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Studies on Extensive Air Showers.

PART I. — Sea Level Observations on the Variation with Shower Size of the Total Number of Nuclear-Interacting Particles in Showers of $(10^4 \div 2.5 \cdot 10^6)$ Particles.

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Summary. — An experiment carried out at sea level on the lateral distribution of nuclear-interacting particles in air showers and the variation of their number with the size of the showers, is described; the showers recorded ranged in size from 10^4 to $2.5 \cdot 10^6$ particles. The results indicate that the number of nuclear-interacting particles is proportional to $N_e^{0.45 \pm 0.05}$ for showers of size N_e less than about $6 \cdot 10^5$ particles, and the number is proportional to $\sim N_e^{1.2}$ for larger sizes. What has been obtained in this experiment and in similar experiments conducted by other groups (NICOL'SKY *et al.* and LEHANE *et al.*), is the *average* number of nuclear-interacting particles in a large number of showers of a given size. The possibility that such an average may not be meaningful if large intrinsic fluctuations (other than normal statistical variations) exist, is pointed out.

1. — Introduction.

NICOL'SKY *et al.* (1) have, in an experiment carried out at a mountain altitude of 650 g cm^{-2} , determined the variation with shower size of the total number of nuclear-interacting particles, (N-particles), in extensive air showers which ranged in size from $4 \cdot 10^3$ to 10^6 particles. A similar study has been carried out by LEHANE *et al.* (2), at sea level, for showers whose sizes ranged from $5 \cdot 10^4$ and $2 \cdot 10^6$ particles. The experiment of NICOL'SKY *et al.* showed

(1) S. I. NICOL'SKY: *Proc. of the Oxford Conf. on Extensive Air Showers* (1956), p. 19.

(2) J. A. LEHANE, D. D. MILLAR and M. H. RATHGEBER: *Nature*, **182**, 1699 (1958).

that the number of N-particles varied with shower size, N_0 , as $N_0^{0.2}$ for showers of size less than $4 \cdot 10^5$ particles, while the variation was given by $N_0^{1.0}$ for larger shower sizes; the change of the exponent was sudden and occurred at a shower size of $\sim 4 \cdot 10^5$ particles (at an altitude of 650 g cm^{-2}). This sudden change of slope, in the curve representing the variation of the total number of N-particles with shower size, has been interpreted by NICOL'SKY *et al.* as indicating that nuclear collisions of particles of energy greater than $6 \cdot 10^{14} \text{ eV}$ are radically different in their characteristics from those due to particles of lower energy.

In this paper we report an experiment carried out at sea level at Bombay, on the variation of the number of N-particles with shower size. The shower size recorded ranged from 10^4 to $2.5 \cdot 10^6$ particles. The results indicate that the number of N-particles is proportional to $N_0^{0.45 \pm 0.05}$ for showers of size less than about $6 \cdot 10^5$ particles, and the exponent becomes ~ 1.2 for larger sizes.

2. - Experimental arrangement.

The extensive air shower array was set up on the terrace of the building of the Tata Institute of Fundamental Research, in Colaba, Bombay. The lay-out of the detectors (Fig. 1), was therefore determined to some extent

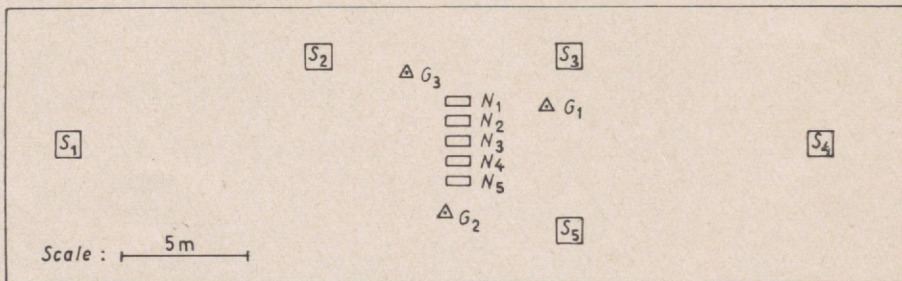
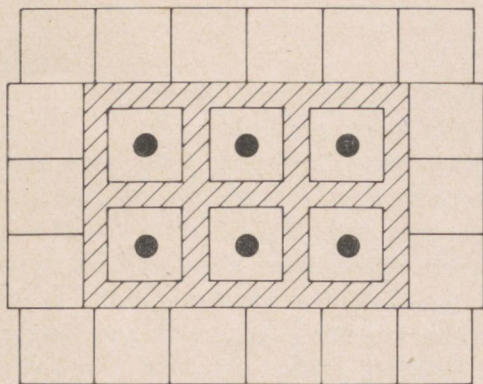


Fig. 1. - Extensive air shower array. S_1 - S_5 are liquid scintillators, each of area 1 m^2 ; N_1 - N_5 are N-detectors, each of area 0.4 m^2 ; G_1 , G_2 , G_3 are Geiger counter trays, each of area 500 cm^2 .

by the dimensions of the terrace. S_1 , S_2 , S_3 , S_4 , S_5 are five liquid scintillation counters each of area 1 m^2 ; they were spread over an area of $30 \text{ m} \times 6 \text{ m}$. N_1 , N_2 , N_3 , N_4 , N_5 are five N-detectors placed close to each other at the centre of the array of scintillation counters. Each N-detector had an effective area of 0.4 m^2 and consisted of enriched BF_3 neutron counters embedded in paraffin and lead (Fig. 2); the BF_3 counters were 75 cm long, 2.5 cm in dia-

meter. Cadmium sheets were inserted between the various N-detectors to prevent slow neutrons produced in one detector from reaching an adjacent detector. The N-particles produce nuclear interactions in the lead and paraffin, and evaporation



□ Paraffin
 ▨ Lead
 ● B^{10}F_3 Counter

0 5 10 cm

Fig. 2. - Cross-section of N-detector.

neutrons from these interactions are slowed down in the paraffin layer and detected as thermal neutrons by the BF_3 counters. A detector of this type has several advantages over the « Jánossy-type » of penetrating-shower detector, in which the N-particle is detected through the shower of penetrating particles it produces. The penetrating-shower detector has a relatively high energy threshold—about 5 to 10 GeV—for detecting nuclear

interactions. On the other hand, the neutron detector responds to interactions of much lower energy, down to about 100 MeV, though with decreasing efficiency. In addition to penetrating showers, the penetrating-shower detector also responds to electron-photon cascades produced by high energy electrons and photons resulting particularly from the electromagnetic interactions of μ -mesons. The neutron detectors respond exclusively to nuclear interactions. Another great advantage is that with a few neutron counters large effective areas can be built up.

Selection and display. - The air showers were selected by a triple coincidence of three Geiger counter trays G_1 , G_2 and G_3 (Fig. 1) each of area 500 cm². Pulses from the photomultipliers looking at the scintillators, and pulses from the neutron counters were displayed on four oscilloscopes. The triple coincidence master-pulse triggered first a 30 μs sweep in each oscilloscope, and then a second sweep 500 μs long, displaced vertically with respect to the first. The photomultiplier pulses, after suitable amplification, delay and attenuation, were displayed on the 30 μs sweeps and the N-detector pulses on the 500 μs sweeps. The non-overloading amplifiers and the display system were designed so that electron densities could be measured over the range of 1 to 1000 particles per m². It was possible to identify the particular N-detectors that were

activated, and also to determine the total number of pulses from the BF_3 counters in each of the N-detectors.

3. - Experimental results.

The experiment was in operation for three months at Bombay (sea level) during which period about 5 000 showers were recorded.

The shower sizes and the core positions were determined assuming a lateral distribution function of the Nishimura-Kamata type ⁽³⁾ for an age parameter $s = 1.25$. The computation was done with the aid of two mechanical analogue computers built for the purpose. The shower sizes recorded ranged from 10^4 to $2.5 \cdot 10^6$ particles, and the core-distances extended up to 25 m from the centre of the array. The error in size was 20 to 30 percent and the error in core position 1 to 2 m up to 10 m from the centre and about 3 to 5 m for cores beyond 10 m. In about 600 out of the 5 000 showers recorded, associated pulses from at least one of the five N-detectors were observed.

Showers were classified according to their size N_0 , and the distance, r , of the core from the centre of the N-detectors. For this purpose, the following six intervals of shower sizes were chosen:

$$\begin{array}{ll} 1.0 \cdot 10^4 \div 2.5 \cdot 10^4 & 1.6 \cdot 10^5 \div 4.0 \cdot 10^5 \\ 2.5 \cdot 10^4 \div 6.3 \cdot 10^4 & 4.0 \cdot 10^5 \div 1.0 \cdot 10^6 \\ 6.3 \cdot 10^4 \div 1.6 \cdot 10^5 & 1.0 \cdot 10^6 \div 2.5 \cdot 10^6 \end{array}$$

For each group of showers a further classification was carried out using the following intervals for r ,

$$\begin{array}{ll} 0 \div 2.5 \text{ m} & 6.3 \div 16 \text{ m} \\ 2.5 \div 6.3 \text{ m} & 16 \div 25 \text{ m} \end{array}$$

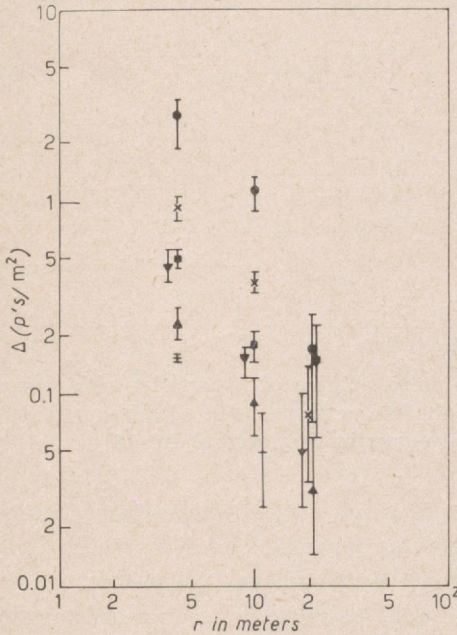
a) *Density of N-particles.* If the assumption is made that there are no large intrinsic fluctuations in the densities of N-particles in showers of same N_0 and r , then the density $\Delta(N_0, r)$ will be the same for all such showers within normal statistical variations, and is given by:

$$(1) \quad \Delta(N_0, r) = \frac{1}{s\epsilon m} \ln \frac{T}{T-Q},$$

⁽³⁾ K. GREISEN: *The Extensive Air Showers*. Chap. I: *Progress in Cosmic Ray Physics*, vol. 3, ed. by J. G. WILSON (Amsterdam, 1956).

where s = area of each N-detector, m = total number of N-detectors, ε = efficiency of the N-detector for the detection of N-particles, T = total number of showers of size N_0 with cores at a distance r from the N-detectors,

Q = number of showers in which one or more N-detectors were activated. For our set-up, $s = 0.4 \text{ m}^2$, $m = 5$ and $\varepsilon = 0.25$.



(\blacksquare $1.6 \cdot 10^4$, \blacksquare $4 \cdot 10^4$, \blacktriangle 10^5)
 (\blacksquare $2.5 \cdot 10^5$, \blacksquare $6.3 \cdot 10^5$, \blacktriangle $1.6 \cdot 10^6$)

Fig. 3. - Lateral distribution of density of N-particles.

distribution curves, we have estimated the total number of N-particles, up to a distance of 16 m from the core, in various groups of showers (of different sizes). Up to this distance, we consider that the data are statistically significant for the individual groups of showers. The variation of the total number of N-particles, N_N , thus obtained, (within 16 m from the core), is plotted in Fig. 4 as a function of the shower size N_0 . It is seen that

$$N_N \propto N_0^{0.45 \pm 0.05}$$

for

$$1.6 \cdot 10^4 \leq N_0 \leq 6.3 \cdot 10^5.$$

The density of N-particles as a function of shower size N_0 and core-distance r , may therefore be expressed as

$$(2) \quad \Delta(N_0, r) \propto N_0^\alpha r^{-\beta},$$

b) *Lateral distribution of N-particles.* The lateral distribution of N-particles has been calculated using equation (1). The results are plotted for different shower sizes in Fig. 3.

If we assume that the lateral distribution function is independent of shower size, then we can combine the lateral distributions obtained for the various groups to arrive at a statistically more significant function. This combined distribution may be expressed by a power-law of the form:

$$f(r) \propto r^{-1.2 \pm 0.05} \text{ for } 2.5 \text{ m} \leq r \leq 25 \text{ m}.$$

c) *Total number of N-particles as a function of shower size.* By integrating the observed lateral density

where

$$\alpha = 0.45 \pm 0.05$$

and

$$\beta = 1.2 \pm 0.05$$

for

$$1.6 \cdot 10^4 \leq N_0 \leq 6.3 \cdot 10^5$$

and

$$2.5 \leq r \leq 16 \text{ m.}$$

The errors indicated above are purely statistical errors (standard deviations). The absolute values of density may, however, be subject to systematic errors owing to uncertainty in the value of ϵ , the efficiency of the N-detectors for detection of N-particles. The absolute values are not likely to be in error by more than 30 per cent.

It may be seen from Fig. 4 that the point corresponding to a shower size of $1.6 \cdot 10^6$ particles lies at a value considerably higher than that expected from the extrapolation of the straight line passing through the other points. The pure chance probability that this last point lies not on this straight line is about 1 per cent.

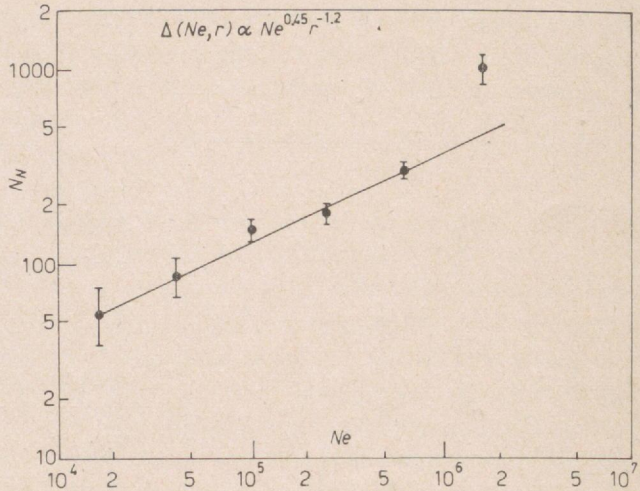


Fig. 4. - Variation of number of N-particles with shower size. (The straight line corresponds to a least-square fit of the experimental points in the interval $1.6 \cdot 10^4 \div 6.3 \cdot 10^5$).

4. - Discussion.

The lateral distribution function and the dependence of the total number of N-particles on shower size determined in this experiment are in very good agreement with the results of WALLACE *et al.* (4), obtained at sea level, using a standard IGY neutron monitor in association with the Australian air shower set-up (2). WALLACE *et al.* have given the relation

$$\Delta(N_0, r) \propto N_0^{0.47 \pm 0.1} r^{-0.92 \pm 0.1}$$

for

$$10^5 \leq N_0 \leq 2 \cdot 10^6 \quad \text{and} \quad 5 \leq r \leq 50 \text{ m.}$$

(4) G. S. WALLACE, M. M. WINN and K. W. OGILVIE: *Nature*, **182**, 1653 (1958).

Our lateral distribution also agrees with that of DMITRIEV *et al.* ⁽⁵⁾ who find a value of 1.1 for β (see eq. (2)) for showers of size $8 \cdot 10^4$ at sea level.

LEHANE *et al.* have determined the lateral distribution function and the variation with shower size of the total number of N-particles, using a penetrating shower detector

in association with an extensive air shower set-up. The lateral distribution function has been given by them as

$$f(r) \propto \exp[-r/20]r^{-0.5}.$$

This function agrees with our lateral distribution function up to 16 m, within the experimental errors. The total number of N-particles has been determined by LEHANE *et al.* by integrating the above function up to 50 m from the core. On the basis of the distribution given by them, about 20 per cent of the particles

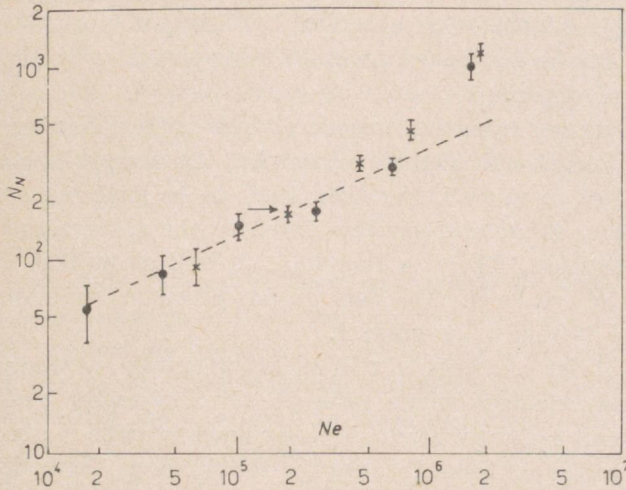


Fig. 5. - Combined plot of variation of N_N with N_0 . (x — LEGANE *et al.*; — present experiment; → indicates the normalization point. The straight line corresponds to the least square fit of the present data.

lie within a distance of 16 m from the core. To compare the results on the variation of the total number of N-particles with shower size, we have normalized the Australian data with ours at a shower size of $3 \cdot 10^5$ particles, where the statistical errors of observation are minimum in both experiments. The combined results are plotted in Fig. 5. Their data are represented by the five points marked with crosses. LEHANE *et al.* state that

$$N_N \propto N_0^{0.6 \pm 0.1}$$

for

$$6 \cdot 10^4 \leq N_0 \leq 8 \cdot 10^5.$$

The data of LEHANE *et al.*, however, extend beyond $N_0 = 8 \cdot 10^5$ particles. The point corresponding to the largest shower size investigated by them, viz, $2 \cdot 10^6$, lies at a higher value of N_N than would be expected on the basis of a

⁽⁵⁾ V. A. DMITRIEV, G. V. KULIKOV and G. B. KRISTIENSEN: as quoted by G. COCONI: *Hand. d. Phys.*, 45 (1958).

straight line passing through the other points, with a slope of 0.6 ± 0.1 . In this respect, their observations are in agreement with ours. A strict comparison of the two sets of results should not, however, be made since the experimental arrays are different, particularly in the type of N-detectors used, and their energy response; also, the lateral distributions have been integrated up to different distances from the core, viz. 16 m and 50 m in the two experiments. Therefore, the slope of 0.6 ± 0.1 obtained by LEHANE *et al.* for showers of size $< 10^6$ need not be considered to be in disagreement with the slope obtained in the present experiment, viz. 0.45 ± 0.05 .

A point of interest that emerges from the two sets of data is the following: *In both the experiments*, the number of N-particles observed at the largest shower size investigated, (at $N_0 > 10^6$), is much larger than that to be expected on the basis of the extrapolation of the straight line fitted to the observations on shower sizes $N_0 < 10^6$ particles. There is therefore an indication that a change of slope occurs even at sea level. The data do not permit us to state whether the change of slope is sudden or gradual.

According to the observations of NICOL'SKY *et al.*, at mountain altitude, (650 g cm^{-2}), the exponent, (α), changes from a value of 0.2 to 1.0 rather suddenly at a shower size of about $4 \cdot 10^5$ particles. If this change of slope is connected with the primary particles, then the change should occur both at mountain altitude and at sea level at shower sizes which correspond to the same primary energy; *i.e.* the change observed at a shower size of $4 \cdot 10^5$ particles at 650 g cm^{-2} should be seen at a shower size of about 10^5 particles at sea level. This is definitely not indicated in Fig. 5. The indication is that it occurs at a shower size larger than 10^5 particles. The fact that it occurs not at the same primary energy but perhaps at about the same shower size, at mountain altitude and at sea level, may be connected with the development of the nucleonic cascade.

However, it should be pointed out that in none of the experiments has it been possible to determine the lateral distribution and the number of N-particles in individual showers. A statistical method has been employed, in which showers lying in the same size interval, with cores striking at the same distance (interval) from the N-detectors are grouped together; assumptions are then made that: *a*) the numbers of N-particles in showers of same size N_0 , *b*) the lateral distributions of N-particles in showers of the same or different sizes and *c*) the ages of the showers, are constant (except for normal statistical variations). Figs. 4 and 5 may therefore be said to correspond to the variation of the *average* number of N-particles with shower size: this average is at present rather *ill*-defined. Intrinsic fluctuations can exist in the various characteristics mentioned above, and selection biases may thus be introduced in the detection and analysis of air showers as a consequence of these fluctuations. Our experimental set-up was designed specifically to check whether the density of N-par-

ticles remains constant in showers of the same size whose cores strike at a given distance from the N-detectors. Preliminary results of this experiment (which will be reported in Part II of this series), indicate the existence of fairly wide fluctuations outside normally acceptable statistical variations. Further experiments are being carried out to determine the exact magnitude and frequency of the fluctuations, and their dependence on the size, zenith angle and age of the showers. This might make it possible to isolate the various causes of these fluctuations and to evaluate their relative contributions. The real meaning that may be attached to the variation of the « *average* » number of N-particles with shower size, when it is obtained after ignoring these fluctuations will be known only when all of the data on fluctuations are available.

* * *

It gives us great pleasure to thank Prof. M. G. K. MENON for his interest in this investigation and for helpful discussions. We are indebted to Prof. S. MIYAKE for his valuable comments. We wish to express our thanks to Dr. M. YASIN of the Atomic Energy Establishment, Trombay, for the large number of neutron counters used in these experiments which have given trouble-free service. We are thankful to Mr. V. S. NARASIMHAM who analysed a considerable part of the shower-data. Our thanks are also due to Mr. S. G. KHARATKAR for the analysis of the showers using the analogue computer, and to Messrs F. GONSALVES, A. R. APTE, K. F. DINSHAW and V. S. PRABHU for their help in the construction and setting up of the air shower array and the associated electronic equipment.

RIASSUNTO (*)

Si descrive un esperimento effettuato al livello del mare sulla distribuzione laterale delle particelle che interagiscono coi nuclei negli sciami dell'aria e sulla variazione del loro numero con la dimensione dello sciame; gli sciami registrati variavano in dimensione da 10^4 a $2.5 \cdot 10^6$ particelle. I risultati indicano che il numero di particelle che interagiscono coi nuclei è proporzionale a $N_e^{0.45 \pm 0.05}$, per sciami di dimensione N_e inferiore a circa $6 \cdot 10^5$ particelle, e proporzionale a $\sim N_e^{1.3}$ per dimensioni maggiori. Ciò che si è ottenuto con questo esperimento ed altri simili eseguiti da altri gruppi (NIKOL'SKY *et al.* e LEHAN *et al.*), è il numero *medio* di particelle che interagiscono coi nuclei in un gran numero di sciami di una data dimensione. Si mette in rilievo la possibilità che una tale media sia priva di significato se esistono ampie fluttuazioni intrinseche (diverse dalle normali variazioni statistiche).

(*) Traduzione a cura della Redazione.

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Studies on Extensive Air Showers.

PART II. — Sea Level Observations on the Fluctuations in the Densities of N-Particles, in Showers of the Same Size.

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Summary. — A study of the fluctuations in the densities of N-particles in extensive air showers has been carried out with a set-up consisting of five N-detectors located at the centre of an array of scintillators. Air showers were grouped according to their size and the distance of the core from the N-detectors. In each group of showers of given size and core distance the observed frequency distribution in the number of N-detectors activated, shows the existence of large fluctuations in the densities of N-particles.

1. — Introduction.

The electron component is the most abundant one in air showers, and accordingly the lateral distribution, as well as the total number of electrons in individual showers, can be estimated fairly accurately for showers of size greater than $\sim 10^4$ particles, with a few detectors each of area $\sim 1 \text{ m}^2$, arranged in a suitable array. On the other hand, the nuclear-interacting particles, (N-particles), constitute less than 1 to 2 per cent of the electrons, and therefore elaborate experimental set-ups are necessary even to detect them; the problem of determining the actual number of N-particles which have traversed any given area is very much more difficult. Consequently, it has not been practicable upto now to measure the densities of N-particles in individual showers. The lateral distribution of N-particles and their total number in showers of given size have therefore been generally obtained by a statistical method. In this procedure showers are grouped according to their size, and according to the distance, r , of the core from the N-detectors.

The density of N-particles at a distance r from the core is then obtained from the ratio of the number of showers (in a given group, of fixed size and core distance r) in which the N-detectors are activated, to the total number of showers in that group; this procedure assumes that the N-particle density is uniquely given, and the probability whether the N-detector is activated or not, is governed by the Poisson law. This method is justifiable if large intrinsic fluctuations (outside the normally acceptable statistical variations) do not exist in the number of N-particles, from shower to shower. It is obvious, however, that this method will lead to erroneous results if wide fluctuations are present with appreciable frequency. The average values of density, and consequently of lateral distribution, and the total number of N-particles so obtained may not then be physically meaningful quantities.

In the experiments of NICOL'SKY *et al.* (1), LEHANE *et al.* (2) and in the work reported from this laboratory (3)—hereafter referred to as I—such a statistical procedure was used to obtain the variation of the total number of N-particles with the size of the shower. It was pointed out in I that the *average* number thus obtained of N-particles in a shower of given size may not be very meaningful, since it is rather ill-defined and since it has been got after ignoring intrinsic fluctuations, and that possible selection biases may have been introduced in the detection and analysis of the air shower, as a consequence of these fluctuations. Large fluctuations in energy flow, in the lateral distribution of electrons, and in the number of μ -mesons in air showers have been reported recently by the Russian (4) and Japanese (5) groups. It is therefore important to determine experimentally whether wide fluctuations are present in the number of N-particles, and also the extent and frequency of such fluctuations, if they exist.

An experiment was designed specifically for this purpose, and some preliminary results have been obtained in a period of operation of three months at Bombay (sea level). In this paper we report the results obtained in this experiment. These results clearly show that large fluctuations occur in the density of N-particles in showers of the same size, with cores striking at a

(1) S. I. NICOL'SKY: *Proc. of the Oxford Conference on Extensive Air Showers* (1956) p. 19.

(2) J. A. LEHANE, D. D. MILLAR and M. H. RATHGEBER: *Nature*, **182**, 1699 (1958).

(3) B. K. CHATTERJEE, G. T. MURTHY, S. NARANAN, B. V. SREEKANTAN and M. V. SRINIVASA RAO: *Nuovo Cimento*, **18**, 1148 (1960).

(4) S. N. VERNOV, G. B. KHRISTIANSEN, A. T. ABROSIMOV, N. N. GORYUNOV, V. A. DMITRIEV, G. V. KULIKOV, YU. A. NECHIN, S. P. SOKOLOV, V. I. SOLOVIEVA, K. I. SOLOVIEV, Z. S. STRUGALSKY and B. A. KHRENOV: *Proc. of the Moscow Cosmic Ray Conference*, vol. 2 (Moscow, 1960), p. 7.

(5) S. FUKUI, H. HASEGAWA, T. MATANO, I. MIUYA, M. ODA, K. SUGA, G. TANAHASHI and Y. TANAKA: *Proc. of the Moscow Cosmic Ray Conference*, vol. 2 (Moscow, 1960), p. 30.

given distance from the N-detectors. The possible sources of these fluctuations and the experiments needed to separate out the relative contributions of the various sources are pointed out.

2. - Experimental details.

Experimental details and the method employed for analyzing the shower data have been given in I. One important difference between other experimental set-ups used (^{1,2}), for the study of N-particles, and the present one is that *five* N-detectors each of area 0.4 m² have been clustered together at the centre of the air shower array of five scintillators: the scintillators were each of area 1 m². It is this feature that has enabled us to study the fluctuations in the densities of N-particles, in the manner discussed in this paper.

The experiment was in operation for three months at Bombay (sea level) and during this period about 5 000 showers were recorded; in about 600 of these, associated pulses from at least one of the five N-detectors were observed. The shower sizes recorded ranged from 10⁴ to 2.5 · 10⁶ particles and the core distances extended upto 25 m from the centre of the array.

The calculations to determine shower size and core position were carried out with a mechanical analog computer; a lateral distribution function of the Nishimura-Kamata type was taken for these computations with a value of 1.2 for «*s*», the age parameter. The error in size was 20 to 30 percent and the error in core position 1 to 2 m upto 10 m from the centre, and 3 to 5 m for cores beyond 10 m, as judged on the analog computer itself, for actual cases picked out arbitrarily from the total sample. About 5% of the showers deviated considerably from the assumed distribution function and those were excluded from the analysis on fluctuations of N-particles.

3. - Fluctuations in the densities of N-particles.

Showers of the same size N_e , with cores striking at a given distance r from the centre of the N-detectors were grouped together. If the density of N-particles, Δ has the same value for all showers which constitute a group, then

$$(1) \quad \Delta(N_e, r) = \frac{1}{s\epsilon m} \ln \frac{T}{T-Q},$$

where s = area of each N-detector, m = total number of N-detectors, ϵ = the efficiency of the detector for detecting N-particles; T = total number of showers in the group, and Q = number of showers in which one or more N-detectors were activated. For our set-up, $s = 0.4$ m², $m = 5$, $\epsilon = 0.25$.

The number, $F(n)$, of showers in which n out of m N-detectors are activated is given by the Poisson distribution:

$$(2) \quad F(n) = T \cdot P(n, \Delta),$$

$$(3) \quad P(n, \Delta) = {}^m C_n (1 - \exp[-s\varepsilon\Delta])^n \exp[(-s\varepsilon\Delta)(m - n)].$$

If the density of N-particles has a unique value for all showers in each group then Δ can be calculated from (1), and the frequency distribution from (2), and compared with the experimental distribution. This comparison is shown in Table I, for various groups of showers. (The grouping of showers according

TABLE I.

Shower size (N_e)	Core distance (in m)	Δ	T	$F(n)$					
				$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$6.4 \cdot 10^4 \div 1.6 \cdot 10^5$	2.6 ÷ 6.3	0.48 (0.56)	277	218 218	47 53.5	10 5.23	0 0.26	2 0.006	0 —
	6.4 ÷ 16	0.146 (0.16)	442	411 411	29 29.7	0 0.86	2 0.012	0 —	0 —
$1.7 \cdot 10^5 \div 4.0 \cdot 10^5$	0 ÷ 2.5	0.54 (0.61)	59	45 45	10 12.5	4 1.38	0 0.07	0 —	0 —
	2.6 ÷ 6.3	0.50 (0.65)	298	231 231	47 60	15 6.2	3 0.3	2 0.001	0 —
	6.4 ÷ 16	0.18 (0.20)	379	347 347	28 30.9	4 1.1	0 0.02	0 —	0 —
$4.0 \cdot 10^5 \div 1.0 \cdot 10^6$	0 ÷ 2.5	1.17 (1.68)	103	58 58	22 35.2	11 8.7	6 1.1	5 0.06	1 0.0016
	2.6 ÷ 6.3	0.94 (1.15)	144	90 90	36 44.4	10 8.7	5 0.85	3 0.04	0 0.001
$1.1 \cdot 10^6 \div 2.5 \cdot 10^6$	2.6 ÷ 6.3	2.76 (3.0)	24	6 6	8 9.6	7 6.1	0 1.94	1 0.31	2 0.02
	6.3 ÷ 16	1.13 (1.42)	58	33 33	16 19.7	6 4.7	1 0.57	0 0.03	2 0.001

In each group, row 1: the observed distribution of n ; row 2: the calculated distribution assuming equation (2).

to size and core-distance, and the intervals involved can be seen clearly from the Table itself.) The observed number of showers in which 2 or more detectors are activated ($n \geq 2$), is consistently higher than that calculated on the assumption of a Poisson distribution; in most of the groups the observed frequency of events with $n = 3, 4$ and 5 , exceeds the expected number by a large factor. The fact that the shower sizes and core-distances have been taken over finite intervals can account only for a small factor (~ 2) between the observed and expected frequencies of showers with $n \geq 3$.

The question may be raised whether cases in which two or more detectors are activated could not be, in the main, the result of cross-interference between detectors, *i.e.* a main interaction taking place in only one detector and the secondaries emitted at large angles from this interaction giving rise to further interactions in the adjacent detectors. This possibility is small as may be seen from the experimental observation that the frequency of events in which two *adjacent* N-detectors were activated, was the same as that of non-adjacent ones (in particular the extreme ones). This shows clearly that the double and multiple events were due to the incidence of separate nuclear active particles on the various detectors. This is essentially because the height of the N-detectors is small compared to their lateral extent and cross-interference can arise only due to secondaries emitted at very large angles which are consequently of lower energies and the efficiency of the N-detectors is low for such small energies.

The discrepancy between the observed and expected frequencies as seen from the Table can be understood only if Δ is not a constant, but fluctuates from shower to shower. Let $G(\Delta)d\Delta$ represent the probability that in a given shower of a given size at a given distance from the core, the density of N-particles has a value between Δ and $\Delta + d\Delta$. The frequency distribution $F(n)$ is then given by

$$(4) \quad f(n) = T \int_0^{\infty} G(\Delta) P(n, \Delta) d\Delta,$$

where $P(n, \Delta)$ is given by (3). This distribution reduces to (2), when $G(\Delta)$ is assumed to be a δ -function, which would be the case if there were no fluctuations. In principle it is possible to determine $G(\Delta)$ if the distribution function $F(n)$ is known accurately. However, the data are too meagre, especially for $n \geq 3$, to justify an attempt at an evaluation of $G(\Delta)$.

The rather close agreement between the observed and the expected number of showers for $n = 1$ and 2 , when the data are normalized for $n = 0$, may, in the first instance, suggest that in a majority of the showers, the value of Δ is a constant, and close to that calculated from (1), and that it is only in a small fraction of showers that appreciable fluctuations exist. To estimate the magnitude and frequency of fluctuations on such a picture, we have drawn

the curves of $P(\Delta)$ vs. Δ for different values of n ($= 0, 1, 2, 3, 4, 5$) in Fig. 1. From these curves, one can estimate that the observed distributions in most of the shower groups of Table I can be accounted for if at least 5 to 10% of the showers fluctuate in density by as much as 10 times the density calculated from eq. (1). However, this interpretation is not unique, and many other types of fluctuations can also reproduce the observed distribution.

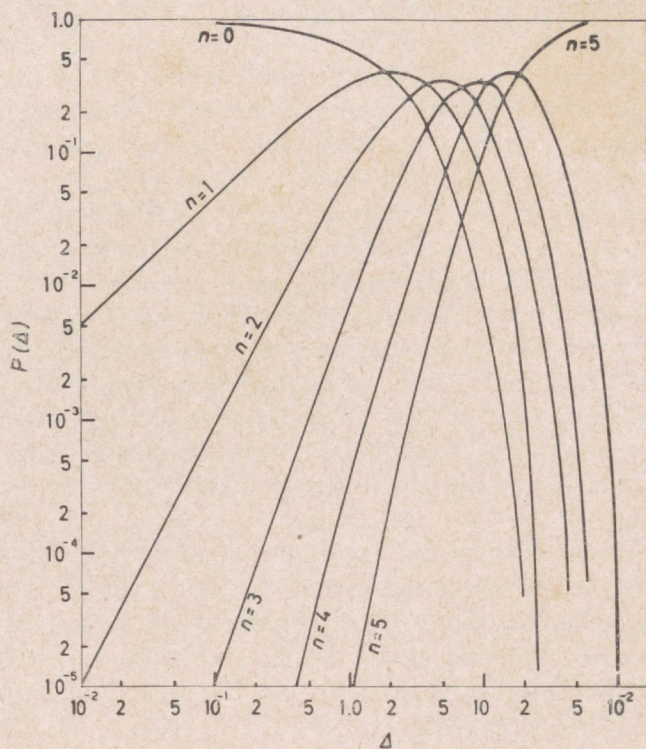


Fig. 1.

The estimation of an *average* density of N-particles, on the basis of eq. (1), presupposes that the distribution $F(n)$ is a Poissonian as represented by eq. (2) and (3). However, the experimental results are not consistent with such an assumption, since the observed number of events in which $n \geq 3$, is larger than that expected from eq. (1), (2) and (3).

It is therefore desirable to compute the average density without making any specific assumption regarding the nature of the distribution $F(n)$, *i.e.* according to the relation

$$(5) \quad \bar{\Delta} = \frac{\sum_{n=1}^{n=m} n F(n)}{s \epsilon m T} .$$

The average densities calculated using (5) are given within brackets in the Table, below the values calculated from eq. (1). A comparison of the two sets of values shows that the densities calculated from (5) are consistently larger than those calculated from (1) by about 10 to 40%.

4. - Sources of fluctuations.

There are three different sources which can contribute to the fluctuations. They are:

1) fluctuations in the characteristics of nuclear interactions, particularly of the first collision; these are fluctuations in the inelasticity of collisions, in the multiplicity of mesons produced, in the energy transferred to π^0 -mesons, etc.;

2) fluctuations in the level of the first collision, and in the inclination of the shower axis — in other words, fluctuations in the amount of atmosphere through which the shower has developed upto the level of observation; and

3) fluctuations due to the initiation of some of the extensive air showers by the heavy nuclei present in the primary cosmic radiation.

Nuclear emulsion data ⁽⁶⁾ on high-energy jets of energy 10^{12} to 10^{13} eV indicate that wide fluctuations exist in the characteristics of high energy interactions. No detailed quantitative data are available to enable an evaluation of the contribution of (1) to the fluctuations in the N-component.

As for (2), because of the fluctuations in the level of the first collision there is no one-to-one correspondence between the size of the shower and the energy of the primary particle. The distribution in the atmosphere of the levels of the first collision (which lead to air shower propagation), for a fixed size of shower observed, depends critically on the interaction mean free path, and on the energy spectrum of the primaries; and also on the longitudinal development and absorption of the electron component as a function of the primary energy. Until data on these parameters are available it is difficult to assess the magnitude of fluctuations in the N-component, due to fluctuations in the levels of shower origin. If the interaction mean-free-path in air is ~ 70 g cm⁻², calculations based on a simple model show that the fluctuations in the levels of shower-origin are not very important, but for larger values of the interaction mean free path ⁽⁷⁾ ($90 \div 100$) g cm⁻² these fluctuations will result in large variations in the N-component.

⁽⁶⁾ D. H. PERKINS: *Progress in Elementary Particle and Cosmic Ray Physics*, vol. 5, Ch. IV (Amsterdam, 1960), p. 257.

⁽⁷⁾ A. E. BRENNER and R. W. WILLIAMS: *Phys. Rev.*, **106**, 1020 (1957).

The important role played by heavy primary nuclei in air shower phenomena has been stressed by PETERS⁽⁸⁾. If one assumes that the primary charge spectrum observed at lower energies ($\sim 10^{13}$ eV) persist at higher energies as well, then one expects that at least 30% of the showers observed at sea level should be produced by α -particles and heavier nuclei. The relative abundance of primaries of different mass number that can produce showers of the same size at sea level is given in Table II (COCCONI)⁽⁹⁾.

TABLE II.

Nature of the primary	H	He	CNO	Ne, Si	A, Fe
Mass number A	1	4	14	24	52
Relative abundance (%)	70	14	8	3	5

The relative abundances in the above Table have been obtained by COCCONI from the relative abundances of these nuclei at the top of the atmosphere after taking into account the decrease in the interaction mean free path with increase in mass number.

If it is assumed that a primary heavy nucleus of mass number A and total energy E_0 is equivalent to A nucleons each of energy E_0/A as far as air shower development is concerned, and that the number N_N , of N-particles is proportional to E^α where E is the energy of the nucleon generating the air shower, then it follows that $N_N \propto A(E_0/A)^\alpha$; or for a given energy E_0 , N_N is proportional to $A^{(1-\alpha)}$. If $\alpha = 1$, then N_N is independent of A , *i.e.* there will be no fluctuations in the number of N-particles as a result of some of the showers being due to heavy nuclei. However, if $\alpha \neq 1$, then there will be a dependence of N_N on the mass number of the primary particle. If $\alpha < 0.5$, then N_N is very sensitive to A , and very large fluctuations in the number of N-particles may be expected, since the mass number of the primary can fluctuate by more than a factor of 50.

5. - Conclusions.

The present experiment has shown that it is not justifiable to assume that the number of N-particles is the same in all showers of the same size. There definitely exist wide fluctuations, though to determine the magnitude and frequency of such fluctuations more experimental observations are necessary.

(8) B. PETERS: *Proc. of the Moscow Cosmic Ray Conference*, vol. 3 (Moscow, 1960).

(9) G. COCCONI: *Extensive Air Showers, Handbuch der Physik*, vol. 45.

Experimental results on the variation of the number of N-particles with shower size, etc. (¹⁻³), which are based on the assumption that the total number of N-particles is always the same in showers of the same size except for normal statistical fluctuations must therefore be treated with caution.

In order to isolate out the various contributions which are due to causes listed previously and which give rise to fluctuations in the N-component, experiments are now being conducted with similar but more extensive air shower arrays at two different altitudes separated by an atmospheric depth of $\sim 200 \text{ g cm}^{-2}$. In these experiments, in addition to data of the type reported in this paper, measurements are also being made on the age, the arrival direction *i.e.* the angle of the shower and the density of μ -mesons. The number of N-detectors has also been doubled compared to the experiment reported herein. If fluctuations of appreciable magnitude in the N-component are seen in *showers of the same size, and of the same age*, then it would indicate that there exist sources of fluctuations other than fluctuations in the level of the first collision. To draw definite conclusions it is necessary to study fluctuations in showers of the same size, and the same age, over a wide range of sizes and of ages, which should be possible through experiments carried out at two atmospheric depths.

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We have great pleasure in expressing our thanks to Professor M. G. K. MENON for his interest in this investigation and many helpful discussions. We are grateful to Professor S. MIYAKE for his valuable comments regarding the analysis of experimental data.

RIASSUNTO (*)

Per mezzo di un'apparecchiatura costituita da cinque rivelatori di particelle N posti al centro di un gruppo di scintillatori si sono studiate le fluttuazioni della densità delle particelle N negli sciami estesi dell'aria. Questi sciami sono stati raggruppati secondo la loro grandezza e la distanza del « core » dai rivelatori di particelle N. In ogni gruppo di sciami di una data grandezza e di una data distanza del « core » la distribuzione di frequenza del numero di rivelatori di particelle N attivati, che si riscontra, mostra l'esistenza di ampie fluttuazioni delle densità delle particelle N.

(*) Traduzione a cura della Redazione.

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