frac{a}{r_1 - a} \times \frac{r_1 - b}{b} = \mu \qquad a = \mathrm{C}L_1 ; \; b = $$

i.e. $b\mu(r_1 - a) = a(r_1 - b)$

$b(\mu r_1 - a\mu + a) = a r_1$

i.e. $$b = \frac{a r_1}{\mu r_1 - \overline{\mu - 1}\, a}$$

Since $a = \dfrac{r_1 t}{r_1 - r_2}$,

$$b = \frac{r_1^2 t / r_1 - r_2}{\mu r_1 - (\mu - 1) \cdot \dfrac{r_1 t}{r_1 - r_2}}$$

$$= \frac{r_1^2 t}{\mu r_1 (r_1 - r_2) - (\mu - 1) r_1 t}$$

$$= \frac{r_1 t}{\mu(r_1 - r_2) - (\mu - 1) t}$$

IIIrly b' the distance of the 2nd nodal point from the 2nd pole

$$b' = \frac{r_2 t}{\mu(r_2 - r_1) - (\mu - 1) t}.$$

When the lens is bounded on both sides by air, $b = \alpha$; $b' = \beta$.

$\alpha = \dfrac{f_1 t}{\mu(f_2 - f_1) + t}$; $\beta = \dfrac{f_2 t}{\mu(f_2 - f_1) + t}$.

Since $\dfrac{1-\mu}{r_2} = \dfrac{1}{f_2}$ $\therefore$ $r_2 = -(\mu - 1) f_2$

& $r_1 = -(\mu - 1) f_1$

Substituting, $\beta = b'_2 \dfrac{r_2 t}{\mu(r_2 - r_1) - (\mu - 1)t}$

$= \dfrac{-(\mu - 1) f_2 t}{\mu(\mu - 1)(f_1 - f_2) - (\mu - 1)t}$.

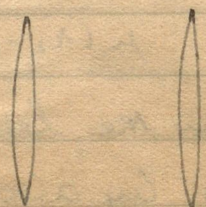
$= \dfrac{f_2 t}{\mu(f_2 - f_1) + t}$.

If $r_1 = r_2$, $a = \dfrac{r_1 t}{r_1 + r_2} = \dfrac{t}{2}$.

$b = \dfrac{rt}{2\mu r - (\mu - 1)t}$.

System of lenses.

For refr. at the first lens, if image is formed at distance v'_1



$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}$ (1) f_1 being the 2nd focal length of the 1st lens.

$\frac{1}{v} - \frac{1}{v'+d} = \frac{1}{f_2}$ (2) for refraction at the 2nd lens.

$v' = \frac{uf_1}{u+f_1}$ & from (2) $v' = \frac{vf_2}{(f_2-v)} - d$

Equating

$$\frac{uf_1}{u+f_1} = \frac{vf_2}{f_2-v} - d.$$

$$uf_1(f_2-v) - vf_2(u+f_1) + d(f_2-v)(u+f_1) = 0$$

$$\therefore uv(-f_1 - f_2 \mp d)$$
$$+ u(f_1f_2 + f_2 d) + v(-f_1f_2 \mp f_1 d)$$
$$+ f_1 f_2 d = 0.$$

This eqn can be expressed in terms of

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{F} \text{ ie } \frac{1}{v-\beta} - \frac{1}{u-\alpha} = \frac{1}{F}$$

$$\therefore F\{(v-\beta)(u-\alpha) - (u-\alpha) + (v-\beta)\} = 0$$
ie $uv(1/F) + u(-(1/F)\beta - 1)$
$$+ v(-(1/F)\alpha + 1) + \alpha\beta((1/F) - 1) = 0$$

$$\therefore \frac{\cancel{-F}}{\cancel{f_1+f_2+d}} = \frac{\cancel{\alpha\beta(F-1)}}{\cancel{f_1f_2d}}$$

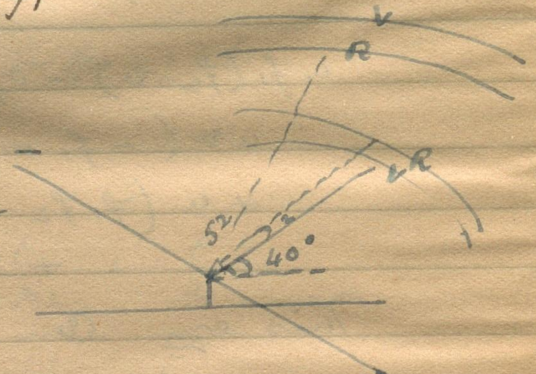
Simplifying

$$F = \frac{f_1 f_2}{f_1 + f_2 + \delta}.$$

The eq^n differs from that of a thickness in this that $\mu = 1$ & f_2 is –ive.

$$\alpha = -\frac{f_1 d}{f_1 + f_2 + d} \quad ; \quad \beta = \frac{f_2 d}{f_1 + f_2 + d}$$

$$\therefore \alpha = -\frac{F\delta}{f_2} \quad ; \quad \beta = \frac{Fd}{f_1}$$



Rainbows

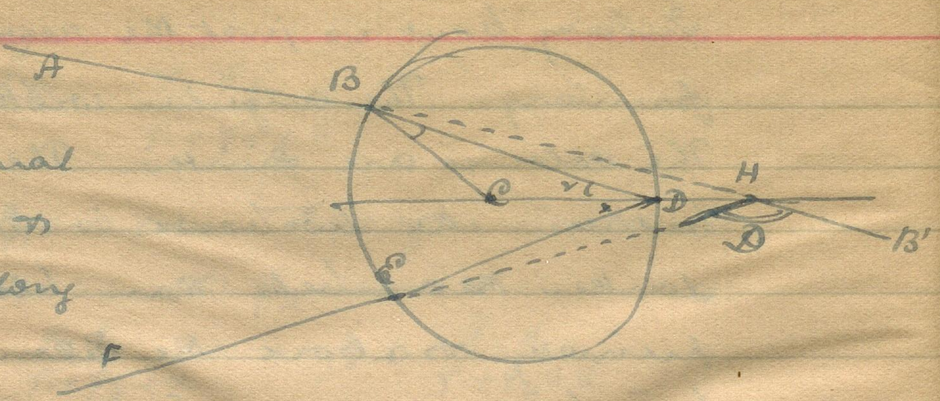
The axis of the bow – the centre of the concentric circles – is on a line joining the observer's eye with the sun. The point is as far below the horizon as the sun is above. The ∠lar radius is 40° & the ∠lar width is 2° for the primary ~~bow~~ bow. The secondary bow has a mean ∠lar radius of 52°. The order in the

-condary bow is just the reverse of the primary. The angular width is 4°. The colours are fainter. The space between the two bows is distinctly darker ~~the as bet~~ than below the primary & above the secondary. These two together are called the principal bows. In addition sometimes a few coloured ~~bands~~ bows are seen above the secondary & below the primary. These are called spurious or supernumerary bows.

From very early times the bows have been associated with the reflection & refraction in raindrops. It is necessary that the sun should be low in the sky, & rainclouds should be on the opposite horizon.

Let a ray AB be incident. On reaching B, it is deviated towards the radius. Let the ray be such as to

Strike at D
It suffers
total internal
reflection &
emerges along
F, as it
starting from



. The ∠ of deviation $E\hat{H}B'$.

Deviation at B $= (i - r)$.

at E $= (i - r)$

at D $= \pi - 2r$.

$\therefore$ Deviation due to n total reflections

$$D = 2(i - r) + n(\pi - 2r). \quad (1)$$

$$dD = 2di - 2dr - 2n\,dr = 0$$

$$di = (n+1)\,dr \quad (2)$$

For any angle of incidence

$$\sin i = \mu \sin r$$

$$\therefore \frac{di}{dr} = \mu \frac{\cos r}{\cos i}$$

$$\text{i.e. } (n+1) = \mu \frac{\cos r}{\cos i}$$

$$\text{i.e. } \cos i = \frac{\mu \cos r}{n+1}.$$

$$\text{i.e. } \cos^2 i = \frac{\mu^2 \cos^2 r}{(n+1)^2} = \frac{\mu^2}{(n+1)^2}(1 - \sin^2 r)$$

$(n+1)^2\cos^2 i = \mu^2 - (1-\cos^2 i)$

$= \mu^2 - 1 + \cos^2 i$

ie $(n^2+2n)\cos^2 i = \mu^2 - 1$

$\cos i = \sqrt{\dfrac{\mu^2-1}{n^2+2n}}$. (3)

$dD = 2\,di - 2\overline{n+1}\,dr$

$dD/di = 2 - 2\overline{n+1}\,dr/di$

$\dfrac{d^2D}{di^2} = -2\,\overline{n+1}\,\dfrac{d^2r}{di^2}$

But $dr/di = \dfrac{\cos i}{\mu\cos r}$

$\therefore \dfrac{d^2r}{di^2} = \dfrac{(\mu\cos r)\sin i - \cos i\,\mu\sin r}{\mu^2\cos^2 r}$

Cf Preston Page 574

$= \dfrac{\cancel{\mu}\cos r\sin i - \cos i\,\cancel{\mu}\sin r\,\frac{\cos i}{\mu\cos r}}{\mu^{\cancel{2}}\cos^2 r}$

$= \dfrac{\mu^2\cos^2 r\sin i - \cos^2 i\sin r/\mu}{\mu^3\cos^3 r}$

Since $\mu = \dfrac{\sin i}{\sin r}$, $\dfrac{d^2r}{di^2} = \dfrac{-(1-\mu^2)\sin i}{\mu^3\cos^3 r}$

Which is -ive ∴ $\dfrac{d^2D}{di^2}$ is ~~min~~ a min

~~Hence~~ The value of Dm can be calculated knowing i from eqn (3)

The value ~~of~~ of n is taken to be unity

It can be shown that min. deviation increases with refractive index

$\sin i = \mu \sin r.$

$dD = 2di - 2\overline{n+1}\, dr.$

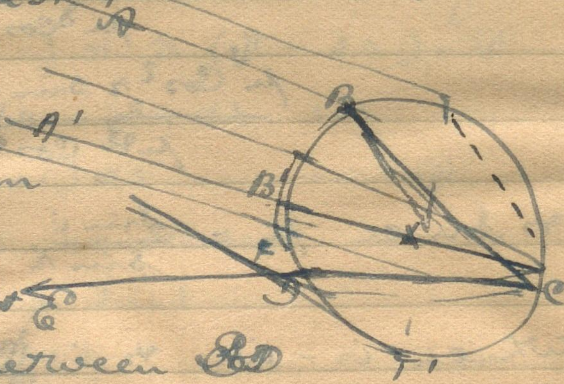
$\cos i\, di = \mu \cos r\, dr + \sin r\, d\mu.$

but $\mu \cos r = (n+1) \cos i$

$\therefore di = (n+1)\, dr$ where $dD = 0$.

Deviation for the violet & red

n	Red	Violet
1	$\pi - 42.1$	$\pi - 40.22$
2	$2\pi - 129.2$	$2\pi - 125.48$
3	$3\pi - 231.4$	$3\pi - 227.08.$



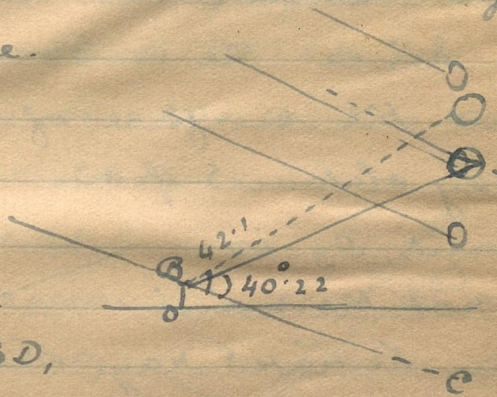
Intensity.

Let ABCDE be the minimum deviated ray.

All other emergent rays will lie between ED & the maximum deviated ray A'B' the deviation of which is $n\pi$. A ray emerging tangentially or from any point below D cuts the ray DE at some point. Thus all the rays which are reflected once lie within a cone whose vertical ∠ is 42°.

At the edges of the cone, there is copious crowding of emergent rays, ~~to~~ while within the cone the rays are more sparse.

Let OB be the observer, BC the direction of the sun's rays. Draw a line BD, inclined at 40·22

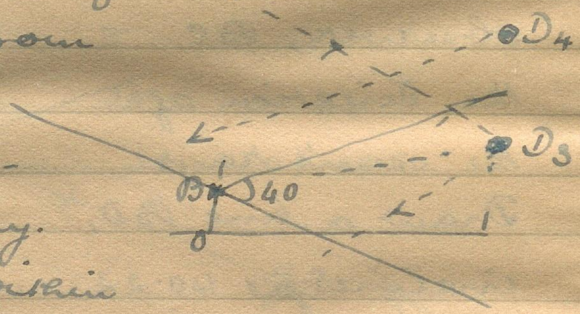


The drops on this line sent in the direction D,B their minimum deviated violet ray. Hence drops at this inclinat an intense violet colour is seen.

At another inclination 42·1 along BD the drops send their red minimum deviated ~~drop~~ ray. Since a cone of lines can be drawn at inclination

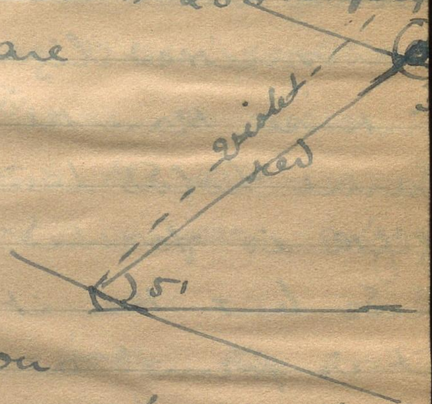
to CD, a bow is seen. Thus the theory explains the existance, the inclination & the angular width of the bow. It also shows why the primary bow is violet within. The whole semicircle can never be seen since the centre of the ☉ is below the surface of the earth.

A ray obtained from a point D_3 is not the minimum deviated ray. But since B is within the cone, drops like D_3 are also seen illumined, but the illumination is feeble. From a drop D_4 the min. dev. ray passes above B, & so D_4 appears to give no illumination water.

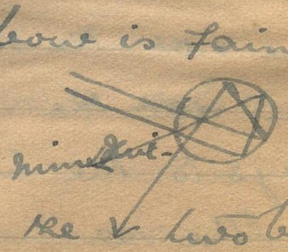


Since the sun is not a point source, the different colours ~~are~~ all form wide bands. Hence there is a certain overlapping of Colours. This effect is exaggerated when a thin cloud Covers the Sun. The source here is very large & hence a white rainbow is seen.

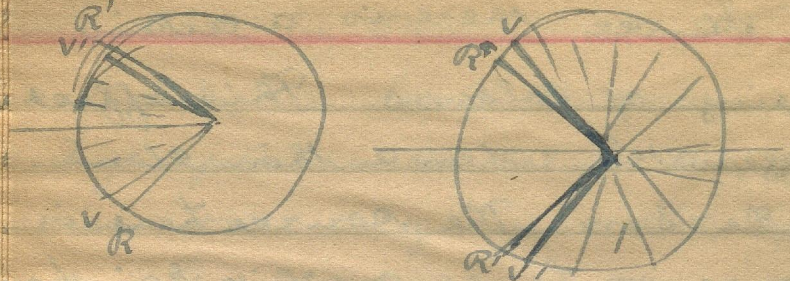
The deviation for a ray reflected twice is $2\pi - 129$ & $2\pi - 125.48$ for red & violet resply, ie 231 & 234 resply
∴ The supplements are 51 & 54. Hence at 51°, the drops appear red. The secondary bow with red on the inner edge & violet on the outer edge is formed by this double reflection. Due to longer path it has to travel in water, the bow is fainter.



No light is reflected by drops below D_5



Hence the darkness bet. the two bows



The third bow is not seen since the drops which have to give this lie behind the observer, on the same side as the Sun; they direct transmitted light is much more intense than the reflected light. The same holds true of the 4th. The fifth is formed on the opposite side but ~~the~~ it is very faint; further it is formed at the same ∠ as the secondary bow.

Bows seen by different people are not the same, since the drops are not the same which send the rays to the two different observers.

The same is true of the bow & its image in a tank.

Supernumerary bows. (Theory devel
by Young, improved by Potter & Airy).

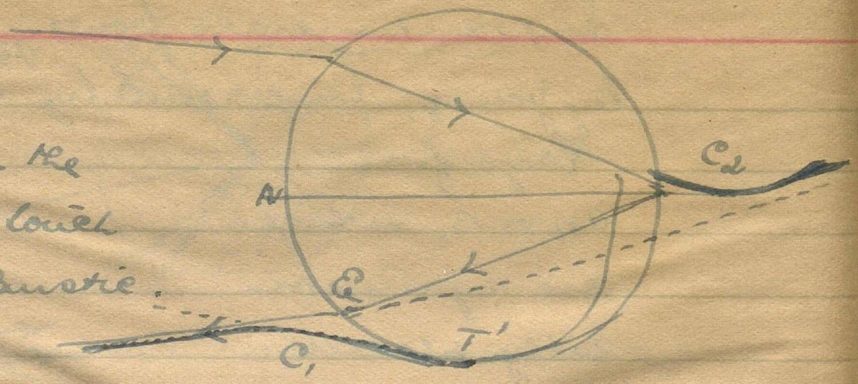
Rays from
B_1 & B_2 which
are symmetri-
cally placed
w.r.t. B (the

point of emergence of min. dev)
are ||l to each other. Hence they
overlap. Interference & reinforceme
are both possible. Hence spurious
bows are seen below the primary.
||ly spurious bows are seen abo
the secondary.

Potter argued that ~~both~~ the ordi
bows are also explained on the
wave theory. He found the eqn for
the involute of the Caustic curve.
Airy calculated the intensity of this
Caustic.

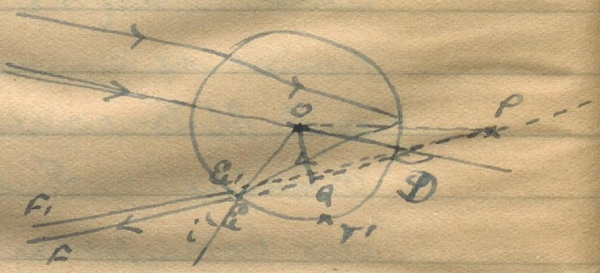
Emergent rays from the portion

i T' all touch
Caustic C_1.
The rays from the
portion NE touch
a similar caustic.



Airy's Expn
for the Caustic.

Let EF the min.
dev. ray & another
E'F' very near it
meet at P. Locus of
P is the caustic.



Let $OP = \rho$ Drop OQ normal to PEF.
OE is the radius. Let i be $\angle$ of emergence.

$$OQ = a \sin i = \rho \sin \gamma. \quad (1)$$

$$a \cos i \, di = \rho \cos \gamma \, d\gamma$$

$$\therefore \frac{d\gamma}{di} = \frac{a}{\rho} \frac{\cos i}{\cos \gamma} = \frac{\tan \gamma}{\tan i}$$

$$D = 2(i - r) + n(\pi - 2r)$$

$$\therefore dD = 2di - 2(n+1)\,dr.$$

$$\therefore \frac{dD}{di} = 2 - 2(n+1)\frac{dr}{di}.$$

$$\sin i = \mu \sin r \quad \therefore \quad \frac{dr}{di} = \frac{\cos i}{\mu \cos r}$$

$$\therefore \frac{dD}{di} = 2 - 2(n+1)\frac{\cos i}{\mu \cos r}.$$

But dD/di & $\frac{d\gamma}{di}$ are equal but opposite in sign.

$$\therefore dD/di = -d\gamma/di$$

$$\therefore \frac{\tan \gamma}{\tan i} = -\left\{2 - 2(n+1)\frac{\cos i}{\mu \cos r}\right\}$$

$$\therefore \tan \gamma = -2\tan i\left\{1 - (n+1)\frac{\cos i}{\mu \cos r}\right\}$$

But from (1) $\rho = \frac{a \sin i}{\sin \gamma} = a \sin i \sqrt{1+\cot^2\gamma}$

$$= a\left[\sin^2 i + \frac{\sin^2 i}{4\tan^2 i\left\{1-(n+1)\frac{\cos i}{\mu \cos r}\right\}^2}\right]$$

$$= a\left[\sin^2 i + \frac{\cos^2 i \; \mu^2\cos^2 r}{4\{-(n+1)\cos i + \mu \cos r\}^2}\right]$$

$\therefore \rho = \infty$ when $\mu \cos r = (n+1)\cos i$

$\cos i = \frac{\mu \cos r}{n+1}$ which is the Con-

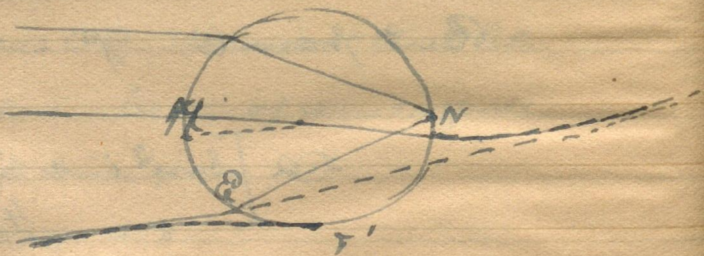
dition for min. deviation.
Hence the min. deviated rays cuts
he consecutive ray at infinite. Hence
e caustic is asymptotic to the
min. deviated.

ii) When $i = 90$ as for the tangen-
tial ray, $\rho = a$. ie the caustic meets
he drop at r_1.

iii) When $i = 0$, $n = 1$; $\mu = \frac{4}{3}$;

$$\rho = a.\left\{\frac{\frac{16}{9}}{4 \times \left(\frac{4}{3} - 2\right)^2}\right\}^{1/2} = a\left\{\frac{16 \times 9}{9 \times 4 \times 4}\right\}^{1/2}$$
$$= a$$

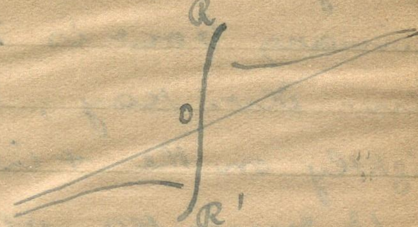
Hence the other
branch of the
caustic meets the
drop at the point
where the ~~axn~~ axial
ray cuts the drop



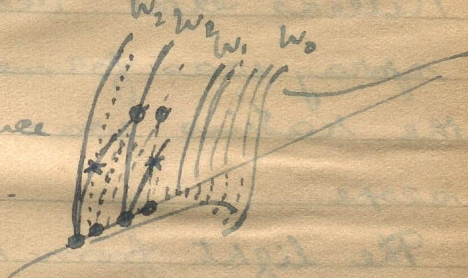
The emergent rays are tangential
to the caustic or normal to the wave
front. ∴ The wavefront is orthogonal

to the caustic. Hence the wavefront is the involute of the caustic.

The involute is concave for one portion & convex is for the other portion.



The crosses show the positions of destructive interference & Os represent reinforcement.



Airy calculated the intensity of the Displacement at point P is given by

$$\left\{\int_{-\infty}^{\infty} \sin(\omega t + 2\pi i + \ldots\ldots)\right\}^2 + \left\{\int_{-\infty}^{+\infty} \cos(\omega t + \ldots\ldots)\right.$$

The integral of the 1st is zero.

& the 2nd becomes

$$4\left\{\int_0^{\infty} \cos(\omega t + 2\pi i + \ldots)\right\}^2.$$

$$= 4\left\{\int_0^{\infty} \cos\frac{\pi}{2}\left(\omega^2 + m\omega\right) d\omega\right\}^2.$$

Airy's computation showed that primary bow is not formed by the min. dev. ray; but the point is slightly on the +ive side. The difference is 13 minutes of arc.

Miller's expt verified these results. A spray was formed ~~from~~ from a jet & the radius was measured with a telescope.

The light from the rainbow is partially polarized. This is explained by the reflections in the drops. The % of polarization has been investigated exptly & theoretically.

$$\cos(\xi - \xi') = \frac{1-(\mu^2+1)\,s^2 + 2\mu^2 s^4}{(\mu^2+1)\,s^2 - 1}.$$

$$F = \frac{\mu f_1 f_2}{\mu(f_2-f_1)+t}. \quad \alpha = \frac{f_1 t}{\mu(f_2-f_1)+t} \quad \beta = \frac{f_2 t}{\mu(f_2-f_1)+t}$$