100001 G_1 ~~100010001, 1000100001000.~~

10000100001 G_2

100000100000100001 - G_3

G_1 is divisible by 11, G_2 by 111, G_3 by 1,111 but G_4 is not divisible by 11,111. Somewhat more sophisticated methods can be used to show that G_5 & all higher G's are composite. An argument John Wade shows that G_4 is the product of

$4,672,725,574,038,601 \times 21,401$
$= 64.$

$2 = 1 + 1$

$3 = 1 + 2 = 1 + 1 + 1.$

$4 = 1+1+1+1 = 1+2+1 = 2+2$

$= 1 + 3 =$

$5 =$

$P(n) = 5($

$$\frac{4}{9} + \frac{1}{2} = \frac{8+9}{18} = \frac{17}{18}$$

The Indian Science Congress Popular Science Lectures

Organisation, Bangalore AND Staff Colloquium

Forest Research Laboratory, Bangalore

Speaker: Dr. B.S. Madhava Rao

Subject: Fascination of Numbers

Date: 25th August, 1972 (Friday)

Time: 3.30 P.M.

You are cordially invited to attend.

TEA: 3 P.M.

S. P. Bhatnagar

FOREST RESEARCH LABORATORY
18TH CROSS, MALLESWARAM
BANGALORE-3

(S.P.Bhatnagar)
Hon.Secretary
Staff Colloquium

pns.

8.3/5 1.6 22 50

"Fascination of Numbers"

Synopsis of Lecture

(1) Thanks to Dr. P. S. Rao.

(2) Presentation of only a few topics possible – Remark on p. 4 of Ogilvy & Anderson.
– omission of works of Aryabhatta, Brahmagupta & Bhaskara but mention of Ramanujan's work.

Topics to be dealt with

(1) Hindu decimal notation using zero & the four processes of arithmetic – Views of Laplace, and Dantzig. – Binary system and its use in modern computers. ~~to [illegible]~~ (Slide 1, O & A, p. 159)

(2) Integers, rational numbers, irrational numbers (algebraic & transcendental). (Slide 2)

(3) Kinds of numbers:

Abundant, Amicable, Binomial, Euclid, Fermat, Mersenne, perfect, Fibonacci, Lucas, prime, Lucky (O & A, p. 100), relatively prime, googol, googolplex, Skewe's number, triangular, ~~round~~, ~~String (of 1's)~~ repunit, Twin primes, Highly composite or round, Deficit, Progression, automorphic, palindromic, factorial, Power, oblong, hexagonal., Reverse (R. p 41), Equivalent numbers (Hardy, note book, p. 114), partition. ~~(30 types)~~
~~(Date written in alphabetical order – Slide 2)~~

(Math. Comp. April, 1970) (Sociable, crowds), prime primes (73939133 by ~~striking~~ dropping the right most digit successively still gives primes, tested by using Lehmer's table of the first ten 10 million primes), curious.. ~~atomic (as in physics)~~, pentagular, infinite, transfinite; ~~(36 in all)~~.

~~(4)~~ multiperfect, narcissistic, perfect digital, recurring digital, & visible representation, giant numbers., Pythagorean numbers (40 in all) (Slide 2)

Book on perfect nos – Vols on primes –

(4) curious numbers:

(a) Printers' errors (b) illegal cancellations / magic squares. (c) Formulae for primes, (d) near miss for Fermat's last theorem, (e) curio on p. 89 of O & A, (f) Prime primes, (g) Repunit nos.
(h) Amicable & sociable numbers, (i) 10th order P.D.I (j) V. R. Nos (8 types marked with red dots on p. 35 of yellow 9.9 Sr. notebook), (k) automorphic nos (red dots), (l) giant nos – table from O & A plus 9th 2 powers of 9. ~~(m) magic squ~~ [Slides 3, 4, 5]

(5) Fibonacci numbers [Slides 7 for trees etc, Slide 6 for log. spiral, pentagram & isosceles △s.] & if need be another slide 8.

(6) Ramanujan's sum in terms of $\mu(n)$, – Theorems involving $\zeta(s)$ – Ramanujan's work on partitions & remarks re. math. prodigies. Ramanujan's continued fractions (Slides 9 and 10)

(7) Concluding remarks.

1. Introduction

(1) Intro

The theory of numbers is one of the most fertile branches of mathematics consisting of numerous unsolved problems continually increasing with time. New problems arise more rapidly than those already solved, and plenty of problems have remained unsolved for centuries. Another remarkable feature about number theory is that many problems are easily understandable by amateurs, although their solution has ~~they have escaped~~ eluded the efforts of professionals of the highest order. ~~It is impossible to present in the small compass of a popular lecture~~ Also most of number theory has very few ~~applications~~ "practical" applications. That does not reduce its importance, and if anything it enhances its fascination. It is impossible to present in this short compass of a popular lecture the really interesting but highly technical results of advanced number theory. I had originally planned to present to you the important aspects of the work of in this field of the famous Indian mathematicians Aryabhatta, Brahmagupta and Bhaskara~~charya~~, ~~but I am afraid time will not permit me to do so. I shall however indicate towards the end~~ and ~~some~~ a few of the beautiful results of Ramanujan, the greatest of our modern mathematicians, but I am afraid time will not permit to do so. ~~Before doing so, I shall place before you some trivial but curious and interesting letters results, and also make a brief survey of the not very trivial Fibonacci numbers.~~ [P.T.O]

2. Hindu decimal ~~notation~~ system. [Stet]

Perhaps the most fascinating achievement of the theory of numbers is the discovery ~~by~~ near about 300 B.C of the decimal number concept including the zero by ~~an unknown~~ the Hindus. The great mathematician Laplace says "It is India - - - produced by antiquity." More than this it ~~is~~ was invention of zero as a basis of the principle of position that ~~was~~ contributed most to the use of the decimal ~~system~~. It is remarkable that during a long period of nearly 5000 years which saw the fall & rise of many ~~a~~ western civilisation, each leaving behind it a heritage of

It is with great reluctance therefore that I shall not be able to talk to you about the theory of numbers at all, but will have to confine myself to some trivial number curiosities understandable by amateurs, ~~cracks~~ quacks, and even school children, but frowned upon by number-theoreticians. Let me assure you, however, that this is not meant to cast any reflection on the audience present ~~thus~~ here this afternoon. I shall also say something about the not too-very-trivial topic of the Fibonacci numbers.

$$\sqrt{2} = 1 + \frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}$$

$$\frac{\pi}{4} = \cancel{R+}\,\frac{1}{1+}\,\frac{1^2}{2+}\,\frac{3^2}{2+}\,\frac{5^2}{2+}\cdots$$

$$e = 2 + \frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{6+}\cdots$$

$$\pi = 2\left\{\frac{2\cdot 2}{1\cdot 3} \times \frac{4\cdot 4}{3\cdot 5} \times \frac{6\cdot 6}{5\times 7} \times \cdots\right\}$$

$$= 2\left[1 + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{5}\right)\left(\frac{1\cdot 3}{2\cdot 4}\right) + \left(\frac{1}{7}\right)\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right) + \cdots\right.$$

literature and philosophy, there should have been a paucity of achievement in the system of reckoning numbers. All that was employed was an inflexible numeration so crude . . . . system! In these circumstances the achievement of the discovery of zero assumes the proportions of a world-event.

Particularly puzzling is the fact . . . . did? Thus the Indian 'Sunya' based on the concept of "nirvana" was destined . . . . human race.

~~How the Indian "Sunya" . . . . zero.~~

— How the Indian "Sunya" - - - - zero. The Arabs later on transferred this decimal system ~~[illegible]~~ to the West through the Italians among whom a prominent part in this transference was played by Leonardo de Pisa (also known as Fibonacci). It was thus that the dark ages in Europe ~~was~~ later underwent a great Renaissance.

3. Real numbers. [Delete]

Before we enter the domain of integers (~~mainly positive integers~~) which constitute ~~is~~ the core ~~backbone~~ of the theory of numbers, we can rapidly examine the other real numbers viz ~~like~~ the rationals and irrationals. The rational numbers are of the form a/b where a & b are integers, and when expressed in the form of decimals, these are either terminating or recurring. When expressed in terms of continued fractions, these are terminating. Numbers which are not rational are ~~is~~ ~~irrat~~ called irrational, as for example $\sqrt{2}$, π & e. ~~These irrationals~~ with e defined by the infinite convergent series $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ The irrationals are represented by non-recurring decimals of infinite length, and by non-terminating continued fractions (simple or otherwise). Among the irrationals we have again the two classes of algebraic and transcendental numbers. The algebraic numbers are those which are roots of a polynomial equation, and the transcendental ones ~~those which~~ are the remaining irrationals. Although the sets of integers, rationals, algebraic numbers and transcendental numbers are infinite, it is possible on the basis of Set theory to allot a measure or a cardinal number to each ~~of~~ of these sets. A set whose elements can be

(1) Abundant nos for eg $12 < 1+2+3+4+6$

Deficit nos " $10 > 1+2+5$

Perfect nos: $6 = 1+2+3$; $28 = 1+2+4+7+14$, — of form $2^{n-1}(2^n-1)$.

(2) Mersenne nos: $M_n = 2^n - 1$, for M_n prime, n prime necessary but not sufficient for eg

$2^9 - 1 = 511 = 7 \times 73$.

Largest n known is M_{11213} a number of 3376 digits

(23rd Mersenne prime).

(3) Fermat nos: $2^{2^n}+1$, $2^{2^5}+1 = 641 \times 4294967297$ (Euler)

(4) Amicable pairs — Each = sum of divisors of the other — simplest example 220 & 284

220 = ~~1×2×4×5×10×11×20×22×44×55×110~~

& ~~sum = 284~~ has factors 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110 whose sum = 284

2×142
2×2×71

284 = ~~1×2×4×~~ has factors 1, 2, 4, 142, 71 whose sum = 220 [400 pairs known]

(5) Sociable nos: divisors of 12496 are 1, 2, 4, 8, ... 6248 whose sum = 14288

divisors of 14288 are 19 in no whose sum = 15472

" 15472 " 9 " " = 14536

" 14536 " 15 " " = 14264

" 14264 " 7 " " = 12496 the original number

We can write this as $s^5(12496) = 12496$

Another sociable number is 14316 repeating after 28 steps ie $s^{28}(14316) = 14316$

If $s^4(N) = N$ (N is called a crowd number ~~for eg s^4~~

For Amicable nos $s^2(N) = s$ & if $s(N) = N$ number is perfect

(6) Prime nos: $N \to \dfrac{N}{\log_e N}$ as $N \to \infty$.

(7) Fermat's last theorem — ~~Yah~~ Prof. Yahya's proof in 1957

(8) Diophantine eqns — Indefinitely many solns of $x^3+y^3+z^3 = w^3$ (eg, 3, 4, 5, 6) — $y^2 - 7 = x^3$: no solns — but 8 solns of $y^2 - 17 = x^3$ (2, 4, 8) — Analysis of $a^b - c^d = 1$ (b & d ≠ 1); one soln is $3^2 - 2^3 = 1$). Are there others? How many? What are they? No one knows

put into a ~~one~~ 1-1 correspondence with the integers ~~is~~ said to be enumerable, and assigned the measure zero or the cardial number denoted in the Cantor notation by $\aleph_0$. The rationals can be ~~written as~~ arranged as

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

ie in the form $x_1, x_2, \cancel{x_3} \dots x_n$ and hence form a set of measure zero ie $\aleph_0$. Similarly it can be proved that the aggregate of algebraic numbers also ~~f~~ is a $\aleph_0$. Thus it follows that almost all real numbers are transcendental and the set consisting of them has a measure > 0 and is assigned the cardinal number $\aleph_1$ of the linear continuum. In spite of this, ~~few~~ very few classes of transcendental numbers are known even now. ~~They are indicated in the adjoing~~ [Slide 1] A mention might

[P.T.O]

4. Number curiosities.

~~Leaving aside those profound set-theoretic considerations, let me~~ now come ~~back~~ to the domain of the integers which is the main playground of number theory, and present some easily understandable curious results. ~~but which nevertheless pose equally challenging problems. I shall first of all present some trivial results of interest to amateurs and, cranks, & children, but and not frowned upon by professional number-theoreticians.~~

(1) Illegal cancellations: It is a well known joke that the following illegal "cancellation" of 6's yields the right answer

$$\frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}.$$

Are there any other fractions with numerator and denominator each of 2 digits that can be "reduced" in this fashion? The answer is that the only other solutions are

$$\cancel{\frac{26}{65}}, \cancel{\frac{19}{95}} \text{ ~~and~~ } \frac{26}{65}, \frac{19}{95} \text{ and } \frac{49}{98}.$$

But it is more difficult to locate a cancellation like

$$\cancel{\frac{143185}{170}} \quad \frac{143\cancel{18}5}{170\cancel{18}56} = \frac{1435}{17056}$$

(2) Printer's errors:

one such famous error giving the right answer is

$$\cancel{2592}\ 2^5 9^2 = 2592$$

and it is believed that this is the only example of this type.

Other examples ~~not~~ involving not only indices, but also simple misplacement of dots in

$2\cancel{5}^{5} \cdot \frac{25}{}$ $\qquad \frac{3200}{4}$ $\qquad \frac{775}{25}$

$441 \cdot \frac{53}{11}$

$32 \cdot \frac{25}{31} = \frac{800}{31} \cdot 25\frac{25}{31} = \frac{800}{31}$ ✓

378×73
1134
2646
27594

1323
$22\cancel{95}$
23373

$121 \cdot 9\frac{1}{3} = 121 \cdot \frac{28}{3} = \frac{3588}{3}$; $1129\frac{1}{3} = \frac{3388}{8}$ ✓

121×28
968
242
$3\cancel{5}88$

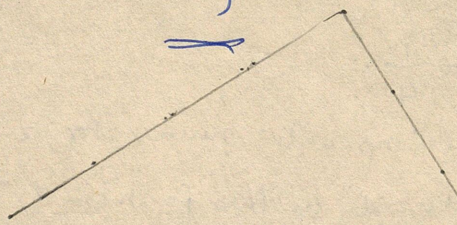
$21^2 \cdot 4\frac{9}{11} = \frac{23373}{11}$; $2124\frac{9}{11} = \frac{23373}{11}$ ✓

$73 \cdot 9 \cdot 42 = 73 \cdot 378 = 27594$; $7 \cdot 3942 = 27594$ ✓

~~also be made of some attempts recently made to differentiate between classes of transcendal numbers themselves, but no success has been achieved. For eg. the machine time required to compute the value of e to 100,000 decimal places was less than one-third of the time required for the corresponding calculation of the value of π. Thus one is led to guess that e is not as "deep" as π, but let one try and prove it !~~

For 2 digit nos. product of number & its reverse is never a perfect square except for $55 \times 55 = 55^2$. This is not true for numbers of more than 2 digits

Thus $169 \times 961 = 433^2$
$1089 \times 9801 = 3267^2$



The formulae $y = x^2 - x + 41$ gives primes up to $x = 40$ & for $x = 41 \to 41^2$

Simly $y = x^2 - 79x + 1601$ " fails for first time $x = 80$.

$5^3 + 6^3 = 341$
$7^3 = 343$ } narrow miss for Fermat's theorem very true

$5^2 = 25$, $5^3 = 125$, $5^4 = 625$, $5^5 = 3125$, $5^6 = 15625$, $5^7 = 78125$, $5^8 = 390625$ fails

Re 3. (3) 18^{18} & 10^{33} are the ~~only~~ highest ones so far known that can be expressed as ~~product~~ of factors with no zeros.
The next one, if there be one, is $> 10^{5000}$

in multiplication are

$$2^5 \cdot \frac{25}{31} = 25\frac{25}{31} \; ; \; 11^2 \cdot 9\frac{1}{3} = 1129\frac{1}{3} \; ; \; 21^2 \cdot 4\frac{9}{11} = 2124\frac{9}{11} \text{, and.}$$

$$73 \cdot 9 \cdot 42 = 7 \cdot 3942 \; ; \cancel{73 \cdot 9 \cdot 420}$$

(3) magic squares: out of the very large number of such squares where the sum of the ~~row~~ digits in all the rows, the columns and the diagonals is the same, we present a few below. It is interesting to note that Ramanujan was interested in such constructions even as a young boy.

96	11	89	68
88	69	91	16
61	86	18	99
19	98	66	81

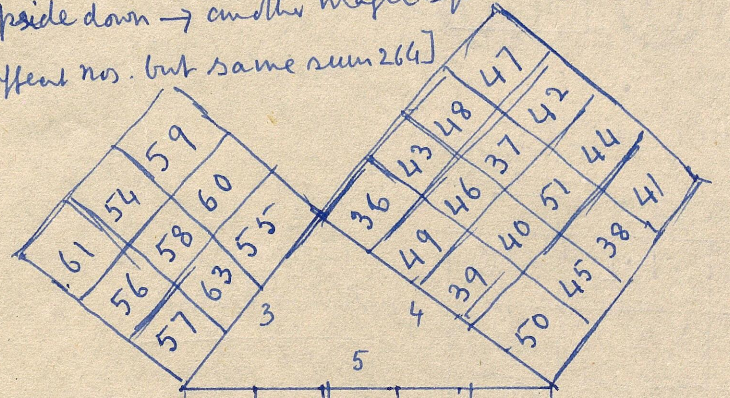
(i)

(upside down → another magic square with different nos. but same sum 264]

2	23	25	7	8
4	16	9	14	22
21	11	13	15	5
20	12	17	10	6
18	3	1	19	24

(ii)

magic square within a magic square.



16	22	28	34	74
33	73	20	21	27
25	26	32	72	19
71	18	24	30	31
29	35	70	17	23

(iii)

Magic square on hypotenuse

= sum of magic squares on the two sides

(All magic squares have total 174, and there is no duplication of numbers)

[P.T.O]

(4) Breaking of integral powers of 10 into factors containing no zeros:

8,589,934,592 × 116,415,321,826,934,814,453,125

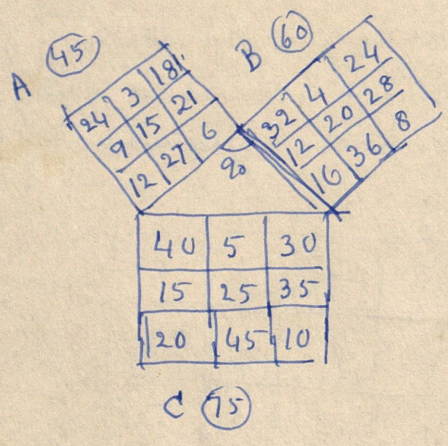
= 1,000,000,000,000,000,000,000,000,000,000,000,000

(5) Repunit & palindromic numbers:

These are numbers which consist entirely of a repetition of the digit 1. The question arises as to which of these is prime. If the number of 1's is even

Simplest 3 square is

8	1	6
3	5	7
4	9	2



(i) Square of any cell of A = sum of squares of corresp. cells of A & B.

(ii) Sum of squares of sum of any two cells of ~~A~~ = sum of squares of sums of corresp. cells of A & B

(iii) " any three

(iv) " any

(This might have been added to the three)

~~45~~, ~~27~~, 36
5, 3, 4
70, 42, 56
10, 6, 8
35, 21, 28
5, 3, 4
36, 48, 60
3, 4, 5
18, 24, 30
3, 4, 5

39, 52, 65
3, 4, 5

Magic multiplication square

1	12	10
15	2	4
8	5	3

not for diagonals

Calculation of π by Computer

Year	Dec. Places	Time (hours)
1949	2000	70
1954	3000	0.2
1958	10,000	33
(IBM) 1958	10,000	1.7
1959	16,000	4.3
1961	20,000	0.7
1961	100,000	9

the number is divisible by 11 and hence not prime. A search has been made for numbers with n 1's up to $n = 199$ have been studied and only ~~the~~ three of these have yielded primes viz.

11 , 1,111,111,111,111,111,111 and 11,111,111,111,111,111,111,111

with $n = 2, 19$ and 23. There are still 7 values viz. $n = 131, 139, 157, 163, 191$ and 197 which have not yielded their nature to the mathematician.

(~~5~~) ~~Repunit numbers lead to what are called~~ Squares of repunit numbers lead to what are called palindromic numbers ie those which are the same ~~when~~ either read forwards or backwards, for eg.

[Also repunit nos are palindromic]

$$11^2 = 121$$
$$111^2 = 12321$$
$$1111^2 = 1234321$$

[From Rouse Ball: 698896 is the same read backwards & forwards & is palindromic
$698896 = 836^2$ (but 836 is not a palindrom
Also $12345679 \times 99999999 = 111111111^2 = 12345678-87654321$]

only up to $(9 \text{ ones})^2 =$

(6) Prime-prime numbers – 73939133 (tested by Lehmer's tables of the first 10 million primes) (largest one known)

(~~8~~) Narcissus numbers: (7) Automorphic

Narcissus, according to Greek mythology, fell in love with his own image seen in a pool of water, and changed into the flower now called by his name. Thus the numbers considered here are those "in love with themselves" or those obtainable in some way by mathematically manipulating the digits of the numbers themselves. The following are some examples

(a) There are just four numbers after 1 which are the sums of the cubes of their digits viz:

$$153 = 1^3 + 5^3 + 3^3 \,;\; 370 = 3^3 + 7^3 + 0^3 \,;\; 371 = 3^3 + 7^3 + 1^3 \,;\; 407 = 4^3 + 0^3 + 7^3$$

(only 4)

(b) the perfect digital invariant number

$$4679307774 = 4^{10} + 6^{10} + 7^{10} + 9^{10} + 3^{10} + 0^{10} + 7^{10} + 7^{10} + 7^{10} + 4^{10}$$

(note number has 10 digits)

(c) the ~~visual~~ visible representation numbers:

(i) $165033 = 16^3 + 50^3 + 33^3$

(ii) $5882353 = 588^2 + 2353^2$ (a prime)

(iii) $40585 = 4! + 0! + 5! + 8! + 5!$

(iv) $4913 = (4+9+1+3)^3$.

(v) $598 = 5^1 + 9^2 + 8^3$, (vi) $2427 = 2^1 + 4^2 + 2^3 + 7^4$

(vii) $3435 = 3^3 + 4^4 + 3^3 + 5^5$ (The only one of its type known)

~~5. Some non-trivial numbers.~~

5. Fibonacci numbers.

From reverse of p. 2

(9) Goldbach's conjecture – Every ~~prime~~ even number = sum of 2 primes

(10) Twin primes – infinite in no.

(11) Problems re. distribn of primes : $(n!+2, \ldots, n!+n)$ i.e. large gaps

(12) Riemann-Zeta fn $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ $(s = \sigma + it$

Euler's product : $\zeta(s)(1-2^{-s}) = \frac{1}{1^s} + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \cdot$

(13) Number prodigies

(1) Colburn – Raising nos. of one digit to 10th power, of 2 digits to 8th power at age of n

Also finding sq. roots & cube roots

(2) Dase – mental multiplication of nos. of 8 digits each in 54 seconds
(German, born 1824)

"	20 "	6 minutes
	40 "	40 "
	100 "	8 3/4 hours.
" square root of no. of 100 "	"	52 minutes

(3) Shanks – π to 707 decimals

Computers in a 20,000th of the time required by prodigies – Shanks result found wrong after 500 decimal places

(14) Big No.

N	Number of digits in N
6-millionth prime	9
F_7	39
F_8	78
Googol	100
~~11213~~ 1000!	2568
$2^{11213}-1$	3376
(Googol)^{100}	10,000
F_{17}	40,000 (approx)
F_{36}	20 billions

23^{rd} Mp

Googolplex	10^{100}
F_{1945}	10^{600} (approx)
Skewes's number	$10^{10^{12}}$ $\rightarrow e^{e^{e^{79}}}$

This arises as follows:
Littlewood has shown that d changes sign infinitely often
How large must x be before d becomes negative for 1st time
Largest such x is Skewes's number S

Of $Li(x) = \int_0^x \frac{du}{\log u}$.

$d = Li(x) - N$ (N = no of primes $\leq x$)

$\frac{d}{N}$ = rel. error

(1) These are numbers ~~form~~ of the sequence given by

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots$$

named after Leonardo da Pisa, a thirteenth century mathematician called Fibonacci, who, as we have mentioned earlier, was to a great extent responsible for taking over the Hindu system of numeration from the East to the West. These Fibonacci ~~numbers~~ are such that, after the first two, every number in the sequence is the sum of the two previous numbers:

$$F_n = F_{n-1} + F_{n-2}$$

Consider now the simplest ~~of the simple~~ ~~all~~ possible simple continued fractions

$$1 + \frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+\cdots}$$

The convergents of the continued fraction are

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \ldots$$

are given by the ratios F_{n+1}/F_n for successive n. Unlike the continued fraction for $\sqrt{2}$ these convergents seem to change rather slowly, and it can be shown that of all ~~periodic~~ possible ~~fraction~~ continued fractions, the convergence of this is the slowest. Denoting ~~these~~ its value by x, we have $x = 1 + \frac{1}{x}$ or $x^2 - x - 1 = 0$ the positive root of which is given by

$$x = \frac{1+\sqrt{5}}{2} = 1.618\ldots$$

called the golden number and usually denoted by ϕ.

(2) ~~Fibona~~ Geometric properties associated with F_n:

(i) Start with two squares each of 1 unit side ~~on a side~~, adjoin to them a 2×2 square, then add to that ~~rectangle~~ picture a 3×3 square and so on, stopping at the 8×8 square. Then we get a rectangle of area 8×13 ie

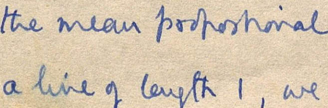
$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 8 \times 13$$

and in general

$$F_1^2 + F_2^2 + F_3^2 + \cdots + F_n^2 = F_n F_{n+1}$$

To this we can add a formula obtained by algebraic means viz $\sum_1^n F_n = F_{n+2} - 1$ [P.T.O]

(ii) Suppose we want to divide a line into two segments such that the longer ~~part~~ segment is the mean proportional between the whole line and the shorter ~~part~~ segment. Starting with a line of length 1, we require the length ~~of~~ x such that ~~$x(1-x)$~~ $x^2 = 1(1-x)$ ie

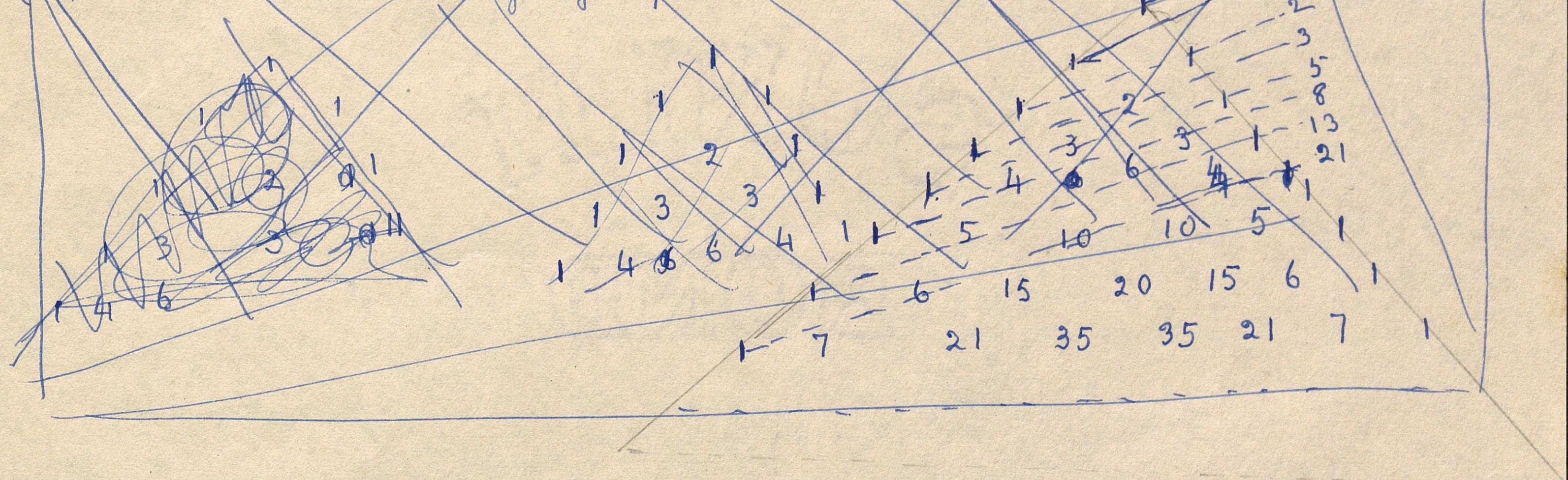


$x^2 + x - 1 = 0$, again a quadratic ~~root~~ eqn whose positive for $$\begin{aligned} F_1 &= F_3 - F_2 \\ F_2 &= F_4 - F_3 \\ &- - \\ F_{n-1} &= F_{n+1} - F_n \\ F_n &= F_{n+2} - F_{n+1} \end{aligned}$$

Adding up these $\underline{n}$ equations, we get ~~[illegible]~~ ΣF_n on the L.H.S, and on the right we have cancellation of the type known to the trade as telescoping leaving $F_{n+2} - F_2 = F_{n+2} - 1$.

is given by $x = \frac{\sqrt{5}-1}{2}$ and $\frac{1}{x} = \frac{1+\sqrt{5}}{2} = \phi$. Thus the ratio of the whole line 1 to x is ϕ and such a division is called the golden section, and ~~this incidentally, is~~ ~~(iii) the most~~ incidentally, the psychologists tell us that the rectangle with its sides in this ratio has the most pleasing shape; Most picture post-cards are approximately of this shape.

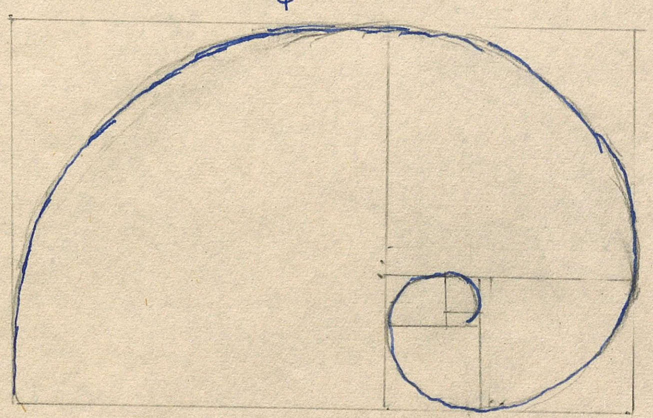
(iii) Consider the Pascal triangle giving the binomial numbers



(iii) Consider a rectangle with sides in ratio $\phi : 1$. If we take away the 1×1 square, the

ϕ

1.62 × 5
8.10



1

remaining rectangle has its sides in the ratio $\frac{1}{\phi-1} = \phi$ since $\phi^2 - \phi - 1 = 0$. This means that the new rectangle to the right is exactly similar to the original ~~tra~~ rectangle. We can therefore repeat the process indefinitely and it can be shown that the curve inscribed in each successive square is ~~the inscribed~~ a logarithmic spiral.

~~(iv)~~ [P.T.O]

(3) <u>Fibonacci sequence encountered in nature:</u>

(i) Suppose a tree grows according to the following not unrealistic formula. Each old branch (including the trunk) puts out one new branch per year; each new branch grows through the next year without branching, after which it qualifies as an old branch. It can be shown that this formula leads to the result that the number of branches after <u>n</u> years is F_n.

(ii) Consider the head of a sunflower. The seeds are distributed over the head in spirals which radiate from the centre of the head to the outside edge, unwinding in both clockwise and counterclockwise directions. Detailed study of these spirals has resulted in the following conclusions:

(a) The spirals are logarithmic spirals;

(b) The number of clockwise spirals and the number of counterclockwise spirals are successive terms of the Fibonacci series.

The normal head, about 5 to 6 inches in diameter, ~~with~~ in general have 34 and 55 spirals. ~~Similar~~ Smaller heads may have (21, 34) or (13, 21) combinations. Abnormally large heads have been grown with (89, 144) combinations. The same phenomenon can be observed, although perhaps not so easily, in the heads of other flowers, such as daisies and asters.

(iii) ~~Phenomenon of phyllotaxis~~ Let us examine the arrangement of leaves on the stalk of a plant. Suppose we fix our attention near the bottom of a stalk on which there is a single leaf at any one point. If we number this leaf 0 and count the leaves up the stalk until we come to one directly over the original one, the number we get is generally some term or other of the Fibonacci series. Again as we work up the stalk, let us count the number of times we revolve about it. This number too is generally a term of ~~these~~ the F-series. ~~This phenomenon is known as phyllotaxis.~~

If the number of revolutions is m and if the number of leaves is n, we ~~shall~~ call the arrangement an "m/n spiral" or a "m/n phyllotaxis". We have examples of different cases in different plants. If the revolutions are contra-clockwise i.e. if ~~the~~ we take the longer path in the revolution, we have in the case of the elm, the twig ~~is has~~ exhibits a 1/2 phyllotaxis, for beech leaves the ratio is 2/3, for the oak 3/5, and 5/8 for the poplar, and 8/13 for the almond, all the ratios being those of successive F-numbers. &

~~This type of spiral nature in growth is to also to be found in~~ [P.T.O]

(4) Fibonacci sequence in relation to art.

The psychologically pleasing $\phi : 1$ rectangle together with the associated logarithmic spiral is fundamental in what has come to be known as "dynamic symmetry". The development of this technique has been used in the design of Greek pottery, and has been extended to the domains of sculpture,

~~stells, horns of animals, human anatomy, nebulae etc. Hence it is~~

Math. Prodigies

(1)

painting, architectural decoration and even to furniture and type display.
The apparent ~~ad~~ aesthetic appeal of dynamical symmetry is perhaps due to the fact of ϕ being a sort of a universal constant of nature. The use of ϕ is to use the words of Goethe, like all the work of an artist, "a revelation working from within, a synthesis of world and mind". Perhaps this appeal is also due to the fact that ~~in man~~ a well-proportioned man himself bears the Fibonacci stamp. Thus, for eg, if the work of a renowned sculptor of the Golden Age of Greece ~~be~~ is used as a model, then ~~if the height of the~~ such a man ~~be~~ of 68 inches in height is built as follows: From the ground to his navel is 42 inches, from the navel to the crown of his head is 26", from the crown ~~of~~ the head to the line of his breasts is 16" and from the breasts to the navel is 10". The ratios of these measurements are

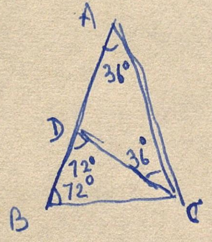
$$10/16 = 5/8\,,\ 16/26 = 8/13\,,\ 26/42 = 13/21 \text{ and } 42/68 = 21/34$$

again ratios of successive terms of an F_n!

(5) Other uses of F_n: We will mention briefly the use of these series in processes of feeding a computer, called the Fibonacci sorting. Also in ~~and~~ numerical analysis a method called the Fibonacci search for locating the maximum or minimum of a function is being used nowadays.

(6) Mystic significance of ϕ and the Fibonacci ~~Society~~ Association.

Consider an ~~equilateral~~ isosceles triangle with base angles 72° each & the vertical angle 36°.



~~Also~~ Bisect C & let the bisector meet AB in D. Then obviously triangles ABC and CBD are similar & hence

$$\frac{AB}{BC} = \frac{BC}{BD} \quad \text{ie} \quad \frac{AB}{AD} = \frac{AD}{BD} \quad (\because AD = BC)$$

or $AD^2 = AB \cdot BD$ which means that D divides AB in a golden section. ~~This~~ property has enables the construction of the 5-pointed

89) 100 (.0112359550506111~~8~~79775280898876404494382022~~02~~

471910

recurs after 46 decimal places

Meyer's Calculation on p. 89 is all wrong

100
89
110
89
210
178
320
267
530
445
850
801
490
445
450
445
500
445
550
534
160
89
710
623
870
801
690
623
670
623
470
445
250
178
720
712
800
712

400
356
440
356
840
801
390
356
340
267
730
712
180
178
200
178
220
178
420
356
640
623
170
89
810
801

90
89
100
89

880
801
790
712
780
712
680
623
570
534
360
356

4
8
1
3
356
5
7
6
2
6
2

star or the Mystic Pentagram known from times immemorial. It ~~was used~~ is one of the Yantras of the Upanishadic period of the Hindus known as the "Smara-Hara", the remover of desire, and according to ~~Vidya~~ the commentator Vidya-Vachaspati "Through the power of this Yantra man can conquer lust (Kama). The seeker who grasps its lesson remains well-guarded, so that no enemy may move him with the weapons of lust, anger, greed, delusion, pain or fear.. Its worshipper can go wherever he pleases in this and other worlds without meeting obstacles. It is truly an instrument of magic accomplishment"

Perhaps these remarks of Vidya-Vachaspati must have prompted a group of enthusiasts in the U.S.A. to form a Fibonacci Association in 1963, and begin publication of a quarterly journal principally devoted to research on Fibonacci numbers. ~~In~~ So far the journal has published nearly 2500 pages covering the results of investigation in this particular field. (Incidentally, it may be remarked that there is not a single Institution in India which subscribes to the Fibonacci Qrl ~~this journal~~) The Fibonacci Association has adopted for its symbol a device consisting of two twisted pentagrams..

One might well ask why people should confine themselves to a narrow field like this which is of no real interest ~~to~~ in the theory of numbers, and which occupies a ~~small~~ small corner of the intellectual landscape. But one might equally well retort by asking what does it matter if a study is not of earth-shaking importance as long as it stimulates the imagination. Even in this narrow field the path is endless, but rewards await on the way. One could do worse than follow the fascination of numbers.

O

1	2	3	4	5	6	7	8	9
I	II	III	IV	V	- -	- - -	- -	- - L, C - -
α	β	- .		α' - -			α'' - - -	
𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
𝍩	𝍪	𝍫	𝍬		𝍮	𝍯	𝍰	𝍱

$$\frac{\cancel{220}4}{5511}$$

Prodigios

$220 = 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110$

$$\begin{array}{r}\cancel{229} \\ 56 \\ \hline \cancel{284}\end{array}$$

$284 = 1 + 2 + 4 + 71 + 142$

$4 \times 71.$

$= \underline{\underline{220}}$

$y^2 = x^3 + 17$

$25, 81,$

64.

512

17

$2\,\overline{)\,529}\quad \cfrac{(2\cdot 3 \times 2)}{9}\; 6$

4

$43)\,129$

(1) Illegal cancellations – Add.

(i) $\dfrac{433\cancel{8}\cancel{9}\cancel{3}385}{443\cancel{8}\cancel{9}\cancel{3}3849} = \dfrac{43385}{443849}$ $\left[L.H.S = \dfrac{5 \times 10001 \times 8677}{10001 \times 443849} ; R.H.S = \dfrac{5 \times 8677}{443849} \right]$.

(ii) $\dfrac{12\cancel{0}12\cancel{0}12\cancel{0}288}{18\cancel{0}18\cancel{0}18\cancel{0}432} = \dfrac{121212288}{181818432}$. $\left[L.H.S = \dfrac{12 \times 10010010024}{18 \times 10010010024} ; R.H.S = \dfrac{12 \times 10101024}{18 \times 10101024} \right]$

(2) ~~Printers~~ fusion one viz $\dfrac{143185}{1701856} = \dfrac{1435}{17056}$ $\left[L.H.S = \dfrac{5 \times 7 \times 4091}{4 \times 8 \times 13 \times 4091} ; R.H.S = \dfrac{5 \times 7 \times 41}{4 \times 8 \times 13 \times 41} \right]$.

2

(2) Printers errors.

~~$2^5 \cdot \frac{25}{31} = \frac{25 \times 25}{31} = \frac{625}{31}$~~ ; $25\frac{25}{31} = \dfrac{775+25}{31}$

$= \dfrac{800}{31}$.

$2^5 \cdot \frac{25}{31} = 32 \cdot \frac{25}{31} = \frac{800}{31}$ ..

5

2 ;

5 . $21^2 \cdot 4\frac{9}{11} = 441 \times \frac{53}{11} = \frac{23373}{11}$; $2124\frac{9}{11} = \frac{23373}{11}$.

2

$$\begin{array}{r} 1323 \\ 2205 \\ \hline 23373 \end{array}$$

2

$73 \cdot 9 \cdot 42 = 657 \times 42$ | 3942×7 |

$$\begin{array}{r} 1314 \\ 2628 \\ \hline 27594 \end{array} \qquad \begin{array}{r} 3942 \times 7 \\ \hline 27594 \end{array}$$

$$\begin{array}{r} 121 \times 28 \\ \hline 968 \\ 242 \\ \hline 3388 \\ \hline 3 \end{array}$$

12 625 31 25

125 x

4 2 | 739 : (27)
4 | 4
4 47 | 339
32 9

3 | 1089 (33
9
63 | 189
189

$(10a+b)(10b+a)$

~~100~~ $101ab + 10(a^2+b^2)$

99×33

$$\begin{array}{r} 297 \\ 297 \\ \hline 3267 \end{array}$$

9 | 9801 (99
81
189 | 1701
1701

8