Illegal cancellation (A)

27/6/74

27/6/74

Domoryad · p. 35 (1)

(1) $\frac{3544}{7531} = \frac{344}{731}$.

(2) $\frac{2666}{6665} = \frac{266}{665} = \frac{26}{65} = \frac{2}{5}$

29×29

261

58

841

(3) $\frac{143185}{17018560} = \frac{1435}{170560}$ 8

(4) $\frac{4251935345}{9181935185} = \frac{425345}{9185185}$

New Books · Developments in O.R Vol. 2.
(Gordon & Breach)
(1970)

Section 9 - Military applications

$2178 \times 4 = 8712$

$8712 - 2178$
$= 3 \times 2178 = 6524$

$$\begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

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(2)

$\frac{3544}{7531} = \frac{344}{731}$ $\quad 4\times86$ $\quad 24|11|74$ $\quad \frac{8\times43}{17\times43}$

from last missed

$\frac{4\times886}{17\times443}$ $\quad \frac{abcd}{ebfg} = \frac{acd}{efg}$. $\quad \frac{3548}{7556}$

Illegal Cancellation

$(1000a + 100b + 10c + d)(100e + 10f + g)$

$= (1000e + 100b + 10f + g)(100a + 10c + d)$

$(1000a + 100b + x)(100e + y)$

$= (1000e + 100b + y)(100a + x)$

$10c + d$
$-10f - g$

$100000ae + 1000ay + 10000be + 100by + 100ex + xy$

$= 100000ae + 1000ex + 10000ba + 100bx + 100ay + xy$

$10ay + 100be + by + ex = 10ex + 100ba + bx + ay$

$10(ay - ex) - (ay - ex) = 100b(a - e)$

$9ay - 9ex = 100b(a-e) + b(x-y)$

$= b\{100(a-e) + (x-y)\}$

$a = 3,\ x = 44,$
$e = 7,\ y = 31$

$b = 5$

$9(93 - 308) = 5\{-400 + 13\}$

$1935 = 1935$

387×5
9.215
387

308
93
215

$$b = \frac{9(ay - ex)}{100(a-e) + (x-y)}.$$

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$$b = \frac{9(ay - ex)}{100(a-e) + (x-y)}$$

$x = 37$ (5)

$y = 12$

$$\frac{9(37a - 12e)}{100(a-e) + 25} = \frac{9}{25}\left\{\frac{37a - 12e}{4(a-e)+1}\right\}$$

$$\frac{9}{25}\left(\frac{37a - 48}{4a - 15}\right)$$

$e = 4$,

$a = 1$, $\frac{9}{25}\left(\frac{-11}{-11}\right)$

$a = 2$, $\frac{9}{25}\left(\frac{26}{-7}\right)$

$a = 3$, $\frac{9}{25}\left(\frac{63}{-3}\right)$

$a = 4$, $\frac{9}{25}\left(\frac{100}{1}\right)$

$a = 5$

$a = 6$

$\frac{9}{25} \cdot \frac{174}{9}$

$b = \frac{9}{100} \cdot \frac{12a - 37e}{100(a-e)+25}$

$\frac{9}{100} \cdot \frac{12a - 37}{100a - 75}$

$e = 1$

wild goose chase:

$\frac{9(ay - ex)}{100(a-e) + (x-y)}$

$9(3y - 7x) = -2000 + 5x - 5y$.

$27y - 63x = -2000 + 5x - 5y$.

$68x - 32y = 2000$

$34x - 16y = 1000$

$17x - 8y = 500$

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$500 = 17x - 8y.$

$100b(e-a) = x(b+9e) - y(b+9a)$ (7)

$b=2$, $e=4$, $a=3$ $\quad 200 = 38x - 29y.$

$\frac{38}{29} = 1 + \frac{9}{19}$ $\quad 1 + \frac{1}{2+\frac{1}{9}}$

$= 1 + \frac{1}{19/9} = 1 + \frac{1}{2+\frac{1}{9}}$

$1 + \frac{1}{2+\frac{1}{1+\frac{1}{8+\frac{1}{1}}}}$

266

$= 1 + \frac{1}{2+\frac{1}{1+\frac{1}{8}}}$ $\quad 1+\frac{1}{2}$ $\quad \frac{3}{2}$

$1 + \frac{1}{2+\frac{9}{8}}$ $\quad 0 + \frac{1}{1}$

$1 + \frac{1}{25/8}$ $\quad \frac{0}{1}, \frac{1}{1}, \frac{3}{4}, \frac{13}{17}.$

$1 + \frac{8}{25}$

$= 1 + \frac{1}{2+\frac{1}{1+\frac{1}{2+\frac{9}{9}}}}$

$= 1 + \frac{9}{26}$

$0 + \frac{1}{1+\frac{1}{3}}$ $\quad 0 + \frac{1}{1+\frac{1}{3+\frac{1}{4}}}$

$0 + \frac{1}{1+\frac{1}{13/4}}$

$0 + \frac{1}{4/13}$ $\quad 38x - 29y = 1.$

$0 + \frac{1}{1+\frac{4}{13}}$

38	29
76	58
114	87
152	116
790	145
228	174
266	203
304	232
342	261

$\frac{1}{1+\frac{1}{1+\frac{1}{4}}}$ $\quad 38 \cdot 9 - 29 \cdot 9 = 5$

$38 \cdot 7 - 29 \cdot 7 = 5$

$38 \cdot 14 - 29 \cdot 8 = 40.$

$\frac{1}{1+\frac{1}{1+\frac{5}{7}}}$ $\quad \frac{12}{7}$

$\frac{1}{12/7}$ $\quad x = 10.$ $\quad y = 20$

$38 \cdot 10 - 29 \cdot 20 = 200.$

$38 \cdot 7 - 29 \cdot 9 = 150.$

$38 \cdot 4 - 29 \cdot 5 = 100.$

$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{2+\frac{1}{2}}}}}$

17	8
34	16
51	24
68	32
85	40
102	48
119	56
136	64
153	72

$b=5$, $e=7$, $a=3$ $\quad 500 \times 4 = x \cdot 68 - y \cdot 32$

$500 = 17x - 8y$

$17 \cdot 2 - 8 \cdot 3 = 10$

$17 \cdot 4 - 8 \cdot 6 = 20$

$17 \cdot 2 - 8 \cdot 3 = 10$

$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}}}}$

$\frac{17}{8} = 2 + \frac{1}{8}$ $\quad \frac{2}{1}, \frac{17}{8}$

$\frac{12}{7} = 1 + \frac{5}{7}$

$0 + \frac{1}{1+1+\frac{1}{1+\frac{1}{2}}}$

$1 + \frac{1}{1+\frac{1}{2}}$ $\quad 1+\frac{5}{7}$ $\quad \frac{7}{5}$ $\quad 1+\frac{2}{5}$ $\quad \frac{7}{12} = 0 + \frac{1}{12/7} = 0 + \frac{1}{1+1}$

$12 \cdot 1 - 7 \cdot 2 = x = 3, y = 5.$

$1+\frac{5}{7}$

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$17x - 8y = 500$ ~~1080~~

$t = -89$.

$29x - 13y$

633
623

~~1080~~ 1055

$\frac{13}{29} = 0 + \frac{1}{2+} \frac{1}{4+} \frac{1}{3}$

$\frac{29}{13}$ $\frac{3}{13}$ $0 + \frac{1}{2} + \frac{1}{1}$ $0 + \frac{1}{9/2}$

$\frac{13}{3}$ $\frac{2}{9}$.

$0 + \frac{1}{2+} \frac{1}{4+3}$

$0 + \frac{1}{2+\frac{3}{13}}$ $\frac{13}{29}$. $\frac{29}{13} = 2 + \frac{3}{13}$

$2 + \frac{1}{1+4}$ $2 + \frac{1}{1+} \frac{1}{4+} \frac{1}{3}$

$\frac{13}{5}$ $2 + \frac{1}{1+} \frac{1}{13/3}$ $1 + \frac{3}{13}$

$2 + \frac{1}{16/13}$ $2 +$ $2 + \frac{1}{1+} \frac{1}{4}$ $\frac{1}{4.5}$ $\frac{3}{13}$

$2 + \frac{1}{1+} \frac{1}{4+} \frac{1}{3}$ $2 + \frac{1}{1+3/13}$ $2 +$ $\frac{11}{}$

$2 + \frac{1}{4+} \frac{1}{3}$ $2 + \frac{1}{83/13}$ $2 + \frac{3}{13}$ $2 + \frac{1}{}$

$1 + \frac{1}{1+} \frac{1}{1+} \frac{1}{4+} \frac{1}{3}$ $4+3$ $1 + \frac{3}{13}$ $\frac{13}{29} = 0 + \frac{1}{2+} \frac{1}{4+} \frac{1}{3}$

$1 + \frac{16}{13}$ $1 + \frac{1}{1+} \frac{1}{4+} \frac{1}{3}$ $\frac{29}{13}$ $\frac{3}{13}$ $\frac{1}{4.3}$ $\frac{1}{13/3}$

$2 + \frac{1}{1+} \frac{1}{4}$ $1 + \frac{1}{1+\frac{3}{13}}$ $\frac{16}{13}$ 29.5 $\frac{13}{}$ $0+$

$2 + \frac{4}{5}$ $\frac{14}{5}$ $1 + \frac{13}{16}$ ~~1416~~ ~~49.5~~

187	88
~~204~~	96
221	104
238	112
255	120
272	128
289	136
306	144
323	152
340	160

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$b=5,\ a=3,\ e=7$

$$b = \frac{9(ay-ex)}{100(a-e)+x-y} \quad ; \quad 5 = \frac{9(3y-7x)}{-400+x-y} \qquad (11)$$

$$-2000 + 5x - 5y = 27y - 63x$$

$$68x - 32y = 2000$$

$$17x - 8y = 500 \quad ;$$

$$\frac{17}{8} = 2 + \frac{1}{8}$$

$17 \cdot 1 - 8 \cdot 2 = 1$

$$x = 1 + 8t \qquad 17x - 8y = 17 + 136t - 16 - 136t = 1$$

$$y = 2 + 17t$$

$$x = 500 + 8t$$
$$y = 1000 + 17t$$

$500 : 58$

456

44

$1000 \quad 9500 \quad 13$

$1054 \quad 488$

$464 \quad 12$

1

$t = -57, \quad x = 500 - 456 = 44$
$\quad\quad y = 1000 - 969 = 31$

$t = -56, \quad x = 500 - 448 = 52$
$\quad\quad y = 1000 - 952 = 48$

$\frac{3552}{7548}$

$t = -55, \quad x = 500 - 440 = 60$
$\quad\quad y = 1000 - 935 = 65$

$t = -54, \quad x = 500 - 432 = 68$
$\quad\quad y = 1000 - 918 = 82$

$t = -53, \quad x = 500 - 424 = 76$
$\quad\quad y = 1000 - 901 = 99$

$t = -52, \quad x = 500 - 416 = 84$
$\quad\quad y = 1000 - 884 = 116$ ✗

$4 \cdot 2 \quad 6$

918

1000
884
116

$37)629(17$
37
259
259

$$\frac{3552}{7548} = \frac{888}{1887} = \frac{296}{629} = \frac{8 \times 37}{17 \times 37} = \frac{8}{17}$$ ✓ Excellent

$$\frac{3552}{7548} = \frac{8 \times 444}{17 \times 444}$$

$$\frac{352}{748} = \frac{8 \times 44}{17 \times 44}$$

$$\frac{352}{748} = \frac{88}{187} = \frac{8}{17}$$ ✓

89) 1513 (17
89
623
623

6 (12)

$\frac{3560}{7565} = \frac{712}{1513} = \frac{8 \times 89}{17 \times 89} = \frac{8}{17}$

$\frac{360}{765} = \frac{72}{153} = \frac{8 \times 9}{17 \times 9} = \frac{8}{17}$ Exell

$\frac{8 \times 445}{17 \times 445} = \frac{8 \times 45}{17 \times 45}$

$\frac{3544}{7531} = 4 \times 886$

$\frac{3568}{7582} = \frac{368}{782} =$

43) 75

10

$\frac{3568}{7582} = \frac{8 \times 446}{17 \times 446} = 8/17$

$\frac{368}{782} = \frac{8 \times 46}{17 \times 46} = 8/17$

Excellent

$\frac{3576}{7599} = \frac{8 \times 447}{17 \times 447}$ another illegal cancellation

$\frac{8 \times 447}{17 \times 447} = \frac{8 \times 47}{17 \times 47}$

$\frac{376}{799} = \frac{8 \times 47}{17 \times 47}$ ✓

This appears hereditary illegal cancellation

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a = 3, e = 4, b = 2 ; 2 = 9 (3y - 4x)

29 (13)

$a=3,\ e=4,\ b=2;\quad 2 = \frac{9(3y-4x)}{-100+x-y}$ (13)

$-200+2x-2y = 27y-36x$

$38x - 29y = 200$

$\frac{38}{29} = 1 + \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{2}}}$

$1 + \frac{9}{13}$ $\frac{17}{13}$

$38\cdot 13 - 29\cdot 17$:

$494 - 493 = 1$

$x = 2600 + 29t$

$y = 3400 + 38t$

$t = -89.$

29×89 = 2581

$x = 19$

$y = 18$

$\frac{3219}{4218} = \frac{29\times 111}{38\times 111} = \frac{29}{38}$ ✓

$\frac{319}{418} = \frac{29\times 11}{38 \times 11}$ ✓

$t = -88 \longrightarrow x = 48,\ y = 56$

$\frac{3248}{4256} = \frac{29\times 112}{38\times 112} = \frac{29}{38}$ ✓

$\frac{348}{456} = \frac{29\times 12}{38\times 12} = \frac{29}{38}$ ✓

$t = -87 \to x = 77,\ y = 94$

$\frac{3277}{4294} = \frac{29\times 113}{38\times 113}$ ✓

$\frac{377}{494} = \frac{29\times 13}{38\times 13}$ ✓

Excellent idea

Side working: 38×89 = 3382; $29 \overline{)3219}(111$; 29×88 = 2552; 38×88 = 3344; 29×87 = 2523; 38×87 = 3306; $38\overline{)4228}(111$; 377; $\frac{29\cdot 13}{38\cdot 13}$

$b = \frac{9(ay - ex)}{100(a-e) + x - y}$ $\quad b = \frac{9(y-6x)}{-500 + x - y}$ (14)

(A)

$a = 1$
$e = 9$
$b = 6$

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$-3000 + 6x - 6y = 9y - 54x$

$60x - 15y = 3000$

$4x - y = 200$ $\quad 4x - y = 1$

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$\frac{4}{1} = 4 + \frac{1}{1}$. $\quad x = 4, y = 15.$ $\quad (4, 15)$ 3

800, 3000.

$x = 800 + t$

$y = 3000 + 4t$

~~3000~~ ~~2996~~ ~~250.~~ ~~02~~ 2

750. $\quad x = 53$ $\quad \frac{1653}{6612} \rightarrow \frac{16666653}{66666612}$ $\quad 6612$ $\quad 3000$ $\quad 2984$

$y = 12$

$t = -749$ ✓

$t = -748$ ✗

Question: $\quad \frac{1654}{6616} \rightarrow \frac{16666654}{66666616}$ (wrong) $\quad \begin{cases} t = -745 \\ x = 55, y = 20 \end{cases}$ 2980

$t = -746$ ✓ $\quad x = 54$ $\quad \frac{1655}{6620} \rightarrow \frac{16666655}{66666620}$ (correct)

$y = 16$

2988

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$t = -744 \rightarrow \begin{matrix} x = 56 \\ y = 24 \end{matrix} \rightarrow \frac{1656}{6624} \rightarrow \frac{16666656}{66666624}$ (correct)

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$t = -743 \rightarrow \frac{1657}{6628}$; $t = -742 \rightarrow \frac{1658}{6632}$ (Both Correct)

$t = -741 \rightarrow \frac{1659}{6636}$; $t = -740 \rightarrow \frac{1660}{6640}$ (Illegal + Illegal Cancellation) (15)

$t=-741 \to \frac{1659}{6636}$; $t=-740 \to \frac{1660}{6640}$ (i.e. 16/64 legal + illegal cancellation) (15)

Successive $\left[\frac{1661}{6644}; \frac{1662}{6648}; \frac{1663}{6652}; \frac{1664}{6656}\right\{\frac{1665}{6660}, \frac{1666}{6664}$ *

$\frac{1667}{6668}\}, \frac{1668}{6672}, \frac{1669}{6676}\Big]$, $\left(\frac{1670}{6680}\right)$, $\frac{1671}{6684}, \frac{1672}{6688}, \frac{1673}{6692}$

$\frac{1674}{6696}$, next are from $y=10c$ not tenable $\left[\frac{168674}{668696} = \frac{1666674}{6666696} = \frac{1}{4}$ formula]

For the cases in [ ] double cancellation of 6 is valid & in this the case (*) is the usual generalisation of 16/64

In cases ◯ double cancellation of 6 & 0, the former illegal and later ~~illegal~~ legal works. Also { } within [ ]

this way we could generalise to four digits in N and D, the middle 2 6's can be cancelled

cases $\frac{26}{65}, \frac{19}{95}$ and $\frac{49}{98}$.

Could there be cases like $\frac{abcd}{ebcf}$ i.e double cancellation 2 digits?

i.e $\frac{abcd}{ebcf} = \frac{ad}{ef}$ i.e $(1000a+100b+10c+d)(10e+f)$

$= 1000e+100b+10c+f)(10a+d)$

$1000be + 100bf + 100ce + 10cf + af + 100ae$

$= 1000ba + 100bd + 100ca + 10cd + 1000ed$

$1000b(e-a) + 100b(f-d) + 100c(e-a) + 10c(f-a) = 0.$

$1000af + 1000be + 100bf + 100ce + 10ef + 10de$

$= 1000ed + 1000ba + 100bd + 100ca + 10cd + 10af$

(16)

$1000(af-de) - 10(af-de) + 1000b(e-a)$

$+ 100b(f-d) + 100c(e-a) + 10c(f-d) = 0$

~~100(af-de)~~ $999(af-de) + (e-a)\{1000b+100c\}$

$\left(\frac{abcd}{ebcf}\right)$ $+ (f-d)\{100b+10c\} = 0.$

$990(af-de) + 100(e-a)(10b+c) + 10(f-d)(10b+c) = 0.$

$990(af-de) + x\{100(e-a) + 10(f-d)\}$

$990(af-de) + 10x\{10(e-a) + (f-d)\} = 0$

$$x = \frac{990(af-de)}{-10\{10(e-a)+(f-d)\}}$$

With $a = 1$, $\frac{1667}{6668}$, we have $a=1$, $e=6$, $d=7$, $f=8$. $\left[\frac{166667}{666668}\right.$ [illegible]?

For $e = 6$

$$\therefore x = \frac{990(8-42)}{-10\{50+1\}} = \frac{990 \times 34}{10 \times 49} = \frac{99 \times 342}{513} = 66$$ Correctly

Similarly for $\frac{1665}{6660}$ & $\frac{1666}{6664}$

& [illegible]

i.e. values of a, d, e, f to be so chosen as to make

x an integer of 2 digits. where

(17)

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x can integer of 2 digits. where

(B) $x = \dfrac{99(af-de)}{10(a-e)+(d-f)}$

$\begin{vmatrix} a & d \\ e & f \end{vmatrix} = \begin{vmatrix} a-e & d-f \\ e & f \end{vmatrix}$

~~$(a-e)(d-f)$~~ $a=1, d=5$

~~$af-(a-f)(d-e)$~~ $e=4, f=6$

$\dfrac{99(6-20)}{-40-1}$, $\dfrac{99\times14}{41}$

~~$a-e=1$~~

~~$e-a=1$~~ $\left.\begin{matrix} \\ f-d=1 \end{matrix}\right\}$ $x = 9(af-de)$.

~~$e=3, a=2$~~

~~$f=9, d=3$~~

~~5~~ 6

$e=7$

$\left.\begin{matrix} e-a=2 \\ f-d=2 \end{matrix}\right\}$ $x = \dfrac{9}{2}(af-de)$

$\left.\begin{matrix} e-a=3 \\ f-d=3 \end{matrix}\right\}$ $x = 3($~~$af-$~~$de-af$

$\left.\begin{matrix} a=1, e=4 \\ f=9, d=6 \end{matrix}\right\}$ $x = 3(24-9)$

$= 45$.

$9-24$.

15 $\dfrac{1456}{4459} = \dfrac{16\times91}{49\times91} \checkmark = \dfrac{16}{49}$

$\dfrac{91\times49}{819}$

364

4459

$\dfrac{16}{4}$

excellent apni

$\dfrac{145456}{445459} =$ $\dfrac{16\times9091}{49\times9091}$ Fantastic

All kinds of possible cases for eg.

$\left.\begin{matrix} e-a=1 \\ f-d=1 \end{matrix}\right\}$ $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ $x = 9(de-af)$

$\dfrac{1638}{2639} = \dfrac{18\times91}{29\times91}$. ✓

$\begin{pmatrix} a & d \\ e & f \end{pmatrix}$ $x = 9\times$

$\begin{pmatrix} 1 & 8 \\ 2 & 9 \end{pmatrix} \rightarrow x = 9(16-9) = 63$.

$\begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix} \rightarrow x = 27$, $\begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix} \rightarrow x = 36$

$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \rightarrow 18$ $\begin{pmatrix} 1 & 6 \\ 2 & 7 \end{pmatrix} \rightarrow x = 45$, $\begin{pmatrix} 1 & 7 \\ 2 & 8 \end{pmatrix} \rightarrow 54$. $\dfrac{1183}{2184}$, $\dfrac{1274}{2275}$,

$\dfrac{1365}{2366}$, $\dfrac{1456}{2457}$, $\dfrac{1547}{2548}$ &

Similarly $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} \rightarrow x = 9 \times \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} \rightarrow x = 18, \begin{pmatrix} 2 & 5 \\ 3 & 6 \end{pmatrix} \rightarrow x = 27$

Also $\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}, \begin{pmatrix} 3 & 6 \\ 4 & 7 \end{pmatrix}, \begin{pmatrix} 3 & 7 \\ 4 & 8 \end{pmatrix} \begin{pmatrix} 3 & 8 \\ 4 & 9 \end{pmatrix}$ Also $\begin{pmatrix} 2 & 7 \\ 3 & 8 \end{pmatrix}, \begin{pmatrix} 2 & 8 \\ 3 & 9 \end{pmatrix}$

$\begin{pmatrix} 4 & 6 \\ 5 & 7 \end{pmatrix}, \begin{pmatrix} 4 & 7 \\ 5 & 8 \end{pmatrix}, \begin{pmatrix} 4 & 8 \\ 5 & 9 \end{pmatrix}; \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix}, \begin{pmatrix} 5 & 8 \\ 6 & 9 \end{pmatrix}$

$\begin{pmatrix} 6 & 8 \\ 7 & 9 \end{pmatrix}$ i.e. with $\left.\begin{matrix} e-a=1 \\ f-d=1 \end{matrix}\right\}$ gives

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$6+5+4+3+2+1 = 21$ cases.

$\frac{6188}{7189} = \frac{68 \times 91}{79 \times 91}$ in every case

91 is common factor in N & D

For this case $\left.\begin{matrix} e-a=2 \\ f-d=2 \end{matrix}\right\}$ $x = \frac{9}{2}(de-af)$.

Question:

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \rightarrow x = 9 \times \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \rightarrow x = 18, \begin{pmatrix} 1 & 4 \\ 3 & 6 \end{pmatrix} \rightarrow x = 27$, etc.

$\begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix} \rightarrow x = 18$, etc. $\begin{pmatrix} 3 & 6 \\ 5 & 8 \end{pmatrix}$

Name & Address: $4+3+2+1 = 10$ cases.

$\left.\begin{matrix} e-a=3 \\ f-d=3 \end{matrix}\right\} \rightarrow x = 3(de-af)$ leaves $4+3+2+1 = 10$ cases

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$\left.\begin{matrix} e-a=4 \\ f-d=4 \end{matrix}\right\} \rightarrow x = \frac{99(de-af)}{44} = \frac{9(de-af)}{4}$

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \rightarrow x = 9 \times \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \rightarrow x = 18 \checkmark \begin{pmatrix} 1 & 4 \\ 5 & 8 \end{pmatrix} \rightarrow x = 27$

$f-d=4$ } $\Rightarrow$ $\frac{}{44} = 4$.

$\begin{pmatrix}1&2\\5&6\end{pmatrix} \to x=9$ ✗ $\begin{pmatrix}1&3\\5&7\end{pmatrix} \to x=18$ ✓ $\begin{pmatrix}1&4\\5&8\end{pmatrix} \to x=27$ ✓ (19)

$\begin{pmatrix}1&5\\5&9\end{pmatrix} \to x=36$] $\begin{pmatrix}2&3\\6&7\end{pmatrix}$ ✗ $\begin{pmatrix}2&4\\6&8\end{pmatrix}$ ✗ $\begin{pmatrix}2&5\\6&9\end{pmatrix}$ ✓

$\begin{pmatrix}3&4\\7&8\end{pmatrix}$ ie $3+2+1 = 6$ cases.

$e-a=5$ } $x = \frac{9}{5}(de-af)$
$f-d=5$ }

$\begin{pmatrix}1&2\\6&8\end{pmatrix}$ ✗ $\begin{pmatrix}1&3\\6&8\end{pmatrix} \to x=18$, $\begin{pmatrix}1&4\\6&9\end{pmatrix} \to x=27$

$\begin{pmatrix}2&3\\7&9\end{pmatrix} \to x=19$. $2+1 = 3$ cass

$e-a=6$ } $x = \frac{3}{2}(de-af)$
$f-d=6$ }

$\frac{99}{66}(de-af)$ $\begin{pmatrix}1&2\\7&8\end{pmatrix}$ ✗ $\begin{pmatrix}1&3\\7&9\end{pmatrix} \to x=18$ ie one case only

$\frac{1182}{7188} = \frac{13\times91}{79\times91}$

$\frac{1183}{7189} = \frac{13\times91}{79\times91}$ ✓ Also

1092 1
1
1183
79
711
7189

ie $21+15+10+6+3+1 = 56$ Cass Always 91 factor in N & D.

Where $10(e-a)+(f-d)$ is a multiple of 11 Cancelling with 99. But in this case, we get $\frac{1667}{6668}$ where $x=66$, we had $\begin{pmatrix}1&7\\6&8\end{pmatrix} \Delta x = \frac{99\times34}{5.1} = 66$

$e-a=5$
$f-d=1$ 5!

not necessarily coming under this above 56 cass.

Special cases when $b=c$ ie 2 digits equal cancelled

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$10x + y - 3$

This $\underline{x}$ is different from (B) p(17): $dx - ay = \lambda\mu$, $10x + y = 3\lambda$

$1 + \frac{1}{1/6}$ $\frac{7}{1} = 7 + \frac{1}{1}$

$x = +1, y = 10$ (21)

$-3\lambda, 10\lambda$

$7x - y = 34$, $10x + y = 51$ $\quad 7x - y = 1.$

$x = -3\lambda + t$

$t = 2, 21 - 10$

$t = 3, 28 - 10$

$x = 1, y = 6.$

$x = 1 + t$, $y = 6 + 7t$

$t = 4$

$42 - 12$

$x = 1 + 34t$, $y = 6 + 168t$

$t = 6, 49 -$

238

$7(1 + 34t) - (6 + 168t) =$

34×10

$x = 34 + t$, $y = 204 + 7t$ $\quad \checkmark t = 1, x = 35, y = 211.$

$dx - ay = 1.$ with $a = 1$: $dx - y = 1$

$t = -29, x = 5, y = 1$

$x = a, y = d - 1.$ $ad - a(d-1).$ $\quad x = 1, y = d - 1.$

$x = \lambda\mu + t$, $y = (d-1)\lambda\mu + dt$ $\quad 10\lambda\mu + 10t + (d-1)\lambda\mu + dt = 3\lambda$

$\lambda\mu(d + 9) + t(10 + d) = 3\lambda$ $\quad (t = -29)$

$\lambda = 17$, $\mu = 2$: $34(16) + t(17) = 3 \times 17.$ $\quad 32 + t = 3:$

$24 + t = 3$

$t = -21.$

$\lambda = 13$, $\mu = 2$, $d = 3$: $26 \times 12 + 13t = 3 \times 13$

$x = 26 - 21 = 5$, $y = 52 - 63 = -9$ $\quad$ ~~$50 - 9 = 39.$~~

$\lambda = 11$, $\mu = 1$, $d = 6$: $x = 11 + t$, $y = 55 + 6t$ $\quad t = -9.$ $y = 2$, $x = 2$ $\leftarrow \begin{pmatrix} 1 & 4 \\ 5 & 6 \end{pmatrix}$ ✓

$\begin{matrix} 1 & 6 \\ 3 & 7 \end{matrix}$ $\quad$ $\frac{99 \times 14}{423}$ $\quad$ 42. 14. $\frac{5336}{5336}$

$\frac{1334}{5336}$

$18 - 7 = 11.$ $\frac{133334}{533336} = \frac{14}{56}$ analogous to $\frac{1666}{6664}$ with scheme $\begin{pmatrix} 1 & 7 \\ 6 & 8 \end{pmatrix}$ ✓

$\frac{1667}{6668}$

Vide (B) on p (17) (22)

$\begin{pmatrix} 1 & 4 \\ 7 & 7 \end{pmatrix} \to x = \frac{99(21)}{63} = 33$. i.e. $\frac{1334}{7337} = \frac{14}{77}$ ✓. $\frac{2}{11}$ 667 7 7 7337

$\frac{1334}{7337} = \frac{2 \times 667}{11 \times 667} = \frac{2}{11}$

$\frac{14}{77} = \frac{2}{11}$. Different from the 56 cases & analogous for to $\frac{1334}{5336}$.

[$\frac{133334}{733337} = \frac{2 \times 66667}{11 \times 66667}$ Can be extended to any no. of 3's or 6's]

Again (1 / 3) So far we have taken a = 1. for all the cases considered, (B) on p. 17 is quite general: Take a = 2. (B) gives $x = \frac{99(de-af)}{10(e-a)+(f-d)}$

2 d / 5 d+6

Take denominator - 63

(3) i.e. $\begin{pmatrix} 2 & d \\ 8 & d+3 \end{pmatrix} \to 8d - 2d - 6 = 6d - 6$. has no factor 7 except for d = 8 (not admissible ; d+3 = 11)

5d - 2d - 12 = 3d - 12 = Let D = 51 $\begin{vmatrix} 2 & d \\ 7 & d+1 \end{vmatrix} \to 5d - 2$. gives no factor 17 for any d up to 8

i.e. 2 6 / 7 leading to

$x = \frac{99 \times 28}{51}$. D = 52 $\begin{vmatrix} 2 & d \\ 7 & d+2 \end{vmatrix} \to 5d - 4$

D must be a factor of 3

D = 42 → $\begin{vmatrix} 2 & D \\ 6 & D+2 \end{vmatrix}$ → 4D − 4

$\frac{2666668}{4666669} = \frac{4 \times 666667}{7 \times 666667} = \frac{4}{7}$ Similar

D = 12, $\begin{vmatrix} 2 & d \\ 3 & d+2 \end{vmatrix} \to d - 4$ ✗ D = 21, $\begin{vmatrix} 2 & d \\ 4 & d+1 \end{vmatrix} \to 2d - 2$, d = 8 → 14. $x = 99 \times 14^2 / 213 = 66$.

→ $\begin{vmatrix} 2 & 8 \\ 4 & 9 \end{vmatrix}$ ⇒ $\frac{2668}{4669} = \frac{4 \times 667}{7 \times 667}$; $\frac{28}{49} = \frac{4}{7}$ ✓ (23)

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$x = \frac{99 \times 44}{213} = 66.$

$\rightarrow \begin{vmatrix} 2 & 8 \\ 4 & 9 \end{vmatrix}$ ✗ $\frac{2668}{4669} = \frac{4 \times 667}{7 \times 667}$; $\frac{28}{49} = \frac{4}{7}$ ✓ (23)

$D = 24$, $\begin{vmatrix} 2 & d \\ 4 & d+4 \end{vmatrix} \rightarrow 2d - 6$ ✗ $D = 33 \rightarrow \begin{vmatrix} 2 & d \\ 5 & d+3 \end{vmatrix} = 3d - 6$ ✗

$D = 30$ $\begin{vmatrix} 2 & d \\ 5 & d \end{vmatrix} \rightarrow 3d$ ✗ $D = 14 \times 3 = 42$, $\begin{vmatrix} 2 & d \\ 6 & d+2 \end{vmatrix} = 4d - 4$ ✗

$D = 15 \times 3 = 45$, $\begin{vmatrix} 2 & d \\ 6 & d+5 \end{vmatrix} = 4d - 10$ ✗ $D = 51$, ✗

$D = 18 \times 3 = 54$, $\begin{vmatrix} 2 & d \\ 6 & d+4 \end{vmatrix} = 4d - 8$, $d = 3 \rightarrow 6\ 4$ 2 3 6 7

$x = \frac{99 \times 6}{54} = 11$.

$d = 5$ gives 12, $x = \frac{99 \times 12}{54} = \frac{99 \times 2}{9} = 22$

ie $\begin{pmatrix} 2 & 5 \\ 6 & 9 \end{pmatrix}$ $\frac{2225}{6229} = \frac{25 \times 89}{69 \times 89}$ 89 × 6

$\begin{vmatrix} 2 & d \\ 7 & d+4 \end{vmatrix} = 5d - 8$, $d = 4 \rightarrow 12 \rightarrow \begin{vmatrix} 2 & 4 \\ 7 & 8 \end{vmatrix}$

$x = 22$

$\frac{2224}{7228} = \frac{4 \times 556}{13 \times 556}$; $\frac{24}{78} = \frac{4 \times 6}{6 \times 13} = \frac{4}{13}$

222224 / 722228

$\frac{4 \times 55556}{13 \times 55556} = \frac{4}{13}$

Can be generalised to any no. of 2's in N & D.

$D = 72 \rightarrow \begin{vmatrix} 2 & d \\ 9 & d+2 \end{vmatrix} = 7d - 4$; $d = 4 \rightarrow 24$

$\frac{99 \times 24}{72} = 33$

$\begin{pmatrix} 2 & 4 \\ 9 & 6 \end{pmatrix}$

$\frac{2334}{9336} = \frac{2334}{4 \times 2334}$; $\frac{24}{96} = \frac{1}{4}$ ✓ quite a good one like 16/64.

2334 / 9336 = 1167 / 9336

$D = 27 \times 3 = 81$. ✗

$D = 75 = 25 \times 3$, $\begin{vmatrix} 2 & d \\ 9 & d+5 \end{vmatrix} = 7d - 10$, $d = 5$ gives 25 but $d + 5 = 10$ ✗

$D = 26 \times 3 = 78$

$\begin{vmatrix} 2 & d \\ 9 & d+8 \end{vmatrix} = 7d - 16$.

no further multiples 3 are eligible

✓ 3 cases with $a = 2$ [Can be generalised to any no. of 3's in N & D]

(24)

$a = 3$

$4 \times 3 = 12 \rightarrow \begin{vmatrix} 3 & d \\ 4 & d+2 \end{vmatrix} = d - 6$ ✗

~~18~~, 15, $\begin{vmatrix} 3 & d \\ 1 & d+5 \end{vmatrix} = d - 15$ ✗

$3 \times 7 = 21 \quad \begin{vmatrix} 3 & d \\ 5 & d+1 \end{vmatrix} = 2d - 3, \ d = 5 \rightarrow 7 \cdot u \quad \begin{vmatrix} 3 & 5 \\ 5 & 6 \end{vmatrix} \quad x = \frac{99 \times 7}{21} = 33$

$\rightarrow \frac{3335}{5336} = \frac{5 \times 667}{8 \times 667}$ ✓

$24 \rightarrow \begin{vmatrix} 3 & d \\ 5 & d+4 \end{vmatrix} = 2d - 12$ ✗, 27 ✗, 30 $\begin{vmatrix} 3 & d \\ 6 & d \end{vmatrix} = 3d$ ✗

$33 \quad \begin{vmatrix} 3 & d \\ 6 & d+3 \end{vmatrix} = 3d - 9$, ~~$d = 4 \rightarrow 3$, $x = \frac{99 \times 8}{33} = 9 \rightarrow \frac{3994}{6997} = \frac{34}{67}$~~

$d = 6 \rightarrow 9, \ x = \frac{99 \times 3}{33} = 27.$

$\rightarrow \frac{3276}{6279} = \frac{12 \times 273}{23 \times 273}, \ \frac{36}{69} = \frac{12}{23}$ ✓

$42 \rightarrow \begin{vmatrix} 3 & d \\ 7 & d+2 \end{vmatrix} = 4d - 6; \ d = 5 \rightarrow 14, \ x = \frac{99 \times 14}{42} = 33.$

$\frac{3335}{7337} = \frac{5 \times 667}{11 \times 667}, \ \frac{35}{77} = \frac{5}{11}$ ✓

$45 \rightarrow \begin{vmatrix} 4 & 9 \\ 7 & d+5 \end{vmatrix} = 3d - 15$ ✗, (48) ✗, $51 \rightarrow \begin{vmatrix} 5 & d \\ 8 & d+1 \end{vmatrix} = 3d - 5$ (no factor 17 for any d) ✗

$63 \rightarrow \begin{vmatrix} 6 & d \\ 9 & d+3 \end{vmatrix} = 3d - 18$ ✗ no further multiplying 3 value

~~72~~ → | i.e. only 2 cases

$a = 4 \quad D = 21; \ \begin{vmatrix} 4 & d \\ 6 & d+1 \end{vmatrix} = 2d - 4$ ✗ $\quad D = 33, \ \begin{vmatrix} 4 & d \\ 7 & d+3 \end{vmatrix} = 3d - 12$ ✗

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$D = 42 \quad \begin{vmatrix} 4 & 9 \\ 8 & d+2 \end{vmatrix} = 4d - 8$ ✗ $\quad D = 51; \ \begin{vmatrix} 4 & d \\ 9 & d+1 \end{vmatrix} = 5d - 4$ ✗ (25)

$D = 52$ $\begin{vmatrix} 4 & d \\ 8 & d+2 \end{vmatrix} = 4d - 8x$ || $D = 51$, $\begin{vmatrix} 4 & d \\ 9 & d+1 \end{vmatrix} = 5d - 4x$ (25)

$D = 12$, $\begin{vmatrix} 4 & d \\ 5 & d+2 \end{vmatrix} = d - 8X$ $D = 54$, $\begin{vmatrix} 4 & d \\ 9 & d+4 \end{vmatrix} = 5d - 16$

$D = 45$, $\begin{vmatrix} 4 & d \\ 8 & d+5 \end{vmatrix} = 4d - 20X$ No possible case

~~D = 04~~

a = 5 $D = 21$ $\begin{vmatrix} 5 & d \\ 7 & d+1 \end{vmatrix} = 2d - 5$ & $d = 6 \to 7$. $\begin{pmatrix} 5 & 6 \\ 7 & 7 \end{pmatrix}$

$x = \frac{99 \times 7}{21} = 33$

✓ $\frac{5336}{7337} = \frac{8 \times 667}{11 \times 667}$; $\frac{56}{77} = \frac{8}{11}$ – Compare this with the

[Cases of a = 3 viz $\frac{3335}{5336}$ and $\frac{3335}{7337}$.

" a = 2 viz $\frac{2668}{4669} = \frac{4 \times 667}{7 \times 667}$ & $\frac{2334}{9336} = \frac{2 \times 1167}{8 \times 1167} = \frac{24}{96} = \frac{1}{4}$

" a = 1 viz $\frac{1334}{7337} = \frac{2 \times 667}{7 \times 667}$

~~$\frac{2 \times 1334}{5 \times 1334}$~~ $\frac{1334}{5336} = \frac{2 \times 667}{8 \times 667} = \frac{14}{56} = \frac{1}{4}$]

~~$\frac{2668}{6660} = \frac{28}{60}$~~ $= \frac{7}{15}$

$D = 33$, $\begin{vmatrix} 5 & d \\ 8 & d+3 \end{vmatrix} = 3d - 15x$ ~~$D = 51$~~

$D = 42$, $\begin{vmatrix} 5 & d \\ 9 & d+2 \end{vmatrix} = 4d - 10$, $d = 6 \to 14$, $x = \frac{99 \times 14}{42} = 33$

6660
9338
208
$\frac{56}{98}$ $\frac{4}{7}$ 9 9

→ ✓ $\frac{5336}{9338} = \frac{4 \times 1334}{7 \times 1334} = \frac{8 \times 667}{14 \times 667} = \frac{56}{98} = \frac{8}{14} = 2/7$.

~~$\frac{2668}{6660}$~~ $\frac{28}{60}$ $\frac{7}{15}$

What about ✓ $\frac{1334}{9338} = \frac{2 \times 667}{14 \times 667}$. || We can continue 1334, 2668, ~~4002~~ 5336, 4669, 9338

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$a=2, x=20$, $\frac{22}{220}$ (trivial) no others

$b=3$, $x=\frac{300a}{9a+3}$, $a=1 \to x=25. \to \frac{13}{325}=\frac{1}{25}$

$a=8 \to x=\frac{300\times 8}{75}=32$

$a=3 \to$ trivial case, $\frac{33}{330}$

$\to \frac{83}{332}=\frac{8}{32}=\frac{1}{4}$.

$b=4$, $x=\frac{400a}{9a+4}$, $a=4 \to$ trivial case, no others

$b=5$, $x=\frac{500a}{9a+5}$, $a=5 \to$ trivial case, no others

$b=6$, $x=\frac{600a}{9a+6}$, $x=0 \to$ trivial case

$a=1 \to x=40 \to \frac{16}{640}=\frac{1}{40}$ (not too trivial & interesting ordn 16/64)

$a=2 \to \frac{1200}{24}=x \to \frac{26}{650}$ (extension of 26/65)

$b=7$, $x=\frac{700a}{9a+7}$, $a=7 \to$ trivial case

$a=2 \to x=\frac{1400}{25}=56 \to \frac{27}{756}=\frac{1}{28}$

27×28
216
54
756

$b=8$, $x=\frac{800a}{9a+8}$, $a=8 \to$ trivial case, no others

$b=9$, $x=\frac{900a}{9a+9}$, $a=1$, $x=50 \to \frac{19}{950}$ (extn of 19/95)

$a=4$, $x=\frac{3600}{45}=80 \to \frac{49}{980}$ (" 49/98)

$a=9$, $x \to$ trivial case.

Barring the trivial cases & the trivial four extensions, the only two new results are

$\frac{13}{325}=\frac{1}{25}$ and $\frac{83}{332}=\frac{8}{32}=\frac{1}{4}$ & $\frac{27}{756}=\frac{1}{28}$

specially the second

(28)

Case (ii) $\frac{abc}{dbe} = \frac{ac}{de}$; Case (iii) $\frac{abc}{cde} = \frac{ab}{de}$

(ii) $(100a+10b+c)(10d+e)$

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$= (100d+10b+e)(10a+c)$

$1000ad + 100ae + 100bd + 10be + 10cd + ce$

$= 1000ad + 100dc + 100ab + 10bc + 10ae + ce$

$10ae + 10bd + be + cd = 10cd + 10ab + ae + bc$

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$9ae + 10bd + be = 9cd + be + ae$

$9ae + 10bd + be = 9cd + 10ab + be$

$9(ae-cd) + 10b(d-a) + b(e-c) = 0$

$9(ae-cd) + b\{10(d-a) + (e-c)\}$

Question: (1) $b = \frac{9(ae-cd)}{10(a-d)+(c-e)}$

$\begin{pmatrix} a & c \\ d & e \end{pmatrix}$ If $a-d=x$, $c-e=y$, Denominator $= xy$

If $xy = 3$, $ae-cd = 2$ or 3 ...

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$(a-d)(c-e)$

$ac - ae - cd + de$

$xy = 12$, $d-a=1$, $e-c=2$

$b = \frac{9(2a-c)}{12} = \frac{3}{12}(2a-c)$

$\begin{pmatrix} a & c \\ a+1 & c+2 \end{pmatrix}$

$2a-c \rightarrow c-2a$.

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$2a = c-4$, $c-8$, $c-12$

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$2a - c = 4$ or 8 or 12

$2a = c-4$ or $c-8$ or $c-12$

$(a,c) = (3,2), (4,4), (5,6), (6,8) \rightarrow b=3$

$a = (5,2), (6,4), (7,6), (8,8) \rightarrow b=9$

$a = (7,2), (8,4), (9,6) \rightarrow b=9$

(29)

$= c+12$

$a = (5,2), (6,4), (7,6), (8,8) \rightarrow b = 6$

$a = (7,2), (8,4), (9,6) \rightarrow b = 9$

$(d, e) = (4,4), (5,6), (6,8)$ (X)

$a = (6,4), (7,6), (8,8)$ (X)

$a = (8,4), (X), (X)$

(29)

2×116

$2 \times 2 \times 7$

ie 7 cases corresponding to $xy = 12$. viz

$\frac{332}{434}, \frac{434}{536}, \frac{536}{638}$ ~~888~~ $b = \frac{9(cd - ae)}{10(d-a) + (e-c)}$ — (D')

$\frac{562}{664}, \frac{664}{766}, \frac{766}{868}$

$2a = c = 4$

$a = c - 8$

$= (c-12)$

$\frac{792}{894}$.

ie only possible case is $c = 8, a = 2$

$\therefore e = 10, d = 3$

$a - d = 1.$

$d = a - 1.$

$c - e = 2$

$e = c - 2$

No possible case!

$c = 2a+4, 2a+8, 2a+12$

$a = 2, c = 8$ only possible case

& back then $d = 1, e = 6, b = 3.$

$\begin{vmatrix} a & c \\ a-1 & c-2 \end{vmatrix} = c - 2a.$

$\frac{238}{136} = \frac{7 \times 119}{7 \times 68} = \frac{7}{4}$.

$\frac{28}{16} = \frac{7}{4}$.

$a = 1, c = 6.$

or 136

238

Start with (D') with denominator $= 12$, $d - a = 1$, $e - c = 2$.

and numerator $= 9\{c(a+1) - a(c+2)\}$

$= 9(c - 2a)$

$\therefore b = \frac{9(c-2a)}{12} = \frac{3}{4}(c-2a)$

$\therefore c - 2a = 4, 8$ or 12 giving $b = 3, 6$ or 9 with

$c - 2a = 4 \rightarrow (a = 1, c = 6); (a = 2, c = 8), -$

$= 4a + 30 -$

$c-2a=8 \rightarrow (a=1, c=9)$ ✗ $c-2a=12$ not possible

only case is $a=1, c=6$ giving $d=2, e=8$ and $b=3$

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ie number req'd $= \frac{136}{238} = \frac{8\times17}{14\times17} = \frac{4}{7}$; $\frac{16}{28} = \frac{4}{7}$ only cases

(a) $xy=15$, $d-a=1$, $e-c=5$, $cd-ae = c(a+1)-a(c+5)$
$= c-5a$

Title of the paper under discussion: $b = \frac{9(c-5a)}{15} = \frac{3}{5}(c-5a)$; $c-5a = 5, 10, 15$

$c-5a=5$, $c=5a+5$ not possible, now the other case. no use

$xy=21$, $d-a=2$, $e-c=1$, $cd-ae = c(a+2)-a(c+1)$
$= 2c-a$

$b = \frac{9(2c-a)}{21} = \frac{3}{7}(2c-a)$, ie $2c-a = 7, 14, 21$ with $b = 3, 6, 9$.

$2c = a+7 \rightarrow (1,4), (3,5), (5,6), (7,7), (9,8)$

$2c = a+14 \rightarrow (2,8), (4,9)$.

$2c = a+21 \rightarrow$ ✗

Question: Mensuration for $(a,e) = (1,4), (3,5), (5,6), (7,7), (9,8)$ (X)
$= (2,8), (4,9)$ (X)

ie 5 cases leading to

Name & Address: $\frac{134}{335}$ $\frac{335}{536}$ $\frac{536}{737}$ $\frac{737}{938}$

and $\frac{268}{469} = \frac{2}{5}, \frac{5}{8}, \frac{8}{11}, \frac{11}{14}$ and $\frac{4}{7}$. respectively.

$\frac{134}{938}$

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$xy=24$, $d-a=2$, $e-c=4$, $cd-ae = c(a+2)-a(c+4)$
$= 2c-4a$

$b = \frac{18(c-2a)}{24} = \frac{3}{4}(c-2a)$

ie $c-2a = 4, 8, 12$. $c = 2a+4$

$b = \frac{9\,18(c-2a)}{24} = \frac{3}{4}(c-2a)$ $= 2c-4a$

i.e. $c-2a = 4, 8, 12.$ $c = 2a+4$, $2a+8$, $2a+12$ (31)

(a, c)

Leavg $b = (1,6), (2,8)$; X, X and $(d,e) = (3,10), (4,12)$

no possible cases.

$xy = 27$ X, $xy = 33$, $d-a = 3$, $e-c = 3$, $cd-ae = c(a+3) - a(c+3)$
$= 3(c-a)$

$b = \frac{9\,27(c-a)}{33} = \frac{9}{11}(c-a)$, $c-a = 11$, which is impossible

i.e. no soln.

$xy = 42$, $d = a+4$, $e = c+2$, $cd-ae = c(a+4) - a(c+2)$
$= 4c-2a = 2(2c-a)$

$b = \frac{9\,18(2c-a)}{42} = \frac{3(2c-a)}{7}$ i.e. $2c-a = 7, 14, 21.$

$2c = a+7 \rightarrow (1,4), (3,5), (5,6)$ with $b=3$

$2c = a+14 \rightarrow (2,8)$, ... & $2c = a+21$ X

and Corresp. $(d,e) = (5,6), (7,7), (9,8)$ and (8,10)

Leavg $\frac{134}{536}, \frac{335}{737}, \frac{536}{938}$, all analogous to cases for $xy = 21$

$xy = 51$, $d = a+5$, $e = c+1$, $cd-ae = c(a+5) - a(c+1) = 5c-a.$

$b = \frac{9(5c-a)}{51} = \frac{3(5c-a)}{17}$, $5c-a = 17$, $5c = a+17$ [$b=3$

$\rightarrow (3,4), (8,5)$ & $(d,e) \rightarrow (8,5)$ only

i.e. $\frac{334}{835} = \frac{2\times 167}{5\times 167} = \frac{34}{85} = \frac{2}{5}$ (Different from previous case) 335 Feb

$xy = 54$, $d = a+5$, $e = c+4$, $cd-ae = c(a+5) - a(c+4) = 5c-4a$

$b = \frac{9}{54}(5c-4a) = \frac{1}{6}(5c-4a)$, $5c-4a = 6, 12, 18, \ldots$

$5c = 4a+6 \rightarrow (1,2), (6,6),$
$= 4a+12 \rightarrow (2,4), (7,8),$
$= 4a+18 \rightarrow (3,6),$
$= 4a+24 \rightarrow (4,8)$
$= 4a+30 \rightarrow$ X

$(d,e) \rightarrow (6,6)$, X | $b=1$
$\rightarrow (7,8)$ X | $b=2$
$\rightarrow$ X
$\rightarrow$ X.

334

167/1169

$\frac{167\times 2}{137\times 5}$

$\rightarrow \frac{112}{616} = \frac{7\times16}{7\times88} = \frac{2}{11}$, $\frac{12}{66} = \frac{2}{11}$ and $\frac{224}{728} = \frac{7\times32}{104\times7} = \frac{4}{13}$

$\frac{24}{78} = 4/13$

ie of a different type from previous results.

$xy = 63$ $\quad d = a+6,\ e = c+3,\ cd - ae = c(a+6) - a(c+3)$

$= 6c - 3a$

$b = \frac{9(6c-3a)}{63} = \frac{3(2c-a)}{7}$, $2c - a = 7, 14, 21$, $b = 3, 6, 9$.

$2c = a+7 \rightarrow (1,4), (3,5), (5,6), (7,7)$ | $(d,e) \rightarrow (7,7), (9,8) \times$

$= a+14 \rightarrow (2,8), (4,9)$. $\quad \rightarrow \times$

$= a+21 \rightarrow \times$

$\rightarrow \frac{134}{737}$; $\frac{335}{938}$ are of the cases obtained for $xy = 21$

$= \frac{2\times67}{11\times67} = \frac{14}{77}$; $\frac{5\times67}{14\times67} = \frac{35}{98} = \frac{5}{14}$

$xy = 66$, $cd - ae = c(a+6) - a(c+6) = 6(c-a)$

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$b = \frac{9\times6(c-a)}{6\cdot11}$ ie $c - a = 11$ $\quad c = a+11, \rightarrow (1,6) \times$

no cases.

$xy = 72$, $cd - ae = c(a+7) - a(c+2) = 7c - 2a$, $\quad \frac{9(7c-2a)}{728} = b$

$7c - 2a = 8, 16, \ldots$

$7c = 2a+8 \rightarrow (3,2) \times$

$= 2a+16 \rightarrow (6,4) \times$

$xy = 81$

$c(a+8) - a(c+1) \rightarrow 8c - a$

$\frac{9(8c-a)}{81} = \frac{8c-a}{9} = 9, 18, \ldots$

no cases.

$8c = a+9 \rightarrow (7,2) \times$

$= a+18 \rightarrow (6,3) \times$

$= a+27 \rightarrow (5,4) \times$

So in all we have in all 11 cases

67

2 67

24

QUESTIONNAIRE

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NON LINEAR BALLISTICS SEMINAR

(21-22 JUNE 74)

Name & Address:

Question:

Title of the paper under discussion:

Note: Please hand over to Chairman after the Technical Session.

Signature

Consecutive composites between
1 and 100

Σ

Illegal cancellations (B)

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$\frac{10}{3} + \frac{16}{6} = 10\left(\frac{1}{3} + \frac{1}{6}\right) = \frac{21}{6}$

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$\frac{17}{2} \times \frac{560}{6480}$

1,000, 999, 998, 997, ~~998, 997~~, 996, 995, 994,
993, 992, 991, 990, 989, 988, 987, 986, 985, 984,
983, 982, 981, 980, 979, 978, 977, 976, 975, 974,
973, 972, 971, 970, 969, 968, 967 (966, 965, 964, 963,
962, 961, 960, 959, 958, 957, 956, 955, 954) 953,
(31²) 7×137, (13 consecutive:
952, 951, 950, 949, 948, 947, 946, 945, 944, 943,
(23×41)
942, 941, 940, 939, 938, 937, 936, 935, 934, 933, 932,
931, 930, 929, 928, 927, 926, 925, 924, 923, 922,
921, 920, 919, 918, 917, 916, 915, 914, 913, 912, 911,
910, 909, 908, 907 (906, 905, 904, 903, 902, 901, 900,
(17×53)
29×31 899, 898, 897, 896, 895, 894, 893, 892, 891, 890,
(19 consecutive
889, 888), 887, 886, 885, 884, 883, 882, 881

880, 879, 878, 877, (876, 875, 874, 873, 872, 871,
870, 869, 868, 867, 866, 865, 861,) 863, 862, 861,
860, 859, 858, 857, 856, 855, 854, 853, (852, 851,
850, 849, 848, 847, 846, 845, 844, 843, 842, 841,
840) 839, 838, 837, 836, 835, 834, 833, 832,
831, 830, 829, 828, 827, 826, 825, 824, 823,
822, 821, 820, 819, 818, 817, 816, 815, 814, 813, 812,
811, 810, 809, 808, 807, 806, 805, 804, 803,
802, 801, 800, 799, 798, 797, 796, 795, 794,
793, 792, 791, 790, 789, 788, 787, (786 785, 784,
783, 782, 781, 780, 779, 778, 777, 776, 775, 774,)
773, 772, 771, 770, 769, 768, 767, 766, 765,
764, 763, 762, 761, 760, 759, 758, 757, 756,
755, 754, 753, 752, 751, 750, 749, 748, 747,
746, 745, 744, 743, 742, 741, 740, 739, 738,
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708, 707, 706, 705, 704, 703 (19), 702, 701, 700, 699,

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581, 580, 579, 578, 577, 576, 575, 574, 573,

572, 571, 570, 569, 568, 567, 566, 565, 564, 563

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435, 434, 433, 432, 431, 430, 429, 428, (5)
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383, 382, 381, 380, 379, 378, 377, 376, 375,
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(13)

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265, 264, 263, 262, 261, 260, 259, 258, 257, 256, 255,

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216, 215, 214, 213, 212, 211, (210, 209, 208, 207, 206,

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175, 174, 173, 172, 171, 170, 169, 168, 167, 166,

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35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22,

21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6,

5, 4, 3, 2, 1.

Between 1 & 1000

Case of 13 Consecutive Composites

(1) 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126.

(2) 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306.

(3) 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330.

(4) 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786.

(8)

(5) 840, 841, 842, 843, 844, 845, 846, 847, 848,
849, 850, 851, 852,

(6) 864, 865, 866, 867, 868, 869, 870, 871,
872, 873, 874, 875, 876,

(7) 954, 955, 956, 957, 958, 959, 960, 961, 962,
963, 964, 965, 966

one case of 17 consecutive composites viz

524, 525, 526, 527, 528, 529, 530, 531,
532, 533, 534, 535, 536, 537, 538, 539, 540.

one case of 19 consecutive composites viz

888, 889, 890, 891, 892, 893, 894, 895,
896, 897, 898, 899, ~~900, 901, 902, 903~~
900, 901, 902, 903, 904, 905, 906.

As we go beyond 1000 we obviously have bigger & bigger gaps between primes. The formula

$n!+2, n!+3, \ldots n!+(n-1)+n!+n$

being a set of n consecutive composites shows that the gaps increase as n increases, but this estimate of the gap falls n too far. Thus for a gap of 19 we need not go to $19!+2$, — (9)

for a gap g < 9 we need not go to 19! + 2, --- (9)

in fact we find it within 1000 < 2·6! << 7!

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~~1, 2, 3, 4, 5~~, 6

Illegal cancellations (B) — continued from (A), p. 32.

The case (iii) mentioned in (A), p. 28. ~~[illegible]~~

~~[illegible]~~ viz! $\frac{abc\!\!\!/}{c\!\!\!/de} = \frac{ab}{de}$

~~[illegible]~~: which gives:

$(100a + 10b + c)(10d + e) = (100c + 10d + e)(10a + b)$

$1000ad + 100ae + 100bd + 10be + 10cd + ce$

$= 1000ac + 100bc + 100ad + 10bd + 10ae + be$

$900ad + 90ae + 90bd + 9be + 10cd + ce$

$= 1000ac + 100bc$. If $x = 10a + b$, $y = 10d + e$,

~~$90ax +$~~ $90ay + 9by + cy = 100cx$.

ie ~~$9yx$~~ $9xy + cy = 100cx$.

$9xy = c(100x - y)$ ie $c = \frac{9xy}{100x - y}$ (E)

$9 = c\left(\frac{100}{y} - \frac{1}{x}\right)$ ~~(E)~~ → Some kind diophantine eqn?

$\frac{9}{c} = \frac{100}{y} - \frac{1}{x}$ — (E′) ie $c = 1, 3, 9.$ (not necessary see next page)

ie $\frac{100}{y} - \frac{1}{x} = 1$, or 3 or 9 where x & y are two digit nos.

$c=9 \rightarrow \frac{100}{y} - \frac{1}{x} = 1$. or $100x - y = xy$.

~~Linear 1st eq diophantine equation~~:

If $\frac{1}{y} = \alpha$, $\frac{1}{x} = \beta$ → indeterminate eqn

$100\alpha - \beta = 1$ $\alpha = 1, \beta = 99$.

(1, 99)

~~$\alpha = 1 + 100t$~~
~~$\beta = 99 + 100t$~~ ✓

$100\alpha - \beta = 100 + 100t$
$-99 = 100t$ $\frac{100}{y} =$

If $x = 10, 11, 12, 13, \ldots$

~~$x = 20$~~, $x = 19$.

$\frac{100}{y} = \frac{11}{10}, \frac{12}{11}, \frac{13}{12}, \frac{14}{13}, \ldots$ ~~$\frac{19}{20}$~~ $\frac{20}{19}$

$\frac{100}{y} = 1\frac{1}{19} = \frac{20}{19}$.

$y = \frac{1900}{20}$.

$y = \frac{1000}{11}, \frac{1100}{12}, \frac{1200}{13}, \ldots$

~~$\frac{1300}{16}$~~

~~y~~ $= \frac{1900}{20} = 95$ ✓, $\frac{2400}{25} = 96$ ✓, $\frac{4900}{50} = 98$ ✓

~~$\frac{3900}{40}$~~ appear to be the only 3 possible cases

Corresponding $x = 19, 24, 49$. leading to with $c = 9$

① ~~$\frac{199}{195}$~~ $\frac{199}{995}$, ~~$\frac{249}{996}$~~, $\frac{249}{996}$, $\frac{499}{998}$

1st & 3rd are trivial extensions of $\frac{19}{95}$ & $\frac{49}{98}$ which

the second $\frac{249}{996}$ ✓ appears new (interesting)

$c = 3 \rightarrow \frac{100}{y} - \frac{1}{x} = 3$.

$\frac{100}{y} = 3 + \frac{1}{x}$ $x = 10, 11, 12, \ldots 99$ (11)

$\frac{100}{y} = 3 + \frac{1}{10}, 3 + \frac{1}{11}, 3 + \frac{1}{12}, \ldots 3 + \frac{1}{99}$

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$\frac{100}{y} = \frac{31}{10}, \frac{34}{11}, \frac{37}{12}, \frac{40}{13}, \ldots$

$y = \frac{1000}{31}, \frac{1100}{34}, \ldots$ (31, 34, 37, 40, ... 76.

$\frac{3300}{100} = 33$ appears to

The only case corresponding to $x = 33$

$y = 33$

$c = 3$

(3/69) giving $\left(\frac{333}{333}\right)$ trivial:

$c = 1 \rightarrow \frac{100}{y} - \frac{1}{x} = 9 ; \frac{100}{y} = 9 + \frac{1}{x}$

$x = 10, 11, 12, \ldots$

$\frac{100}{y} = \frac{91}{10}, \frac{100}{11}, \frac{109}{12}, \ldots$

$y = \frac{1000}{91}, \frac{1100}{100}, \frac{1200}{109}, \ldots$

$c = 6$

$x = 26$

$y = 65$

$\frac{100}{65} - \frac{1}{26} = \frac{2000-50}{1300} = \frac{1950}{1300} = \frac{3}{2}$

118, 127, 136, 145, 154, 163, 172, 181, 190,

$x = 11, y = 11 \rightarrow \frac{111}{111}$ trivial

restriction of c to 1, 3, 9 not necessarily justified. for eg

$\frac{266}{665}$ fails $x = 26, y = 65, c = 6.$

$\Delta \frac{100}{65} - \frac{1}{26} = \frac{2000-50}{1300} = \frac{1950}{1300} = \frac{3}{2} = \frac{9}{6}$ //

So we have 1 Again $\frac{166}{664}$ where $x=16, y=64$.

Same $\frac{100}{64} - \frac{1}{64} = \frac{100-1}{64} = \frac{96}{64} = \frac{3}{2} = \frac{9}{6}$.

ie for $c=6$ we get the two known cases.

So we have to consider all values $x = 1, 2, \ldots 9$.

$\frac{9}{1} = \frac{100}{y} - \frac{1}{x}$. $x=11, y=11$ $\frac{100-1}{11} = \frac{99}{11} = 9$.

(E) is $10xy - xy = 100cx - cy$. $9xy = c(100x - y)$

$10x(10c-y) = y(c-x)$ $\frac{y}{10x} = \frac{10c-y}{c-x}$

$10x(y-10c) = y(x-c)$. $\frac{y}{y-10c} = \frac{10x}{x-c}$.

$\frac{y}{10c} = \frac{10x}{9x+c}$

$9xy + cy = 100cx$ $y = \frac{100xc}{9x+c}$

$y(9x+c)$ $c=1$, $y = \frac{100x}{9x+1}$. $\frac{600x}{9x+6}$

Possible values for x are 11, 21, 31, .. ,91 only

common case is $x=11 \rightarrow y=11$ ie trivial case

$c=2$, $y = \frac{200x}{9x+2}$. , $x = 12, 22, 32, 42, 52, 62, 72, 82, 92$

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x	12	22	32	42	52	62	72	82	92
$9x+2$	110	200	280	380	470	560	650	740	830

only case is $x=22 \rightarrow y=22$ ie $222/222$ trivial

$c = 3$, $y = \frac{300x}{9x+3} = \frac{100x}{3x+1}$

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$x =$	13	23	33	43	53	63	73	83	93
$3x+1 =$	40	70	100	130	160	190	220	250	280

only case is $x = 33$, $y = 33$ ie trivial $\frac{333}{333}$.

$c = 4$. $y = \frac{400x}{9x+4}$.

$x =$	14	24	34	44	54	64	74	84	94
$9x+4$	130	220	310	400	490	580	670	760	850

only case is $x = 44$ giving $y = \frac{400 \times 44}{400} = 44$ → trivial case.

$c = 5$, $y = \frac{500x}{9x+5}$

$x =$	15	25	35	45	55	65	75	85	95
$9x+5 =$	140	230	320	410	500	590	680	770	860

Again only case is $x = 55$, $y = 55$ trivial case.

$c = 6$, $y = \frac{600x}{9x+6} = \frac{200x}{3x+2}$

$x =$	16	26	36	46	56	66	76	86	96
$3x+2 =$	50	80	110	140	170	200	230	260	290

$x = 16 \rightarrow y = \frac{200 \times 16}{50} = 64. \rightarrow \frac{166}{664}$ well known case (16)

$x = 26 \rightarrow y = \frac{200 \times 2613}{8040} = 65 \rightarrow \frac{266}{665}$ another well known case.

$x = 66 \rightarrow y = 66 \rightarrow \frac{666}{666}$ found

$c = 7, \; y = \frac{700x}{9x+7} \rightarrow \frac{777}{777}$

$c = 8, \; y = \frac{800x}{9x+8}.$

$x =$	18	28	38	48	58	68	78	88	98
$9x+8 =$	170	260	350	440	530	620	710	800	890

(42) $\rightarrow \frac{888}{888}$ found

Left $c = 9, \; y = \frac{900x}{9x+9} = \frac{100x}{x+1}.$

$x =$	19	29	39	49	59	69	79	89	99
$x+1 =$	20	30	40	50	60	70	80	90	100

$x = 19 \rightarrow y = 95 \rightarrow \frac{199}{995}$ well known

$x = 49 \rightarrow y = 98 \rightarrow \frac{499}{998}$ — "

Here we further have the possible case $x = 24$

leading to $y = \frac{100 \times 24}{25} = 96$

i.e. $\frac{249}{996}$ new case on p. 10.

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$\frac{700x}{9x+7}$

$9x+8$ $x=13$, $9x+13=125$

19 $y = \frac{\cancel{800}^{16} \times 13}{\cancel{1255}} \times$

$9x+4$, 175, 265, 355, 435, 525, 615, 705, 795, 885, 975

12 205 295 385 475 565 655 745 835 925

215, 305, 395, 485 575, 665, 755, 845, 935

So case (iii) yields no really new result except $\frac{249}{996}$

— 0 —

$\left(\frac{abcd}{efgh}\right)$

Four digits as in N & D, we have considered

(i) Cancellation of 2nd digits ⇒ 2nd eqn of 1st order

(ii) " 2nd & 3rd digits & also case where these digits are the same in N & D.

We can further consider several other cases for eg

(iii) 2nd of N with 1st of D ✓

(iv) 2nd & 3rd of N with 1st & 2nd of D ✓

(v) 3rd of N with 1st or 2nd or 3rd of D (3 cases)

(vi) 4th of N with 1st ~~of D~~ or 2nd or 3rd of D (3 cases). ✓

(vii) 3rd & 4th of N with 1st & 2nd of D. ✓

Let us first consider only two cases of the above viz (vi) 1st case & (vii)

ie (vi) $\frac{abcd}{defg} = \frac{abc}{efg}$ and $\frac{abcd}{cdef} = \frac{ab}{ef}$ · vii

(vi) $(1000a + 100b + 10c + d)(100e + 10f + g) = (1000d + 100e + 10f + g)(100a + 10b + c)$

$\{10(100a + 10b + c) + d\}(100e + 10f + g)$
$= \{1000d + (100e + 10f + g)\}\{(100a + 10b + c)\}$

$x = abc,\ y = efg.$

$(10x + d)y = (1000d + y)x$

$9xy = d(1000x - y)$ ie $d = \frac{9xy}{1000x - y}$

$\frac{1}{d} = \frac{1000x - y}{9xy}$ $\quad 1000dx - dy = 9xy$

$y(9x + d) = 1000dx$

$y = \frac{1000dx}{9x + d}$ [x & y being 3-digit nos]

$d = 1,\ y = \frac{1000x}{9x+1}$, $x = 101, 111, 121, 131, \ldots$ 499

$9x + 1 = 1000 \rightarrow$ ~~x = 101~~ $x = 111$

$9x + 1 = 100x, = 200, 500,$
x x

It appears that these will lead only to the cases

$\frac{1666}{6664}, \frac{2666}{6665}, \frac{1999}{9995}, \frac{4999}{9998}$ and $\frac{2499}{9996}$

(vii) $\frac{abcd}{cdef} = \frac{ab}{ef}$. $(1000a + 100b + 10c + d)(10e + f)$
$= (1000c + 100d + 10e + f)(10a + b)$

$10a + b = x$
$10e + f = y$
$10c + d = z$

$(100x + z)y = (100z + y)x$

$99xy + yz = 100zx$

$z(100x - y) = 99xy$

$z = 99xy/(100x - y)$

$99xy = 2(100x - y)$

$y(99x + 2) = 100x \cdot 2$, $y = \dfrac{100x2}{99x+2}$.

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The above 4 cases with $z = 66$ & 99 must obviously come under this

$z = 66 \rightarrow y = \dfrac{100x \times 66}{99x + 66} = \dfrac{200x}{3x+2}$.

$x = 16 \rightarrow y = \dfrac{200 \times 16}{50} = 64$

$x = 26 \rightarrow y = \dfrac{200 \times 26}{80} = \dfrac{200 \times 13}{40} = 65$

$z = 99$, $y = \dfrac{100x \times 99}{99x + 99} = \dfrac{100x}{x+1}$.

$x = 19 \rightarrow y = 95$, $x = 49 \rightarrow y = 98$

$x = 24 \rightarrow y = 96$.

310
420
530
640
750
860
970
1080

$y = \dfrac{100x2}{99x+2}$

To find if there be any others try for z multiples of 9 and 11

$z = 18$, $y = \dfrac{100x \times 18}{99x+18} = \dfrac{200x}{11x+2}$, $x = 18 \rightarrow y = \dfrac{200 \times 18}{200} = 18$

Leads to $\dfrac{1818}{1818}$ trivial, $x = 28$, ..98 don't work.

$x = 13$, $11x + 2 = 145$, 255, 365, 475, 585, 695, 805, 915. ✗

$z = 11$, $\dfrac{100x}{99x+1} = y$, $x = 11 \rightarrow y = 11$ trivial case

$x = 21, 31, 51, \ldots 91$ ✗

$z = 22$, $y = \dfrac{200x}{99x+2}$ $99x + 2 = 50, 100, 200$ but $x = 22 = y$ found

$z = 27$, $y = \dfrac{100x \times 27}{99x+27} = \dfrac{300x}{11x+3}$. $11x + 3 = 300 \rightarrow x = 27 = y$ trivial

$11x + 3 = 150$ ✗

~~$x = 33$~~, $z = 33$, $y = \frac{100x \times 33}{99x+33} = \frac{100x}{3x+1}$.

$3x+1 = 25 \to x = 8$ ✗, $3x+1 = 50$ ✗, $3x+1 = 100 \to x = 33$ ✓ (trivial)

$z = 36$, $y = \frac{100x \times 36}{99x+36} = \frac{400x}{11x+4}$

$11x+4 = 100x$, $11x+4 = 200x$, $11x+4 = 400 \to x = y = 36$ (trivial)

$z = 44$, $y = \frac{100x \times 44}{99x+44} = \frac{400x}{9x+4}$.

$9x+4 = 100x$, $9x = 200x$, $9x+4 = 400 \to$ trivial

$z = 45$, $y = \frac{500x}{11x+5} \to$ only trivial case $\Rightarrow x = y = 45 = z$.

$z = 55$, $y = \frac{500x}{9x+5} \to$ again trivial.

$z = 54$, $y = \frac{600x}{11x+6} \to$ Do.

$z = 63$, $y = \frac{700x}{11x+7} \to$ Do

$z = 66 \to$ already covered $\to \frac{600x}{9x+6} = \frac{200x}{3x+2}$, $x = 16, 26$

$z = 72 \to y = \frac{800x}{11x+8} \to$ trivial

$z = 77 \to y = \frac{700x}{9x+7} \to$ trivial

$z = 88 \to y = \frac{800x}{9x+8} \to$ trivial

$z = 81 \to y = \frac{900x}{11x+9} \to$ trivial

$z = 90 \to y = \frac{1000x}{11x+10} \to$ trivial

$z = 99 \rightarrow y = \frac{1000x \times 99}{99x+99} = \frac{100x}{x+1}$, $x+1 = 20, 25, 50$

gives 3 cases already considered

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So the cases (vi) and (vii) have led to wildgoose chases, but confirmed the fact that the valid ones are really the trivial extensions, but $\left(\frac{2499}{996}\right)$ & $\left(\frac{24999}{9996}\right)$. Let us try the other cases mentioned on p. 15.

100,000

Case (iii) $\frac{abcd}{befg} = \frac{acd}{efg}$

$(1000a+100b+10c+d)(100e+10f+g)$

$= (1000b+100e+10f+g)(100a+10c+d)$.

$\left[\begin{array}{l} x = 10c+d \\ y = 100e+10f+g \end{array}\right.$

$(1000a+100b+x)\,y = (1000b+y)(100a+x)$

$1000ay + 100by + xy = 100,000ab + 1000bx + 100ay + xy$

$10ay + by = 1000ab + 10bx + ay$.

$9ay + by = 10b(100a+x)$

$y(9a+b) = 10b(100a+x)$.

$y = \frac{10b(100a+x)}{9a+b}$.

~~$x = 10efg$~~

~~$y = cd$~~

interchange x & y $\rightarrow$ $(9a+b)x - 10by = 1000ab$.

which gives an indeterminate equation of 1st order

$y(9a+b) - 10bx = 1000ab$

Simplest case $a=1, b=1$ leads to:

$10y - 10x = 100$ or $y - x = 10$

$y = 101$ } $y = 102$ } 103 | ... 109 | are the only
$x = 91$ } $x = 92$ } 93 | ... 99 |

x = possible solutions leading to

1191 / 1101 ; 1191 / 1101 → 191 / 101

1191 × 101 → 1191 / 1191 / 9009 / 119111 / 120291

$\frac{11}{99} = \frac{1}{9}$; 1199 / 1109

1101 | 1111 ; 1101 × 191 → 1101 / 101 / 9909 / 11011 / 210291

$y(9a+b) - 10bx = 1000ab$.

$a=1, b=1$ leads to $y - x = 100$.

$y = 110, x = 10 \rightarrow \frac{1110}{1110}$

$y = 111, x = 11 \rightarrow$ ⊢

$x = \alpha\beta$, $y = 1\alpha\beta$, $\frac{11\alpha\beta}{11\alpha\beta}$ trivial case.

~~$a=2, b=1$~~ ⟹ In fact $a = b$ leads to

$y(10a) - 10ax = 1000a^2$ i.e. $y - x = 100a$

all leading to trivial cases

$a=1, b=2 \rightarrow y(11) - 20x = 2000$ ——

$11y - 20x = 2000$ (an indeterminate eqn)

$\frac{11}{20} = 0 + \frac{1}{1+} \frac{1}{1+} \frac{1}{4+} \frac{1}{1+} \frac{1}{1}$ $\quad 0 + \frac{1}{1+} \frac{1}{1+} \frac{1}{4} + \frac{1}{1}$

$1 + \frac{9}{11}$ $\frac{11}{9}$ $0 + \frac{1}{1+} \frac{1}{1+} \frac{1}{4}$

$0 + \frac{1}{1+} \frac{1}{1+} \frac{1}{5} = 0 + \frac{1}{1+} \frac{1}{6/5}$

$0 + \frac{1}{1+} \frac{1}{4\frac{1}{5}}$ $5/9$.

$0 + \frac{1}{1 + 5/6} = 0 + \frac{6}{11}$

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$11 \cdot 11 - 6 \cdot 20 = 1$. General soln of eqn $11y - 20x = 1$ is

~~$y = 6 + 20t,\ x = 11 + 11t$~~

$y = 11 + 20t$
$x = 6 + 11t$

$11y - 20x = 121 + 220t$
$- 120 - 220t = 1$

∴ Gen soln of eqn $11y - 20x = 2000$ is

$y = 22000 + 20t$
$x = 12000 + 11t$ ✓

~~$11y - 20x = 242,000 - 240,000 = 2$~~

t is to be chosen that x is a 2 digit & y a 3-digit number

$\frac{12000}{11} = 1091$ ~~$1090 \times 11 = 11990$~~

Let $t = -1089$, $x = 12,000 - 11979 = 21$

$y = 22000 - 21780 = 220$ 3 digit

$\frac{22000}{20} = 1100$

~~$1099 \quad 22000 - 21980$~~

leaving to $\frac{1221}{2220} = \frac{121}{220} = \frac{11}{20}$.

$\frac{1221}{} = \frac{11 \times 111}{111 \times 20} = \frac{11}{20}$. Looks trivial, but really not so

$\frac{198}{360} = \frac{11 \times 18}{20 \times 18}$

Suppose $t = -1088$, then

$x = 12,000 - 11968 = 32$ $\quad y = 22000 - 21760 = 240$

21
32
43
54
65
76
87
98

leaving to $\frac{1232}{2240} = \frac{11 \times 112}{20 \times 112}$

$\frac{132}{240} = \frac{11 \times 12}{20 \times 12}$

For $a=1, b=2$, we have further cases taking

$t = -1087, -1086, -1085, -1084, -1083, -1082.$

giving $x = 43, 54, 65, 76, 87, 98$ resp

& corresponding $y = 260, 280, 300, 320, 340, 360,$

i.e. 8 cases viz (not necessary Sch. 24)

$\frac{1221}{2220}, \frac{1232}{2240}, \frac{1243}{2260}, \frac{1254}{2280}, \frac{1265}{2300}, \frac{1276}{2320},$

$\frac{1287}{2340}, \frac{1298}{2360}$. the factors in the N & D of the fractions being $\frac{11 \times 111}{20 \times 111}$,

i.e. cancelling factor being 111, 112, 113, — — 118. the illegally cancelled fractions having common factor in N & D as

11, 12, — — 18

& the fractions being = 11/20.

It looks therefore we have ~~a~~ all together for a and b

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$9 \times 9 = 81$ cases, for each case ~~a~~ an indeterminate equation and for each indeterminate eqn number of solutions (Is it 8 as above?) for x and y leading to

~~$81 \times 8 = 64$~~ 8 illegal cancellation examples:

Let us try 2 cases: (i) $a = 3$, $b = 5$

$$y(9a+b) - 10bx = 1000ab$$

$$32y - 50x = 15000 \text{ i.e. } 16y - 25x = 7500$$

$\frac{25}{16} \quad 1 + \frac{9}{16}$

$\frac{16}{25} = 0 + \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{1+} \frac{1}{2}$

$\frac{7}{9} \quad \frac{9}{7} \quad \frac{7}{2}$

$1 \quad 2 \quad 7$

$0 + \frac{1}{1+} \frac{1}{1+} \frac{3}{4}$ $\quad \frac{7}{11}$

$0 + \frac{1}{1+} \frac{4}{7}$

$0 + \frac{7}{11}$ $\quad 176$

$175 \quad 15 \quad 130$

$16 \cdot 11 - 25 \cdot 7 = 1.$

$y = 11 \times 7500 + 25t$

$x = 7 \times 7500 + 16t$ } general soln

$\frac{52500}{16} = 3345 \frac{16}{16} = 3281 \frac{4}{4}$

$t = -3345, \quad x = 52500 - 53520$

$= -3344 \quad 52500 - 53504$

$t = -3343, \quad 52500 - 53488$ ✗

$t = -3342 \quad 52500 - 53372$ ✗

$t = -3340 \quad 52500 - 440$ ✗

$t = -3320 \quad 20$

$t = -3300, \quad 52500 - 52800$ ✗

$t = -3281, \quad 52500 - 52496 \quad 04$

$t = -3280, \quad 52500 - 52480 \quad 20$

$t = -3280,\ x = 20,\ y = \begin{array}{r} 82500 \\ 82000 \\ \hline 500 \end{array}$

810

$\rightarrow \dfrac{3520}{5500} = \dfrac{352}{550} = \dfrac{32}{50} = \dfrac{16}{25}$; $\dfrac{320}{500} = \dfrac{16}{25}$.

Successively $t = -3279, -3278, -3277, -3276, -3275, -3274, -3273$.

$t = -3272 \rightarrow x = \begin{array}{r} 52500 \\ 52352 \\ \hline 148 \end{array}$ X $\qquad t = -3273 \rightarrow x = \begin{array}{r} 52500 \\ 52368 \\ \hline 132 \end{array}$ X

$t = -3274 \rightarrow \begin{array}{r} 52500 \\ 52384 \\ \hline 116 \end{array}$ X $\qquad t = -3275 \rightarrow \begin{array}{r} 52500 \\ 52400 \\ \hline 100 \end{array}$ X

$t = -3276 \rightarrow x = \begin{array}{r} 52500 \\ 52416 \\ \hline 84 \end{array} \quad \left| \quad y = \begin{array}{r} 82500 \\ 81900 \\ \hline 600 \end{array} \right.$ 12

9

There are not 8 cases as ~~was~~ guessed on p. 22, but only 5

corresponding to $t = -3280, -3279, -3278, -3277, -3276$

giving ~~$x = 20, 36, 52, 68, 84$~~ $x = 20, 36, 52, 68, 84$

and $y = 500, 525, 556, 575, 600$. respectively

Leading to $\dfrac{3520}{5500}, \dfrac{3536}{5525}, \dfrac{3552}{5550}, \dfrac{3568}{5575}, \dfrac{3584}{5600}$ respectively.

each reducing to $\dfrac{16}{25}$. $\dfrac{16 \times 221}{25 \times 221}$; $\dfrac{336}{525} = \dfrac{16 \times 21}{25 \times 21}$, etc.

22100/4

(ii) $a = 9, b = 7. \rightarrow 88y - 70x = 63000$ (25)

$44y - 35x = 31500 \qquad 44 \cdot 4 - 35 \cdot 5 = 1$

$y = 4 \times 31500 + 35t; \; x = 5 \times 31500 + 44t.$

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$t = -3579, -3578, -3577, -3576$ gives

$x = 24, 68,$ ~~60, 90~~ 2 cases only

$y = 735, 770,$ ~~805, 840~~ t

$\frac{9724}{7735}, \frac{9768}{7770}, \frac{9768}{7805}, \frac{9790}{7840}$ each $= 44/35$ as also.

The cancelled fractions

(iii) $a = 8, b = 9, \quad 81y - 90x = 72000$

$9y - 10x = 8000. \qquad 9 \cdot 9 - 10 \cdot 8 = 1$

~~$y = 80000 + 10t$~~ $\quad y = 72000 + 10t$

~~$x = 80000 + 9t$~~ $\quad x = 64000 + 9t.$

$t = -7110 \rightarrow x = 10, \; y = 900$

values for x are $10, 19, 28, 37, 46, 55, 64, 73, 82, 91$

" y $900, 910, 920, 930, 940, 950, 960, 970, 980, 990.$ } 10 values.

$\frac{8910}{9900}, \frac{8919}{9910}, \ldots \frac{8991}{9990}$ each equal to $9/10$

Also $\frac{810}{900}, \frac{819}{910}, \ldots \frac{891}{990}$ " $9/10$.

For each 81 cases of possible choices of a & b, we have not necessarily 8 but varying number of cases — ~~have to~~ not need to consider other cases.

Case (iv) of p. 15 viz $\frac{abcd}{bcef} = \frac{ad}{def}$

$(1000a + 100b + 10c + d)(10e + f) = (1000b + 100c + 10e + f)(10a + d)$

(a) $x = 10b + c$, $y = 10e + f$, $z = 10a + d$.

$(1000a + 10x + d)y = (100x + y)(10a + d)$

~~$10000ax + 10$~~ $1000ay + 10xy + dy = 1000ax + 100xd + 10ay + dy$

$100ay + xy = 100ax + 10dx + ay$

$99ay + xy = 10x(10a + d)$

$y = \frac{10x(10a + d)}{99a + x}$

(b) $a = 1$, ~~$x = 23$~~, ~~$y = \frac{230(10+d)}{99+23} = \frac{230(10+d)}{122}$~~

$y = \frac{10x(10+d)}{99+x}$

Now for $a = 1, 2, 3, \ldots 9$, we get

$y = \frac{10x(10+d)}{99+x}, \frac{10x(20+d)}{198+x}, \frac{10x(30+d)}{297+x}, \ldots, \frac{10x(90+d)}{891+x}$ — (A)

(c) $y = \frac{10x(10+d)}{99+x}$, $x = 11 \rightarrow y = \frac{110(10+d)}{110} = 10 + d$

$\therefore y$ may be $11, 12, 13, \ldots 19$.

ie $\frac{1111}{1111}, \frac{1112}{1112}, \ldots \frac{1119}{1119}$ all trivial ✗✗

$x = 21$, $y = \frac{210(10+d)}{120} = \frac{7(10+d)}{4}$

$\therefore d = 2, 6, \rightarrow y = 21, 28$ ie $\frac{1212}{2121}, \frac{1216}{2128}$ Each equal to $4/7$

$x = 26$, ~~Each~~ $y = \frac{260(10+d)}{125} = \frac{52(10+d)}{25}$ ✗

$x = 31$, $y = \frac{310(10+d)}{130} = \frac{31(10+d)}{13}$ ie $d = 3 \rightarrow y = 31$

ie $\frac{1313}{}$ 13×101

ie $\frac{1313}{3131} = \frac{13 \times 101}{31 \times 101} = \frac{13}{31}$

$x = 41,\ y = \frac{410(10+d)}{140} = \frac{41}{14}(10+d),\ d = 4 \rightarrow \frac{1414}{4141}$

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99
33
132

Simly $x = 51, 61, \ldots 91$ gives $\frac{1515}{5151}, \ldots \frac{1919}{9191}$

~~N.A. — Now first we can cancel 4th digit N with 1st digit D and 3rd ... 3rd ...~~

$x = 33,\ y = \frac{330(10+d)}{132} = \frac{10(10+d)}{4} = \frac{5}{2}(10+d)$

$d = 2, 4, 6, 8$, give $y = 30, 35, 40, 45$

$\frac{1332}{3330}, \frac{1334}{3335}, \frac{1336}{3340}, \frac{1338}{3345}$ each $= 2/5$

$x = 22,\ y = \frac{220(10+d)}{121} = \frac{20(10+d)}{11}$ $d = 1 \rightarrow y = 20$

$\rightarrow \frac{1221}{2220} = \frac{11 \times 111}{20 \times 111} = \frac{11}{20}$

Simlarly $x = 44, 55, 66, 77, 88, 99$ lead to ~~(with d =~~

$y = \frac{440(10+d)}{143}, \frac{550(10+d)}{154}, \frac{660(10+d)}{165}, \frac{770(10+d)}{176},$

$\frac{880(10+d)}{187}, \frac{990(10+d)}{198}$

ie $\frac{40(10+d)}{13}, \frac{50(10+d)}{14}, \frac{60(10+d)}{15}, \frac{70(10+d)}{16}, \frac{80(10+d)}{17}, \frac{90(10+d)}{18}$

$x = 44 \rightarrow y = 40$ with $d = 3 \rightarrow \frac{1443}{4440}$

$x = 55 \rightarrow y = \frac{25(10+d)}{7}$ with $d = 4 \rightarrow \frac{1554}{5550}$

$x = 66$, $y = \frac{660(10+d)}{15} = 4(10+d)$ ie 44, 48, 52, 56, 60, .. 76
with $d = 1, 2, \ldots$

giving $\frac{1661}{6644}, \frac{1662}{6648}, \frac{1663}{6652}, \ldots \frac{1669}{6676}$ ✓✓

The first remaining one of 16/64 ✓

$x = 77$, $y = \frac{70(10+d)}{16}$ & $d = 6 \to y = 70 \to \frac{1776}{7770}$ ✓

$x = 88$, $y = \frac{80(10+d)}{17}$ & $d = 7 \to y = 80 \to \frac{1887}{8886}$ ✓

$x = 99$, $y = \frac{90(10+d)}{18} = 5(10+d)$, $d = 1, 2, \ldots 9$

leading to $y = 55, 60, 65, 70, 75, 80, 85, 90, 95$

$\to \frac{1991}{9955}, \frac{1992}{9960}, \ldots \frac{1999}{9995}$ ✓✓

The first one remaining one of $\frac{19}{95}$ & the last one a direct Generalisation of the same.

So there should be some other values of x giving analogues of $\frac{26}{65}$ and $\frac{49}{98}$. for eg $\frac{2666}{6665}$. Perhaps this come under $a = 2$ and $a = 4$. So it appears we have a still closed no. of illegal cancellations under Case(iv) of p. 15

$x = 27$, $y = \frac{270(10+d)}{126} = \frac{45(10+d)}{21} = \frac{15(10+d)}{7}$

99 / 19 / 118 / 27 / 118

& $d = 4$ gives $y = 30$ ie ✓ $\frac{1274}{2730} = \frac{7 \times 182}{15 \times 182} = \frac{14}{30} = \frac{7}{15}$.

$x = 57$, $y = \frac{570(10+d)}{156}$ ✗

Data seems $x = 12, 22, 32, 42, 52, 62, 72, 82, 92$

Take series $x = 12, 22, 32, 42, 52, 62, 72, 82, 92$ (29)

$x = 72 \rightarrow y = \frac{720(10+d)}{171} = \frac{80(10+d)}{19}$ $d = 9 \rightarrow y = 80$

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86

ie $\frac{1729}{7280} = \frac{19 \times 91}{80 \times 91} = \frac{19}{80}$ — Cf. with $\frac{1274}{2730}$

192

Take series 13, 23, 33, 43, 53, 63, 73, 83, 93

$x = 63 \rightarrow y = \frac{630(10+d)}{162} = \frac{70(10+d)}{18} = \frac{35(10+d)}{9}$

and $d = 8$ gives $y = 70 \rightarrow \frac{1638}{6370} = \frac{9 \times 182}{35 \times 182} = \frac{18}{70} = \frac{9}{35}$

281 Series 14, 24, 34, 44, 54, 64, 74, 84, 94

184

Series 15, 25, 35, 45, 55, 65, 75, 85, 95

$x = 45 \rightarrow y = \frac{450(10+d)}{144} = \frac{50(10+d)}{16}$ & $d = 6 \rightarrow y = 50$

ie $\frac{1456}{4550} = \frac{8 \times 182}{25 \times 182} = \frac{16}{50} = \frac{8}{25}$

158
195
39

205 Series 16, 26, 36, 46, 56, 66, 76, 86, 96.

$x = 36, y = \frac{360(10+d)}{135} = \frac{40(10+d)}{15}$ & $d = 5 \rightarrow y = 40$

$\rightarrow \frac{1365}{3640} = \frac{3 \times 455}{8 \times 445} = \frac{15}{40} = \frac{3}{8}$

196

Series 17, 27, 37, 47, 57, 67, 77, 87, 97

49

Seris 18, 28, 38, 48, 58, 68, 78, 88, 98
✓ ✗ ✗ ✗ ✗ ✗ ✗ ✓ ✗

$x = 18,\ y = \frac{180(10+d)}{117} = \frac{20(10+d)}{13}$ & $d = 3 \to y = 20$

$\to \frac{1183}{1820} = \frac{13 \times 91}{20 \times 91} = \frac{13}{20}$

Seris 19, 29, 39, 49, 59, 69, 79, 89, 99
✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✓

$x = 39 \to y = \frac{390(10+d)}{138}$

ie we have considered all possible cases for $a = 1$

(2) $a = 2$, $x = 20, 30, 40, 50, 60, 70, 80, 90$

$y = \frac{10x(20+d)}{198+x}$

$x = 90,\ y = \frac{900(20+d)}{288} = \frac{100(20+d)}{32} = \frac{25(20+d)}{8}$

& $d = 4$ gives $y = 75$ ie $\frac{2904}{9075} = \frac{8 \times 363}{25 \times 363} = \frac{24}{75}$

(Good one which ~~does~~ D ending with 0 as in all cases of $a = 1$)

$x = 11, 21, 31, 41, 51, 61, 71, 81, 91$
✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗

$x = 81 \to y = 8$ ~~$x = 11 \to y = \frac{110(20+d)}{209} = \frac{10(20+d)}{19}$~~ ✗

$x = 81,\ y = \frac{810(20+d)}{279} = \frac{910(20+d)}{31}$ ✗

$x = 12, 22, 32, 42, 52, 62, 72, 82, 92$
✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

$x = 12 \to y = \frac{120(20+d)}{210} = \frac{4}{7}(20+d)$

$d = 3 \to y = 12,\ d = 8 \to y = 16$

$\frac{2121}{2128}$ & (31)

$d = 4 \rightarrow y = 12$, $d = 8 \rightarrow y = 16$

$\frac{2121}{1212}$ & $\frac{2128}{1216}$ (31)

$x = 22 \rightarrow y = \frac{220(20+d)}{220} = 20 + d$, $d = 1, 2, \ldots 9$ (9 cases)

$y = 21, 22, \ldots 29$

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$\frac{2221}{2211}, \frac{2222}{2222}, \frac{2223}{2223}, \ldots \frac{2229}{2229}$ (all trivial)

$x = 32$, $y = \frac{320(20+d)}{230} = \frac{32}{23}(20+d)$

$d = 3 \rightarrow y = 32 \rightarrow \frac{2323}{3232} = \frac{23 \times 101}{32 \times 101} = \frac{23}{32}$

interesting in that D is reverse of N.

$x = 42$, $y = \frac{420(20+d)}{240} = \frac{21(20+d)}{12} = \frac{7(20+d)}{4}$

$d = 4 \rightarrow y = 42$ & $d = 8 \rightarrow y = 49$

$\rightarrow \frac{2424}{4242}$ & $\frac{2428}{4249}$

(D reverse of N)

$x = 52$ $y = \frac{520(20+d)}{250} = \frac{52}{25}(20+d)$

$d = 5 \rightarrow y = 52 \rightarrow \frac{2525}{5252}$

$x = 62$, $y = \frac{31}{13}(20+d)$ $d = 6$, $y = 62 \rightarrow \frac{2626}{6262}$

$x = 72$, $y = \frac{720(20+d)}{270} = \frac{8(20+d)}{3}$

$d = 1, 4, 7 \rightarrow y = 56, 64, 72$

$\rightarrow \frac{2721}{7256}, \frac{2724}{7264}, \frac{2727}{7272}$

$3/8 \quad 3/8 \quad 3/8$

$x=82, \; y = \frac{820(20+d)}{280} = \frac{82(20+d)}{28}, \; d=8 \rightarrow \frac{2828}{8282}$ $\quad y=82$

$x=92, \; y = \frac{920(20+d)}{290} \rightarrow \frac{2929}{9292}$

Sens $x = 13, 23, 33, 43, 53, 63, 73, 83, 93.$

$\quad \times \quad \times \quad \checkmark \quad \times \quad \times \quad \checkmark \quad \times \quad \times \quad \times$

$x=33, \; y = \frac{330(20+d)}{231} = \frac{30(20+d)}{21} = \frac{10(20+d)}{7}$

$d = 1, 8 \rightarrow y = 30, 40 \rightarrow \frac{2331}{3330}, \frac{2338}{3340}$

$x=63 \rightarrow y = \frac{630(20+d)}{261} = \frac{70(20+d)}{29}$

$d=9 \rightarrow y = 70 \rightarrow \frac{2639}{6370} = \frac{29\times 91}{70\times 91} = \frac{29}{70}$

Sens, $x = 14, 24, 34, 44, 54, 64, 74, 84, 94$

$\quad \times \quad \times \quad \times \quad \checkmark \quad \checkmark \quad \times \quad \times \quad \times \quad \times$

$x=34, \; y = \frac{340(20+d)}{232} = \frac{85(20+d)}{58}$

$x=44, \; y = \frac{440(20+d)}{242} = \frac{40(20+d)}{22}, \; d=2 \rightarrow y=40$

$\rightarrow \frac{2442}{4440}$

$x=54, \; y = \frac{540(20+d)}{252} = \frac{30(20+d)}{14} = \frac{15(20+d)}{7}$

$d=8 \rightarrow y=60 \rightarrow \frac{2548}{5460} = \frac{182\times 14}{390\times 14} = \frac{91}{195} = \frac{7}{15}.$

$\frac{28}{60} = \frac{7}{15}$

$x=74, \; y = \frac{740(20+d)}{273} = \frac{185(20+d)}{68}$

$x = 74,\ y = \frac{740(20+d)}{272} = \frac{185}{68}(20+d)$ ✗ (33)

ie 50 cases examine & still 50 more left & similarly for

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each of the cases $a = 3, \ldots 9$ we have to examine 10 nos ie 700 cases — No point in this. Let us take one each for $a = 3, -9$.

Now for $a = 2$, $x = 66$ gives $y = \frac{660(60+a)}{594+x}$

$\frac{660(60+d)}{660} = 60+d$.

$y = 61, 62, \ldots 69$

$y = \frac{660(20+d)}{264} = \frac{30(20+d)}{12}$

$= \frac{5}{2}(20+d)$, $d = 2, 4, 6, 8$

$\rightarrow y = 55, 60, 65, 70$

ie $\frac{2662}{6655}, \frac{2664}{6660}, \frac{2666}{6665}, \frac{2668}{6670}$ each $= 2/5$

The third one is the general solution of $26/65$

(21) $a = 3$, $y = \frac{10x(30+d)}{297+x}$ — (A), p. 26.

Take the case $x = 18, 28, 38, \ldots 98, 63$,

$y = \frac{63(30+d)}{378} = \frac{21(30+d)}{126}$

$x = 18$, $\frac{264}{315} \frac{+80(30+d)}{7}$

$d = 5$, $y = 20$

$4(30+d)/7$

(3) a = 3 $\quad y = \frac{10x(30+d)}{297+x}$

342

~~39~~3

$x = 45,\ y = \frac{450(30+d)}{342} = \frac{50}{38}(30+d)$

$= \frac{25}{19}(30+d),\ d = 8,\ y = 50.$

$\frac{3458}{4550} = \frac{19 \times 182}{25 \times 182} = \frac{38}{50} = \frac{19}{25}$ ✓

(4) ~~225~~ a = 4, $y = \frac{10x(40+d)}{396+x}$

~~$x = 24,\ y = \frac{240(40+d)}{420} = \frac{4}{7}(40+d)$~~

~~390~~

~~$d = 2 \Rightarrow y = 24, \Rightarrow \frac{4242}{2424}$~~

428

430
~~438~~
396
34

~~73~~

~~$x = 34,\ y = \frac{340x(40+d)}{430}$~~

~~$= \frac{34}{43}(40+d),\ d = 3,\ y =$~~ 34

~~4343~~ Cases 4, 5, 6, 7, 8, 9

Give this up – To be taken up when necessary

~~Case of 5 digits~~ Before leaving 4 digits let us consider one

more case not mentioned in p. 15 viz 3rd of N with 3rd of D

[Damoraya takes 2nd of N with 2nd of D – vide Book (A), p. 3]

Case (viii) $\frac{abcd}{efcg} = \frac{abd}{efg}$ $\quad ab = x,\ ef = y.$

$(100x + 10c + d)(10y + g)$

$= (100y + 10c + g)(10x + d)$

$100xg + \cancel{100xg} + 100cg + 10cg + 10dy$

$= 100yd + 100cx + 10cd + 10xg$

$10gx + 10cy + cg + dy = 10dy + 10cx + cd + gx$

$= 100yq + 100cx \quad q + 10cd + 10xg$

$10gx + 10cy + cg + dy = 10dy + 10cx + cd + gx$ (35)

$9gx + 10cy + cg = 9dy + 10cx + cd + gx.$

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$q(gx - dy) = 10c(x - y) + c(d - g)$

$x(9g - 10c) - y(9d - 10c) = c(d - g)$

ie an ind. eqn of 1st degree.

Choose c, d, g suitably & solve for x & y.

~~$d = 4, g = 3, c = 5$~~

$d = 5, g = 4, c = 3 \rightarrow 6x - 15y = 3$

$2x - 5y = 1.$

$2 \cdot 3 - 5 \cdot 1 = 1.$

$x = 3 + 5t$
$y = 1 + 2t$

$t = 12 \rightarrow x = 63$
$y = 25$

$\frac{2}{5} = 0 + \frac{1}{2 + \frac{1}{2} + \frac{1}{1}}$

$0 + \frac{1}{3}$

$\frac{6335}{2534} = \frac{635}{254}.$

$6335 = 5 \times 1267$
$2534 = 2 \times 1267$

$\frac{5 \times 127}{2 \times 127}.$

$\frac{6335}{2534} = \frac{635}{254}$ so easily obtained

for $t = 5, 6, 7, 8, \ldots 19$ we get x & y with double digits

$t = 5, x = 28, y = 11$
$t = 6, x = 33, y = 13$
$t = 7, x = 38, y = 15$
$t = 8, x = 43, y = 17$

find (28,11), (33,13), (38,15), (43,17)
(48,19), (53,21), (58,23), (63,25)
(68,27), (73,29), (78,31), (83,33)
(88,35), (93,37), (95,39) (15 cases)

~~last case with t = 19~~ → for $t = 13$ → $\frac{6835}{2734}$ with all digits different

$$\frac{6835}{2734} = \frac{5 \times 1367}{2 \times 1367} = \frac{685}{274} = \frac{5 \times 137}{2 \times 137}$$

15 cases for a particular choice of a, g and c.

ie perhaps $9 \times 9 \times 9$ ~~x 15 cases~~ $\times n$ ie 729 n cases

Let us now go to 5 digits. — Domoryan has none with 5 digits.

5 digits (1) Consider ~~only~~ the case with middle digit cancelled

$$\frac{a_1 a_2 a_3 a_4 a_5}{b_1 b_2 a_3 b_4 b_5} = \frac{a_1 a_2 a_4 a_5}{b_1 b_2 b_4 b_5}$$

Let $a_1 a_2 = x$, $a_4 a_5 = y$

$b_1 b_2 = y$, $b_4 b_5 = x$

$$(10^4 a_1 + 10^3 a_2 + 10^2 a_3 + 10 a_4 + a_5)(10^3 b_1 + 10^2 b_2 + 10 b_4 + b_5)$$
$$= (10^4 b_1 + 10^3 b_2 + 10^2 a_3 + 10 b_4 + b_5)(10^3 a_1 + 10^2 a_2 + 10 a_4 + a_5)$$

$$(10^3 x + 10^2 a_3 + y)(10^2 x' + y')$$
$$= (10^3 x' + 10^2 a_3 + y')(10^2 x + y)$$

$$10^5 xx' + 10^3 xy' + 10^4 a_3 x' + 10^2 a_3 y' + 10^2 xy' + yy'$$
$$= 10^5 xx' + 10^3 x'y + 10^4 a_3 x + 10^2 a_3 y + 10^2 xy' + yy'$$

$$10 xy' + 10^2 a_3 x' + a_3 y' + x'y$$
$$= 10 x'y + 10^2 a_3 x + a_3 y + xy'$$

$$9 xy' + 10^2 a_3 x' + a_3 y' = 9 x'y + 10^2 a_3 x + a_3 y$$

$$9(xy' - x'y) = 10^2 a_3 (x - x') + a_3 (y - y')$$

$$a_3 = \frac{9(xy' - x'y)}{\qquad\qquad} \qquad (X)$$

$9(xy' - x'y) = 10a_3(x - x') + a_3(y - y')$

$$a_3 = \frac{9(xy' - x'y)}{100(x - x') + (y - y')} \qquad \text{——(X)}$$

(37)

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$\begin{vmatrix} x & x' \\ y & y' \end{vmatrix}$

$x - x' = 1, \quad y - y' = -1$ 5

$\begin{vmatrix} 23 & 22 \\ 17 & 18 \end{vmatrix}$ ~~414 - 374 = 40~~ X

$\begin{vmatrix} x & x-1 \\ y & y+1 \end{vmatrix} = x + y$

18 → 17
17 → 18

If $x + y$ ~~is~~ has a factor 11.

$x = 18, \quad y = 26$

44
18
26

$\begin{vmatrix} 18 & 17 \\ 26 & 27 \end{vmatrix} = 486 - 442 = 44.$

i.e. Choose $x = 18, y = 26, x' = 17, y' = 27.$

Setting $a_3 = \frac{9(44)}{99} = 4.$

$\rightarrow \frac{18426}{17427} = \frac{1826}{1727}$ ✓ $\frac{2 \times 9213}{3 \times 5809} = \frac{2 \times 3 \times 3071}{3 \times 5809}$

$\frac{2 \times 3071}{5809}$

$\frac{1826}{1727} = \frac{11 \times 166}{11 \times 157}$

157) 1742 7 (111
157
172
157
157

166) 18426 (111
166
182
166
166

i.e. $\frac{18426}{17427} = \frac{111 \times 166}{111 \times 157} = \frac{18}{} \frac{166}{157}$

$\frac{1826}{1727} = \frac{11 \times 166}{11 \times 157} = \frac{166}{157}.$

There must be heaps of other examples.

(ii) 4 digits in N and 5 in D with 3^{rd} of N & 3^{rd} of D cancelled

$$\frac{a_1 a_2 c a_3}{b_1 b_2 c b_3 b_4} = \frac{a_1 a_2 a_3}{b_1 b_2 b_3 b_4}$$

$a_1 a_2 = x$
$b_1 b_2 = y$
$b_3 b_4 = z$

$$(10^2 x + 10c + a_3)(10^2 y + z) = (10^3 y + 10^2 c + z)(10x + a_3)$$

$$10^4 xy + 10^2 xz + 10^3 cy + 10cz + 10^2 a_3 y + a_3 z$$
$$= 10^4 xy + 10^3 a_3 y + 10^3 cx + 10^2 a_3 c + 10xz + a_3 z$$

$$10xz + 10^2 cy + cz + 10a_3 y = 10^2 a_3 y + 10^2 cx + 10a_3 c + xz$$

$$9xz + 10^2 cy + cz = 90 a_3 y + 10^2 cx + 10a_3 c$$

$$90 a_3 y - 9xz = c\{10^2 y + z - 10^2 x - 10a_3\}$$

$$c = \frac{9(10a_3 y - xz)}{100(y-x) + (z - 10a_3)} \qquad (B)$$

$y > x$, $10a_3 - z = 19$ ~~> 81~~.

$$10a_3 \cdot a_3 = \frac{19 + z}{10}.$$

y	x		$x+1$	x
z	a_3	→	z	$(19+z)/10$

$$11 \frac{(19+z)(x+1)}{10} - xz \text{ be a multiple of } 9$$

$$209z + (209 + 11z)(x+1) - 10xz \text{ "}$$
$$209x + 209 + 11xz + 11z - 10xz \text{ "}$$
$$11(19x + z) + 209 + xz + 209 \text{ "}$$

Putting $c = 9$, (B) $\longrightarrow 10a_3 y - xz = 10(y-x) + (z - 10a_3)$

Putting $c=9$, (B) $\rightarrow 10a_3 y - xz = 10(y-x) + (z-10a_3)$ (39)

$x(100-z) - y(100-10a_3) = z - 10a_3$

Taking $z=31$, $a_3 = 3$ to make R.H.S = 1, we have the indeterminate eqn

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$69x - 70y = 1$ i.e. $x = 69 + 70t$, $y = 68 + 69t$ | start $t=0$ $x=69, y=68$

$[69 \cdot 69 - 70 \cdot 68 = 1]$

i.e. $\frac{6993}{68931} = \frac{693}{6831}$ | L.H.S $= \frac{7 \times 999}{69 \times 999}$, R.H.S $= \frac{7 \times 99}{69 \times 99}$.

8
Other ~~9~~ solns viz all with $c=9$, can be found for (z, a_3) giving

$(11,1), (21,2), (41,4), (51,5), (61,6), (71,7), (81,8), (91,9)$

The last gives the ind. eqn $9x - 10y = 1 \rightarrow x = 9 + 10t$, $y = 8 + 9t$

$t = 1, 2, \ldots 9$ gives 9 values for x & y.

$t=9 \rightarrow \frac{9999}{89991} = \frac{999}{8991} = \frac{1}{9}$ || $t=8 \rightarrow \frac{8999}{80991} = \frac{899}{8091} = \frac{1}{9}$

$t=7 \rightarrow \frac{7999}{71991} = \frac{1}{9}$ || $t=6 \rightarrow \frac{6999}{62991} = \frac{1}{9}$ || $t=5 \rightarrow \frac{5999}{53991}$ ||

$t=4 \rightarrow \frac{4999}{44991} = \frac{1}{9}$ || $t=3 \rightarrow \frac{3999}{35991} = \frac{1}{9}$ || $t=2 \rightarrow \frac{2999}{26991}$ || $t=1 \rightarrow \frac{1999}{17991} = \frac{1}{9}$

Note that the last is a only alternate for $\frac{1999}{9995}$

Hoping to handle cases using $c = 3, 6, 9$ & (z, a_3) suitably & letting ind. eqn in x & y etc. [See Book (C)]

(18) Doctor to burly woman patient: "Follow that diet & in a couple of months I want to see those three-quarters of you back here for a check up".

(19) U.S. Army custom of presenting silver baby cup for officers with baby born - once went to a British liaison officer who said in acceptance speech: "I am very pleased to receive this cup, but I must say it's the first time I've ever been awarded a trophy for this particular sport"

(20) Wing-Commander in aerospace division disagrees with the brass hat about making frequent flights in aircraft.

(21) "I never felt happy as a Colonel"? motto to Bongarts (81 year old mother)

(22) For a perfect fit at the bust line there are two points to consider - Clothier's advertisement.

(23)

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(1) 12 girls in Chorus "April Fool"

✓(2) waste can without a bottom – for people who have artificial flowers.

✓(3) the wrist watch without numerals on the dial – for the man who does' Nt care what time it is

(4) Grandfather brought home in policeman's car (Sd lost the as the police man said" – "How can you get lost in the dark, you are going there for 40 year" – "No, not exactly lost, I was tired of walking"

(5) Telephone – "buzz" – "dig" – went on my last.

(6) "When Jacob leaned on his staff he died" – Churchman to Dean Rusk

✓(7) Poets in jacket or nude – Cecil to Martin

✓(8) Denizens of other planets to entend themselves & discover us

(9) Boy to policeman "Have you seen a lady without me?"

✓(10) Bookshop sign "We don't mind your reading there books, but we wish you would do it at home"

✓(11) Sign behind a car "See you later, overtaker, I am an undertaker"

(12) Vicary Reading's Fiji shooting – Triples

(13) "It is too late to agree with me. I have changed my mind"

(14) Churchill's personal physician watching his meals to see what he eats & then wearing the same for himself

✓(15) Adenauer "I want to go on getting older"

(16) Martins & Martians.

(17) Best definition of Christianity "Give, give, give"