RELATIVITY AND UNIFIED FIELD THEORIES*

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There is a great deal of popular enthusiasm for any work of Einstein. Any little addition to his theories (and there have been many since 1919) receives a great popular ovation and is broadcast all over the world. Even the daily newspapers show interest in his scientific work. This ~~enviable~~ ~~pos~~ ~~pos~~ position of Einstein, unique among the living physicists of the age, is due to his theory of gravitation. ~~In this~~ The time-honoured concepts of space and time were revised and, what was more important, the Newtonian theory of gravitation, so well-established by facts (observed and predicted) of Astronomy, was displaced from its high position, and a more ~~sond~~ sound and logical

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theory was built by Einstein in ~~his~~ its place. In the following, we shall first describe (briefly) the ~~st~~ mathematical structure of this Einstein's theory of gravitation with its novel features. Then we shall feel the need to generalize it to ~~obt~~ obtain a theory of, what Einstein calls, the total field. After mentioning briefly the field equations of the new theory we shall see what expectations people have from this ultimate theory of the total field. And lastly we shall see how work is at present going on in different directions, to see if these high expectations are fulfilled by that mathematically beautiful and logically simple theory.

To describe gravitational situations, general relativity uses Riemannian geometry which is a generalization of the Euclidean geometry studied in ~~the~~ the schools. In this geometry, we begin with a fundamental tensor g_{ik} which is symmetric in the suffixes i and k

$$g_{ik} = g_{ki} .$$

This tensor, in its turn, generates the non-tensor quantities $\left\{ {l \atop ik} \right\}$ by the rule

$$\left\{ {l \atop ik} \right\} = \frac{1}{2} g^{lm} \left(g_{im,k} + g_{mk,i} - g_{ik,m} \right).$$

A comma indicates a differentiation. These curly brackets were first introduced by Christoffel in 1869. These ~~bracks~~ brackets, in their turn, generate the symmetric Einstein tensor

$$R_{ik} = - \left\{ {s \atop ik} \right\}_{,s} + \left\{ {t \atop is} \right\} \left\{ {s \atop tk} \right\} + \left\{ {s \atop is} \right\}_{,k} - \left\{ {s \atop ik} \right\} \left\{ {t \atop st} \right\}$$

The field equations of Einstein's theory of ~~gravit~~ gravitation, ~~are~~ given by him in 1917, are

$$R_{ik} = 0 .$$

[The brief mathematical intrusion (the non-mathematician-reader may excuse me for it) is hereby ended.]

The popular interest in this theory was aroused by the three so-called crucial tests. ~~Our present~~ We are interested in this theory for the generality of its concepts and for its physical impli-cations. For the first time field-concept embraced ~~s~~ gravitation. Gravitation laws are expressed as field laws. The field equations are invariant under a the group of continuous Gaussian ~~tras~~ ~~transforma~~ transformations. This group is more general than the Lorentz group of special theory of relativity. One consequence of the invariance under this more general group was that the equations were non-linear in the derivatives. This means that the sum of two solutions does not become a solution. This non linearity is a peculiar ~~fea~~ feature of general relativity. Its direct consequ-ence is that the inter-action between two particles is included in it. Linear theories will describe fields of individual particles and the fields of two particles will be simply added up without allowing for any interaction between them. The inter-action has to be postulated separately for linear field theories. Thus the non-linea~~ry~~rity of the general relativity field equations is of great advantage. In ~~an~~ a non-linear theory the equations of motion are included in the theory. The linear theory of Maxwell ~~can~~ does not give the motion of a charged particle in the field. The additional law of Lorentz has to be postulated to get the equations of motion ~~A~~ Newton's law of gravitation does not give motion. His laws of motion are to be taken as additional postulates. But the non-linear theory of gravitation obtained in general relativity, generates the equations of motion ~~for the field~~ from the field equations.

The mass particles are represented by the singularities in the field. The equations of motion are obtained as prescribing the motions of these singularities. In a purely gravitation theory. singularities will always remain at points occupied by particles. (Please ~~se~~ note that we ~~h~~ are dealing with a classical macroscopic non-quantum theory). This is because the closer we approach the particles the more pronounced will the forces other than gravitation become and so the gravitational field must break down near mass-particles. Only when we have a field theory which describes not only the gravitational field, but the total physical field (gravitational, electromagnetic, nuclear and what not!) shall we have a field free from singularities. ~~When~~ looked at from this point, it was ~~found~~ necessary to generalize the gravitational theory to get the total field theory. This leads us to what is generally known as the Unified Field Theory.

Since 1923 Einstein has been trying to generalize his theory of gravitation so as to ~~des~~ get a description of the total field. The invariance of his field equations with respect to the general continuous group must be maintained as it has led to non-linear equations which has a great advantage over linear equations in as much as the field equations themselves contain the interaction of particles. The most satisfactory generalization was reached by Einstein in 1946, and proceeding in quite a different manner, Schrödinger, in in 1947, also reached the same generalization. The new theory, or the theory of the total field is described by the non-symmetric tensor g_{ik}.

$$g_{ik} \neq g_{ki};$$

so that we can split it into symmetric and skew parts:

$$g_{ik} = a_{ik} + s_{ik}$$

$$a_{ik} = a_{ki} \quad \text{but} \quad s_{ik} = - s_{ki}.$$

Here again from g_{ik} we can derive the christoffel affinities Γ^{l}_{ik} ~~(as they are called)~~. These are also non-symmetric. They are connected with the tensor g_{ik} by the relations

$$g_{ik,l} - g_{is} \Gamma^{s}_{lk} - g_{sk} \Gamma^{s}_{il} = 0.$$

The Einstein tensor R_{ik} can again be derived. It is non-symmetric in i and k, so that it can be split up into symmetric and skew parts

$$R_{ik} = A_{ik} + S_{ik}$$

$$A_{ik} = A_{ki}, \quad S_{ik} = - S_{ki}.$$

The new field equations are

$$A_{ik} = 0,$$

$$S_{ik,l} + S_{kl,i} + S_{li,k} = 0$$

$$\left\{ g^{ik} \sqrt{-g} \right\}_{,k} = 0.$$

Einstein's idea in deriving these new ~~equat~~ equations ~~was~~ is to ~~see that they~~ get a representation of the total field. ~~Whether these~~ One criterion for deciding whether these equations represent the total field or not would be to see if they allow significant solutions which would be regular everywhere (i.e. which would have no singularities). Hence he is not interested in approximate solutions of these equations. He wants to examine rigorous, non-approximate solutions. After describing these equations in his autobiographical notes, he ends with these remarks: "What are the everywhere regular solutions of these equations?" One direction in which investigations are proceeding at present on this theory leads to a search for such "everywhere-regular solutions".

~~I am at present interested in the work in this particular direction.~~

Schrödinger's approach to these ~~&~~ problems is, however, different from that of Einstein. He does not believe that such rigorous solutions will reveal anything as regards the nature of ultimate particles. These particles are not dainty little toys which can be described by beutiful rigorous solutions. of complicated field equations. Even the rigorous solutions of general relativity, according to him, have disclosed only the ingenuity of the mathematicians who discovered them and nothing more. ~~St~~ Still, however, he feels ~~that~~ that ~~is~~ this theory is so much the simplest generalization of the theory of pure gravitation that it becomes imperative to study its consequ--ences. Again he feels that the unified field-theory will form a better foundation for the quantum-mechanical treatment of fields, which is at present based on a number of classical and pseudo-classical theories of independent origin, cemented to-gether by "inter-action terms". Macroscopic experience based on Maxwell's theory provides the guidance for the choice of field equations and inter-action terms of the quantum theory. The choice is still much arbitrary the only general restriction being Lorentz invariance. Is it too much to expect safer guidance from a unified field theory based from the ~~a~~ outset on the principle of general invariance? So he asks. He, therefore, proceeds to study the consequences of the theory without waiting to get a "rigorous" exact solution. This is another direction in which the work is proceeding at present.

In the end we shall just mention a third direction of work. This direction leads to investigations in pure mathematics. For example, Eisenhart has developed the geometrical theory of generalized Riemann spaces based on a non-symmetrical fundamental tensor g_{ik}. In our ~~own~~ country Professor S. N. Bose has taken up the algebraic problem of working with the sixtyfour ~~ab~~ simultaneous equations for the affinities Γ^{l}_{ik}. At the Lucknow sessions of the Science Congress (~~Januar~~ Jan. 1953) he took about two hours simply to mention ~~his~~ the main results of his investigation into the Modern Algebra part of these simultaneous equations.