Book No. 27

100 With Wrapper

Book No. 27

S_1	S_2
S_3	S_4

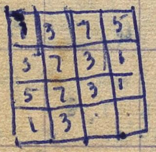
1	5	3	7	6	8	4	2
3	7	1	5	2	4	6	8
5	3	7	1	8	6	2	4
7	1	5	3	4	2	8	6
6	8	2	4	3	5	1	7
4	2	6	8	1	7	3	5
8	6	4	2	5	1	7	3
2	4	8	6	7	3	5	1

(A)

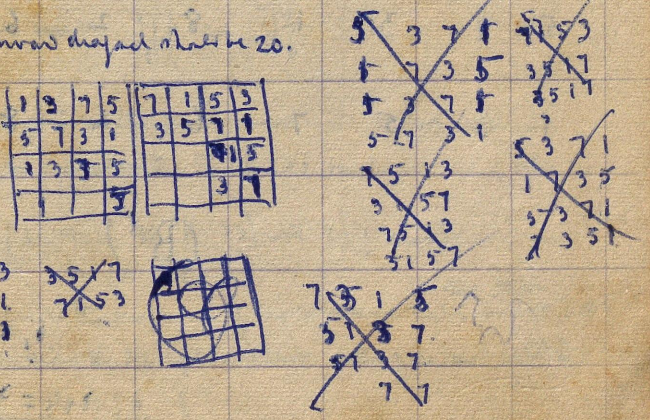
S_1 is not in any good form.

(S_1)
1, 3, 5, 7
7, 5, 3, 1
7, 5, 3, 1
1, 3, 5, 7

Magic S = 16.



(S_4) downward diagonal should be 20.



(2)

In the figure (A) on p. 1., 8^{th}, 7^{th}, 6^{th} & 5^{th} cols are reverses of 1^{st}, 2^{nd}, 3^{rd} & 4^{th} cols similarly 8^{th}, 7^{th}, 6^{th} & 5^{th} rows are reverses of 1^{st}, 2^{nd}, 3^{rd} & 4^{th} rows.

Suppose we ~~take~~ form (B') by taking 8^{th} col as 1^{st} row, 7^{th} col as 2^{nd} row & so on

2	8	4	6	7	5	3	1
4	6	2	8	1	3	7	5
8	4	6	2	5	7	1	3
6	2	8	4	3	1	5	7
7	5	1	3	4	8	2	6
3	1	7	5	2	6	4	8
5	7	3	1	8	2	6	4
1	3	5	7	6	4	8	2

(B')

$\begin{matrix} 1 & 6 \\ 7 & 4 \end{matrix}$ $\quad$ $\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$

1, 2, 3, 4, 5, 6, 7, 8

$1+4 = 3+2$ $\quad$ $2+3 = 4$

$1+4 \equiv 7+6 \pmod 8$ $\quad$ $5+7 = 1+3 \pmod 8$

This method of constructing ~~the~~ ~~of~~ Narayana's square with even & odd numbers occupying quadrants appears extremely difficult :. Give this up for the present

Consider some more 8×8 squares from Andrews other than 74·449. Analyse them fully.

(1) 74·449, p. 256 of Andrews – This has been split up into (A), (a) & (α) (B) & (B') on p. 98 of Book. No. 20. The first two rows of the top left hand quadrant are

1	6	7	4
7	4	1	6

The ~~se~~ two ~~sub~~ 2×2 subsquares ~~are~~ viz $\begin{array}{c|c} 1 & 6 \\ 7 & 4 \end{array}$ and $\begin{array}{c|c} 7 & 4 \\ 1 & 6 \end{array}$

are exactly of the same formation as the subsquares used to form magic squares ~~from the~~ in the case $n = 4$ with the top left 2×2 subsquare being of the form $\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}$ ie type (iii) [See Main Bk. p. 133] with complementaries 74·(A):96.
along diagonals. In fact we can in the same sense of modular arithmetic ~~assume~~ see that $7 + 6$ $\equiv 1 + 4 \pmod 8$: Could we consider the top left hand ~~corner of the~~ quadrant of the 8×8 square starting with the 2×2 subsquare $\begin{array}{c|c} 1 & 6 \\ 7 & 4 \end{array}$?

(4) Putting complementaries along diagonal, for nasik square the bottom right raw square is given by

8	3
2	5

1	6		
7	4		
		8	3
		2	5

If $S = 18$, the 3rd element of 1st row should be 3, 4, 5, 6, 7, or 8

8 is ruled out since 3rd col would be > 16

If we prescribe that no elements be repeated in rows or cols, 4, & 6 are eliminated leaving 3, 5, 7

5 is ruled out since this would necessitate 6 being repeated in first row. Leaving only 3, 8.

with 3 square would be magic as in (i) ~~[illegible]~~ say to be n

1	6	3	8
7	4	5	2
6	1	8	3
4	7	2	5

(i)

1	6	7	4
7	4	1	6
2	5	8	3
8	3	2	5

(ii)

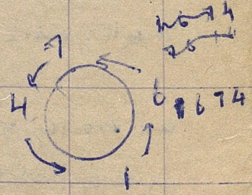
with 7 the condn for nasik requires element 2 as 3rd element in 1st col. from top. & completing we get (ii) which is same as that in fig. 449 of Andrews

7 4 6 6 4 7

(i) & (ii) have the same elements (not in same order) in rows & cols.

Interchanging rows & cols in (i) we get

$$\begin{vmatrix} 1 & 7 & 6 & 4 \\ 6 & 4 & 1 & 7 \\ 3 & 5 & 8 & 2 \\ 8 & 2 & 3 & 5 \end{vmatrix}$$



Also in (ii) & (i) 1st & 4th rows (cols) are complementary & 2nd & 3rd rows (cols) are complementary. thus (i) & (ii) are trivially different.

So the top left quadrant is formed ~~[illegible]~~ by means of 2 x 2 subsquares just as in case of $n = 4$.

~~The bottom right square is similarly formed with $\begin{array}{c|c} 4 & 7 \\ \hline 6 & 1 \end{array}$ (ie by interchange of diagonal elements in $\begin{array}{c|c} 1 & 6 \\ \hline 7 & 4 \end{array}$) in exactly the same way as the top left quadrant is formed.~~

The top right square is formed starting from $\begin{array}{c|c} 3 & 8 \\ \hline 5 & 2 \end{array}$ (ie taking complements of $\begin{array}{c|c} 1 & 6 \\ \hline 7 & 4 \end{array}$ and reflection about vertical) in exactly the same way as the top left quadrant as in nasik formation. The bottom two quadrants are now completed by using the condn that the whole square be **associated**. This makes not only the two bottom quadrants nasik, but the whole square nasik.

~~Next to obtain the next square (B)~~

Looking at p. 98 of notebook 20, we note that (B') has no ~~relat~~ simple relationships to (A)

(B') is not magic

$$\begin{array}{|c|c|c|c|} \hline 1 & 7 & 2 & 8 \\ \hline 6 & 4 & 5 & 3 \\ \hline 7 & 1 & 8 & 2 \\ \hline 4 & 6 & 3 & 5 \\ \hline \end{array} \rightarrow \begin{vmatrix} 1 & 7 & 2 & 8 \\ 7 & 1 & 8 & 2 \\ 6 & 4 & 5 & 3 \\ 4 & 6 & 3 & 5 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 2 & 7 & 8 \\ 7 & 8 & 1 & 2 \\ 6 & 5 & 4 & 3 \\ 4 & 3 & 6 & 5 \end{vmatrix}$$

$2^{nd} \rightleftarrows 3^{rd}$ to (A) $\qquad 2^{nd}$ row $\rightleftarrows 3^{rd}$ row $\qquad 2^{nd}$ col $\rightleftarrows 3^{rd}$ col

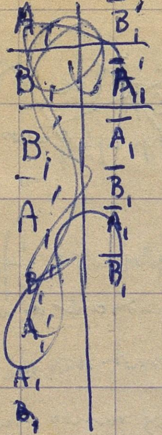
does not give top left quadrant of (B') which has all the elements 1, 2, 7, 8

1 — 2 — 7 8
7 — 8 — 1 2
2 — 1 — 8 7

~~Top left quadrant (A) top left quadrant (B)~~ looks difficult.

Denoting the rows of the quadrants by $A_1, A_2 - B_1, B_2 -$ etc we have for the primary square (A) the scheme written vertically downwards for the left and right quadrants given by

(b)



A_1	$\bar{B}_1'$
B_1	$\bar{A}_1'$
B_1'	$\bar{A}_1$
A_1'	$\bar{B}_1$
B_1'	$\bar{A}_1$
A_1'	$\bar{B}_1$
A_1	$\bar{B}_1'$
B_1	$\bar{A}_1'$

(A)

C_1	F_1'
D_1	E_1'
C_1'	F_1
D_1'	E_1
E_1	D_1'
F_1	C_1'
E_1'	D_1
F_1'	C_1

(B')

$A_1 = (1, 6, 7, 4)$

$B_1 = (7, 4, 1, 6)$

$C_1 = (1, 2, 7, 8)$

$D_1 = (2, 1, 8, 7)$

$E_1 = (3, 4, 5, 6)$

$F_1' = (4, 3, 6, 5)$.

where $C_1' = \bar{C}_1$, $D_1' = \bar{D}_1$,

$E_1' = \bar{E}_1$, $F_1' = \bar{F}_1$

where the dashdents the complement & the bar the reverse

(2) Fig. 468, p. 266 of Andrews (a & n) (See p. 99 of note book 20)

~~T_L~~ Q_1	~~T_R~~ Q_2
B_L Q_3	B_R Q_4

Squares (A) & (B) are obtained on p. 99.

8 5 3 4

1	4	8	5	2	3	7	6	A	B
2	3	7	6	1	4	8	5	B	A
6	7	3	2	5	8	4	1	$\bar{B}$	$\bar{A}$
5	8	4	1	6	7	3	2	$\bar{A}$	$\bar{B}$
7	6	2	3	8	5	1	4	B'	A'
8	5	1	4	7	6	2	3	A'	B'
4	1	5	8	3	2	6	7	$\bar{A}'$	$\bar{B}'$
3	2	6	7	4	1	5	8	$\bar{B}'$	$\bar{A}'$

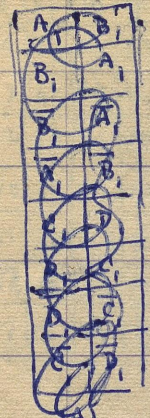
(A)

1	6	4	7	1	6	4	7	C	C
8	3	5	2	8	3	5	2	C'	C'
5	2	8	3	5	2	8	3	D	D
4	7	1	6	4	7	1	6	D'	D'
3	8	2	5	3	8	2	5	$\bar{D}$	$\bar{D}$
6	1	7	4	6	1	7	4	$\bar{D}'$	$\bar{D}'$
7	4	6	1	7	4	6	1	$\bar{C}$	$\bar{C}$
2	5	3	8	2	5	3	8	$\bar{C}'$	$\bar{C}'$

(B')

The squares T_L, T_R, B_L & B_R are not magic squares in (A). In (B'), however, all these quadrants are 4 x 4 nasiks & these can be constructed as (A) of Fig. 449 of Andrews.

11. Row-schemes for (A) & (B) is one



where $A_1' \equiv D_1$

$B_1' \equiv C_1$

A_1	B_1
B_1	A_1
$\bar{B}_1$	$\bar{A}_1$
$\bar{A}_1'$	$\bar{B}_1$
B_1'	A_1'
A_1'	B_1'
$\bar{A}_1'$	$\bar{B}_1'$
$\bar{B}_1'$	$\bar{A}_1'$

(A)

C_1	C_1
C_1'	C_1'
D_1	D_1
D_1'	D_1'
$\bar{D}_1$	$\bar{D}_1$
$\bar{D}_1'$	$\bar{D}_1'$
$\bar{C}_1$	$\bar{C}_1$
$\bar{C}_1'$	$\bar{C}_1'$

(B')

$A_1 = (1,4,8,5)$
$B_1 = (2,3,7,6)$

$C_1 = (1,6,4,7)$
$C_2 = (8,3,5,2)$.

Fig. 567, p. 296 (a neg)

1	2	3	4	5	6	7	8	$A\ \bar{A}'$
8	7	6	5	4	3	2	1	$A'\ \bar{A}$
1	2	3	4	5	6	7	8	$A\ \bar{A}'$
8	7	6	5	4	3	2	1	$A'\ \bar{A}$
8	7	6	5	4	3	2	1	$A'\ \bar{A}$
1	2	3	4	5	6	7	8	$A\ \bar{A}'$
8	7	6	5	4	3	2	1	$A'\ \bar{A}$
1	2	3	4	5	6	7	8	$A\ \bar{A}'$

(A) (a)

1	8	8	1	1	8	8	1
2	7	7	2	2	7	7	2
3	6	6	3	3	6	6	3
4	5	5	4	4	5	5	4
5	4	4	5	5	4	4	5
6	3	3	6	6	3	3	6
7	2	2	7	7	2	2	7
8	1	1	8	8	1	1	8

(B') (B) (R_0) defines:

1	8		
2	7		
		6	3
		5	4

on (A) T_L, B_L, T_R, B_R are not magic squares, and ~~they are so in~~ they are not magic in (B') also ~~both quadrants~~ ~~& B' can't be constructed~~ ~~as a squares using~~ ~~(next B) technique~~

(A) obtained from (B') by interchange of rows & cols

but not (B') from (A) rather peculiar (?)

The row schemes are

(8)

(†)

A_1	$\bar{A}_1'$
A_1'	$\bar{A}_1$
A_1	$\bar{A}_1'$
A_1'	$\bar{A}_1$
A_1'	$\bar{A}_1$
A_1	$\bar{A}_1'$
A_1'	$\bar{A}_1$
A_1	$\bar{A}_1'$

(A)

A	A
B	B
C	C
D	D
$\bar{D}'$	D'
$\bar{C}'$	C'
$\bar{B}'$	B'
$\bar{A}'$	A'

(B')

C_1	C_1
D_1	D_1
E_1	E_1
F_1	F_1
F_1'	F_1'
E_1'	E_1'
D_1'	D_1'
C_1'	C_1'

(B')

A
A'
A'
A
A
A'
A'
A

In my earlier method for associate squares, I had taken

A_1	$\bar{A}_1'$
A_1'	$\bar{A}_1$
A_1'	$\bar{A}_1$
A_1	$\bar{A}_1'$
A_1	$\bar{A}_1'$
A_1'	$\bar{A}_1$
A_1'	$\bar{A}_1$
A_1	$\bar{A}_1'$

(A)

& (B') by merely interchanging rows & cols of (A).

Eg $A = (1, 2, 3, 4)$:

The (A) would be different from the (A) above (†)

but interchange of rows & cols would give above (B') correctly

The difference in the (A) does not matter since in both cases there is symmetry read downwards & upwards so that an associate square will result.

Suppose we take the (A) ~~coin....~~ corresp. to (p. 56) ~~out~~ of Andrews then reverse ~~...~~ its rows & cols, for primitive & add to (A), what would be the square like? Corresp. (B') would have 1st row

0, 56, 0, 56, 56, 0, 56, 0

Adding to first row (A) → 1, 58, 3, 60, 61, 6, 63, 8

1	58	3	60	61	6	63	8
16	55	14	53	52	11	50	9
17	42	19	44	21	46	23	48
32	39	30	37	28	35	26	33
40	31	38	29	36	27	34	25
41	20	43	22	45	24	47	26
56	15	54	13	52	11	50	9
57	2	59	4	~~61~~ 5	62	63	64

1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8

(A)

0	56	0	56	0	56	0	56
8	48	8	48	8	48	8	48
16	40	16	40	16	40	16	40
24	32	24	32	24	32	24	32
32	24	32	24	32	24	32	24
40	16	40	16	40	16	40	16
48	8	48	8	48	8	48	8
56	0	56	0	56	0	56	0

(B')

+

1	58	3	60	5	62	7	64
16	55	14	53	12	51	10	49
17	42	19	44	21	46	23	48
32	39	30	37	28	35	26	33
40	31	38	29	36	27	34	25
41	18	43	20	45	22	47	24
56	15	54	13	52	11	50	9

0	56	0	56	56	0	56	0
8	48	8	48	48	8	48	8
16	40	16	40	40	16	40	16
24	32	24	32	32	24	32	24
32	24	32	24	24	32	24	32
40	16	40	16	16	40	16	40
48	8	48	8	8	48	8	48
56	0	56	0	0	56	0	56

=

1	58	3	60	61	6	63	8
16	55	14	53	52	11	50	9
17	42	19	44	45	22	47	24
32	39	30	37	36	27	34	25
40	31	38	29	28	35	26	33
41	18	43	20	21	46	23	48
56	15	54	13	12	51	10	49
57	2	59	4	5	62	7	64

[illegible] O4C

(C)

(10)

an associated one allright but different from Fig. 567 p. 296 of Andrews which has been obtained by Planck using method of reversions. In fact we might get this by taking 1st two & last cols. the same, and then reversing the remaining columns.

If we had taken $A_1, A_1', A_1', A_1, A_1, A_1', A_1', A_1$ in my usual scheme for $\underline{a}$ squares then also we would get some similar square.

~~What would be the exact scheme~~

~~Obtaining (B')~~

Fig. 695, p. 377 of Andrews. (A) & (B) are given on this page (only $\underline{n}$)

	A_1	B_1
	A_1'	B_1'
	A_1	B_1
(A)	A_1'	B_1'
	A_1	B_1
	A_1'	B_1'
	A_1	B_1
	A_1'	B_1'

(B') is obtained from (A) by interchange of rows & columns

This is a generalisation of my case with the second set of rows starting with B_1, B_1'. instead of A_1', A_1. not so simple

B_1 contains elements 1, 2, .. 8 other than those in A_1

i.e. my case is obtained by taking $B_1 = A_1'$.

(ornate properties are given)

A_1 has 2 complementary pairs

In this figure 695, odd and even numbers are not separated in the rows, but the sequence is e, o, o, e, e, o, o, e

o, e, e, o, o, e, e, o

e
o
e
o
e
o

Fig 716, p. 3919 shows $\underline{a}$ & $\underline{n}$ and 4-ply (i.e. any 2x2 subsquare elements sum to 130)

1	8	1	8	1	8	1	8
7	2	7	2	7	2	7	2
6	3	6	3	6	3	6	3
4	5	4	5	4	5	4	5
4	5	4	5	4	5	4	5
6	3	6	3	6	3	6	3
7	2	7	2	7	2	7	2
1	8	1	8	1	8	1	8

(A) ($\underline{a}$ & $\underline{n}$)

1	7	6	4	4	6	7	1
8	2	3	5	5	3	2	8
1	7	6	4	4	6	7	1
8	2	3	5	5	3	2	8
1	7	6	4	4	6	7	1
8	2	3	5	5	3	2	8
1	7	6	4	4	6	7	1
8	2	3	5	5	3	2	8

(B') ($\underline{a}$ & $\underline{n}$)

Beautiful squares. Each is obtained from the other by interchange of rows & cols.

A_1	A_1
B_1	B_1
C_1	C_1
D_1	D_1
D_1	D_1
C_1	C_1
B_1	B_1
A_1	A_1

(A)

E_1	$\overline{E}_1$
E_1'	$\overline{E}_1'$
E_1	$\overline{E}_1$
E_1'	$\overline{E}_1'$
E_1	$\overline{E}_1$
E_1'	$\overline{E}_1'$
E_1	$\overline{E}_1$
E_1'	$\overline{E}_1'$

(B')

(A) written in cols

$E_1\ E_1'\ E_1\ E_1'\ E_1\ E_1'\ E_1\ E_1'$

$\overline{E}_1\ \overline{E}_1'\ \overline{E}_1\ \overline{E}_1'\ \overline{E}_1\ \overline{E}_1'\ \overline{E}_1\ \overline{E}_1'$

is identical with (B') in rows

& above is [illegible] (B') written in cols.

B''

A	$\overline{A}$
A'	$\overline{A}'$
A	$\overline{A}$
A'	$\overline{A}'$
A	$\overline{A}$
A'	$\overline{A}'$
A	$\overline{A}$
A'	$\overline{A}'$

~~1,8,2,1~~ 1,8,7,2

(12) Fig. 720, p. 391 of Andrews. ($\underline{a}$, $\underline{n}$ & 4-ply)

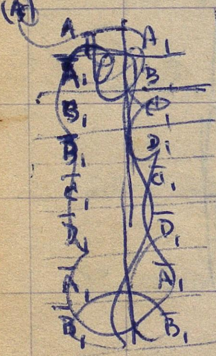
1	8	5	4	1	8	~~8~~5	4
8	1	4	5	8	1	4	5
3	6	7	2	3	6	7	2
6	3	2	7	6	3	2	7
2	7	6	3	2	7	6	3
7	2	3	6	7	2	3	6
4	5	8	1	4	5	8	1
5	4	1	8	5	4	1	8

(A) ($\underline{a}$ & $\underline{n}$)

1	6	3	8	4	7	2	5
4	7	2	5	1	6	3	8
5	2	7	4	8	3	6	1
8	3	6	1	5	2	7	4
5	2	7	4	8	3	6	1
8	3	6	1	5	2	7	4
1	6	3	8	4	7	2	5
4	7	2	5	1	6	3	8

(B') ($\underline{a}$ & $\underline{n}$)

(A)



(A)

E_1	F_1
F_1	E_1
$\bar{F}_1 = F_1'$	$\bar{E}_1 = E_1'$
$\bar{E}_1 = E_1'$	$\bar{F}_1 = F_1'$
$\bar{F}_1 = F_1'$	$\bar{E}_1 = E_1'$
$\bar{E}_1 = E_1'$	$\bar{F}_1 = F_1'$
E_1	F_1
F_1	E_1

$(B^{(1)})$

A_1	A_1
A_1'	A_1'
B_1	B_1
B_1'	B_1'
$\bar{B}_1$	$\bar{B}_1$
$\bar{B}_1'$	$\bar{B}_1'$
$\bar{A}_1$	$\bar{A}_1$
$\bar{A}_1'$	$\bar{A}_1'$

(A)

(A) & (B') are not $r \rightleftarrows c$ interchanges & producing getting (B') from (A) remains

1854
~~248~~
5274 Fig. 732, p. 396 of Andrews (4-ply, franklin, nasik)

& Each corner subsquare of order 4 a ~~map~~ nasik

1	8	1	8	1	8	1	8
6	3	6	3	6	3	6	3
8	1	8	1	8	1	8	1
3	6	3	6	3	6	3	6
2	7	2	7	2	7	2	7
5	4	5	4	5	4	5	4
7	2	7	2	7	2	7	2
4	5	4	5	4	5	4	5

(A) (M a 4-ple)

1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4
1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4
1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4
1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4

(B') (π a 4-ple)

A_1	A_1
B_1	B_1
A_1'	A_1'
B_1'	B_1'
C_1	C_1
D_1	D_1
C_1'	C_1'
D_1'	D_1'

(A)

E_1	F_1
E_1'	F_1'
E_1	F_1
E_1'	F_1'
E_1	F_1
E_1'	F_1'
E_1	F_1
E_1'	F_1'

(B')

(1, 7, 6, 4)

(i_1, 6_1)

By cols (A) is $\left.\begin{matrix} G_1 & G_1' & G_1 & G_1' & G_1 & G_1' & G_1 & G_1' \\ H_1 & H_1' & H_1 & H_1' & H_1 & H_1' & H_1 & H_1' \end{matrix}\right\}$ i.e. of same type as (B')

(A) micos / Again a generalisation of my (M)

(M) scheme

A	A'
A'	A
A	A'
A'	A
A	A'
A'	A
A	A'
A'	A

with $A = G_1$, $A' = H_1$

(B') in rows is also a generalisation with $A \to E_1$, $A' \to F_1$

(A) $\rightleftarrows$ (B') is not $\gamma \rightleftarrows$ C

Mittag Planets Fig. 695, p. 377

(14) Fig. 732, p. 397 of Andrews (4-ply, Franklin, Knight-nasik) nasik

1, 5, 3, 2

1	8	1	8	1	8	1	8
4	5	4	5	4	5	4	5
2	7	2	7	2	7	2	7
3	6	3	6	3	6	3	6
8	1	8	1	8	1	8	1
5	4	5	4	5	4	5	4
7	2	7	2	7	2	7	2
6	3	6	3	6	3	6	3

(A)

1	5	3	7	8	4	6	2
8	4	6	2	1	5	3	7
1	5	3	7	8	4	6	2
8	4	6	2	1	5	3	7
1	5	3	7	8	4	6	2
8	4	6	2	1	5	3	7
1	5	3	7	8	4	6	2
8	4	6	2	1	5	3	7

(B')

A_1	A_1
B_1	B_1'
C_1	C_1'
D_1	D_1'
A_1'	A_1'
B_1'	B_1'
C_1'	C_1'
D_1'	D_1'

(A)

E_1	E_1'
E_1'	E_1
E_1	E_1'
E_1'	E_1
E_1	E_1'
E_1'	E_1
E_1	E_1'
E_1'	E_1

(B')

(B') is exactly of my form for rs only! by rows

(A) also is of same form by cols

1st row of (A) = 1st col of (B'),
2nd " = [2nd col of B']'
3rd " = [4th col]'
4th " = 3rd col
5th " = 5th row
6th " = [6th col]'
7th " = [8th col]'
8th " = 7th col.

That Fig 734 is Knight nasik is verified for eg.

for knight move (2, 1) starting from 49 → 49 + 38 + 15 + 29 + 56 + 35 + 10 + 28 = 260

" — " Similarly other knight moves can be verified in Fig 734

Fig. 750, p. 410 of Andrews (results a central 4^2 also results from 4^2-ply. (15)
concentric

5	4	3	6	7	2	1	8
1	8	7	2	3	6	5	4
7	2	1	8	5	4	3	6
3	6	5	4	1	8	7	2
6	3	4	5	8	1	2	7
2	7	8	1	4	5	6	3
8	1	2	7	6	3	4	5
4	5	6	3	2	7	8	1

(A)

6	4	5	3	6	4	5	3
7	1	8	2	7	1	8	2
4	6	3	5	4	6	3	5
1	7	2	8	1	7	2	8
6	4	5	3	6	4	5	3
7	1	8	2	7	1	8	2
4	6	3	5	4	6	3	5
1	7	2	8	1	7	2	8

(B')

A_1	C_1
B_1	D_1
C_1	A_1
D_1	B_1
D_1'	B_1'
C_1'	A_1'
B_1'	D_1'
A_1'	C_1'

(A)

E_1	E_1
F_1	F_1
G_1	G_1
H_1	H_1
E_1	E_1
F_1	F_1
G_1	G_1
H_1	H_1

(B')

Good for ornate

Very funny forms.

Earlier squares from Andrews

(16). Fig. ~~[illegible]~~ 53, p. 27 of Andrews. (associated)

1	7	6	4	5	3	2	8
8	2	3	5	4	6	7	1
8	2	3	5	4	6	7	1
1	7	6	4	5	3	2	8
1	7	6	4	5	3	2	8
8	2	3	5	4	6	7	1
8	2	3	5	4	6	7	1
1	7	6	4	5	3	2	8

(A)

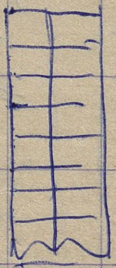
1	8	8	1	1	8	8	1
7	2	2	7	7	2	2	7
6	3	3	6	6	3	3	6
4	5	5	4	4	5	5	4
5	4	4	5	5	4	4	5
3	6	6	3	3	6	6	3
2	7	7	2	2	7	7	2
8	1	1	8	8	1	1	8

(B′)

This is the usual simple type

A	$\bar{A}'$
A'	$\bar{A}$
A'	$\bar{A}^2$
A	$\bar{A}'$
A	$\bar{A}'$
A'	$\bar{A}$
A'	$\bar{A}^2$
A	$\bar{A}'$

(A)



(B′)

& (B′) being (A) with $r \rightleftarrows c$.

Figs 56, 58, 60 are of the same types. Lap.

In my usual scheme for ~~A~~ a squares, suppose A has 2 elements repeated, or three repeated. Does the result hold?

Eg $A = (1, 2, 3, 1)$ no

~~1, 2, 3~~, 8, 6, 7, 8

8,

1, 2, 3, 1 8, 6, 7, 8

8, 7, 6, 3, 1, 3, 2, 1

8, 7, 6, 3, 1, 3, 2, 1

1, 2, 3, 1, 8, 6, 7, 8

1, 2, 3, 1, 8, 6, 7, 8

8, 7, 6, 3, 1, 2, 3, 1

1, 2, 3, 1 8, 7, 6, 3

8, 7, 6, 3 1, 2, 3, 1

1	8	1	8
8	1	8	1
8	1	8	1
1	8	1	8
8	1	8	1
1	8	1	8
8	1	8	1
1	8	1	8

1, 58, 59, 1, 8, 62, 7, 64

0 56 56 0 0 56 0 56.

DO's not hold for this choice

0, 56, 56, 0, 0, 56, 56, 0
1, 64, 57, 1, 8, 57.

1 8 1 8

Nor for this case also.

1 8 8 1	1 8 8 1	A
7 2 2 7	7 2 2 7	B
6 3 3 6	6 3 3 6	C
4 5 5 4	4 5 5 4	D
5 4 4 5	5 4 4 5	D'
3 6 6 3	3 6 6 3	C'
2 7 7 2	2 7 7 2	B'
8 1 1 8	8 1 1 8	A'

4	7	2	8	1	7	2	5
5	2	7	1	8	2	7	4
5	2	7	1	8	2	7	4
4	7	2	8	1	7	2	5
4	7	2	8	1	7	2	5
5	2	7	1	8	2	7	4
5	2	7	1	8	2	7	4
4	7	2	8	1	7	2	5

(P)

28	39	34	32	25	39	34	29
52	15	16	56	49	56	10	53

Repetition

for A $\bar{A}$,
A'
A'
A
A
A'
A'
A

for f(R) to be P with $r \rightleftarrows c$

A should not contain a complementary pair or pairs

(18) 3,7,6,8 | 1,3,2,6 14, 47, 46, 24, 17, 43, 42, ~~22~~
~~2 6~~
6,2,3,1
6,2,3,1
3,7,6,8
.3,7,6,8
.6,2,3,1
6,2,31
3,7,68

7368.1,3,6,2 55, 11, 14, 56, 49, 11, 14, 50 X
2631
2631
7368
7368
.7368
2631
2631
7638

3786 3126 → 19, 47, 48, 24, 19, 41, 42, 22 ✗
6213
6213
3786
.3786
6213
6213
3786

4° 74281752 → 55, 12, 10, 56, 49, 15, 13, 50 ✓
2571
2571
7428
7428
2571
2571
7428

3768 1326 → 19, 47, 46, 24, 17, 43, 42, 22 ✓
6231
6231
3768
.3768
6231
6231
3768

3762 7326 → 19, 47, 46, 18, 23, 43, 42, 22 ✓
6237 2673
6237 2673 → 46,
3762 7326
.3762
6237
6237
3762

4238. 1764
1357.
1234.
1246

Try 468 of Aurora [illegible] & use my prescription for a

1485 4158 → 1, 60, 64, 5, 4, 57, 61, 8
8514 5841 → ~~32~~ 37, 33, 28, 29, 40, 36, 25
8514 5841 → 64, 5,
1485 4158
.1485 4158
8514 5814
8514 5814
1485 4158

(19)

4	2	3	8	1	6	7	5
5	7	6	1	8	3	2	4
5	7	6	1	8	3	2	4
4	2	3	8	1	6	7	5
4	2	3	8	1	6	7	5
5	7	6	1	8	3	2	4
5	7	6	1	8	3	2	4
4	2	3	8	1	6	7	5

(P)

24	32	32	24	24	32	32	24
8	48	48	8	8	48	48	8
16	40	40	16	16	40	40	16
56	0	0	56	56	0	0	56
0	56	56	0	0	56	56	0
40	16	16	40	40	16	16	40
48	8	8	48	48	8	8	48
32	24	24	32	32	24	24	32

(R)

28	34	35	32	25	38	39	29
13	55	54	9	16	51	50	12
21	47	46	17	24	43	42	20
60	2	3	64	57	6	7	61
4	58	59	8	1	62	63	5
45	23	22	41	48	19	18	44
53	15	14	49	56	11	10	52
36	26	27	40	33	30	31	37

(M)

Q ✓

matrices Fig. 60, p. 28

1, 7, 3, 4, 5, 6, 2, 8
8, 2, 6, 5, 4, 3, 7, 1
8, 7, 3, 5, 4, 6, 2, 1
1, 2, 6, 4, 5, 3, 7, 8
1, 2, 6, 4, 5, 3, 7, 8
8, 7, 3, 5, 4, 6, 2, 1
8, 2, 6, 5, 4, 3, 7, 1
1, 7, 3, 4, 5, 6, 2, 8

(P)

A $\bar{A}'$
A $\bar{A}'$
A' $\bar{A}$
A' $\bar{A}$
A' $\bar{A}$
A' $\bar{A}$
A $\bar{A}'$
A $\bar{A}'$

Fig. 60, p. 28

A $\bar{A}'$
A' $\bar{A}$
A $\bar{A}'$
A' $\bar{A}$
A $\bar{A}'$
A' $\bar{A}$
A $\bar{A}'$
A' $\bar{A}$

1, 8, 8, 1, 1, 8, 8, 1
7, 2, 7, 2, 2, 7, 2, 7
3, 6, 3, 6, 6, 3, 6, 3
4, 5, 5, 4, 4, 5, 5, 4
5, 4, 4, 5, 5, 4, 4, 5
6, 3, 6, 3, 3, 6, 3, 6
2, 7, 2, 7, 7, 2, 7, 2
8, 1, 1, 8, 8, 1, 1, 8

(R)

A, A', A", A', A, A', A, A' | A', A

1, 7, 3, 4, 5, 6, 2, 8	0, 56, 0, 56, 56, 0, 56, 0	1, 63, 3, 60, 61, 6, 58, 8
8, 2, 6, 5, 4, 3, 7, 1	48, 8, 48, 8, 48, 8, 48, 8	56, 10, 54, 13, 52, 51, 15, 49
1, 7, 3, 4, 5, 6, 2, 8	16, 40, 16, 40, 40, 16, 40, 16	17, 47, 19, 44, 45, 22, 42, 24
8, 2, 6, 5, 4, 3, 7, 1	24, 32, 24, 32, 32, 24, 32, 24	32, 34, 30, 37, 36, 27, 39, 25
1, 7, 3, 4, 5, 6, 2, 8	32, 24, 32, 24, 24, 32, 24, 32	33, 31, 35, 28, 29, 38, 26, 40
8, 2, 6, 5, 4, 3, 7, 1	40, 16, 40, 16, 16, 40, 16, 40	48, 18, 46, 21, 20, 43, 23, 41
1, 7, 3, 4, 5, 6, 2, 8	8, 48, 8, 48, 48, 8, 48, 8	9, 55, 11, 51, 53, 14, 50, 16
8, 2, 6, 5, 4, 3, 7, 1	56, 0, 56, 0, 0, 56, 0, 56	64, 2, 62, 5, 4, 59, 7, 57

not magic in diagonals

(A, A, A', A', A', A', A, A)

1, 2, 6, 5, 4, 3, 7, 8	0, 0, 56, 56, 56, 56, 0, 0	1, 2, 62, 61, 60, 59, 7, 8
1, 2, 6, 5, 4, 3, 7, 8	8, 8, 48, 48, 48, 48, 8, 8	9, 10, 54, 53, 52, 51, 15, 16
8, 7, 3, 4, 5, 6, 2, 1	40, 40, 16, 16, 16, 16, 40, 40	48, 47, 19, 20, 21, 22, 42, 41
8, 7, 3, 4, 5, 6, 2, 1	32, 32, 24, 24, 24, 24, 32, 32	40, 39, 27, 28, 29, 30, 34, 33
8, 7, 3, 4, 5, 6, 2, 1	24, 24, 32, 32, 32, 32, 24, 24	32, 31, 35, 36, 37, 38, 26, 25
8, 7, 3, 4, 5, 6, 2, 1	16, 16, 40, 40, 40, 40, 16, 16	24, 23, 43, 44, 45, 46, 18, 17
1, 2, 6, 5, 4, 3, 7, 8	48, 48, 8, 8, 8, 8, 48, 48	49, 50, 14, 13, 12, 11, 55, 56
1, 2, 6, 5, 4, 3, 7, 8	56, 56, 0, 0, 0, 0, 56, 56	57, 58, 6, 5, 4, 3, 63, 64

a ✓

(A, A', A, A, A, A, A', A)

1, 7, 6, 4, 5, 3, 2, 8	0, 56, 0, 0, 0, 0, 56, 0	1, 63, 6, 4, 5, 3, 58, 8 X
8, 2, 3, 5, 4, 6, 7, 1	48, 8, 48, 48, 48, 48, 8, 48	
1, 7, 6, 4, 5, 3, 2, 8	40, 16, 40, 40, 40, 40, 16, 40	
1, 7, 6, 4, 5, 3, 2, 8	24, 32, 24, 24, 24, 24, 32, 24	
1, 7, 6, 4, 5, 3, 2, 8	32, 24, 32, 32, 32, 32, 24, 32	
1, 7, 6, 4, 5, 3, 2, 8	16, 40, 16, 16, 16, 16, 40, 16	
8, 2, 3, 5, 4, 6, 7, 1	8, 48, 8, 8, 8, 8, 48, 8	
1, 7, 6, 4, 5, 3, 2, 8	56, 0, 56, 56, 56, 56, 0, 56	

Does not work with (R) obtain by ...

1 8 8 1 1 8 8 1
1 1 8 8 8 8 1 1
7 2 7 2 2 7 2 7
2 2 7 7 7 7 2 2

Fig 56, p. 27 Andrews

1, 7, 3, 4, 5, 6, 2, 8
8, 2, 6, 5, 4, 3, 7, 1
8, 7, 3, 5,
8
8
8

1, 1, 8, 8, 8, 8, 1, 1
2, 2, 7, 7, 7, 7, 2, 2
6, 6, 3, 3,

Fig 56, p.27 Andrews

1	7	3	4	5	6	2	8
8	2	6	5	4	3	7	1
8	7	3	5	4	6	2	1
1	2	6	4	5	3	7	8
1	2	6	4	5	3	7	8
8	7	3	5	4	6	2	1
8	2	6	5	4	3	7	1
1	7	3	4	5	6	2	8

(P)

1	1	8	8	8	8	1	1
2	2	7	7	7	7	2	2
6	6	3	3	3	3	6	6
5	5	4	4	4	4	5	5
4	4	5	5	5	5	4	4
3	3	6	6	6	6	3	3
7	7	2	2	2	2	7	7
8	8	1	1	1	1	8	8

(R_0)

1	2	6	5	4	3	7	8	A	$\bar{A}'$
1	2	6	5	4	3	7	8	A	$\bar{A}'$
8	7	3	4	5	6	2	1	A'	$\bar{A}$
8	7	3	4	5	6	2	1	A'	$\bar{A}$
8	7	3	4	5	6	2	1	A'	$\bar{A}$
8	7	3	4	5	6	2	1	A'	$\bar{A}$
1	2	6	5	4	3	7	8	A	$\bar{A}'$
1	2	6	5	4	3	7	8	A	$\bar{A}'$

(R_0) $(r \rightleftarrows c)$

Same as (P) of p.20

Substitute Fig. 60, p.28, (P) is contains A & B

(P) in Fig.56		(P) in Fig 60		(R) in Fig.56	
A	$\bar{A}'$	A	A	C	$\bar{C}$
A'	$\bar{A}$	A'	A	D	$\bar{D}$
B	$\bar{B}'$			E	$\bar{E}$
B'	$\bar{B}$			F	$\bar{F}$
B'	$\bar{B}$	← do —		F'	$\bar{F}'$
B	$\bar{B}'$			E'	$\bar{E}'$
A'	$\bar{A}$			D'	$\bar{D}'$
A	$\bar{A}'$			C'	$\bar{C}'$

Same as $r \rightleftarrows c$ of (P) of Fig. 56.

↑

(R) in Fig 60

A	E	D
B	F	C
C	G	B
D	H	A
A'	D'	D
B'	C'	C
C'	B'	B
D'	A'	A

A	$\bar{A}'$
B	$\bar{B}'$
C	$\bar{C}'$
D	$\bar{D}'$
D	$\bar{D}'$
C	$\bar{C}'$
B	$\bar{B}'$
A	$\bar{A}$

A	$\bar{A}'$
A'	$\bar{A}$
A'	$\bar{A}$
A	$\bar{A}'$
A	$\bar{A}'$
A'	$\bar{A}$
A'	$\bar{A}'$
A	$\bar{A}'$

Fig.58, p.28 Andrews

1	7	6	5	4	3	2	8
1	2	6	5	4	3	7	8
8	2	3	4	5	6	7	1
8	7	3	4	5	6	2	1
8	7	3	4	5	6	2	1
8	2	3	4	5	6	7	1
1	2	6	5	4	3	7	8
1	7	6	5	4	3	2	8

(P)

1	1	8	8	8	8	1	1
7	2	2	7	7	2	2	7
6	6	3	3	3	3	6	6
5	5	4	4	4	4	5	5
4	4	5	5	5	5	4	4
3	3	6	6	6	6	3	3
2	7	7	2	2	7	7	2
8	8	1	1	1	1	8	8

(R_0)

Answer

Fig. 53 (22)

A, A', A, A', A, A', A, A'

Fig. 53

A	$\bar{A}'$
A'	$\bar{A}$
A'	$\bar{A}$
A	$\bar{A}'$
A	$\bar{A}'$
A'	$\bar{A}$
A'	$\bar{A}$
A.	$\bar{A}'$

(P)

& R_0 is $r \rightleftarrows c$ of (P)

Fig. 56

A	$\bar{A}'$
A'	$\bar{A}$
B'	$\bar{B}$*
B*	$\bar{B}'$
B*	$\bar{B}'$
B'	$\bar{B}$*
A'	$\bar{A}$
A	$\bar{A}'$

(P)

2 types R_0 not $r \rightleftarrows c$ of P.

Fig. 58

A	$\bar{A}'$
B	$\bar{B}'$
A'	$\bar{A}$
B'	$\bar{B}$
B'	$\bar{B}$
A'	$\bar{A}$
B	$\bar{B}'$
A	$\bar{A}'$

& R_0 is (P) $r \rightleftarrows c$

Fig. 60

A	$\bar{A}'$
A'	$\bar{A}$
B'	$\bar{B}$
B	$\bar{B}'$
B	$\bar{B}'$
B'	$\bar{B}$
A'	$\bar{A}$
A	$\bar{A}'$

& R_0 is (P) $r \rightleftarrows c$

1357
8642
1357
8642
8642
13

(P) of 56 ≡ (P) of (60)

In (P) of Fig. 58: ~~then~~ interchange 2^{nd} & 7^{th} rows & cols } ; " 4^{th} & 5^{th} "

(P) →

1	7	3	4	5	6	2	8	1	2	3	4	5	6	7	8
8	2	6	5	4	3	7	1	8	7	6	5	4	3	2	1
8	7	3	5	4	6	2	1	8	2	3	5	4	6	7	1
1	2	6	4	5	3	7	8	1	7	6	4	5	3	2	8
1	2	6	4	5	3	7	8	1	7	6	4	5	3	2	8
8	7	3	5	4	6	2	1	8	2	3	5	4	6	7	1
8	2	6	5	4	3	7	1	8	7	6	5	4	3	2	1
1	7	3	4	5	6	2	8	1	2	3	4	5	6	7	8

(2^{nd}) & (7^{th}) rows $\rightleftarrows$; 2^{nd} & 7^{th} cols $\rightleftarrows$

~~Alternate~~ interchange of 4^{th} & 5^{th} rows & cols will not lead to (R_0) of Fig. 58.

1	1
8	1
8	8
1	8
1	8
8	1
8	1
1	

Even for Fig. 56, we might use the (R_0) obtained from (P) by $r \rightleftarrows c$

& get a magic square, but in 56 the (R_0) obtained from a

different scheme viz

1265 4378
1265 4378
8734 5621
8734
8734
8734
1265

is used

Scheme for 7y √6 is (A, A', B', B, B, B', A', A) or equivalent to
(A, A', B, B', B', B, A', A).

And the R_0 used is got from (A, A, A', A', A', A', A, A) by $r \rightleftarrows c$.

Can we use for (A, A', A', A, A, A', A', A) the (R_0) got from
(A, A, A', A', A', A', A, A) by $r \rightleftarrows c$? Yes

1, 3, 5, 7, 2, 4, 6, 8	0, 0, 56, 56, 56, 56, 0, 0	1, 3, 61, 63, 59, 60, 6, 8
8, 6, 4, 2, 7, 5, 3, 1	8, 48, 48, 48, 48, 48, 8, 8	16, 14, 52, 50, 55, 53, 11, 9
8, 6, 4, 2, 7, 5, 3, 1	40, 40, 16, 16, 16, 16, 40, 40	48, 46, 20, 18, 23, 21, 43, 41
1, 3, 5, 7, 2, 4, 6, 8	+ 32, 32, 24, 24, 24, 24, 32, 32	33, 35, 29, 31, 26, 28, 38, 40
1, 3, 5, 7, 2, 4, 6, 8	24, 24, 32, 32, 32, 32, 24, 24	25, 27, 37, 39, 34, 36, 30, 32
8, 6, 4, 2, 7, 5, 3, 1	16, 16, 40, 40, 40, 40, 16, 16	24, 22, 44, 42, 47, 45, 19, 17
8, 6, 4, 2, 7, 5, 3, 1	48, 48, 8, 8, 8, 8, 48, 48	56, 54, 12, 10, 15, 13, 51, 49
1, 3, 5, 7, 2, 4, 6, 8	56, 56, 0, 0, 0, 0, 56, 56	57, 59, 5, 7, 2, 4, 62, 64

a ✓

Can we use for (A, A', B, B', B', B, A', A) the R_0 from (A, A', A', A, A, A', A', A)?

1, 2, 3, 4, 5, 6, 7, 8		3, 2, 4, 8, 1, 5, 7, 6		~~[illegible]~~
8, 7, 6, 5, 4, 3, 2, 1		6, 7, 5, 1, 8, 4, 2, 3		17, 42, 43, 20, 19, 46, 47, 24
3, 4, 2, 8, 1, 7, 5, 6		6, 7, 5, 1, 8, 4, 2, 3		16, 55, 54, 13, 12, 51, 50, 9
6, 5, 7, 1, 8, 2, 4, 3	⊕	3, 2, 4, 8, 1, 5, 7, 6	=	27, 36, 34, 32, 25, 39, 37, 30
6, 5, 7, 1, 8, 2, 4, 3		3, 2, 4, 8, 1, 5, 7, 6		62, 5, 7, 57, 64, 2, 4, 59
3, 4, 2, 8, 1, 7, 5, 6		6, 7, 5, 1, 8, 4, 2, 3		6, 61, 63, 1, 8, 58, 60, 3
8, 7, 6, 5, 4, 3, 2, 1		6, 7, 5, 1, 8, 4, 2, 3		35, 28, 26, 40, 33, 31, 29, 38
1, 2, 3, 4, 5, 6, 7, 8		3, 2, 4, 8, 1, 5, 7, 6		56, 15, 14, 53, 52, 11, 10, 49
		+ 0, 8, 16 … of the above in each row		41, 18, 19, 44, 45, 22, 23, 48

a ✓

Yes. In fact in the above two cases we have used A, B, different from the A & B of 7y p 53, 56, 58 & 60

(24) What about (a) $(A, B, B', A', A', B', B, A)$? Take $A = (1, 4, 7, 3)$

(b) $(A, B, A', B', B', A', B, A)$ $B = (2, 6, 5, 8)$

(with A, & B having no common elements – in Fig 56, A & B have <u>2</u> common elements).

~~1, 4, 7, 3~~

~~2, 5, 6, 8~~

(a)

1, 4, 7, 3, 6, 2, 5, 8	0, 8, 48, 56, 56, 48, 8, 0	1, 12, <u>55</u>, 59, 62, 50, 13, 8
2, 6, 5, 8, 1, 4, 3, 7	24, 40, 16, 32, 32, 16, 40, 24	26, 46, 21, 40, 33, 20, 43, 31
7, 3, 4, 1, 8, 5, 6, 2	48, 32, 16, 8, 8, 16, 32, 48	<u>55</u>,
8, 5, 2, 6, 3, 7, 4, 1	16, 56, 0, 40, 40, 0, 56, 16	
8, 5, 2, 6, 3, 7, 4, 1	40, 0, 56, 16, 16, 56, 0, 40	
7, 3, 4, 1, 8, 5, 6, 2	8, 24, 32, 48, 48, 32, 24, 8	
2, 6, 5, 8, 1, 4, 3, 7	32, 16, 40, 24, 24, 40, 16, 32	
1, 4, 7, 3, 6, 2, 5, 8	56, 48, 8, 0, 0, 8, 48, 56	

+

$\checkmark$ ZC.

Does not work

In Fig. 56, $A = (1, 7, 3, 4)$
$B = (8, 7, 3, 5)$

With the A, B in (a) above
$A = (1, 4, 7, 3)$
$B = (2, 6, 5, 8)$

i.e. every element of A has a complement in B, while in Fig 56 only two elements have complements in B. Suppose we make a similar choice in (a), say

(a) $A = (1, 4, 7, 3)$, $B = (6, 4, 7, 2)$ or $B = (1, 5, 2, 3)$ or $B = (2, 1, 5, 3)$

1, 4, 7, 3, 6, 2, 5, 8	1 12 55 59, 62, 50, 13, 8
2, 1, 5, 3, 6, 4, 8, 7	26, 1 does not work.
7, 8, 4, 6, 3, 5, 1, 2	
8, 5, 2, 6, 3, 7, 4, 1	
8, 5, 2, 6, 3, 7, 4, 1	
7, 8, 4, 6, 3, 5, 1, 2	
2, 1, 5, 3, 6, 4, 8, 7	
1, 4, 7, 3, 6, 2, 5, 8	

X

Take (1, 5, 2, 3)

1, 4, 7, 3, 6, 2, 5, 8	1, 4, 63, 59, 62, 58, 5, 8
1, 5, 2, 3, 6, 7, 4, 8	25, 37, 26, 35, 38, 31, 36, 32
8, 4, 7, 6, 3, 2, 5, 1	56, 12, 55, 14, 11, 50, 13, 49
8, 5, 2, 6, 3, 7, 4, 1	24, 21, 42, 46, 43, 47, 20, 17
8, 5, 2, 6, 3, 7, 4, 1	48, 45, 18, 22, 19, 23, 44, 41
8, 4, 7, 6, 3, 2, 5, 1	16, 52, 15, 54, 51, 10, 53, 9
1, 5, 2, 3, 6, 7, 4, 8	33, 29, 34, 27, 30, 39, 28, 40
1, 4, 7, 3, 6, 2, 5, 8	57, 60, 7, 3, 6, 2, 61, 64

$\checkmark$ $\checkmark$ a

So it looks as if in choice of $\underline{A}$ & $\underline{B}$ $\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_1 & a_2' & a_3' & a_4 \end{pmatrix}\begin{matrix} A \\ B \end{matrix}$

or (a_1, a_2, a_3, a_4) $(a_1' a_2 a_3 a_4'$

so that $(A, A', A, A, A, A, A', A)$ although symmetrically placed does not work. i.e. should have four with s four without dashes. What about

$(A, \bar{A}, A, \bar{A}, \bar{A}, A, \bar{A}, A)$. Or

1, 7, 6, 5, 4, 3, 2, 8	1, 39, 6, 37, 4, 35, 2, 40 X
5, 6, 7, 1, 8, 2, 3, 4	
1, 7, 6, 5, 4, 3, 2, 8	Does not work.
5, 6, 7, 1, 8, 2, 3, 4	
1, 7, 6, 5, 4, 3, 2, 8	X Does (A, B, C, D, D, C, B, A) work for Q_1, Q_2)
5, 6, 7, 1, 8, 2, 3, 4	Q No
1, 7, 6, 5, 4, 3, 2, 8	
5, 6, 7, 1, 8, 2, 3, 4	

1, 2, 6, 5, 4, 3, 7, 8	1 2 6 13, 12 X
1, 7, 6, 5, 4, 3, 2, 8	Suppose we choose $A = (a_1, a_2, a_3, a_4)$
8, 4, 7, 3, 6, 2, 5, 8	$B = (a_1', a_2, a_3, a_4')$
8, 4, 5, 3, 6, 4, 8, 7	$C = (a_1, a_2', a_3', a_4)$
8, 4, 5, 3	$D = (a_1', a_2', a_3', a_4')$
8, 4, 7, 3	
1, 7, 6, 5	
1, 2, 6, 5	

1, 2, 6, 5, 4, 3, 7, 8	1, 58, 6, 61, 60, 3, 63, 8	
8, 2, 6, 4, 5, 3, 7, 1	16, 10, 54, 52, 53, 51, 15, 9	
1, 7, 3, 5, 4, 6, 2, 8	41, 47, 19, 21, 20, 22, 42, 48	
8, 7, 3, 4, 5, 6, 2, 1	40, 31, 35, 28, 29, 38, 26, 33	Q
8, 7, 3, 4, 5, 6, 2, 1	32, 39, 27, 36, 37, 30, 34, 25	
1, 7, 3, 5, 4, 6, 2, 8	17, 23, 43, 45, 44, 46, 18, 24	
8, 2, 6, 4, 5, 3, 7, 1	56, 50, 14, 12, 13, 11, 55, 49	
1, 2, 6, 5, 4, 3, 7, 8	57, 2, 62, 5, 4, 59, 7, 64	

A
A'
A'
A
A
A'
A
A

Possible cases distinct

(1) Single $\underline{A}$ – (A, A', A', A) Q_1, $(A, A', \underline{A'}, A)$ (A, A', A, A') Q_2

(A, A', A', A) (A', A, A', A) Q_2

Q_3, Q_4 determined automatically

(2 b) (2) <u>A</u> & <u>B</u> to be such that if $A = (a_1, a_2, a_3, a_4)$, $B = (a_1, a_2', a_3', a_4)$ or (a_1', a_2, a_3, a_4)

(A, B, A', B'), (A, A', B, B'), ~~(A, B', A, B)~~ (A, B, B', A') three distinct types.

~~(3) A, B, C, D~~
~~$A = (a_1, a_2, a_3, a_4)$, $B = (a_1, a_2, a_3, a_4)$~~

(3) A, B, C, D — $A = (a_1, a_2, a_3, a_4)$, $B = (a_1', a_2, a_3, a_4)$, $C = (a_1, a_2', a_3, a_4)$
$D = A' = (a_1', a_2', a_3', a_4')$.

In all these cases (R_0) obtained from (P) by $r \rightleftarrows c$.

Of these six cases, any (P) ~~with~~ and the (R_0) obtained by $r \rightleftarrows c$ from the other cases gives an associated magic square.

Take the (P) from (A, B, C, D) & R_0 from (A, A', A', A).

1, 2, 6, 5, 4, 3, 7, 8	0, 56, 56, 0, 0, 56, 56, 0	1, 58, 62, 5, 4, 59, 63, 8
8, 2, 6, 4, 5, 3, 7, 1	48, 8, 8, 48, 48, 8, 8, 48	56, 10, 14, 52, 53, 11, 15, 49.
1, 7, 3, 5, 4, 6, 2, 8	40, 16, 16, 40, 40, 16, 16, 40	41, 23, 19, 45, 44, 22, 18, 48
8, 7, 3, 4, 5, 6, 2, 1	24, 32, 32, 24, 24, 32, 32, 24	32, 39, 35, 28, 29, 38, 34, 25
8, 7, 3, 4. 5, 6, 2, 1	32, 24, 24, 32, 32, 24, 24, 32	40, 31, 27, 36, 37, 30, 26, 33
1, 7, 3, 5, 4, 6, 2, 8	16, 40, 40, 16, 16, 40, 40, 16	17, 47, 43, 21, 20, 46, 42, 24
8, 2, 6, 4, 5, 3, 7, 1	8, 48, 48, 8, 8, 48, 48, 8	16, 50, 54, 12, 13, 51, 55, 9
1, 2, 6, 5, 4, 3, 7, 8	56, 0, 0, 56, 56, 0, 0, 56.	57, 2, 6, 61, 60, 3, 7, 64

$\sqrt{}$ <u>a</u>

from Fig. 53 //

I think these cases will do for <u>8 × 8</u> associated squares.

$A \quad \bar{A}'$
$A' \quad \bar{A}$
$A' \quad \bar{A}$
$A \quad \bar{A}'$

1, 7, 59, 60, 61, 62, 2, 8.
49, 10, 14, 52, 53, 11, 15, 56.
34, 42, 46, 21, 20, 43, 47, 17
32, 31, 35, 28, 29, 38, 26, 25
40, 39, 27, 36, 37, 30, 34, 33
48, 18, 22, 45, 44, 19, 23, 41
9, 50, 54, 12, 13, 51, 55, 16.
57, 63, 3, 4, 5, 6, 58, 64

Remarks: Does choosing A' is restriction of no complementary pair in it necessary ?

(1) (A, A', A, A')

T1

1, 2, 7, 5, 8, 7, 2, 4	1, 58, 7, 61, 8, 63, 2, 60
8 7 2 4, 1, 2, 7, 5	16, 55, 10, 52, 9, 50, 15, 53
1 7 7, 5, 8, 7, 2, 4	1, 58, 7, 61, 8, 63, 2, 60
8, 7, 2, 4, 1, 2, 7, 5	16, 55, 10, 52, 9, 50, 15, 53.
1, 2, 7, 5, 8, 7, 2, 4	49, 10, ~~[illegible]~~
8, 7, 2, 4, 1, 2, 7, 5	This condition is necessary here also
1, 2, 7, 5, 8, 7, 2, 4	
8, 7, 2, 4, 1, 2, 7, 5	

1, 3, 7, 5, 8, 6, 2, 4	0, 56, 0, 56, 0, 56, 0, 56	1, 59, 7, 61, 8, 62, 2, 60
8, 6, 2, 4, 1, 3, 7, 5	16, 40, 16, 40, 16, 40, 16, 40	24, 46, 18, 44, 17, 43, 23, 45
1, 3, 7, 5, 8, 6, 2, 4	48, 8, 48, 8, 48, 8, 48, 8	49, 11, 55, 13, 56, 14, 50, 12
8, 6, 2, 4, 1, 3, 7, 5	32, 24, 32, 24, 32, 24, 32, 24	40, 30, 34, 28, 33, 27, 39, 29
1, 3, 7, 5, 8, 6, 2, 4	56, 0, 56, 0, 56, 0, 56, 0	57, 3, 63, 5, 64, 6, 58, 4
8, 6, 2, 4, 1, 3, 7, 5	40, 16, 40, 16, 40, 16, 40, 16	48, 22, 42, 20, 41, 19, 47, 21
1, 3, 7, 5, 8, 6, 2, 4	8, 48, 8, 48, 8, 48, 8, 48	9, 51, 15, 53, 16, 54, 10, 52
8, 6, 2, 4, 1, 3, 7, 5	24, 32, 24, 32, 24, 32, 24, 32	32, 38, 26, 36, 25, 35, 31, 37

N ✓ & also [illegible]

$32 + 15 + 41 + 58 + 40 + 55 + 17 + 2 = 260$

This is interesting as 0 4 8 no [illegible]

This appears Knight — result like Fig. 262, p. 165 of Andrews & also like Fig on p. 96 Bk. 20.

(2) (A, A', A', A); A, A', A', A | $33 + 22 + 49 + 6 + 25 + 46 + 9 + 62 = 252$

Doesn't appear Knight-nasik (29

1, 2, 3, 4, 8, 7, 6, 5	0, 56, 56, 0, 0, 56, 56, 0	1, 58, 59, 4, 8, 63, 62, 5
8, 7, 6, 5, 1, 2, 3, 4	8, 48, 48, 8, 8, 48, 48, 8	16, 55, 54, 13, 9, 50, 51, 12
8, 7, 6, 5, 1, 2, 3, 4	16, 40, 40, 16, 16, 40, 40, 16	24, 47, 46, 21, 17, 42, 43, 20
1, 2, 3, 4, 8, 7, 6, 5	24, 32, 32, 24, 24, 32, 32, 24	25, 34, 35, 28, 32, 39, 38, 29
1, 2, 3, 4, 8, 7, 6, 5	56, 0, 0, 56, 56, 0, 0, 56	57, 2, 3, 60, 64, 7, 6, 61
8, 7, 6, 5, 1, 2, 3, 4	48, 8, 8, 48, 48, 8, 8, 48	56, 15, 14, 53, 49, 10, 11, 52
8, 7, 6, 5, 1, 2, 3, 4	40, 16, 16, 40, 40, 16, 16, 40	48, 23, 22, 45, 41, 18, 19, 44
1, 2, 3, 4, 8, 7, 6, 5	32, 24, 24, 32, 32, 24, 24, 32	33, 26, 27, 36, 40, 31, 30, 37

$\underline{n}$ ✓ ~~not gaina~~ only 16 subsquares 4 × 4 add to 130. (*)

interesting re $\underline{0}$ & e.

(A, A, A', A') (A, A, A', A')

1, 7, 6, 4, 8, 2, 3, 5	0, 0, 56, 56, 0, 0, 56, 56	1, 7, 62, 60, 8, 2, 59, 61
1, 7, 6, 4, 8, 2, 3, 5	48, 48, 8, 8, 48, 48, 8, 8	49, 55, 14, 12, 56, 50, 11, 13
8, 2, 3, 5, 1, 7, 6, 4	40, 40, 16, 16, 40, 40, 16, 16	48, 42, 19, 21, 41, 47, 22, 20
8, 2, 3, 5, 1, 7, 6, 4	24, 24, 32, 32, 24, 24, 32, 32	32, 26, 35, 37, 25, 31, 38, 36
1, 7, 6, 4, 8, 2, 3, 5	56, 56, 0, 0, 56, 56, 0, 0	57, 63, 6, 4, 64, 58, 3, 5
1, 7, 6, 4, 8, 2, 3, 5	8, 8, 48, 48, 8, 8, 48, 48	9, 15, 54, 52, 16, 10, 51, 53
8, 2, 3, 5, 1, 7, 6, 4	16, 16, 40, 40, 16, 16, 40, 40	24, 18, 43, 45, 17, 23, 46, 44
8, 2, 3, 5, 1, 7, 6, 4	32, 32, 24, 24, 32, 32, 24, 24	40, 34, 27, 29, 33, 39, 30, 28

$\underline{n}$ ✓ not gaina

$40 + 43 + 16 + 3 + 32 + 19 + 56 + 59 = 268$ not Knight-nasik

interesting re $\underline{0}$ & $\underline{e}$

(A & B) Q_1 Q_3

(A, B, A', B') (A, B, A', B')

(A, A', B, B') (A, A', B, B')

(A, B, B', A') (A, B, B', A')

Are the special choices of A & B as in the case of $\underline{a}$-squares.

Perhaps (a_1, a_2, a_3, a_4) & (a_1', a_2', a_3, a_4') or $(a_1' a_2 a_3' a_4)$ will are necessary in this case $\Big\}$ n-type

Anyway try (i) general A, B
(ii) A, B of $\underline{a}$ type ~~squares~~
(iii) A, B of n-type.

(*) This may also for a square which is $\underline{a}$ & not $\underline{n}$

See fig. 105 main book, p. 143

(*) Such 16 subsquares adding to 130 may hold for a square which is not $\underline{n}$ nor $\underline{a}$ ~~nor e~~

See fig. 268, p. 165 of Andrews

[the Bk. p. 51

1, 7, 3, 4, 8, 2, 6, 5	1, 7, 59, 60, 8, 2, 62, 61
1, 5, 2, 6, 8, 4, 7, 3	49, 37, 10, 38, 56, 36, 15, 27
8, 2, 6, 5, 1, 7, 3, 4	24, 10, 46, 53, 17, 15, 43, 52
8, 4, 7, 3, 1, 5, 2, 6	X
1, 7, 3, 4, 8, 2, 6, 5	
1, 5, 2, 6, 8, 4, 7, 3	Any two A & B do not work
8, 2, 6, 5, 1, 7, 3, 4	
8, 4, 7, 3, 1, 5, 2, 6	a.

256

1, 7, 3, 4, 8, 2, 6, 5	1, 7, 59, 60, 8, 2, 62, 61
1, 2, 6, 4, 8, 7, 3, 5	49, 10, 14, 52, 56, 15, 11, 53.
8, 2, 6, 5, 1, 7, 3, 4	24, 42, 46, 21, 17, 47, 43, 20
8, 7, 3, 5, 1, 2, 6, 4.	32, 31, 35, 37, 25, 26, 38, 36.
1, 7, 3, 4, 8, 2, 6, 5	57, 63, 3, 4, 64, 58, 6, 5
1, 2, 6, 4, 8, 7, 3, 5	9, 50, 54, 12, 16, 55, 51, 13
8, 2, 6, 5, 1, 7, 3, 4	47, 18, 22, 45, 41, 23, 19, 44.
8, 7, 3, 5, 1, 2, 6, 4	40, 39, 27, 29, 33, 34, 30, 28

1	7	59	60	8	2	62	61
49	10	14	52	56	15	11	53
24	42	46	21	17	47	43	20
32	31	35	37	25	26	38	36
57	63	3	4	64	58	6	5
9	50	54	12	16	55	51	13
48	18	22	45	41	23	19	44
40	39	27	29	33	34	30	28

works with $A = (a_1, a_2, a_3, a_4)$, $B = (a_1, a_2', a_3', a_4)$.

$(40 + 22 + 16 + 6 + 32 + 46 + 56 + 62 = 260)$

int. re 0 & e n ✓

not Jaina but affirms knight-move

(a_1, a_2', a_3, a_4')

1, 7, 3, 4, 8, 2, 6, 5
1, 2, 3, 5, 8, 7, 6, 4
8, 2, 6, 5, 1, 7, 3, 4
8, 7, 6, 4, 1, 2, 3, 5
1, 7, 3, 4, 8, 2, 6, 5.
1, 2, 3, 5, 8, 7, 6, 4
8, 2, 6, 5, 1, 7, 3, 4
8, 7, 6, 4, 1, 2, 3, 5

1	7	59	60	8	2	62	61
49	10	11	53	56	15	14	52
24	18	46	45	17	23	43	44
32	39	38	28	25	34	35	29
57	63	3	4	64	58	6	5
9	50	51	13	16	55	54	12
48	42	22	21	41	47	19	20
40	31	30	36	35	26	27	37

n ✓

always with $B = (a_1, a_2', a_3, a_4')$

not Jaina nor knight-move

int. re 0 & e

(30)

A, B, C, D.

X

$A = (a_1, a_2, a_3, a_4),\ B = (a_1, a_2', a_3, a_4'),\ C = (a_1, a_2', a_3', a_4),\ D = (a_1', a_2', a_3', a_4')$

$C' = (a_1', a_2, a_3', a_4')$

1, 2, 6, 4, 8, 7, 3, 5	1, 2, 6, 60, 8, 7, 3, 61 X. Does not work.
1, 7, 6, 5, 8, 2, 3, 4	1, 2, 62, 60, 8, 7, 59, 61
8, 2, 3, 4, 1, 7, 6, 5	9, 55, 14, 53, 16, 50, 11, 52
8, 7, 3, 5, 1, 2, 6, 4.	48, 42, 19, 20, 41, 47, 22, 21
1, 2, 6, 4, 8, 7, 3, 5	32, 39, 27, 37, 25, 34, 30, 38
1, 7, 6, 5, 8, 2, 3, 4	57, 58, 6, 4, 64, 63, 3, 5
8, 2, 3, 4, 1, 7, 6, 5	49, 15, 54, 13, 56, 10, 51, 12
8, 7, 3, 5, 1, 2, 6, 4.	24, 18, 43, 44, 17, 23, 46, 45
	40, 31, 35, 29, 33, 26, 38, 28

not Jaina

not knight needs

η ✓ with (a_1, a_2, a_3, a_4)

(a_1, a_2, a_3, a_4)

(a_1', a_2', a_3, a_4')

(a_1', a_2, a_3', a_4)

(a_1', a_2', a_3', a_4')

(a_1, a_2', a_3', a_4)

(a_1', a_2, a_3, a_4')

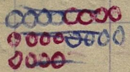
(a_1', a_2', a_3', a_4')

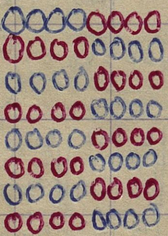
Try only 93. Bk 20 $\quad 40 + 43 + 56 + 3 + 32 + 19 + 16 + 59 = 268$

not knight's move

1, 8, 1, 8, 1, 8, 1, 8 $\quad$ 1, 2, 3, 4, 8, 7, 6, 5.

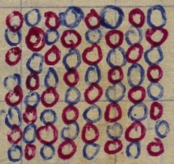
8, 7, 6, 5,

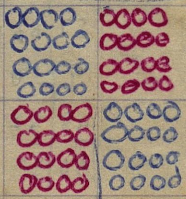




1	3	5	7	8	6	4	2
3	1	7	5	6	8	2	4
5	7	1	3	4	2	8	6
7	5	3	1	2	4	6	8
8	6	4	2				
6	8	2	4				
4	2	8	6				
2	4	6	8				

1, 19, 37, 55, 64, 46, 28, 10





5 → 1, 1 ← 5

1, 59, 5, 63, 8, 62, 4, 58 → 264 260
17, 43, 21, 47, 24, 46, 20, 42
33, 27, 37, 31, 40, 30, 36, 26.
49, 11, 53, 15, 56, 14, 52, 10
64, 6, 60, 2, 57, 3, 61, 7
48, 22, 44, 18, 41, 19, 45, 23
32, 38, 28, 34, 25, 35, 29, 39.
16, 54, 12, 50, 9, 51, 13, 55 → 256 260

12 16

Q_1	Q_2
Q_3	Q_4

	5	59	1	63	8	62	4	58	
o	17	43	21	47	24	46	20	42	e
	33	27	37	31	40	30	36	26	
	49	11	53	15	56	14	52	10	
	64	6	60	2	57	3	61	7	
	48	22	44	18	41	19	45	23	
e	32	38	28	34	25	35	29	39	o
	12	54	16	50	9	51	13	55	

(M) neither a nor n

Taking square on p. 96 of Bk. 20 ie the nasik square, I found that exchanging all odd ~~cols~~ rows in Q_2 with the even rows in Q_1 & similarly in Q_3 & Q_4, I found a magic square magic in both rows & cols, but the upper diagonal summed to 264 and the lower diagonal to 256. So by a mere fluke, I found that exchange the 16 & 12 in the bottom row & 5 and 1 in the top row made the diagonals correct and also kept the cols correct & obviously the rows are correct. But the resulting square above is no longer nasik nor is it associated. But it does not matter. What I wanted to find out ie (Q_2, Q_3) all e & (Q_1, Q_4) all o

(P)

5	3	1	7	8	6	4	2
1	3	5	7	8	6	4	2
1	3	5	7	8	6	4	2
1	3	5	7	8	6	4	2
8	6	4	2	1	3	5	7
8	6	4	2	1	3	5	7
8	6	4	2	1	3	5	7
4	6	8	2	1	3	5	7

+

(R)

0	56	0	56	0	56	0	56
16	40	16	40	16	40	16	40
32	24	32	24	32	24	32	24
48	8	48	8	48	8	48	8
56	0	56	0	56	0	56	0
40	16	40	16	40	16	40	16
24	32	24	32	24	32	24	32
8	48	8	48	8	48	8	48

(32) This splitting can be written for (P) & (R_0)

B	A'	E	E
A	A'	F	F
A	A'	G	G
A	A'	H	H
A'	A	E'	E'
A'	A	F'	F'
A'	A	G'	G'
B'	A	H'	H'

(P) (R_0)

The (R_0) is the same as (R_0) of p. 96, Bk. 20 which was obtained by the (P) of that page by $r \rightleftarrows c$.

So the adjoining scheme cannot be derived by any rational scheme or formula. The only thing we have done is a kind reversion trick due to the Hindus i.e. after getting the result (P) with A = (1, 3, 5, 7) & taking the scheme (A, A', A, A') & getting (R) by $r \rightleftarrows c$, we interchange the odd & even rows in Q_1 & Q_2 and in Q_3 & Q_4, and finally do the reversions of 1 & 5 in the first row of (Q_1) and of (8, 4) in the last row of Q_3.

Suppose in the (P) of p. 96, Bk. 20, we do the reverse interchange of so as to bring all even nos in Q_1 and odd ones in Q_2 & similarly odd ones in Q_3 & even ones in Q_4 & keep (R) the same. We have (P_1) as below in which rows & cols sum correctly to 36 but the upper diagonal sums to 32 while the lower diagonal sums to 40. If we interchange the 8 & 4 in first row of Q_1 of (P_1) and 1 & 5 in the last row of Q_3 of (P_1) we get a magic (P_1). Taking the same (R) as on p. 96, Bk. 20 we get the magic square

8, 6, 4, 2,	1, 3, 5, 7
8, 6, 4, 2,	1, 3, 5, 7
8, 6, 4, 2,	1, 3, 5, 7
8, 6, 4, 2,	1, 3, 5, 7
1, 3, 5, 7,	8, 6, 4, 2
1, 3, 5, 7,	8, 6, 4, 2
1, 3, 5, 7,	8, 6, 4, 2
1, 3, 5, 7,	8, 6, 4, 2.

(P_1)

4	62	8	58	1	59	5	63
24	46	20	42	17	43	21	47
40	30	36	26	33	27	37	31
56	14	52	10	49	11	53	15
~~64~~ 2	~~4~~ 27	~~60~~ 3	[illegible]	57 6	[illegible]	61	7
48	22	44	18	4[illegible]	19	45	23
32	38	28	34	25	35	29	39
13	51	9	55	16	54	12	50

4	62	8	58	1	59	5	63
24	46	20	42	17	43	21	47
40	30	36	26	33	27	37	31
56	14	52	10	49	11	53	15
57	3	61	7	64	6	60	2
41	19	45	23	48	22	44	18
25	35	29	39	32	38	28	34
13	51	9	55	16	54	12	50

We get a square in which Q_2 & Q_3 contain all odd nos & Q_1 & Q_4 contain the even numbers. This is a new analogue of the case mentioned by Cammann [[illegible] note book, p. 186] who remarks that a study of the basic patterns of such squares as above can serve as a clue to their construction method & also will be useful in classifying or identifying unfamiliar squares. [Ref. G.K, p. 378]. I don't think Cammann would have ever guessed this method of getting an n-square using (A, A', A, A', A, A', A, A') and getting (P) & (R) and doing the unbroken reversions & the two broken reversions.

It looks as if we can take for A four odd numbers, construct (P) with choices (A, A', A, A'), (A, A', A', A), (A, A, A', A') & perform the above reversions. We have done this above for only one choice (A, A', A, A') & using the nasik scheme. Does this work for associated also?

Try A = (1, 3, 5, 7) for associated square.

(34) $A = (1,3,5,7)$ to get associated square with $(A, A', A, A', A', A, A', A)$ for Q_1, Q_3

and $(\bar{A}', \bar{A}, \bar{A}', \bar{A}, \bar{A}, \bar{A}', \bar{A}, \bar{A}')$ for Q_2, Q_4.

1	3	5	7	2	4	6	8
8	6	4	2	7	5	3	1
1	3	5	7	2	4	6	8
8	6	4	2	7	5	3	1
8	6	4	2	7	5	3	1
1	3	5	7	2	4	6	8
8	6	4	2	7	5	3	1
1	3	5	7	2	4	6	8

(P)

0	56	0	56	56	0	56	0
16	40	16	40	40	16	40	16
32	24	32	24	24	32	24	32
48	8	48	8	8	48	48	8
8	48	8	48	48	8	8	48
24	32	24	32	32	24	32	24
40	16	40	16	16	40	16	40
56	0	56	0	0	56	0	56

(R)

1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
8	6	4	2	7	5	3	1
2	4	6	8	1	3	5	7
8	6	4	2	7	5	3	1
2	4	6	8	1	3	5	7

(P_1)

Change (P) to (P_1) by unbroken reversions. (P_1) is magic in rows & cols. Upper diagonal sums to 40 & lower diagonal to 32. The broken reversion of 1 & 5 in first row of Q_1 ~~and~~ has no analogous broken reversion to last row of Q_3 to keep the cols. magic & certainly other such reversion in this last row does not make l.d magic. Suppose we interchange 4 & 8 in first row of Q_2. It appears that no such reversions are possible for this case. Try some other case of $\underline{\Pi}$ & weight with (A, A', A', A)

$A, A', A', A, A, A', A', A$

(P)

4	6	8	2	5	3	1	7
5	3	1	7	4	6	8	2
5	3	1	7	4	6	8	2
4	6	8	2	5	3	1	7
4	6	8	2	5	3	1	7
5	3	1	7	4	6	8	2
5	3	1	7	4	6	8	2
4	6	8	2	5	3	1	7

(R)

24	32	32	24	24	32	32	24
40	16	16	40	40	16	16	40
56	0	0	56	56	0	0	56
8	48	48	8	8	48	48	8
32	24	24	32	32	24	24	32
16	40	40	16	16	40	40	16
0	56	56	0	0	56	56	0
48	8	8	48	48	8	8	48

5	3	1	7	4	6	8	2
5	3	1	7	4	6	8	2
5	3	1	7	4	6	8	2
5	3	1	7	4	6	8	2
4	6	8	2	5	3	1	7
4	6	8	2	5	3	1	7
4	6	8	2	5	3	1	7
4	6	8	2	5	3	1	7

(P_1)

~~5~~ 1	~~3~~ 7	~~1~~ 5	~~7~~ 3	4	6	8	2
5	7	1	3	8	6	4	2
5	3	1	7	4	6	8	2
5	3	1	7	4	6	8	2
~~4~~ 8	~~6~~ 2	~~8~~ 4	~~2~~ 6	5	3	1	7
4	6	8	2	5	3	1	7
4	6	8	2	5	3	1	7
4	2	8	6	1	3	5	7

(P_2)

	35						
29	37	33	31	28	38	40	26
45	23	17	43	48	22	20	42
61	3	1	63	60	6	8	58
13	51	49	15	12	54	56	10
36	30	32	34	37	27	25	39
20	46	48	18	21	43	41	23
4	62	64	2	5	59	60	7
52	10	16	54	49	11	13	55

making changes in (P_2) & adding to (R) gives different result

~~long repetitions~~

~~unless the bottom reversions are symmetrical there is no use~~

28	38	40	26	29	35	33	31
45	19	17	47	44	22	24	42
61	3	1	63	60	6	8	58
12	54	56	10	13	51	49	15
36	30	32	34	37	27	25	39
21	43	41	23	20	46	48	18
5	59	57	7	4	62	64	2
52	14	16	50	53	11	9	55

	33	31	29	35					
	29	35	33	31	28	38	40	26	
	45	19	17	47	44	22	24	42	
	61	3	1	63	60	6	8	58	
	13	51	49	15	12	54	56	10	
32, 36	36	30	32	34	37	27	25	39	~~34, 38~~
	20	46	48	18	21	43	41	23	no result
	4	62	64	2	5	59	57	7	
	52	14	16	50	53	11	9	55	

Note — from (P) set (P_1). Add (P_1) to (R) and then make the [illegible] reversions. Don't make these in (P_1) itself a set (P_2). Let us

$A = (1, 3, 5, 7)$

try the associative case again with $(A, A', A, A', A', A, A', A)$ &

1	3	5	7	2	4	6	8
8	6	4	2	7	5	3	1
1	3	5	7	2	4	6	8
8	6	4	2	7	5	3	1
8	6	4	2	7	5	3	1
1	3	5	7	2	4	6	8
8	6	4	2	7	5	3	1
1	3	5	7	2	4	6	8

(P)

0	56	0	56	56	0	56	0
16	40	16	40	40	16	40	16
32	24	32	24	24	32	24	32
48	8	48	8	8	48	8	48
8	48	8	48	48	8	48	8
24	32	24	32	32	24	32	24
40	16	40	16	16	40	16	40
56	0	56	0	0	56	0	56

(R)

1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
7	5	3	1	8	6	4	2
1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
1	3	5	7	2	4	6	8

~~(P_1)~~

~~In (P_1) rows & diagonals sum to 36 but not cols.~~

(256 | (256 | 256 | 256

1	59	5	63	58	4	62	8
23	45	19	41	48	22	44	18
33	27	37	31	26	36	30	40
55	13	51	9	16	54	12	50
15	53	11	49	56	14	52	10
25	35	29	39	34	28	38	32
47	21	43	17	24	46	20	42
57	3	61	7	2	60	6	64

256

$(P_1) + (R) = (M_4)$

1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
1	3	5	7	2	4	6	8
7	5	3	1	8	6	4	2
8	6	4	2	7	5	3	1
2	4	6	8	1	3	5	7
8	6	4	2	7	5	3	1
2	4	6	8	1	3	5	7

(P_1)

1	59	5	63	58	4	62	8
23	45	19	41	48	22	44	18
33	27	37	31	24	36	30	40
55	13	51	9	16	54	12	50
16	54	12	50	55	13	51	9

$(P_1) + (R)$

X Repetition

So it is clear that associate square does not give the G-K type

Let us try one more nasik case with $A = (1, 7, 7, 1)$ & (A, A, A', A')

1	7	7	1	8	2	2	8
1	7	7	1	8	2	2	8
8	2	2	8	1	7	7	1
8	2	2	8	1	7	7	1
1	7	7	1	8	2	2	8
1	7	7	1	8	2	2	8
8	2	2	8	1	7	7	1
8	2	2	8	1	7	7	1

(P)

0	0	56	56	0	0	56	56
48	48	8	8	48	48	8	8
48	48	~~56~~ 8	~~56~~ 8	48	48	8	8
0	0	56	56	0	0	56	56
56	56	0	0	56	56	0	0
8	8	48	48	8	8	48	48
8	8	48	48	8	8	48	48
56	56	0	0	56	56	0	0

(R)

(37)

1	7	7	1	8	2	2	8
1	7	7	1	8	2	2	8
1	7	7	1	8	2	2	8
1	7	7	1	8	2	2	8
8	2	2	8	1	7	7	1
8	2	2	8	1	7	7	1
8	2	2	8	1	7	7	1
8	2	2	8	1	7	7	1

(P_1)

This leads to repetition

Choose some other A, say $A = (1, 5, 7, 3)$, scheme (A, A, A', A').

(P)

1	5	7	3	8	4	2	6
1	5	7	3	8	4	2	6
8	4	2	6	1	5	7	3
8	4	2	6	1	5	7	3
1	5	7	3	8	4	2	6
1	5	7	3	8	4	2	6
8	4	2	6	1	5	7	3
8	4	2	6	1	5	7	3

(P_1)

1	5	7	3	8	4	2	6
1	5	7	3	8	4	2	6
1	5	7	3	8	4	2	6
1	5	7	3	8	4	2	6
8	4	2	6	1	5	7	3
8	4	2	6	1	5	7	3
8	4	2	6	1	5	7	3
8	4	2	6	1	5	7	3

(R)

0	0	56	56	0	0	56	56
32	32	24	24	32	32	24	24
48	48	8	8	48	48	8	8
16	16	40	40	16	16	40	40
56	56	0	0	56	56	0	0
24	24	32	32	24	24	32	32
8	8	48	48	8	8	48	48
40	40	16	16	40	40	16	16

264

$(P_1) + (R) = (M_1)$

1	5	63	59	8	4	58	62
33	37	31	27	40	36	26	30
49	53	15	11	56	52	10	14
17	21	47	43	24	20	42	46
64	60	2	6	57	61	7	3
32	28	34	38	25	29	39	35
16	12	50	54	4	13	55	51
48	44	18	22	41	45	23	19

256

(M) is given by interchanging 1 & 5 in first row of (M_1), and 4 & 44 in last row of (M_1)

Let us now go to cases $n = 12$ and $n = 16$.

But before doing this find out ways in which odd & even numbers are distributed in 4^{th} order squares.

(a) Associated.

(i) (P)

1	4	3	2
2	3	4	1
4	1	2	3
3	2	1	4

(ii) (P)

1	4	4	1
2	3	3	2
3	2	2	3
4	1	1	4

(iii) (P)

1	4	2	3
4	1	3	2
3	2	4	1
2	3	1	4

(iv) (P)

1	4	3	2
4	1	2	3
2	3	4	1
3	2	1	4

(v) (P)

1	2	4	3
3	4	2	1
4	3	1	2
[illegible]	1	3	4

~~[illegible] R_2, R_3, [illegible] R_4, R_5 [illegible].~~

~~[illegible]~~

Take a [illegible] [illegible]

(a) (P)

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4

(R)

0	12	12	0
8	4	4	8
4	8	8	4
12	0	0	12

(P_1)

1	3	2	4
3	1	4	2
4	2	3	1
2	4	1	3

$(P_1) + (R)$

1	15	14	4
11	5	8	10
8	10	11	5
14	4	1	15

repeated.

to make n

(P)

1	3	2	4
4	2	3	1
3	1	4	2
2	4	1	3

(R)

0	12	8	4
8	4	0	12
4	8	12	0
12	0	4	8

(P_1)

1	3	2	4
3	1	4	2
4	2	3	1
2	4	1	3

1	15	10	8
11	5	4	14
8	10	15	1
14	4	5	11

repeated - as such forms for 4th order squares

(39) Possible types are

n^2+1

(a) For associative squares.

(i)	(ii)	(iii)
o e e o	o e o e	o o e e
e o o e	e o e o	e e o o
o e e o	e o e o	e e o o
e o o e	o e o e	o o e e

(b) nasik squares.

also same three types for q → of type (iii) above

1	15	6	12
14	4	9	7
11	5	16	2
8	10	3	13

(N)

(c) Other types:

Plane 3, p. 179 of Andrews.

1, 13, 4, 16
8, 12, 5, 9
14, 2, 15, 3
11, 7, 10, 6.

(P)

1	1	4	4
4	4	1	1
2	2	3	3
3	3	2	2

(R)

0	12	0	12
4	8	4	8
12	0	12	0
8	4	8	4

(P_1)

1	1	4	4
1	1	4	4
2	2	3	3
2	2	3	3

$(P_1)+(R)$

1	13	4	16
5	9	8	12
14	2	15	3
10	6	11	7

Both cols & diagonals not magic – impossible

Plane 4

X
1, 7, 14, 12
5, 3, 10, 16
4, 6, 15, 9
8, 2, 11, 13

14 12, 15 9
10 16 11, 13
4 6
8 2

Plane 9

X
5, 1, 12, 16
3, 7, 10, 14
15, 11,
13, 9

Plane 10

12, 4, 3, 5
16, 8, 1, 9 X
15, 7, 2, 10
3, 11, 6, 14.

o o e e	e o e o
e o e o	e o e o
o e o e	o e o e
e e o o	o e o e

no such type as in 8×8.

$n = 12$ Andrews Fig. 98, p. 45. Splitting this up into (P) & (R) is a colossal job. $(1, 2, 3, \ldots 10, 11, 12)$
$0, 12, 24, \ldots 108, 120, 132$

A	B	C
D	E	F
G	H	I

Splitting A, B, C ... I separately

$$B = \begin{vmatrix} 1, & 15, & 14, & 4 \\ 12, & 6, & 7, & 9 \\ 8, & 10, & 11, & 5 \\ 13, & 3, & 2, & 16. \end{vmatrix}$$ an associated square

$$B = \begin{vmatrix} 1, & 3, & 2, & 4 \\ 12, & 6, & 7, & 9 \\ 8, & 10, & 11, & 5 \\ 1, & 3, & 2, & 4 \end{vmatrix} + \begin{vmatrix} 0 & 12 & 12 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 12 \end{vmatrix} \quad I = \begin{vmatrix} 5, & 7, & 6, & 8 \\ 4, & 10, & 11, & 1 \\ 12, & 2, & 3, & 9 \\ 5, & 7, & 6, & 8 \end{vmatrix} + \begin{vmatrix} 12, & 24, & 24, & 12 \\ 24, & 12, & 12, & 24 \\ 12, & 24, & 24, & 12 \\ 24, & 12, & 12, & 24 \end{vmatrix}$$

$$D = \begin{vmatrix} 9, & 11, & 10, & 12 \\ 8, & 2, & 3, & 5 \\ 4, & 6, & 7, & 1 \\ 9, & 11, & 10, & 12 \end{vmatrix} + \begin{vmatrix} 24, & 36, & 36, & 24 \\ 36, & 36, & 36, & 36 \\ 36, & 36, & 36, & 36 \\ 36, & 24, & 24, & 36 \end{vmatrix}, \quad G = \begin{vmatrix} 1, & 3, & 2, & 4 \\ 12, & 6, & 7, & 9 \\ 8, & 10, & 11, & 5 \\ 1, & 3, & 2, & 4 \end{vmatrix} + \begin{vmatrix} 48, & 60, & 60, & 48 \\ 48, & 48, & 48, & 48 \\ 48, & 48, & 48, & 48 \\ 60, & 48, & 48, & 60 \end{vmatrix}$$

$$E = \begin{vmatrix} 5, & 7, & 6, & 8 \\ 4, & 10, & 11, & 1 \\ 12, & 2, & 3, & 9 \\ 5, & 7, & 6, & 8 \end{vmatrix} + \begin{vmatrix} 60, & 72, & 72, & 60 \\ 72, & 60, & 60, & 72 \\ 60, & 72, & 72, & 60 \\ 72, & 60, & 60, & 72 \end{vmatrix}; \quad C = \begin{vmatrix} 9, & 11, & 10, & 12 \\ 8, & 2, & 3, & 5 \\ 4, & 6, & 7, & 1 \\ 9, & 11, & 10, & 12 \end{vmatrix} + \begin{vmatrix} 72, & 84, & 84, & 72 \\ 84, & 84, & 84, & 84 \\ 84, & 84, & 84, & 84 \\ 84, & 72, & 72, & 84 \end{vmatrix}$$

$$F = \begin{vmatrix} 1, & 3, & 2, & 4 \\ 12, & 6, & 7, & 9 \\ 8, & 10, & 11, & 5 \\ 1, & 3, & 2, & 4 \end{vmatrix} + \begin{vmatrix} 96, & 108, & 108, & 96 \\ 96, & 96, & 96, & 96 \\ 96, & 96, & 96, & 96 \\ 108, & 96, & 96, & 108 \end{vmatrix}; \quad A = \begin{vmatrix} 5, & 7, & 6, & 8 \\ 4, & 10, & 11, & 1 \\ 12, & 2, & 3, & 9, \\ 5, & 7, & 6, & 8 \end{vmatrix} + \begin{vmatrix} 108, & 120, & 120, & 108 \\ 120, & 108, & 108, & 120 \\ 108, & 120, & 120, & 108 \\ 120, & 108, & 108, & 120 \end{vmatrix}$$

(41)

$$H = \begin{vmatrix} 9, & 11, & 10, & 12 \\ 8, & 2, & 3, & 5, \\ 4, & 6, & 7, & 1, \\ 9, & 11, & 10, & 12 \end{vmatrix} + \begin{vmatrix} 120, & 132, & 132, & 120 \\ 132, & 132, & 132, & 132 \\ 132, & 132, & 132, & 132 \\ 132, & 120, & 120, & 132, \end{vmatrix}$$

5	7	6	8	1	3	2	4	9	11	10	12
4	10	11	1	12	6	7	9	8	2	3	5
12	2	3	9	8	10	11	5	4	6	7	1
5	7	6	8	1	3	2	4	9	11	10	12
9	11	10	12	5	7	6	8	1	3	2	4
8	2	3	5	4	10	11	1	2	6	7	9
4	6	7	1	12	2	3	9	8	10	11	5
9	11	10	12	5	7	6	8	1	3	2	4
1	3	2	4	9	11	10	12	5	7	6	8
12	6	7	9	8	2	3	5	4	10	11	1
8	10	11	5	4	6	7	1	12	2	3	9
1	3	2	4	9	11	10	12	5	7	6	8

(P)

108	120	120	108	12	24	24	12	72	84	84	72
120	108	108	120	24	12	12	24	84	84	84	84
108	120	120	108	12	24	24	12	84	84	84	84
120	108	108	120	24	12	12	24	84	72	72	84
24	36	36	24	60	72	72	60	96	108	108	96
36	36	36	36	72	60	60	72	96	96	96	96
36	36	36	36	60	72	72	60	96	96	96	96
36	24	24	36	72	60	60	72	108	96	96	108
48	60	60	48	120	132	132	120	12	24	24	12
48	48	48	48	132	132	132	132	24	12	12	24
48	48	48	48	132	132	132	132	12	24	24	12
60	48	48	60	132	120	120	132	24	12	12	24

(R)

This is good enough only to go under "Composite squares"

$n = 16$

$n^2+1 = 257$

$S = \frac{1}{2} 16 \cdot 257$
$= 2056$

Q_1	Q_2
Q_3	Q_4

1, -- 64
65, -- 128
129, -- 192
193, -- 256

But what about nasik & associated squares of order 8 & squares neither associated nor nasik.

(43) 8 × 8

~~Q.S.~~ n squares from Andrews are. More systematic search

(A) Earlier associated ones.

(1) Fig. 53, p. 25 – Constructed by some ad hoc method

(2) Fig. 56, p. 27 – "

(3) Fig. 58, p. 28 – "

(4) Fig. 60, p. 28 – "

(B) Earlier ones by the N – method – associated

(1) Fig. 91, p. 43

(2) Fig. 94, "

(3) Fig. 95 "

(C) Franklin squares.

(1) Fig. 183, p. 90

(2) Fig. 191, p. 97

(3) Fig. 197, p. 101

(4) Fig. 203, p. 106

Not magic squares at all

(D) Chap. IV – Reflections on magic squares

(1) Fig. 218, p. 120

(2) Fig. on p. 121.

(E) Chap. VII – Curious magic squares

(1) Fig. 261, p. 165 – neither a nor n
(2) Fig. 262, p. 165 – n
(3) Fig. 265, p. 167 – neither a nor n
(4) Fig. 267, p. 169 – n
(5) Fig. 268, p. 169 – neither a nor n
(6) Fig. 269, p. 170 – neither a nor n
(7) Fig. 270, p. 170 – neither a nor n
× ~~(8) Fig. 281, p. 175 – not magic in diagonals but continuous knight's tour~~

~~(b) Chap. V – Various kinds of magic squares.~~

(F) Chap XI – Summary methods

~~(1) Fig. ... p. 255~~
(1) Fig. 449, p. 256. – a & n
(2) Fig. 468, p. 266. – a & n

(G) Chap. XII – Theory of reversions (C.P)

(1) Fig. 567, p. 296 – a

(H) Chap XV – Ornate magic squares

(1) Fig. 695, p. 377 – n
× (2) Fig. 715, p. 390 × nor n
(3) Fig. 716, p. 391 a & n
× (4) Fig. 717, " × nor n
(5) Fig 720, " a & n
(6) Fig. 732, p. 396 – n
(7) Fig. 734, p. 397 n
(8) Fig. 750, p. 410 n

Altogether 32 squares! to be examined.

(45) (A)

(1) 1, 7, 6, 4, 5, 3, 2, 8   0, 56, 56, 0, 0, 56, 56, 0
8, 2, 3, 5, 4, 6, 7, 1   48, 8, 8, 48, 48, 8, 8, 48
8, 2, 3, 5, 4, 6, 7, 1   40, 16, 16, 40, 40, 16, 16, 40
1, 7, 6, 4, 5, 3, 2, 8   24, 32, 32, 24, 24, 32, 32, 24
— — —   — — —
(P)   (R)

(P) is (A, A', A', A, A A' A' A) with A = (1, 7, 6, 4) & (R) from (P) by r ⇄ c

(2) 1, 7, 3, 4, 5, 6, 2, 8   0, 0, 56, 56, 56, 56, 0, 0
8, 2, 6, 5, 4, 3, 7, 1   8, 8, 48, 48, 48, 48, 8, 8
8, 7, 3, 5, 4, 6, 2, 1   40, 40, 16, 16, 16, 16, 40, 40
1, 2, 6, 4, 5, 3, 7, 8   32, 32, 24, 24, 24, 24, 32, 32
1, 2, 6, 4, 5, 3, 7, 8   24, 24, 32, 32, 32, 32, 24, 24
8, 7, 3, 5, 4, 6, 2, 1   16, 16, 40, 40, 40, 40, 16, 16
8 — (P) — — —   48 — (R) — — —
1   56,

P is (A, A', B, B', B', B, A', A) where A = (1, 7, 3, 4), B = (8, 7, 3, 5)
$= (a_1, a_2, a_3, a_4)$ $= (a_1', a_2, a_3, a_4')$

~~R is~~
R_0 with rows & cols. reversed is

1, 2, 6, 5, 4, 3, 7, 8
1, 2, 6, 5, 4, 3, 7, 8
8, 7, 3, 4, 5, 6, 2, 1
8, 7, 3, 4, 5, 6, 2, 1

R_0 is

1, 1, 8, 8, 8, 8, 1, 1
2, 2, 7, 7, 7, 7, 2, 2
6, 6, 3, 3, 3, 3, 6, 6
5, 5, 4, 4, 4, 4, 5, 5
4, 4, 5, 5, 5, 5, 4, 4

i.e. (A, A, A', A', A', A', A, A) where A = (1, 2, 6, 5)

~~Instead~~ Suppose we add the (R) obtained from (R_0) by r ⇄ c to (P) i.e.

0, 8, 40, 32, 24, 16, 48, 56
0, 8, 40, 32, 24, 16, 48, 56
56, 48, 16, 24, 32, 40, 8, 0
56, 48, 16, 24, 32, 40, 8, 0
56, 48, -
56, 48. -
0, 8 -
0, 8. —

1, 15, 43, 36, 29, 22, 50, 64 not an associate square

Suppose we add to (P), (R) obtained from (P) by r ⇄ c.

1, 7, 3, 4, 5, 6, 2, 8	1, 63, 59, 4, 5, 62, 58, 8
8, 2, 6, 5, 4, 3, 7, 1	56, 10, 54, 13, 12, 51, 15, 49
8, 7, 3, 5, 4, 6, 2, 1	24, 47, 19, 45, 44, 22, 42, 17
1, 2, 6, 4, 5, 3, 7, 8	25, 34, 38, 28, 29, 35, 39, 32
1, 2, 6, 4, 5, 3, 7, 8	33, 26, 30, 36, 37, 27, 31, 40
8, 7, 3, 5, 4, 6, 2, 1	48, 23, 43, 21, 20, 46, 18, 41
8, 2, 6, 5, 4, 3, 7, 1	16, 50, 14, 53, 52, 11, 55, 9
1, 7, 3, 4, 5, 6, 2, 8	57, 7, 3, 60, 61, 6, 2, 64

we get an associate square all right

If for (P_1), we get (R_1) by interchange of rows & cols. & substitution of right no., $(P_1) + (R_1) = (M_1)$. Similarly for (R_2) & (R_2). But we can take $(P_1) + (R_2)$ or $(P_2) + (R_1)$ also give associated squares if (P_1) & (P_2) & hence (R_1) & (R_2) are also associated.

(3)

1, 7, 6, 5, 4, 3, 2, 8	0, 0, 56, 56, 57, 56, 0, 0.
1, 2, 6, 5, 4, 3, 7, 8	48, 8, 8, 48, 48, 8, 8, 48
8, 2, 3, 4, 5, 6, 7, 1	40, 40, 16, 16, 16, 16, 40, 40
8, 7, 3, 4, 5, 6, 2, 1	32, 32, 24, 24, 24, 24, 32, 32
8	
8	
1	
1	

(R1)

(P)

i.e. case of $(A, B, A', B', B', A', B, A)$ & (R_0) obtained from (P) by r ⇄ c.

(4)

(47)

(4) 1, 7, 3, 4, 5, 6, 2, 8
8, 2, 6, 5, 4, 3, 7, 1
8, 7, 3, 5, 4, 6, 2, 1
~~1, 2, 6, 4, 5, 3, 7, 8~~
1, 2, 6, 4, 5, 3, 7, 8
8,
8,
1

0, 56, 56, 0, 0, 56, 56, 0
48, 8, 48, 8, 8, 48, 8, 48
16, 40, 16, 40, 40, 16, 40, 16
24, 32, 32, 24, 24, 32, 32, 24
32, 24, 24, 32, 32, 24, 24, 32

ie (A, A', B, B', B', B, A', A) & R_0 is (P) r ⇄ c
A = (1, 7, 3, 4), B = (8, 7, 3, 5)

(B)

(1) (A, A', A', A, A, A', A', A) where A = (1, 7, 6, 4) & (R_0) from P by r ⟷ c.

(2) (A, A, A', A', A', A', A, A) where A = (1, 2, 6, 5) & (R_0) "

(3) (P) of Ex (1) combines with (R) of Ex (2).

(C) (1) 4, 5, 4, 5, 4, 5, 4, 5
6, 3, 6, 3, 6, 3, 6, 3

48, 56, 0, 8, 16, 24, 32, 40
8, 0, 56, 48, 40, 32, 24, 16.

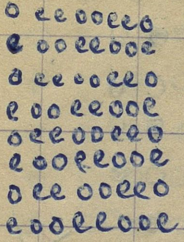
(2)
(3)
(4)
These Franklin squares are not magic squares at all. The diagonals are not magic.

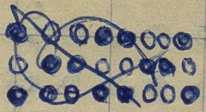
(D) (1) This is merely (A, A', B, B', B', B, A', A) with A = (1, 7, 3, 4), B = (8, 7, 3, 5).
& (R) from (P) by r ⟷ c.

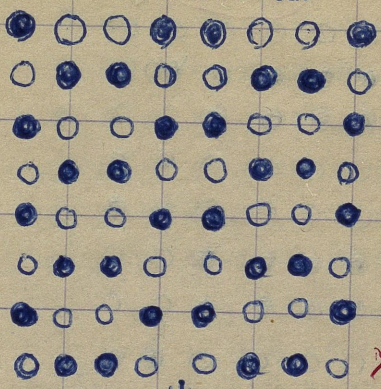
~~Fig 121 is~~ & this is equivalent to changing the original square entries by exchanging complementaries

Fig on p. 120 showing the schemes of 0, ro, i, & ri may be compared with the scheme for distribution of ~~p~~ odd & even numbers

Take square on p. 95, Fig. 20 & interchange rows & cols & taking black ⊙ for odd nos & white for even numbers giving the scheme below (48)

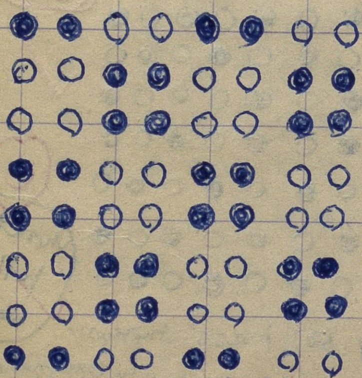






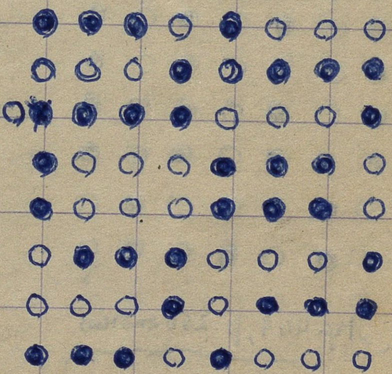
& this scheme is not exactly the same as that Fig. 218, p. 120 of Andrews, but differs from it in the orientation of the black & white ⊙ in the columns.

Figs. 91 & 95, p. 43 of Andrews



(2) Choose

Fig. 56, p. 27 of Andrews.



(3)

Figs. 261 & 262 on p. 165 of Andrews are also very interesting

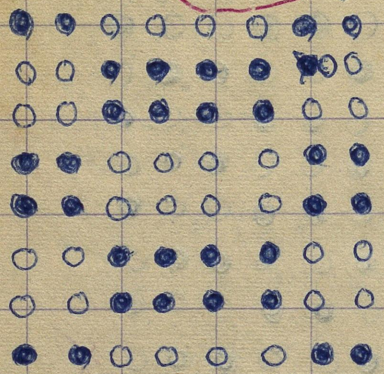
See next page.

Chosen Fig. 106. Main Book — or Fig. 133, p. 157 Main Book

(49)

Choose

Chosen Fig. 119, p. 150 main book



M Fig. 261, p. 165, Andrews (4)

~~Also fig p. 99, Bk. 20~~ (Bottom fig. p. 28, Bk. 4 rotated)

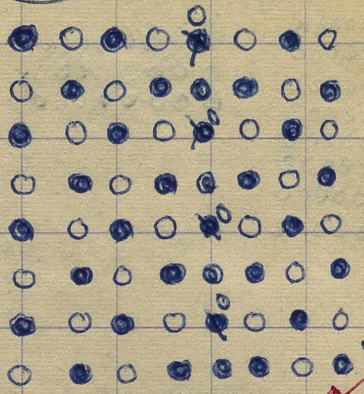
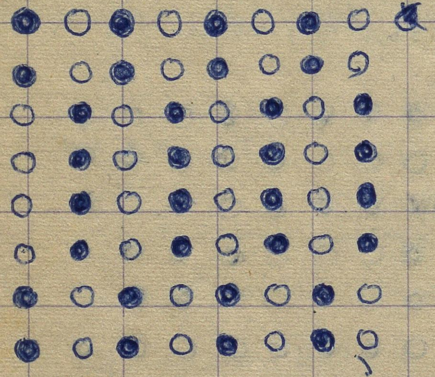


Fig. 262, p. 165 Andrews (5)

C

alternate o & e completely

Choose



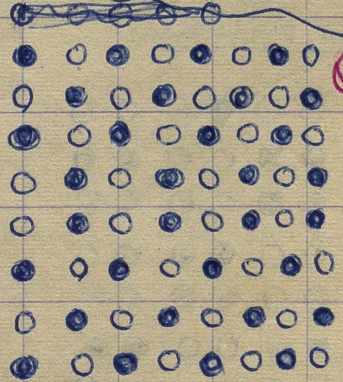
Choose Fig. 449, p. 256 Andrews

M Also fig. p. 99, Bk. 20, r ↔ c

(6)

Chosen under asm squares

Also fig 716, p. 391 Andrews



C

Chose fig. 115 p. 148.2 main book

Fig. 567, p. 206. Andrews

This is of the type mentioned by Cammann.

Also fig. on p. 93, Bk. 20.

Also lower fig p. 23, Bk. 21 (this book)

Also '

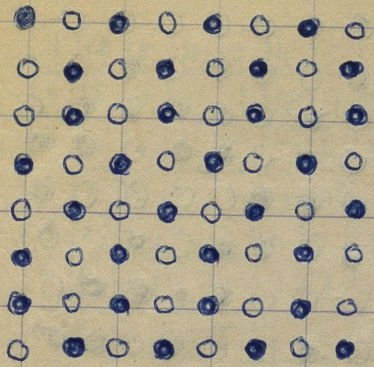
(7)

Also Chose fig. 126 ~~p. 152 main book~~ as an option

(50)

Choose

Fig. 132 p157 mainbook



Figs 732 & 734, pp. 396-97 Andrews.

(8)

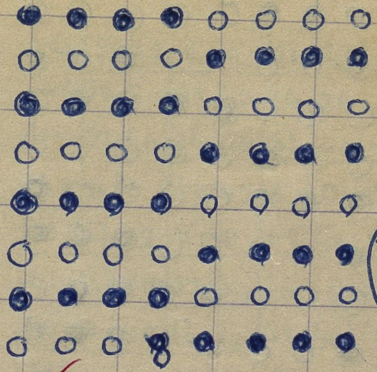


Fig. 750, p. 410 Andrews with r ↔ c. (9)

Same as Fig. p. 96, Bk. 20.

Also Fig. p. 23, Bk. 21 (top fig)

Also Fig. p. 27, " (bottom right fig

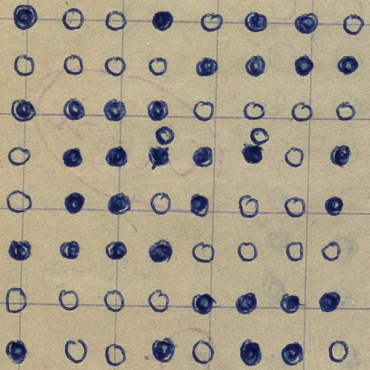
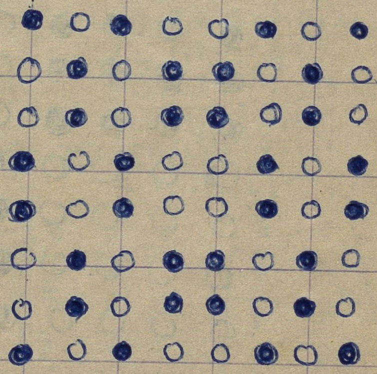


Fig. p. 53, Bk. 21 (10)

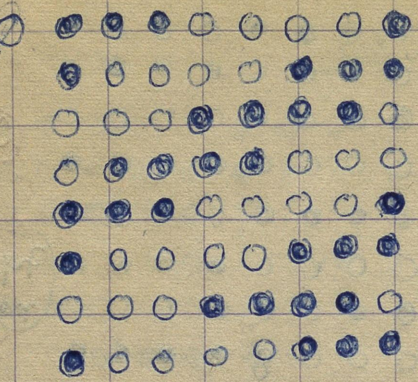
Also Fig p. 26 "



Top Fig p. 28, Bk. 21. (11)

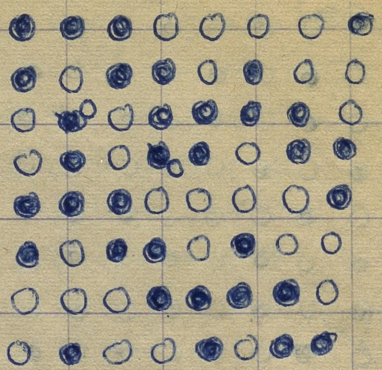
Many of these are not entirely different. They appear to be the same when rows & cols. are interchanged. Pick them out & arrange. Some more follow.

(51)



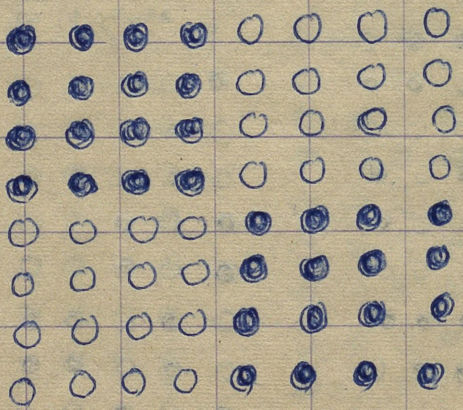
Top Fig. p. 29 – Bk. 21

(12) X



Bottom Fig. p. 29 – Bk. 21

(13) X



Choose

C

Fig. p. 33, Bk. 21

(14)

~~[Choose – Nos. 14, 5, 7, 9, 4, 6, 2]~~

E (1) The square is neither a nor n fig. 261, p. 165 – algebraic sum

Splitting it up. a, a_1, a_2, a_3, a_4 to 520.

1	3	8	6	2	4	5	7
6	8	3	1	7	5	4	2
8	6	1	3	7	5	4	2
3	1	6	8	2	4	5	7
3	1	6	8	4	2	7	5
8	6	1	3	5	7	2	4
6	8	3	1	5	7	2	4
1	3	3	6	4	2	7	5

(P)

0	56	48	8	0	56	48	8
40	16	24	32	40	16	24	32
24	32	40	16	24	32	40	16
48	8	0	56	48	8	0	56
0	56	48	8	0	56	48	8
40	16	24	32	40	16	24	32
24	32	40	16	24	32	40	16
48	8	0	56	48	8	0	56

(R)

5, 7, 2, 4

Scheme for P: $(A, \bar{A}, A', \bar{A}', \bar{A}', A', \bar{A}\ A)$ $A = 1, 3, 8, 6$, $B = (2, 4, 5, 7)$ 8, 6, 1, 3.

$\cancel{(B, \bar{B}, \bar{B}, B}, (B, B', B', B, C, C', C', C)$ $C = (4, 2, 7, 5)$; $\bar{C} = 5, 7, 2, 4 = C'$.

" R_0: (1, 8, 7, 2 | 1, 6, 4, 7,
6, 3, 4, 5 | 8, 3, 5, 2
4, 5, 6, 3. | 7, 4, 6, 1
7, 2, 1, 8. | 2, 5, 3, 8

1, 6, 4, 7, 1, 6, 4, 7
8, 3, 5, 2, 8, 3, 5, 2
7, 4, 6, 1, 7, 4, 6, 1
2, 5, 3, 8, 2, 5, 3, 8
1, 6, 4, 7, 1, 6, 4, 7
8, 3, 5, 2, 8, 3, 5, 2
7, 4, 6, 1, 7, 4, 6, 1
2, 5, 3, 8, 2, 5, 3, 8

R_0 with $r \leftrightarrow c$ is $(D, D', \bar{D}, \bar{D}', D, D', \bar{D}, \bar{D}')$

(P) written in full $\begin{pmatrix} A & B \\ \bar{A} & B' \\ A' & B' \\ \bar{A}' & B \\ \bar{A}' & C \\ A' & C' \\ \bar{A} & C' \\ A & C \end{pmatrix}$ (R_0) with $r \leftrightarrow c$. $\begin{pmatrix} D & D \\ D' & D' \\ \bar{D} & \bar{D} \\ \bar{D}' & \bar{D}' \\ D & D \\ D' & D' \\ \bar{D} & \bar{D} \\ \bar{D}' & \bar{D}' \end{pmatrix}$ R_0 is $\begin{pmatrix} E & E \\ F & F \\ \bar{F}' & F' \\ \bar{E}' & \bar{E}' \\ E & E \\ F' & F' \\ \bar{F}' & \bar{F}' \\ \bar{E}' & \bar{E}' \end{pmatrix}$

There seems to no way to find out ~~whether (P) is associative~~ unless the method. (P) is neither $\underline{n}$ or $\underline{G}$. (R) is associative).

Suppose we take one of our simple $\underline{G}$ form and add to it a $\underline{n}$-form with $r < x$.

$$
\begin{vmatrix}
1,2,3,4,5,6,7,8\\
8,7,6,5,4,3,2,1\\
8,7,6,5,4,3,2,1\\
1,2,3,4,5,6,7,8\\
1,2,3,4,5,6,7,8\\
8,7,6,5,4,3,2,1\\
8,7,6,5,4,3,2,1\\
1,2,3,4,5,6,7,8
\end{vmatrix}
+
\begin{vmatrix}
1,7,6,4,8,2,3,5\\
8,2,3,5,1,7,6,4\\
1,7,6,4,8,2,3,5\\
8,2,3,5,1,7,6,4\\
1,7,6,4,8,2,3,5\\
8,2,3,5,1,7,6,4\\
1,7,6,4,8,2,3,5\\
8,2,3,5,1,7,6,4
\end{vmatrix}
$$

(P) (a) (R)

+

$$
\begin{vmatrix}
0,56,0,56,0,56,0,56\\
48,8,48,8,48,8,48,8\\
40,16,40,16,40,16,40,16\\
24,32,24,32,24,32,24,32\\
56,0,56,0,56,0,56,0\\
8,48,8,48,8,48,8,48\\
16,40,16,40,16,40,16,40\\
32,24,32,24,32,24,32,24
\end{vmatrix}
=
$$

1	58	3	60	5	62	7	64
56	15	54	13	52	11	50	9
48	23	46	21	44	19	42	17
25	34	27	36	29	38	31	40
57	2	59	4	61	6	63	8
16	55	14	53	12	51	10	49
24	47	22	45	20	43	18	41
33	26	35	28	37	30	39	32

(d)

neither $\underline{\alpha}$ nor $\underline{\underline{\pi}}$ but maybe all right

In this square also the property possessed by (E)(1) of the sixteen subsquares summing to 130 holds, but the property of Q_1, Q_2, Q_3, Q_4 being associated squares is special which does not hold in this case. In fact in this case choosing $A = (1,2,3,4)$ for the associated (P) & $(1,7,6,4)$ for the Nasik i.e. (a_1, a_2, a_3, a_4) in (P) & (a_1, a_2', a_3', a_4) in (R) perhaps is responsible for the property of the 16 subsquares adding to 130.

In (E)(1) the nos. in α are ~~1, 14, 19~~ 1, 6, 9, 14, 19, 24, 27, 32, 33, 38, 41, 46, 51, 56, 59, 64

Instead of taking 17, 6, 4 for A in the result (R) suppose I take the same 1, 2, 3, 4

Does it work?

1, 2, 3, 4, 8, 7, 6, 5	0, 56, 0, 56, 0, 56, 0, 56
8, 7, 6, 5, 1, 2, 3, 4	8, 48, 8, 48, 8, 48, 8, 48
1, 2, 3, 4, 8, 7, 6, 5	16, 40, 16, 40, 16, 40, 16, 40
8, 7, 6, 5, 1, 2, 3, 4	24, 32, 24, 32, 24, 32, 24, 32
1, 2, 3, 4, 8, 7, 6, 5	56, 0, 56, 0, 56, 0, 56, 0
8, 7, 6, 5, 1, 2, 3, 4	48, 8, 48, 8, 48, 8, 48, 8
1, 2, 3, 4, 8, 7, 6, 5	40, 16, 40, 16, 40, 16, 40, 16
8, 7, 6, 5, 1, 2, 3, 4	32, 24, 32, 24, 32, 24, 32, 24

1	58	3	60	5	62	7	64
16	55	14	53	12	51	10	49
24	47	22	45	20	43	18	41
25	34	27	36	29	38	31	40
57	2	59	4	61	6	63	8
56	15	54	13	52	11	50	9
48	23	46	21	44	19	42	17
33	26	35	28	37	30	39	32

(β)

Here again it works also having the 16 subsquares adding up to 130. The rows are same as rows in (α) but in different order. The cols. all contain the same elements? Only the diagonals except for the end terms are different. The subsquares property is not special for (1, 2, 3, 4) & (1, 7, 6, 4)

Suppose I take $A = (1, 2, 6, 4)$ for the a-square and

$A = (3, 1, 4, 5)$ for the n-square

Here the A's chosen arbitrarily.

(55)

(P) for a		(P) for n			(P)	(R)
1, 2, 6, 4,	5, 3, 7, 8	3, ~~1~~ 2, 4, ~~5~~ 8,	6, ~~8~~ 7, 5, ~~4~~ 1		A	A
8, 7, 3, 5,	4, 6, 2, 1	6, ~~8~~ 7, 5, ~~4~~ 1,	3, ~~1~~ 2, 4, ~~5~~ 8		A'	A'
1, 2, 6, 4,	5, 3, 7, 8	6, ~~8~~ 7, 5, ~~4~~ 1,	3, ~~1~~ 2, 4, ~~5~~ 8		A	A'
8, 7, 3, 5,	4, 6, 2, 1	3, ~~1~~ 2, 4, ~~5~~ 8,	6, ~~8~~ 7, 5, ~~4~~ 1		A'	A
8, 7, 3, 5,	4, 6, 2, 1	3, ~~1~~ 2, 4, ~~5~~ 8,	6, ~~8~~ 7, 5, ~~4~~ 1		A'	A
1, 2, 6, 4,	5, 3, 7, 8	6, ~~8~~ 7, 5, ~~4~~ 1,	3, ~~1~~ 2, 4, ~~5~~ 8		A	A'
8, 7, 3, 5,	4, 6, 2, 1	6, ~~8~~ 7, 5, ~~4~~ 1,	3, ~~1~~ 2, 4, ~~5~~ 8		A'	A'
1, 2, 6, 4,	5, 3, 7, 8	3, ~~1~~ 2, 4, ~~5~~ 8,	6, ~~8~~ 7, 5, ~~4~~ 1		A	A

(P) for a (P) for n (R) for n

16,	40,	40,	16,	16,	40,	40,	16
~~0~~ 8,	~~56~~ 48,	~~56~~ 48,	~~0~~ 8,	~~0~~ 8,	~~56~~ 48,	~~56~~ 48,	~~0~~ 8
24,	32,	32,	24,	24,	32,	32,	24
~~32~~ 56,	~~24~~ 0,	~~24~~ 0,	~~32~~ 56,	~~32~~ 56,	~~24~~ 0,	~~24~~ 0,	~~32~~ 56
40,	16,	16,	40,	40,	16,	16,	40
~~56~~ 48,	~~0~~ 8,	~~0~~ 8,	~~56~~ 48,	~~56~~ 48,	~~0~~ 8,	~~0~~ 8,	~~56~~ 48
32,	24,	24,	32,	32,	24,	24,	32
~~24~~ 0,	~~32~~ 56,	~~32~~ 56,	~~24~~ 0,	~~24~~ 0,	~~32~~ 56,	~~32~~ 56,	~~24~~ 0

17	42	46	20	21	43	47	24
~~8~~ 16	~~63~~ 55	~~59~~ 51	~~5~~ 13	~~4~~ 12	~~62~~ 54	~~58~~ 50	~~1~~ 9
25	34	38	28	29	35	39	32
~~48~~ 64	~~31~~ 7	~~27~~ 3	~~37~~ 61	~~36~~ 60	~~30~~ 6	~~26~~ 2	~~33~~ 57
~~48~~ [illegible]	~~23~~ [illegible]	~~19~~ [illegible]	~~45~~ [illegible]	~~44~~ [illegible]	~~22~~ [illegible]	~~18~~ [illegible]	~~41~~ [illegible]
~~57~~ 49	~~2~~ 10	~~6~~ 14	~~60~~ 52	~~61~~ 53	~~3~~ 11	~~7~~ 15	~~64~~ 56
40	31	27	37	36	30	26	33
1	58	62	4	5	59	63	8

repetition

This repetition is not ~~because~~ in $\underline{n}$-(R) we have chosen 3 odd & 1 even number, ~~suppose~~ that ~~had taken (3, 2, 4, 8) instead of (3, 1, 4, 5)~~ because choice of 4 & 5 which are complementary. What a fool? Take 3, 2, 4, 8. Here again it works with subsquare property.

So my simple scheme with single A working with an a-P & a n-R & getting a square neither a nor n concerns to Conserves the 16 sub-square property. To get other ornate properties as in (E)(1) requires the choice of further alternatives other than my simple scheme.

Suppose we choose for Q_1, $A = (1, 2, 3, 4)$; for Q_3, $B = (17, 6, 4)$

for Q_2, $C = (1, 2, 6, 5)$; for Q_4, $D = (1, 3, 5, 7)$. & take for Q_1 (A, A', A', A) & for $Q_3 = (D, D', D', D)$, etc.

These won't make Q_1, Q_2, Q_3, Q_4 associated at all. Suppose I take for Q_1: $(A, \bar{A}, A', \bar{A}')$. This makes Q_1 associated. Do this similarly for Q_2, Q_3, Q_4

1	2	3	4	2	1	6	5
4	3	2	1	5	6	1	2
8	7	6	5	7	8	3	4
5	6	7	8	4	3	8	7
4	7	6	1	3	1	5	7
1	6	7	4	7	5	1	3
5	2	3	8	6	8	4	2
8	3	2	5	2	4	8	6

(P)

8	32	16	56	56	16	32	8
40	24	48	0	0	48	24	40
56	8	32	16	16	32	8	56
0	40	24	48	48	24	40	0
56	16	32	8	8	32	16	56.
0	48	24	40	40	24	48	0
16	32	8	56	56	8	32	16
48	24	40	0	0	40	24	48

9	34	19	60	58	17	38	13
44	27	50	1	5	54	25	42
64	15	38	21	24	39	11	60

Reference

I think (P) is not properly chosen

1, 2, 3, 4	2, 1, 6, 5
4 3 2 1	5 6 1 2
8 7 6 5	7 8 3 4
5 6 7 8	4 3 8 7

Better give this up & come back.

(E) (2) Fig. 262, p. 165 of Andrews (nasik) — It is also famous in that all 2×2 subsquares add to 130 & hence algebraic

Splitting this up has been done on p. 92 of Book No. 20 & is of the form

(57)

A, A'	with $A = (1,2,3,4)$	
A', A	is a nasik square.	
A, A'		
A', A'		
A, A	& Q_s is merely (P) $r \leftrightarrow c$	
A', A		
A, A'		
A', A	being the simplest of my type.	
(P)		

It is mentioned that this "Knight-nasik with a few exceptions." I wonder what those exceptions are.

$40+43+49+62+32+19+9+6 = 260$. Checking up & finding exceptions would mean examining 64 cases & would be too tedious. Is this ~~true of ever~~ Knight-nasik property" true only for $A = (1,2,3,4)$ or any other (A) of mine? Take the one where $A = (1,3,5,7)$

(See Bk. 20, p. 96). $16+29+41+60+56+37+17+4 = 260$ ✓
$18+6+15+27+42+62+55+35 = 260$ ✓

Looks correct again.

Also all 2×2 squares (not merely the 16 subsquares) sum to 130.

~~16+39+45+3+49+26+5+51+~~ [Note – This square is of my ~~[illegible]~~ (ie Jaina) special form for nasiks ~~[illegible]~~ ~~[illegible]~~]

For nasiks I have put the condn on A of not having a pair of complementaries. Can we modify this by taking

$A, \bar{A}, A', \bar{A}'$?

1 3 6 8 . 8, 6, 3, 1
8 6 3 1 . 1, 3, 6, 8
8 6 3 1 . 1, 3, 6, 8
1 3 6 8 . 8, 6, 3, 1
1 3 6 8 . 8, 6, 3, 1
8 6 3 1 . 1, 3, 6, 8
8 6 3 8 . 1, 3, 6, 8
1 3 6 8 . 8, 6, 3, 1

0 56 56 0 0 56, 56, 0.

0, 57, 57, 1, 1, 57, 57, 1 X

[illegible]

X

4, 3, 2, 1
1, 2, 3, 4, 8, 7, 6, 5
4, 3, 2, 1, 5, 6, 7, 8
8, 7, 6, 5, 1, 2, 3, 4
✓ 5, 6, 7, 8, 4, 3, 2, 1
× 1, 2, 3, 4, 8, 7, 6, 5
? 4, 3, 2, 1, 5, 6, 7, 8
● 8, 7, 6, 5, 1, 2, 3, 4
^ 5, 6, 7, 8, 4, 3, 2, 1
8, 7, 6, 5

0, 24, 56, 32, 0, 24, 56, 32
8, 16, 48, 40, 8, 16, 48, 40
16, 8, 40, 48, 16, 8, 40, 48
✓ 24, 0, 32, 56, 24, 0, 32, 56
× 24, 0, 32, 56, 24, 0, 32, 56
? 16, 8, 40, 48, 16, 8, 40, 48
● 8, 16, 48, 40, 8, 16, 48, 40
32, 56, 24, 0, 32, 56, 24, 0
^ 0, 24, 56, 32, 0, 24, 56, 32

1, 26, 59, 36, 8, 31, 42, 37
12, 19, 50, 41, 13, 20, 55, 48
24, 15, 46, 53, 17, 10, 43, 52
29, 6, 39, 64, 28, 3, 34, 57
~~57, 34, 3, 28, 64, 39, 6, 29~~
25, 2, 34, 60, 28,

Not a magic square at all

This bar business fails for n

Reflection:

1, 26, 59, 36, 4, 27, 58, 33
12, 19, 50, 41, 9, 18, 51, 44
24, 15, 46, 53, 21, 14, 47, 56,
29, 6, 39, 64, 32, 7, 38, 61
25, 2, 35, 60, 28, 3, 34, 57
20, 11, 42, 49, 17, 10, 43, 52
16, 23, 54, 45, 13, 22, 55, 48
5, 30, 63, 40, 8, 31, 62, 37

X

(59) (E)(3) – Fig. 265, p. 167. (Frierson) (neither a norm) algebraic & sum of 2,2,2,2,4 = 520

1,1,8,8,2,2,7,7	0,24,48,40,0,24,48,40	1,4,7,6,1,4,7,6
8,8,1,1,7,7,2,2	32,56,16,8,32,56,16,8	5,8,3,2,5,8,3,2
1,1,8,8,2,2,7,7	56,32,8,16,56,32,8,16	8,5,2,3,8,5,2,3
8,8,1,1,7,7,2,2	24,0,40,48,24,0,40,48	4,1,6,7,4,1,6,7
3,3,6,6,4,4,5,5	0,24,48,40,0,24,48,40	1,4,7,6,1,4,7,6
6,6,3,3,5,5,4,4	32,56,16,8,32,56,16,8	5,8,3,2,5,8,3,2
3,3,6,6,4,4,5,5	56,32,8,16,56,32,8,16	8,5,2,3,8,5,2,3
6,6,3,3,5,5,4,4	24,0,40,48,24,0,40,48	4,1,6,7,4,1,6,7
(P)	(R)	(R_0)

R_0 with $r \leftrightarrow c$,

1,5,8,4,1,5,8,4	A, A
4,8,5,1,4,8,5,1	$\bar{A}, \bar{A}$
7,3,2,6,7,3,2,6	B, B
6,2,3,7,6,2,3,7	$\bar{B}, \bar{B}$
1,5,8,4,1,5,8,4	A, A
4,8,5,1,4,8,5,1	$\bar{A}, \bar{A}$
7,3,2,6,7,3,2,6	B, B
6,3,2,7,6,3,2,7	$\bar{B}, \bar{B}$

$A = (1,5,8,4)$

$B = (7,3,2,6)$

~~1,5,8~~,4

(P) is C, E

$C' = \bar{C}, \bar{E} = E'$

$= C, E$

$C' = \bar{C}, \bar{E} = E'$

D, F

$D' = \bar{D}, \bar{F} = F'$

D, F

$D' = \bar{D}, \bar{F} = F'$

$C = (1,1,8,8), \bar{C} = C'$

$D = (3,3,6,6), \bar{D} = D'$

$E = (2,2,7,7), \bar{E} = E'$

$F = (4,4,5,5), \bar{F} = F'$

[· Any 4 × 4 square has summation 520 – Can we call this a ~~Jaina~~ Sengalese? square? There are only 25, 2×2 squares adding to 130 & so no Jaina in ordinary sense] ✓

There is no hint how Frierson arrived at this square

(E)(4) Fig. 267, p. 169 – this square is (nasik) n – algebraic but not Jaina because 4 exceptions of 2×2 squares not summing up to 130. These four are in the 4th and 5th rows of the square

Splitting it up

[note – this is an example of a n-square not of my standard form]

8	1	4	5	8	1	4	5
3	6	7	2	3	6	7	2
5	4	1	8	5	4	1	8
2	7	6	3	2	7	6	3
8	1	4	5	8	1	4	5
3	6	7	2	3	6	7	2
5	4	1	8	5	4	1	8
2	7	6	3	2	7	6	3

(n) (P) 8, 1, 4, 5 $\boxed{1, 3, 5, 4}$

56	56	0	0	48	48	8	8
0	0	56	56	8	8	48	48
56	56	0	0	48	48	8	8
0	0	56	56	8	8	48	48
40	40	16	16	32	32	24	24
16	16	40	40	24	24	32	32
40	40	16	16	32	32	24	24
16	16	40	40	24	24	32	32

(R) (n) ~~48, 8, 2, 4~~

These suggest entirely new ways of constructing n-squares.

A, A	~~8, 8~~ R_0 with $r \leftrightarrow c$ is	
B, B	8, 1, 8, 1, 6, 3, 6, 3	C, E
$\bar{A}$, $\bar{A}$	8, 1, 8, 1, 6, 3, 6, 3	C, E
$\bar{B}$, $\bar{B}$	1, 8, 1, 8, 3, 6, 3, 6	C′, E′
A, A	1, 8, 1, 8, 3, 6, 3, 6	C′, E′
B, B	7, 2, 7, 2, 5, 4, 5, 4	D, F
$\bar{A}$, $\bar{A}$	7, 2, 7, 2, 5, 4, 5, 4	D, F
$\bar{B}$, $\bar{B}$	2, 7, 2, 7, 4, 5, 4, 5	D′, F′
(P)	2, 7, 2, 7, 4, 5, 4, 5	D′, F′

$C' = \bar{C}$, $D' = \bar{D}$, $E' = \bar{E}$, $F' = \bar{F}$

Is this knight-result? no

$18 + 17 + 27 + 28 + 2 + 1 + 14 + 12 \neq 260$ ✗

$64 + 6 + 10 + 52 + 41 + 19 + 31 + 37 = 260$ ✓

$42 + 46 + 6 + 2 + 34 + 38 + 14 + 10 = 292$ ✗

All the A, B, C, D, E, F have two pairs of complementaries which are forbidden in my constructions.

(E)(5) - Ap. 268, p. 169. (neither a nor n) (Frierson) - algebraic

(61)

1, 5, 4, 8, 1, 5, 4, 8	0, 56, 56, 0, 8, 48, 48, 8
2, 6, 3, 7, 2, 6, 3, 7	0, 56, 56, 0, 8, 48, 48, 8
7, 3, 6, 2, 7, 3, 6, 2	56, 0, 0, 56, 48, 8, 8, 48
8, 4, 5, 1, 8, 4, 5, 1	56, 0, 0, 56, 48, 8, 8, 48
8, 4, 5, 1, 8, 4, 5, 1	16, 40, 40, 16, 24, 32, 32, 24
7, 3, 6, 2, 7, 3, 6, 2	16, 40, 40, 16, 24, 32, 32, 24
2, 6, 3, 7, 2, 6, 3, 7	40, 16, 16, 40, 32, 24, 24, 32
1, 5, 4, 8, 1, 5, 4, 8	40, 16, 16, 40, 32, 24, 24, 32
(P)	(R)

$$\begin{pmatrix} A & A \\ B & B \\ \bar{B} & \bar{B} \\ \bar{A} & \bar{A} \\ \bar{A} & \bar{A} \\ \bar{B} & B \\ B & B \\ A & A \end{pmatrix}$$

(P)

$A = (1,5,4,8)$

$B = (2,6,3,7)$

$\bar{A} = A'$, $\bar{B} = B'$

$$\begin{pmatrix} 1,8,8,1, & 2,7,7,2 \\ 1,8,8,1, & 2,7,7,2 \\ 8,1,1,8, & 7,2,2,7 \\ 8,1,1,8, & 7,2,2,7 \\ 3,6,6,3, & 4,5,5,4 \\ 3,6,6,3, & 4,5,5,4 \\ 6,3,3,6, & 5,4,4,5 \\ 6,3,3,6, & 5,4,4,5 \end{pmatrix} \begin{pmatrix} C & E \\ C & E \\ C' & E' \\ C' & E' \\ D & F \\ D & F \\ D' & F' \\ D' & F' \end{pmatrix}$$

$\bar{C} = C$

$\bar{D} = D$

$\bar{E} = E$

$\bar{F} = F$

Again special type.

(E) (6), Fig 269, p. 170 (Frieson) (neither a nor n) – almost algebraic

1, 2, 4, 3, 5, 6, 8, 7	0, 0, 40, 40, 16, 16, 56, 56
3, 4, 2, 1, 7, 8, 6, 5	0, 0, 40, 40, 16, 16, 56, 56
8, 7, 4, 3, 5, 6, 1, 2	48, 48, 48, 48, 8, 8, 8, 8
6, 5, 2, 1, 7, 8, 3, 4	48, 48, 48, 48, 8, 8, 8, 8
1, 2, 5, 6, 4, 3, 8, 7	24, 24, 0, 0, 56, 56, 32, 32
3, 4, 7, 8, 2, 1, 6, 5	24, 24, 0, 0, 56, 56, 32, 32
8, 7, 5, 6, 4, 3, 1, 2	40, 40, 24, 24, 32, 32, 16, 16
6, 5, 7, 8, 2, 1, 3, 4	40, 40, 24, 24, 32, 32, 16, 16

Again special type.

(E)(7).74.270, p.170 (Frierson) - neither $\underline{a}$ nor $\underline{n}$ — algebraic & also half of each row sums to 130.

1, 8, 8, 1, 8, 1, 1, 8	0, 24, 32, 56, 48, 40, 16, 8
2, 7, 7, 2, 7, 2, 2, 7	0, 24, 32, 56, 48, 40, 16, 8
3, 6, 6, 3, 6, 3, 3, 6	0, 24, 32, 56, 48, 40, 16, 8
4, 5, 5, 4, 5, 4, 4, 5	0, 24, 32, 56, 48, 40, 16, 8
5, 4, 4, 5, 4, 5, 5, 4	56, 32, 24, 0, 8, 16, 40, 48
6, 3, 3, 6, 3, 6, 6, 3	56, 32, 24, 0, 8, 16, 40, 48
7, 2, 2, 7, 2, 7, 7, 2	56, 32, 24, 0, 8, 16, 40, 48
8, 1, 1, 8, 1, 8, 8, 1	56, 32, 24, 0, 8, 16, 40, 48
(P)	(R)

(P) with r ↔ c | (R_0) is

1, 2, 3, 4, 5, 6, 7, 8	1, 4, 5, 8, 7, 6, 3, 2
8, 7, 6, 5, 4, 3, 2, 1	1, 4, 5, 8, 7, 6, 3, 2
8, 7, 6, 5, 4, 3, 2, 1	1, 4, 5, 8, 7, 6, 3, 2
1, 2, 3, 4, 5, 6, 7, 8	1, 4, 5, 8, 7, 6, 3, 2
8, 7, 6, 5, 4, 3, 2, 1	8, 5, 4, 1, 2, 3, 6, 7
1, 2, 3, 4, 5, 6, 7, 8	8, 5, 4, 1, 2, 3, 6, 7
1, 2, 3, 4, 5, 6, 7, 8	8, 5, 4, 1, 2, 3, 6, 7
8, 7, 6, 5, 4, 3, 2, 1	8, 5, 4, 1, 2, 3, 6, 7

$$
\underset{(P)}{\begin{pmatrix} A, & \bar{A}' \\ A', & \bar{A} \\ A', & \bar{A} \\ A, & \bar{A}' \\ A', & \bar{A} \\ A & \bar{A}' \\ A & \bar{A}' \\ A' & \bar{A} \end{pmatrix}}
\qquad
\underset{(R_0)}{\begin{pmatrix} C & D \\ C & D \\ C & D \\ C & D \\ D & C \\ D & C \\ D & C \\ D & C \end{pmatrix}}
$$

$C = \bar{C}'$

$D = \bar{D}'$

Again a special type.

(F)(1) Fig. 449, p. 256. – (a and n) – Split up on p. 98, Bk. 20.
(Frierson) – Algebraic, half of each row sums to 130 and

Half of each col. also sums to 130 – Not Jaina since some 2 x 2 squares

do not sum to 130, although many others do. – not knight-nasik

$(39 + 31 + 62 + 6 + 56 + 16 + 45 + 21 = 276)$ [Also a sq. on p. 99, Bk. 20
got by me similar to Fig 449)

All 4 x 4 squares sum to 520. So it is Jaina in the generalised sense for
ie (4^2-ply)

8 x 8 squares.

F(2) Fig. 468, p. 266 (Frierson) (a & n) – Split up on p. 6 of this book

Half of each row sums to 130 & hence square is algebraic – Half of each

col. does not sum to 130 as in Fig. 449 – not Jaina ie not 4-ply.

But it is 4^2-ply ie general Jaina – not knight-nasik

$[11 + 45 + 47 + 9 + 29 + 59 + 57 + 31 = 288]$

[Planck's rule for converting assoc a-squares of order 4p into n-squares

15 4
10 5
is quite interesting ie (1) Leave Q_1 as it is (2) Reflect Q_2 about central vertical
10 5
15 4
(3) Reflect Q_3 about central horizontal (4) Rotate D thro' 180°. or reflect about central vertical
& about central horizontal

~~But~~ Verify if this be true for my 8×8 a squares for,

1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
8	7	6	5	4	3	2	1
1	2	3	4	5	6	7	8

(P) a

→

1	2	3	4	8	7	6	5
8	7	6	5	1	2	3	4
8	7	6	5	1	2	3	4
1	2	3	4	8	7	6	5
1	2	3	4	8	7	6	5
8	7	6	5	1	2	3	4
8	7	6	5	1	2	3	4
1	2	3	4	8	7	6	5

ie

$$\begin{array}{cc} A & A' \\ A' & A \\ A' & A \\ A & A' \\ A & A' \\ A' & A \\ A' & A \\ A & A' \end{array}$$

ie nasik alright

In this special case the inverse change from n to a also holds good, although, in general, it may not be effective as Planck remarks.

vertical reflexion reverses ~~horizontals~~ rows ie $A \to \bar{A}$

inversion ie horizontal refl^n reverses cols ie

Suppose we take

$$\left|\begin{array}{cc} A, & \bar{A}' \\ A' & \bar{A} \\ A & \bar{A}' \\ A' & \bar{A} \\ A' & \bar{A} \\ A & \bar{A}' \\ A' & \bar{A} \\ A & \bar{A}' \end{array}\right| \to \left|\begin{array}{cc} A & A' \\ A' & A \\ A & A' \\ A' & A' \\ A & A' \\ A' & A \\ A & A' \\ A' & A \end{array}\right| \qquad \left(\begin{array}{cc} A & \bar{A}' \\ A & \bar{A}' \\ A' & \bar{A} \\ A' & \bar{A} \\ A' & \bar{A} \\ A' & \bar{A} \\ A & \bar{A}' \\ A & \bar{A}' \end{array}\right) \quad \left(\begin{array}{cc} A & A' \\ A & A' \\ A' & A \\ A' & A \\ A & A' \\ A & A' \\ A' & A \\ A' & A \end{array}\right)$$

(a) ✓ (n) a (n)

8, 7, 6, 5

1, 2, 3, 4

So the rule & its inverse holds for ~~any~~ all special cases of single A.

(G) (1) 7 p. 567, p. 296 (only a not n).

Split up on p. 7 of this book & other remarks made.

Square is algebraic – The sixteen 2×2 squares add to 130 each as happens for

(65) as happens for square in Fig. 261, p. 125 (See pp. 51–55 of this book).

((H) (1) Fig. 695, p. 377. _ ~~cf.~~ p. 10, this book
(only $\underline{n}$)

This is given by C.P. as an example of a general rule for construction of ornate magic squares of orders $\equiv 0 \pmod 4$.

For $\underline{n = 8}$, he takes Q_1 to be of the form

Q_1

a_1	b_1	a_2	b_2
a_2	b_2	a_1	b_1
a_1	b_1	a_2	b_2
a_2	b_2	a_1	b_1

& calls this a Jaina square (first type) & says the first Jaina type can be obtained by using the pattern (1, 2), (2, 1)(?)

Here (a_1, a_2) are complementaries & so also (b_1, b_2). For Q_1 he takes in particular $(a_1, a_2) = (2, 7)$ & $(b_1, b_2) = (3, 6)$. Next for Q_2 he takes (4, 5) & (1, 8). Q_3 is repeat of Q_1 and Q_4 is repeat of Q_2. This is his (P). He obtains (R) by taking (P) with $r \longleftrightarrow c$. & get finally Fig. 695 which is nasik.

In view of this let us reexamine our taking A to be of the special form where there is no pair of complementaries & modify it so as to include such a pair or pairs. Suppose A = (1, 2, 7, 4) with the pair (2, 7) appearing.

If we take A' as first row in Q_2 repetitions occur. So we shall take (8, 3, 6, 5) & call it B or (8, 6, 3, 5) as first row in Q_2 (Here 1 & 4 are replaced by 8 and 5 here) - (3, 6) or (6, 3) is chosen & not (1, 8) or (4, 5) because 1 & 4 appear already in A. The other rows in Q_2 are B', B, B' and Q_3 & Q_4 are repeats of Q_1 & Q_2 respectively. This should give a result by taking (R) as (P) $r \leftrightarrow c$.

1	2	7	4	8	3	6	5
8	7	2	5	1	6	3	4
1	2	7	4	8	3	6	5
8	7	2	5	1	6	3	4
1	2	7	4	8	3	6	5
8	7	2	5	1	6	3	4
1	2	7	4	8	3	6	5
8	7	2	5	1	6	3	4

(P)

X

0	56	0	56	0	56	0	56
8	48	8	48	8	48	8	48
48	8	48	8	48	8	48	8
24	32	24	32	24	32	24	32
56	~~56~~ 0	56	~~56~~ 0	56	~~56~~ 0	~~0~~ 56	~~56~~ 0
16	40	16	40	16	40	16	40
40	16	40	16	40	16	40	16
32	24	32	24	32	24	32	24

(R)

X

$48+47+53+50+41+46+52 = 337$
$+57 \neq u.$

(2, 1) $48+36+17+61+56+28+9+5 = 260$ ✓
(2, −1) $48+4+9+29+56+60+17+37 = 260$ ✓

There will be repetition. X

1	58	7	60	8	59	6	61
16	55	10	53	9	54	11	52
49	10	55					

every 3×3 square is magic in diagonals ~~Further~~ ~~constant~~

diag & corner elements of every 5×5 square ~~contains~~ two pairs ~~2 consecutive~~ elements in of P

So we should take
A in Q_1 as (1, 2, 4, 7) replacing (2, 7) by (3, 6)
B in Q_2 as (8, 3, 5, 6). & taking complements of 1 & 4

1, 2, 4, 7, 8, 3, 5, 6 — 0, 56, 0, 56, 0, 56, 0, 56.
8, 7, 5, 2, 1, 6, 4, 3 — 8, 48, 8, 48, 8, 48, 8, 48
1, 2, 4, 7, 8, 3, 5, 6 — 24, 32, 24, 32, 24, 32, 24, 32
8, 7, 5, 2, 1, 6, 4, 3 — 48, 8, 48, 8, 48, 8, 48, 8
1, 2, 4, 7, 8, 3, 5, 6 — 56, 0, 56, 0, 56, 0, 56, 0
8, 7, 5, 2, 1, 6, 4, 3 — 16, 40, 16, 40, 16, 40, 16, 40
1, 2, 4, 7, 8, 3, 5, 6 — 32, 24, 32, 24, 32, 24, 32, 24
8, 7, 5, 2, 1, 6, 4, 3 — 40, 16, 40, 16, 40, 16, 40, 16

(P) $48+55+53+42+41+54+57+43$
$\neq 260$.

(M)

1	58	4	63	8	59	5	62
16	55	13	50	9	54	12	51
25	34	28	39	32	35	29	38
56	15	53	10	49	14	52	11
57	2	60	7	64	3	61	6
24	47	21	42	17	46	20	43
33	26	36	31	40	27	37	30
48	23	45	18	41	22	44	19

n

(37) If we had taken for B as $(8, 6, 5, 3)$ only the 6th & 8th cols would be interchanged & this will still keep the square magic & again : Re. properties of this square cf. with Planck's Fig. 695, p. 377, we have

(1) magic in rows & cols – holds ✓

(2) pandiagonal i.e. magic in all 16 diagonals – holds ✓

(3) Hawkins's property of bent diagonals – not quite clear whether it holds or not

(4) great square is 4-ply – holds & consequences – holds ✓

(5) Subsquares being balanced Jaina squares i.e. each of them has the 36 summations of a Jaina and in each case the magic sum is four times the mean number of the great square – what does this mean? 16 for rows, cols & diagonals + 20 for ~~sub~~ 2×2 subsquare sums & other sums. – This does not hold here since the Q_1, Q_2, Q_3, Q_4 are not magic squares at all, let alone Jaina squares.

2,4,1
2,7,8,7

(6) Property of subsidiary minors – looks trivial & not clear – Anyway it does not hold here.

⟵

8, 7, 5, 2
2, 5, 7, 8

What about getting an associated square with $A = (1, 2, 4, 7)$? We might take $(A, A', A', A, A, A', A', A)$ in Q_1 & Q_3 and in Q_3 & Q_4? originally we took in Q_2 $(\bar{A}', \bar{A}, \bar{A}, \bar{A}' \ldots$

8, 7, 5, 2   2, 5, 7, 8

suppose we take $\bar{A}'$ from A. This gives 2, 5, 7, 8 and replacing 2 & 7 by 3, 6 we get 3, 5, 6, 8. Anything (6, 8)

1, 2, 4, 7, 3, 5, 6, 8 + 0, 56, 56, 0, 0, 56, 56, 0
8, 7, 5, 2, 6, 4, 3, 1 + 8, 48, 48, 8, 8, 48, 48, 8
8, 7, 5, 2, 6, 4, 3, 1 + 24, 32, 32, 24, 24, 32, 32, 24
1, 2, 4, 7, 3, 5, 6, 8 + 48, 8, 8, 48, 48, 8, 8, 48
1, 2, 4, 7, 3, 5, 6, 8 + 16, 40, 40, 16, 16, 40, 40, 16
8, 7, 5, 2, 6, 4, 3, 1 + 32, 24, 24, 32, 32, 24, 24, 32
8, 7, 5, 2, 6, 4, 3, 1 + 40, 16, 16, 40, 40, 16, 16, 40
1, 2, 4, 7, 3, 5, 6, 8 + 56, 0, 0, 56, 56, 0, 0, 56

X Refeth

1	58	60	7	3	61	62	8
16	55	53	10	15	52	50	9
32	39	37	26	31	36	34	25
49	10	52	15				
57	2	4	63	59	5	6	64
1	58	4	63	59	5	61	8
16	51	13	54				
25	34	28	39				
56	41	53	40				
				11	52	46	49
57	2	60	3	62	61	7	64

(P) 3, 5, 6, 8 (R)

1, 2, 4, 7, 2, 5, 7, 8 | 0, 56, 0, 56, 56, 0, 56, 0
8, 3, 5, 6, 7, 4, 2, 1 | 8, 48, 8, 48, 48, 8, 48, 8
1, 2, 4, 7, 2, 5, 7, 8 | 24, 32, 24, 32, 32, 24, 32, 24
8, 3, 5, 6, 7, 4, 2, 1 | 48, 40, 48, 40, 8, 48, 8, 48
8, 7, 5, 2, 7, 4, 2, 1 | 16, 40, 16, 40, 40, 16, 40, 16
1, 2, 4, 7, 2, 5, 7, 8 | 32, 24, 32, 24, 24, 32, 24, 32
8, 7, 5, 2, 7, 4, 2, 1 | 40, 16, 40, 16, 16, 40, 16, 40
1, 2, 4, 7, ~~2, 5, 7, 8~~ 3, 5, 6, 8 | 56, 0, 56, 0, 0, 56, 0, 56

X Again fails.

7 4 2 1
2 5 7 8 3, 5, 6, 8

I think for associated squares my original prescription is correct

~~Let us now look at another square with A having two pairs of complementaries.~~

1, 4, 2, 7, 3, 6, 5, 8 0, 56, 0, 56, 56, 0, 56, 0 | 1, 60, 2, 63, 59, 6, 61, 8
8, 5, 7, 2, 6, 3, 4, 1 | 1, 2, 4, 7, 2, 5, 7, 8
1, 4, 2, 7, 3, 6, 5, 8 | 8, 7, 5, 2
8, 5, 7, 2, 6, 3, 4, 1 | 1, 2, 4, 7
8, 5, 7, 2, 6, 3, 4, 1 | 8, 7, 5, 2
1, 4, 2, 7, 3, 6, 5, 8
8, 5, 7, 2, 6, 3, 4, 1 X | 7 4 2 1
1, 4, 2, 7, 3, 6, 5, 8 X 56, 0, 56, 0, 0, 56, 0, 56 | 1 2 4 7, 2, 5, 7, 8

57, 4, 58, 7,

(69)

$\bar{B}'$
$\bar{B}$ 5, 4, 53, 63, 8

8, 7, 2, 5
1, 4, 2, 7
5, 2, 7, 8
3, 3, 6, 5

1, 2, 7, 4, ~~5, 2, 7, 8~~ | 0, 56, 56, 0, 0, 56, 56, 0 | 1, 58, 63, 4, 5, 58, 63, 8 X rpt
8, 7, 2, 5, 4, 6, 3, 4 | 8, 48, 48, 8, 8, 48, 48, 8
8, 7, 2, 5, 4, 6, 3, 4 | 48, 8, 8, 48, 48, 8, 8, 48
1, 2, 7, 4, 8, 3, 6, 5 | 24, 32, 32, 24, 24, 32, 32, 24
1, 2, 7, 4, 8, 3, 6, 5 | 56, 0, 0, 56, 56, 0, 0, 56 | Nothing appears to work for associative squares
8, 7, 2, 5, 4, 6, 3, 4 | 16, 40, 40, 16, 16, 40, 40, 16
8, 7, 2, 5, 4, 6, 3, 4 | 40, 16, 16, 40, 40, 16, 16, 40
1, 2, 7, 4, 8, 3, 6, 5 | 32, 24, 24, 32, 24, 32, 32, 24 | 57, 2, 7, 60, 61 X rpt.

8, 3, 6, 5
5, 3, 6, 8

59, 62

1, 2, 7, 4, 5, 2, 7, 8 | 0, 56, 56, 0, 0, 56, 56, 0 | 1, 58, 63, 4, 5, 58, 63, 8
8, 7, 2, 5, 4, 7, 2, 1
8, 7, 2, 5, 4, 7, 2, 1 | $a_1\ a_2\ a_2'\ a_3$
1, 2, 7, 4, 5, 2, 7, 8
1, 2, 7, 4, 5, 2, 7, 8
8, 7, 2, 5, 4, 7, 2, 1 | $a_3'\ a_2\ a_2'\ a_1'$
8, 7, 2, 5, 4, 7, 2, 1
1, 2, 7, 4, 5, 2, 7, 8 | 56, 0, 0, 56, 56, 0, 0, 56. | 57, 3, 6, 60, 61, 2, 7, 64
5, 3, 6, 8

I think ~~There~~ perhaps that there is no point in trying to construct an $\underline{a}$ from A in which two elements are complementary say $A = (a_1, a_2, a_2', a_4)$

56 8 56 0 56
1, 3, 4, 6, 3, 5, 6, 8 | 0, 56, 56, 0, 0, 56, 56, 0 | ~~1, 59, 60, 6, 3, 61, 62, 8~~
8, 6, 5, 3, 6, 4, 3, 1 | | 57, 3, 4, 62, 59, 5, 6, 64
8, 6, 5, 3, 6, 4, 3, 1
1, 3, 4, 6, 3, 5, 6, 8 | | 3, 6, 6 are repeated
1, 3, 4, 6, 3, 5, 6, 8 | | X
5, 6, 5, 3, 6, 4, 3, 1
8, 6, 5, 3, 6, 4, 3, 1
1, 3, 4, 6, 3, 5, 6, 8 | | 1, 59, 60
| 56, 0, 0, 56, 56, 0, 0, 56 | 57, 3, 4, 62, 59, 5, 6, 64
| 0, 56, 56, 0, 0, 56, 56, 0

2, 3, 7, 6, 4, 1, 5, 8
7, 6, 2, 3, 5, 8, 4, 1
2, 3, 7, 6, 4, 1, 5, 8
7, 6, 2, 3, 5, 8, 4, 1
7, 6, 2, 3, 5, 8, 4, 1
2, 3, 7, 6, 4, 1, 5, 8
7, 6, 2, 3, 5, 8 4 1
2, 3, 7, 6, 4, 1, 5 8

8, 48, 8, 48, 48, 8, 48, 8

2, 7 – 3, 6
4, 5 – 1, 8
1, 8 – 4, 5 | 2, –

56, 0, 56, 0, 0, 56, 0, 56

10, 51, 15, 54, 52, 9, 53, 16 (70)

8 + 1 = 9
18

58 1, 2, 3, 4
8, 7, 6, 5

~~2, 3, 6, 7, 1~~
1, 1, 4, 5, 8, 2 3, 6, 7
8, 5, 4, 1 8
8, 5, 4, 1 8
1, 4, 5, 8 1
1, 4, 5, 8 1
8, 8 5 4 1 8 8
8, 8, 5, 4, 1 8 8
8 4, 4, 5, 8, 7 1

0, 56, 56, 0, 0, 56, 56, 0

0, 56, 56, 0, 0, 56, 56, 0
~~48, 8, 8, 48, 48, 8, 8, 48~~

~~1, 60, 61, 8, 2, 59, 62, 7~~
1, 60, 61, 8, 8, X

No associate square

Knight paths – Planck mentions 64 ~~no~~ Knight paths for his nasik square of order 16 (Andrews, p. 383)

What does this mean? Take for eg. a nasik square of order 8 (say, Fig. 108, p. 149, main Bk)

Through each of the 64 elements there are 8 Knight paths given by $(\pm 2, \pm 1)$ & $(\pm 1, \pm 2)$. Of these elements in $(2, 1)$, $(-2, -1)$ are ~~identical~~ the same (See Fig. ~~403~~, p. 742, p. 404 of Andrews) & similarly (~~[illegible]~~, i.e. $(2, -1)$ & $(-2, 1)$ are the same. Again $(1, 2)$ & $(-1, -2)$ and $(1, -2)$ & $(-1, 2)$ are the same. So there are really 4 distinct Knight paths from each cell of the square. That would mean we should examine 64×4 Knight paths. But this number is again reduced if we note that for each of the 8 elements of a particular

(71) Knight path from a cell, if taken as the starting path will consist of the same elements. ~~As~~ Thus for eg. in Fig. 108, p. 149 starting from element 40, the paths for the Knight move (2, 1) ~~are 40~~ give $40 + 43 + 49 + 62 + 32 + 19 + 9 + 6 = 260 \ (= S)$. If we start from 43, or 49 or 62, or 32, or 19 or 9 or 6, we get the same elements traversed in the Knight path. Thus the number 64×4 is reduced to $\frac{64 \times 4}{8} = 32$ Knight paths.

For a square of order 16, the distinct Knight paths $= \frac{16 \times 16 \times 4}{16} = 64$ as mentioned by Planck. He has also mentioned on the same page (Andrews, p. 383) that for all orders > 8 and $\equiv 0 \pmod 8$, this ~~nasik~~ Knight-nasik property can be easily derived by taking suitable ones in the ~~quadrant~~ of quadrants (2 in case of $n = 8$ & 4 in case of $n = 16$). So this means that we have to examine in the case of $n = 8$ the several paths for the Knight nasik property. This also explains why ~~for~~ Fig. 262, p. 165 of Andrews it is mentioned (p. 166) that the Knight-nasik property has a few exceptions. However what the exceptions are, is not meant to point out. Anyway let us examine my Fig. 108 for these 32 Knight paths. For each particular Knight move there should only be 8 paths.

For (2, 1) starting from 40, the elements other than this are 43, ~~19~~, 49, 62, 32, 19, 9, 6.
Take the next element in the bottom row, 31, $40+43+49+62+32+19+9+6 = 260$

$40 + 43 + 49 + 62 + 32 + 19 + 9 + 6$
from 31 → $31 + 20 + 10 + 5 + 39 + 44 + 50 + 61 = 260$.
from 38: $38 + 48 + 51 + 57 + 30 + 24 + 11 + 1 = 260$.
from 29: $29 + 23 + 12 + 2 + 37 + 47 + 52 + 58 = 260$
from 33: $33 + 46 + 56 + 59 + 25 + 22 + 16 + 3 = 260$. (X)
from 26: $26 + 21 + 15 + 4 + 34 + 45 + 55 + 60 = 260$
from 35: $35 + 41 + 54 + 64 + 27 + 17 + 14 + 8 = 260$
lastly from 28: $28 + 18 + 13 + 7 + 36 + 42 + 53 + 63 = 260$.

These Knight paths which are nasik consist of all the 64 elements 1 to 8^2 (7.2)
without repetition. So it is easy to check up this property since we need not make an arbitrary enumeration but take the cells of any one row of the square.

For (2, −1). Let us start with the same bottom row.

$$\left.\begin{array}{l}
\underline{40}+3+\underline{9}+22+\underline{32}+59+\underline{49}+46=260 \\
\underline{31}+60+\underline{50}+45+\underline{39}+4+\underline{10}+21=260. \\
38+8+11+17+30+64+51+41=260. \\
29+63+52+42+37+7+12+18=260. \\
33+6+16+19+25+62+56+43=260. \\
26+61+55+44+34+5+15+20=260 \\
35+1+14+24+27+57+54+48=260 \\
28+58+53+47+36+2+13+23=260
\end{array}\right\} (Y)$$

[Note that the 64 elements in (X) or (Y) constitute again nasik magic squares. The cols will be the rows of the original square but the diagonals
40 + 60 + 11 + 42 + 25 + 5 + 54 + 23 = 260 is not a diagonal central or broken of the original square, while the upper diagonal
28 + 1 + 55 + 19 + 37 + 64 + 10 + 46 = 260 is the same as the l. d. of the original square. The broken diagonals of (X) and (Y) can be similarly examined]

After having for (X) & Y, perhaps there is no need to examine all the 8 cases for the remaining knight moves viz (1, 2) & (1, −2). Enough to examine only one
(1, 2) → 40 + 15 + 30 + 53 + 33 + 10 + 27 + 52 = 260 ✓
(1, −2) → 40 + 55 + 30 + 13 + 33 + 50 + 27 + 12 = 260 ✓

So square in Fig. 108 of p. 149 of main book is Knight nasik in all the 32 Knight paths.
So let us see what are the exceptions in Fig. 262, p. 165 of Andrews.

(73) ~~For this figure~~ There is no need to do this over this figure is the same as my ~~(Σ, 1) + 40 + 43 + 49 + 62 + 32 + 19 + 9 + 6~~ fig 108.

Perhaps the exceptions arise when we consider moves of a bishop, but what is this move in chess? I must look up chess literature.

H(3) fig. 716, p. 391 — a & n - See p. 11 of this book

This is the one with the scheme $(A, A', A, A', A, A', A, A')$ in Q_1, Q_3

and scheme $(\bar{A}, \bar{A}', \bar{A}, \bar{A}', \bar{A}, \bar{A}', \bar{A}, \bar{A}')$ in Q_2, Q_4

and (R) from (P) by $r \leftrightarrow c$ — only new rule to make a & n squares

(5) fig. 720, p. 391 — a & n - see p. 12 of this book - for the splitting

Here both (P) & (R_0) are a & n.

In (P) ~~Q~~ Q_1 is formed from (A, A', B, B'), Q_3 is $(\bar{B}, \bar{B}', \bar{A}, \bar{A}')$ with $A = (1, 8, 5, 4)$, $B = (3, 6, 7, 2)$

R Q_2, Q_4 are Q_1 & Q_3 repeated.

(R_0) is formed having Q_1 $(A, B, \bar{B}, \bar{A})$

Noted from

$A = (1, 2, 3, 4)$ is knight nasik as in Fig. 262, p. 165 of Andrews

Also $A = (1, 6, 5, 2)$ gives R-n $[A = a_1, a_2, a_3, a_4]$ $a_1 + a_4 = a_2 + a_3$

or $a_1 + a_2 = \cancel{a_2 + a_3}$ $= a_3 + a_4$ Are these prescriptions correct for K-n squares

Try $A =$ (~~1, 2, 4, 7, 6~~, (1, 3, 2, 4)

1, 3, 2, 4, 8, 6, 7, 5	0, 56, 0, 56, 0, 56, 0, 56	1, 59, 2, 60, 8, 62, 7, 61
8, 6, 7, 5, 1, 3, 2, 4	16, 40, 16, 40, 16, 40, 16, 40	22, 46, 13, 45, 17, 43, 18, 44
1, 3, 2, 4, 8, 6, 7, 5	8, 48, 8, 48, 8, 48, 8, 48	9, 51, 10, 52, 16, 54, 15, 53
8, 6, 7, 5, 1, 3, 2, 4	24, 32, 24, 32, 24, 32, 24, 32	32, 38, 31, 37, 25, 35, 26, 36
1, 3, 2, 4, 8, 6, 7, 5	56, 0, 56, 0, 56, 0, 56, 0	57, 3, 58, 4, 64, 6, 63, 5
8, 6, 7, 5, 1, 3, 2, 4	40, 16, 40, 16, 40, 16, 40, 16	48, 22, 47, 21, 41, 19, 42, 20
1, 3, 2, 4, 8, 6, 7, 5	48, 8, 48, 8, 48, 8, 48, 8	49, 11, 50, 12, 56, 14, 55, 13
8, 6, 7, 5, 1, 3, 2, 4	32, 24, 32, 24, 32, 24, 32, 24	40, 30, 39, 29, 33, 27, 34, 28

$a_i + a_j = a_k + a_l$

or

$a_1 + a_4 = a_2 + a_3$

$\longrightarrow$ K.n

it is Knight - numb

$40 + 50 + 41 + 63 + 32 + 10 + 17 + 7 = 260$ ✓

$40 + 2 + 17 + 15 + 32 + 58 + 41 + 55 = 260$ ✓

$40 + 22 + 31 + 45 + 33 + 19 + 26 + 44 = 260$ ✓

$40 + 46 + 31 + 21 + 33 + 43 + 26 + 20 = 260$ ✓

4, 2
2 3, 2

take $(a_1, a_2, a_3, a_4) =$ ~~(1, ... 3~~ ~~(... 0, 2~~ ~~(1, 4,~~ (1, 7, 5, 3).

1, 7, 5, 3, 8, 2, 4, 6	0, 56	1, 63, 5, 59, 8, 58, 4, 62
8, 2, 4, 6, 1, 7, 5, 3	48, 8	56, 10, 52, 14, 49, 15, 53, 11
1, 7, 5, 3, 8, 2, 4, 6	32, 24	33, 31, 37, 27, 40, 26, 36, 30
8, 2, 4, 6, 1, 7, 5, 3	16, 40	24, 42, 20, 46, 17, 47, 21, 43
1, 7, 5, 3, 8, 2, 4, 6	56, 0	57, 7, 61, 3, 64, 2, 60, 6
8, 2, 4, 6, 1, 7, 5, 3	8, 48	16, 50, 12, 54, 9, 55, 13, 51
1, 7, 5, 3, 8, 2, 4, 6	24, 32	25, 39, 29, 35, 32, 34, 28, 38
8, 2, 4, 6, 1, 7, 5, 3	40, 16	48, 18, 44, 22, 41, 23, 45, 19

260

~~48 + 29 + 64 + 21 + 33 + 57 + 8~~

$(2, 1): 48 + 29 + 9 + 60 + 24 + 37 + 49 + 4 = 260$ ✓

$(2, -1): 48 + 5 + 49 + 36 + 24 + 61 + 9 + 28 = 260$ ✓

$(1, 2): 48 + 50 + 20 + 14 + 41 + 35 + 21 + 11 = 260$ ✓

$(1, -2)\ 48 + 10 + 20 + 54 + 41 + 15 + 21 + 51 = 260$ ✓

(75)

For 16 × 16 what is cond^n for knight-nasiks: Planck (Andrews p. 381)

has given a scheme for Q_1 & Q_2

1, 2, 16, 15, 13, 14, 4, 3 | 12, 11, 5, 6, 7, 9, 10

$$1 + 13 + 12 + 8 = 34 = 2 \times 17$$

$$16 + 4 + 5 + 9 = 34/4$$

But here he uses complementaries in A ~~which is forbidden~~ in my simpler method

for getting nasiks ~~on the~~ we have not taken complementaries. In the case of

$n = 8$ using such simple cases of form $a_i + a_j = a_k + a_l$ we have for k-n's

is there such a counterpart for $n = 16$? for $A = (1, 2, 3, 4, 5, 6, 7, 8)$

$1 + 2 + 7 + 8 = 3 + 4 + 5 + 6$ If this should give a k-n of my prescription for

$n = 8$ be correct. [yes it does, see, thin sheets collection near page]

$\frac{1}{4}$ (+5)

$k = 30$

910

$$An + \beta k = 130$$

$$4A + 30\beta = 130$$

$$2A + 15\beta = 65 = 65(1 \cdot 15 - 2 \cdot 7)$$

$$\frac{2}{15} = 0 + \frac{1}{15/2} = 0 + \frac{1}{7+\frac{1}{2}} = 0 + \frac{1}{7+}\frac{1}{2}$$

$$2(A + 455) = 15(65 - \beta)$$

$65 - \beta = 2t$

$$\frac{A + 455}{15} = \frac{65 - \beta}{2} = t$$

$$\frac{0}{1}, \frac{1}{7}, \frac{2}{15}$$

$2(455 + 15t) + 15(65 - 2t)$

910, 975

$A = ~~455 + 15t~~ \; 15t - 455$

$\beta = 65 - 2t$

910, 975, 65

$$\left.\begin{aligned} A &= 15t - 455 \\ \beta &= 65 - 2t \end{aligned}\right\}$$

$\frac{455}{15} = 30\frac{1}{3}$.

$\frac{65}{2} = 32\frac{1}{2}$

$t = 30, 31, 32.$

$(A, \beta) = (10, 3), (25, 1)$

~~10, 15, 20, 25~~ 10, 13, 16, 19, .. 25.

25, 26, .. , 40

	1,	2,	3,	4.	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16
0	1	2	3	4	5, 6, 7, 8		
A 16	17	18	19	20			
B 17							
C 18							

TIME TABLE

PERIODS	1	2	3	4		5	6	7	8
MON					RECESS				
TUE									
WED									
THU									
FRI									
SAT									

Bhaskara Says "The difference between the longitudes of a planet found at the Same time on two successive days is called its rough motion during the interval of time and its tatkalika motion is its exact motion".

The tatkalika or instantaneous motion of a planet is the motion which it would have in a day had its velocity at any given instant of time remained uniform.

... "Tatkalika motion can be no other than the differential of the longitude of a planet" ... — Bapu

Let x, x' be the mean longitude of a planet on two successive days.

y, y' — the mean anomalies ; u, u' = true longitudes

a = eccentricity or the Sine of the greatest equation of the orbit.

$(x'-x)$ = mean motion, $(y'-y)$ = motion of mean anomaly

$(u'-u)$ = true motion.. Then according to Bhaskara the equation of the orbit on the 1st day $= \frac{a \operatorname{Sin} y}{rad}$.

On the next day $\frac{a \operatorname{Sin} y'}{rad}$.

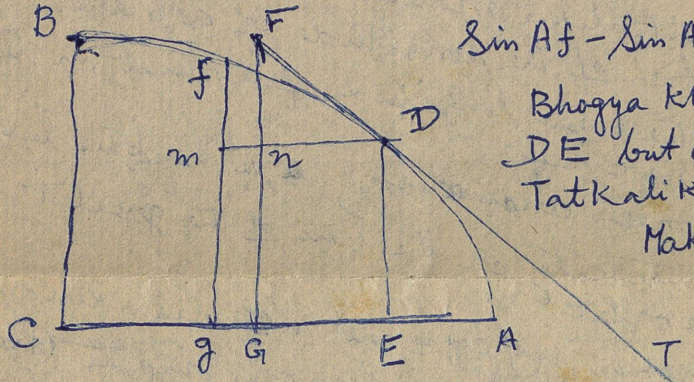
$$u = x - \frac{a \operatorname{Sin} y}{rad} \qquad u' = x' - \frac{a \operatorname{Sin} y'}{rad}$$

$$\therefore u' - u = (x' - x) - \frac{a(\operatorname{Sin} y - \operatorname{Sin} y')}{rad}.$$

Difference between two successive Sines can be found from Bh's Table of Sines (Hindu) such as $\operatorname{Sin} A - \operatorname{Sin} 0$; $\operatorname{Sin} 2A - \operatorname{Sin} A$, $\operatorname{Sin} 3A - \operatorname{Sin} 2A$ ... are Bhogya khandas

P.T.O

"These are not equal to each other but gradually decrease. Hence the difference between two Successive Sines is not Tatkalika Bhogya-khanda; but if the arc instead of being deflected be increased in the direction of the tangent, then the increase which would take place in the Sine is the Tatkalika-Bhogyakhanda.



Let arc $Df = A$

$\sin Af - \sin AD = fm$

Bhogya khanda of Sine DE but this is not Tatkalika –

Make $DF = Df$ (arc) along tangent –

$FG \perp$ to CE

Then $Dn \perp FG$, $\therefore Fn =$ Tatkalikha Bhogyakhanda.

Bhaskara has determined that Fn varies as the cosine of the arc, ie when arc $= 0$ its cosine $=$ radius, and then $A =$ Tatkalika Bhogya khanda.

As R (or the Cosine when the arc is 0) : Tatkalika Bhogyakhanda (A)

: : Cosine y : Tatkalikha Bhogya of Sine y.

$\therefore$ Sine $y = \dfrac{A\cos y}{R}$ (Comparing Similar Δs DFn and DCE)

Then using the proportion –

$A : \dfrac{A\cos y}{R} : : (y'-y) : \dfrac{(y'-y)\cos y}{R}$ (= the instantaneous value of $\sin y' - \sin y$)

Then $u'-u = (x'-x) - \dfrac{a}{R}\,\dfrac{(y'-y)\cos y}{R}$ = Instantaneous motion of planet.

This equation is Compared to

$$d(u) = d\left(x - \frac{a\sin y}{R}\right)$$

$$= dx - \frac{a}{R}\,\frac{\cos y}{R}\,dy.$$

Tatkalikha Bhogyakhanda is the velocity of the planet

Hence it is plain that Bhaskara was fully acquainted with the principles of Diffl calculus But incidentally and briefly treated by him His followers neglected it!

(0, 0) (0, 1), (0, 3), (0, 4)

(1, 0), (1, 1), (1, 2), (1, 3), (1, 4)

(2, 2)

Srirangapuram
25. 2. 80

Sir

Venkatachaliyan & myself will call on you on Thursday (28.2.80) at about 4 p.m.

Trust this finds you keeping fine

Respectfully yours

[illegible]

p. 232

p. 44 -

New recreations with Magic squares by William H. ~~Besor~~ Benson & Oswald Jacoby

Dover Publications

Write to Ashok about this book

BM
10/4/80

Dr B. S. Madhava Rao

4, Kanakapura Road

Bangalore

560004

~~पिन PIN~~

23-6-81 Good God

1. There was once a little birdie,
Living in a forest tree,
And it sang a song one morning
That was sweet, as sweet could be

2. Would you know what sang the birdie,
Living in the forest tree?.
Joyously it sang one morning,
God is Good, he cares for me

3). Little children, join the music,
Of the birdie in the tree
Sing again this happy morning
God is good, He cares for me.

	3	1
Brinjal	—	00 = 70
Beans	—	00 = 30
Tomato	—	2 = 00
Snake gourd	—	1 = 75
Lady finger	—	00 = 75
Lemon	—	00 = 50
Charcoal	—	4 = 00
		10 = 00

(1) Pauling & Wilson – Introdⁿ to Quant. Mech. – McGraw Hill, 1935.

(2) Heisenberg – Physical principles of quantum theory – Chicago Press, 1930.

(3) D. Bohm – Quantum Theory – Prentice Hall, 1951.

(4) E. C. Kemble – Fund. principles of Quant. Mech. – McGraw Hill, 1937.

(5) P. A. M. Dirac – Principles of Quant. Mech – Oxford, N.Y, ~~1937~~ 1947.

(6) J. V. Neumann – Math. Grund. der Quantmech. – Dover, N.Y, 1932.

(7) Feenberg & Pake – Notes on Quant. Th. of Ang. Mom – Addison-Wesley, 1953.

(8) Morse & Feshbach – Methods of Th. Phys – McGraw Hill, 1953.

(9) R. E. Marshak – Meson Physics – McGraw Hill, 1952.

(10) W. Heitler – Quantum Theory of radiation – Oxford, N.Y, 1954.

(11) Blatt & Weisskopf – Theoretical nuclear physics – Wiley, N.Y, 1952.

(12) Condon & Shortley – Theory of atomic spectra – Cambridge, 1935

(13) H. A. Bethe – Elem. nuclear Theory – Wiley, N.Y, 1947

(14) L. Rosenfeld – Nuclear forces – Interscience, N.Y, 1948.

(15) Gamow & Critchfield – Theory of atomic nucleus etc. – Oxford, N.Y, 1949

(16) E. Fermi – Nuclear Physics – Univ. of Chicago Press, 1950

(17) R. G. Sachs – Nuclear Theory – Addison-Wesley, 1953.

(18) G. Wentzel – Einführung in die Quant th. der Wellenfelder. – English trⁿ – Interscience 1949.

(19) F. J. Dyson – Advanced Quant. Mech – Cornell Univ, 1954.

(20) Pauli's article in the Enzy. Math. Wiss.

(21) Landau & Lifschitz – Quant. Mech.

(1) cyclic method (Stern)
& enumeration. n prime

11.70
2.05
7.90
16.09

36.75

4.25+7

29.75

3
38.25
1.7?

8
10
11
5

34

			15
8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	15

S = 15

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

				34
16	2	3	13	34
5	11	10	8	34
9	7	6	12	34
4	14	15	1	34
34	34	34	34	34

S = 34

$$a + o + h + y = t + k + n + d.$$

$a - l = p - t$	$a - v = s - d.$
$v - o = k - b$	$l - o = n - q$
$s - n = h - c$	$p - k = h - m$
$d - g = m - y$	$t - b = c - y$

$$(a-v)-(l-o) = (a+o)-(v+l) = s-d-n+q$$

$$(a-v)-(l-o) = (a+o)-(v+l) = p-k-t+b$$

$$(h-m)-(c-y) = (h+y)-(m+c) = p-k-t+b$$

$$a+o+h+y = (s+q+p+b)-(n+b+t+b)$$

$$(t+k)-(b+p)$$

$h+n = 5-(g+m)$

$o+p = 5-(k+s)$

$n+q = \quad 5+2(h+o)+2(n+p)+5 = 45$

$h+o+n+p = \frac{4}{2} \cdot 25 = 5$

x	15	4	24	16			
	13	20	15	17	14	16	
$\frac{264}{4}$	15	4	5	8	9	12	
66			x	x-7	y	y-7	
$\frac{256}{14}$ = 16			x+10	x-16	y-5	y-11	

$x-10+16 = y+9 \quad y+15 = x-7+9$