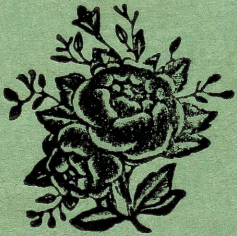
NOTE BOOK



(With Wraper 100 Pages)

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Class Book. No. 16

Subject

School/Collage

(1)

(18/4/76) <u>old ratio of squares problem</u>

$$l^2 - (4\lambda - 1)3n^2 = (\lambda - 1)^2.$$

$\boxed{\lambda = 16}$ $l^2 - 189n^2 = 225$

$l = 3l', \; 9l'^2 - 189n^2 = 225$

$l'^2 - 21n^2 = 25.$

$\sqrt{21}$

0	4	1	3	3	1	4	4	1
1	5	4	3	4	5	1	5	4
4	1	1	2	1	1	8	1	1
	*					*		

$c = 6, \; (p_6, q_6).$

$$4 + \frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{1} = 4 + \frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{2}$$

$$= 4 + \frac{1}{1+}\frac{1}{1+}\frac{2}{5} = 4 + \frac{1}{1+}\frac{5}{7} = 4\frac{7}{12} = \frac{55}{12}.$$

Same C.F. as for $\lambda = 2$ ie $l^2 - 21n^2 = 1$

~~$l = 165, n = 12.$~~ $l =$

~~$m = (495 - 48)/63 = 447/63$~~

for $l'^2 - 21n^2 = 25$

$l' = 165, n = 60$ for $l^2 - 189n^2 = 225$

$l = 495, n = 60$

495
48
447

$(3 \cdot 165)^2 - 189 \cdot 60^2 = (165^2 - 21 \cdot 60^2)9$

$= ~~81(225)^2~~ \; (55^2 - 21 \cdot 12^2)225 = 225.$

$\frac{497 - 48}{63}$ not an integer

$\lambda = 16$ is a flop.

(2)

$$l^2 - (4\lambda - 1) 3 n^2 = (\lambda - 1)^2$$

$$m = (l - 3\lambda)/(4\lambda - 1)$$

$$l - 3\lambda = m(4\lambda - 1), \quad l = m(4\lambda - 1) + 3\lambda$$

$$\{m(4\lambda - 1) + 3\lambda\}^2 - 3(4\lambda - 1) n^2 = (\lambda - 1)^2$$

$$m^2(4\lambda - 1)^2 + 9\lambda^2 + 6\lambda(4\lambda - 1) m - 3(4\lambda - 1) n^2 = (\lambda - 1)^2$$

$$\frac{q^2 + (q+1)^2 + \cdot + (q+m-1)^2}{p^2 + (p+1)^2 + \ldots + (p+m-1)^2} = \lambda$$

$$\frac{1}{6}(q+m-1)(q+m)(2q+2m-1) - \frac{1}{6}(q-1)q(2q-1)$$

$$= \lambda\left\{\frac{1}{6}(p+m-1)(p+m)(2p+2m-1) - \frac{1}{6}(p-1)p(2p-1)\right\}$$

$$(q+\overline{m-1})(q+m)(2q+\overline{2m-1}) - q(q-1)(2q-1)$$

$$= \lambda\{(p+m-1)(p+m)(2p+2m-1) - p(p-1)(2p-1)\}$$

$$L.H.S = (q+m)\{2q^2 + q(2m-1+2m-2) + m(2m-1)(m-1)$$

$$= \cancel{2q} - q\{2q^2 - 3q + 1\}$$

$4m^2 - 3m + 2m^2 - 3m + 1$

$(m-1)$

$$= 2q^3 + q^2(4m - 3 + 2m) + q\{(4m-3)m + m(2m-1)\}$$

$$+ m^2(2m-1)(m-1)$$

$$-2q^3 + 3q^2 - q$$

$$= q^2(6m - 3 + 3) + q(6m^2 - 6m) + m^2(2m-1)(m-1)$$

$$6mq^2 + (6m^2 - 6m + 1)q + m^2(2m-1).(m-1)$$

$$R.H.S. = \lambda\{6mp^2 + (\qquad)p + \text{"}$$

~~$6m(q^2 - \lambda p^2) + (6m^2 - 4m - 1)(q - \lambda p) + m^2(2m-1)(1-\lambda) = 0$~~

$$\text{Put } \lambda = 2, \quad 6m(q^2 - 2p^2) \cancel{+ (\qquad)(q - 2p)} - m^2(2m-1) = 0$$

$$\text{If } q = 2p, \quad 12mp^2 - m^2(2m-1) = 0$$

8 x 15

$$12p^2 = m(2m-1).$$

441.

$q = \lambda p.$

$6mp^2\lambda(\lambda-1) + m^2(2m-1)(1-\lambda) = 0$

$6\lambda mp^2 = m^2(2m-1)$, $6\lambda p^2 = m(2m-1)$

$m = 6n$, $6\lambda p^2 = 6n(12n-1)$, $\lambda p^2 = n(12n-1)$ X

8 × 95 $12n-1 = \lambda = 2$, $6m^2(q^2-2p^2) + (6m^2-4m-1)(q-2p) - m^2(2m-1) = 0.$

$m = 10$, $60(q^2-2p^2) +$

600
41
559

$6mq^2 + 6m(m-1)q + (m-1)m(2m-1)$

$= \lambda\{6mp^2 + 6m(m-1)q + (m-1)(m)(2m-1)\}$

$6q^2 + 6(m-1)q + (m-1)(2m-1)$

$= \lambda\{6p^2 + 6(m-1)p + (m-1)(2m-1)\}$

$6(q^2-\lambda p^2) + 6(m-1)(q-\lambda p) + (m-1)(2m-1)(1-\lambda). = 0$

Putting $\lambda = 2$, $6(q^2-2p^2) + 6(m-1)(q-2p) - (m-1)(2m-1) = 0$

$2m^2 - 3m + 1 - 6(q^2-2p^2) - 6(m-1)(q-2p) = 0$

$2m^2 - 3m\{2(q-2p)+1\} - 6(q^2-2p^2) + 6(q-2p) \cancel{=0}$ $+1 = 0$ — Ⓐ

If $q = 2p$, $2m^2 - 3m - 12p^2 = 0$ x ~~$2q-4p+1=$~~ ~~$2q-4p+1$~~.

~~$2m^2 - 6m(q-2p) - 3m$~~. $12p^2 = m(2m-3)$ $2m-3 = n$

If $m =$ $12p^2 = \frac{1}{2}n(n+3)$. $m = \frac{1}{2}(n+3)$

$n(n+3) = 24p^2$. $15 \times 18 = 24p^2$ $24p^2 = (m-1)(2m-1)$

$m = 4p+1$
$2m = 8p+2$
$4p(8p+1)$
$= 24p^2$
$8p^2 + 4p$

144
169
196
509

If $q = \mu p.$ $2m^2 - 3m\{2p(\mu-2)+1\} - 6p^2(\mu^2-2) + 6p(\mu-2) = 0$

$m = 3$, $18 - 9\{2p(\mu-2)+1\} - 6p^2(\mu^2-2) + 6p(\mu-2) = 0$

$2m^2 - 3m - 24p^2 + 1 = 0$

~~$6p^2(\mu^2-2) + 3p$~~

$9 - 18p(\mu-2) - 6p^2(\mu^2-2) + 6p(\mu-2) = 0$

$(2m^2-3m+1) = 24p^2$

$6p^2(\mu^2-2) + 12p(\mu-2) - 9 = 0$

$(2m-1)(m-1) = 24p^2$

$\dfrac{12^2+13^2+14^2}{6^2+7^2+8^2}$

$2p^2(\mu^2-2) + 2p(\mu-2) - 3 = 0$

36
49
64
149

298

$p = \dfrac{-2(\mu-2) + 24(\mu^2-2)}{4(\mu^2-2)}$

If $\mu = 2$, $p = \dfrac{48}{8} = 6.$

(4)

$(m-1)(2m-1) = 24p^2$ 24, 96, 24·9, 24·25
12·18 4·6·5·5

$2m^2 - 3m + 1 =$

$2m^2 - 3m + (24p^2 - 1) = 0$ $\Delta = 9 + 8(24p^2 - 1)$

~~$\frac{1}{4}[9 + 8(24p^2-1)]$~~

~~$96p^2 + 1 = 4l^2$~~ $= 1 + 192p^2 = l^2$

~~$4l^2 - 96p^2 = 4.$~~

$4\cdot 25 - 96\cdot 1 = 1.$

$\sqrt{96}$

96
36
6

$$\begin{array}{c|c|c|c|c|c} 0 & 9 & 6 & 6 & 9 & 9 \\ 1 & 15 & 4 & 15 & 1 & 15 \\ 9 & 1 & 3 & 1 & 18 & 1 \\ & \times & & & \times & \end{array}$$

~~$c = 4$~~

~~$9 + \frac{1}{1+}\frac{1}{3}+\frac{1}{1}$~~

$4 + \frac{1}{1+}\frac{1}{8+}$

$5/1.$

$(-1)^4 r_5 = 1.$

$\sqrt{\frac{96}{4}}$

$\sqrt{\frac{96\times 4}{4}}$ $\sqrt{24}$

$$\begin{array}{cccc|} 0 & 4 & 4 & 4 \\ 1 & 8 & 1 & 8 \\ 4 & 1 & 8 & 1 \\ & \times & \times & \end{array}$$

$(-1)^2 r_3 = 1.$

$l^2 - 192p^2 = 1$

14×14
4

192
169
123 92

10

0 3
1 189
3

$$\begin{array}{c|c|c|c|c|c} 0 & 13 & 10 & 10 & 13 & 13 \\ 1 & 23 & 4 & 23 & 1 & 23 \\ 13 & 1 & 5 & 1 & 26 & 1 \\ & \times & & & \times & \end{array}$$

$13 + \frac{1}{1+}\frac{1}{5+}\frac{1}{1} = 13 + \frac{1}{1+}\frac{1}{6} = 13\frac{6}{7} = \frac{97}{7}.$

192×49
588   12

$97^2 - 192\cdot 7^2 = 1$ // $l = 97, p = 7$ //

28×4 7×24
38 81
194
23 28.

9409
9408

~~$2m^2 - 3m - 2327 = 0$~~

$2m^2 - 3m - 1175$

235
47 2.

24×49.
196
98
1176

$m = \frac{3 + 97}{4} = 25.$

$(m-25)(2m+47) = 0$

$p = 7,\ m = 25$ }
$q = 14$

14
24
8

~~$\frac{14^2+15^2+ - +38^2}{7^2+8^2+ - +31^2}$~~

~~$\frac{1}{6}(38)(39)(77) - \frac{1}{6}(13)(14)(27)$~~

~~$= 2\{\frac{1}{6}(31)(32)(63) - \frac{1}{6}(6)(7)(13)\}$~~

~~$(19)(13)(77) - (13)(7)(9)$~~

~~$= 2\{(31)(16)(21) - (7)(13)$~~

~~$(19)(13)(77) - (13)(7)(9) + 2(7)(13)$~~

~~$= 2(31)(16)(21)$~~ 1

13 ~~$(13)\{(19)(77) - (7)(9) + 14\} = 2(31)(16)(21)$~~ 2

~~$13\{1463 - 63 + 14\} = 2\,2(16)(651)\}$~~

~~$13 \cdot 1414 = 2(16)(651)..\times$~~ ~~1302~~

5 ~~$18382 = 20832$~~ //

$\ell \quad 2m^2 - 3m - (12p^2-1) = 0$

$\Delta = \cancel{9}\ 9 + 8(12p^2-1) = \ell^2$

$96p^2+1 = \ell^2$ ~~$\ell^2 - 1 = 96p^2$~~

$\ell^2 - 96p^2 = 1$

$\sqrt{96}.$

0	9	6	6	9	9
1	15	4	16	7	15
9	1	3	1	18	1
	×		⊗	×	

~~$\sqrt{96} = 9+\frac{1}{1+}\frac{1}{7+}\frac{1}{1+}\frac{1}{8+}$~~

$9+\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{18+}$
× ×

15
64
96 32
36 60
60

~~$c, (p_2, q_2)\ 6_1^0 = (10,$~~ $C=4$

$9+\frac{1}{1+}\frac{1}{4} = 9\frac{4}{5} = \frac{49}{5}$

$p=5, \ell=49$ //

9600
4

$49^2 - 96\cdot 5^2 = 2401 - 2400 = 1$ ✓

(b)

~~$2m^2-3m-299=0$~~ ~~13·23~~

~~2m~~ $m=(3+49)/4=13$ ✓

$(m-13)(2m+23)=$ $p=5,\ q=10.$

$$\boxed{\frac{10^2+11^2+\cdots+22^2}{5^2+6^2+\cdots+17^2}=2.}$$ ✓✓

$(22)(23)(45)-9(10)(19)=2\{(17)(18)(35)-(4)(5)(9)\}$

$(22)(23)(5)-(10)(19)=2\{(17)(2)(35)-(4)(5)\}$

$(22)(23)-38=2\{(17)(14)-4\}$

25 3 $506-38=2\{238-4\}.$

$468=2\cdot 234=468$ ✓

Try $\lambda=5$; $6(q^2-\lambda p^2)+6(m-1)(q-\lambda p)+(m-1)(2m-1)(1-\lambda)=0.$

$3\cancel{6}(q^2-5p^2)+\underset{3}{\cancel{6}}(m-1)(q-5p)-\overset{2}{\cancel{4}}(m-1)(2m-1)=0$

$\cancel{4}(\ 2(m-1)(2m-1)-3(q^2-5p^2)-3(m-1)(q-5p)=0$

Put $q=\mu p$. $2(m-1)(2m-1)-3\cancel{p}\,3p^2(\mu^2-5)-3p(m-1)(\mu-5)=0.$

$4m^2-6m+2-3pm(\mu-5)-3p^2(\mu^2-5)+3p(\mu-5)=0$

$4m^2-3m\{2+p(\mu-5)\}-3p^2(\mu^2-5)+3p(\mu-5)+2=0$

If $\mu=2\rightarrow$ $4m^2-3m(2-3p)+(3p^2-9p+2)=0.$

$\Delta=9(2-3p)^2-16(3p^2-9p+2)$

$=9(4-12p+4p^2)-16(3p^2-3p+2)$

108
48 $=-12p^2-60p+4=-4(3p^2+15p-1)=-ve$

$\mu=3,$ $4m^2-3m\{2-2p\}\cancel{+q}-12p^2-6p+2=0$

$2m^2+3m(p-1)-(6p^2+3p-1)=0$

$1+19t^2=l^2$

$19p^2+2p - t^2=0$

~~4~~ $+1$

$4p^2 \cdot 4p^2 + 4 \cdot t^2 \cdot 19p^2$

17

$-18p$

$$\Delta = 9(p-1)^2+8(6p^2+3p-1)$$

$$= 57p^2+6p+1, = 3(19p^2+2p)+1$$

$l^2-19t^2=1$

$$\Delta = 3t^2+1, = l^2, \quad l^2-3t^2=1.$$

$l=2, t=1.$ $4p^2(1+19t^2)$

156

$\sqrt{19}$	0	4	2	3	3	2	4	4
	1	3	5	2	5	3	1	3
	4	2	1	3	1	2	8	2
		*					*	

$$4+\frac{1}{2+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{2} = 4+\frac{1}{2+}\frac{1}{1+}\frac{1}{3+}\frac{2}{3}$$

$$= 4+\frac{1}{2+}\frac{1}{1+3/11} = 4+\frac{1}{2+\frac{11}{14}} = 4\frac{14}{39} = \frac{\cancel{160}\;170}{39}$$

$170^2-19\cdot 39^2$ $\quad 19.1521$

$28900-28899$

$=1$

$$p = \frac{-2+2l}{38}$$

$$p = 338/38 \quad \text{not an integer}$$

$\sqrt{4+4\cdot 19t^2}$

~~86700~~

2 ‖ 86701(29

4

49 | 467

441

58 | 2601

4563

4563

$-2\pm\sqrt{\ }$ $\quad \frac{338}{38} \quad \frac{169}{19}$

$\mu=3$ does not work

$$\mu=4 \rightarrow 4m^2-3m(2-p)-33p^2-3p+2=0$$

$$4m^2+3m(p-2)-(33p^2-3p-2)=0$$

$$\Delta = 9(p-2)^2+16(33p^2-3p-2)$$

$$= 9(p^2-4p+4)+16(33p^2-3p-2) \quad \times$$

$$\mu=5 \rightarrow 4m^2-6m-60p^2+2=0, \quad \text{~~4m^2-6m-~~}$$

$$\cancel{Q} = 2m^2-3m-(30p^2-1)=0$$

$$\Delta = 9+8(30p^2-1) = 240p^2+1.$$

$$l^2-240p^2=1. \qquad m=(3+l)/4.$$

$\sqrt{240}$	0	15	15	15
	1	15	1	15
	15	2	30	2
		*	*	

$c=2,$ $\quad \sqrt{240} = 15+\frac{1}{2+}\frac{1}{30+}$

$(p_2, q_2) = (31, 2)$ $\quad 31/2$ $\quad l=31, p=2$

$31^2-4\cdot 240 = 961-960 = 1.$ $\quad (31+3)/4$ not an integer

(p_4, q_4), $p_4 = 31^2+240\cdot 4 = 1921$, $(1921+3)/4 = \frac{1924}{4} = \underline{\underline{481}}.$

10(8)

1.2.3

$$\frac{10^2 + 19^2 + \quad - \quad + 490^2}{2^2 + 3^2 + \quad - \quad + 482^2} = 5.$$

$(490)(491)(981) - (9)(10)(19)$

$= 5\{(482)(483)(965) - 6\}$

$(98)(491)(981) - (18)(19) = (482)(483)(965) - 6.$

$(49)(491)(327) - 57 = (241)(161)(965) - 1.$

9. $(49)(491)(327) - 56 = (241)(161)(965)$

$(7)(491)(327) - 8 = (241)(23)(965)$

ends with 1 ends with 5

$2m^2 - 3m - (30p^2 - 1) = 0$

$2m^2 - 3m - 119 = 0$ $\quad 9 + 8(30p^2 - 1)$

952
9
961.

~~$9 + 8.119. = 19^2 = \square.$~~ $\quad 240p^2 + 1 = l^2$

λ=4 ~~$m = (3 \pm 19)/4 = 22/4.$~~

$m = \{3 + \sqrt{9 + 8(30p^2 - 1)}\}/4.$

$(3 + \sqrt{240p^2 + 1})/4 = 34/4.$

~~$l^2 - 144p^2 = 1$~~ $\quad l^2 - 192p^2 = 1$

$\sqrt{144} = 11$ ~~0 12~~

~~1 0~~

~~12~~

$13 + \frac{1}{1+}\frac{1}{5+}\frac{1}{1}$

$q = 28$

$\sqrt{192}$
169
23

0	13	10	10	13	13
1	23	4	23	1	23
13	1	5	1	26	1
	×			×	

$13 + \frac{1}{1+}\frac{1}{6}$

14

$13\frac{6}{7} = \frac{97}{7}.$

49
784

$97^2 - 192.7^2 = 1$ ✓

$\frac{97+3}{4} = 25 = \underline{\underline{m}}$

$9409 - 9408 = 1$

145

$$\frac{28^2+29^2+ \quad - \quad +52^2}{7^2+8^2+ \quad - \quad +31^2} = 4.$$

$$(52)(53)(105) - (27)(28)(55) = 4\{(31)(32)(63) - 6\cdot7\cdot13\}$$

$$(13)(53)(105) - (7)(27)(55) = (31)(32)(63) - (6)(7)(13)$$

4

$$(13)(53)(15) - (27)(55) = (31)(32)(9) - (6)(13)$$

13^5 6 12

$$(13)(795) - (1485) = (279)(32) - 78$$

558 12 9

$$\begin{array}{r} 10335 \\ 1485 \\ \hline 8850 \end{array} \checkmark \qquad = \begin{array}{r} 8928 \\ 78 \\ \hline 8850 \end{array} \checkmark$$

$$l^2 - 48\lambda p^2 = 1$$

$\lambda = 6.$ $\qquad l^2 - 288 p^2 = 1.$

$\sqrt{288}$

$$\begin{array}{r} 288 \\ 256 \\ \hline 32 \end{array}$$

$$\begin{array}{c|c|c|c|} 0 & 16 & 16 & 16 \\ 1 & 32 & 1 & 32 \\ 16 & 1 & 32 & 1 \\ & \ast & \ast & \end{array}$$

$$16 + \frac{1}{1+} \Big| \frac{1}{32+} - \quad l = 17, p = 1$$

$$\frac{17}{1}. \qquad m = \frac{17+3}{4} = 5.$$

$$\boxed{\frac{6^2+7^2+8^2+9^2+10^2}{1^2+2^2+3^2+4^2+5^2} = 6}$$

$$36+49+64+81+100 = 330$$

$18^2 = 324$

$$\tfrac{1}{6}\cdot5\cdot6\cdot11 = 55 \checkmark$$

✓ $\lambda = 6$ $\qquad l^2 - 336 p^2 = 1$ $\qquad 18 + \frac{1}{3} \qquad \frac{55}{3}. \qquad \frac{55+3}{4} = \text{?}$

5

$\sqrt{336}$

$$\begin{array}{c|c|c||c|} 0 & 18 & 18 & 18 \\ 1 & 12 & 1 & 12 \\ 18 & 3 & 36 & 3 \\ & \ast & \ast & \end{array}$$

$$55^2 - 336\cdot9$$

$$3025 - 3024 = 1 \checkmark$$

(10)

37

$\sqrt{384}$

$\begin{array}{r} 384 \\ 361 \\ \hline 23 \end{array}$

$\begin{array}{r} 384 \\ 16 \\ \hline 368 \end{array}$

$\begin{array}{r} 384 \\ 144 \\ \hline 240 \end{array}$

7

$\begin{array}{r} 48 \times 9 \\ \hline 472 \end{array}$

$\lambda = 8 \rightarrow l^2 - 384 p^2$

0	19	4	12	18	18	12	4	19	19
	23	16	15	4	~~60~~ 15	16	23	1	23
1	1	1	2	9	2	1	1	38	1
19	*							*	

$C = 8, \quad 19 + \frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{9+}\frac{1}{2+}\frac{1}{1+}\frac{1}{1}.$

$19 + \frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{9+}\frac{1}{2+}\frac{1}{2}$

$19 + \frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{9+}\frac{2}{5}$

$2\frac{5}{47} \quad 1\frac{47}{99}$

$1\frac{99}{146} \quad 146$

245

$19\frac{245}{146}.$

$\frac{34}{9}$

$\boxed{\lambda = 9}$

$\sqrt{432}$

0	20	12	15	8	8	15	12	20	20
1	32	9	23	16	23	9	32	1	32
20	1	3	1	1	1	3	1	40	1
	*							*	

$21^2 = 441$

$\begin{array}{r} 432 \\ 144 \\ \hline 288 \end{array}$

$C = 8.$

$\frac{1}{1+}\frac{1}{3+}\frac{1}{1} \quad \frac{1}{1+}\frac{1}{4}$

$20\frac{57}{65}$

$\frac{1357}{65}$

$\boxed{\lambda = 10}$

$\begin{array}{r} 432 \\ 225 \\ \hline 207 \end{array} \quad \begin{array}{r} 432 \\ 64 \\ \hline 368 \end{array}$

$22^2 = 484)$

$l^2 - 480 p^2 = 1$

$\frac{1354}{4}$ X

$2310^2 - 480 \cdot 11^2$

$11^2 \cdot 210^2 - 11^2 \cdot 480.$

0	21	18	18	21	21
1	39	4	39	1	39
21	1	9	1	42	1
	*			*	

$21^2 = 441$

$\begin{array}{r} 480 \\ 441 \\ \hline 39 \end{array} \quad \begin{array}{r} 480 \\ 324 \\ \hline 156 \end{array}$

$21 + \frac{1}{1+}\frac{1}{9+}\frac{1}{1}$

10

$21 + \frac{1}{1+}\frac{1}{10}$

$21\frac{10}{11}.$

$\frac{\cancel{2310}}{11}$

$\frac{241}{11}$

$(170)(171)(341) - (109)(110)(219)$

$= 10\{(71)(72)(143) - (10)(11)(21)\}$

$(17)(171)(341) - (109)(11)(219) = (71)(72)(143) - (10)(11)(21)$

~~$(17)(171)(117) - (109)(11)(73) = (71)(24)(143) - (10)(11)(7)$~~

~~$340119 - 527$~~

$(17)(171)(31) - (109)(219) = (71)(72)(13) - (10)(21)$

$(17)(57)(31) - (109)(73) = (71)(24)(13) - 70$

$30039 - 7957 = 16952 - 70$

~~220082~~

$$\begin{array}{r} 30039 \\ 7957 \\ \hline 22082 \end{array} \checkmark \qquad \begin{array}{r} 22952 \\ 70 \\ \hline 22082 \end{array} \checkmark \qquad \begin{array}{r} 284 \\ 142 \\ \hline 1704 \\ 9 \end{array}$$

~~$(m-1)(2m-1)$~~$(1-\lambda)$

~~mq~~ $(m-1)(2m-1)(\lambda-1) - 6(q^2 - \lambda p^2) - 6(m-1)(q-\lambda p) = 0.$

$q \neq \lambda p$, $q = \lambda p - \mu$.

$(m-1)(2m-1)(\lambda-1) - 6\{(\lambda p - \mu)^2 - \lambda^2 p^2\} + 6(m-1)\mu = 0.$

$(m-1)(2m-1)(\lambda-1) - 6\{-2\lambda\mu p + \mu^2\} + 6\mu(m-1) = 0.$

~~$(2m^2 - 3m + 1)$~~ $($ $\lambda = 2$

$(m-1)(2m-1) + 6(4\mu p - \mu^2) + 6\mu(m-1) = 0$

$2m^2 - 3m + 1 + 6\mu m + 24\mu p$ ~~$= 0$~~ $- 6\mu(\mu+1)$

~~$2m^2 - 3m (1 -$~~ $2m^2 + 3m(2\mu - 1) + 6\mu(4p - \mu - 1)$ ~~$+ 0$~~ $= 0$

$\Delta = 9(2\mu-1)^2 - 8\{6\mu(4p - \mu - 1) + 1\}$

$$\begin{array}{r} 1197 \\ 171 \\ 171 \\ \hline 20007 \\ 327 \\ 763 \\ \hline 7957 \end{array}$$

$$\begin{array}{r} 57 \\ 171 \\ \hline 1767 \\ 327 \\ 763 \\ \hline 7957 \\ 284 \\ 142 \\ \hline 1304 \\ 1767 \times 17 \\ \hline 12369 \\ 1767 \\ \hline 30039 \end{array}$$

$$\begin{array}{r} 57 \\ 171 \\ \hline 1767 \end{array}$$

(12) $(m-1)(2m-1)(\lambda-1) - 6(q^2-\lambda p^2) - 6(m-1)(q-\lambda p) = 0$

$\lambda = 2$ $\rightarrow (m-1)(2m-1) - 6(q^2-2p^2) - 6(m-1)(q-2p) = 0$

$q = p+1$, $q^2-2p^2 = (p+1)^2-2p^2$, $q-2p = p+1-2p = -(p-1)$

$(2p+1-p^2) = -(p^2-2p-1)$

$(m-1)(2m-1) + 6(p^2-2p-1) + 6(m-1)(p-1) = 0$

$2m^2 - 3m + 1 + 6m(p-1) + 6(p^2-2p-1) + 6(p-1) = 0$

$2m^2 + 3m(2p-2-1) + 6p^2 - 12p - 6 + 6p - 6 + 1 = 0$

$2m^2 + 3m(2p-2-1) + (6p^2-6p-11) = 0$

$\Delta = 9(2p - \overset{3}{\cancel{2p-1}})^2 - 8(6p^2-6p-11)$

$= 9(4p^2-12p+9) - 8(6p^2-6p-1)$

$= 36p^2 - 48p^2 - 108p + 48p + 89.$

$= -12p^2 - 60p + 89.$ ×

$q = p+2$, $(m-1)(2m-1) - 6(-p^2+4p+4) - 6(m-1)(-p+2) = 0$

$(p+2)^2-2p^2$ $2m^2 - 3m + 1 + 6(p^2-4p-4) + 6(m-1)(p-2) = 0$

$p+2-2p$ $2m^2 \cancel{-3(m}\ -3m\{1 \mp 2(p-2)\} + 6(p^2-4p-4) - 6(p-2) \cancel{+1} = 0$

$2m^2 + 3m(2p+5) + (6p^2-30p-11) = 0$

$9(2p+5)^2 - 8(6p^2-30p-11)$

~~$-12p^2 - 30p + 321$~~ ×

-180
150

225
96
$\overline{721}$

$36p^2 + 180p - 8(6p^2-30p-11)$

-240

$\Delta = -12p^2 + 420p + 88$

321
42
$\overline{279}$

$q = 3p]$ $(m-1)(2m-1) - 6(q^2 - \cancel{2}p^2) - 6(m-1)(q - \cancel{2}p) = 0$

420 508
88 12
$\overline{508}$ $\overline{496}$
12
$\overline{496}$

$(m-1)(2m-1) - 6(7p^2) - 6(m-1)p$

$2m^2 - 3m + 1 - 6mp - 42p^2 + 6p$

$2m^2 - 3m(2p+1) - (42p^2 - 6p - 1)$

$\Delta = 9(2p+1)^2 + 8(42p^2 - 6p - 1)$

336 $\quad = 9(4p^2+4p+1) + 8(42p^2 - 6p - 1)$

36 $\quad = 472p^2 - 12p + 1. \qquad p = 1,$

473

12

461 $\quad 6(q^2 - \lambda p^2) + 6(m-1)(q - \lambda p) + (m-1)(2m-1)(1-\lambda) = 0.$

$\lambda = 2 \quad 6(q^2 - 2p^2) + 6(m-1)(q - 2p) - (m-1)(2m-1) = 0$

$q = 3p. \quad 6(7p^2) + 6(m-1)p - (m-1)(2m-1) = 0.$

$m = 7, p = 1 \longrightarrow 42 + 36 - 6 \cdot 13 = 0$

6. $\quad 2m^2 - 3m + 1 - 6mp + 6p - 42p^2 = 0$

$2m^2 - 3m(2p+1) - (42p^2 - 6p - 1) = 0$

$\Delta = 9(2p+1)^2 + 8(42p^2 - 6p - 1)$

36 $\quad = 9(4p^2+4p+1) + 8(42p^2 - 6p - 1).$

336

372 $\quad = 372p^2 - 12p + 1.$

373

12 $\quad$ If $p = 1, \Delta = 361 = 19^2.$

361 $\quad m = \{3(2p+1) \pm 19\}/4 = 28/4 = \underline{\underline{7}}.$

$q = p + 2. \quad 6\{(p+2)^2 - 2p^2\} + 6(m-1)(p+2-2p) - (m-1)(2m-1) = 0$

$6(4p + 4 - p^2) + 6(m-1)(2-p) - (m-1)(2m-1) = 0$

$6(p^2 - 4p - 4) + 6(m-1)(p-2) + (m-1)(2m-1) = 0$

$2m^2 - 3m + 1 + 6m(p-2) - 6(p-2) + 6(p^2 - 4p - 4) = 0$

$2(p-2) \quad 2m^2 - 3m(1 - 2p + 4) + (6p^2 - 24p - 24 - 6p + 12 + 1)$

393

12 $\quad 2m^2 + 3m(2p-5) + (6p^2 - 30p - 11)$

381

-180 $\quad \Delta = 9(2p-5)^2 - 8(6p^2 - 30p - 11)$

225

89 $\quad = 9(4p^2 - 20p + 25) - (48p^2 - 240p - 88)$

313 $\quad = -12p^2 + 60p + 313 \quad$ If $p = 1, \Delta = 361.$

(14) $$6(q^2-\lambda p^2)+6(m-1)(q-\lambda p)+(m-1)(2m-1)(1-\lambda)=0.$$

$q=p+\mu$ $$6\{(p+\mu)^2-\lambda p^2\}+6(m-1)\{(p+\mu-\lambda p)\}+(m-1)(2m-1)\overset{(1-\lambda)}{(\cancel{\lambda-1})}=0$$

$$6\{p^2(1-\lambda)+2p\mu+\mu^2\}+6(m-1)\{p(1-\lambda)+\mu)+\quad '' \quad =0$$

$$(m-1)(2m-1)(\lambda-1)+6\{(\lambda-1)p^2-2p\mu-\mu^2\}+6(m-1)\{(\lambda-1)p-\mu\}=0$$

$$(2m^2-3m+1)(\lambda-1)+6\{\qquad\}+6m\{(\lambda-1)p-\mu)\}$$
$$-6\{(\lambda-1)p-\mu\}=0$$

$$2m^2-3m\{(\lambda-1)-2(\overline{\lambda-1}p-\mu)\}+6\{(\lambda-1)p^2-2p\mu-\mu^2\}$$
$$-6\{(\lambda-1)p-\mu\}+(\lambda-1)=0$$

$$2m^2-3m\{(\lambda-1)(1-2p)+2\mu\}+[6(\lambda-1)p^2-6p\{2\mu+(\lambda-1)\}$$
$$-6\mu^2+6\mu+(\lambda-1)=0$$

$$2m^2+3m\{(\lambda-1)(2p-1)-2\mu\}+\{6(\lambda-1)p^2-6p(2\mu+\lambda-1)$$
$$-(6\mu^2-6\mu\underset{+\lambda}{-1})\}=0$$

$$\Delta=9\{(\lambda-1)(2p-1)-2\mu\}^2-8\{\qquad\}$$
$$=9\{(\lambda-1)2p-(2\mu+\lambda-1)\}^2-8\{\qquad\}$$

$-(\lambda-1)-2\mu$

$-\lambda+1-2\mu$

$$=9\{4(\lambda-1)^2p^2-4p(\lambda-1)(2\mu+\lambda-1)+(2\mu+\lambda-1)^2\}$$
$$-8\{6(\lambda-1)p^2-6p(2\mu+\lambda-1)-(6\mu^2-6\mu\underset{+\lambda}{-1})\}$$

$$=\cancel{p^2\{36(\lambda-1)^2}\ 12p^2\{3(\lambda-1)^2-4(\lambda-1)\}$$
$$-12p\{3(\lambda-1)(2\mu+\lambda-1)-4(2\mu+\lambda-1)\}$$
$$+\{9(2\mu+\lambda-1)^2+8(6\mu^2-6\mu\underset{+\lambda}{-1})\}$$

$-36p(\ )$

$+48p(\ )$

$$12p^2(3\nu^2-4\nu)-12p\{3\nu(2\mu+\nu)-4(2\mu+\nu)\}$$
$$+\{9(2\mu+\nu)^2+8(6\mu^2-6\mu\underset{-\lambda+1.}{\cancel{-1}})\}$$

(15)

$= 12p^2 \{ \cancel{3\lambda^2 - 6\lambda + 3 - 4\lambda + 1} \} - 12p \{$

$\nu = \lambda - 1]$ $\quad 12p^2(3\nu^2 - 4\nu) - 12p\{ \overset{(2\mu+\nu)(3\nu-4)}{\cancel{6\mu\nu + 3\nu^2 - 8\mu + 4\nu}} \}$

$+ \{ 9(4\mu^2 + 4\mu\nu + \nu^2) + 8(6\mu^2 - 6\mu \overset{-\nu}{\cancel{+2\nu}}) \}$

If μ & ν be chosen such that coeff of p = 0, we get a Pellian eqn

$$6\mu\nu + 3\nu^2 - 8\mu + 4\nu = 0$$

If $\nu = 1$, ~~$12\mu + 12 - 8\mu + 8$~~ $6\mu + 3 - 8\mu + 4 = 0$, ~~$\mu = 7$~~ ✗

But soln to Pellian eqn is sufficient but not necessary. $\quad 36 \ 4 \ / 4$

If $p = 1$, $\nu = 1$, $\mu = 2$, the above Δ becomes

$\Delta = \cancel{12} - 12 - 12(12 + 3 - 16 + 4) + \{ 9(16 + 8 + 1) + 8(24 - 12 \bar{\cancel{2}} + 1) \}$

~~$= -12 - 144 - 36 + 192 - 48 + 225 + 104$~~

~~$= 521 - 240 = 281$ ✗ Something wrong.~~

Or ($\Delta = -12 - 12(5)(-1) + \{ 9(16 + 8 + 1) + 8(24 - 12 \bar{7} 1)$

$= -12 + 60 + 225 + \overset{\cancel{104}}{88}$ ~~$= 385 - 12 = 373$~~

$104 \ 88 \ / 16$ $\quad = 373 - 12 = 361 = 19^2$ ✓ Correct

$12p^2(3\nu^2 - 4\nu) - 12p(2\mu + \nu)(3\nu - 4) + (84\mu^2 + 36\mu\nu + 9\nu^2 - 48\mu - 8\nu)$

$[84\mu^2 - 48\mu = 12(7\mu^2 - 4\mu \cancel{-3}) + 36$

$= 12(\mu - 1)(7\mu + 3) + 36.$

$3\nu - 4$

$9\nu^2 + 12\nu + 16$ $\quad 9\nu^2 - 8\nu \cancel{\ } - 1 + 1 = (\nu - 1)(9\nu + 1) + 1]$

$16 + 28\lambda.$ $\quad \nu = 2;$ ~~$12p^2$~~ ~~$48p^2 - 48p$~~ $\quad 84\mu^2 + 72\mu + 36 - 48\mu - 16$

~~84~~ $64 + 36\lambda$ $\quad 48p^2 - 48p(\mu + 1) + (84\mu^2 + 24\mu + 20)$ ~~20~~

$12p^2 - 12p(\mu + 1) + (21\mu^2 + 6\mu + 5)$ $\quad 21 \ 6 \ 5 \ / \ \cancel{28}$

~~$12p^2 - 24p + \cancel{28}\ 32$~~

~~36~~ $\quad$ ~~$4(3p^2 - 6p + 8)$~~

(16)

$\Delta' = 144(\mu+1)^2 - 48(21\mu^2+6\mu+5)$

$= 48\{3(\mu+1)^2 - (21\mu^2+6\mu+5)$

$= 48\{-18\mu^2 - 2\} = 96 - 96(2\mu^2+1)$

$12p^2\nu(3\nu-4) - 12p(3\nu-4)(2\mu+\nu) + \{12(\mu-1)(7\mu+3) + (\nu-1)(9\nu+1)$
$+ 36\mu\nu + 37\}$

~~$144(3\nu-4)^2(2\mu+2\nu)^2 - 48\nu(3\nu-4)\{\quad\} = 0$~~

57

$\nu = 3 \rightarrow 180p^2 - 60p(2\mu+3) + \{84\mu^2 + 108\mu + 81 - 48\mu - 24\}$

$180p^2 - 60p(2\mu+3) + (84\mu^2 + 60\mu + 57)$

$\Delta' = 144(3\nu-4)^2(2\mu+\nu)^2 - 48\nu(3\nu-4)(84\mu^2 + 36\mu\nu + 9\nu^2$
$- 48\mu - \nu)$

~~$\frac{1}{16}\Delta' = 9\{$~~

$144(2\mu+1)^2 + 48(84\mu^2 + 36\mu + 9 - 48\mu - 1\}$

$3\nu - 4 = x^2$

$48(84\mu^2 - 12\mu + 8) + 144(2\mu+1)^2$ 3

$48[84\mu^2 - 12\mu + 8 + 3(4\mu^2+4\mu+1)]$ $\frac{84}{12}$

$=$ ~~$48[72\mu^2 - 24\mu + 5]$~~ $= 48(96\mu^2+11) = 3l^2$

$= 16 \cdot$ ~~$3(72\mu^2 - 24\mu + 5)$~~ $2(n+1)^2 - 2(n+1) + 1$

$576 - 20 \cdot 72 = 576 - 1440 = -ve$

$288\mu^2 + 33 = l^2$ $l^2 - 288\mu^2 = 33.$ $2n^2 + 2n + 1$

$96\mu^2 + 11 = 3l^2$

~~16 17~~

$3l^2 - 96\mu^2 = 11$

0	16	16	16
1	32	1	32
16	1	32	1
	*	*	

0	5	2	2	4
1	7	4	7	
5	1	1	1	

$\sqrt{\frac{16}{3}}$ $\sqrt{32}.$

$2n^2 - 2n$
$98 - 14$
$2n^2 - 2n$
$+ n - 1$
$\sqrt{288}$
$2n^2 - n.$
$17^2 = 289$ 84
288 $2n^2 - n - 1$
256 $98 - 7 - 1$
32 (99)
$-2n^2 + 2n - 1$
$-(2n^2 - n - 1)$
$+ n - 2.$ $-(2n^2 - n - 1)$
$-2n^2 + n + 2$
$+2n^2 - n - 2)$

$2n^2 - 3n - 1$
$2n^2 - 2n$
$2(n+1)^2 - 2(n+1) + 2$ $2n^2 - 2n - \overline{n-1}$
$2n^2 - 3n + 1$
$2(n+1)^2 - (n+1) + 1$ ~~$n+1$~~ $2n$
$2n^2 + 4n + 2 - n - 1 + 1$
$2n^2 + 2n + 2$ $2n^2 - n - 1$ $+ n - 2$ $2n^2 - 2n$

$\mu = 1,$ $\qquad 2(n+1)^2 - 3(n+1) + 1 \qquad 2-3+1$

$144\cancel{(3\nu-4)^2(\nu+2)^2} - 48\nu(\nu-4)$ $\qquad \cancel{2n^2+2n}$ $\quad 2n^2+n$

$\nu = 4]\quad \Delta' = 144\cdot 64(2\mu+4)^2 - 48\cdot 4\cdot 8(84\mu^2 + 288\mu + 36 - 48\mu - 4)$

$= 144\cdot 64(4\mu^2+16\mu+16) - 48\cdot 32(84\mu^2+240\mu+32)$

$2^4\cdot 2^6\cdot 2^2 \qquad = 144\cdot 64\cdot 4(\mu^2+4\mu+4) - 48\cdot 32\cdot 4(21\mu^2+60\mu+8).$

$= 9\cdot 2^{12}(\mu^2+4\mu+4) - 3\cdot 2^{11}(\qquad)$

$2^4\cdot 2^5\cdot 2^2$

$= 3\cdot 2^{11}\{3\mu^2+12\mu+12-21\mu^2-60\mu-8\} = -ve$ ✗

$2(n+1)^2-(n+1)-1$

$2n^2+4n+2$

Vigdergau

20/4/76.

$m = n-2. \qquad m_1 = \frac{1}{2}(n-2)(n-1); \; N_1 = \frac{1}{2}n(n+1)$

$m_1 - N_1 = \cancel{0} - \frac{1}{2}\{n(n+1)-(n-2)(n-1)\}$

$= -\frac{1}{2}\{\cancel{n^2}+n-\cancel{n^2}+3n-2\} = \cancel{2n-1} = -(2n-1)$

$m_2 - N_2 = \cancel{6} - \frac{1}{6}\{n(n+1)(2n+1) - (n-2)(n-1)(2n-3)\}$

$= -\frac{1}{6}\{n(2n^2+3n+1) - (n-1)(2n^2-7n+6)\}$

$m-n+1$

$n-2-n+1$

$= -\frac{1}{6}(2\cancel{n^3}+3n^2+n-2\cancel{n^3}+9n^2-13n+6)$

$= -\frac{1}{6}(12n^2-12n+6) = -(2n^2-2n+1)$

$(-1)k^2 + 2k\{-(2n-1) - n(n-2)\} + \{-(2n^2-2n+1) - n(n-2)(2n-1)\} = 0$

$n-2+n+1$

$-k^2 - 2k(n^2-1) + \{-2n^2+\cancel{2n}-1-2n^3+5n^2-\cancel{2n}\} = 0$

$k^2 + 2k(n^2-1) + (2n^3-3n^2+1) = 0$

$\cancel{k = -2(n^2-1)} \pm \quad k^2 + 2k(n^2-1) - (n-1)(\cancel{n(2n+1+2n^2-n-1}\; 2n^2+n-1)$

$(k+n-1)(k+2n^2-n-1) = 0$

$k = -(2n^2-n-1)$ $\qquad 2(n+1)^2-(n+1)-1$

$(2n^2-n-1)^2 + (2n^2-n-2)^2 + \cdots + (2n^2-2n+1)^2$ $\qquad 2n^2+3n+1$

$2(n+1)^2-2(n+1)+1$

$2n^2+4n+2-2n$

$-2n+1$

$-(2n^2-n-1)$

$+1$

$= (2n^2-2n+2)^2 + (2n^2-2n-1)^2 + + (2n^2-3n+1)^2$

$-(2n^2-n-2)$

$-(2n^2-n-1)$

$+n-2$

$2n^2-2n-n-1.$

$-2n^2+2n-1$

$2n^2-2n+2$

$+n-1$

$-(n+1-1)$

$-n;$

$-\{2(n+1)^2-(n+1)-1\}$

$-(2n^2+4n+2-n-1-1)$

$(2n^2+n)$

$2n^2+2n+1$

(18)

$k = -(n-1)$ leads

$-n+1$
$+n-2$

$$(n-1)^2 + (n-2)^2 + - \quad + 1^2 = 0^2 + 1^2 + \cdots + (n-1)^2$$

$+$
$n-2.$

$$m = n-1 \quad -1$$

$m-n+1$

$$2k\left\{\tfrac{1}{2}(n-1)n - \tfrac{1}{2}n(n+1) - n(n-1)\right\}$$
$$+\left\{\tfrac{1}{6}(n-1)n(2n-1) - \tfrac{1}{6}n(n+1)(2n+1) - n(n-1)(2n) = 0\right.$$

$\frac{1}{2}(n-1)n$
$-\frac{1}{2}n(n+1)$
$n-1+n+1$

$$2k(-n^2) + \tfrac{1}{6}n\{(n-1)(2n-1) - (n+1)(2n+1) - 2n(n-1)\} = 0$$
$$-2n^2k + \tfrac{1}{6}n\{2n^2-3n+1-2n^2-3n-1-12n^2+12n\} = 0$$
$$-2n^2k + \tfrac{1}{6}n\{-12n^2+6n\} = 0 \qquad -2n^2k = -n^2(2n-1)$$
$$-2n^2k + \tfrac{1}{3}n^2(n+2) = 0 \qquad k = -(n+2)/6. \quad k = -\frac{2n-1}{2}$$

$n-\cancel{1}+n+\cancel{1}$

$$\left(-\frac{n+2}{6}\right)^2 + \left(-\frac{n+2}{6}+1\right)^2 + \quad + \left(-\frac{n+2}{6}+n-1\right)^2$$
$$= \left(-\frac{n+2}{6}+n\right)^2 + - \quad + \left(-\frac{n+2}{6}+2n-1\right)$$

$-(n+2)+6$
$-(n-4)$

$$(n-2)^2 + (n-4)^2 + - + (5n-8)^2 = (5n-2) + -$$

$-(n+2)$
$+6(n-1)$
$5n-8$

$$\left(\frac{2n-1}{2}\right)^2 + \left(\frac{2n-3}{2}\right)^2 + - + \left(\tfrac{1}{2}\right)^2 = \left(\tfrac{1}{2}\right)^2 + - + \left(\frac{2n-1}{2}\right)^2.$$

$-n-2+6n$
$-n-2+12n-6$
$-n+\frac{1}{2}+1$
$-n+3/2$
$-\frac{2n-1}{2}+(n-1)$
$\frac{-2n+1+2n-2}{2}$
$-\frac{1}{2}+n-1$

$n=1,\ m=0,\ m-n+1=0,$
$$2k(-1)+\{0-1\}=0,\ k=-1/2.$$
$n=1,\ m=1 \quad k^2+2k(-1)+(-3)=0$
$$k^2-2k-3=0,\ k=3 \text{ or } -1$$
$$(k-3)(k+1)$$
$$3^2+4^2=5^2$$
$$(-1)^2+0^2=1^2$$

$$k^2 m + 2k(M_1 - 1 - m) + \{M_2 - 1 - m(m+2)\} = 0$$

$m - n + 1$

$m + 1 + 1$

$$mk^2 + 2k(M_1 - m - 1) + \{m(m+2) + 1 - M_2\} = 0$$

$$mk^2 + 2k\left\{\tfrac{1}{2}m(m+1) - (m+1)\right\} - \left\{m(m+2) + 1 - \tfrac{1}{6}m(m+1)(2m+1)\right\} = 0$$

$$mk^2 + k(m+1)\{m~~(\ldots)~~ - 2\} - \frac{1}{6}\{6m(m+2) + 6 - m(m+1)(2m+1)\}$$

$\frac{1}{2}m - 1$

$$mk^2 + k(m+1)(m-2) - \frac{1}{6}\{6m^2 + 12m + 6 - 2m^3 - 3m^2 - m\}$$

$$mk^2 + k(m+1)(m-2) - \frac{1}{6}\{-2m^3 \overset{+3m^2}{} + 11m + 6\} \qquad 2m^3 - 11m - 6$$

$$mk^2 + k(m+1)(m-2) + \frac{1}{6}(2m^3 \underset{-3m^2}{} - 11m - 6)$$

$$\Delta = (m+1)^2(m-2)^2 - \frac{4m^2}{\cancel{6}\,3}(2m^3 \underset{-3m^2}{} - 11m - 6)$$

$$3\Delta = 3(m^2 - m - 2)^2 - 2m(2m^3 \underset{-3m^2}{} - 11m - 6)$$

m

$$= 3(m^4 - 2m^3 - 3m^2 + 4m + 4) - (4m^4 \underset{-6m^3}{} - 22m^2 - 12m)$$

$$= -m^4 - \cancel{6m^3} + 13m^2 + 24m + 12 + \underset{+\cancel{6m^3}}{} \quad (?)$$

$$-m^4 + 13m^2 + 24m + 12 = -m^4 + 13m^2 + 24m + 12$$

$$\text{~~}24m - (m^4 - 13m^2 - 12)\text{~~} \quad (24m + 24) - (m^2 - 1)(m^2 - 12)$$

$$24m - (m^4 - 13m^2 + 12) + 24. \qquad 24(m+1) - (m^2-1)(m^2-12)$$

18

$3^2 \cdot 6^2$

336
12
324

$$3\Delta = 24(m+1) - (m^2-1)(m^2-12)$$

(48 + 18)

$m = 1$, $3\Delta = 48$ or $\Delta = 16$, usual config $m = n$

$m = 2$, $3\Delta = ~~48 + 10~~ = 72 + 3 \cdot 8 = 96$. or $\Delta = 32$ ×

$m = 3$, $3\Delta = 96 + 8 \cdot 3 = 120$, $\Delta = 40$ ×

$m = 4$, $3\Delta = 120 - 15 \cdot 4 = 60$, $\Delta = 20$

$m = 5$, $3\Delta = 144 - 24 \cdot 13 = 144 - 312 = -4$ //×

How does it happen that for $m < n$ there are no solutions so far found?

Suppose $m = n - j \quad (j < n)$

$(m-n+1) = n-j-n+1 = -j+1.$

$$M_1 - N_1 - mn = \frac{1}{2}(n-j)(n-j+1) - \frac{1}{2}n(n+1) - n(n-j)$$

$$= \frac{1}{2}\left\{(n-j)^2 + (n-j) - n(n+1) - 2n(n-j)\right.$$

$$= \frac{1}{2}\left\{\cancel{n^2} - 2\cancel{n}j + j^2 + \cancel{n} - j - \cancel{n^2} - \cancel{n} - 2n^2 + 2\cancel{n}j\right\}$$

= ~~$\frac{1}{2}(2n^2 + 2nj - j^2 + j)$~~ $= -\frac{1}{2}(2n^2 - j^2 + j)$

$M_2 - N_2 - mn(m+n+1)$

$$= \frac{1}{6}(n-j)(n-j+1)(2n-2j+1) - \frac{1}{6}n(n+1)(2n+1)$$

$$- n(n-j)(2n-j+1)\}$$

$n-j+n+1$

$$= \frac{1}{6}\left\{(n-j)(n-j+1)\overset{2(n-j)}{(2n-2j+1)} - n(n+1)(2n+1)\right.$$

$$\left. - 6n(n-j)(2n-j+1)\right\} \qquad n\{2n^2+3n+1$$

$$= \frac{1}{6}\left\{(n-j)\{2(n-j)^2 + 3(n-j) + 1 - n(n+1)(2n+1)\right.$$

$$-6(n-j)(2n^2 - nj + n)\}$$

$$= \frac{1}{6}\Big[2(n-j)^3 + 3(n-j)^2 + (n-j) - n(n+1)(2n+1).$$

$$-12n^2(n-j) + 6nj(n-j) - 6n(n-j)\Big]$$

= $\frac{1}{6}\Big[$ ~~$2n^2 - 4nj + 2j^2$~~ $- \cancel{2n^3} - \cancel{6n^2j} + \cancel{6nj^2} - 2j^3 + \cancel{3n^2} - \cancel{6nj} + 3j^2$

$- \cancel{2n^3} - \cancel{3n^2} - n - 12n^3 + 12n^2j + \cancel{6n^2j} - \cancel{6nj^2} - 6n^2 + \cancel{6nj}\Big]$

$$= \frac{1}{6}\left[-12n^3 + 12n^2j - 6n^2 - 2j^3 + 3j^2 - n\right]$$

~~$= \frac{1}{6}[-12$~~

(21)

$$\Delta = -(j-1)k^2 + 2k\cdot\left(-\tfrac{1}{2}\right)(2n^2-j^2+j) + \tfrac{1}{6}\big(-12n^3+12n^2j-6n^2$$
$$-2j^3+3j^2-n)$$
$$)$$

$$\Delta = (2n^2-j^2+j)^2 + \tfrac{2}{3}(j-1)(\cdot$$

$$3\Delta = 3\,(4n^4+j^4+j^2-4n^2j^2+4n^2j-2j^3)$$
$$+2\big(-12n^3j+12n^2j^2-6n^2j-2j^4+3j^3-nj$$
$$+12n^3-12n^2j+6n^2+2j^3-3j^2+n)$$
$$= 12n^4-24n^3j+12n^3-12n^2j^2+\cancel{12n^2j}+24n^2j^2-\cancel{12n^2j}-2nj+n$$
$$\cancel{+j^4}+3j^4-4j^4-6j^3+6j^3\,\cancel{+3j^2}+4j^3-3j^2$$

$$= 12n^4-12n^3(2j-1)+12n^2j^2-n(2j-1)-j^4+4j^3-3j^2.$$

What to do about this subclash?

$961-960=1$

—

No guidelines about taking values of $m<n$. — yes. If $m<n-1$ i.e. $m+1<n$, the no. of terms on LHS < no. of terms on RHS. & every term in RHS is > every term in LHS & hence impossible

—

$p_6 = \cancel{124}+15\cdot8$
120
$\cancel{44}$

$\left.\begin{matrix} n=8 \\ j=3 \end{matrix}\right\}$ $4\{n^2(n+9)^2-540\}$ & Let $n^2(n+9)=x$.

& let $x^2-540=y^2$ $\quad x^2-y^2=540.$ $\quad 270\cdot2$

576
136
440

$(x+y)(x-y)$ $\quad \left.\begin{matrix} x+y=270 \\ x-y=2 \end{matrix}\right\}$

272

268

$x=136,\ y=134.$

$p_4 = 16+15$
$=31$
$q_4=8$

576
$\cancel{546}$
36

$n^2(n+9)=136 \quad n^2+9n-136=0.$

36×15 $\quad 9$
540 $\quad 9$

$\frac{4}{1}$

$31^2-15\cdot8^2$
$15\cdot64$

$(n-8)(n+17)=0.$

$4\cdot34$
$8\cdot17$

180
$540 \cdot 4$
$\cancel{36}$
$\sqrt{135}$

~~$x^2-13\cdot2^2=y$~~
~~$x^2-15\cdot6^2=y^2$~~

6

$3+\frac{1}{1} \quad \frac{4}{1}$

0	3	3	3
1	6	1	6
3	1	6	1
	*	*	

$4^2-15\cdot1^2=1.$

$\sqrt{1576}$
480

$24^2-15\cdot6^2=6^2$

$136^2-15\cdot6^2=134^2$

~~$8^2-15\cdot6^2$~~ $=4.$

(22)

34
1156.16
187

9 $136^2 - ~~50~~\,15\cdot6^2$ $x^2-15\cdot6^2=y^2$.

18496
540
$\overline{17956} = 134$ 4489

$\sqrt{15}$

0	3	3	3
1	6	1	6
3	1	6	1
	x	x	

$\sqrt{135}$

121
126
9

0	11	3	6	5	5	6	3	11	11
1	14	9	11	10	11	9	14	1	14
11	1	1	1	1	1	1	1	22	1
	x							x	

$c = 8$ $\frac{4}{3}$.

6.

$\sqrt{60}\cdot3^2$

$4^2-15\cdot1^2=1^2$

$11+\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{1}$

$11\frac{13}{21}$ $\frac{254}{21}$

241
13
$\overline{254}$

60

0	7	4	8	7	11
1	11	4	11	1	
7	1	3	1	18	

0	7	4	4	7	7
1	11	4	11	1	11
7	1	2	1	14	1
	x			x	

$7+\frac{1}{1+}\frac{1}{2+}\frac{1}{1}$

$7+\frac{1}{1+3}$ $7\frac{3}{4}$ $\frac{31}{4}$

$23^2=529$.

$\sqrt{540}$.

$\frac{21}{1}$ 225
$\overline{315}$

44
$\frac{23}{1}$

540
441
$\overline{99}$

0	23	21	15	20	20	15	21	23	23
1	11	9	35	4	35	9	11	1	11
23	4	4	1	10	1	4	4	46	4
	*							x	

187165
155155
$\overline{32010}$

294
294
$\overline{3234}$

3234
2100
$\overline{1134}$

$23+\frac{1}{4+}\frac{1}{4}\frac{95}{104}\frac{1}{1}+\frac{9}{95}\frac{1}{10}\frac{5}{9}\frac{1}{1}\frac{4}{5}4\frac{1}{4+}\frac{1}{4}$ X

$\frac{5}{4}+\frac{104}{511}$

416
95
$\overline{511}$

2044
104
$\overline{2148}$

For $n=77$, $M_1=3003$, $N_2=155155$.

2) $(m-n+1)k^2+2k(M_1-N_1-mn)+\{M_2-N_2-mn(m+n+1)\}=0$

~~$(m-76)k^2+2k(M_1-3003-77m)+\{$~~

$n=77)35$

$36k^2+2k(3003-903-77\cdot42)+\{155155-25585-77\cdot42\cdot120\}$

$n=42,$
$m=77$

$36k^2-2k\cdot1134+-~~2\ldots0~~\,258510$

$6k^2+378k-43085=0$ $378^2+24\cdot43085$.

189

4.

4

3234 × 120

388080

25585

413665

155 55

258 510

$\underline{l = 6}$ in $4n^2(n+l^2)^2 - \frac{1}{3}l^4(l^4-1)$

$4n^2(n+36)^2 - \frac{1}{3}\cdot\overset{424}{\cancel{1296}}\cdot 1295$

$= 4\{n^2(n+36)^2 - 106\cdot 1295\}$

$4\{n^2(n+36)^2\} - 106\cdot 1295$

530 × 259

106·1295 has no factors both of which are even.

530 × 7 × 37

$3l^4(l^4-1)$. $4\{n^2(n+4)^2\} - 20\}$

10·2

$x+4=10$

$x-4=2$

3.16. $l = 2$, $\frac{1}{3}\cdot 16\cdot\overset{5}{\cancel{15}}$ 80 ↗ $n(n+4) - 6$ ×

$l = 3$, $\frac{1}{3}\cdot 81\cdot 80 = 27\cdot 80$.

$\frac{9\ \ 40}{\cancel{27\cdot 80}}$ / 6

2160 = 27·20

2, 1080	12, 180	40, 54
4, 540	24, 90	60, 36
6, 360		
8, 270	30, 72	
10, 216	36, 60	

366 ~~$n^2 + 9n - 541$~~ ×

1082 ~~$n^2 + 9n - 272$~~ ~~81 + 1088 = 1164~~ ×

544 ~~$n^2 + 9n - 183$~~ ~~81 + − 2~~ ×

27·20 = 540

1,68

16,17 ~~$n^2 + 9n - 139$~~ ~~81 + − 6~~ ×

$n^2 + 9n - 136$.

278 ~~$n^2 + 9n - 113$~~ ~~81 + − 2~~ ×

226

192 ~~$n^2 + 9n - 96$~~ ~~81 + 384 = 465~~ ×

81 + 136

544

625

114 ~~$n^2 + 9n - 57$~~ ~~81 + 228 = 309~~ ×

102

148 ~~$n^2 + 9n - 51$~~ ~~81 + 204 = 285~~ ×

85 × 64

340

510

5440

282

28

94 ~~$n^2 + 9n - 48$~~ ~~81 + 192~~ ×

130 120

80

~~$n^2 + 9n - 47$~~ ~~81 + 188 = 269~~ ×

2722

1361

554

277

$\boxed{l = 4}$, $\frac{1}{3}\cdot 256\cdot\overset{85}{\cancel{255}} \to 64\cdot 85. = 5440$

$4\{n^2(n+16) - 1360\}$

2, 2720	80, 68
4, 1360	160, 34
8, 680	316, 340
10, 544	32, 170
20, 272	356 102
40, 136	178 101

1364

682

188

344

584

$n^2 + 16n - 1361 \to 256 + 5444 = 5700$ ×

256 + 2728 = 2984 ×

256 + − 6 ×

632

~~256 + 1128 = 1384~~ ×

194

256 + 1108 = 1364

47 358

RW

178 356
178

88
148 74
$\frac{194}{2} = 97$

197

n^2. 256 + ‥ 2 ×
256 + ‑ 6 ×
256 + 388 = 644 ×
256 + ‑ 2 ×

256 + 404 = 660 ✓

l = 4 is a flop —

$\frac{2}{3}\cdot 6^4(6^4-1)$
$\frac{2}{3}\cdot 1296\cdot 1295$

l = 5 → Vigder's case

l = 6 flop ?

800
$\frac{2}{3}\cdot 2401\cdot$ ~~2400~~
432·1295

$\frac{2}{3}\cdot 129$

230

$$l = 7 \quad A = 4\{n^2(n+49)^2\} - \frac{2}{3}\cdot 49^2(49^2-1)$$

$$= 4\{n^2(n+49)^2 - 2401\cdot 200\}$$

480200

480200
2401

4802
100
4902
2451

2·240100
4.120050
~~8.60~~
5·96040 ×
10.48020
20.24010
~~25.~~
~~40.12005~~

70·68600
14·34300
28·17150
140·3430
98·4900
196, 2450

240102
120054
48030
24030
6930
34314
17178
3570
4998
2646
x

120051 + ~~19208~~ ×
60027. ×
24015 + 2401 = 26416
12015 + 2401 = 14416
3465 + 2401 = 5866 ×
17157 + 2401 — ×
8589 + 2401 = ^
1785 + 2401 = 4186 ^
2499 + 2401 = 4900 ×
1323 + 2401 = 3724 ×

2450
196
2646

$n^2 + 9n - 136$

all any should be
$49^2 + 4\cdot x$:

~~$n^2 + 49n - 480200$~~

~~$n^2 + 49n - 4900\cdot 98$~~

~~$n^2 + 49n - 4998 = 0$~~

~~x = 4998~~. 13860
~~y = 4802~~ 2401
16261

4900
98
4802
2

136

$n^2 + 49n - 2499$.

4·1785 + 2401 = 7140 + 2401 = 9541 ×

81
544
625

240108
2401

$x^2 - 2401\cdot 200 = y^2$

$x^2 - y^2 = 98\cdot 4900$

$x + y = 4900$
$x - y = 98$
$2x = 4998$

x = 2499.

y = 4802
y = 2401.

1 | 16261 (12
22 | 62
44
249 | 1861

41386
2401
16261

8 4 | 242509 (49
16
89 | 825
801
98 2409

$n^2 - 49n - 2499 = 0$

$\frac{49 \pm \sqrt{79}}{2} = \frac{119}{2}$.

7140
2401
9541

48060
2401
2 | 50461 (21
4
81 | 104 ~~2361~~
81
412

96060
2401
3 | 98461 (31
9
61 | 84 2361
61
62

~~$l = 4$ again~~ ~~$\frac{1}{3} l^4 (l^4 - 1) = \frac{1}{3} . 256 . 255 = 256 . 85$~~

$l = 6$ $\frac{1}{3} l^4 (l^4 - 1) = \frac{1}{3} . 1296 . 1295 = 432 . 1295$

$\Delta = 4n^2 (n+16)^2 - 432 . 1295$ 108 . 1295

$= 4 \{ n^2 (n+16)^2 - 108 . 1295 \}$ 4 . 27 . 5 . 259

1295 × 54
5180
6475
69930

139864
256
~~140120~~

2 . 69930

$\frac{69932}{2} = 34966$ 3

~~3894~~ ~~11658 × 4 + 256~~ 6 . 18 . 1295

7770
~~1558~~

23310
6
23316

$n^2 + 64n - \lambda = 0$

$l = 8$ $\frac{1}{3} l^4 (l^4 - 1) = \frac{1}{3} . 4096 . 4095 = 4096 . 1365$ ~~4096 + 4λ~~ $= l^2$

$4096 + 4\lambda = l^2$

$\Delta = 4n^2 (n+64)^2 - 4096 . 1365$

$= 4 \{ n^2 (n+64)^2 - 1024 . 1365 \}$ 1024 . 5 . 273

512, 2730

256, 5460

~~3242~~
~~1621~~
~~6484~~
4096

$2^{10} . 3 . 5 . 7 . 13$

5112
2858

35 × 32

$2x = 2560$

8836
4096
~~5140~~
~~1441~~

160 × 39

14 91 × 16

1456
960

1248
1120

2368

$\frac{2368}{2} = 1184$

39 × 32 15 × 64
624 ~~130~~

32 × 32 × 15 × 7 × 13

4

32 × 3 × 5 × 13

195 × 7 × 32 × 32

4356
4096
260

1560
1896
2456
~~1228~~

480 × 13

7 × 32 × 4
128

7 × 64 × 5
5 7 × 320

2240
6240
8480

4240

10

16960
4096
21056

4 × 5264

9216
4096
5120:
1280:

(26)

$1024 \cdot 1365 = 32 \cdot 32 \cdot 35 \cdot 91$ $\frac{1}{2}(32 \cdot 35 + 32 \cdot 91) = 16 \cdot 126$

$= 2016$

9
4
4
2

~~8064~~
~~9096~~

$64 \cdot 35 + 16 \cdot 91$ $7 \cdot 5 \cdot 13 \cdot 7$

3696
~~14784~~
~~4096~~
80

$64 \cdot 91 + 16 \cdot 35$

$16(91 + 140) = 16 \times 231$

~~1848~~

7

$\frac{1}{2} \cdot 16(35 + 364) = 8 \cdot 399$

3192

12768
4096
16864 $= 16 \cdot 1054$

49
~~09~~ 1824
114

~~7296~~
~~4096~~
~~2~~

$x^2 - 15 \cdot 6^2 = y^2$. $n^2 + 9n - \lambda = 0$

$(\mu + 9)(\mu - 9) = 4\lambda$. $81 + 4\lambda = \mu^2$

$l = 9, \Delta = 4n^2(n+81)^2 - \frac{1}{3} \cdot 9^4(9^4 - 1)$ $(81^2 = 6561)$

$= 4n^2(n+81)^2 - \frac{1}{\cancel{3}} \cdot \overset{2187}{\cancel{6561}} \cdot 6560$

81
324
3321

$= 4\{n^2(n+81)^2 - 2187 \cdot 1640\}$ $27 \cdot 81 \cdot 41 \cdot (40)$

~~2.4~~ 10
2.20, 4.10 2
8.5
1107

540, 6642
7182
3591
14364
6561
20925
837

820, 4374
5194
2597
10388
6561
16949
~~9881~~

1
23
26

16949 (13
1
69
69
49

410, 17496
17906
8953
~~6561~~
~~15214~~
~~2~~
~~35812~~
35812

2

810, 4428
5238
6561
11799
9.1311

2
1

10.3333000
3333010

1666505

23332
10000
33332
4.8333

11.1.71.

$l = 10, \Delta = 4n^2(n+100)^2 - \frac{1}{3} \cdot 100^2(9999)$

$4n^2(n+100)^2 - 100^2 \cdot 3333$

6666020
1000000

10000
5000
6666
11666
5833

100, 333300
100
0333400
166700

666800
100000
~~766800~~

6766020
4.1691505
~~4.5.338301~~
4.5.3.11.2.767

2500
13332
15832
7916

31684
10000
41664

46.2604

$100^2 \cdot 33 \cdot 101$

$50 \cdot 200 \cdot 33 \cdot 101$ 500

6600
5050
11650
5825
23300
X
40.250

3300
10100
13400
6700
26800

2020
16500
18520
9260
37040
825 7

1650
20200
21850
10925
43700
X

4040,
8250
12290
6145
24580
X

~~202400~~

nothing works! $4\{(n^2+9n)^2-540\}$

$n^2(n+9)^2-540$

$n^2(n^2+18n+81)-540$

$n^4+18n^3+81n^2-540=\square$

$(n^2+9n+\lambda)^2-540-\lambda^2-2\lambda n^2-18n\lambda$

$(n^2+9n+\lambda)^2-\{\lambda^2+2\lambda n(n+9)+540\}$

$\lambda = \cancel{-2n(n+9)}$

$\Delta = 4n^2(n+9)^2-4 \cdot 540$

$325 \cdot 2$
650
50
700

$2x=700$

625×52
1250
3125
32500

$4\{n^2(n+25)^2-32500\}$ $625 \cdot 52$ $25 \cdot 25 \cdot 13 \cdot 4$

$(n-10)(n+35)$

$x^2-y^2=32500$ $x+y=650$

$n(n^2+25n-350)$ $=650 \cdot 50$ $x-y=50$ $x=350$

$2x=700$

Δ

32500

$\Delta = 4\{1$

$100 \cdot 128$

$100 \cdot 35^2$

6

$25k^2+k(625-25-200)+\frac{1}{6}\{50 \cdot 625-3 \cdot 625-25(1199)$
$-600(19)\}$

1730
1250
480

$25k^2+400k+\frac{25}{6}\{1250-75-1199-456$

24.19 " $+\frac{25}{6}\{1250-1730) = \frac{25}{6} \cdot \overset{50}{\cancel{48}}$

22/4/76 Case of $n = 10$ for $m > 34$ – not done earlier in Bk. No. 15

Do up to $m = 5 \cdot 10 - 1 = 49$

& a few more cases.

(28) $(m-9)^2 k^2 + 2k(M_1 - 55 - 10m)$

$- \{10m(m+11) + 385 - M_2\}$

$10m(m+11)$

$m = 35$ → $26k^2 + 2k(630 - 55 - 350) - \{350 \cdot 46 + 385 - 14910\} = 0$

$26k^2 + 2k(225) - (16100 + 385 - 14910) = 0$

$26k^2 + 450k - 1575 = 0,$

~~$\Delta = 490^2 + 4 \cdot 26 \cdot 1575$~~ ~~$450^2 + 4 \cdot 26 \cdot 1575 = 100(45^2 + 26 \cdot 63)$~~

~~$= 4(245^2 + 26 \cdot 1575) = 4 \cdot 25(49^2 + 26 \cdot 63)$~~

~~$= 100(2401 + 1638) = 100 \cdot 4039$~~ ✗

630
405
225

76 26 7

315 $45^2 \cdot 55$ 63

210
140
16100
385
16485
14910
1575

2025
~~1638~~ ✗

126×13

$m = 36$ $27k^2 + 2k(666 - 55 - 360) - \{360 \cdot 47 + 385 - 16206\} =$

$27k^2 + 2k(251) - 1099 = 0$ ✗

252
144
16920
385
17305
16206
1099

346 666
415
251

$m = 37$ $28k^2 + ~~278~~ + 2k(278) - \{370 \cdot 48 + 385 - 17575\}$

$28k^2 + 556k - \{370 \cdot 48 - 17190$ ✗

~~740~~
1480

$m = 38$ $29k^2 + 2k(306) - (380 \cdot 49 + 385 - 19019)$ ✗

$m = 39$ $30k^2 + 2k(335) - (390 \cdot 50 + 385 - 20540) = 0$

$6k^2 + 134k - (3900 + 77 - 4128) = 0$

67

$6k^2 + 134k - 151 = 0$ ✗

4128
3977
151

$m = 40$ $31k^2 + 2k(365) - (400 \cdot 51 + 385 - 22140) = 0$ ✓

23821
385
23438

$m = 41$ $32k^2 + 2k(396) - (410 \cdot 52 + 385 - 23821) = 0$

$8k^2 + 198k - (410 \cdot 13 - 5859) = 0$

99

$8k^2 + 198k - 529 = 0$ ✗

5859
5330
529

$m = 42$ $33k^2 + 2k(428) - (420 \cdot 53 + 385 - 25585) = 0$ ✗

~~10~~ 5·10·1

m = 43 $34k^2 + 2k(461) - (\quad) = 0$ X

m = 44 $35k^2 + 2k(495) - (440 \cdot 55 + 385 - 29370) = 0$

99 $7k^2 + 198k - (440 \cdot 11 + 77 - 5874)$

11.

534
447
87·11
29·3·11·7 7

$7k^2 + 198k - 957 = 0$

$198^2 + 28 \cdot 957 = 4(99^2 + 7 \cdot 957) = 4 \cdot 11(99 \cdot 9 + 7 \cdot 87)$ X

m = 45 $36k^2 + 2k(530) - (\quad)$ X

60·1 ·1

m = 46 $37k^2 + 2k(566)$ X

m = 47 $38k^2 + 2k(603) -$ X

m = 48 $39k^2 + 2k(641) -$ X

40425
385
40040

m = 49 $40k^2 + 2k(690) - (490 \cdot 60 + 385 - 40425) = 0$

4004 5
2940
1064

$4k^2 + 2k(69) - (49 \cdot 60 - 4004)$

$4k^2 + 138k - 1064 = 0.$

$138^2 + 16 \cdot 1064 = 4(69^2 + 4 \cdot 1064) = 4(4761 + \;\; 6)$ X

m = 50 $41k^2 + 2K(730) -$ X

m = 51 $42k^2 + 2k(771) -$ X

m = 52 $43k^2 + 2k(813) - (\quad)$ X

51049
385
50664 4
12666
8480
4186

m = 53 $44k^2 + 2k(856) - (530 \cdot 64 + 385 - 51049)$

~~$44k^2 +$~~ $11k^2 + 428k - (530 \cdot 16 - 12666) = 0$ X

m = 54 $45k^2 + 2k(900) - (540 \cdot 65 + 385 - 53955) = 0$

53955
385
53370 3

$45k^2 + 2k(900) - (540 \cdot 65 - 53370) = 0$

5930
3900
1030

$5k^2 + 200k - (60 \cdot 65 - 5930) = 0$

5·10·1 $k^2 + 40k + 206. = 0$

400
206

$40^2 + 4 \cdot 206 = 4(20^2 + 206)$

$= 4 \cdot 606$ X

w

(30) $m = 55$, $46k^2 + 2k(945) + (550 \cdot 66 + 385 - 56980)$

330
336
36304
9
2104

$m = 56$, $47k^2 + 2k(991) \pm (560 \cdot 67 + 385 - 60116)$

$46k^2 + 2k(945) +$

$\frac{\cancel{4}}{\cancel{6}} \cdot \overset{26}{\cancel{52}} \cdot 53 \cdot \frac{\cancel{105}}{35}$ 2

265
159
1855
3710
48230

$n = 17, m = 50$ $34k^2 + 2k(1275 - 153 - 50 \cdot 17) - (50 \cdot 17 \cdot 68 + 1785 - 42925)$ $= 0$

$34k^2 + 544k - 16660 = 0$

$2k^2 + 32k - 980 = 0$

$k^2 + 16k - 490 = 0$

$256 + 1960 = 2216$ X

$\boxed{\text{Sq } 46^2 = 2116}$

48240
2809
51049

850
3400
57800
1785
59585
42925
16660

~~1275~~
850
153
1003
1275
272

~~136~~
1960
256
2216

46×46
276
184
2116

$9k^2 - 34k$

1785
324
2109

473
2365
2494
29
~~46~~
~~65~~

$\frac{\cancel{4}}{3} \cdot 53 \cdot \overset{9}{\cancel{54}} \cdot 107$

86
49
37

n = (15) m = (29) . $N_2 \neq 15$,

$15k^2 + 2k(435 - 120 - 29 \cdot 15) - (29 \cdot 15 \cdot 45 + 1240$

477
60
47
48177

~~16, 31~~. ~~n = 31~~,

$\frac{\cancel{4}}{\cancel{6}} \cdot \overset{5}{\cancel{15}} \cdot \overset{8}{\cancel{16}} \cdot 31$

$n = 16, m = 31$, $16k^2 + 2k(496 - 136 - 16 \cdot 31) - (16 \cdot 31 \cdot 48 + 1496 - 10416)$

$16k^2 - 272k - 14888 = 0$, ~~$2k^2 - 136k$~~ $- 7$

159 6

$2k^2 - 34k - 1861$ $4(17^2 + 3722) = 4(289 + 3722)$ 4
$= 4 \cdot 4011$

477
3339
000
477
57039

496×16
7936
23808
1496
25304
10416
14888

$\boxed{18, 26}$ $9k^2 + 2k(~~190~~ - 351 - 171 - 18 \cdot 26)$
$- (18 \cdot 26 \cdot 45 + 2109 - 6201)$ X

$k^2 + 2k(39 - 19 - 52) - (18 \cdot 26 \cdot 5 + 2$

~~$\frac{4}{6} \cdot 18 \cdot 19 \cdot 31$~~

19019
1521
20540
1600
22140
1681
23821
1764
25585
1849
27434
1936
29370
2025
31395
2116
33511
2209
35720
2304
38024
2401
40425
2500
42925
2601
45526
2704
48230
2809
51039
2916
53955

$\overset{n}{27}, \overset{m}{53}$ $27k^2+2k(1431-378-53\cdot 27)$

$-(53\cdot 27\cdot 81+6930-51039)=0$

[31)

60.

26

$3k^2+2k(159-42-53\cdot 3)-(53\cdot 3\cdot 81+770-5671)$

$3k^2 + -84k -$

7

4

159×81
159
1272
12879
770
13649
6 5671
7978

$\boxed{19, 38}$ $20k^2+2k(741-190-19\cdot 38)$

159
42
201

$-(19\cdot 38\cdot 58+2470-19019)$

$20k^2+38k(39-10-38)$

7

$-(38\cdot 58+130-1001)$

$20k^2-342k-333=0$

30 4.
190
1204
130

1334
1001
-333

$4(171^2+20\cdot 333)$ X

4.20.

$\boxed{n=11}$ $4(75^2+2590)=4(5625+2540)=4(8165$

4

$n=11, m=$ ~~32~~

$2k^2+2k(78-66-132)-(132\cdot 24+506-650)=0$

$2k^2-240k-3019=$ X

3168
506
3669
650
3019

$\boxed{11, 16}$ $6k^2+2k(136-66-16\cdot 11)-(16\cdot 11\cdot 28+506-1496)$

198
78

$6k^2$ $3k^2+2k(91-66-13\cdot 11)$ X

176 $\boxed{11, 14}$ $4k^2+2k(105-66-14\cdot 11$ X

66
242
136
106

143
66
209
91
118

165
66
231
120
111

$5k^2+2k(120-66-15\cdot 11)$ X

~~$6k^2+2k(136$~~

187
66
253

154
66
220
105
115

$7k^2+2k(153-66-17\cdot 11)$ O

11
187.
209
66
275
190
35

198
66
264
171
93

$8k^2+2k(171-66-18\cdot 11)$ X

$9k^2+2k(190-66-19\cdot 11)$

(32)

(m=20) $10k^2+2k(210-66-20\cdot11)$ ✗

21 $11k^2+2k(231-66-21\cdot11)$ ✗

22 ~~17+~~ $12k^2+2k(253-66-22\cdot11)$ ✗

242
66
308
253
55

231
66
297
231
66

286
210
76

264 13 20

23 $13k^2+2k(276-66-23\cdot11)$ ✗

264
66
340

253
66
319
276

23
40
5406

24 $14k^2+2k(300-66-24\cdot11)-(24\cdot11\cdot35+506-4900)$ ✗

9240
566
9806
4900
5406

25 $15k^2+2k(325-66-25\cdot11)$ ✗

286
66
352
351

26 $16k^2+2k(351-66-26\cdot11)-(26\cdot11\cdot38+506-6201)$ ✗

286 572 3 13

27 $17k^2+2k(378-66-27\cdot11$ ✗

10868
506
11374
6201
5173

28 $18k^2+2k(406-66-28\cdot11)-(28\cdot11\cdot40+506-7714)$

297
66
363
378
15

$18k^2+64k-5112=0$

12320
506
12826
7714
5112

$9k^2+32k-2556$

$32^2+36\cdot2556$ 9·639 8 3

32·32 $=16(64+5751)=16\cdot5815$ ✗

308
66
374
406
32

29 $19k^2+2k(435-66-29\cdot11)$ ✗

330
66
396
465

319
66
385
435
50

30 $20k^2+2k(465-66-30\cdot11)$ ✗

69

31 $21k^2+2k(496-66-31\cdot11$ ✗

341
66
407
496

89

32 $22k^2+2k(528-66-32\cdot11)-(32\cdot11\cdot42+506-11440)$

352
66
418
528
110

$$(m-n+1)k^2+2k(M_1-N_1-mn)+\{M_2-N_2-mn(m+n+1)\}$$

$$A=4(M_1-N_1-mn)^2-4(m-n+1)\{M_2-N_2-mn(m+n+1)\}$$

$$=4\left[(M_1-N_1-mn)^2-\cancel{4}(m-n+1)\{M_2-N_2-mn(m+n+1)\}\right]$$

$$4\left[(M_1-N_1-mn)^2-(m-n+1)\left\{\tfrac{1}{3}M_1(2m+1)-3N_1(2n+1)-mn(m+n+1)\right\}\right.$$

$M_2=\frac{1}{6}\cdot 2M_1\cancel{m(m+1)}(2m+1)$

$\frac{1}{3}M_1(2m+1)$

(n = 12) (m = 47) (48) (49)

$37k^2 + 2k(1176 - 78 - 48\cdot 12) - (48\cdot 12\cdot 61$ ⟨

768
78
846

12
1176
846
336

$M_1 - N_1 - mn$

$= \frac{1}{2}m(m+1) - \frac{1}{2}n(n+1) - mn$

$= \frac{1}{2}\{(m^2-n^2) + (m-n)\} - mn$

$= \frac{1}{2}(m-n)(m-n+1) - mn$

$\frac{1}{2}n(n-1) - \frac{1}{2}n(n+1)$

$-\frac{n}{2} = -n$

$M_2 - N_2 = \frac{1}{6}m(m+1)(2m+1) - \frac{1}{6}n(n+1)(2n+1)$

$m(2m^2 + 3m+1)$

$= \frac{1}{6}\{2(m^3 - n^3) + 3(m^2-n^2) + (m-n)\}$

$= \frac{1}{6}(m-n)\{2m^2 + 2mn + 2n^2 + 3(m+n) + 1\}$

$m - n = q.$ $\quad M_1 - N_1 - mn = \frac{1}{2}q(q+1) -$

$\lambda = 8$

$\mu = 7.$

$23n^4 + 10n^3 - 2n^2 - n$

256

3584
2048
24064
2816
56
26936

$\left.\begin{matrix}\mu = 3\\ \lambda = 4\end{matrix}\right\}$ $34 = 47n^4 + 44n^3 + 7n^2 - 2n$

$n = 8,$ $47\cdot 8^4 + 44\cdot 8^3 + 7\cdot 8^2 - 2\cdot 8$

$= 47\cdot 4096 + 44\cdot 512 + 7\cdot 64 - 2\cdot 8$

$= 8(47\cdot 512 + 44\cdot 64 + 7\cdot 8 - 2)$

26934.

~~16(13467~~

$\sqrt{13467}$ 1, 34, 21, 21

$1134 = 8(26934)$

$= 16(13467)$

$\Delta = 16(4489) = \underline{\underline{17}}$

$(n-1)(19n+2)$

$19n^2 - 17n - 2$

$-\mu^4 + 12\mu^2 + 24\mu + 12$

$-(\mu^4 - 12\mu^2 + 36) + 24\mu + 48$ $\quad 4n(11n^3 - 34n^2 - 17n - 2)$

$-(\mu^2 - 6)^2 + 24(\mu + 2).$ $\quad 11n^4 - 34n^3 - 17n^2 - 2n = 3\cdot 17$

$11n^4 - 17(2n^3 + n^2) - 2n$

$(2n+1)(n^2 + \ldots)$

$11n^4 - 17n^2(2n+1) - 2n$

$17n(2n^2+1)$

(34) $11n^4 - 17n^2(2n+1) - 2n$ $\quad 11n^4 - 34n^3 - 17n^2 - 2n$

~~$n(11n^3 - 2) - 17n^2(2n+1)$~~

$n = 3,\ 11 \times 81 - 34 \cdot 27 - 17 \cdot 9 - 6$

$891 - 918$

6

$n = 4,\ 11 \cdot 256 - 34 \cdot 64 - 17 \cdot 16 - 8$

$= 2816 - 2176 - 272 - 8$

$$\begin{array}{r} 2816 \\ 2456 \\ \hline 360 \end{array}$$

$$\begin{array}{r} 192 \\ 25 \\ \hline 2176 \\ 128 \end{array}$$

$11n^2 - 34n - 17 - \frac{2}{n}$

$11n^4 - 34n^3 - 17n^2 - 2n$

~~$x_1 = y = \frac{17}{22}$~~ $\cdot 11 \quad 11\left(n^4 - \frac{34}{11}n^3 - \frac{17}{11}n^2 - \frac{2n}{11}\right)$

~~$(n^2+pn+s)^2 - (mn+n)^2$~~

$(n^2+pn+q)^2 - (rn+s)^2$

$n^4 + 2pn^3 + 2n^2q + 2pqn + p^2n^2 + q^2$

$-r^2n^2 - 2rsn - s^2$ $\qquad q^2 - s^2 = 0.$

$2p = -\frac{34}{11}$ $\quad p = -17/11.$ $\qquad 2pq - 2rs = -\frac{2}{11}$

$2pq + p^2 - r^2 = -17/11$

$-\frac{34}{11}q + \frac{189}{121} - r^2 = -17/11$

$12n^4 + 24n^3 + 12n^2 \quad \Delta = 4n^4 + 8n^3 + 4n^2 = 4n^2(n^2+2n+1) = 4n^2(n+1)^2$

$\frac{34}{4} =$ $\quad 11n^4 - 17n^2(2n+1) - 2n$

$11n^4 - 17n^2 - 2n(17n^2+1)$

~~$289n^4 - 17n^2$~~

~~$12n^4 - n^2$~~ $(\cdot 300n^4 - 289n^4 - 17n^2 - 2n(17n^2+1)$

$$\begin{array}{r} 289 \\ 11 \\ \hline 300 \end{array}$$

$300n^4 - 17n^2(17n^2+1) - 2n(17n^2+1)$

$300n^4 - n(17n+2)(17n^2+1)$

$17n^2+1 = l^2.$ $\quad 300n^4 - ($

$47n^4+44n^3+7n^2-2n$

$~~4n^3~~\ 47n^3+44n^2+7n-2.$

$a=47,\ b=44/3,\ c=7/3,\ d=-2$

$\Delta_{cub} = 4\cdot 47^2-6\cdot 47\cdot\frac{44}{3}\cdot\frac{7}{3}(-2)$

$8j(j+1)$ 8 8

$j(j+1)=2l^2$

$j^2+j-2l^2 = 289-2\cdot 8$

$1+8l^2=m^2.$

$m^2-8l^2=1.$

0 2 2 2 $l+\frac{1}{1}$

1 4 1 4 $\frac{3}{1}$.

2 1 4 "

x x

$9+8=\frac{17}{6}$

j^2+j-72 $(j+9)(j-8)$ $\boxed{2(l^2-1)}$ $2j+1.$

~~8A6~~.

$j=8,$

17^2-1

$2\cdot 288$

$\begin{Bmatrix} n \\ 2n \end{Bmatrix}$ $n+1=l^2$ $2j+1$ $\frac{288\times 2}{576.}$ 4. 108+1

$2n=2(l^2-1)$ 2

$=2(17^2-1)$

$n=288$

$m=576$

$289\cdot k^2+~~2k(288\cdot 577-144\cdot 289-288\cdot 576)~~$

$-~~(288\cdot 576\cdot 865~~$

$\frac{576\cdot 577}{2}$

864

$289k^2+2k(288\cdot 577-\overset{48}{\underset{144}{\cancel{144}}}\cdot 289-288\cdot 576)$

$-(238\cdot 576\cdot 865+\underset{48}{\cancel{144}}\cdot 289\cdot 577$

$-96\cdot 577\cdot 1153)=0$

~~288~~ 1154

$\begin{array}{r} 289 \\ 1152 \\ \hline 1441 \\ 1154 \\ \hline 287 \end{array}$

24 $24+1!$

48 2^3

$\frac{1}{6}\cdot 288\cdot 289\cdot 577$

$\frac{1}{6}\cdot 576\cdot 577\cdot$ $289k^2+574k$

colossal [illegible]

Cursed

$144\cdot\ ~~577\ 48~~(1153-578$

~~$288\cdot 576\cdot 865-96\cdot 577\cdot 575$~~

$57~~5\quad 96\cdot 5~~\}$

$577\cdot 48(2306-289)$

$577\cdot 48\cdot 2017$ 3 3

$48(6\cdot 576\cdot 865$

$-577\cdot 2017)$

$5190\cdot(577-1)$

$-577\cdot 2017$

$\begin{array}{r} 5190 \\ 2017 \\ \hline 3173 \end{array}$

$577\cdot 3173.$

-5190

Colossal X

6

(39, 64),

$13(160-60$

-192

$\cancel{25}2$
160

$$24k^2+2k(2080-780-64\cdot39)$$

103

$$-(64\cdot39\cdot104+20540-89440)$$

89440
20540
68900

$$24k^2-184k-190684=0$$

7

$$6k^2-46k-47672=0$$

53

68900
259584
190684

$$3k^2-23k-23836$$

$13\cdot192\cdot104$

$20\cancel{8}$
936
104
19968
8 10.

$23^2+12\cdot\cancel{308}\ 28603\overset{2}{2}$
529

5 | 286561 (53
25
103 | 365
309
1069 | 5661

99.100

$990\cancel{1}50$

198

50, 99.
50

$$50k^2+2k(4950-1275-50\cdot99)$$

$$50k^2-2550k-(50\cdot99\cdot150+42925-328350)=0$$

149

7

$$50k^2-2550k-457075=0$$

$495\cdot15$
742500
328350
414150
42925
457075

14

$$2k^2-102k-18283=0$$

2401 00
2
240102

$$4n^2(n+l^2)^2-3l^4(l^4-1).$$

49.$\cancel{50}$ 20
49.$\cancel{2}$ 10 120051

(l = 7)

$$4n^2(n+49)^2-\frac{4}{\cancel{3}}\cdot2401\cdot\overset{800}{\cancel{2400}}$$

$$x^2-y^2=49\cdot49\cdot200$$

$$\Delta=4\{n^2(n+49)^2-2401\cdot200\}$$

$\cancel{200}$

48020
10
48030
24015

x

$$x=n^2+49n.$$

$$x^2-y^2=480200.$$

$$49^2+96060$$

96060
2401
9846$\cancel{1}$

118951

~~$n^2+49n-1470=0$~~

$n^2+49n-735=0$

(37)

~~2401~~
~~480204~~
~~482605~~

2401
4880
8281

7.105
35,21

2401
735
3136

2401
2940
5341

980 245
490
1470

$-49+56 = \frac{7}{2}$

2401
2

6

196
2450
2646

1323

29

7140
2041
8181

4900
98
4998

~~13860~~
~~2401~~
~~16261~~

3444
2041
5485

6
2499 343

6860
70
6930

7.2323

1785

19308
2401
21709

1 | 21709 (14
1
2 | 117
96
~~2109~~
2004

287

4
6

3465

1404
3430
3570

700
686
1386

7

693

350
1372
1722
861

48060
2401
50461

2 | 50461 (82
4
42 | 104
84
2061

44

~~89~~
~~983~~

242509 (49
16

825
80

~~2409~~

8

50.4

49.7.7.2.100

343

2.100

24010
20
24030

44.49
6

7.7.7.7.200

7.7.49.200

7.49.4

2401.200

4827

12015

240108
2401
242509

7.49.16

9604
50
9654

120050
4
120054

60027

$n = 11$

$-3m^2$ 7
79 2

$$(m-10)k^2 + 2k(M_1 - 66 - 11m) - \{11m(m+12) + 506 - M_2\} = 0$$

$$(m-10)k^2 + 2k\{\tfrac{1}{2}m(m+1) - 66 - 11m\} - \{11m(m+12) + 506 - \tfrac{1}{6}m(m+1)(2m+1)\}$$

$$(m-10)k^2 + k\{m(m+1) - 66 - 11m\} - \frac{1}{6}\{66m(m+12) + 3216 - m(2m^2+3m+1)\}$$

$$(m-10)k^2 + k(m^2 - 10m - 66) - \frac{1}{6}\{-2m^3 + 63m^2 + 791m + 3216\} = 0$$

$$\Delta = (m^2 - 10m - 66)^2 + \frac{\cancel{4}^{2}}{\cancel{6}_{3}}(m-10)(-2m^3 + 63m^2 + 791m + 3216)$$

$$3\Delta = 3(m^2 - 10m - 66)^2 - 2(m-10)(2m^3 - 63m^2 - 791m - 3216)$$

$= 3(m^4 - 20m^3 - 32m^2 - 1320m + 4356)$

$- 2(2m^4 -$ ~~$83m^4$~~ $- 83m^3 - 161m^2 + 4694m + 32160)$

$= -m^4 + 106m^3$ (?).

-132
170

-791
630
-3216
7910
4694

-60
166
106

$n = 11, m = 43.$

$$33k^2 + 2k(946 - 66 - 11\cdot 43) - (11\cdot 43\cdot 55 + 506 - 27434) = 0$$

$$3k^2 + 2k(86 - 6 - 43) - (43\cdot 55 + 46 - 2494)$$

$$3k^2 + 74k + 83 = 0.$$

$$\cancel{4(37^2 + 249)}$$

1369
249

606488
380963
225525

86
49
37

215
215
2365
46
2411
2494
83

$m = 3n - 1$, for the case $n = 202163$; $k = -380963$

$m = 606{,}488$

~~-360~~ $(-380963)^2 + (-380962)^2 + \cdot + 1^2 + 0^2 + 1^2 + 2^2 + \cdot + (225525)^2$

$= (225526)^2 + \cdot + (427688)^2$

225526
202162
427688

$(380963)^2 + \cdot + (225526)^2 + (225525)^2 + \cdot + 1^2 + 0^2 + (1^2 + 2^2 + \cdot + 225525^2)$

$2(1^2+2^2+ \cdot + 225525^2) = 380964^2 + \cdots 427688^2.$

$$\begin{array}{r} 427688 \\ 380964 \\ \hline 46724 \end{array}$$

~~$p_4 = 16 + 11\cdot 1,\ q_4 = 2\cdot 4\cdot 1$~~ $p_4 = 16 + \overset{47}{\cancel{141}}\cdot 1 = \cancel{157}\ 63$

~~$p_4 = 27,\ q_4 = 8$~~ $q_4 = 8$

~~$p_6 = 27\cdot 4 + 11\cdot 8 = 108 + 88 = 196$~~

$p_6 =$ ~~$157\cdot 4 + 141\cdot 8\cdot 1$~~ $157\cdot 1 + 141 \cdot 8\cdot 4$

$$\begin{array}{r} 1128 \\ 628 \\ \hline 1756 \end{array}$$

$p_6 = p_4 p_2 + \overset{6}{\cancel{141}}\cdot q_4 q_2 =$ ~~$628 + 141\cdot 8\cdot 63$~~.

$= 63\cdot 4 + 47\cdot 8\cdot 1$

$$\begin{array}{r} 192 \\ 81 \\ \hline 111 \end{array}$$

$3\Delta = n^2(-81 + 108 + 72 + 12) + 9.$

$\Delta = 37n^2 + 3 = l^2 \quad l^2 - 37n^2 = 3.$

Case $m = 5n-1$ ~ vide Bu. No. 15

$\mu k^2 + k\{(\mu^2-2)n - \mu\} + - \quad = 0$

$4k^2 + k\{14n - 4\} \quad 4k^2 + 24k + \quad = 0$

$k = (-24 \pm 8)/8 = -2$ or -4.

$l = 8,\ n = 2,\ m =$ ~~$4n$~~ $5n-1 = 9$

$(-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$

$= 8^2 + 9^2$

$(-2+9)^2$

$2(1^2+2^2) + (3^2+4^2+5^2+6^2+7^2) = 8^2 + 9^2$

$$\begin{array}{rr} 2\cdot 7\cdot 8+5 & \\ 9 & 64 \\ \hline 145 & 81 \end{array}$$

(p_6, q_6) also satisfying $3l'^2 - 44n'^2 = 4.$

$$\begin{array}{r} 8836 \\ 8836 \\ \hline 97196 \end{array}$$

$3 + \frac{1}{1+}\frac{1}{4+}\frac{1}{1+}\frac{1}{6+}\frac{1}{1} = 3 + \frac{1}{1+}\frac{1}{4+}\frac{1}{1+}\frac{1}{7} = 3 + \frac{1}{1+}\frac{1}{4+7/8}$

$= 3 + \frac{1}{1+8/39} = 3 + \frac{39}{47}$

$\frac{39}{8}$

$\frac{14}{3}\ \frac{1}{9} \quad \left(\frac{180}{47}\right)$

$$\begin{array}{r} 97200 \\ 97196 \\ \hline 4 \end{array}$$

$3\cdot 180^2 - 44\cdot 47^2 = 12\cdot 8100 - 44\cdot 2209 = \boxed{4}$

$(180, 47) \to 360, 94.$ $l = 360, n = 94$

~~$\frac{-24 \pm 360}{8} = 42, -48$~~ //

~~$k = 42, n = 94$~~, $m = 469.$

~~$42^2 + 43^2 + \cdots + 511^2 = 512^2 + 513^2 + \cdots + 605^2.$~~

~~$\frac{1}{6}\cdot 511\cdot 512\cdot 1023 - \frac{1}{6}\cdot 41\cdot 42\cdot 83$~~

~~$= \frac{1}{6}\cdot 605\cdot 606\cdot 1211 - \frac{1}{6}\cdot 511\cdot 512\cdot 1023$~~

~~$605\cdot 606\cdot \overset{1211}{\cancel{1209}} + 41\cdot 42\cdot 83 = 2\cdot 511\cdot 512\cdot 1023$~~

$4k^2 + k(14n-4) + \cdots = 0$

$\frac{-1312 \pm 360}{8}$

1312
360
952

119

1312
360
1672

$(-119)^2 + (-118)^2 + \cdots + (-119+118)^2 + (-119+119)^2 +$

$+ \cdots + (-119+469)^2 = (-119+470)^2 + \cdots +$

$119^2 + 118^2 + \cdots + 1^2 + 0^2 + 1^2 + \cdots + 350^2$

$= 351^2 + \cdots + 644^2.$

$2(1^2 + 2^2 + \cdots 119^2) + (120^2 + \cdots + 350^2) = 351^2 + \cdots + \cancel{644}\, 644^2.$

$2\cdot\frac{1}{\cancel{6}}\cdot 119\cdot 120\cdot 239 + \frac{1}{\cancel{6}}\cdot 350\cdot 351\cdot 701 - \frac{1}{\cancel{6}}\cdot 119\cdot 120\cdot 239$

$= \frac{1}{\cancel{6}}\cdot 644\cdot 645\cdot 1289 - \frac{1}{\cancel{6}}\cdot 350\cdot 351\cdot 701.$

$119\cdot 120\cdot 239 + 2\cdot 350\cdot 351\cdot 701 = 644\cdot 645\cdot 1289$

$17\cdot 120\cdot 239 + 2\cdot 50\cdot 351\cdot 701 = 92\cdot 645\cdot 1289$

$17\cdot \cancel{40}\cdot 239 + 2\cdot 50\cdot 117\cdot 701 = 92\cdot 215\cdot 1289$

$17\cdot 10\cdot 239 + 25\cdot 117\cdot 701 = 23\cdot 215\cdot 1289$

$17\cdot 2\cdot 239 + 5\cdot 117\cdot 701 = 23\cdot 43\cdot 1289$

42

360
24
336

$\frac{-384}{8}$

469
42
511

512
93
605

6
$\frac{0}{6}$

605
512
93+2

1210 7n−2
−658+360
4

469
119
350

351
93
644

300
256
44

470

1358

97×14
388
97
1358

9

93

126
126
5.

14×94
1316
1312

376
94
1316

261
93
354

4

239 × 34
956
717
8126

701 × 117
4907
701
701
82017

82017
8126

410.085
8126
418211

1289 × 43
3867
5156
55427

~~55427 × 23~~
~~166281~~
~~110854~~
~~1274821~~

$(1^2 + \dots + 119^2) + (1^2 + 2^2 + \dots + 350^2) = 351^2 + \dots + 444^2.$

357
93
444

$\frac{1}{6}\cdot 119\cdot 120\cdot 239 + \frac{1}{6}\cdot 350\cdot 351\cdot 701 = \frac{1}{6}\cdot 444\cdot 445\cdot 889 - \frac{1}{6}\cdot 350\cdot 351\cdot 701.$

$119\cdot 120\cdot 239 + 2\cdot 350\cdot 351\cdot 701 = 444\cdot 445\cdot 889$

$17\cdot 120\cdot 239 + 2\cdot 50\cdot 351\cdot 701 = 444\cdot 445\cdot 127$

$17\cdot 30\cdot 239 + 25\cdot 351\cdot 701 = 111\cdot 445\cdot 127$

$17\cdot 2\cdot 239 + 5\cdot 117\cdot 701 = 37\cdot 89\cdot 127$

127 × 89
1143
1016
11303 × 37
79121
33909
418211 ✓

8126 + ~~82017~~
~~418~~ 410085

410085
8126
418211 ✓

Correct

$(-209)^2 + (-208)^2 + \dots + (-209+208)^2 + (-209+209)^2$

$+ (-209+210)^2 + \dots + (-209+469)^2$

$= (-209+470)^2 +$

470
93
563

$209^2 + 208^2 + \dots + 1^2 + 0^2 + 1^2 + \dots + 209^2 + 210^2 + \dots + 260^2$

$= 261^2 + 262^2 + \dots + 354^2$

−209 + 418

$\frac{1}{6}\cdot 209\cdot 210\cdot 419 + \overset{2}{\frac{1}{3}}\cdot 260\cdot 261\cdot 521 = \frac{1}{6}\cdot 354\cdot 355\cdot 709$

209
261
93
354

$209\cdot 210\cdot 419 + 2\cdot 260\cdot 261\cdot 521 = 354\cdot 355\cdot 709$

469
209
260

$209\cdot 14\cdot 419 + 2\cdot 52\cdot 87\cdot 521 = \cancel{354}\,118\cdot 71\cdot 709.$

$209\cdot 7\cdot 419 + 52\cdot 87\cdot 521 = 59\cdot 71\cdot 709$

209 × 2933

627
627
1881
418
612997

521 × 87

3647
4168
45327

45327 × 52

90654
226635
2357004
612997
2970001 ✓

209 × 59

6381
3545
41831

41831 × 71

41831
292817
2970001 ✓

Correct

2

$$\Delta = 4\left\{n^2(n+l^2)^2\right\} - \frac{1}{3}\,l^4(l^4-1) \qquad l=3,\ n=8.$$

$$4\left\{64\cdot\cancel{289}\right\} - \frac{1}{\cancel{3}}\cdot\overset{27}{\cancel{81}}\cdot 80.$$

17

$$= 4\left\{64\cdot 289\right\} - 27\cdot 80 = \cancel{64\left\{4\cdot 289\right\} - 5\cdot 27}$$

4

~~16 (16.~~

1

$$= 16\,(16\cdot 289 - 27\cdot 5)$$

Sub of $n = 288$, $l = 17$.

$$4\left\{288^2(577)^2\right\} - \frac{1}{3}\cdot 289\cdot 289\,(289\cdot 289 - 1) = \square$$

$$4\left\{(288)^2\cdot(577)^2\right\} - \frac{1}{3}\,289\cdot 289\cdot 290\cdot 288$$

$$4\left\{\qquad\right\} - 289\cdot 289\cdot 290\cdot 96\,\cancel{(289^2+1)}$$

$$16\left\{(144)^2\cdot(577)^2 - 289^2\cdot 290\cdot 6\cdot\cancel{(289^2+1)}\right\}$$

~~16 {(144)^2 (288 + 289)^2 − 289^2 · 1740}~~

~~16 {(144)^2 (288^2 + 2·288 · 2.2~~

288
289
577

$(144\cdot 2)^2$

3

1156
135
1041
1021

14
14
14

4624
135
4489

5

$289^2 - 1$

$(289^2-1)(289^2+1)$

$(289^2 - 1)$

$(17^2 - 1)$

6

6

8

577 × 144

2308
2308
577

83088

83088 × 83088

664704
664704
000000
249264
664704
~~103088~~

~~10,627,135,744~~ 6903615744
~~14532654~~0 ~~1453~~

289 · 289

2601
2312
578

83521

83521 × 174

334084
584647
83521

14532654

184818092 04

5 | 262045230 i (5119
25
101 | 120
101
1021 | 1985
1021
10229 | 92423
92061
10238 | 36201

6903615744
1453265 40

6758289 204

8

4 × 1658957230 1 ✓

3 | 112979 (337
9
63 | 229
189
667 | 4079
4669

4 4

3
3

14
14
14

11532654 × 83522

29065308
29065308
72663270

2

n $\quad$ $n+1 = l^2$

$2n$ $\quad$ $n = l^2 - 1$

$2(l^2-1) = m^2$

$= 2n.$

$2l^2 - 2 = m^2$

$2l^2 - m^2 = 2$

$n + 1 = l^2$

$n = l^2 - 1, 288.$

m	m
24	48.

25
$l = 5$

$4\{(24)^2(29)^2\} - \frac{1}{3} \cdot 5^4(5^4-1)$

208

$4\{(24)^2(29)^2\} - \frac{1}{3} \cdot 625 \cdot 624$

$4\{(24)^2(29)^2\} - 625 \cdot 16 \cdot 13$

$10\{(12)^2(29)^2 - 13 \cdot 625\}$

841 × 144

3364
3364
841

121104
8125

112979

6
3

(44)

$4\{(24)^2(49)^2\} - 625\cdot 208$

$16\{(12)^2(49)^2\} - 16\cdot 625\cdot 13$

$= 16\{(12)^2(49)^2 - 13\cdot 625\}$

2401 × 144
9604
9604
2401
345744
8125
337619

6
3
138
9

5 | 337619 (58
25
108 | 876
864
116 | 1219

147 | 2
160 | 0
163 | 2
9
23
621
90
711
7

8

(704)

$47n^{\overset{4}{3}} + 44n^3 + 7n^2 - 2n$

②$n(23n^3 + 10n^2 - 2n - 1)$

184
40
224
5
219

16

343 × 23
1029
686
7889
490

8379
15
8364

729 × 23
2187
1458

16767
810
17577
19

9

27 × 23
621
90
711
7
704

64 × 23
1472
160
1632
21

1611

125 × 23
2875
250
3125
11

(2114)

216 × 23
4968
360
5328
13
5315

13
3
7
2
18
6
16
69

343 × 23
7889
490
8379
15
8364

512 × 23
11776
640
12416
17
12309

729 × 23
16767
820
17577
91
17486

22 6
23000
500
23500
21
23479

1331 × 23
30613
1210
31823
23
31800

1728 × 23
39744
1440
41184
25
41159

2197 x 23
50531
1690
52221
27
84 52194

16 17 9
2744 x23
63112
1960
65072
29
65043
19 7227

3375 x23
77625
2250
79875
31
79844
19961

20 13 19 (863)

17
1 19961 114
99
24 196
28 361

4096 x23
94208
2560
96768
33
96735
32245
6449

~~4913 x 23~~
~~109399~~
~~2890~~
~~112289~~
35
111254
(45)

2
92
20
92
4

6 2 14

5832 x 23
134136
3240
137376
37
137339
18

6859 x 23
157757
3610
161367
39
161328
16

4913 x 23
112999
2890
115749
35
115764

6
2
20
28941
9647
X

~~13441~~
9647

184000
4000
188000
41
187959

9261 x 23
213003
8820
221823
43
221780

13
17
13

10648 x 23
244904
4840
249744
43
249701

12167 x 23
279841
5290
285131
47
285084

31676
7914

323 663

13824 x 23
317952
5760
323712
49

4 20

15625 x 23
359375
6250
365625
51
365574
121858

17576 x 23
404248
6760
411008
53
410955
136985 X
18
20

3

19683 x 23
452709
7290
459999
55
459944

18
6
16

21952 x 23
504896
7840
512736
57
512679

170893 x
24389 x 23
558647
8410
567057
59
566998

20
6 27000 x 23
621000
9000
630000
61
629939
X

16
29791 x 23
685193
9610
694803
63
X 40

6 18 15 17

16 115 118 118 105 92

32768 x 23
753664
10240
763904
65

763839
x

35937 x 23
827011
11560
838571
47
838524

13
~~1~~9 69877
X

n = 1 to 33 for p=23

no soln

m = 211 is a flop.

㊽ $m = \lambda n, \ \lambda = 4.$

225 × 37

74
27
148
259

$\Delta = n(37n^3 - 4n^2 - 11n - 2)$

225
900
165
2
1067
18
9

$n = 14$ 4225

2744 × 37
101528
940
100588
25147
×

784
940

3375 × 37
124875
33709
91166

~~8325~~
33300
407
2

124875
1067

148

8
9

4225 × ~~256~~ 16 × 16
67600
1081600
9
12

64
16
48

㊾ $A = 4\{n^2(n+l^2)^2\} - \frac{4}{3}l^4(l^4-1)$

$n = 16$

$4\{16^2(65)^2\} - \frac{1}{3} \cdot 2401 \cdot \overset{800}{\cancel{2400}}$

$4\left[\{16^2 \cdot 65^2\} - 2401.200\right]$

1081600
480200
601400

296
7
11
185
30
30

$n^2(n+2{,}9)^2 - 480200$

148 4
148 11
33

22
20

4913 × 37
181781
1345
180436

45109
8
×

1156
187
2

94102

5832 × 37
215784
1498
214288
×

1296
198
2
1496

~~1400~~
1600
220
2
1822

6859 × 37
254883
1655
253228

111
33
185
22
296
32 9
222

1444
209
2
1655

8000 × 37
296000

17 29
148

14
626

1764
231
2
1997

10648 × 37
393976
2169
391807
×

1936
231
2
8 4

3

11.2744
30184
6904
23280

196 × 34
392 × 17
6664
238
2
6904
15
6

9261 × 37
339657
1997
337660
×

22
275

4900
625
3275

$n = 1, \ m = 24$ $24k^2 + 2k.(300 - 1 - 24) - (24.1 \ 26 + 1 - 4900)$

$24k^2 + 450k + 3275 = 0$ $450^2 - 4.24.3275 .. 4(225^2 - 24.3275$

$m = \lambda n$, In $\lambda = 5.$, $3\Delta = 4n(11n^3 - 34n^2 - 17n - 2).$

$n = 14$ to 20]

(n=14) $3\Delta = 4\cdot 14(23280)$ ✗

(n=15) $3\Delta = 4\cdot 15(37125 - 7650 - 255 - 2)$

$= 4\cdot 15(\qquad\qquad)$ ✗

(n=16) $3\Delta = 4\cdot 16(45056 - 8704 - 272 - 2)$

$= 4\cdot 16(36078) = 4\cdot 16\cdot 3\cdot 12026$ ✗

(n=17) $3\Delta = 4\cdot 17(54043 - 9826 - 289 - 2)$

$= 4\cdot 17\cdot 3\cdot 14642$ ✗

(n=18) $3\Delta = 4\cdot 18(64152 - 11154)$

$= 4\cdot 18(52998)$

$= 4\cdot 18\cdot 3\cdot 17666$ ✗ 8833

(n=19) $3\Delta = 4\cdot 19(75449 - 12274 - 323 - 2)$

$= 4\cdot 19(75449 - 12599)$

$= 4\cdot 19\cdot 62850 = 4\cdot 19\cdot 3\cdot 20950$ ✗

(n=20) $3\Delta = 4\cdot 20(88000 - 13600 - 340 - 2)$ ✗

The damned ② makes everything impossible for all λ

($m = \lambda n$) appears a flop. Better drop it:

$m = \lambda n + \mu$, $p = (\lambda+1)n + \mu$

$2m - p = 2\lambda n + 2\mu - (\lambda+1)n - \mu = \lambda n - n + \mu = n(\lambda-1) + \mu.$

$2m^2 - p^2 = 2(\lambda n + \mu)^2 - \{(\lambda-1)n + \mu\}^2$

$= 2\lambda^2 n^2 + 4\lambda\mu n + 2\mu^2 - (\lambda-1)^2 n^2 - 2\mu(\lambda-1)n - \mu^2$

$= 2\lambda^2 n^2 + 4\lambda\mu n + 2\mu^2 - \lambda^2 n^2 + 2\lambda n^2 - n^2 - 2\lambda\mu n + 2\mu n - \mu^2$

$= \lambda^2 n^2 + 2\lambda\mu n + \mu^2 + 2\lambda n^2 - n^2 + 2\mu n$

$= n^2(\lambda^2 + 2\lambda - 1) + 2\mu n(\lambda + 1) + \mu^2.$

$m = \lambda n + 1$

$m = \lambda n - 1$

$\lambda(n-1) + 1$

$\lambda n - (\lambda - 1)$

11 × 3375

2450 × 17

8

4096

096

~~256~~

~~512~~ × 17

80

16

640

289

4913

4913 × 2

5832 × 11

648 × 17

6859 × 11

722 × 17

$$\begin{array}{r} 45056 \\ 8978 \\ \hline 36078 \end{array}$$

$$\begin{array}{r} 54043 \\ 10117 \\ \hline 43926 \end{array}$$

7321 13 8

$$\begin{array}{r} 11016 \\ 136 \\ 2 \\ \hline 11154 \end{array}$$

$6k^2(\alpha+1)+6k(\alpha+\beta)+(\alpha+3\beta+2\gamma)=0.$

$36(\alpha+\beta)^2-24(\alpha+1)(\alpha+3\beta+2\gamma) = \square$

$4\left[\{9(\alpha^2+2\cancel{\alpha\beta}+\beta^2)-6(\alpha^2+3\cancel{\alpha\beta}+2\alpha\gamma+\alpha+3\beta+2\gamma)\right]$

$4\left[(3\alpha^2+9\beta^2-12\alpha\gamma)-6(\alpha+3\beta+2\gamma)\right]$

$\eta\,\gamma=0 \rightarrow 4\left[3\alpha^2+9\beta^2-6\alpha-18\beta\right] = 4\left[3(\alpha+3\beta)^2-6(\alpha+3\beta)-18\alpha\beta+3-3\right]$

$4\left[3\alpha(\alpha-2)+9\beta(\beta-2)\right] = 4\left[3(\alpha+3\beta-1)^2-3(6\alpha\beta+1)\right.$

$\alpha=2,\ \beta=$ $\quad 6\alpha\beta+1=0,\ \alpha+3\beta-1=3l^2$

$2m^3=p^3;\quad 6(2m-p)(2m^2-p^2)+1=0,\quad 2m-p+3(2m^2-p^2)=3l^2$

$6(4m^3+p^3-2mp^2-2m^2p)+1=0$

$6(3p^3-$

$\alpha=1.$

$\boxed{2m-p=1}$

$2m\cdot m+n$

$m-n=\alpha.$

$2\alpha^2+4\alpha n+\cancel{n^2}2n^2$

$-4n^2-4\alpha n-\alpha^2$

$\cancel{\alpha^2-2n^2}$

$2m^3=(m+n)^3.$ ~~$2m^3=m^3+3m^2n+3mn^2+n^3$~~

~~$m^3=3m^2n+3mn^2+n^3.$~~

$m=\alpha+n$

$\alpha=m-n.$

~~$2p-$~~

$2m-p$

$\boxed{2m-m-n}$

$p=m+n$

$p=2n+\alpha$

$\beta=2m^2-(m+n)^2=m^2-2mn-n^2$

$=(\alpha+n)^2-2n(\alpha+n)-n^2.$

$=\alpha^2+2\cancel{\alpha n}+n^2-2\cancel{\alpha n}-2n^2-n^2$

$=\alpha^2-2n^2$ ✓

$3n(\alpha^2+2\alpha n+n^2)$

$2(\alpha+n)^2$

$-(2n+\alpha)^2$

$\gamma=2m^3-(m+n)^3=m^3-3m^2n-3mn^2-n^3$

$=(\alpha+n)^3-3n(\alpha+n)^2-3n^2(\alpha+n)-n^3.$

$=\alpha^3+3\cancel{\alpha^2 n}+3\cancel{\alpha n^2}+\cancel{n^3}-3\cancel{\alpha^2 n}-6\alpha n^2-3n^3.$

$-3\cancel{\alpha n^2}$ $\qquad -3n^3\quad -\cancel{n^3}$

$\alpha^3-4\alpha n^2$

$-2\alpha n^2$

$=\alpha^3-6\alpha n^2-6n^3.$

$=\alpha(\alpha^2-6n^2)-6n^3$ ✓

~~$2(\alpha+p)^3$~~ $-($

$(2n+\alpha)^3$

$2(\alpha+n)^3-(2n+\alpha)^3$

$2\alpha^3+6\alpha^2n+6\alpha n^2+2n^3-8n^3$

$-12\alpha^2 n$ ~~$-12\alpha n$~~

~~$-6(12)\alpha n^2$~~ $-\alpha^3.$

$3\cdot 4n^2$

$-3\cdot 4n\alpha^2$

$3\cdot\alpha\cdot 2n$

$6k^2(\alpha+1) + 6k\{\alpha+\alpha^2-2n^2\} + \{\alpha + 3\alpha^2 - 6n^2 + 2(\alpha^3 - 6\alpha n^2 - 6n^3)\}$

$6k^2(\alpha+1) + 6k(\alpha^2+\alpha-2n^2) + \{2\alpha^3+3\alpha^2+\alpha - 6n^2 - 12\alpha n^2 - 12n^3\}$

$6k^2(\alpha+1) + 6k\{\alpha(\alpha+1) - 2n^2\} + \{\alpha(\alpha+1)(2\alpha+1) - 6n^2(2n+2\alpha+1)\}$

$\alpha(2\alpha^2+3\alpha+1)$

$-6n^2(2n^3+2\alpha+1)$

$36\{\alpha(\alpha+1)-2n^2\}^2 - 24(\alpha+1)\{\alpha(\alpha+1)(2\alpha+1) - 6n^2(2n+2\alpha+1)\}$
$= \square.$

~~$6k^2(\alpha+1) + 2k(A_1$~~

$6k^2(\alpha+1) + 12k(A_1-n^2) + \{6A_2 - 6n^2\underline{(2m+1)}\} = 0$

$(\alpha+1)k^2 + 2k(A_1-n^2) + (A_2 - \overline{2m+1}\,n^2) = 0$ ——— (B)

$\Delta = 4\left[(A_1-n^2)^2 - (\alpha+1)(A_2 - \overline{2m+1}\,n^2)\right]$ ——— (C)

$= 4\left[(A_1^2 - 2A_1n^2 + n^4) - (\alpha+1)\{A_2 - n^2(2n+2\alpha+1)\}\right]$

$\Delta' = \frac{1}{2}\Delta,\ \Delta' = n^4 + 2n^3(\alpha+1) + n^2\{-2A_1 + 2\alpha(\alpha+1) + (\alpha+1)\}$
$+ A_1^2 - (\alpha+1)A_2.$

$= n^4 + 2(\alpha+1)n^3 + n^2(2A_1 + \alpha+1) + A_1^2 - (\alpha+1)A_2$

$= (A_1+n^2)^2 + n^4 + (\alpha+1)(2n^3 + n^2 - A_2)$

~~$=\{n^2 + (A_1+n^2)\}^2 - 2n^2(A_1+n^2)$~~

~~$\{(A_1+n^2) - n^2\}^2 + 2n^2(A_1+n^2)$~~

$\boxed{\alpha = n}$

$\boxed{2n = m-n}$ $\boxed{m = 3n}$

$\boxed{2m+1 = n^2}$

$(2n+2\alpha+1) = n$

1024
64
1638

$m = 8, n = 16.$ $\boxed{\alpha = 8}.$ ~~$A_2 = 204.$~~

$m = 2n$

512

$(36+64)^2 + 4096 + 9(1024 - 204)$

10000 + 4096 + 7380

3

11476
10000
21476

5369.

28

1024
204
820

$(36+64)^2 + 9$
$+9\{1024 + 64 - 204\}$

1088
204
884

10000
7956
3044

17956, 4489 ✓

$$\Delta' = (A_1 - n^2)^2 - (\alpha+1)\{A_2 - n^2(2n+2\alpha+1)\}$$

$$= A_1^2 - 2A_1n^2 + n^4 - (\alpha+1)A_2 + (\alpha+1)(2n^3+2\alpha n^2+n^2).$$

$$= A_1^2 - 2A_1n^2 + (\alpha+1)(2\alpha+1)n^2 + n^4 - (\alpha+1)A_2$$

$$+2(\alpha+1)n^3$$

$$(A_1^2 - 2A_1n^2 + n^4) + 2\alpha(\alpha+1)n^2 + (\alpha+1)n^2 + 2(\alpha+1)n^3$$

$$-(\alpha+1)A_2$$

$$\underbrace{A_1^2 - 2A_1n^2 + n^4 + 4A_1n^2} + (\alpha+1)n^2 + 2(\alpha+1)n^3$$

$$-(\alpha+1)A_2$$

$$\Delta' = (A_1+n^2)^2 + (\alpha+1)(2n^3+n^2-A_2)$$

If $\alpha = n$ i.e. $m = 2n$; ~~$\frac{1}{2}n(n+1)+m$~~

$$(A_1+n^2)^2 + (n+1)n^2(2n+1) - (n+1)A_2$$

$n=5$
$m=10$

$$(A_1+n^2)^2 + 6A_2n - (n+1)A_2$$

$$(A_1+n^2)^2 \cancel{+A_2}\,(\cancel{5n}).\ A_2(5n-1).$$

$$\cancel{+A_2 n+6}.$$

~~850~~
855
385
470

~~1055~~
~~385~~
~~670~~

If $n=5$, $\Delta' = \square$.

$m=10.$

$$6k^2 + 2k(55-15-50) - (50\cdot 16+55-385) = 0$$

$$6k^2 - 20k - 470 = 0 \quad 3k^2 - 10k - \cancel{335}\ 235$$

$$100 + 12\cdot 235 \quad 2820 \quad 2920 \neq \square$$

4

6

$n=8$

~~300+576~~
876

$$\Delta' = (A_1+n^2)^2 + (5n-1)A_2.$$

$$(36+64)^2 + 39\cdot 204$$

1838
612
7956

10000
7956
17956

6

$$\cancel{\left\{\tfrac{1}{2}n(n+1)+n^2\right\}^2 + (5n-1)\cdot\tfrac{1}{6}n}(n+1)(2n+1)$$

1024
24
1048
261.

$A_2 = \frac{1}{6}\cdot n(n+1)(2n+1)$

$\frac{1}{3}\cdot A_1(2n+1)$

$$3\Delta' = 3(A_1+n^2)^2 + (2n+1)(5n-1)A_1$$

$$= 3A_1^2 + A_1\{6n^2 + (2n+1)(5n-1)\} + 3n^4$$

$$= 3A_1^2 + A_1(16n^2+3n-1) + 3n^4$$

$$3(A_1^2 + A_1$$

$10n^2+3n-1 = -6n^2$

$10n^2+3n-1 =$

$$c^2 - A_2(5n-1) = (A_1+n^2)^2$$

$i^2 - A_2(5n-1) = 1.$ (51)

Let $n = 24$, $A_2 = 70^2$

4
5
4
6

$n = 24$] $(300 + 576)^2 + 4900 \cdot 119.$

876 × 876
5256
6132
7008
767376
583100
1250476

105
110

1071
476
5831

312619 (~~205~~ 5
25
626
525
10119

91 819.

260^2
67600
52416
120016

15
819 × 64
3276
4914
16.7501

$n = 13,$ $(A_1 + n^2)^2 + 9A_1(5n-1)$

$(91 + 169)^2 + 9 \cdot 91 \cdot 64$

$\Delta' = (A_1 + n^2)^2 + (\alpha + 1)(2n^3 + n^2 - A_2)$

6

$(A_1 + n^2)^2 + (5n-1)A_2.$

$(1+1)^2 + 4 \cdot 1 = 8$ ×

$(3+4)^2 + 9 \cdot 5 = $ ×

$(6+9)^2 + 14 \cdot 16.$

$15^2 + 14 \cdot 16$

225
224
449

4

39
195

51

$(36+64)^2 + 39 \cdot 204$

$4[(18+32)^2 + 39 \cdot 51.$

2500
1989
4489 = □ ✓

~~$(A_1 + n^2)^2 + 9A_1(5n-1)$~~

~~$A_1^2 + 2A_1(n^2 + 10n - 2) + n^4.$~~

Δ

$m = 3n - 1.$
$\alpha = m - n$
$= 2n - 1.$

$(\alpha+1)k^2 + 2k(A_1 - n^2) + (A_2 - \overline{2m+1}\, n^2)$

$\Delta' = (A_1 + n^2)^2 + (5n-1)A_2$

$n(n+2) = x.$

~~$x^2 + 1 = y^2$~~
~~$y^2 - x^2 = 1.$~~
~~$y + x = 1$~~
~~$y - x = 1.$~~
~~$y = 1, x = 0.$~~

$\Delta' = (A_1 + n^2)^2 + (\alpha+1)(2n^3 + n^2 - A_2)$

Let $\alpha = 1$] $\Delta' = (n^2+1)^2 + 2(2n^3 + n^2 - 1).$

$= n^4 + 4n^3 + 4n^2 + 1. = n^2(n+2)^2 + 1$

$= n^2$ ~~$(n^2+2)^2 + 1.$~~ $= (n^2 + 2n)^2 + 1.$

(52)
$(\alpha = 8)$

$(36 + n^2)^2 + 9 \cdot (2n^3 + n^2 - 204)$

$n^4 + 72n^2 + 1296 + 18n^3 + 9n^2 - 1836$

$$\begin{array}{r} 1836 \\ 1296 \\ \hline 540 \end{array}$$

$n^4 + 18n^3 + 91n^2 - 540 = 0$

$n^2(n + 9)^2 - \underline{540} = 0.$

$x^2 - 6 = y^2$

$x^2 - 15 = y^2$

$x^2 - y^2 = 15$

$1 \cdot 15$

$3 \cdot 5$

$x + y = 5$

$x - y = 3$

$x = 4, y = 1$

$x + y = 15$

$x - y = 1$

$x = 8, y = 7$

$\alpha = 2,$ $(3 + n^2)^2 + 3(2n^3 + n^2 - 5)$

~~$n^4 + 9$~~ $n^4 + \overset{6}{~~12~~}n^3 + 9n^2 - 15 + 9$

$\Delta' = n^2(n + 3)^2 - 15 + 9$

~~$n^2 \cdot n +$~~ $n^2 + 3n - 4 \therefore (n + 4)(n - 1)$

$n = 1, 3$ $\quad n = 1, m = 3 \quad 3k^2 + 2k(3 - 1^2) + (5 - 7)$

$n(n + 3) - 8$

$3k^2 + 4k - 2 = 0$

$\frac{16 + 24}{6} \quad \frac{20}{3}.$

$-8 \pm \frac{24}{6}$

$\frac{16}{6}$ $\quad 8/3 \quad -\frac{16}{3}.$

$\frac{16 + 24}{40}$

$-8 \pm \frac{16 + 24}{6}$

88

$\frac{8}{3} + 1$

$\left(\frac{8}{3}\right)^2 + \left(\frac{11}{3}\right)^2 + \left(\frac{14}{3}\right)^2 + \left(\frac{17}{3}\right)^2$

$= \left(\frac{20}{3}\right)^2$

$\boxed{16 + 24}$

270

$$\begin{array}{r} 64 \\ 121 \\ 196 \\ 289 \\ \hline 670 \\ 324 \\ \hline 994 \end{array}$$

10

$\frac{17}{3} + 1$

$8^2 + 11^2 + 14^2 + 17^2 = 20^2.$

$n^2(n + 2)^2 - (2 - 1)$

$n^2(n + 1)^2 - 1 = 0$

$n^2 + n - 1 =$

~~1000~~ $(64 \cdot 17)^2 + 36^2 - 9 \cdot 204$

$\Delta' = (A_1 + n^2)^2 n^2(n + \alpha + 1)^2 + (A_1 - \alpha + 1 \; A_2)^2$

$\alpha = 3.$ $\Delta' = (6+n^2)^2 + 4\,(2n^3+n^2-14)$

$= n^4 + 8n^3 + 16n^2$

$\Delta' = (A_1+n^2)^2 + (\alpha+1)(2n^3+n^2-A_2)$

$A_1^2 - (\alpha+1)A_2$

$\frac{1}{4}\alpha^2(\alpha+1)^2 - (\alpha+1)\cdot\frac{1}{6}\alpha(\alpha+1)(2\alpha+1)$

$\frac{1}{12}\{3\alpha^2(\alpha+1)^2 - 2\alpha(\alpha+1)^2(2\alpha+1)\}$

$\frac{1}{12}\alpha(\alpha+1)^2\{3\alpha - 2(2\alpha+1)$

$= n^4 + 2n^2A_1 + A_1^2 + 2(\alpha+1)n^3 + (\alpha+1)n^2 - (\alpha+1)A_2$

$-\alpha-2$

$= n^4 + 2(\alpha+1)n^3 + (\alpha+1)n^2 + n^2\alpha(\alpha+1) + A_1^2 - (\alpha+1)A_2$

$= n^4 + 2(\alpha+1)n^3 + (\alpha+1)^2n^2 + \{A_1^2 - (\alpha+1)A_2\}$

$\Delta' = n^2(n+\alpha+1)^2 + \{A_1^2 - (\alpha+1)A_2\} = \Delta'.$

10·2
5·4.

$(\alpha=3)$ $\Delta' = n^2(n+4)^2 + (36-56) = \{n(n+4)\}^2 - 20.$

$x^2 - y^2 = 20$
$x + y = 10$
$x - y = 2$
$x = 6,\ y = 4.$

$n^2(n+4) = 6$ X

$20 = 2^2\cdot5$ $\frac{1}{2}(1)\cdot2.$ ① 10
10
20.

$\alpha = 4.$ $n^2(n+5)^2 + (100 - 150)$

$n^2+5n = 6$ $(n-1)(n+6)$ $n = 1.$ $m = 5.$ $\}$:

$36 - 20 = ⑯.$

$50 = 2\cdot5^2$ // $n = 9$ $\frac{1}{2}$ 320

$A_2 = \frac{1}{3}(2\alpha+1)A_1$

~~$(\alpha+1)k^2 + 2k(A_1 - n^2) + (A_2 - 2m+1\,n^2) = 0$~~

2.25
2·25
10·5

25 2

X ⑪

~~$5k + 18k + (30 - 11) = 0$~~ ⑲ $(n^2+5n)^2 - 50.$

330
225
~~33~~0

7, 15

~~$324 - 20\cdot19.$~~ X

105. 3 5 3

$(\alpha=5)$ $\Delta' = n^2(n+6)^2 + (15^2 - 6\cdot55)$ 4 cases

$n^2(n+6)^2 - 105 = X = 3\cdot5\cdot7$ (3)(5·7) (3·7)(5) (3·5)(7) 1·(3·5·7)

5, 21 ⑬: ⑲

63 7

19, 23
5·23
11 25
$\frac{1}{2}$.1. 13 ② 105.

1·105
5·23 ⑩ ⑭

441
196

5, 21
3·7 5·7

9·49

2·98 2·98
4·49 4·14
5·21 14·14
10·150 4·3·3·2 2 50

$\alpha = 6$ $\Delta' = n^2(n+7)^2 + (21^2 - 7\cdot91)$

~~n^2+7n~~ $n^2(n+7)^2 - 14^2$

⑨

2·168 2·3·2·2 2
4·84
8·42
14·24.
12·28
17, 5 85. 170

1120
784
336 2·2·3·2

T·48 ⑥ 88 45
44.
2·3·5
2·3²·5

$n^2+7n-50.$

$(\alpha=7)$ $\Delta' = n^2(n+8)^2 + (28^2 - 8\cdot140)$

$2^2\cdot89$ $2,2\cdot89$
$\frac{1}{2}(2n)(1+1)$ ①

$n^2(n+8)^2 - 336$ $2^4\cdot78$ $2^4\cdot3\cdot7$ 180 ⑥ cases

$n^2(n+8)^2 - \overset{28\cdot 12}{\cancel{168\cdot 2}} = \square$　　　$\frac{1}{2}(168+2) = \underline{170}$　85.

20.　　$\boxed{17.5}$

$n(n+8) - 20$

$= (n-2)(n+10)$

$n=2,\ m=9$

$(\alpha+1)k^2 + 2k(A_1 - n^2) + (A_2 - \overline{2m+1}\,n^2) = 0.$　　140 − 76 = 64

$8k^2 + 2k(28-4) + (140-76)$

$8k^2 + 48k + 64$　　144 −

~~$2k^2 + 12k + 19$~~ 20.　~~36 − 5~~　$\underline{36 - 32 = 4}$

$k^2 + 6k + 8 = 0$　　~~$k =$~~ $\dfrac{-6 \pm 2}{2} = -2, -4.$

−21

$(-2)^2 + (-1)^2 + 0^2 + (1)^2 + 2^2 + \quad + 7^2 = 8^2 + 9^2.$

$2(1^2+2^2) + 3^2 + \cdot \quad + 7^2 = 8^2 + 9^2$　　81 + 64 = 145　　1120 − 784 = 36

$140 + 5 = \cancel{15}145$ ✓

$(-4)^2 + \qquad + 5^2 = 6^2 + 7^2$　　49 + 36 = 85

$60 + 25 = 85$

same as (R_2) & (R_3) on p. 47 for $n = 2, m = 9$.

$\alpha = 8.$

$\Delta' = n^2(n+9)^2 + (36^2 - 9\cdot 204)$　　1836 − 1296 = 540　　2.270, 4.135, 20.27, 10.54

$= n^2(n+9)^2 - 540.$　　270.2　　27^2　136　8, 17

540 =

$2^2\cdot 135$

$2^2\cdot 3^3\cdot 5$

$\frac{1}{2}(3)(4)(3)$

$\textcircled{12}$ $\textcircled{4}$

$n^2(n+9) - 136 = 0$

$(n-8)(n+17) = 0$　　$n = 8,\ m = 16.$　　33.64　192 + 192 = 2112

$9k^2 + 2k(36-64) + (204 - 2112) = 0$

$9k^2 - 56k - 1908 = 0.$

28

~~$k = \frac{28 \pm 3}{}$~~ same eq^{n} as for $n = 8, m = 16$

Bk 15, p. 16　✓

But these eq^{ns} are simpler

$\alpha = 9$

$\Delta' = n^2 (n+10)^2 + (45^2 - 10 \cdot 285)$

$n^2(n+10)^2 - 825$ $3 \cdot 5^2 \cdot 11$ $\frac{1}{2} \cdot 2 \cdot 3 \cdot 2$ ⑥

$n^2 + 10n$ X X

275
3
278

139

77
15
92

$\alpha = 10$ $n^2 (n+11)^2 + (55^2 - 11 \cdot 385)$

~~$n^2 + 11n$~~

216
231

$3 \cdot 5^2 \cdot 11$ 55
$(3 \cdot 5)(5 \cdot 11)$ 15
70
$(3 \cdot 11)(5 \cdot 5)$ 35
$(3 \cdot 5)(5 \cdot 11)$

2025 2850
2025
825

85
17 5

4235
55
4180

$2 \cdot 2090$.

$3 \cdot 5 \cdot 5 \cdot 11$

$5 \cdot 165$
$5 \cdot 5 \cdot 33$

86 43

1046 523

2850
45
2805

$5 \cdot 561$

566
283

566
283 (55)

$3 \cdot 5 \cdot 187$
$3 \cdot 5 \cdot 7 \cdot 11$

33

105
11
116

58 (29.2)

231
5
236

(2, 97)

33
35
68

77
15

118
2, 59

187
15
202

193 194

21
55
76

38
165
7
172

38 86

388

21 × 11
187
15

202

(4 · 1045)

95

826
413

$2 \cdot 605$
$2 \cdot 5 \cdot 11$

11×25
$3 \cdot 5 \cdot (3 \cdot 5 \cdot 11)$
$5^2 (3 \cdot 11)$

$\alpha = 10$. $\Delta' = n^2(n+11)^2 + (55^2 - 11 \cdot 385)$

$n^2(n+11)^2 - 1210$.

$(1210 = 2 \times 605)$ X

4235
3025
1210

$10 \cdot 11 \cdot 11$
$2 \cdot 5 \cdot 11 \cdot 11$

55

$\alpha = 11$. $n^2(n+12)^2 + (66^2 - 12 \cdot 506)$

$n^2(n+12)^2 - 2716$ only $2 \cdot 1358$

$\frac{1}{2}(1360) = 680$

~~$n(n+12)$~~ $- 680 = 0$

144 + 2520
144
2664

$n = 680$
only value

$x^2 - 680 = y^2$
$x^2 - y^2 = 680$

$8 \cdot 85$

6072
4356
2716

$4 \cdot 679$
$2 \cdot 1358$
1360.
680.

68
10

$8 \cdot 85$
$8 \cdot 5 \cdot 17$
$2 \cdot 2 \cdot 2 \cdot 5 \cdot 17$

$4 \cdot 679$

22 49

$n^2 + 16n$ $n^2 + 9n - 540$

81 + 2160 X

2160
81
2141

0 6 7

$\alpha = 12$. Δ' $n^2(n+13)^2 + (78^2 - 13 \cdot 650)$

$n^2(n+13)^2 - $ ~~3016~~

X

$2 \cdot 2 \cdot 2 \cdot 13 \cdot 29$

26, 116,
58
52, 1508

8450
6084
2366

9100
6084
3016

1510
755
151

158
379

$4 \cdot 754$
$8 \cdot 377$
$8 \cdot 13 \cdot$

58
52
110

55

$2 \cdot 1181$ X

116
26
142

⑦X

$4 \cdot 754$ ~~8.377~~

$\alpha = 13$

$n^2(n+14)^2 + (91^2 - 14 \cdot 819)$.

$14 \cdot 91 \cdot 9 - 91^2$ 4×7

$91(126-91)$

$91 \cdot 35$.

5, 7, 7, 13

ab given $2 \cdot 3 \cdot 19$ X

124
4.31.
2.62

126
65
65
49
114
65 2.
243
49
16
23.5
13
27,3

637
3185

631
5
836
92.53
220
2
272
136

3.531
3.9.59.
27

$\alpha = 14$

$n^2(n+15)^2 + (105^2 - 15 \cdot 1015)$

$n^2(n+15)^2 - 4200$

105×105
525
000
105
11025
15225

91.
321
3.107
142
(71)

637
5
642

706
4200 352
6.700 ~~9.39~~
4.88
706
351

~~9.13.3~~
22.16
9.39

4.11.8

27,13

231
455 11.21.
7 33 1
462
245
22,21
13
258

42.100
210
20
7.30.20

10. ~~2102~~ 1054
1051 527
4 × 1050 17×31

Given ab. to find c, d

$\frac{a+b}{2}$ such that such that c+d =

ab = ~~x~~ y a+b = 2cd

c+d = y

$(a-b)^2 = (a+b)^2 - 4ab$.

$= 4(c^2d^2 - ab)$.

84
50
134 67

50
28
178

84
50
134

21

67

129
3.43

8, 21, 25
3, 7, 8, 5, 5

2, 2, 2, 3, 7, 8, 5, 5

210
20
2102
1051

232
4.58 8,29

323

$'\Delta' = 49 \cdot 25^2 - 8721$
4

5+2 81.

490000
16

341 =
= ~~16.1~~ 9

21904

19840 → 16.1369
14,400
5440

110
116

This also appears impossible//

495

30625
8721

150
28
70
60
130
65.
13,5

140
30
170
85 17,5

420
10
430
43,10

$\alpha = 15$ $n^2(n+16)^2 + (120^2 - 16 \cdot 1240) = \Delta'$

2.2720

2722
1366
2.683

$2 \cdot 2 \cdot 1360 = 2 \cdot 2 \cdot 2 \cdot 680 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 340$

$= 2^5 \cdot 170 = 2^6 \cdot 85 = 2^6 \cdot 5 \cdot 17 = 64 \cdot 5 \cdot 17$

64.85

11.51 5
33.17
561
1122

85
64
340
510
5440

$4 \cdot 1360 \rightarrow \frac{1364}{2} = 682 = 11 \cdot 62 = 2 \cdot 11 \cdot 31$ x

$8 \cdot 680 \rightarrow \frac{688}{2} = 344 = 4 \cdot 86 = 8 \cdot 43$. x

$\frac{194}{2} = 97$ x

$16 \cdot 340 \rightarrow \frac{356}{2} = 178 = 2 \cdot 89$ x

~~54~~, 34, 160

~~68 × 80~~ ~~$\frac{148}{2}$~~ = 74

472
4.88 34,2

$32 \cdot 170 \rightarrow \frac{202}{2} = 101$ x ~~80 340×2 = 332 x~~

~~136.40~~ $\frac{176}{2}$ = 88.

$\frac{148}{2} = 74$

$80 \cdot 68 \rightarrow \frac{148}{2} = 74$ x

~~68.20~~ 680.8 688 8.43

40+136
176 88

$160 \cdot 34 \rightarrow 194/2 = 97$ x

18.33.17 34.18
61 2 1224

7
17.72

344.
4.38

16, (17,1)
18, 2
19, 3
20, 4.
21, ~~5~~

~~42~~ 22, 6
23, 7.
24, 8
25, 9
26, 10

27, 11
28, 12
29, 13
30, 14
31, 15
32, 16

33, 17
34, 18
68, 52
136, 120
170, 154
340, 324

27·17
459
19

11
27·19
513
17
530

13

57
153
210

$\alpha = 15$ also appears difficult

$(\cancel{2} \times 68)^2$

27
323 350 478

25432
18496
6936.

18
4 × 4624

$\alpha = 16$

~~$\alpha = 16$~~

4·1734

1738
869.
11×79 } 238

3468 ~~590~~
3472
1735 10×59
~~8×217~~ 118
~~170·51~~ 170
5·347.
578 17·11·2
1156 = 17·17·4
6 11
17×27
578

57 513.
5.53 17
530

57 171
2 51.

4·25·2·21
$2^3 \cdot 3 \cdot 5^2 \cdot 7$

$\alpha =$
$m = \alpha + n$

53
49 104
104

$k = -14$

148
104
252

148
104
44

$n^2(n+17)^2 - (17 \cdot 1496 - 136^2)$

$\cancel{69} \cdot n^2(n+17)^2 - 6936$

$6936 = 8 \cdot 867 = 8 \cdot 3 \cdot 289$

$= 8 \cdot 3 \cdot 17 \cdot 17$

868
217

171
51
222

4 × 289
4·7·31
347 347
13

$4 \cdot 1734 \to 1738 \to 869 = 11 \cdot 79 \times$

$6 \cdot 1156 \to 1162 \to 2 \cdot 581 \times$

$12 \cdot 578 \to 590 \to 10 \cdot 59 \times$

$102 \cdot 68 \to 85 \to 17 \cdot 5 \times$

$204 \cdot 34 \to 119 \to 17 \cdot 7 \times$

This appears difficult again

5· 2810] 2807
323 478
969 +9
13

$\alpha = 17$ $n^2(n+18)^2 - (18 \cdot 1785 - 153^2)$

$n^2(n+18)^2 - 153 \cdot 57$

$153 \cdot 57 = 3 \cdot 3 \cdot 3 \cdot 17 \cdot 19$

$459 \cdot 19 \to 289 = 17 \cdot 17 \times$

$153 \cdot 57 \to \cancel{106} = \cancel{370}\ 3 \cdot 35 \times$

$513 \cdot 17 \to 265 = 5 \cdot 53 \times$

$171 \cdot 51 \to 111 = 37 \cdot 3 \times$

$969 \cdot 9 \to 489 = 3 \cdot 163 \times$

$323 \cdot 27 \to 175 = 25 \cdot 7$ ✓

$17(18 \cdot 105 - 153 \cdot 9)$
$= 153(210 - 153)$
$= 153 \cdot 57$
$= 9 \cdot 17 \cdot 3 \cdot 19$

2261
646
8721
222

8722

350
323 170
27 34
210 5
105
226
459
19
478

513
17
530 171
51

92·239
966 323 15
969
9
978 489. 350
3·163

49
175
25, 87.
462

$49^2 \cdot 25^2 - 8721$

16 × 289 ·10

126
258

3468
1734
867

2401
1785
616

3186
578

$n(n+18) - 175 = (n+25)(n-7)$

$n = 7,\ m = 24.$

$(\cancel{17}+1)k^2 + 2k(153 - 49) + \cancel{9} \cdot (1785 - ~~3549~~ - 49 \cdot 49)$

126
22
104

$18k^2 + 208k - 616 = 0.$

$9k^2 + 104k - 308 = 0.$ $\boxed{(9k - 22)(k + 14) = 0}$ ✓

2·11·31

34
51

$\Delta = 104^2 + 4 \cdot 9 \cdot 308 = 16(26^2 + 9 \cdot 77) = 16(676 + 9 \cdot 77)$

$= 16(676 + 693) = 16 \cdot 1369 = 16(37)^2 = (148)^2$

✓ $k = \dfrac{-104 \pm 148}{18} = -14,\ \dfrac{22}{9}.$

(Note) mistake is made for $n = 7, m = 24$ on p. 70 of Bk. 14

(58)

Since the +ve root is not an integer, we don't get a pure Vigder type relation. Anyway after all got one with $\Delta' = \square$ using this new method involving α & n.

$$K = -14 \longrightarrow (-14)^2+(-13)^2+ \dots 1^2+0^2+1^2+ \cdot +(-14+24)^2$$

$$= (-14+25)^2 + \quad + (-14+31)^2$$

11 17

ie $2(1^2+2^2+ \dots +10^2)+11^2+12^2+13^2+14^2$

$$= 11^2 + \quad \cdot \quad +17^2$$

or $\boxed{2\,(1^2+2^2+ \cdot +10^2) = 15^2+16^2+17^2.}$

$L.H.S = 2\cdot 385 = 770$, $R.H.S = 225+256+289 = \underline{770}$ ✓

or $\boxed{\dfrac{15^2+16^2+17^2}{1^2+2^2+ \quad +10^2} = 2}$

$$K = \frac{22}{9} \text{ gives } \left(\frac{22}{9}\right)^2+\left(\frac{31}{9}\right)^2+ \quad \cdot \left(\frac{22}{9}+24\right)^2$$

$$= \left(\frac{22}{9}+25\right)^2 + \cdot \quad + \left(\frac{22}{9}+31\right)^2$$

$\frac{22}{9}$ +

22
216
238

22
225
247

22
279
301

$$\boxed{22^2+31^2+ \cdot \quad +238^2 = 247^2+256^2+ \cdot +301^2.}$$

quite a beautiful relation generalising Vigder's result by numbers in A.P with C.D = 9 instead of consecutive integers. [A relation like this is got for $K = -\frac{106}{9}$ in Bk. 15, p. 16 ($n = 8$, $m = 16$), but here K is +ve]

Verification

$$L.H.S = 24\left(\frac{22}{9}\right)^2 + 2\cdot\frac{22}{9}(1+2+ \quad +24) + (1^2+ \quad +24^2)$$

$$= 25\cdot\left(\frac{22}{9}\right)^2 + \frac{44}{9}\cdot 300 + 4900$$

(using table for M_1 & M_2)

$$R.H.S = 7\cdot\left(\frac{22}{9}\right)^2 + 2\cdot\frac{22}{9}(25+ \cdot +31) + (25^2+ \cdot \quad +31^2)$$

$$= 7\left(\frac{22}{9}\right)^2 + 2\cdot\frac{22}{9}(496-300) + (10416-4900)$$

121

25 83
7
25 90
25
1295

~~10 37~~

Using

i.e. to show that $24\cdot(22)^2 + 9\cdot44\cdot300 + 81\cdot4900 = 7\left(\frac{22}{9}\right)^2 + \frac{9\cdot44}{9}(196) + 81(5516)$

$(22)^2(25-7) + 9\cdot44\cdot104 = 81\cdot616$

$(484)(\overset{18}{\cancel{168}}) + (396)(104) = (81)(616)$

$(11)(\overset{18}{\cancel{22}}) + (9)(104) = (81)(\cancel{14})(14)$ $\div 11\cdot 8$

~~$(11)(28) + (3)(52) = (27)(7)$~~

~~308 +~~ $(11)(18) + (9)(104) = (81)(14)$

~~77 + 936~~ $198 + 936 = 1134$ ✓

10416
4900
5516

3004900
196
104

8 × 8
44 × 36
8

190 70 260
450 14 ~~6~~ 56
380 14 266 394 50 316
1330 10 1340
350 38 388
133 6650 2 6652
964
5326
$2\cdot1663$
$171\times\cancel{17}\ 9$
1
13

$\boxed{\alpha = 18}$ $n^2(n+19)^2 - (19\cdot2109 - 171^2)$

$19(2109 - 1539)$

$19(570)$

$\Delta' = n^2(n+19)^2 - 19\cdot570$ $\quad 19\cdot570 = 30\cdot19\cdot19 = 2\cdot15\cdot19\cdot19$

Since 2 is the only possible evenfactor, X 19·6 ~~14~~ 95

$\boxed{\alpha = 19}$ $n^2(n+20)^2 - (20\cdot2470 - \cancel{20}\cdot190^2)$

49400
36100
13300

$n^2(n+20)^2 - 7\cdot19\cdot100$

$7\cdot19\cdot100 = 2\cdot2\cdot5\cdot5\cdot7\cdot19$

18. No choice appears possible

950
14
964

1330
10
1340
670
67, 14
260
130
~~140~~ 350
38
388
287
210
194
2·97
190
70
266
50
316
158
2×79
161
121
282
870

482 $\boxed{\alpha = 20}$ $n^2(n+21)^2 - (21\cdot2870 - 210^2)$ $\quad 210\cdot287 - 210^2$

$n^2(n+21)^2 - 210.77$

$210\cdot77 = \cancel{2\cdot5}\cdot2\cdot3\cdot5\cdot7\cdot7\cdot11$

Only evenfactor is ~~for~~ 2 : So X

106
9
2·241

161×11
1771
11
1782
253
77
330
847
23
121×23
363
2·42
2790
2783

$\boxed{\alpha = 21}$ $n^2(n+22)^2 - (22\cdot3311 - 231^2)$ $\quad \left[2\cdot11\cdot11\cdot301 - 11\cdot11\cdot21^2\right.$

$n^2(n+22)^2 - 7\cdot11\cdot11\cdot23$ $\quad 19\frac{487}{2}$ $\quad 11\cdot11(602 - 441)$

$\left.11\cdot11\times161\right]$

$77\cdot253 \rightarrow 165$ x

$121\cdot161 \rightarrow 141$ x

$847\cdot23 \rightarrow 435$ x

$2783\cdot7 \rightarrow 1395$ x

$1771\cdot11 \rightarrow 891$ x

no possible cases

$5\cdot279 = 5\cdot9\cdot31$

5·3·11
161
121
282
15·29
4282 435
~~20~~ 141 5, 87.
121 870
~~29, 30~~
3
7
90
295

253
77
330
165.
33, 5
b
~~29~~ 161
1771
11
1782

7·185
35·37
7·5·37
11·81 811.

$\boxed{\alpha = 22}$

$n^2(n+23)^2 - (23 \cdot 3795 - 253^2)$

$n^2(n+23)^2 - 11 \cdot 23 \cdot 92$

$11(23 \cdot 345 - 253 \cdot 231$

$11 \cdot 23(345 - 253)$

$11 \cdot 23(92)$

22 6 9 3

20.291.
$\cancel{23 \cdot 13}$ 2013.97

$11 \cdot 23 \cdot 92 = \boxed{2 \cdot 2 \cdot 11 \cdot 23 \cdot 23.}$

529

$\cancel{22}(23 \cdot 2 \cdot 23 \cdot 11 \cdot 2)$ $46 \cdot 506 \rightarrow 276$

~~5786~~
5819

$\boxed{23 \cdot \cancel{23} \cdot 2 \cdot 11 \cdot 2 = 1058 \cdot 22}$

23.12
253 69
506
46 23.7
552
23.4 (23.4
23.3
23.14
52, 69 3.4.23

$\cancel{23 \cdot 22}, 23 \cdot \cancel{4}$
23 (22+2)
23.12.

529 253
506 552
46 $\boxed{22 \cdot 23}$
22, 1058 1080

4 6 9
23

276 4,69
2.2.2.3.3
36
7
27

540 . 135 4 80 .
30.18
15.36.
~~240~~. 1080
~~10 1209~~

2.2.3.23. 46

10×54
9×60 ~~10~~.27

1080 540
2.3.3.5.
2.2.3.3.3.5

9, 60 18, 30 45, 12 30 18

10 529
45
12
33
17
540

17 $\boxed{92}$ $\cancel{\tfrac{1}{6}n(n+1)(2n+1) = 6t^2}$

$\cancel{2n+1 = t^2 \quad t^2 - 2n = 1}$

~~appears~~ no obvious case

253
23
276

$\boxed{\alpha = 23}$ $n^2(n+24)^2 - (24 \cdot 4324 - 276^2)$

$24 \cdot 4324 - 276^2 = 16(6 \cdot 1081 - 69^2)$

$= 16(6486 - 4761)$

$= 16 \cdot 1725 = 16 \cdot 23 \cdot 75$ 5

$= \boxed{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 23}$

6486
4761
1725
345.
69.

334

276
100
376

150
184

138

4.49 196. 392

184
150
334
167

~~164~~
~~190~~
254
330 27

120
230
350

22 ~~240~~

392
196
7.28
14, 14
188
4.47

100
276
376

460
60
520
= 4 × 65

60
460
520

32 5 26 10

260 690 . 30
260.
360
2.8.15.5.23

12
30. 40
9

4600 30 4630

120
230
350

~~350~~ 175

600 646
46 323
17.19

115
345 690

5,35
25,7

529.2
~~108~~
1058.

36 43 89

46.
23

4

69
46.

115
230

690
40
730

= 23.46
365

$\boxed{5 \cdot 73.}$ × 690 184
40 150
730 174

334

$\boxed{167}$ ×

552 50 602

230
120
350

3458

2312

2300 12

587.

1156 = 4.289

660
276136
10
115
27600 10
5275
460 × 200

1150
24

13800 20

4600 6
4606

27600
10
27610

19.17.17

2303

2.2303
2.7.329
2.7.7.47

13820

46
690
323.

288

138 20 338

nothing obvious?

1150
24
~~2 ×~~ 281

~~2760~~
13802
6904
4606

646 300 92 392 25 × 5 5 × 43 1174 2 × 263 ~~2760~~ ~~231~~ 138

$\boxed{\alpha = 24}$ ✓ $n^2(n+25)^2 - (25 \cdot 4900 - 300^2)$ $25(4900 - 3600)$ $\boxed{190}$ $\boxed{19, 10}$

$n^2(n+25)^2 - 25 \cdot 1300$ 65 × 40 2600

250
130
$\cancel{3}80$

$25 \cdot 13 \cdot 100.$ 5200.

650
$\cancel{500}$
1150
675

650
50
700

350 $\boxed{35 \times 10}$ ✓ obvious case.

390.

6

$2 \cdot 2 \cdot \cancel{25} \cdot 25 \cdot 13$

825
1250
26
1276
638
$2 \cdot 319$
$2 \cdot 11 \cdot 29$

$n(n+25) - 350.$ $(n+35)(n-10).$

$n = 10, m = 34.$ – Vigdus case.

$\boxed{2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 13}$

$25k^2 + 2k(300 - 100) + (4900 - 6900)$

69
100 × 20
2000
25

$25k^2 + 400k - 2000 = 0$

$k^2 + 16k - 80 = 0.$ $(k+20)(k-4).$

$n = -35 \to m = -11,$ $25k^2 + 2k(300 - 1225)$
$+ (4900 + 21 \cdot 1225)$

-22+1

$\Delta' = -ve$

2
5

$25(5746 - 4225)$
1521

$\boxed{\alpha = 25}$ ✓ $n^2(n+26)^2 - (26 \cdot 5525 - 325^2)$

$n^2(n+26)^2 - 25 \cdot 1521.$ $25(26 \cdot 221 - 65^2)$

975

$25 \cdot 1521 = 25 \cdot 9 \cdot 169 = 3^2 \cdot 5^2 \cdot 13^2$

6

$\boxed{3} \cdot 3 \cdot 5 \cdot 5 \cdot 13 \cdot \boxed{13}$

39 × 25
195
78
975.

11.

13 × 25
325

507
75
582

117
325
442

$\boxed{\begin{array}{r} 675 \\ 39 \\ \hline 714 \end{array}}$

$\cancel{n^2(n+26)^2 - 675 \cdot 39}$

$n(n+26) - 42 \cdot 17$

507

$\boxed{1014}$

291.

221
13·17

$2 \cdot 357$
$2 \cdot 7 \cdot 51$
$2 \cdot 7 \cdot 3 \cdot 17$

975
39
1014

$(n+39)(n-13) = 0.$

169 × 3
507

$\boxed{m = 38}$:

$n = 13, m = 38$ Case

175

$\boxed{\begin{array}{r} 42 \\ 17 \\ \hline 25 \end{array}}$ 65 × 5

$2 \cdot 3 \cdot 13 \cdot 13$ $2 \cdot 507$ $2 \cdot 3 \cdot 169$

39, 2
39.

757
525

(62) 169 $3\cdot3\cdot5\cdot5\cdot13\cdot13$

845
45
445 ~~890~~

5·84

507
75
582 | 291 = 3·97 ×

169·9

1521
25
1546
773 ×

169 × 25
5225
9
5234

2617 Δ

16275
3
16278
8139 = 3·2713 ×

195 975

65
585
550
275

5·55 5·5·11 ×

No other cases except (975, 39)

39.
27·13

$d = 26$ $n^2(n+27)^2 - (27\cdot6201 - 351^2)$

$27\cdot6201 - 351^2$

689

$= 27\cdot9\cdot689 - 27\cdot27\cdot169)$

$= ~~243~~\ 81(2067 - 1521) = 81(546) = 81\cdot2\cdot3\cdot91$

$2\cdot3\cdot9\cdot9\cdot7\cdot13.$

$2\cdot3\cdot3\cdot3\cdot3\cdot3\cdot7\cdot13$ only one $\underline{2}$ impossible

$d = 27$ $n^2(n+28)^2 - (28\cdot6930 - 378^2)$

$28\cdot6930 - 378^2 = 4(7\cdot6930 - 189^2)$

$= 4(7\cdot630\cdot11 - 9\cdot63^2)$

49

$= 36(7\cdot11\cdot70 - 63^2) = 36(5390 - 3969)$

$= 36\cdot1421 = 36\cdot7\cdot203$

$= 6\cdot6\cdot7\cdot7\cdot29.$

174
294
468

$(29\cdot6)(49\cdot6)$ ×; ~~294~~

$(29\cdot42)(7\cdot6)$ ×

$(~~29\cdot7~~)\cdot$ $2\cdot2\cdot3\cdot3\cdot7\cdot7\cdot29$

234
2·117
2·9·13
39
2·3·3·13

18
2842
2860

1430 2·5·11·13

406
126
538

98 × 29
882
196
2842

268

261 406 147

1218
42
1260
630
~~2·3·5~~
2·3·3·5·7

4·67

58
882
940
470

174
294
468
4.117

~~622~~
98
522
620
310 x

~~522~~
98
622
3110
x

14
3654
3668
1834
2.917

2 x 7 x 131

58
882
940

~~470~~
~~10 x 2 x 31~~
2 x 2 x 5 x 31

12790
2.6395
2.5.1279
x

no obvious choice

---

18 x 49 x 29 + 1
9.49.29 + 1
261 x 49
2349
1044
12789

α $\quad$ Q $n^2 + n\alpha$

$$n^2(n+\overline{\alpha+1})^2 - (\overline{\alpha+1}\,A_2 - A_1^2)$$

$\boxed{\alpha = 28}$ $\quad n^2(n+29)^2 - (29\cdot 7714 - 406^2)$

$$n^2(n+29)^2 - (29\cdot 29\cdot 266 - 29^2\cdot 14^2)$$

$$n^2(n+29)^2 - 29^2(266 - 196)$$

$n^2(n+29)^2 - 29^2\cdot 70$ x only one factor $\underline{2}$.

$\boxed{\alpha = 29}$ $\quad n^2(n+30)^2 - (30\cdot 8555 - 435^2)$

$n^2(n+30)^2 - (30\cdot 29\cdot 295 - 29^2\cdot 15^2)$ Q.

" $\quad - 25\cdot 29(6\cdot 59 - 29\cdot 9)$

$$= 3\cdot 25\cdot 29\cdot 31 = 3\cdot 5\cdot 5\cdot 29\cdot 31$$

2
191
174
174

14
275
261
14
70
275 14
295

5
8
354
261
93
93
97

93
465
145
610
305 x
61

145

435
155
~~608~~ 590
~~300~~
295
5.59
x

725
93
818
~~409~~ x

899
75
974
487 x

675
87 x
762
381 ~~= 3.127~~ x

6745
5.1349

13490

155
87 x 31
87
261
2697
13485

~~no p~~ no obvious choice

$\boxed{\alpha = 30}$ $\quad n^2(n+31)^2 - (31\cdot 9455 - 465^2)$

$$n^2(n+31)^2 - (31^2\cdot 305 - 31^2\cdot 15^2)$$

$n^2(n+31)^2 - 31^2(80)$ $\quad$ 2.2.2.2.5.31.31

$$= 2^4\cdot 5\cdot 31^2$$

305
225
80

31.10, 31.8
31.18
155
31.3.2.3
155.124
124 x 155 = 31^2 x 20
31^2 x 20

620
124
744
2.2.186
~~2.2.2.3.31~~

8.93
4.3.31

$\frac{1}{2}(4+1)(1+1)(2+1)$ 31. ~~8~~.31.10

4.5.2.3
2.2.3.31 $\quad$ ⑮

31.4.31.20
31.12
31.24

$\frac{1}{2}$ (124^2)

$2^2\cdot 5$ (20) $\quad$ $2, 2\cdot 2\cdot 2\cdot 5\cdot 31\cdot 31$ has 15 decompositions $= \frac{1}{2}(5)\cdot 2\cdot 3 = 15$ into 2 factors.

$x^2-y^2=20$ $\quad$ no. of solns $= \frac{1}{2}(3)(2)(3) = \underline{\underline{9}}$:

$\frac{1}{2}\cdot 1\cdot 2$ $\frac{1}{2}$

(2,10)

28|4|76

$(2)\cdot(2^3\cdot 5\cdot 31^2)$	~~(5)(2^2·31)~~ $(2\cdot 5)(2^3\cdot 31^2)$	$(2\cdot 31)(2^3\cdot 5\cdot 31)$
$(2)^2(5\cdot 31^2\cdot 2^2)$	$(2^2\cdot 5)(2^2\cdot 31^2)$	$(2^2\cdot 31)(2^2\cdot 5\cdot 31)$
$(2^3)\cdot(5\cdot 31^2\cdot 2)$	$(2^3\cdot 5)(2\cdot 31^2)$	$(2^3\cdot 31)(2\cdot 5\cdot 31)$
× $(2^4)\cdot(5\cdot 31^2)$	× $(2^4\cdot 5)(31^2)$	× $(2^4\cdot 31)(5\cdot 31)$

ie 9 solns. [Vrepicelli's result that the number of solns of $x^2-y^2=g$ where $g = 2^\mu h_1^\alpha \ldots h_k^\gamma$ is given by

$$\nu_1 = \frac{1}{2}(\mu-1)(\alpha+1)\ldots(\gamma+1) \text{ or } \nu_1 - 1/2$$

according as at least one of the exponents $\mu, \alpha, \ldots \gamma$ is odd or all are even. Here the former case holds since $\alpha = 1$. since $g = 2^4\, 5^1\cdot 31^2$ ie

961·8·5 / 40 $\quad$ $\nu_1 = \frac{1}{2}(4-1)(1+1)(2+1) = 9$. Dickson II, p. 403]

961·8 $\quad$ These 9 are given above viz (2)(38440), (4)(19220), (8)(9610),

(10)(7688), (20)(3844), (40)(1922),

(31)(~~[illegible]40~~ (62)(2440), (124)(1220), (248)(310)

38442 $\quad$ | 19224

19221. = 3·6407 × $\quad$ | $9612 = 12\cdot 801 = 12\cdot 9\cdot 81 = 12\cdot 9\cdot 9\cdot 9 = 4\cdot 3^7 = 2^2\cdot 3^7$

$\frac{1}{2}\cdot 3\cdot 8 = 12$ cases ~~(2)(2·3^7), 2^2~~

108 / 81 / 27 $\quad$ $(2)(2\cdot 3^7)$, $(2^2)(3^7)$, $(2)(3)(3^6)$, $(2)(3^2)(3^5)$, $(2\cdot 3^3)3^4$ ×

$(2\cdot 3^4)(3^3)$, $(2\cdot 3^5)(3^2)$, $(2\cdot 3^6)(3)$

27 34 / 36 161 / 81 $\quad$ $\frac{9618}{2} = 4809 = 3\cdot 1603 = 3\cdot 7\cdot 229$ ×

$\frac{7698}{2} = 3849 = 3\cdot 1283$ × | $\frac{3864}{2} = 1932 = 4\cdot 483 = 3\cdot 4\cdot 161$

$= 3\cdot 4\cdot 7\cdot 23 = 2\cdot 2\cdot 3\cdot 7\cdot 23$ | $\frac{1344}{2} = 672 = 16\cdot 42 = 32\cdot 21$

42 / 16 / 26 $\quad$ $= 4\cdot 8\cdot 21 = 4\cdot 8\cdot 3\cdot 7$ | $\frac{558}{2} = 279 = 9\cdot 31$ | $\frac{1962}{2} = 981 = 9\cdot 109$ X

$(9 = 3^2) \rightarrow$ X

$\boxed{\alpha = 31}$ $n^2(n+32)^2 - (32\cdot 10416 - 496^2)$. 651

21

$n^2(n+32)^2 - (32\cdot 651\cdot 16 - 16^2\cdot 31^2)$

$n^2(n+32)^2 - (16^2\cdot 2\cdot 21\cdot 31 - 16^2\cdot 31^2)$ ~~$= n^2(n+32)^2$~~

$= n^2(n+32^2) - 16^2\cdot 31(42-31) = n^2(n+32)^2 - 16^2\cdot 11\cdot 31$

$16^2\cdot 11\cdot 31 = 2^8\cdot 11\cdot 31.$

No of solns ~~are~~ no of decompositions into two even factor $= \frac{1}{2}(8-1)(1+1)(1+1)$

$= \frac{1}{2}\cdot 7\cdot 2\cdot 2 = \underline{14}$.

$(2)(2^7\cdot 11\cdot 31)$	$(2\cdot 11)(2^7\cdot 31)$	$(2^6\cdot 11\cdot 31)+1$	$(2^6\cdot 31)+11$
$(2^2)(2^6\cdot 11\cdot 31$	$(2^2\cdot 11)(2^6\cdot 31)$	$(2^5\cdot 11\cdot 31)+2$	$(2^5\cdot 31)+11\cdot 2$
$(2^3)(2^5\cdot 11\cdot 31)$	$(2^3\cdot 11)(2^5\cdot 31)$	$(2^4\cdot 11\cdot 31)+2^2$	$(2^4\cdot 31)+11\cdot 2^2$
$(2^4)(2^4\cdot 11\cdot 31)$	$(2^4\cdot 11)(2^4\cdot 31)$	$(2^3\cdot 11\cdot 31)+2^3$	$(2^3\cdot 31)+11\cdot 2^3$
$(2^5)(2^3\cdot 11\cdot 31)$	$(2^5\cdot 11)(2^3\cdot 31)$	$(2^2\cdot 11\cdot 31)+2^4$	$(2^2\cdot 31)+11\cdot 2^4$
$(2^6)(2^2\cdot 11\cdot 31)$	$(2^6\cdot 11)(2^2\cdot 31)$	$(2\cdot 11\cdot 31)+2^5$	$(2\cdot 31)+11\cdot 2^5$
$(2^7)(2\cdot 11\cdot 31)$	$(2^7\cdot 11)(2\cdot 31)$	$(11\cdot 31)+2^6$	$(31)+11\cdot 2^6$

341 + 64	704 + 31	405	735	5·81 ×	15·49 ×
682 + 32	352 + 62	714	414	6·7·17 ×	18·46 ×
1364 + 16	176 + 124	1380	300	6·10·23 ×	30·10 ×
2728 + 8	88 + 248	2736	336	16·9·19 ×	16·21 ×
5456 + 4	44 + 496	5460	540	6·10·91 ×	18·30 ×
10912 + 2	22 + 992	10914	1014	6·17·107 ×	$6\cdot 13^2$ ×
21824 + 1	11 + 1984	21825	1995	25·9·97 ×	15·7·19 ×

All the ~~cases~~ 14 cases carefully examined & no factors with diff i.e 32 obtained

38

341

6·10·7·13

64

2·2·3·3·3·5

4×4×3×7

14·18

169

~~198~~ 18 45

1819

107

133

4365

485

97

(66) $\boxed{q=32}$ $n^2(n+33)^2-(33.~~528~~\,11440-528^2)$

$33\cdot11440-528^2=11^2(3\cdot1040-48^2)=8\cdot11^2(390-288)$

$=8\cdot11^2\cdot102=~~2^4\cdot18~~\;2^4\cdot3\cdot11^2\cdot17.$

48·48 390
288
102

No of cases to be considered $=\frac{1}{2}(4-1)(1+1)(2+1)(1+1)=3\cdot3\cdot2=18$

$(2)(2^3\cdot3\cdot11^2\cdot17)$	$(2\cdot11^2)(2^3\cdot3\cdot17)$	$(2\cdot11\cdot3)(2^3\cdot11\cdot17)$
$(2^2)(2^2\cdot3\cdot11^2\cdot17)$	$(2^2\cdot11^2)(2^2\cdot3\cdot17)$	$(2^2\cdot11\cdot3)(2^2\cdot11\cdot17)$
$(2^3)(2\cdot3\cdot11^2\cdot17$	$(2^3\cdot11^2)(2\cdot3\cdot17)$	$(2^3\cdot11\cdot3)(2\cdot11\cdot17)$
$(2\cdot3)(2^3\cdot11^2\cdot17)$	$(2\cdot17)(2^3\cdot3\cdot11^2)$	$(2\cdot11\cdot17)(2^3\cdot3\cdot11)$
$(2^2\cdot3)(2^2\cdot11^2\cdot17)$	$(2^2\cdot17)(2^2\cdot3\cdot11^2)$	$(2^2\cdot11\cdot17)(2^2\cdot3\cdot11$
$(2^3\cdot3)(2\cdot11^2\cdot17)$	$(2^3\cdot17)(2\cdot3\cdot11^2)$	$(2^3\cdot11\cdot17)(2\cdot3\cdot11)$

$2^2\cdot3\cdot11^2\cdot17+1.$	$2^2\cdot3\cdot17+11^2$	$(2^2\cdot11\cdot17)+11\cdot3$
$2\cdot3\cdot11^2\cdot17+2$	$2\cdot3\cdot17+11^2\cdot2$	$2\cdot11\cdot17+11\cdot3\cdot2$
$3\cdot11^2\cdot17+2^2$	$(3\cdot17)+11^2\cdot2^2$	$11\cdot17+11\cdot3\cdot2^2$
$2^2\cdot11^2\cdot17+3$	$(2^2\cdot3\cdot11^2)+17$	$2^2\cdot3\cdot11+11\cdot17$
$2\cdot11^2\cdot17+3\cdot2$	$(2\cdot3\cdot11^2)+17\cdot2$	$2\cdot3\cdot11+11\cdot17\cdot2$
$(11^2\cdot17)+3\cdot2^2$	$(3\cdot11^2)+17\cdot2^2$	$3\cdot11+11\cdot17\cdot2^2$

187
4
748
33

132
187
374
66

24685	325 ~~446~~	781	5 × 4937 x	15·23 x	11·71 x
12344	344	440	8·1543 x	8·43 x	22·20 x
6175	535	319	25·13·19 x	5·107 x	11·29 x
8231 x	1469	319	Prime x	Prime x	11·29 x
4120	760	440	40 × 103 x	19·40 x	22·20 x
2069	431	781	Prime x	Prime	11·71 x

363 × 17
6171
24684
12342
121 × 17
2057
8228
4114
~~121~~
204
~~127~~
51 325
484
283 187
535 132
319
102 51
242 484
344 535
4.86 2
187.4
43 748
363 × 4 33
1452
17
726 363
34 68
760 431
374 11·29
66
440

No cases. Seems more & more that there are no cases.

$$\Delta' = n^2(n+\overline{\alpha+1})^2 - (\overline{\alpha+1}\,A_2 - A_1^2)$$

$\left.\begin{matrix} \eta\ m = 2n \\ u\ \alpha = 1 \end{matrix}\right\}$ $\Delta' = n^2(2n+1)^2 - (\overline{n+1}\,A_2 - A_1^2)$

$$= n^2(2n+1)^2 - \left\{\frac{(n+1)n(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4}\right\}$$

~~$= \frac{1}{12}\left\{12n^2(2n+1)^2 - 2n(n+1)^2(2n+1) - 3n^2(n+1)^2\right\}$~~

??

?

~~$= \frac{1}{12}(n+1)^2\left\{12n^2 - 2n(2n+1) - 3n^2\right\}$~~

//

~~$= \frac{1}{12}(n+1)^2\left\{17n^2 - 2n\right\} = \frac{1}{12}n(n+1)$~~

~~$\frac{1}{12}n(17n-1) = \square.$~~ $n = 8 \rightarrow \frac{1}{12}\cdot 8\cdot$

$$\frac{1}{12}n\left\{12n(2n+1)^2 - 2(n+1)^2(2n+1) - 3n(n+1)^2\right\}$$

$$= \frac{1}{12}n\left[12n(4n^2+4n+1) - (n+1)^2\left\{2(2n+1) - 3n\right\}\right]$$

$$= \frac{1}{12}n\left[12n(4n^2+4n+1) - (n+1)^2(n+2)\right]$$ $\frac{1}{2}n(n+1)$

This again is a cubic in n // Same cubic fit before

1, 1+3, 1+8,

1, 1+4, 1+4+8, 1+4+8+12 $\quad n^2(2n+1)^2 - \frac{1}{3}(n+1)(2n+1)A_1 + A_1^2$

1, 1+5, 1+5+9 $\quad n^2(2n+1)^2 - \frac{1}{3}(n+1)(2n+1)A_1 +$

1, 1, 1

$$\Delta' = n^2(2n+1)^2 - \frac{1}{6}n(n+1)^2(2n+1) + \frac{1}{4}n^2(n+1)^2$$

1, 2

1, 3

$$= n^2\left\{n^2 + 2n(n+1) + (n+1)^2\right\} - \frac{1}{6}n(n+1)^2\left\{n + (n+1)\right\}$$

n(n+1)

$n(n+\overline{n+1})$

$$\left\{\frac{1}{2}n(n+1)^2 - n(2n+1)\right\}^2 + n^2(n+1)^2(2n+1)$$

$$\left\{n(2n+1) - A_1\right\}^2 + 2n(n+1)A_1 - \frac{1}{3}(n+1)(2n+1)A_1$$

$$+ (n+1)A_1\left\{2n - \frac{1}{3}(2n+1)\right\}$$

$\frac{1}{3}(n+1)A_1(4n - n)$

(68)

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$

$m = 3n-1.$ ~~$m-n = 2n-1$~~

$m - n = \alpha$

$n + \alpha = 3n-1 \quad \alpha = 2n-1.$

$A_2 = \frac{1}{6}\alpha(\alpha+1)(2\alpha+1)$

$A_1 = \frac{1}{2}\alpha(\alpha+1)$

$A_2 = \frac{1}{3}(2\alpha+1)A_1$

$n^2(n+2n)^2 - (2n\cdot A_2 - A_1^2)$

$\alpha+1$

$9n^4 - \{2n\cdot\frac{1}{6}(2n-1)(2n)(2n-1) + \frac{1}{4}(2n-1)^2 2n^2\}$

$9n^4 - \frac{4}{3}n^2(2n-1)^2 + \frac{1}{4}n^2(2n-1)^2$

$\Delta' = 9n^4 + \frac{1}{3}n^2(2n-1)^2$

$= n^2\{9n^2 + \frac{1}{3}(2n-1)^2\}$

$3\Delta' = n^2\{27n^2 + \underline{4n^2 - 8n + 1}\}$?

$3\Delta = 47n^4 + 44n^3 + 7n^2 - 2n.$

$n(47n^3 + 44n^2 + 7n - 2) = 3\square.$

$n = 2j+1, \; 3\Delta = 47(2j+1)^4 + 44(2j+1)^3 + 7(2j+1)^2 - 2(2j+1)$

$= 47(16j^4 + 32j^3 + 24j^2 + 8j + 1)$

$+ 44($

22

169 × 25
4225
9
4234

12678   6339 ✓

4225
169
38025
1
38026

1521
25
1546

169 × 9
1521 × 5
7605
5
7610

19013

25

8
6

25 × 13
325 × 9
2925
13
2938

$m = 3n-1, \; \alpha = m-n = 2n-1.$

$\Delta' = n^2(n+2n)^2 - (2n\cdot A_2 - A_1^2)$

$= 9n^4 - \{2n\cdot\frac{1}{6}\cdot(2n-1)(2n)(4n-1) - \frac{1}{4}(2n-1)^2(2n)^2\}$

$\frac{1}{6} - \frac{1}{4}$

$\frac{2-3}{12}$

$= 9n^4 - \{4n^2$ ~~$9n^4 - \frac{1}{12}(2n)^2(2n-1)^2 = 9n^4 - \frac{1}{3}n^2(2n-1)^2$~~

~~$3\Delta' = 27n^4 - n^2(4n^2 - 4n + 1) = 23n^4 + 4n^3 - n^2.$~~

~~$= n^2(23n^2 + 4n - 1)$~~

$\Delta' = 9n^4 + \frac{1}{3} n^2 (2n-1)^2 = y$ $\quad x = 3n.$

~~$y^2 - x^2 = \frac{1}{3} n^2 (2n-1)^2.$~~

~~$9n^2 + \frac{1}{3}(2n-1)^2 = y^2$~~

~~$y^2 - 3n^2 = \frac{1}{3}(2n-1)^2$~~

$2n-1$ $\quad \frac{1}{3}(2n-1) \quad 2n-1$

$\frac{1}{3}(2n-1+6n-3$

$\frac{1}{3}(8n-4)$

$\frac{1}{3}(2n-1)$

3.169

+3.25

$\Delta' = 9n^4 - \left\{\frac{2n^2}{3}(2n-1)(4n-1) - \frac{1}{2}n^2(2n-1)^2\right\}$

$= 9n^4 - n^2 (2n-1)\left\{\frac{2}{3}(4n-1) - (2n-1)\right\}$

$= 9n^4 - \frac{n^2(2n-1)}{3}\{8n - 2 - 6n + 3\}$

$= 9n^4 - \frac{1}{3}n^2(4n^2-1) = n^2\{9n^2 - \frac{1}{3}(4n^2-1)$

~~$3n^4 - y^4 = \frac{1}{3}n^2$~~ $3\Delta' = 27n^4 - n^2(4n^2-1)$

~~$3\Delta'' = 9n^4 - 4n^2 + 1 = 23n^2 + 1. = 3y^2$~~

~~$3y^2 - 5n^2 = 1.$~~

$27n^2 - 4n^2 + 1$

$23n^2 + 1.$

23.169.

1521
225
169 1296
282

507
225

4.169 $\quad ^3_2$

676-1
675
225 $\quad$ 432

4

169
845
846
283×

$3 \cdot 13^4$

507
338
3887
1

3888 $\quad$ 1944
972
481

$x = 3n^2.$

$9n^4 - 4n^2 (2n-1)\left\{\frac{1}{3}(4n-1) - \frac{1}{4}(2n-1)\right\}$

30/4/78 Inst – Scripta Mathematica – Vol. 24, No. 2, June 1959

p. 179 – On a chain of eqns by C. E. Britton

Vietas method – to find r positive integral solns $x_r^3 \mp y_r^3 = N.$

Hardy & Wright say that to find r solns one must have at least $x_1 > 4^{r-1} y_1.$

Gives one soln of $x^3 + y^3 = u^3 + v^3 = s^3 + t^3$

$2406810^3 + 2166129^3 = 2879081^3 + 622072^3 = 2888172^3 + 240681^3.$

Another soln of $x^3 + y^3 = u^3 + v^3 = s^3 + t^3 = h^3 + k^3$ due to Fauquembergue is

$969360^3 + 121170^3 = 955512^3 + 312378^3 = 956305^3 + 336455^2 = 908775^3 + 545265^3.$

(70) **Ibid · No·3**, Sept 59

p·257 – Fun with lattice points by H·D. Grossman.

Something to do with the series $\pi/4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + —$

Ibid Vol. 25, No·2

p. 125 – On some unsolved problems of arithmetic by Sierpinski

Very interesting article

$$x^3 - y^3 = 7.$$

It has been shown that the number of solutions of this equation is finite, but we don't know how many it is

p. 165. Eqn $x^3+y^3 = u^3+v^3 = N$ by C. E. Britton

Gives a table for values of $N < 5,000,000$ – Colossal

1st Ex. $1729 = 12^3+1^3 = 10^3+9^3$

Last one $-4931101 = 165^3+76^3 = 157^3+102^3$

In fact the first 30 could be verified using tables

p. 167 – Diophantine eqns by A. Gloden

I. Eqn $x^2-3y^4 = z^3$.

Putting $y^2=t$, $x^2-3t^2 = Z^3$ which is satisfied by

$$x = m(m^2+9n^2),\ \cancel{t} = y^2 = 3n(m^2+n^2),\ z = m^3-3n^2$$

The Diophantine equation, $y^2 = 3n(m^2+n^2)$ can be solved by putting $m^2+n^2 = y_1^2$, $n = 3y_2^2$. The first of these is solved by $m = p^2-q^2$, $n = 2pq$. To satisfy the second we put $p = 3r^2$, $q = 2s^2$ or $p = 6r^2$, $q = s^2$. We then have

$$m = 9r^4-4s^4,\ n = 12r^2s^2$$

or $m = 36r^4 - s^4,\ n = 12r^2s^2$.

Ex: Putting $r = 2,\ s = 1$ we get

$$88235^2 - 3.222^4 = 793^3 = 498677257$$

$$202032575^2 - 3.6924^4 = 323713^3 = 33921919703028097$$

II The eqn $x^2 - y^4 = z^3$. Putting $y^2 = t$

$x^2 - t^2 = z^3$ with $x = m^2 + 3mn^2,\ t = 3m^2n + n^3,\ z = m^2 - n^2$

Now t must be a square, hence $n(n^2+3m^2) = y^2$ which is satisfied by $n^2 + 3m^2 = y_1^2,\ n = y_2^2$

For the first of these eqns we have

$$n = p^2 - 3q^3,\ m = 2pq,\ y_1 = p^2 + 3q^2$$

Combining the two reps, we get

$$p^2 - 3q^2 = y_2^2$$

$$p = r^2 + 3s^2,\ q = 2rs,\ y_2 = r^2 - 3s^2.$$

97)~~3135~~(3
~~291~~
~~165~~

Ex. with $r = 2,\ s = 1$

$$175784^2 - 97^4 = 3135^3.$$

III The equation is $x_1^3 + x_2^3 + x_3^3 - 3x_1x_2x_3 = y_1^3 + y_2^3 + y_3^3 - 3y_1y_2y_3$
$= z_1^3 + z_2^3 + z_3^3 - 3z_1z_2z_3 = u^3$

For the equation $x_1^3 + x_2^3 + x_3^2 - 3\boxed{y_1y_2y_3} = u^3$, we have the 2-parameter soln $\times\ \boxed{x_1x_2x_3}$

$$x_1 = (r^2+3s^2)^2 + 2r$$

$$x_2 = (r^2+3s^2)^2 - r - 3s$$

$$x_3 = (r^2+3s^2)^2 - r + 3s$$

$$u = 3(r^2+3s^2)$$

~~Ex:~~ Find 3 ~~solutions~~ possibilities of $r^2 + 3s^2$

Ex: $67228 = 25^2 + 3\cdot149^2 = 49^2 + 3\cdot147^2 = 119^2 + 3\cdot133^2$

Hence the three expns in x, y, z.

Editor's note – ~~xa~~ Gloden's 2-parameter soln remains valid if $r \to -r$ and $s \to -s$, and using this additional solns using the same 67228 are given

Vol. 26. No. 3, p. 197 – 150th Anniversary of Galois – Tribute by I. Malkin – Calls him one of the greatest geniuses the human race has ever produced ?

Vol. 22. No. 1. p. 14 – Recreational Geometry – The triangle by Victor Thebault.

pp. 25–26 – Angle trisectors. – Gives proof of Morley's theorem – Other theorems ✓

(Many interesting articles on recreational mathematics)

$\boxed{q=100}$ $n^2(n+101)^2-(101.338350-5050^2)$

$101.338350-5050^2$

$=101(338350-5050.50)$

$=1010(338350-25250)$

$=10\cdot101^2(335-250)$

$=10\cdot101^2(85)=2\cdot5\cdot17\cdot101^2$

$101)33835(335$
303
353
303
505

$\boxed{v_1=0}$.

$\frac{1}{2}\cdot1\cdot5\cdot2\cdot2\cdot2$

$\boxed{20}$ Cases

$\boxed{q=99}$ $n^2(n+100)^2-(100.328350-4950^2)$

$100(328350-495^2)$

$=2^2\cdot5^4\cdot3\cdot11\cdot101$

$100\cdot25(13134-99^2)$

$50^2\cdot(11\cdot(1194-891)=11\cdot50^2\cdot303$

$99\cdot5$

65670

13134

$99\cdot9$

$x^3 - y^3 = 7.$ $x^3 = y^3 + 7.$

$(x-y)(x^2+xy+y^2) = 7$

$49 - 48 = 1.$

$y^2 = t]$ $x^2 - 3t^2 = z^3.$ ①

$1 + \frac{1}{1+}\frac{1}{3}$ $\frac{7}{4}$

$1 + \frac{3}{2_1}$

$\sqrt{3}$

n	1	2	3	4	5
b	0	1	1	1	1
r	1	2	1	2	1
a	1	1	2	1	2

x 3 x

$1 + \frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{2}$

$\frac{2}{1}$

$(-1)^2 r_3 = 1.$

$(4 - 3 = 1)$

$2^2 - 3 \cdot 1^4 = 1^3$

$7^2 - 3 \cdot 2^4 = 1.$

$(q = 33)$ $n^2(n+34)^2 - (34 \cdot 12529 - 561^2)$

$2 \cdot 17 \cdot 11 \cdot 1139 - 11^2 \cdot 51^2$

$2 \cdot 17^2 \cdot 11 \cdot 67 - 11^2 \cdot 3^2 \cdot 17^2 = 11 \cdot 17^2 (134 - 99) = 11 \cdot 17^2 \cdot 35$

$= 5 \cdot 7 \cdot 11 \cdot 17^2$ $\nu_1 = \frac{1}{2}(1+1)(1+1)(1+1)(2+1) = \underline{\underline{12}}$

$(5)(7 \cdot 11 \cdot 17^2)$	$(11 \cdot 5)(7 \cdot 17^2)$	11129 x	1039 x	
$(5 \cdot 7)(11 \cdot 17^2)$	$(11 \cdot 5 \cdot 17)(7 \cdot 17)$	1607 x	527	$527 = 17 \cdot 31$ x
$(5 \cdot 7 \cdot 11)(17^2)$	$(17 \cdot 5)(11 \cdot 7 \cdot 17)$	337 x	1197	$1197 = 3 \cdot 3 \cdot 7 \cdot 19$ x
$(5 \cdot 7 \cdot 11 \cdot 17)(17)$	$(17 \cdot 11)(5 \cdot 7 \cdot 17)$	3281 x	391	$391 = 17 \cdot 23$ x
$(7)(5 \cdot 11 \cdot 17^2)$	~~$(11 \cdot 17^2)(5$~~ $(11)(5 \cdot 7 \cdot 17^2)$	7951 x	5063 x	
$(7 \cdot 11)(5 \cdot 17^2)$	$(1)(5 \cdot 7 \cdot 11 \cdot 17^2)$	761 x	55633 x	no cases

$(q = 34)$ $n^2(n+35)^2 - (35 \cdot 13685 - 595^2)$

$n^2(3+35)^2 - 2^2 \cdot 5^2 \cdot 7 \cdot 179$ $\nu_1 = \frac{1}{2}(2-1)(2+1)(1+1)(1+1) = \underline{6}$

x	$(2)(2 \cdot 5^2 \cdot 7 \cdot 179)$	$(2 \cdot 7)(5^2 \cdot 179)2)$	x
x	$(2 \cdot 5)(2 \cdot 5 \cdot 7 \cdot 179)$	$(2 \cdot 7 \cdot 5)(2 \cdot 5 \cdot 179)$	x
x	$(2 \cdot 5^2)(2 \cdot 7 \cdot 179)$	$(2 \cdot 179)(2 \cdot 5^2 \cdot 7)$	x

no cases found

(74) $\boxed{\alpha = 35}$ $n^2(n+36)^2 - (36 \cdot 14910 - 630^2)$

" $- 2^2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 37 \rightarrow$ 16 cases // – not tried

$\boxed{\alpha = 36}$ $n^2(n+37)^2 - (37 \cdot 16206 - 666^2)$ 47

" $- 2 \cdot 3 \cdot 19 \cdot 37^2$ ie $\nu_1 = 0$. no cases

$\boxed{\alpha = 37}$ $n^2(n+38)^2 - (38 \cdot 17575 - 703^2)$

" $- (2 \cdot 19 \cdot 19 \cdot 925 - 19^2 \cdot 37^2)$

" $- (2 \cdot 19^2 \cdot 37 \cdot 25 - 19^2 \cdot 37^2)$

" $- (13 \cdot 19^2 \cdot 37)$. $\nu_1 = \frac{1}{2}(1+1)(2+1)(1+1) = \underline{\underline{6}}$

$(13)(19^2 \cdot 37)$	$(13 \cdot 19 \cdot 37)(19)$	3170 x	4579 x	495 = 5.9.11 x
$(13 \cdot 19)(19 \cdot 37)$	$(13 \cdot 37)(19^2)$	495	421 x	
$(13 \cdot 19^2)(37)$	$(1)(13 \cdot 19^2 \cdot 37)$	1130 x	20321 x	no cases

$\boxed{\alpha = 38}$ $n^2(n+39)^2 - (39 \cdot 19019 - 741^2)$

" $- 2^2 \cdot 3 \cdot 5 \cdot 13^2 \cdot 19$; $\nu_1 = 12$

$(2)(2 \cdot 3 \cdot 5 \cdot 13^2 \cdot 19)$	$(2 \cdot 5)(2 \cdot 3 \cdot 13^2 \cdot 19)$
$(2 \cdot 3)(2 \cdot 5 \cdot 13^2 \cdot 19)$	$(2 \cdot 5 \cdot 13)(2 \cdot 3 \cdot 13 \cdot 19)$
$(2 \cdot 3 \cdot 5)(2 \cdot 13^2 \cdot 19)$	$(2 \cdot 5 \cdot 13 \cdot 19)(2 \cdot 3 \cdot 13)$
$(2 \cdot 3 \cdot 5 \cdot 13)(2 \cdot 13 \cdot 19)$	$(2 \cdot 5 \cdot 13^2)(2 \cdot 3 \cdot 19)$
$(2 \cdot 3 \cdot 5 \cdot 13^2)(2 \cdot 19)$	$(2 \cdot 5 \cdot 19)(2 \cdot 3 \cdot 13^2)$
$(2 \cdot 3 \cdot 5 \cdot 19)(2 \cdot 13^2)$	$(2 \cdot 13)(2 \cdot 3 \cdot 5 \cdot 13 \cdot 19)$

No cases – nt

systematically tried

$\boxed{\alpha = 39}$ $n^2(n+40)^2 - (40 \cdot 20540 - 780^2)$

" $- 2^2 5^2 \cdot 13 \cdot 41$ $\nu_1 = \underline{6}$

Let us try this

$(2)(2\cdot5^2\cdot13\cdot41)$	~~2·13~~ $(2\cdot5\cdot13)(2\cdot5\cdot41)$	13326 ×	540 ×
$(2\cdot5)(2\cdot5\cdot13\cdot41)$	$(2\cdot5^2\cdot41)(2\cdot5\cdot13)$	2670 ×	1038 ×
$(2\cdot5^2)(2\cdot13\cdot41)$	$(2\cdot5^2\cdot13)(2\cdot41)$	558 ×	366 ×

no cases

$\boxed{\alpha=40}$ $n^2(n+41)^2-(41.22140-820^2)$

" $-2^5\cdot5\cdot11\cdot41^2$ —— 24 cases. – too many

$\boxed{\alpha=41}$ $n^2(n+42)^2-(42.23821-861^2)$

" $-(6\cdot7\cdot7\cdot3403-7^2\cdot123^2)$

$6\cdot7\cdot7\cdot41.83-7^2\cdot41^2\cdot3\cdot3$

$3\cdot7^2\cdot41(166-123)=3\cdot7^2\cdot41\cdot43$ — 72 cases.

$\frac{1}{2}(1+1)(2+1)(1+1)(1+1)$

~~42~~ $123 = 41\cdot3$ 41)3403(83 328 123

43

2)~~301~~ 301

$\boxed{\alpha=42}$ $n^2(n+43)^2-(43\cdot25585-903^2)$

$43\cdot5\cdot5117-9\cdot7^2\cdot43^2$

$5\cdot43^2\cdot7\cdot17-9\cdot7^2\cdot43^2$

$=7\cdot43^2(85-63)=2\cdot7\cdot11\cdot43^2$ $\nu_1=0$ no cases

43)5117(119 43 81 43

387 7

22

$\boxed{\alpha=43}$ $n^2(n+44)^2-(44\cdot27434-946^2)$.

$\boxed{\alpha=50}$ $n^2(n+51)^2-(51\cdot42925-1275^2)$

$51.5.8585-25^2\cdot51^2$

$51\cdot25\cdot1717-25^2\cdot51^2$

$51\cdot25\cdot17.101-25^2\cdot51^2$

$51.17.25(101-75)=2\cdot3\cdot13\cdot17^2\cdot5^2$. $\Sigma\nu_1=0$

no cases

5 255 51

$25\cdot51.25$

(76) $m = 2n$ again $\underline{\alpha = n}$.

$A_{n+1} = \frac{1}{6}$ $_3(n+1)$ A_1

$$\Delta' = n^2(n+n+1)^2 - \left\{(n+1)\cdot A_{n,2} - A_{n_1}^2\right\}$$

$$= n^2(2n+1)^2 - \left\{\tfrac{1}{6}(n+1)^2 n(2n+1) - \tfrac{1}{4}\cdot n^2(n+1)^2\right\}$$

$$= n^2(2n+1)^2 - n(n+1)^2\left\{\tfrac{1}{6}(2n+1) - \tfrac{1}{4}n\right\}$$

$$= n^2(2n+1)^2 - n(n+1)^2\left\{\tfrac{1}{12}(4n+2-3n)\right.$$

$$= n^2(2n+1)^2 - \tfrac{1}{12}n(n+1)^2(n+2).$$

$n=8$, $\Delta' = ~~64~~\,8^2\cdot17^2 - \frac{1}{12}\cdot8\cdot9^2\cdot10$ $\qquad \frac{289\times64}{6}$ $\qquad \frac{135}{14}$

$8\cdot9\cdot9\cdot10$

$8^2\cdot17^2 - 540 = 8^2\cdot17^2 - 4\cdot135$

$= 4(289\cdot16 - 135) = 4(4624 - 135)$

$= 4\cdot4489 = 4\cdot67^2.$

$\Delta = 16\cdot67^2 = (268)^2.$

$12\Delta' = 12n^2(2n+1)^2 - n(n+1)^2(n+2).$

$= 12n^2(4n^2+4n+1) - (n^2+2n)(n^2+2n+1)$

$\Delta' = \frac{\Delta}{4}$ $\qquad = 48n^4 + 48n^3 + 12n^2 - n^4 - 4n^3 - 5n^2 - 2n$

or $3\Delta = 47n^4 + 44n^3 + 7n^2 - 2n.$ (The usual stuff)

Where is the $x^2 - y^2 = g$.

Here $x = n(2n+1)$., $g = \frac{1}{12}n(n+2)(n+1)^2.$

$n^2(n+9)^2$

~~$\frac{1}{2}n(n+1)$, $+\frac{1}{3}(n+1)(n+2)$~~

~~$3n(n+1) + 4(n+1)(n+4) = \frac{7n^2 + 15n + 8}{12}$~~

~~12~~

$n=4$, $x = 36$. $\frac{1}{12}\cdot4\cdot6\cdot25$ $g = 50$ $\qquad \frac{1}{12}\cdot8\cdot10\cdot9\cdot9$

$3\cdot12$

$2\ 27^2 \quad 270\cdot2$

$136. \quad 27^2 = \boxed{136}$

$n=8$, $x = 136$, $g = 540$.

$n(n+9)^2 = 136.$ $\qquad 64$ $\qquad \frac{1}{6}(2048 + 1408 + 280$

$\frac{1}{6}\,3750 = 625$ ✓ $\qquad 128\times11$ $\qquad +141$

$\frac{17}{8}$

$\boxed{\alpha = 190}$

$25n^4 + 22n^2(n+1)^2 - 15n^2 - 2n$ $\qquad 2\cdot381 \quad 2\cdot3\cdot127 \quad 381 \quad 762$

$n^2(n+191)^2 - (191\cdot95\cdot191\cdot127 - 95^2\cdot191^2)$. $\qquad \frac{381}{191}$ $\quad 190 = \alpha$. $\quad 191\cdot4 \quad 190\cdot4$

$4(191+190) \quad 381 \quad \cancel{191+190+}$

$95 \quad 127 \quad n^2(n+191)^2 -$

$\frac{1}{2}\cdot\cancel{190}\cdot191\cdot\cancel{381}$.

$95\cdot191^2(127-95) = 95\cdot191^2\cdot32$. $\qquad 190\cdot16\cdot191 \quad 19\cdot5\cdot32\cdot191^2$

$\frac{1}{2}\cdot190\cdot191$

$2^5\cdot5\cdot19\cdot\cancel{191^2}$ $\qquad \frac{1}{4}(5-1)(\cancel{1+1})(1+1)(2+1)$

$\boxed{24 \text{ cases}}$.

Colossal, but we know the method.

$n(n+191) =$ ______

$n = 8)$

$n^2(n+9)^2 - (9\cdot204 - 36^2) \qquad 9\cdot4\cdot51 - 36^2$

$36(51-36) = 36\cdot15 = 540$ //

$(\alpha+1)\frac{1}{6}\cdot\alpha(\alpha+1)(2\alpha+1) - \frac{1}{4}\alpha^2(\alpha+1)^2$. $\qquad 8\cdot91$.

$\alpha(\alpha+1)^2\cdot\{\frac{1}{6}(2\alpha+1) - \frac{1}{4}\alpha\} = \alpha(\alpha+1)^2(\frac{1}{12}\alpha + \frac{1}{6})$

$\frac{1}{\cancel{2}}\cdot\overset{2}{\cancel{8}}\cdot81\cdot10 \quad 27 \quad 3$

$= \frac{1}{12}\alpha(\alpha+1)^2(\alpha+2)$. $\qquad \underline{270\cdot2}$

$\{\frac{1}{3}(\alpha+1)^2(\alpha+2)\}\{\frac{1}{4}\alpha\} \qquad \frac{1}{3}(\alpha+1)^2(\alpha+2) + \frac{1}{2}\alpha$.

$\frac{1}{6}(\alpha+1)^2(\alpha+2) + \frac{1}{8}\alpha = rs \qquad r - s = \alpha+1$.

$\frac{1}{2}\cdot81\cdot\frac{5}{10} \quad 27$

$135 + 1 = \underline{136}$.

$(r+s)^2 = (r-s)^2 + 4rs$

$\cancel{2(\alpha+2)(\alpha^2+2\alpha+1)}$

$= (\alpha+1)^2 + \frac{2}{3}(\alpha+1)^2(\alpha+2) + \frac{1}{2}\alpha \qquad 2(2\alpha+7)(\alpha^2+2\alpha+1)$

$2(2\alpha^3 + 11\alpha^2 + 16\alpha + 7$

486

$\cancel{394}$ $\quad \cancel{4536}$

$= \frac{1}{6}\{6(\alpha+1)^2 + 4(\alpha+1)^2(\alpha+2) + 3\alpha\}$

$81\cdot40 + 24$

$= \frac{1}{6}\{(\alpha+1)^2(6 + 4\alpha + 8) + 3\alpha\}$

3240 3726

24 24

$= \frac{1}{6}\{(\alpha+1)^2(4\alpha + 14) + 3\alpha\}. = \frac{1}{6}\{4\alpha^3 + 22\alpha^2 + 3\cancel{2}\alpha + 14\}$

$\cancel{3264} \quad 3752$

$\cancel{\frac{1}{6}\{4\alpha^3 + 18\alpha^2 + 30\alpha + 14 + 3\alpha\}} = \cancel{\frac{1}{3}(2\alpha^3 + 11\alpha^2 + 16\alpha + 7)}$

$\cancel{804}$ $\boxed{625}$

544 $\boxed{25}$ $\quad 64\cdot18$

$= \cancel{\frac{1}{6}(2\alpha^3 + 15\alpha^2 + 33\alpha + 14)} = 17$. $\qquad 1$

$81\cdot\cancel{57} + 24 \quad 4536 \quad 4560 \quad 24 \quad 760$

$\frac{1}{6}(1024 + 1152 + 2\cancel{6}4 + 14) = \frac{1}{2}(\quad \frac{1}{6}(1024 + 704 + 12\cancel{8} + 7)$

$\cancel{\frac{1}{3}\ 1838\ 3 > 2}$

(78)

$$(r+\delta)^2 = \frac{1}{6}\left(4\alpha^3 + 22\alpha^2 + 35\alpha + 14\right)$$

$$r\cancel{+}\delta \Rightarrow r-\delta = \alpha+1$$

$$r\delta = \frac{1}{8}(\alpha+1)^2(\alpha+2) + \frac{1}{8}\alpha,$$

cubic

$ax^3 + \quad + d$

$$a = 4,\ b = \frac{22}{3},\ c = \frac{35}{3},\ d = 14$$

$$A = a^2d^2 - 6abcd + 4ac^3 + 4b^3d - 3b^2c^2$$

$3b = 22$

$b = \frac{22}{3}$

~~βl~~ $\alpha = \beta - {}^{22}/_{12} = \beta - {}^{11}/_{6}.$

~~$\frac{1}{6}\left\{4\left(\beta - {}^{11}/_{8}\right)^3 + 22\left(\beta - {}^{11}/_{8}\right)^2 + 35\left(\beta - {}^{11}/_{8}\right) + 14\right\}$~~

~~$\frac{1}{6}\left\{4\left(\beta^3 - 3\beta^2 \cdot {}^{11}/_{8} + 3\beta\cdot\frac{121}{64} - \frac{1331}{512}\right)\right.$~~

~~$\left. + 22\left(\beta^2 - 2\beta\cdot\frac{11}{8} + \frac{121}{64}\right) + 35\left(\beta - {}^{11}/_{8}\right) + 14\right\}$~~

~~$\frac{1}{6}\left\{4\beta^3 - \beta^2\left\{\frac{33}{2} - 22\right.\right.$~~

~~$\frac{1}{6}\left[4\left(\beta - {}^{11}/_{6}\right)^3 + 22\left(\beta - {}^{11}/_{6}\right)^2 + 35\left(\beta - {}^{11}/_{6}\right) + 14\right]$~~

~~$\frac{1}{6}\left[4\beta^3 + \beta^2\left(-\frac{11}{2}\right.\right.$~~

$\frac{22}{12}$

$$\frac{1}{6}\left[4\alpha^3 + \cancel{3}\cdot\frac{22}{3}\cdot{}^{2} + \frac{35}{3}\cancel{3}\alpha + 14\right]$$

$$\left[\alpha = \beta - {}^{11}/_{6}\right] \to \frac{1}{6}\left[4\left(\beta - {}^{11}/_{6}\right)^3 + \cancel{3}\cdot\frac{22}{3}\left(\beta - {}^{11}/_{6}\right)^2\right.$$

$$\frac{1}{6}\left[4\beta^3 - \cancel{4\cdot 3\beta^2\cdot\frac{11}{6}} + 4\cdot 3\beta\cdot\frac{121}{36} - 4\cdot\frac{1331}{216}\right.$$

$4\beta^3 - 4\cdot 3\cdot\frac{11}{6}$

$$\cancel{+22\beta^2} - 22\cdot 2\beta\cdot\frac{11}{6} + 22\cdot\frac{121}{36}$$

$$\left. + 35\beta - 35\cdot\frac{11}{6} + 14\right]$$

$$\frac{1}{6}\left[4\beta^3+\beta\left\{\frac{121}{3}-\frac{242}{3}+35\right\}+\left\{14+\frac{1331}{18}-\frac{1331}{54}\right\}\right.$$

$$\frac{1}{6}\left[4\beta^3-\frac{16\beta}{3}+\frac{1709}{27}\right]$$

$14+\frac{1331}{54}(3-1)$

$14+\frac{1331}{27}$ 9

378
1331
1709

$$6y^2 = 4\alpha^3+22\alpha^2+35\alpha+13.$$

$$81\cdot6y^2 = 81\cdot6\cdot4\alpha^3+81\cdot6\cdot22\alpha^2+81\cdot6\cdot35\alpha+81\cdot6\cdot14.$$

$(\alpha\Rightarrow x)$

$y_1 = 9\cdot6y.,\ x_1 = 3x$

$$y_1^2 = 72x_1^3+9\cdot6\cdot22x_1^2+27\cdot6\cdot35x_1+81\cdot6\cdot14$$

$t = 144y_1,\ s = 72x_1+18\cdot22\,b_1$

12 8
128

$\frac{121}{3}$

$35-\frac{121}{3}$

$\frac{105-121}{3}$

16

$\frac{x_1^3\ 4\cdot81\cdot6}{27}$

$\frac{81\cdot6\cdot22x_1^2}{9}$

$\frac{81\cdot6\cdot35x_1}{3}$

$4\cdot8^3+22\cdot8^2+35\cdot8+14$

$=4\cdot512+22\cdot64+35\cdot8+14$

2048
1408
280
14

$3750 = 6\cdot25^2$

(625) ✓

Mordell – D-Eqns – h.1, p.255

$ey^2 = ax^3+bx^2+cx+d \quad (a\neq0)$

has only a finite number of solns.

We know that $6y^2 = 4x^3+22x^2+35x+14$ —— (M)

has the soln $x = 8,\ y = 25$. But what are the other solns & how to find them

---

[In Bk. 15, we ~~got of~~ considered general case of $m = \lambda n^4$

Later — sol 4th order diophantine eqns for case of $\lambda = 2, 3, 4$.]

Getting (M) dependence on taking particular factorisations of $q = \frac{1}{12}n(n+1)^2(n+2)$. viz $q = \frac{1}{3}.n(n+1)^2(n+2)$ and ~~α~~ $\alpha = \frac{1}{4}n$.

Suppose we take ~~α~~ $= \frac{1}{3}(n+1)(n+2)$, and $= \frac{1}{4}n(n+1)$..

~~with the cond^n~~

so that $x = \frac{1}{6}(n+1)(n+2) + \frac{1}{8}n(n+1)$.

and if $x = \gamma\delta$ such that $\gamma - \delta = n+1$.

$$(\gamma+\delta)^2 = (n+1)^2 + \frac{2}{3}(n+1)(n+2) + \frac{1}{4}n(n+1).$$

say $$12y^2 = 12(n+1)^2 + 8(n+1)(n+2) + 3n(n+1)$$

$$= (n+1)\{12(n+1) + 8(n+2) + 3n\}$$

$$= (n+1)(23n + 28).$$

12
16
8
226

$$= 23n^2 + 51n + 28$$

~~[illegible] 23 28~~ $= 23\left(n^2 + \frac{51}{23}n + \frac{28}{23}\right)$

$$= 23\left\{\left(n + \frac{51}{46}\right)^2 - \frac{51^2}{46^2} + \frac{28}{23}\right.$$ 46·46.

$$= 23n'^2 - \frac{25}{46^2}.$$

$$\frac{-51^2 + 92\cdot 28}{46^2}$$

$$\boxed{12y^2 - 23n'^2 = -\frac{25}{46^2}}$$ n' integer

51×51
51
255
736
184
2576
2601

25

10

64×23
192
128
1472

1472
408
28
1908

159

$\underline{m = \lambda n}$ (Bk. 15, p. 111)

$\left.\begin{array}{l}\lambda = 2,\\ \mu = 3\end{array}\right\}$ $3\Delta = 111n^4 - 12n^3 - 33n^2 - 6n$

or $\Delta = 37n^4 - 4n^3 - 11n^2 - 2n$.

If $m = 4n$, $\alpha = m - n = 3n$.

$$n^2(n + \overline{3n+1})^2 - \left\{(3n+1)\cdot A^2_{2,3n} - A^2_{1,3n}\right\}$$

$$n^2(4n+1)^2 - \left[(3n+1)\cdot\frac{1}{\cancel{6}\,2}\cancel{3}n(3n+1)(6n+1) - \frac{1}{4}(3n)^2(3n+1)^2\right]$$

$$\Delta' = n^2(4n+1)^2 - n(3n+1)^2\left\{\frac{6n+1}{2} - \frac{9n}{4}\right\}$$ $12n + 2 - 9n$

$$= n^2(4n+1)^2 - \frac{n}{4}(3n+1)^2(3n+2)$$

$$g = \tfrac{1}{4}n(3n+1)^2(3n+2)$$

Take factors as $\frac{1}{2}(3n+1)^2(3n+2)$ & $\frac{1}{2}n$. $x^2 - y^2 = g$.

$= \frac{1}{4}(3n+1)^2(3n+2) + \frac{1}{4}n = r\delta, pq$

such that $r - \delta = 3n+1$.

$$(r+\delta)^2 = (3n+1)^2 + (3n+1)^2(3n+2) + n$$

$= (3n+1)^2(3n+3) + n$. —— a cube

Taking factors: $\frac{1}{2}n(3n+1)$, $\frac{1}{2}(3n+1)(3n+2)$

$r\delta = \frac{1}{4}n(3n+1) + \frac{1}{4}(3n+1)(3n+2)$ $r - \delta = 3n+1$

$$(r+\delta)^2 = (3n+1)^2 + n(3n+1) + (3n+1)(3n+2)$$

$$= (3n+1)\{3n+1 + n + 3n+2\}$$

$= (3n+1)(\underline{7n+3})$ —— quadratic (1)

Taking factors $\frac{1}{2}(3n+1)^2$, $\frac{1}{2}n(3n+2)$

$\boxed{\begin{array}{c}3n^2+6n+1\\ 3n^2+2n\end{array}}$

$$r\delta = \tfrac{1}{4}(3n+1)^2 + \tfrac{1}{4}n(3n+2)$$

$$(r+\delta)^2 = (3n+1)^2 + (3n+1)^2 + n(3n+2)$$

$7\cdot 7,^2$ $= 2(3n+1)^2 + n(3n+2) = 21n^2 + 14n + 2$

1253
7×179

(80)

$\frac{1}{4}$ $\frac{7}{21} \cdot \frac{}{3}$

$-\frac{4}{9}$

R. Ramachandra,
1253, T. Nyappa Block
Bangalore – 21.

$21n^2 + 14n + 2.$

$21\left(n^2 + \frac{2}{3}n\right) + 2$

$21\left(n + \frac{1}{3}\right)^2 - \frac{7}{3} + 2$

$= 21\left(n + \frac{1}{3}\right)^2 - \frac{1}{3}$

$y^2 = \frac{7}{3\cdot 7}(3n+1)^2 - \frac{1}{3}$

$7(3n+1)$

$7(3n^2+2n$

$3y^2 = 7(3n+1)^2 - 1.$

$7l^2 - 3y^2 = 1$

$r + s = 8.333$

$r - s = 218$

$\sqrt{\frac{3}{7}}$

$\frac{\sqrt{21}}{7}$

$\frac{\sqrt{21}}{3}$

179
179 154
36
144 143
179 179
35 81
98
35
144
35

~~n~~	~~1~~	~~2~~
~~b~~	~~0~~	~~0~~
~~r~~	~~7~~	
~~a~~	~~0~~	

$\sqrt{179}$

4

n	1	2	3	4	5	6	7	8	9	10
b	0	13	7	6	5	9	12	13	13	12
r	1	10	13	11	14	7	5	2	5	7
a	13	2	1	1	1	3	5	13	5	3

$3y^2 - 7l^2 = -1$

n	11	12	13
b	13	12	13
r	5	7	5
a	5	13	5

(Ramachandra's
closed continued fraction)

$\sqrt{\frac{7}{3}}$ $\frac{\sqrt{21}}{3}.$

	1	2	3	4					
b	0	3	1	4	4	1	3	3	1
r	3	7	5	1	5	4	3	4	5
a	0	1	1	8	1	1	2	1	1
		*						*	

$1 + \frac{1}{1+}\frac{1}{1+}\frac{1}{8+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2}$

$1 + \frac{1}{1+}\frac{1}{1+}\frac{1}{8+}\frac{1}{1+2/3)}$

$1\frac{42}{58}$ $\frac{1}{8\,3/5}$ $1+\frac{5}{8}$

$7l^2 - 3y^2 = 1$

$1\frac{48}{91}$ $\frac{139}{91}$

1	2	3	4	5	6	7	8	9	10	11
0	0	3	1	4	4	1	3	3	1	4
7	3	4	5	1	5	4	3	4	5	1
0	1	1	1	8	1	1	2	1	1	8
		*					*			

$(-1)^4 r_5 = 1.$

$0, \frac{1}{1+}\frac{1}{1+}\frac{1}{1}$

$0, \frac{1}{1+}\frac{1}{2}$ $2/3$

$\sqrt{\frac{3}{7}}$

$\frac{\sqrt{21}}{7}$

$28 - 27 = 1$

284
71
213

$3n+1 = 2.\ n = 1/7x$

$n = 71$

$m = 284$

$3n+1 = 218$

$3n = 217$

$1+\frac{1}{1+}\frac{1}{1+}\frac{1}{8+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{1}$

$1+\frac{1}{1+}\frac{12}{115}$

$1+\frac{115}{137}$ $\frac{1}{8}+\frac{7}{12}$

$\frac{252}{137}$ $\frac{103}{115}$

$\frac{218}{115}$ $1\frac{115}{218}$

333

$\frac{87}{12}$

$\frac{103}{12}$

~~1~~ $0+\frac{1}{1+}\frac{1}{1+}\frac{1}{1+}\frac{1}{8+}\frac{1}{1+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{1}$

$\boxed{\frac{218}{333}}$

$7\cdot 218^2 - 3\cdot 333^2$

218 × 218
1744
218
436
47524

332668
332667

~~$243k^2 + 2k$~~

$214k^2 + 2k(35470 - 2556 - 284\cdot 71)$

$(\alpha+1)k^2 + 2k(A_1 - n^2) + (A_2 - 2m + n^2)$

$214k^2 + 2k(213\cdot\overset{107}{\cancel{137}} - 71^2)$

$+\left(\frac{1}{6}\cdot 213\cdot 214\cdot 427 - 569\cdot 71^2\right)$

333 × 333
999
999
999
110889

142
$\frac{1}{2}\cdot\cancel{284}\cdot 285$
570
1140568
285
35470

146

$\alpha = 213$

$A_1 = \frac{1}{2}\cdot 213\cdot 214$

$= \frac{1}{6}$ ~~273~~ $\cdot 137\cdot 214$

1911
514
273
37401

$\frac{37401}{6}$

71|3 ~~250~~ 569
3983
40399

$A_2 = \frac{1}{6}\cdot 273\cdot 274\cdot 547$

$= \frac{1}{3}\cdot 273\cdot 137\cdot 547$

$= 12467 \times 547.$

$214k^2 + 2k\cdot 71(321-71) + 71\left(\frac{1}{6}\cdot 3\cdot 214\cdot 427 - 569\cdot 71\right) = 0.$

$214k^2 + 500\cdot 71k + 5290\,\cancel{71} = 0$

$\frac{-500\cdot 71 \pm \sqrt{500^2\cdot 71^2 - 4\cdot 71\cdot 5290\cdot 214}}{428}$

288

107 × 427
749
214
428
45689
40399
5290

107

21·4·214

213

$\Delta' = 71^2(289)^2 - \frac{71}{4}(214)^2(215)$

$= 71^2(289)^2 - 71\cdot(107)^2(215)$

(82)

8 8
71

$71\{71.289^2 - (107)^2.(215)$

289 x 289

2601
2312
578

83521 x 71

83521
584647

5929991
2461535

3460456

4,865119

107 x 107

749
000
107

11449 x 215

57245
11449
22898

2461535

4

2

$\boxed{3n+1 = 218}$ does not give $n = 71$ as I put down

) alternative form

$$(r+\delta)^2 = (3n+1)(7n+3)$$
$$= 21n^2 + 16n + 3$$
$$= 21\left(n^2 + \frac{16}{21}n\right) + 3$$
$$= 21\left(n + \frac{8}{21}\right)^2 + 3 - \frac{64}{21^2}$$

L

441
1323
64
9

Rather complicated

for $\lambda = 2n$, ~~the~~ the factors of q are [illegible]

$$\tfrac{1}{4}(n+1)^2 \text{ and } \tfrac{1}{3}n(n+2)$$

$$r\delta = \frac{1}{8}(n+1)^2 + \frac{1}{6}n(n+2).$$

$$r - \delta = n+1$$

$$y^2 = (\gamma+\delta)^2 = (n+1)^2 + \frac{1}{2}(n+1)^2 + \frac{2}{3}n(n+2)$$

$$6y^2 = 6(n+1)^2 + 3(n+1)^2 + 4n(n+2)$$

$$= 9(n^2+2n+1) + 4n(n+2)$$

$$= 13n^2 + 26n + 9.$$

$$= 13(n+1)^2 - 4.$$

$$13(n+1)^2 - 6y^2 = 4.$$

$$13c^2 - 6y^2 = 4.$$

$\sqrt{6/13}$

$\frac{\sqrt{78}}{13}.$

n	1	2	3	4	5	6	7
b	0	0	6	8	8	6	6
r	13	6	7	2	7	6	7
a	0	1	2	8	2	2	2
			*			*	⊚

~~n84~~ [illegible] the r's

q a 4

$$0 + \frac{1}{1+}\frac{1}{2+}\frac{1}{8+}\frac{1}{2+}\frac{1}{2}$$

78
36
42

78
64
614

78
84
44

78
36
42

11

144·13

43×43
129
172
1849
2

~~13·1296 − 6·1849~~

8 8

16848 − 11094

5754

$13 \cdot 89^2 - 6 \cdot 131^2$

7921 ×13

102973
102986

47.

$13 \cdot 36^2 - 6 \cdot 43^2$

~~18772 − 1109~~4

$\frac{89}{131}$

$8\frac{2}{3}$ $\frac{5}{42}$ $\frac{842}{89}$

$1\frac{42}{84}$

$\frac{1}{84_5}$ $2\frac{5}{42}$

131×131
131
393
131
17161

$$7x^2-3y^2=1$$

$$\sqrt{\frac{3}{7}}=\frac{\sqrt{21}}{7}$$

n	1	2	3	4	5	6	7	8	9
b	0	0	3	1	4	4	1	3	3
r	7	3	4	5	1	5	4	3	4
a	0	1	1	1	8	1	1	2	1
			*					*	

$$(-1)^4 r_5 = 1.$$

$$0+\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1}$$

What I have already done p. 88

$$(\alpha+1)\cdot\tfrac{1}{6}\alpha(\alpha+1)(2\alpha+1)-\tfrac{1}{4}\alpha^2(\alpha+1)^2$$

$$\tfrac{1}{6}(\alpha+1)^2\alpha(2\alpha+1)-\tfrac{1}{4}\alpha^2(\alpha+1)^2$$

$$=\alpha(\alpha+1)^2\left\{\tfrac{1}{6}(2\alpha+1)-\tfrac{1}{4}\alpha\right\}$$

$$=\alpha(\alpha+1)^2\left\{\tfrac{1}{12}\alpha+\tfrac{1}{6}\right\}$$

$$=\tfrac{1}{12}\alpha(\alpha+1)^2(\alpha+2)$$

$\frac{1}{12}\cdot17\cdot18^2\cdot19$

$r-\delta=13$

$r\delta=175$

$(r+\delta)^2=324+700$

$=1024$

$r+\delta=32$

$r-\delta=18$

$r=25$

$\delta=7$

$\frac{1}{\cancel{12}_{2}}\cdot\overset{3}{\cancel{18}}\cdot\overset{9}{\cancel{18}}$ 18

$$\tfrac{1}{12}(\alpha+1)^2\cdot\alpha(\alpha+2)$$

$$g=\tfrac{1}{12}\alpha(\alpha+1)^2(\alpha+2) \text{ for every } \alpha$$

$\frac{18\times18}{12}$

$9\times9\times$

$$\Delta'=n^2(n+\overline{\alpha+1})^2-\tfrac{1}{12}\alpha(\alpha+1)^2(\alpha+2)$$

$$x^2-y^2=g_1(g_1\cdot g_2)$$

$x+y=g_1$

$x-y=g_2$

$$(r+\delta)^2=(\alpha+1)^2+2(g_1+g_2)$$

$\cancel{2}x=\frac{1}{2}(g_1+g_2)=r\delta$

$r-\delta=\alpha+1$

$$(g_1-g_2)^2=(g_1+g_2)^2\cancel{+}-4g_1g_2$$

$(g_1 - g_2)^2 = \frac{1}{2}\{(r+\delta)^2 - (\alpha+1)^2\}^2 - 4g$

$\frac{1}{12}\alpha(\alpha+1)^2(\alpha+2)$ $\quad g_1 = \frac{1}{3}(\alpha+1)^2(\alpha+2)$

$g_2 = \frac{1}{2}\alpha.$

$\left\{\frac{1}{3}(\alpha+1)^2(\alpha+2) + \frac{1}{4}\alpha\right\}^2 - \frac{1}{3}\alpha(\alpha+1)^2(\alpha+2). = \square$

$\boxed{F_1\ \alpha = 8}$

---

factors $\frac{1}{12}(\alpha+1)^2$, $\alpha(\alpha+2)$

$\frac{1}{24}(\alpha+1)^2 + \frac{1}{2}\alpha(\alpha+2) = r\delta$

$r - \delta = 1\cancel{7} = \alpha+1.$

$(r+\delta)^2 = (\alpha+1)^2 + \frac{1}{6}(\alpha+1)^2 + 2\alpha(\alpha+\cancel{2})$

$6y^2 = 7(\alpha+1)^2 + 12\alpha(\alpha+\cancel{2})$

$= 19\alpha^2 + \cancel{3}8\alpha + 7.$

$19(\alpha+\cancel{1})^2 - 12. = 6y^2.$

$19l^2 - 6y^2 = 12.$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14
l	0	0	6	7	8	2	9	9	2	8	7	6	6.	7
r	19	6	13	5	10	11	3	11	10	5	13	6	13	5
a	0	1	1	3	1	1	6	1	1	3	1	2	!	3

x $\qquad\qquad$ λ

$19\cdot18^2 - 6\cdot32^2$ $\quad 0 + \frac{1}{1+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{7}$ ✓

$19l^2 - 6y^2 = 3$ $\quad 19\cdot9^2 - 6\cdot16^2 = 3$

$(-1)^6 r_7 = 3.$

So only $19l^2 - 6y^2 = 3$ is $(9, 16)$.

✓ ∴ of $19l^2 - 6y^2 = 12$ is $(18, 32)$. **Correct:**

13

27
~~3~~23
350

For

$\alpha = 17$

3 9
18 . 18 . 18
+2 2

$3^3 . 17\cdot19$

$\frac{27}{2} + \frac{17\cdot19}{2}$

18

(195)

323
27
350

175
−14 + 7

32~~4~~
700
1024

√114
19

110

114
49
78
65

8

64
114
81
33

114
81
33

1024

114
49
(2 65)

324

$\frac{18\cdot18}{24} + \frac{1}{2}\cdot17\cdot19.$

$\frac{27}{2} + \frac{17\cdot19}{2} = \frac{27+32}{2} = 50$

= (175)

19 · 289

5491

64 6
−7
6144

~~990~~ 1024 ✓

13
3
3
19
16
10
13

4, 7

324 ×19
6156
15 6144
17
12

18 . 18 3
+2 2

16/9 51 257 1

1539
1136

$n^2(n+18)^2 -$

(86)

$g = \frac{2}{12}(\alpha+1)^2\alpha(\alpha+2)$ (i) $\frac{1}{12}(\alpha+1)^2 \cdot \alpha(\alpha+2) \longrightarrow \alpha = 17$

by Pell's eqn.

(ii) $\frac{1}{12}\alpha(\alpha+1) \times (\alpha+1)(\alpha+2)$

$\frac{1}{24}\alpha(\alpha+1) + \frac{1}{2}(\alpha+1)(\alpha+2) = rs, \quad r-s = \alpha+1.$

$(r+s)^2 = (\alpha+1)^2 + \frac{1}{6}\alpha(\alpha+1) + 2(\alpha+1)(\alpha+2)$

$6y^2 = (\alpha+1)\{12(\alpha+1) + 2\alpha + 12(\alpha+2)\}$

$= (\alpha+1)\{20\alpha + 30)$

$= 10(\alpha+1)(2\alpha+3)$ $\quad 3y^2 = 5(2\alpha^2+5\alpha+3)$

$3y^2 = 10\alpha^2 + 25\alpha + 15$

$= 10\left(\alpha + \frac{5}{2}\right)^2 - \frac{125}{2} + 15$

$= 10\left(\alpha + \frac{5}{2}\right)^2 - \frac{95}{2}$

$= \frac{\cancel{10}\,5}{\cancel{4}\,2}(2\alpha+5)^2 - \frac{95}{2}$

$\frac{\cancel{10}\,5 \cdot 25}{\cancel{4}\,2}$

$\frac{-125 \quad 30}{95}$

$6y^2 = 5(2\alpha+5)^2 - 95$

$5t^2 - 6y^2 = 95$

$\sqrt{6/15}$

$\frac{\sqrt{30}}{5}$

$6y^2 = (\alpha+1)(19\alpha+30)$

$= 19\alpha^2 + 49\alpha + 30.$ ugly quadratic

$\frac{1}{3}\alpha(\alpha+1), \quad \frac{1}{4}(\alpha+1)(\alpha+2)$

$\frac{(r+s)^2}{4} = (\alpha+1)^2 + \frac{2}{3}\alpha(\alpha+1) + \frac{1}{2}(\alpha+1)(\alpha+2)$

$6y^2 = 6(\alpha+1)^2 + 4\alpha(\alpha+1) + 3(\alpha+1)(\alpha+2)$

$6y^2 = 13^2 - 1$

$= 6(\alpha^2+2\alpha+1) + 4(\alpha^2+\alpha) + 3(\alpha^2+3\alpha+2)$

$13(\alpha+1)^2$

$= 13\alpha^2 + 25\alpha + 12 = 13(\alpha+1)^2 - \alpha - 1$

$= 13(\alpha+1)^2 - (\alpha+1)$

$$13\left(\alpha^2 + \frac{25}{13}\alpha + \frac{25^2}{26^2}\right)$$

$$p_6 = 16,\ q_6 = 9$$

$$13\left(\alpha + \frac{25}{26}\right)^2 - \frac{25^2}{26^2} + 12$$

$3\frac{15}{34}$

$$\begin{array}{r} 102 \\ 25 \\ \hline 127 \\ \hline 34 \end{array}$$

$$0 + \frac{1}{1+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{1+}\frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{2+}\frac{1}{1+}\frac{1}{3+}\frac{1}{1+}\frac{1}{1}$$

[illegible]

All possible factors leading to quadratics.

(i) $\frac{1}{3}(\alpha+1)^2,\ \frac{1}{4}\alpha(\alpha+2)$ (ii) $\frac{1}{4}(\alpha+1)^2,\ \frac{1}{3}\alpha(\alpha+2)$

(iii) $\frac{1}{12}\alpha(\alpha+1),\ (\alpha+1)(\alpha+2)$, (iv) $\alpha(\alpha+1),\ \frac{1}{12}(\alpha+1)(\alpha+2)$

(iv) $\frac{1}{3}\alpha(\alpha+1),\ \frac{1}{4}(\alpha+1)(\alpha+2)$, (vi) $\frac{1}{4}\alpha(\alpha+1),\ \frac{1}{3}(\alpha+1)(\alpha+2)$

(vii) $(\alpha+1)^2,\ \frac{1}{12}\alpha(\alpha+2)$

(i) $\rightarrow (r+\theta)^2 = (\alpha+1)^2 + \frac{2}{3}(\alpha+1)^2 + \frac{1}{2}\alpha(\alpha+2)$

$\frac{1}{2}$ $\frac{1}{8}$

$$6(r+\delta)^2 = 6(\alpha+1)^2 + 4(\alpha+1)^2 + 3\alpha(\alpha+2)$$

$$= 10(\alpha+1)^2 + 3\alpha(\alpha+2) = 13\alpha^2 + 26\alpha + 10.$$

(ii) $\rightarrow (r+\theta)^2 = (\alpha+1)^2 + \frac{1}{2}(\alpha+1)^2 + \frac{2}{3}\alpha(\alpha+2)$

$\frac{1}{3}$ $\frac{1}{6}$

$$6(r+\theta)^2 = 9(\alpha+1)^2 + 4\alpha(\alpha+2) = 13\alpha^2 + 26\alpha + 9.$$

(iii) $(r+\theta)^2 = (\alpha+1)^2 + \frac{1}{6}\alpha(\alpha+1) + 2(\alpha+1)(\alpha+2)$

$$6(r+\theta)^2 = 6(\alpha+1)^2 + \alpha(\alpha+1) + 12(\alpha+1)(\alpha+2)$$

$$= (\alpha+1)\{6(\alpha+1) + \alpha + 12(\alpha+2)\} - (\alpha+$$

$$= (\alpha+1)(19\alpha+38) = 19\alpha^2 + 139\alpha + 38.$$

(iv) $(r+\theta)^2 = (\alpha+1)^2 + 2\alpha(\alpha+1) + \frac{1}{6}(\alpha+1)(\alpha+2)$

$$6(r+\theta)^2 = (\alpha+1)\{6(\alpha+1) + 12\alpha + (\alpha+2)\}$$

$$= (\alpha+1)(19\alpha+8) = 19\alpha^2 + 27\alpha + 8.$$

(v) $(r+\theta)^2 = (\alpha+1)^2 + \frac{2}{3}\alpha(\alpha+1) + \frac{1}{2}(\alpha+1)(\alpha+2)$

$$6(r+\theta)^2 = 6(\alpha+1)^2 + 4\alpha(\alpha+1) + 3(\alpha+1)(\alpha+2)$$

$= (\alpha+1)\{6(\alpha+1)+4\alpha+3(\alpha+2) = (\alpha+1)(13\alpha+9)$

$= 13\alpha^2+22\alpha+4$

(vi) $(r+5)^2 = (\alpha+1)^2+\frac{1}{2}\alpha(\alpha+1)+\frac{2}{3}(\alpha+1)(\alpha+2)^2$

$6(r+5)^2 = 6(\alpha+1)^2+3\alpha(\alpha+1)+4(\alpha+1)(\alpha+2)$

$= (\alpha+1)\{6(\alpha+1)+3\alpha+4(\alpha+2)\}$

$= (\alpha+1)(13\alpha+14) = 13\alpha^2+27\alpha+14.$

(vii)

(i) $6y^2 = 13\alpha^2+26\alpha+10 = 13(\alpha+1)^2-3$

$l = \alpha+1]$ $13l^2-6y^2 = 3.$

$\sqrt{\frac{6}{13}} = \frac{\sqrt{78}}{13}$

n	1	2	3	4	5	6	7
b	0	0	6	8	8	6	6
r	13	6	7	2	7	6	7
a	0	1	2	8	2	2	2

r is nowhere 3.

78 42 7 64 14 576 42

$6y^2-13l^2 = -3.$

$\frac{\sqrt{78}}{6}$

n	1	2	3	4	5	6
b	0	6	8	8	6	6
r	6	7	2	7	6	7
a	1	2	8	2	2	2
		x			x	

r is again nowhere 3

11 3.4

(i) does not work.

(ii) $6y^2 = 13\alpha^2+26\alpha+9 = 13(\alpha+1)^2-4$

$13l^2-6y^2 = 4.$

(ii) also does not work since r is nowhere 1

$38+49$

(iii) $6y = 19\alpha^2+49\alpha+30 =$

$\alpha = \beta-1$ $\alpha+1 = 1]$ $= 19(\beta-1)^2+49(\beta-1)+30 = 19\beta^2$ ~~+111~~ β

$6y^2 = {}^{19}\ 19\left(\alpha + \frac{49}{38}\right)^2 - \frac{49^2}{38^2}\cdot 19 + 30$ ~~$+30$~~

$30 - \frac{49^2}{38^2}\cdot 19$

$(30\cdot 38^2 - 49^2\cdot 19)/38^2$

~~$= 19\left(\alpha + \frac{49}{38}\right)^2 + \frac{40919}{38^2}$~~

$30\cdot 1444 - 2401\cdot 19$

$$\begin{array}{r} 43320 \\ 2401 \\ \hline 40919 \end{array} \qquad \begin{array}{r} 43320 \\ 45619 \\ \hline 2299 \end{array}$$

$= \frac{19}{38^2}(38\alpha+49)^2 - \frac{2299}{38^2}$

~~$6\cdot 38^2 y^2 = 19(38\alpha+49)^2 + 40919$~~

~~$6z^2 = 19l^2 + 40919$~~

~~$6z^2 - 19l^2 = 40919$.~~

~~$60\cdot 19$~~ ~~$\begin{array}{r} 2401 \\ 1140 \\ \hline 1561 \end{array}$~~ $\quad$ ~~109~~ 209 $\quad 11^2\cdot 19.$

$z = 38y]$ $\quad 6z^2 = 19l^2 - 2299$ $\quad \frac{19}{2\cdot 38^2}(38\alpha+49)^2 - \frac{11^2\cdot 19}{2\cdot 38}$ $\quad 38\alpha + 49 = 11$ $\quad \frac{76\cdot 6}{456}$ $\quad 3$ $\quad \boxed{\alpha = 1}$

$\to 456y^2 = l^2 - 11^2.$

$19l^2 - 6z^2 = 2299. = 11^2\cdot 19$

$\sqrt{\frac{l}{19}}$ $\quad \frac{\sqrt{114}}{19}$

n	1	2	3	4	5	6	7	8	9	10	11	12	13
l	0	0	6	7	8	2	9	9	2	8	~~10~~ 7	6	6
r	19	6	13	5	10	11	3	11	10	5	~~13~~ 3	6	13
a	0	1	1	3	1	1	6	1	1	3	1	2	1
			×									×	✗

$\begin{array}{r} 114 \\ 36 \\ \hline 78 \end{array} \quad \begin{array}{r} 114 \\ 49 \\ \hline 65 \end{array} \quad \begin{array}{r} 110 \end{array} \quad \begin{array}{r} 114 \\ 64 \\ \hline 50 \end{array} \quad 110 \quad \begin{array}{r} 114 \\ 81 \\ \hline 33 \end{array} \quad 110 \quad 110$

no $\underline{r}$ is a 19 except @ $r_1 = 19$, $(-1)^0 r_1 = 19$.

ie $\alpha = 1$ $\quad n^2(n+2)^2 = 0$ $\quad (n-0)(n+2) = 0$ $\quad n = 0 \cdot m = 1 \cdot$ [illegible]

<u>trivial</u>. $\quad (1^2 = 1^2)$.

(iv) $6(r+5)^2 = 19\alpha^2 + 27\alpha + 8 = 19\left(\alpha + \frac{27}{38}\right)^2 - 19\cdot\frac{27^2}{38^2} + 8$

$6y^2$

$6\cdot 38^2\cdot y^2 = 19(38\alpha+27)^2 - 11^2\cdot 19.$ $\qquad 8\cdot 38^2 = 8\cdot 1444$ $\quad 17$

$= 11552$ $\quad 5$

$8\cdot 38^2 - 19\cdot 27^2$ $\quad$ ~~12~~ $\cdot 19^2 y^2 = (38\alpha+27)^2 - 11^2$ $\qquad 38\alpha + 16$

$19\cdot 27^2 = 19\cdot 729$

$= 38(\alpha+1)\cdot 2(19\alpha+8).$

$$= \begin{array}{r} 13851 \\ 11552 \\ \hline 2299 \end{array}$$

$3\cdot 19y^2 = (\alpha+1)(19\alpha+8)$

$19l^2 - 6z^2 = 11^2\cdot 19$ // same eqn as above /

(v) $6y^2 = 13\alpha^2 + 22\alpha + 9$

$= 13\left(\alpha + \frac{11}{13}\right)^2 - \frac{121}{13} + 9 - 4/13$

$6y^2 = \frac{(13\alpha+11)^2}{13} - \frac{4}{13}$

$78y^2 = c^2 - 4.$

$6 \cdot 13^2 \cdot y^2 = 13(13\alpha+11)^2 - 4 \cdot 13$

$6 \cdot 13 \cdot y^2 = c^2 - 4$

$6z^2 = 13c^2 - 52$

$13c^2 - 6z^2 = 4 \cdot 13$

$c^2 - 78y^2 = 4$

$\sqrt{\frac{6}{13}}$

$\frac{\sqrt{78}}{13}$

n	1	2	3	4	5	6	7
b	0	0	6	8	8	6	6
r	13	6	7	2	7	6	7
a	0	1	2	8	2	2	2

$6y^2 =$

no r is a 13

n	1	2	3	4	5	6	7	8	9
b	0	8	6	6	8	8	6	6	8
r	1	14	3	14	1	14	3	14	1
a	8	1	4	1	16	1	4	1	16
		*			*				

r = 1.

$(7)^{8,9} = 1.$

78
36
42
$\sqrt{78}$
$\frac{36}{42}$

78
64
14

4

$8 + \frac{1}{1} + \frac{1}{4} + \frac{1}{1} = 3\frac{5}{6} = 53/6$ ✓

$53^2 - 78 \cdot 6^2$ $\quad 2809 - 2808 = 1$

78×36
468
234
2808

$\left(\frac{106}{12}\right)$

$13\alpha + 11 = 106$

106
11
95

$8 + \frac{1}{1} + \frac{1}{4} + \frac{1}{1} + \frac{1}{16} + \frac{1}{1} + \frac{1}{4} + \frac{1}{1}$

$\frac{1}{16\,5/7}$

117
7
3
2

$4\frac{117}{124}$ $\quad \frac{1}{1\,7/117}$ $\quad 1\frac{124}{613}$ $\quad \frac{737}{613}$ $\quad 1\frac{124}{613}$ $\quad 8 \quad \frac{613}{737}$ $\quad 8 \quad \frac{5896}{613}$ $\quad 6509$ $\quad \frac{124}{117}$

496
117
613

$13\alpha + 11 = 6509$

$13\alpha = 6498$

129

~~54~~

(91)

$p_4 = 53, \; q_4 = 6.$

$p_8 = 53^2 + 78 \cdot 6^2$

$p_{12} = 5617 \cdot 53 + 78 \cdot 436 \cdot 6$

$q_8 = 2 \cdot 53 \cdot 6 = 636$

2809
2808
5617

5606
431
40
3

~~11234~~
~~11~~
~~11223~~
86

$13\alpha + 11 =$ ~~5606~~ $5617.$

~~$p_{18} = 5617 \cdot 53 + 78 \cdot 6 \cdot 436$~~

A great pity it does not work.

26·26

$\frac{5606}{73} = 43$

(vi) $6y^2 = 13\alpha^2 + 27\alpha + 14$

$$= 13\left(\alpha + \frac{27}{26}\right)^2 - \frac{27^2 \cdot 13}{26^2} + 14$$

$\frac{27^2}{52} + 14$

4 ~~8~~ 2
728

$$= 13\left(\alpha + \frac{27}{26}\right)^2 - \frac{729}{52} + 14$$

11234
11
11223
86

52
52

$$= 13\left(\alpha + \frac{27}{26}\right)^2 - \frac{1}{52}$$

13
27
14
$6y = 54$

$\boxed{y^2 = 9}$

$y^2 = 9.$

$$= \cancel{13}\frac{(26\alpha + 27)^2}{26^2 \; 52} - \frac{1}{52}$$

$r + \delta = 8$
$r - \delta = 2$

$6 \cdot 52 y^2 = \underbrace{(26\alpha + 27)}_{t}^2 - 1$

(11)

$t^2 - 312y^2 = 1.$

$y = 3.$ (2)

$\sqrt{312}$

n	1	2	3	4	5	6
b	0	17	6	6	17	17
r	1	23	12	23	1	23
a	17	1	1	1	34	1
		*			*	

$r - \delta = 216$
$r + \delta = 318$
$(-1)^4 r_5 = 1$
$\delta = 102$

312
289
23

312
36
276

$53^2 - 312 \cdot 9$

$\boxed{n = 51}$

53/3 $2809 - 2808 = 1$ ✓

$\alpha = 215$

$(n + \alpha + 1)$

$17 + \frac{1}{1+}\frac{1}{1+}\frac{1}{1}$

$17\frac{2}{3}$

$26\alpha + 27 = 53$ $\quad \alpha = 1.$

318
216
102
8

5

$n + \alpha + 1$

$n(n+1)$

$p_8 = 53^2 + 312 \cdot 9 = 5617$

5617
27
5590
26

$\frac{430}{2} = \alpha.$

53

$215 = \alpha$

$\boxed{528}$

318
216
534

$q_8 = 2 \cdot 53 \cdot 3 = 954$
318

$\boxed{159}$

(92)

$n = \underset{215}{51}$, $m = \boxed{265^2}$, $\alpha = 215$.

$216k^2 + 2k(A_1 - n^2) + (A_2 - \overline{2m+1}\, n^2)$

$216k^2 + 2k\left\{\cancel{\tfrac{1}{2}}.215.216 - 51^2\right\} + \left\{\cancel{\tfrac{1}{6}}.215.\overset{36}{\cancel{216}}.431 - \boxed{531}\,51^2\right\} = 0$

$216k^2 + k(215.216 - 2.51^2) + (215.36.431 - 531.51^2) = 0$

$24k^2 + k(215.24 - 2.17^2) + (215.4.431 - 531.17^2) = 0$

$\div 9$

2.289

$24k^2 + 4582k$

$A_2 =$

5166
518
4532
430
36

$\cancel{\tfrac{1}{6}}.215.\overset{36}{\cancel{216}}.431$

$\Delta' = 51^2(51+216)^2 - (216.3335940 - 108^2.215^2)$

$= 51^2(267)^2 - 108(6671880 - 108.215^2)$

$= 51^2(267)^2 - 108(6671880 - 108.46225)$

256
2586
1293
15516

15516 × 215
77580
15516
31032
3335940

$\tfrac{1}{\cancel{2}}.215.\overset{108}{\cancel{216}}$

$5.43.2.54$

~~1849 × 2916~~
4

18490
4

46225 × 108
369800
00000
46225
4992300

6671880
4992300
1679580

7921 × 2601
7921
0000
47526
15842
20602521 × 9
185422689
181394640
4028049

215
108 × 215
540
108
216
23220

1679580 × 108
13436640
0000000
1679580
181394640

23220
2601
20619

$\Delta' = 2007^2$

$4\Delta' = 4\cdot 2007^2$

=

2 | 4028049 (2007
4
40 | 002
00
400 | 280
000
4007 | 28049
28049

2007

215·216
1290
215
430 (41238) 46440
5202
41238
4014
36824

41238
4014
36224

$216k^2 + 2k\cdot 20619.$

$\frac{36824}{432} = \frac{9206}{108}$

$\frac{4603}{54}$

−41238 ~~+82476~~

82476
41238
41238

41238
8028
33210
?

24
~~41238~~
~~432) 41238 (9~~
~~3888~~
~~2358~~

41238
4014
~~37224~~

432) 37224 (8
3456
2664

45252 10
432
20

$\frac{33210}{432}$

$\frac{3690}{48}$

$-\frac{1845}{24}$

~~23220~~ 432) 49266 (11
432
606
432
1746

$-\frac{49266}{432} = \frac{5474}{48}$

$\frac{2737}{24}$

100+100 = 99
1912
210

$k = -\frac{1845}{24}, -\frac{2737}{24}$

$= -\frac{615}{8}$

4

(1=0) 2120
615
1505

100+100
= 101.
100 = 101

2528
615
1913
28

100
100
100

$\left(-\frac{615}{8}\right)^2 + \left(-\frac{615}{8}+1\right)^2 + - + \left(-\frac{615}{8}+265\right)^2$

$= \left(-\frac{615}{8}+266\right)^2 + - + \left(-\frac{615}{8}+316\right)^2.$

$-\frac{615}{8}+77$

4 266
50
316
−615
$-\frac{1}{8}+\frac{1608}{1}$

$615^2 + 607^2 + \cdot + 1^2 + 9^2 + - + 1913^2 (1505)^2$

$= ~~1921~~^2 + ~~1929~~^2 + - +$

$= 1506^2 + 1507^2 + - + 1913^2$

100
100
001
101

$2(1^2+9^2+17^2+ + \quad + 615$

Taking (ii) $6(r+\delta)^2 = 6y^2 = 19\alpha^2+38\alpha+18$

614

$$6y^2 = 19(\alpha+1)^2-1.$$

$$19l^2-6y^2=1.$$

In the scheme of C.F's for $\sqrt{6/19} = \frac{\sqrt{114}}{19}$ there is no $r_m = 1$, Hence no solution.

(iii) reduces to $l^2-456y^2=121$ $\quad (l = 38\alpha+49)$

$\sqrt{456}$

$21^2=441$
$22^2=484$

n	1	2	3	4	5	6	7	8
b	0	21	9	16	16	9	21	21
r	1	15	25	8	25	15	1	15
a	21	2	1	4	1	2	42	2

* *

Here $(-1)^6 r_7 = 1$

$456 - 441 = 15$

$456 - 81 = 375 \quad 456 - 196 = 260$

$456 - 256 = 200$

$456 - 441 = 15$

(p_6, q_6) is a sol^n of $l^2-456y^2=1.$

$$21+\frac{1}{2+}\frac{1}{1+}\frac{1}{4+}\frac{1}{1+}\frac{1}{2}$$

$21\frac{17}{48}.$ $\quad 1025^2-456.48^2$

$1050625-1050624=1.$

$$\begin{array}{r} 48 \\ 96 \\ \hline 1008 \\ 17 \\ \hline 1025/48 \end{array}$$

$$\begin{array}{r} 5125 \\ 2050 \\ 0000 \\ 1025 \\ \hline 1050625 \end{array}$$

$\frac{14}{3}$ $\quad 2\frac{14}{17}$ $\quad \frac{17}{14}$ $\quad 48$

$$\begin{array}{r} 2304 \times 456 \\ \hline 13824 \\ 11520 \\ 9216 \\ \hline 1050624 \end{array}$$

$1025 - 49 = 976$ X

181

171

103

$19\alpha = 5613$

$\alpha = 29$

$$\begin{array}{r} 1025 \times 11 \\ \hline 11275 \end{array}$$

$38\alpha = 11275 - 49 = 11226$