

$$\begin{aligned}
 V_I &= \frac{m}{PS} + \frac{i}{PI} + \frac{i_1}{PI_1} + \frac{i_2}{PI_2} + \frac{i_3}{PI_3} + \dots \\
 V_{II} &= \frac{m'}{PS} + \frac{i'_1}{PI_1} + \frac{i'_2}{PI_2} + \frac{i'_3}{PI_3} + \dots \\
 &\quad + \frac{j'_1}{PJ_1} + \frac{j'_2}{PJ_2} + \frac{j'_3}{PJ_3} + \dots \\
 V_{III} &= \frac{m''}{PS} + \frac{j_1}{PJ_1} + \frac{j_2}{PJ_2} + \frac{j_3}{PJ_3} + \dots
 \end{aligned}
 \left. \begin{aligned}
 V_I &= V_{II} \\
 \mu_1 \frac{\partial V_I}{\partial v} &= \mu_2 \frac{\partial V_{II}}{\partial v}
 \end{aligned} \right\} A$$

$$\left. \begin{aligned}
 V_{II} &= V_{III} \\
 \mu_2 \frac{\partial V_{II}}{\partial v} &= \mu_3 \frac{\partial V_{III}}{\partial v}
 \end{aligned} \right\} B.$$

$AI = AS, BI_1 = BS, AJ_1 = AI_1, BI_2 = BJ_1, AJ_2 = AI_2, BI_3 = BJ_2, \dots$

(A)

$$\frac{m+i}{PS} + \frac{i_1}{PI_1} + \frac{i_2}{PI_2} + \frac{i_3}{PI_3} = \frac{m'}{PS} + \frac{i'_1 + j'_1}{PI_1} + \frac{i'_2 + j'_2}{PI_2} + \frac{i'_3 + j'_3}{PI_3} + \dots$$

$$\mu_1 \frac{\partial}{\partial v} \left(\frac{1}{PS} \right) + \mu_1 i_1 \frac{\partial}{\partial v} \left(\frac{1}{PI_1} \right) + \mu_1 i_2 \frac{\partial}{\partial v} \left(\frac{1}{PI_2} \right) + \mu_1 i_3 \frac{\partial}{\partial v} \left(\frac{1}{PI_3} \right) + \dots$$

$$= \mu_2 m' \frac{\partial}{\partial v} \left(\frac{1}{PS} \right) + \mu_2 (i'_1 - j'_1) \frac{\partial}{\partial v} \left(\frac{1}{PI_1} \right) + \mu_2 (i'_2 - j'_2) \frac{\partial}{\partial v} \left(\frac{1}{PI_2} \right) + \mu_2 (i'_3 - j'_3) \frac{\partial}{\partial v} \left(\frac{1}{PI_3} \right)$$

(B)

$$\frac{m'+i'_1}{PS} + \frac{i'_2 + j'_1}{PI_2} + \frac{i'_3 + j'_2}{PI_3} + \dots = \frac{m''}{PS} + \frac{j_1}{PI_2} + \frac{j_2}{PI_3} + \dots$$

$$\mu_2 (m' - i'_1) \frac{\partial}{\partial v} \left(\frac{1}{PS} \right) + \mu_2 (i'_2 - j'_1) \frac{\partial}{\partial v} \left(\frac{1}{PI_2} \right) + \mu_2 (i'_3 - j'_2) \frac{\partial}{\partial v} \left(\frac{1}{PI_3} \right)$$

$$= \mu_3 m'' \frac{\partial}{\partial v} \left(\frac{1}{PS} \right) - \mu_3 j'_1 \frac{\partial}{\partial v} \left(\frac{1}{PI_2} \right) - \mu_3 j'_2 \frac{\partial}{\partial v} \left(\frac{1}{PI_3} \right)$$

$$\left. \begin{array}{l} m+i = m' \\ \mu_1(m-i) = \mu_2 m' \end{array} \right\} \left| \begin{array}{l} i_1 = i_1' + j_1' \\ \mu_1 i_1 = \mu_2 (i_1' - j_1') \end{array} \right| \left| \begin{array}{l} i_2 = i_2' + j_2' \\ \mu_1 i_2 = \mu_2 (i_2' - j_2') \end{array} \right| \left| \begin{array}{l} i_3 = i_3' + j_3' \\ \mu_1 i_3 = \mu_2 (i_3' - j_3') \end{array} \right|$$

$$\left. \begin{array}{l} m_0' + i_1' = m'' \\ \mu_2(m_0' - i_1') = \mu_3 m'' \end{array} \right\} \left| \begin{array}{l} i_2' + j_1' = j_1 \\ \mu_2(i_2' - j_1') = \mu_3 j_1 \end{array} \right| \left| \begin{array}{l} i_3' + j_2' = j_2 \\ \mu_2(i_3' - j_2') = \mu_3 j_2 \end{array} \right|$$

$$\left. \begin{array}{l} m' = \frac{2\mu_1 m}{\mu_1 + \mu_2} \\ i = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} m \end{array} \right\} \left| \begin{array}{l} m'' = \frac{2\mu_2 m'}{\mu_2 + \mu_3} \\ i_1' = \frac{\mu_2 - \mu_3}{\mu_2 + \mu_3} m' \end{array} \right| \left| \begin{array}{l} i_1 = \frac{2\mu_2 i_1'}{\mu_1 + \mu_2} \\ j_1' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu_2} i_1' \end{array} \right| \left| \begin{array}{l} j_1 = \frac{2\mu_2 j_1'}{\mu_2 + \mu_3} \\ i_2' = \frac{\mu_2 - \mu_3}{\mu_2 + \mu_3} j_1' \end{array} \right|$$

$$\left. \begin{array}{l} i_2 = \frac{2\mu_2 i_2'}{\mu_1 + \mu_2} \\ j_2' = \frac{\mu_2 - \mu_1}{\mu_1 + \mu_2} i_2' \end{array} \right\} \left| \begin{array}{l} j_2 = \frac{2\mu_2 j_2'}{\mu_2 + \mu_3} \\ i_3' = \frac{\mu_2 - \mu_3}{\mu_2 + \mu_3} j_2' \end{array} \right|$$

$$p = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}, \quad p' = \frac{\mu_3 - \mu_2}{\mu_3 + \mu_2}$$

$$\frac{(1-p')(1+p)m}{pS} + \frac{(1-p')pp'(1+p)m}{pJ_1} + \frac{(1-p')pp'pp'(1+p)m}{pJ_2}$$

$$= m(1-p')(1+p) \left\{ \frac{1}{pS} + \frac{pp'}{pJ_1} + \frac{(pp')^2}{pJ_2} + \dots \right\}$$

$$SJ_1 = AJ_1 - AS = AI_1 - AI_0 = SI_1 - SI = 2(SB - SA) = 2BA = 2c$$

$$\frac{1}{pS} = \int_0^{\infty} e^{-xt} J_0(yt) dt \approx, \quad \frac{1}{pJ_1} = \int_0^{\infty} e^{-(x+2c)t} J_0(yt) dt,$$

$$\frac{1}{pJ_2} = \int_0^{\infty} \dots$$

∴ in the given example, $\mu_1 = \mu_3 = \mu$, $\mu_2 = 1$; $p = p'$.

$$\therefore \Omega = m(1-p^2) \int_0^{\infty} \frac{e^{-xt} J_0(yt)}{1-p^2 e^{-2ct}} dt$$

4/1

7/3 - Tuesday - 7:30 PM - Ramesh K. Kalayathil
6/9/78 - Wednesday - 5:30 PM - Vishwanath

Then can be made by this my system

Proof

- (1) Fig. 262 on p. 165 of Andrews - Same as my Fig. 108 (n) ~~alternately 0 & 1 with 4's~~
- (2) ~~Fig. 261, p. 165 of Andrews (neither a nor n)~~ Fig. bottom Fig. p. 28, Bl. 21. ~~(2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100)~~
- (3) Fig. 95 ~~Fig. 95~~, p. 43, Andrews (annals) ~~replace (1, 7, 6, 4) by (1, 3, 2, 1)~~
- (4) ~~Lower Fig. p. 23, Bl. 21 (Carmichael's form of Narayana's)~~ ~~(40, 42, 44, 46)~~ ~~with some form~~ ~~this is not Carmichael's form~~
- (5) Lower Fig. p. 27, Bl. 21. (n) ~~Fig. 95~~ ~~with some form~~ ~~to~~
- (6) Finally Narayana's Q_2, Q_3 even & Q_1, Q_3 odd (neither a nor n)

Carmichael's form neither a nor n.

$$\frac{1}{\pi} = \frac{1}{12} \left(\frac{1}{2}\right)^2 + \frac{2}{3^2} \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 + \frac{3}{5^2} \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 + \frac{4}{7^2} \left(\frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\right)^2$$

$$\frac{2}{\pi^2} = \frac{1 \cdot 3}{3^3} \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^4 + \frac{2 \cdot 7}{3^3} \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^4 + \frac{3 \cdot 11}{5^3} \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^4 + \dots$$

0	1, 2, 6, 7	1, 37, 59, 7	0, 1, 3, 10, 12
7	8, 9, 15, 16	2, 9, 15, 23	4, 5, 7, 14, 16
22	23, 24, 28, 29	16, 24, 28, 29	6
29	30, 31, 37, 38	30, 31, 37, 38	10

Fig. 119 main form

40 + 43 + 16 + 3 + 32 + 19 + 56 + 59 = 268
 40 + 62 + 56 + 22 + 32 + 6 + 16 + 46 = 280 X
 40 + 15 + 35 + 12 + 33 + 10 + 35 + 13 X

114
146
200
222
122
138
138

A	1, 3, 2, 4, 5, 7, 6, 8	0, 56, 56, 0, 0, 56, 56, 0
M	8, 6, 7, 5, 4, 2, 3, 1	16, 40, 40, 16, 16, 40, 40, 16
M	8, 6, 7, 5, 4, 2, 3, 1	8, 48, 48, 8, 8, 48, 48, 8
A	1, 3, 2, 4, 5, 7, 6, 8	24, 32, 32, 24, 24, 32, 32, 24
A	1, 3, 2, 4, 5, 7, 6, 8	32, 24, 24, 32, 32, 24, 24, 32
A	8, 6, 7, 5, 4, 2, 3, 1	48, 8, 8, 48, 48, 8, 8, 48
A	8, 6, 7, 5, 4, 2, 3, 1	40, 16, 16, 40, 40, 16, 16, 40
A	1, 3, 2, 4, 5, 7, 6, 8	56, 0, 0, 56, 56, 0, 0, 56

1, 59, 58, 4, 5, 63, 62, 8
24, 46, 47, 21, 20, 42, 43, 17
16, 52, 55, 13, 12, 50, 51, 9
25, 35, 34, 28, 29, 39, 38, 32
33, 27, 26, 36, 37, 31, 30, 40
56, 14, 15, 53, 52, 10, 11, 49
48, 22, 23, 45, 44, 18, 19, 41
57, 3, 2, 60, 61, 7, 6, 64

A+a, A+b, A+c, A+d
B+a, B+b, B+c, B+d
C+a, C+b, C+c, C+d
D+a, D+b, D+c, D+d
E+a, E+b, E+c, E+d
F+a, F+b, F+c, F+d
G+a, G+b, G+c, G+d
H+a, H+b, H+c, H+d
I+a, I+b, I+c, I+d
J+a, J+b, J+c, J+d
K+a, K+b, K+c, K+d
L+a, L+b, L+c, L+d
M+a, M+b, M+c, M+d
N+a, N+b, N+c, N+d
O+a, O+b, O+c, O+d
P+a, P+b, P+c, P+d
Q+a, Q+b, Q+c, Q+d
R+a, R+b, R+c, R+d
S+a, S+b, S+c, S+d
T+a, T+b, T+c, T+d
U+a, U+b, U+c, U+d
V+a, V+b, V+c, V+d
W+a, W+b, W+c, W+d
X+a, X+b, X+c, X+d
Y+a, Y+b, Y+c, Y+d
Z+a, Z+b, Z+c, Z+d

17, 10, 27, 60, 5, 38, 55, 48
49

1, 2, 3, 4, 5, 6, 7, 8	0, 56, 56, 0, 0, 56, 56, 0
8, 7, 6, 5, 4, 3, 2, 1	8, 48, 48, 8, 8, 48, 48, 8
8, 7, 6, 5, 4	16, 40, 40, 16, 16, 40, 40, 16
1, 2, 3, 4, 5	24, 32, 32, 24, 24, 32, 32, 24
1, 2, 3, 4, 5	32, 24, 24, 32, 32, 24, 24, 32
8, 7, 6, 5, 4	40, 16, 16, 40, 40, 16, 16, 40
8, 7, 6, 5, 4	48, 8, 8, 48, 48, 8, 8, 48
1, 2, 3, 4, 5	56, 0, 0, 56, 56, 0, 0, 56

1, 58, 59, 4, 5, 62, 63, 8
16, 55, 54, 13, 12, 51, 50, 9
49, 44, 42, 24, 17, 47, 45, 22
30, 37, 39, 25, 32, 34, 36, 27
35, 28, 26, 40, 33, 31, 29, 38
43, 20, 18, 48, 41, 23, 21, 46
56, 15, 14, 53, 52, 11, 10, 49
57, 2, 3, 60, 61, 7, 6, 64

2 + 28 + 63 + 37
 64 + 38 + 1 + 27 = 260
 31 + 60 + 34 + 5
 36 + 7 + 29 + 58 = 260.

1, 2, 3, 4, 5, 6, 7, 8
2, 1, 4, 3, 6, 5, 8, 7

49 + 43 + 45 + 55
 51 + 41 + 47 + 53

The question whether q has any meaning beyond the limit n of our observations is of no consequence to our practical problems.

Coming back to "Dust thou art, and dust returneth"; if mathematics is a tool to investigate physical reality, "computer mathematics" is all that we need ^{for describing physics}. All quantities used in this manipulation always exist - including the integer zero. Every number has a binary representation, and finite integers and arithal fractions having a finite number of arithal places are all that we need. Any mathematical equation can be solved by a computer, using a finite q number of steps by a computer. ~~Should we~~ shall we shall call this


FINITE MATHEMATICS

Infinite Numbers and arithal fractions with inf. places

10. Finite mathematics, I believe, is logical and self-consistent, and everything we have in mathematical analysis that is relevant to solving practical problems is contained in it. So, we will leave it at that, and go to the question of how the concepts of "infinity" and the "infinitesimal" can be sought to be defined in a ^(accepted) sufficiently precise manner for us to put ^{the} the accepted domain of pure mathematics (particularly in functional analysis) in a form readily comprehensible from the above viewpoint.

arguing

1, 2, 3, 4, 5, 6, 7, 8	0, 56, 56, 0, 0, 56, 56, 0	1, 58, 59, 4, 5, 62, 63, 8
8, 7, 6, 5, 4, 3, 2, 1	8, 48, 48, 8, 8, 48, 48, 8	16, 55, 52, 13, 12, 51, 50, 9
8, 7, 6, 5, 4, 3, 2, 1	16, 40, 40, 16, 16, 40, 40, 16	24, 47, 46, 21, 20, 43, 42, 17
1, 2, 3, 4, 5, 6, 7, 8	24, 32, 32, 24, 24, 32, 32, 24	25, 34, 35, 28, 29, 38, 39, 32
1, 2, 3, 4, 5, 6, 7, 8	32, 24, 24, 32, 32, 24, 24, 32	33, 26, 27, 36, 37, 30, 31, 40
8, 7, 6, 5, 4, 3, 2, 1	40, 16, 16, 40, 40, 16, 16, 40	48, 23, 22, 45, 44, 19, 18, 41
8, 7, 6, 5, 4, 3, 2, 1	48, 8, 8, 48, 48, 8, 8, 48	56, 15, 14, 53, 52, 11, 10, 49
1, 2, 3, 4, 5, 6, 7, 8	56, 0, 0, 56, 56, 0, 0, 56	57, 2, 3, 60, 61, 6, 7, 64

Fig. 716 p. 391 

1, 58, 3, 60, 61, 6, 63, 8

57, 2, 59, 4, 5, 62, 7, 64

1, 2, 3, 4, 5, 6, 7, 8
8, 7, 6, 5, 4, 3, 2, 1
1, 2, 3, 4, 5, 6, 7, 8
8, 7, 6, 5, 4, 3, 2, 1
1, 2, 3, 4, 5, 6, 7, 8
8, 7, 6, 5, 4, 3, 2, 1
1, 2, 3, 4, 5, 6, 7, 8
8, 7, 6, 5, 4, 3, 2, 1

1, 58, 3, 60, 5, 62, 7, 64
 16, 55, 14, 53, 12, 51, 10, 49
 17, 42, 19, 44, 21, 46, 23, 48
 32, 39, 30, 37, 28, 35, 26, 33
 33, 26

for A & n

(1)

1, 2, 3, 4, 4, 3, 2, 1
8, 7, 6, 5, 5, 6, 7, 8
1, 2, 3, 4, 4, 3, 2, 1
8, 7, 6, 5, 5, 6, 7, 8
1, 2, 3, 4, 4, 3, 2, 1
8, 7, 6, 5, 5, 6, 7, 8
1, 2, 3, 4, 4, 3, 2, 1
8, 7, 6, 5, 5, 6, 7, 8

1, 58, 3, 60, 4, 59, 2, 57
16, 55, 14, 53, 13, 54, 15, 56
17, 42, 19, 44, 20, 43, 18, 41
32, 39, 30, 37, 29, 38, 31, 40
25, 34, 27, 36, 28, 35, 26, 33
24, 47, 22, 45, 21, 46, 23, 48
9, 50, 11, 52, 12, 51, 10, 49
8, 63, 6, 61, 5, 62, 7, 64

26, 28, 31, 21, 26
 32+19+13+...
 Difference from what is in 716 p. 391

~~1, 3, 6, 8, 8, 6, 3, 1~~

1, 3, 6, 8, 8, 6, 3, 1
8, 6, 3, 1, 1, 3, 6, 8
1, 3, 6, 8, 8, 6, 3, 1
8, 6, 3, 1, 1, 3, 6, 8
1, 3, 6, 8, 8, 6, 3, 1
8, 6, 3, 1, 1, 3, 6, 8
1, 3, 6, 8, 8, 6, 3, 1
8, 6, 3, 1, 1, 3, 6, 8

1, 59, 6, 64, 8, 62, 3, 57
 24, 46, 19, 41, 17, 43, 22, 48
 41, 19

Must state not have complementary pairs

1, 6, 3, 8, 8, 6, 3, 8

15051
 14661
 161051
 177656
 1771361



A B A B A B A B
 A B A B A B A B

1, 7, 6, 4, 5, 3, 2, 8
x 8 2 3 5 4 6 7 1
2 8 2 3 5 4 6 7 1
x 1 7 6 4 5 3 2 8
x 1 7 6 4 5 3 2 8
e 8 2 3 5 4 6 7 1
e 8 2 3 5 4 6 7 1
1 7 6 4 5 3 2 8

1, 6, 3, 8, 4, 7, 2, 5
 4, 7, 2, 5, 1, 6, 3, 8

1	6	3	8	4	7	2	5
---	---	---	---	---	---	---	---

4
 3
 2
 1

be listed, and that we cannot prove that q is not one of the a 's. Hence, every q must have an ordinal representation.

[The proof can readily be extended to the decimal system, instead of the binary system; but this is trivial. The essence of the argument is there in the above].

Now, we shall try to prove Theorem 2; namely that the set of ordinals defined above ^{is equivalent} ~~is~~ continuous to the physical notion of a continuous variation of a quantity q .

Here, we come back to what we stated earlier ~~th~~ namely that the ^{notion} ~~idea~~ of ~~a~~ continuity can only be given in an intuitive way to begin with, which can then be made more and more precise, with rules as to what makes q a continuous variable, and when it is not so, by the technique of the block diagram in page 5.

So, the a priori definition of continuity which we shall use is by having ^{the object in question and taking} a distant vision of the object concerned, and then focussing the instrument used better and better, ending up with an electron microscope view. This means that, initially, defining q to better than the first ^{ordinal} ~~ordinal~~ place is no good. Thus

$$0.0 < q < 0.1 \tag{12a}$$

Next, go to higher and higher ordinal places & write

$$0.01 < q < 0.11 \tag{12b}$$

$$0.011\dots 0 < q < 0.011\dots 1 \tag{12c}$$

1	6	3	8	4	7	2	5
4	7	2	5	1	6	3	8
5	2	7	4	8	3	6	1
8	3	6	1	5	2	7	4
5	2	7	4	8	3	6	1
8	3	6	1	5	2	7	4
1	6	3	8	4	7	2	5
4	7	2	5	1	6	3	8

(P)

$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$
 $a_5, a_6, a_7, a_8, a_1, a_2, a_3, a_4$
 $a_5', a_6', a_7', a_8', a_1', a_2', a_3', a_4'$
 $a_1', a_2', a_3', a_4', a_5', a_6', a_7', a_8'$

$a_8, a_7, a_6, a_5 = a$

$a_1', a_2', a_3', a_4' = a_1, a_2, a_3, a_4$
 $a_5', a_6', a_7', a_8' = a_5, a_6, a_7, a_8$
 $b_1', c_2', c_3', c_4' = b_1, b_2, b_3, b_4$
 $b_1', b_2', b_3', b_4' = c_1, c_2, c_3, c_4$
 $c_5', c_6', c_7', c_8' = a_5, a_6, a_7, a_8$
 $b_5', b_6', b_7', b_8' = c_5, c_6, c_7, c_8$

$b_1, b_2, b_3, b_4 = a_5, a_6, a_7, a_8$

$b_5, b_6, b_7, b_8 = a_1, a_2, a_3, a_4$

Ex. 267, p. 169 Andrews - n but not of the form on p. 92 of Bk. 20?

Further Q_1 is 64, 57, 4, 5 (a_1, a_2, a_3, a_4)

and further Q_2 is 40, 33, 28, 29 in d complements of (a_1, a_2, a_3, a_4) as required

$64 - 40 = 24$
 $57 - 33 = 24$
 $4 - 28 = -24$
 $5 - 29 = -24$

Smig 3, 6, 63, 58
 27, 30, 39, 34

differences are $-24, -24, 24, 24$

$a_5, a_6, a_7, a_8 = 56, 49, 12, 13$ further in R_2
 $48, 41, 20, 21$ - differences = 8, 8, -8, -8.

$11, 14, 55, 50$ } 53, 52, 9, 16 } 2, 7, 63, 59
 $19, 22, 47, 42$ } 37, 36, 25, 32 } 45, 44, 17, 24 } 26, 31, 38, 34
 $-8, -8, 8, 8$ } 24, 24, -24, -24 } 8, 8, -8, -8 } -24, -24, 24, 24

$10, 15, 54, 51$
 $18, 23, 46, 43$
 $-8, -8, 8, 8$

In the (P) ~~(R)~~ of above Ex. 267, we have ^{in the usual nark scheme (ie p. 92, Bk. 20)} in the usual nark scheme

a_1, a_2, a_3, a_4 instead of a_1', a_2', a_3', a_4' etc. in const. rows of Q_1, Q_4 , etc.

In (R) we have the above differences between rows of (Q_1, Q_4) & (Q_2, Q_3) instead of being complements

Ans for $a \Delta n$ agrees on p. 92, Bk. 20 can be written shortly as

A, E	A, E
B, F	B, F
C, G	C, G
D, H	D, H
H', D'	E', A'
G', C'	F', B'
F', A'	G', C'
E', B'	H', D'
A	B

in Ex. 267, p. 169 which is n , the scheme is

A	A
B	B
A	A
B	B
A	A
B	B

(P)

8, 1, 4, 5
1, 8, 5, 4

$a_1 + d_6' + c_7' + b_8'$
 $+ a_1' + d_6 + c_7 + b_8 = 4 \times 65 = 260$

multiplication and division of these, extended to the ordinal system for this purpose).

9. The third defines an irrational number, and we assert that all irrational numbers from 0 to 1 are given by ~~ordinal~~ non-terminating ordinal ~~dec~~ fractions. (XXV) 7

Proof: Suppose x lies between two non-terminating ordinal fractions a_1 and a_2 , which may, or may not, be repeating. Then, they must agree upto an ordinal place n and disagree beyond. (Before comparing them, convert both a_1 and a_2 into form A, if either is repeating.) Thus, if $a_1 > a_2$, the $(n+1)$ st place will be 1 for a_1 and 0 for a_2 . Thereafter, look for the first opportunity of x there being 0, on a_2 and change it to 1, to get a new ordinal less than a_1 and greater than a_2 . \square

Theorem 1.

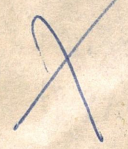
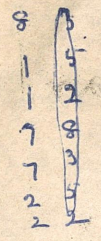
(XXV) First, we suppose that all quantities between 0 and 1 can be represented by an ordinal fraction of types (i), (ii) or (iii). So, we can generalise and say that all numbers between -2^m and $+2^m$ can be represented by a binary representation with m integer displaces and any number of ordinal places). We shall prove this for x between 0 and 1.

"digit"

Thus, we have shown that, if $a_1 < x < a_2$, and a_1 and a_2 are ordinal fractions, then ~~it is~~ ^{we show} ~~impossible~~ ^{not possible} to find, at least, a contradiction that there is another ordinal fraction, say b , between a_1 and a_2 . The conclusion we draw from this is that the a 's cannot

8 8, 1, 4, 5, 8, 1, 4, 5

4, 1, 7, 2, 6, 1, 3



64, 57, 1, 4, 53, 56, 12, 13

1, 2, 3, 4, 1, 2, 3, 4
2, 7, 6, 4, 1, 7, 6, 4
8, 2, 3, 5, 8, 2, 3, 5
4, 6, 7, 1, 4, 6, 7, 1
5, 3, 8, 2, 5, 3, 8, 2
1, 7, 6, 4, 1, 7, 6, 4
8, 2, 3, 5, 8, 2, 3, 5
4, 6, 7, 1, 4, 6, 7, 1
5, 3, 2, 8, 5, 3, 2, 8

1, 4, 8, 5, 1, 2, 4, 5
3, 2, 6, 7, 3, 2, 6, 7
5, 8, 2, 1, 5, 8, 2, 1
7, 6, 2, 3, 7, 6, 2, 3
1, 4, 8, 5, 1, 4, 8, 5
3, 2, 6, 7, 3, 2, 6, 7
5, 8, 2, 1, 5, 8, 2, 1
7, 6, 2, 3, 7, 6, 2, 3

1, 8, 4, 5, 1, 8, 4, 5
3, 6, 7, 2, 3, 6, 7, 2
5, 4, 8, 1, 5, 4, 8, 1
2, 7, 6, 3, 2, 7, 6, 3
1, 8, 4, 5, 1, 8, 4, 5
3, 6, 7, 2, 3, 6, 7, 2
5, 4, 8, 1, 5, 4, 8, 1
2, 7, 6, 3, 2, 7, 6, 3

8, 2, 1, 1
6, 4, 3, 3
7, 7, 2, 2
5, 5, 4, 4
How to get (R₆)



1, 7, 3, 4, 4, 3, 7, 1
8, 2, 6, 5, 5, 6, 2, 8
1, 7, 3, 4, 4, 3, 7, 1
8, 2, 6, 5, 5, 6, 2, 8
1, 7, 3, 4, 4, 3, 7, 1
8, 2, 6, 5, 5, 6, 2, 8
1, 7, 3, 4, 4, 3, 7, 1
8, 2, 6, 5, 5, 6, 2, 8

0, 56, 0, 56, 0, 56, 0, 56
48, 8, 8, 48, 48, 8, 8, 48
16, 40, 40, 16, 40, 40, 16
24, 32, 32, 24, 32, 32, 24
24, 32, 32, 24, 24, 32, 32, 24
16, 40, 40, 16, 16, 40, 40, 16
48, 8, 8, 48, 48, 8, 8, 48
0, 56, 56, 0, 0, 56, 56, 0

1, 63, 59, 4, 1, 63, 3, 60, 4, 59, 7, 57

391
Defy 716, p. 109, we get a 4n

because we start with 1, 7, 6, 4 (minimum) to 18
Suppose we start with 1, 3, 8, 6.

1, 3, 8, 6, 6, 8, 3, 1
8, 6, 1, 3, 3, 1, 8, 6
1, 3, 8, 6, 6, 8, 3, 1
8, 6, 3, 1, 3, 1, 8, 6
1, 3, 6, 8, 6, 8, 3, 1
8, 6, 3, 1, 3, 1, 8, 6
1, 3, 6, 8, 6, 8, 3, 1
8, 6, 3, 1, 3, 1, 8, 6

0, 56, 0, 56, 0, 56, 0, 56
16, 40, 16, 40, 16, 40, 16, 40
56, 0, 56, 0, 56, 0, 56, 0
40, 16, 40, 16, 40, 16, 40, 16
40, 16, 40, 16, 40, 16, 40, 16
56, 0, 56, 0, 56, 0, 56, 0
16, 40, 16, 40, 16, 40, 16, 40
0, 56, 0, 56, 0, 56, 0, 56

1, 59, 8, 62, 6, 64, 3, 57

1, 7, 6, 4, 2 permutations
2, 5, 3, 8, 2
3, 5, 8, 2
1, 4, 6, 7
8, 5, 3, 2

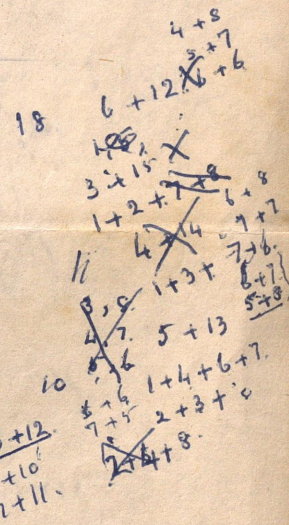
5, 62, 3, 57, 3
appears to be the only two cases.

2, 3, 5, 8, 8, 5, 3, 2
7, 6, 4, 1, 1, 4, 6, 7
Even these do not appear to give a 4n square
inherently different from Def. 716, p. 391

2, 3, 5, 8, 8, 5, 3, 2
7, 6, 4, 1, 1, 4, 6, 7
2, 3, 5, 8, 8, 5, 3, 2
7, 6, 4, 1, 1, 4, 6, 7
2, 3, 5, 8, 8, 5, 3, 2
7, 6, 4, 1, 1, 4, 6, 7
2, 3, 5, 8, 8, 5, 3, 2
7, 6, 4, 1, 1, 4, 6, 7

10, 57, 53, 56, 16, 53, 11, 50
23, 46, 20, 41, 17, 46, 22, 47
34, 27, 37, 32, 40, 29, 35, 26
63, 6, 60, 1, 57, 4, 62, 7
58, 3, 61, 8, 64, 5, 59, 2
39, 30, 36, 25, 33, 28, 38, 31
18, 43, 21, 48, 24, 45, 19, 42
15, 54, 12, 49, 9, 52, 14, 55

not inherently
different from
Def. 716, p. 391
but this appears unique



Brief outline of further discussion

(a) If ^{the} unit can be subdivided into halves again and again, we can measure any quantity q in the binary system [for convenience take $q < 1$], to as much accuracy as we want

$$q = 0.1011101101001011 \dots \quad (15)$$

This is a practically applicable process to determine q — obviously, "whatever it may be," q can be determined.

(b) This way of defining q shows that the "measure" of q is a continuous variable — by "continuous," we mean that there is no value that cannot be measured and defined this way. $\square \quad \neq$

(c) The subdivision of the unit may terminate, or may go on forever — Just as we use the word "decimal" ⁱⁿ the scale of ten, we may use the word "ardhal" ^{fraction} for the scale of two (3-tes = half). Ardhal ^{fraction} of 3 types:

(i) Terminating ~~at~~ after n ardhal places, all 0 beyond

(ii) ^{Having a finite} Repeating sequence of 1's or 0's after n .

(iii) Non-repeating, non-terminating sequence

(Note) do not use the term "infinite sequence" ^{terminating} but non-~~terminating~~

The first is a rational number, being ^{an integer} multiple of $0.000 \dots 1$ ^{the n th} with 1 at ^{the n th} ardhal place, etc.

The second is also rational, and ~~it~~ ^{it} can be obtained by generalizing the equality

$$\begin{aligned} 0.1000 \dots & \dots \dots \dots \quad \} \text{(Form A)} \quad (17) \\ = 0.011111 \dots & \dots \dots \dots \quad \} \text{(Form B)} \end{aligned}$$

[We have to use the rules of arithmetic and the usual techniques of dealing with fractions, including

\odot $\begin{matrix} 1, 2, 3, 4, 5, 6, 7, 8 \\ 2, 3, 4, 5, 6, 7, 8, 1 \\ \sqrt{3}, 4, 5, 6, 7, 8, 1, 2 \\ \times 4, 5, 6, 7, 8, 1, 2, 3 \\ \odot 8, 6, 2, 3, 4, 5, 6, 7 \\ \odot 7, 8, 1, 2, 3, 4, 5, 6 \\ \odot 6, 7, 8, 1, 2, 3, 4, 5 \\ \odot 5, 6, 7, 8, 1, 2, 3, 4 \end{matrix}$

\odot $\begin{matrix} 1, 10, 19, 28, 37, 46, 55, 64 \\ 57, 2, 11, 20, 29, 38, 47, 56 \\ 50, 59, 4, 13, 22, 31, 40, 49 \\ 43, 52, 61, 6, 15, 24, 33, 42 \\ 36, 45, 54, 63, 8, 9, 18, 27 \\ 29, 38, 47, 56, 57 \\ 5, 14, 23, 32, 41, 50, 59, 60 \\ 5, 14, 23, 32, 33, 42, 51, 60 \\ 30, 39, 48, 49, 58, 3, 12, 21 \\ 29, 32, \end{matrix}$

Rep

Better

$8^{th} \text{ col} + 1^{st} \text{ row}$
 $7^{th} \text{ col} + 2^{nd} \text{ row}$
 $6^{th} \text{ col} + 3^{rd} \text{ row}$
 $5^{th} \text{ col} + 4^{th} \text{ row}$
 $4^{th} \text{ col} + 5^{th} \text{ row}$
 $3^{rd} \text{ col} + 6^{th} \text{ row}$
 $2^{nd} \text{ col} + 7^{th} \text{ row}$
 $1^{st} \text{ col} + 8^{th} \text{ row}$

$1, 10, 19, 28, 61, 54, 47, 40$
 $1, 2, 3, 4, 5, 6, 7, 8$
 $5, 6, 7, 8, 1, 2, 3, 4$
 $1, 2, 3$
 $1, 7, 6, 4, 17, 6, 4$
 $8, 2, 3, 5, 8, 2, 3, 5$
 $1, 7, 6, 4, 8, 7, 6, 4$
 $8, 2, 3, 5, 8, 2, 3, 5$
 $8, 2, 3, 5, 8, 2, 3, 5$
 $1, 7, 6, 4, 1, 7, 6, 4$
 $8, 2, 3, 5, 8, 2, 3, 5$
 $1, 7, 6, 4, 1, 7, 6, 4$

$1, 2, 3, 4, 8, 7, 6, 5$
 $8, 7, 6, 5, 1, 2, 3, 4$
 $1, 2, 3, 4, 8, 7, 6, 5$
 $8, 7, 6, 5, 1, 2, 3, 4$
 $4, 2, 2, 1, 5, 6, 7, 8$
 $5, 0, 7, 8, 4, 3, 2, 1$
 $4, 3, 2, 1, 5, 6, 7, 8$
 $5, 6, 7, 8, 4, 3, 2, 1$

1, 63, 6, 60	57, 7, 62, 4
56, 10, 51, 13	16, 50, 11, 53
41, 23, 46, 20	17, 47, 22, 44
32, 34, 27, 37	40, 26, 35, 29
8, 58, 3, 61	64, 2, 59, 5
49, 15, 54, 12	9, 55, 14, 52
48, 18, 21, 26, 42, 19, 45	
25, 39, 30, 36, 33, 31, 38, 28	

$1, 58, 3, 60, 32, 39, 50, 37$
 $16, 55, 11, 53, 17, 42, 19, 44$
 $17, 42, 19,$

a new rank.
 all 16 subgroups 2×2
 add to 130
 hence algebraic
 $45 + 43 + 9 + 59$
 $32 + 46 + 16 + 62 = 292 \times \text{hd } k-n$

$1, 7, 6, 4, 4, 6, 7, 1$
 $8, 2, 3, 5, 5, 3, 2, 8$
 $1, 7, 6, 4, 4, 6, 7, 1$
 $8, 2, 3, 5, 5, 3, 2, 8$
 $8, 2, 3, 5, 5, 3, 2, 8$
 $1, 7, 6, 4, 4, 6, 7, 1$
 $8, 2, 3, 5, 5, 3, 2, 8$
 $1, 7, 6, 4, 4, 6, 7, 1$

$1, 5, 4, 6, 6, 6, 8, 1$
 $1, 3, 6, 8, 1, 3, 6, 8$
 $8, 6, 3, 1, 8, 6, 3, 1$
 $1, 3, 6, 8, 1, 3, 6, 8$
 $8, 6, 3, 1, 8, 6, 3, 1$
 $1, 3, 6, 8, 1, 3, 6, 8$
 $8, 6, 3, 1, 8, 6, 3, 1$
 $1, 3, 6, 8, 1, 3, 6, 8$

new rank.
 X rep
 $1, 2, 3, 4, 8, 7, 6, 5$
 $0, 8, 16, 24, 56, 43, 40, 32$

study 734, p. 397
 Anon
 (P) right form A, A', A, \dots in CS with $A = (1, 4, 2, 3)$
 A', A, \dots in mm with $A = (1, 5, 3, 7)$
 A, A', A, A' in mm with $A = (1, 2, 3, 4)$ with (P) AR
 opetelle $A = (1, 7, 4, 6)$

$1, 8, 1, 8, 1, 8, 1, 8$
 $2, 7, 2, 7, 2, 7, 2, 7$
 $3, 6, 3, 6, 3, 6, 3, 6$
 $4, 5, 4, 5, 4, 5, 4, 5$
 $8, 1, 8, 1, 8, 1, 8, 1$
 $7, 2, 7, 2, 7, 2, 7, 2$
 $6, 3, 6, 3, 6, 3, 6, 3$
 $5, 4, 5, 4, 5, 4, 5, 4$

$1, 8, 1, 8, 1, 8, 1, 8$
 $2, 7, 2, 7, 2, 7, 2, 7$
 $3, 6, 3, 6, 3, 6, 3, 6$
 $4, 5, 4, 5, 4, 5, 4, 5$
 $8, 1, 8, 1, 8, 1, 8, 1$
 $7, 2, 7, 2, 7, 2, 7, 2$
 $6, 3, 6, 3, 6, 3, 6, 3$
 $5, 4, 5, 4, 5, 4, 5, 4$

$0, 48, 24, 40, 56, 8, 32, 16$
 $56, 8, 32, 16, 0, 48, 24, 40$
 $0, 48, 24, 40, 56, 8, 32, 16$
 $56, 8, 32, 16, 0, 48, 24, 40$
 $0, 48, 24, 40, 56, 8, 32, 16$
 $56, 8, 32, 16, 0, 48, 24, 40$
 $0, 48, 24, 40, 56, 8, 32, 16$
 $56, 8, 32, 16, 0, 48, 24, 40$

56	25	48	57	16	33	24
58	34	23	2	55	26	47
3	54	27	46	59	14	35
60	13	36	21	4	53	28
8	49	32	41	64	9	40
63	10	39	18	7	50	31
6	51	30	43	62	11	38
61	12	37	20	5	52	29

min liberty 734
 p. 397
 also rank
 $4 + 4 + 4 + k-n$
 new one

$(2,1) \rightarrow 61 + 30 + 7 + 40 + 60 + 27 + 2 + 33 = 260 \checkmark$
 $(3,1) \rightarrow 61 + 25 + 2 + 35 + 60 + 32 + 7 + 38 = 260 \checkmark$
 $(1,2) \rightarrow 61 + 10 + 36 + 23 + 5 + 50 + 28 + 47 = 260 \checkmark$
 $(1,2) \rightarrow 61 + 15 + 36 + 18 + 5 + 55 + 28 + 42 = 260 \checkmark$

The n 's appearing in the (a, a, n) & (n, a, n) appear typical

we have the values of q_1, q_2, q_3 , in terms of u , as

$$\left. \begin{aligned} q_1 &= a_1 b_2 b_3 u \quad \text{---} = n_1 u \\ q_2 &= a_2 b_1 b_3 u \quad \text{---} = n_2 u \\ q_3 &= a_3 b_1 b_2 u \quad \text{---} = n_3 u \end{aligned} \right\} (19)$$

where n_1, n_2, n_3 are ^{finite} integers. If the unit u is "infinitesimally small", that is, b_1, b_2, b_3 are infinitely large in (8), then n_1, n_2, n_3 are also infinitely large, and we must have a precisely defined way of multiplying an infinitely small quantity by an infinitely large integer to go anywhere forward from here.

So, we need logical, rigorous definitions of infinity, and methods of dealing with ratios of infinite numbers, so that we do not commit errors in manipulating them. (Of course, there are rules of the type in accepted rigorous pure mathematics, but they ~~and~~ have one defect, as we see it, in that "the set of all rational numbers" can be thought of, without including in it the ideas of continuity and of the infinitesimally small unit.) By saying that we limit ourselves to integers and their manipulations in finite arithmetic on one side and that we discuss what are the properties of continuous real variables and their algebra on the other side, we bring out the ~~real~~ precise nature and notion of the latter to represent any "quantity" in physics. We will find that continuity is an ~~arbitrary~~ intuitive primitive notion in physics, and all that rigorous mathematics can do is to clothe it

1, 7, 3, 4, 5, 6, 2, 8
 1, 2, 6, 4, 5, 3, 7, 8
 8, 2, 6, 5, 4, 3, 7, 1
 8, 7, 3, 5, 4, 6, 2, 1
 8, 7, 3, 5, 4, 6, 2, 1
 8, 2, 6, 5, 4, 3, 7, 1
 1, 2, 6, 4, 5, 3, 7, 8
 1, 7, 3, 4, 5, 6, 2, 8

1, 63, 59, 4, 5, 62, 58, 8
 17, 42, 46, 20, 21, 43, 47, 24
 16, 50, 52, 13, 12, 51, 55, 9
 32, 39, 35, 29, 28, 38, 34, 25
 40, 31, 27, 37, 36, 30, 26, 33
 56, 10, 14, 53, 52, 11, 15, 49
 41, 18, 22, 44, 45, 19, 23, 48
 57, 7, 3, 60, 61, 6, 2, 64

(6)

(P) of Fig. 107 + (R) of Fig. 105 → another square

Find an alternative to Fig. 262, p. 165 of Andrews but having the same properties.

1, 4, 3, 1, 6, 5, 2.

1, 6, 5, 2, 8, 3, 4, 7	0, 56, 0, 56, 0, 56, 0, 56	1, 62, 5, 58, 8, 59, 4, 63
8, 3, 4, 7, 1, 6, 5, 2	40, 16, 40, 16, 40, 16, 40, 16	48, 19, 42, 23, 41, 22, 45, 18
1, 6, 5, 2, 8, 3, 4, 7	32, 24, 32, 24, 32, 24, 32, 24	33, 30, 37, 26, 40, 27, 36, 31
8, 3, 4, 7, 1, 6, 5, 2	8, 48, 8, 48, 8, 48, 8, 48	16, 57, 12, 55, 9, 52, 13, 50
1, 6, 5, 2, 8, 3, 4, 7	56, 0, 56, 0, 56, 0, 56, 0	57, 6, 61, 2, 64, 3, 60, 7
8, 3, 4, 7, 1, 6, 5, 2	16, 40, 16, 40, 16, 40, 16, 40	24, 43, 20, 47, 17, 46, 21, 42
1, 6, 5, 2, 8, 3, 4, 7	24, 32, 24, 32, 24, 32, 24, 32	25, 38, 29, 34, 32, 35, 28, 39
8, 3, 4, 7, 1, 6, 5, 2	48, 8, 48, 8, 48, 8, 48, 8	56, 11, 52, 15, 49, 14, 53, 10

4-ply

56 + 29 + 17 + 60 + 16 + 37 + 41 + 4 = 260 ✓
 56 + 5 + 41 + 36 + 16 + 61 + 17 + 28 = 260 ✓
 56 + 43 + 12 + 23 + 49 + 46 + 13 + 18 = 260 ✓
 56 + 19 + 12 + 47 + 49 + 22 + 13 + 42 = 260 ✓
 also knight nasik

56 + 29 + 17 + 60 + 16 + 37 + 41 + 4
 11 + 34 +

39 + 61 + 27 + 1 + 34 + 60 + 30 = 260 ✓
 39 + 43 + 2 + 52 + 31 + 19 + 58 = 260 ✓
 + 14

For (2,1):
 56 + 29 + 17 + 60 + 16 + 37 + 41 + 4 = 260
 11 + 34 + 46 + 7 + 51 + 26 + 22 + 63 = 260
 52 + 32 + 21 + 57 + 12 + 40 + 45 + 1 = 260
 15 + 35 + 42 + 6 + 55 + 27 + 18 + 62 = 260
 49 + 28 + 24 + 61 + 9 + 36 + 48 + 5 = 260
 14 + 39 + 43 + 2 + 52 + 31 + 19 + 58 = 260
 53 + 25 + 20 + 64 + 13 + 33 + 44 + 8 = 260
 10 + 38 + 47 + 3 + 50 + 30 + 23 + 59 = 260

8, 5, 1, 4, 8, 5, 1, 4
 3, 2, 6, 7, 3, 2, 6, 7
 4, 8, 5, 1, 4, 8, 5, 1
 7, 3, 2, 6, 7, 3, 2, 6
 1, 4, 8, 5, 1, 4, 8, 5
 6, 7, 3, 2, 6, 7, 3, 2
 5, 1, 4, 8, 5, 1, 4, 8
 2, 6, 7, 3, 2, 6, 7, 3

This fits another way getting a nasik

For (2,-1):
 56 + 5 + 41 + 36 + 16 + 61 + 17 + 28 = 260
 11 + 58 + 22 + 31 + 51 + 2 + 46 + 39 = 260
 52 + 38 + 45 + 33 + 12 + 64 + 21 + 25 = 260
 15 + 59 + 18 + 30 + 55 + 3 + 42 + 38 = 260
 49 + 4 + 48 + 37 + 9 + 60 + 24 + 29 = 260
 14 + 63 + 19 + 26 + 54 + 7 + 43 + 34 = 260
 53 + 1 + 44 + 40 + 13 + 57 + 20 + 32 = 260
 10 + 62 + 23 + 27 + 50 + 6 + 47 + 35 = 260

Putty this ← (P)

(19)

(R) is A₁, A₁
 A₁, A₁
 A₁, A₁
 A₁, A₁
 A₁, A₁
 A₁, A₁
 A₁, A₁
 A₁, A₁

8, 3, 4, 7, 1, 6, 5, 2
 5, 2, 8, 3, 4, 7, 1, 6
 1, 6, 5, 2, 8, 3, 4, 7
 4, 7, 1, 6, 5, 2, 8, 3
 8, 3, 4, 7, 1, 6, 5, 2
 5, 2, 8, 3, 4, 7, 1, 6
 1, 6, 5, 2, 8, 3, 4, 7
 4, 7, 1, 6, 5, 2, 8, 3

(P) is A, A₁
 A, A₁
 B, B₁
 A₁, A
 B₁, B
 A, A₁
 B, B₁
 A₁, A
 B₁, B

10 + 20 + 54 + 48 + 15 + 21 + 57 + 41 = 260 ✓
 10 + 39 + 42 + 7 + 50 + 31 + 18 + 63 = 260 ✓
 33, 6, 29, 58, 40, 3, 28, 63
 12, 47, 49, 22, 13, 42, 50, 19

with A₁ = A
 64, 35, 4, 31, 57, 38, 5, 26
 21, 10, 48, 51, 20, 15, 41, 54
 25, 62, 37, 2, 32, 59, 36, 7
 52, 23, 9, 46, 53, 18, 16, 43
 8, 27, 60, 39, 1, 30, 61, 34
 45, 50, 24, 5, 44, 55, 17, 14

they had in having an infinite subdivision of matter (or of anything) was that there could be no difference in the number of units, which could be readily conceived of, ^{in the masses of} between a pin and a mountain, because both would be infinitely large. (I am using the word 'infinite' quite freely without defining it, but the idea is quite clear). So, we must have a precise, logical, understanding of infinity — the infinitely large and the infinitely small. The latter is intimately connected with our notions of "continuity". & Note that we do not need to consider functions and their behaviour at all to discuss continuity. It is necessary even in the very notion of a quantity in physics, or a variable in mathematics — what do we mean when we say that x is a real variable, as distinct from x being an integer. There are only these two possibilities, as we shall show below. There is no such thing between them, as "the set of all rational numbers". In fact, this set cannot be defined at all, as far as physics is concerned, except in terms of integers.

We may illustrate this as follows. Suppose we have three quantities Q_1, Q_2, Q_3 ^{in terms of some unit U} such that

$$Q_1 = \frac{q_1}{l_1} U \quad Q_2 = \frac{q_2}{l_2} U \quad Q_3 = \frac{q_3}{l_3} U, \quad q_i \text{ finite integers} \quad (7)$$

Then, multiplying by l_1, l_2, l_3 and (assume $(l_1, l_2, l_3) = 1$ for convenience), and calling the new unit

$$U = U / l_1 l_2 l_3 \quad (8)$$

kg, gram, or mg is immaterial. If now, the unit can be made as small as we ~~please~~^{like}, every object concerned will have a "very large number" of units, and the process of comparison of the quantities concerned becomes more and more difficult. ~~Ultimately~~, if the unit is so small the question is not one of practicality, but that of concept. In principle, can the unit be made so small that ~~any~~^{any} quantity concerned for any object can be expressed mathematically. Intuitively we feel that this ~~is~~^{is} possible - it requires for this purpose the process of "infinite division". (We have not yet defined ~~the~~^{the} terms "infinite", or "division". However, let us say that division is understood for rational numbers, which are ratios of ^{finite} integers p/q . The steps needed for these are fairly simple, and can be rigorously stated, & we shall not discuss them.

6. On the other hand, the concept of something being "infinite", even in the domain of integers, requires careful consideration, or definitions. It is intimately connected with the precise knowledge of the "infinitesimal" also. One cannot think of one without thinking of the other. ~~Any~~

अतः अणुसूत्रम् अणुं न अणुं न

The idea of an atom, or an ^{quantum} infinitesimal unit which must be finite, in order that everything we see or use can be expressed as a finite number of multiples of this unit, has come even in the B.C.'s. (Democritus in Greece, and Kanada in India). The real difficulty

To find an o.e.t. 49 (table) using (A, B, A', B' etc & words)

$$38 + 29 + 40 + 31 + 35 + 28 + 33 + 26 = 260 \checkmark$$

$$38 + 37 + 40 + 39 + 35 + 36 + 33 + 34 = 272 \times$$

(8)

1, 5, 8
 (A) 1, 7, 4, 6
 8, 2, 3
 5, 5, 6
 6, 4, 7, 1
 (B) 4, 6, 1, 7
 1, 7, 4, 6, 8, 2, 5, 3
 6, 4, 7, 1, 3, 5, 2, 8
 8, 2, 5, 3, 8, 7, 4, 6
 3, 5, 2, 8, 6, 4, 7, 1
 1, 7, 4, 6, 8, 2, 5, 3
 6, 4, 7, 1, 3, 5, 2, 8
 8, 2, 5, 3, 1, 7, 4, 6
 3, 5, 2, 8, 6, 4, 7, 1

0, 40, 56, 16, 0, 40, 56, 16
 48, 24, 8, 32, 48, 24, 8, 32
 24, 48, 32, 8, 24, 48, 32, 8
 40, 0, 16, 56, 40, 0, 16, 56
 56, 16, 0, 40, 56, 16, 0, 40
 8, 32, 48, 24, 8, 32, 48, 24
 32, 8, 24, 48, 32, 8, 24, 48
 16, 56, 40, 0, 16, 56, 40, 0

1, 47, 60, 22, 8, 42, 61, 19
 52, 28, 15, 33, 51, 29, 10, 40
 32, 50, 37, 11, 25, 55, 36, 14
 43, 5, 18, 64, 46, 4, 23, 57
 57, 23, 4, 46

repetitive
 does not work

$$32 + 17 + 56 + 57 + 40 + 41 + 16 + 1 = 260$$

$$20 + 53 + 60 + 37 + 44 + 13 + 4 + 29 = 260$$

$$54 + 59 + 38 + 43 + 11 + 3 + 30 + 19 = 260$$

1, 7, 6, 4
 6, 4, 8, 2
 6, 4, 8, 2
 1, 7, 6, 4
 6, 4, 8, 2
 6, 4, 8, 2

1, 7, 6, 4, 8, 2, 3, 5
 8, 2, 3, 5, 1, 7, 6, 4
 6, 4, 8, 2, 3, 5, 1, 7
 3, 5, 2, 7, 6, 4, 8, 2
 1, 7, 6, 4, 8, 2, 3, 5
 8, 2, 3, 5, 1, 7, 6, 4
 6, 4, 8, 2, 3, 5, 1, 7
 3, 5, 1, 7, 6, 4, 8, 2

1, 63, 46, 20, 8, 58, 43, 21
 56, 10, 27, 37, 49, 15, 30, 36
 46, 20, 64, 2, 43, 21

repetitive
 does not work

with B = (a₃, a₄, a₁, a₂)

$$50 + 14 + 53 + 16 + 55 + 11 + 52 + 9 = 260$$

$$20 + 28 + 4 + 12 + 44 + 36 + 60 + 52 = 256$$

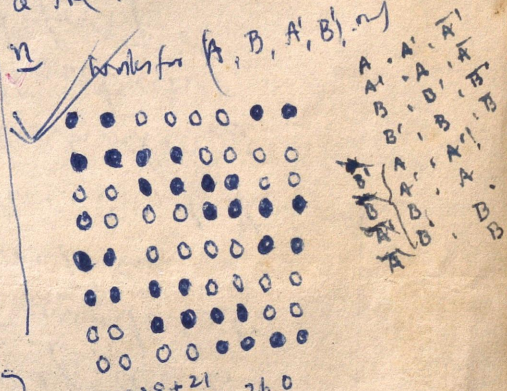
A(a₁, a₂, a₃, a₄), B = (a₃, a₄, a₁, a₂)

A(1, 7, 6, 4)
 B(3, 5, 1, 7)
 (Cn 3)
 A(a₁, a₂, a₃, a₄)
 B(a₃, a₄, a₁, a₂)

1, 7, 6, 4, 8, 2, 3, 5
 3, 5, 1, 7, 6, 4, 8, 2
 8, 2, 3, 5, 1, 7, 6, 4
 6, 4, 8, 2, 3, 5, 1, 7
 1, 7, 6, 4, 8, 2, 3, 5
 3, 5, 1, 7, 6, 4, 8, 2
 8, 2, 3, 5, 1, 7, 6, 4
 6, 4, 8, 2, 3, 5, 1, 7

0, 16, 56, 40, 0, 16, 56, 40
 48, 32, 8, 24, 48, 32, 8, 24
 40, 0, 16, 56, 40, 0, 16, 56
 24, 48, 32, 8, 24, 48, 32, 8
 56, 40, 0, 16, 56, 40, 0, 16
 8, 24, 48, 32, 8, 24, 48, 32
 16, 56, 40, 0, 16, 56, 40, 0
 32, 8, 24, 48, 32, 8, 24, 48

1, 23, 62, 44, 8, 19, 59, 45
 57, 37, 31, 52, 36, 16, 26
 48, 2, 19, 61, 41, 7, 22, 60
 30, 52, 40, 10, 27, 53, 33, 15
 57, 47, 6, 20, 64, 42, 3, 21
 11, 29, 49, 59, 14, 28, 56, 34
 24, 58, 43, 5, 17, 63, 46, 4
 38, 12, 32, 50, 35, 13, 25, 55



Ex 2 mix words

like A = (a₁, a₂, a₃, a₄), B = (a₁, a₂, a₃, a₄) Δ (A, A', B, B')

A(1, 2, 3, 4)
 B(1, 7, 3, 5)
 (Cn 2)
 A(a₁, a₂, a₃, a₄)
 B(a₁, a₂, a₃, a₄)

A 1, 2, 3, 4, 8, 7, 6, 5
 A' 8, 7, 6, 5, 1, 2, 3, 4
 B 1, 7, 3, 5, 8, 2, 6, 4
 B' 8, 2, 6, 4, 1, 7, 3, 5
 1, 2, 3, 4, 8, 7, 6, 5
 8, 7, 6, 5, 1, 2, 3, 4
 1, 7, 3, 5, 8, 2, 6, 4
 8, 2, 6, 4, 1, 7, 3, 5

0, 56, 0, 56, 0, 56, 0, 56
 8, 48, 48, 8, 8, 48, 48, 8
 16, 40, 16, 40, 16, 40, 16, 40
 24, 32, 32, 24, 24, 32, 32, 24
 56, 0, 56, 0, 56, 0, 56, 0
 48, 8, 8, 48, 48, 8, 8, 48
 40, 16, 40, 16, 40, 16, 40, 16
 32, 24, 24, 32, 32, 24, 24, 32

1, 58, 3, 60, 8, 63, 6, 61
16, 55, 54, 13, 9, 50, 51, 12
17, 47, 19, 45, 24, 42, 22, 44
32, 34, 38, 28, 25, 39, 35, 29
57, 2, 59, 4, 64, 7, 62, 5
56, 15, 14, 53, 49, 10, 11, 52
41, 23, 43, 21, 48, 18, 46, 20
40, 26, 36, 36, 33, 31, 27, 37

$$38 + 43 + 14 + 3 + 30 + 19 + 56 + 59 = 260$$

$$38 + 62 + 52 + 22 + 30 + 6 + 14 + 46 = 272 \times$$

$$12 + 5 + 28 + 21 + 52 + 61 + 36 + 45 = 260$$

$$33 + 46 + 56 + 59 + 24 + 51 + 1 + 30 = 300 \times$$

$$40 + 43 + 49 + 62 + 32 + 19 + 9 + 6 = 260 \text{ only - pb}$$

$$33 + 10 + 35 + 12 + 40 + 15 + 38 + 13 = 196 \times$$

$$40 + 15 + 38 + 13 + 33 + 26 + 14 + 28 + 9 + 31 + 11 + 29 + 16 = 260$$

$$33 + 46 + 56 + 59 + 25 + 22 + 16 + 3 = 260$$

1, 4, 3, 2, 8, 5, 6, 7
 3, 2, 8, 5, 6, 7, 1, 4
 6, 7, 1, 4, 3, 2, 8, 5
 8, 5, 6, 7, 1, 4, 3, 2
 1, 4, 3, 2, 8, 5, 6, 7
 3, 2, 8, 5, 6, 7, 1, 4
 6, 7, 1, 4, 3, 2, 8, 5
 8, 5, 6, 7, 1, 4, 3, 2

1, 20, 43, 58, 8, 21, 46, 63
 27, 10, 56, 37, 30, 15, 49, 36
 22, 63

does not work
 So cancel the last alternative under A, B, C, D
 for next question

$$50 + 63 + 34 + 47 + 10 + 7 + 26 + 23 = 260 \checkmark$$

$$64 + 11 + 41 + 30 + 8 + 51 + 17 + 38 = 260 \checkmark$$

$$38 + 62 + 54 + 22 + 30 + 6 + 14 + 46 = 272 \times$$

$$40 + 3 + 9 + 22 + 32 + 59 + 49 + 46 = 260 \checkmark$$

$$12 + 49 + 10 + 52 + 13 + 56 + 55 = 260 \checkmark$$

(P.T.O) for Ex 1

$$2 + 31 + 18 + 55 + 58 + 39 + 42 + 15 = 260 \checkmark$$

$$+ 51 = 260 \checkmark$$

have ~~as~~ a "quantum", with all quantities of this type having an integral number of quanta. [Note that no multiplication is involved in this, but only the most elementary concept of "counting"].

Thus, if the quantum exists for the physical entity we are considering, then, as far as that is concerned, integer arithmetic will do. [I am jumping ahead and assuming that we have arrived at the laws of arithmetic - addition, subtraction, multiplication and division. I shall not consider the structure of the rules of arithmetic and find out which are fundamental, or basic, and which can be derived from them. For the present, I shall not take into account the extension of division to define rational numbers; but shall use it where necessary without any special comment, unless one is called for.]

Therefore one naturally asks the question - is "everything" a multiple of a unit, or can they be different? In the former case, life is easy, and, if it is not so, some ^{apparently} logical inconsistencies arise. For example, if there is no unit of mass or weight, how would one ever test that two objects have the same mass? - The same with the length of two lines, although, in this case, the two lines may be superposed and compared (as with two masses by a balance). However, ^{no} physics is possible unless we can have a method of measuring things, and this requires always a unit - whether it is a

$A = (1, 2, 6, 5)$
 $B = (8, 2, 3, 5)$
 $C = (1, 7, 6, 4)$
 $D = (8, 7, 3, 4)$

1, 2, 6, 5, 8, 7, 3, 4
 8, 2, 3, 5, 1, 7, 6, 4
 1, 7, 6, 4, 8, 2, 3, 5
 8, 7, 3, 4, 1, 2, 6, 5
 1, 2, 6, 5, 8, 7, 3, 4
 8, 2, 3, 5, 1, 7, 6, 4
 1, 7, 6, 4, 8, 2, 3, 5
 8, 7, 3, 4, 1, 2, 6, 5

0, 56, 0, 56, 0, 56, 0, 56
 8, 8, 48, 48, 8, 8, 48, 48
 40, 16, 40, 16, 40, 16, 40, 16
 32, 32, 24, 24, 32, 32, 24, 24
 56, 0, 56, 0, 56, 0, 56, 0
 48, 48, 8, 8, 48, 48, 8, 8
 16, 40, 16, 40, 16, 40, 16, 40
 24, 24, 32, 32, 24, 24, 32, 32

1, 58, 6, 61, 8, 63, 3, 60
 16, 10, 51, 53, 9, 15, 54, 52
 41, 23, 46, 20, 48, 18, 43, 21
 40, 39, 27, 28, 33, 34, 30, 29
 57, 2, 62, 5, 64, 7, 59, 4
 56, 50, 11, 13, 49, 55, 14, 12
 17, 47, 22, 44, 24, 42, 19, 45
 32, 31, 35, 36, 25, 26, 38, 37

$32 + 22 + 49 + 30$
 $41 + 51 + 8 + 38 = 7(9)$
 $(2+1) \cdot 32 + 6 + 9 + 43 + 40 + 62 + 49 + 19 = 240$
 $2 + 4 + 7$
 $32 + 50 + 27 + 53$
 $25 + 55 + 30 + 52 = 1$
 $32 + 10 + 27 + 13 + 25$
 $15 + 30 + 45 + 12 = 9$
 2, 61, 23, 44
 47, 20, 58, 5
 60, 7, 45, 18
 21, 42, 41, 63

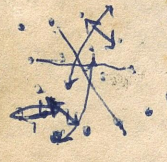
The 15 (1, 3, 5, 7) & R from (8, 6, 4, 2) & scheme (A, A', A, A')

1, 3, 5, 7, 8, 6, 4, 2
 8, 6, 4, 2, 1, 3, 5, 7
 1, 3, 5, 7, 8, 6, 4, 2
 8, 6, 4, 2, 1, 3, 5, 7
 1, 3, 5, 7, 8, 6, 4, 2
 8, 6, 4, 2, 1, 3, 5, 7
 1, 3, 5, 7, 8, 6, 4, 2
 8, 6, 4, 2, 1, 3, 5, 7

128 132
 57, 3, 61, 7, 64, 6, 60, 2
 48, 23, 44, 18, 41, 19, 48, 22
 25, 35, 29, 39, 32, 38, 28, 34
 16, 54, 12, 50, 9, 51, 13, 55
 7, 59, 5, 63, 8, 62, 4, 58
 24, 46, 26, 42, 17, 43, 21, 47
 33, 27, 37, 31, 40, 30, 36, 26
 56, 14, 52, 10, 49, 11, 53, 15

$56 + 37 + 17 + 4 + 16 + 29 + 41 + 60 = 260$
 $56 + 61 + 41 + 28 + 16 + 5 + 17 + 36 = 260$
 $56 + 26 + 12 + 18 + 49 + 43 + 13 + 23 = 260$
 $56 + 22 + 12 + 42 + 49 + 19 + 13 + 47 = 260$

also Knight - knight also 0,0,0,0,2,2,2,2
 2,2,2,2,0,0,0,0



to make this
 find the knight's path in a square

~~60, 2, 61, 7, 64, 6, 57, 3~~
~~45, 23, 44, 18, 41, 19, 48, 22~~
~~28, 34, 29, 39, 32, 38, 25, 35~~
~~13, 55, 12, 50, 9, 51, 16, 54~~
~~13, 55, 12, 50, 9, 51, 16, 54~~

60, 2, 61, 7, 64, 6, 57, 3
 45, 23, 44, 18, 41, 19, 48, 22
 28, 34, 29, 39, 32, 38, 25, 35
 13, 55, 12, 50, 9, 51, 16, 54
 4, 58, 5, 63, 8, 62, 1, 59
 21, 47, 20, 42, 17, 43, 24, 46
 36, 26, 37, 31, 40, 30, 33, 27
 53, 15, 52, 10, 49, 11, 56, 14

60, 2, 61, 7, 64, 6, 57, 3
 45, 23, 44, 18, 41, 19, 48, 22
 21, 47, 20, 42, 17, 43, 24, 46
 4, 58, 5, 63, 8, 62, 1, 59
 13, 55, 12, 50, 9, 51, 16, 54
 28, 34, 29, 39, 32, 38, 25, 35
 36, 26, 37, 31, 40, 30, 33, 27
 53, 15, 52, 10, 49, 11, 56, 14

166	182	102
60	2	61
45	23	44
21	47	20
4	58	5
13	55	12
28	34	29
36	26	37
53	15	52
166	182	102

8, 6, 4, 2
 1, 3, 5, 7
 8, 6, 4, 2
 1, 3, 5, 7
 8, 6, 4, 2
 1, 3, 5, 7
 8, 6, 4, 2
 1, 3, 5, 7

0, 12, 0, 12
 4, 8, 4, 8
 12, 0, 12, 0
 8, 4, 8, 4

1, 14, 4, 15
8, 11, 5, 10
13, 2, 16, 3
12, 7, 9, 6

3, 48, 29, 50
 8, 43, 26, 53
 14, 2, 61, 49
 64, 2, 61, 3
 45, 19, 48, 18
 21, 47, 20, 42, 17, 43, 24, 46
 5, 58, 5, 63

53, 26, 29, 50
 4, 47, 44, 7
 53, 26, 29, 50
 8, 43, 48, 3
 60, 23, 20, 63
 9, 38, 33, 14
 49, 30, 25, 54
 8, 43, 48, 3
 49, 30, 25, 54
 4, 47, 44, 7
 64, 19, 24, 59
 34, 37, 10

2, 4, 5, 7
 18, 20, 21, 23
 42, 44, 45, 47
 58, 60, 61, 63

4, 11, 14, 16
25, 27, 30, 32
33, 35, 38, 40
41, 51, 54, 56

14, 2, 61, 49, 10, 6, 57, 53
 45, 33, 30, 18, 41, 37, 26, 22
 21, 47, 20, 42, 17, 43, 24, 46
 4, 58

1, 62, 59, 8
 48, 19, 22, 41
 24, 43, 46, 17
 51, 6, 3, 64
 1, 62, 24, 43
 48, 19, 59, 8
 59, 8, 46, 17
 2, 41, 3, 64

14, 2, 61, 49, 10, 6, 57, 53
 21, 47, 20, 42, 17, 43, 24, 46
 4, 58, 5, 63, 8, 62, 1, 59
 13, 55, 12, 50, 9, 51, 16, 54
 28, 34, 29, 39, 32, 38, 25, 35
 36, 48, 19, 31, 40, 24, 27, 27
 3, 15, 52, 64, 7, 11, 56, 60

8

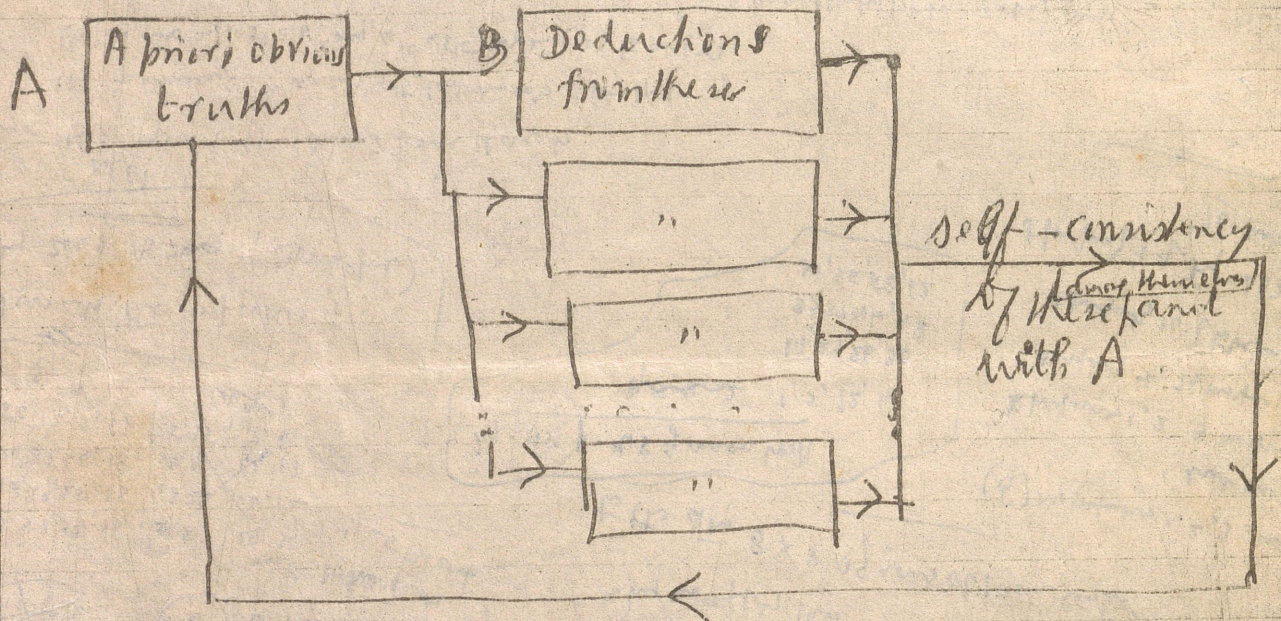
5. The first a priori concept that I would think of is "equality", or sameness - e.g. two fruits, each a "fruit"; two blocks of wood of the same size (in which the size may be neglected if the emphasis is on the "block of wood"). The integers 1, 2, 3... follow by abstracting the concept of number from n fruits, n books, n blocks etc.

The next a priori concept is that of "larger" or "smaller". The simplest way of knowing this is to compare two sets of same things -
 n books and m books, $n > m$ or $m < n$, (6)
 where n, m are integers.

In fact this obvious concept of larger or smaller is there in everybody's mind for continuous matter also - e.g. one book heavier than another. We can always settle the question of which is larger, or smaller, without any weights or "measure", by $\$$ a balance. In the same way, we can draw two lines, and by superposing them, find out which is longer. (The question of accuracy of this is irrelevant, for we are thinking of the concept of $>$ and $<$, and not quantitate it). In fact, immediately this idea becomes quantized and definite if we ~~to~~ compare collections of identical objects, by finding out if $n > m$ or $m > n$; in this ~~case~~ case $m = n$ has a meaning, because the least difference permitted is the integer 1.

So, the idea of equality of any two similar quantities (of properties like length, mass) becomes possible only if they are "quantized", and we

in any field of knowledge, including mathematics, there are first ^{of all some} "self-evident truths" or "axioms" if we would like to call them ^{that way}. Often, these are used ^{to build} a body of knowledge ~~build up~~, and afterwards one comes back to the axioms and tries to justify them or at least show that ~~they are~~ ^{they are} not inconsistent with anything that follows. So, the method of expanding knowledge has the following block diagram:



Obviously, it will be a stupendous task to make this type of analysis for mathematics. So, we shall take the ^{original} structure of mathematics as it ^{exists} now for granted, and only ~~see~~ try to verify where the theorems, or postulates, of type A and deductions, of ^{type} B, in the above block diagram, need any modification, improvement, or clarification, if they must fit the intuitive ideas of physicists, or the requirements of them. So, of physics.

88'97
91'55
21'15' 87'84
26'85'1
86
05
75
81
81
91'21'50'1
91

59'45'55
23'44'45'18
47'20'21'42
2'61'60'7
2'61'23'44

59'19'09'85
47'54'44'24
23'20'08'81
2'5'4'2
1'5'1'7

Getting (8) with Q's associated with a fluke & hence getting (6) with Q's with also a fluke

It would be wonderful if we could get a square which is $n \times n$ with Q's n - so this possible?

Yes. See Planet's Fig 695, p. 377

Fig. 115, p. 1489, main book.

Splitting up (8)

g. Andrews

Q ₁ is	1, 5, 12, 16	1, 58, 55, 16
	19, 23, 26, 30	48, 23, 26, 33
	33, 37, 44, 48	30, 37, 44, 19
	51, 55, 58, 62	51, 12, 5, 62

magic but
not a mn

2	5	7	4	1	6	8	3	0	56	16	40	0	56	16	40
7	4	2	5	8	3	1	6	40	16	56	0	40	16	56	0
4	7	5	2	3	8	6	1	56	0	40	16	56	0	40	16
5	2	4	7	6	1	3	8	16	40	0	56	16	40	0	56
1	6	8	3	2	5	7	4	8	48	24	32	8	48	24	32
8	3	1	6	7	4	2	5	32	24	48	8	32	24	48	8
3	8	6	1	4	7	5	2	48	8	32	24	48	8	32	24
6	1	3	8	5	2	4	7	24	32	8	48	24	32	8	48

Andrews Fig. 94, p. 43.

Q ₂ is	1, 2, 9, 10	1, 61, 54, 10
	19, 20, 27, 28	48, 20, 27, 39
	29, 28, 35, 36	28, 40, 47, 19
	27, 28, 39, 40	53, 9, 20, 62
	42, 43, 61, 62	39, 46, 47, 48
	39, 46, 47, 48	53, 54, 61, 62

not
magic

Does not appear to
work for associated
square

Fig. 449, p. 256, Andrews.

Q ₁	1, 2, 3, 4	1, 59, 58, 4
	13, 14, 15, 16	56, 14, 15, 53
	58, 57, 55, 56	16, 57, 55, 13
	59, 58, 59, 60	57, 3, 2, 60

Indiscernible square
and

(2,3), (1,3), (1,4), (1,1)
(2,4), (2,3), (2,4)
(2,3), (2,4)

$$32 + 56 + 39 + 15 + 23 + 63 + 48 + 8 = 284 \times (2,1)$$

$$32 + 7 + 48 + 64 + 23 + 16 + 39 + 55 = 284 \times (2,-1)$$

$$32 + 27 + 18 + 21 + 31 + 28 + 17 + 22 = 196 \times (1,2)$$

$$32 + 20 + 18 + 30 + 31 + 19 + 17 + 29 = 196 \times (1,-2)$$

Suppose in this we change the name Q's into associated Q's by Planet's rule

10, 51, 54, 15	12, 49, 56, 13
23, 46, 43, 18	21, 48, 41, 20
47, 22, 19, 42	45, 24, 17, 44
50, 11, 14, 55	52, 9, 16, 53
26, 35, 38, 31	28, 33, 40, 29
7, 62, 59, 2	5, 64, 57, 4
63, 6, 13, 58	61, 8, 1, 60
34, 27, 30, 39	36, 25, 32, 37

this is no longer magic or associated

we can transfer the total square 695 also into associated one by Planet's rule because Fig. 695 is combined in my rule (A, A', A, A' -) (B, B', B, B')

done this by quadrant Q's

10	51	15	14	56	13	49	12
23	46	18	43	41	20	48	21
50	11	55	14	16	53	9	52
47	22	42	19	17	44	24	45
63	6	58	3
34	27	39	30
7	62	2	59
26	35	31	38

no because this applies only in my rule

it applies to quadrant although because Q's have (A, A', A, A') & (B, B', B, B') separated

Also applies to (A, B', B, B') in Q₁ (force)

In Fig. 120 of main book, p. 153
Can we make the Q's associated?

Q ₁	1, 3, 13, 16
	17, 19, 28, 32
	34,

no

1, 4, 4, 1	0 12 0 12	1, 16, 4, 13	1, 4, 2, 3	0 4 0 4	1, 8, 2, 7	1, 4, 3, 2	1, 12, 15, 6
4, 1, 1, 4	12 0 12 0	16, 13, 4	2, 3, 1, 4	12 8 12 8	3, 2, 1, 4	3, 2, 1, 4	3, 2, 1, 4
1, 4, 4, 1	12 0 12 0		1, 4, 2, 3	4 0 4 0	4, 1, 2, 3	4, 1, 2, 3	4, 1, 2, 3
4, 1, 1, 4	0 12 0 12		2, 3, 1, 4	8 12 8 12	2, 3, 4, 1	2, 3, 4, 1	2, 3, 4, 1

1, 4, 1, 4	1, 12, 13, 8
3, 2, 3, 2	15, 6, 3, 10
4, 1, 4, 1	4, 9, 16, 5
2, 3, 2, 3	14, 7, 2, 11
1, 4, 1, 4	1, 12, 13, 8
3, 2, 3, 2	15, 6, 3, 10
4, 1, 4, 1	4, 9, 16, 5
2, 3, 2, 3	14, 7, 2, 11

1, 4, 1, 4	1, 12, 13, 8
3, 2, 3, 2	15, 6, 3, 10
4, 1, 4, 1	4, 9, 16, 5
2, 3, 2, 3	14, 7, 2, 11
1, 4, 1, 4	1, 12, 13, 8
3, 2, 3, 2	15, 6, 3, 10
4, 1, 4, 1	4, 9, 16, 5
2, 3, 2, 3	14, 7, 2, 11

finally must come back to physics. "Dust thou art, to dust returnest" — "was not spoken of the soul"? — Is there ^{valley} true of mathematics? — Can it reign supreme without the background of knowledge from the sense organs? Pure mathematicians say so. — That they are allowed to follow their nose anywhere they want, and, once the idea of number and measure and operations are introduced, they are at liberty to allow their minds to roam unfettered ^{by restrictions of reality.} I am not saying that they shouldn't; but the purpose of this series of talks is to analyse what limitations should be put on the liberties that they can take, if the quantities they measure should pertain to real objects, that is, correspond to "physical reality."

In this, I shall, for the first part, limit myself to the idea of "measure" and of "functions." Afterwards, if possible, we shall expand the range to include "operators" as well — the only exceptions are the operations of differentiation and integration, which are so fundamental to the idea of a function, so that we must introduce them right in the beginning.

4. So, let us get down to brass tacks. The first topic I shall take up is — what is the mathematical nature of physical "quantity"? Before discussing it in detail, I will first state a logical proposition, which I think is true, and which must be accepted, if we wish to avoid running in circles. This is that,

1, 4, 10, 13
 15, 16, 25, 28
 34, 39, 50, 53
 55, 56, 58, 63

1, 58, 56, 138
 53, 16, 25, 31
 28, 39, 50, 162
 55, 10, 4, 63
 137, 123, 135, 125

1, 58, 56, 15
 55, 16, 25, 34
 28, 39, 50, 13
 53, 10, 4, 63
 137 | 123, 135, 125

1, 2, 3, 4, 5, 6, 7, 8

(12)

figs. 138, 139 & 140

$63 + 39 + 5 + 29 + 47 + 55 + 21 + 13 = 272X$

1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4
1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4
1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4
1	2	8	7	3	4	6	5
8	7	1	2	6	5	3	4

fig. 138

0	56	0	56	0	56	0	56
8	48	8	48	8	48	8	48
56	0	56	0	56	0	56	0
48	8	48	8	48	8	48	8
16	40	16	40	16	40	16	40
24	32	24	32	24	32	24	32
40	16	40	16	40	16	40	16
32	24	32	24	32	24	32	24

fig. 139

1	58	8	63	3	60	6	61
16	55	9	50	14	53	11	52
57	2	64	7	59	4	62	5
56	15	49	10	54	13	51	12
17	42	24	47	19	44	22	45
32	39	25	34	30	37	27	36
41	18	48	23	43	20	46	21
40	31	33	26	38	29	35	28

fig. 140

1, 8 - 2, 7, 3, 6 - 4, 5
 3, 6 - 4, 5

1, 8 - 2, 7, 5, 4 - 6, 3

$40 + 48 + 30 + 22 + 56 + 64 + 14 + 6 = 280$

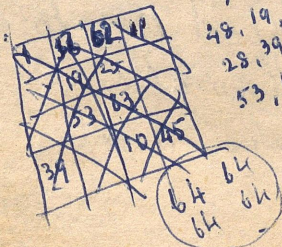
1, 58, 63
 57, 2, 7
 56, 15, 10

$\frac{8 \times 17}{2} = 68$
 $\frac{8 \times 19}{2} = 76$
 $\frac{36 \times 36}{4} = 324$

mark, 4 ply,

0	11	4	10	11
17	18	21	27	28
37	38	41	47	48
52	53	56	62	63

1, 4, 10, 11
 18, 19, 25, 28
 38, 39, 45, 48
 53, 56, 62, 63



1, 62, 56, 11
 48, 19, 25, 38
 28, 39, 45, 18
 53, 10, 4, 63

1, 58, 8, 63
 16, 55, 9, 50
 57, 2, 64, 7

1, 58, 63, 8
 16, 55, 50, 9
 56, 15, 10, 49
 57, 2, 7, 64

1, 2, 9, 10
 19, 20, 27, 28
 39, 40, 47, 48
 53, 54, 61, 62

1, 2, 9, 10
 19, 20, 27, 28
 39, 40, 47, 48
 53, 54, 61, 62

1	2	9	10	
18	19	20	27	28
38	39	40	47	48
52	53	54	61	62

1, 61, 54, 10
 48, 20, 27, 39
 28, 40, 47, 19
 53, 9, 2, 62

1, 63, 8, 58
 16, 50, 9, 55
 57, 7, 64, 2
 56, 10, 49, 15

1, 63, 58, 8
 56, 10, 15, 49
 16, 50, 55, 9
 57, 7, 2, 64

1, 63, 8, 58
 56, 10, 15, 49
 57, 7, 64, 2
 16, 50, 9, 55

1, 58, 8, 63
 16, 55, 9, 50
 57, 2, 64, 7
 56, 15, 49, 10

So, we reformulate the equations connecting physically ^{relevant} interesting quantities ^{variables} into different forms in terms of Q , a complex number, equal to $a + ib$. Now, what I now say may look trivial - but it is a mystery that always makes me stand in awe - the equations in terms of ~~the~~ complex Q , when ^(they) solved by mathematical methods of complex variables, finally give results as a function of $(a + ib)$; but then these, when "decoded" in terms of q and x , agree with observation, or facts.

In other words, once a problem in physics is formulated in terms of mathematics, hereafter mathematical rules can be applied to them, without any rechecking with the facts, and results can be derived for application to practical things. Thus, the "laws" of mathematics, ^{pure} products of the human mind, ~~a~~ ⁱⁿ fact, of course, are in full consonance with the interrelationships between different aspects of Nature, which become available to man via his sense organs. It is an unstated postulate of science, that the two must agree. In fact, even prior to mathematics, "reason" and "logic", ~~for~~ which occur in the human mind, are consistent with what occurs in natural phenomena in the external world.

3. I shall not dwell further on these philosophical problems, but I have mentioned them to show the intimate relation ^{ship} between mathematics and physics. Maths arose out of physics and

Fig. 20, p. 1659 Andrews - Count quadrants into hands

1	59	24	56	2	60	15	53
46	24	33	27	47	21	34	28
51	9	64	6	50	12	63	5
32	38	19	41	31	37	18	44
53	51	46	54	44	58	13	55
38	22	35	25	45	23	36	26
44	31	62	28	52	10	61	7
30	40	17	43	29	39	20	42

1, 5, 9, 13, 17, 21, 25

Ranking square with

Fig. 96, Andrews - associated square.

The Q's are not magic squares, but can be made magic by Heuristics table.

(13)

(Q1) 1, 2, 9, 10 19, 20, 27, 28 39, 40, 47, 48 53, 54, 61, 62	1, 61, 28, 40 48, 20, 53, 9 54, 10, 47, 19 27, 39, 2, 62	(Q2) 7, 8, 15, 16 21, 22, 29, 30 33, 34, 41, 42 51, 52, 59, 60	7, 59, 30, 34 42, 22, 51, 15 52, 16, 40, 41 29, 33, 8, 60
(Q3) 5, 6, 13, 14 23, 24, 31, 32 35, 36, 43, 44 49, 50, 57, 58	5, 57, 32, 36 44, 24, 49, 13 50, 14, 43, 23 31, 35, 6, 58	(Q4) 3, 4, 11, 12 17, 18, 25, 26 37, 38, 45, 46 55, 56, 63, 64	3, 63, 26, 38 46, 18, 55, 11 56, 12, 45, 17 25, 37, 4, 64

Now the square becomes

3, 5, 4, 8, 7, 3, 6, 2

1	61	28	40	7	59	30	34
48	20	53	9	42	22	51	15
54	10	47	19	52	16	41	21
27	39	2	62	29	33	8	60
5	57	32	36	3	63	26	38
44	24	49	13	46	18	55	11
50	14	43	23	56	12	45	17
31	35	6	58	25	37	4	64

Again associated (1) corners of the 3x3 squares within quadrants add to 130 with quadrants

all magic squares which might be called semi-associated since elements in complementary cells sum to 63467.

Very interesting this is 16-ply but not 4-ply only 16 sub-squares & quadrants add to 130

a	→	n
A	, A'	A, A'
A	, A'	A, A'
A'	, A	A', A
A'	, A	A', A
A'	, A	A', A
A'	, A	A', A
A	, A	A, A
A	, A	A, A

Suppose we transform the H-squares Fig. 96 into a rank square by Planck's rule

126
134
134

98
98
2

1, 2, 62, 61	8, 7, 59, 60
9, 10, 54, 53	16, 15, 51, 52
48, 47, 19, 20	41, 42, 22, 21
40, 39, 27, 28	33, 34, 30, 29
57, 58, 6, 5	64, 63, 3, 4
49, 50, 14, 13	56, 55, 11, 12
24, 23, 43, 44	17, 18, 46, 45
32, 31, 35, 36	25, 26, 38, 37

rank alright (Y)

- (1) corner elements of all 3x3 squares add to 130
- (2) corner elements of all 5x5 squares add to 130
- (3) " all 7x7 squares add to 130.

16-ply but not 4-ply.

Suppose we transform (X) into a rank by Planck's rule

rank, quadrants all magic squares which are semi-magic associated

126	134	98	98	162	162	98	98	98	98
126	134	162	162	98	98	162	162	162	162
134	126								
134	126								
126	134								
126	134								
134	126								
126	134								

3	63	64
2	11	55
45	46	18
37	38	26

92(X) K-n-2 no

31 + 43 + 46 + 26 + 27 + 47 + 42 + 30 = 277

92(Y) K-n-2

32 + 43 + 56 + 3 + 40 + 19 + 16 + 59 = 268

1, 61, 28, 40	34, 30, 59, 7
48, 20, 53, 9	15, 51, 22, 42
54, 10, 47, 19	21, 41, 16, 52
27, 39, 2, 62	60, 8, 33, 29
31, 35, 6, 58	64, 4, 37, 25
50, 14, 43, 23	17, 45, 12, 56
44, 24, 49, 13	11, 55, 18, 46
5, 57, 32, 36	38, 26, 63, 3

rank (Z)

92(Z) K-n-2 no

5 + 49 + 17 + 37 + 27 + 47 + 15 + 59 = 256

(1) Corner of 3x3 squares inside quadrants add to 130 16-ply but not 4-ply

1, 61, 40, 28	1, 7, 10, 16
48, 20, 9, 53	19, 21, 28, 30
27, 39, 62, 2	33, 39, 47, 48
54, 10, 19, 47	51, 53, 60, 62
	1, 60, 53, 16
	48, 21, 28, 33
	30, 39, 42, 19
	51, 10, 7, 62

1, 61, 40, 28	semi-magic
48, 20, 9, 53	
27, 39, 62, 2	
54, 10, 19, 47	

which I wish to discuss further. In this case, the idea of an algebraic ^{polynomial} equation with integral coefficients was considered in a general way, e.g. ~~a~~ many quadratic eqⁿ: $x^2 - 2x + 3 = (x-3)(x+1)$ has $x=3, -1$ (1)

as roots. We know the general solution for $ax^2 + bx + c = 0$ as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (2).

In the school, we were told that, if $b^2 - 4ac$ is $-ve$, there is "no solution", and we believed it. But, can we generalize the rule that a quadratic equation always has 2 roots? This leads to i in the soln of the equation

$$x^2 = -1, \quad x = +i, -i \quad (3)$$

& the wonder is that, just this one new entity, with the rule, $i^2 = -1$, is sufficient for the solution of all algebraic equations (polynomials) of degree n , if n is finite. ~~One~~

Once we learn about i , hereafter we take it for granted in mathematics. But what is its relation to physics? or "reality". In Nature, all measured quantities are real, - so, if i occurs in

$$q = a + ib \quad (4a)$$

then $|q|^2 = a^2 + b^2$ (4b)

is often the physically measured quantity. In vibs. & wave motion, the phase α , given by

$$\alpha = \tan^{-1}(b/a) \quad (4c)$$

is very useful and significant. But, both q & α (given by a and b) are used in the combination

$$a + ib = q / \exp i\alpha = q (\text{say}) \quad (5)$$

In 79 (148) change quadrants from north to around square.

1, 58, 63, 8	3, 60, 61, 6
16, 55, 50, 9	14, 53, 52, 11
57, 15, 10, 49	54, 13, 12, 51
57, 2, 57, 64	59, 4, 5, 62
17, 42, 47, 24	19, 44, 45, 22
32, 39, 34, 25	30, 37, 36, 27
40, 31, 26, 33	38, 29, 28, 35
41, 18, 23, 48	43, 20, 21, 46

but the square is neither a nor n
 but it has properties 1, 2, 3, 4 (but will add 3+3 squares)
 7 79, 261, p. 165 of Andrews

To obtain a square neither a nor n

Method (P) $\frac{1}{2}n$ to (R) $\frac{1}{2}n$ Changing semi-orthogonal squares

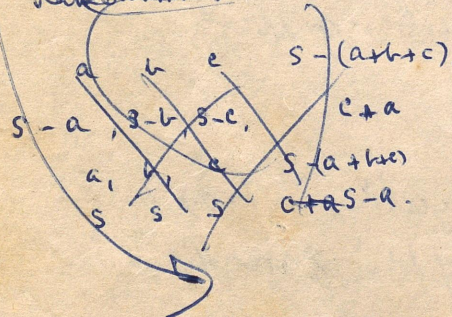
transforming 144 into a square with quadrants semi-norith (Vol 1 p 13)

1, 61, 40, 28	7, 59, 34, 30
48, 20, 9, 53	42, 22, 15, 51
27, 39, 62, 2	29, 33, 60, 8
52, 10, 19, 47	52, 16, 21, 41
5, 57, 36, 32	3, 63, 38, 26
44, 24, 13, 49	46, 18, 11, 55
31, 35, 58, 6	25, 37, 64, 4
50, 14, 23, 43	56, 12, 17, 45

neither a nor n, 2's are semi-norith
 (1) At 50 in 2x2 square out 130 with many other odd squares

$50 + 58 + 3 + 21 + 27 + 9 + 7 + 17 \times 2 \times 2 = n$

Relationships between determinants of 4 semi-norith squares



- 1, 2, 4, 6, 3, 5, 7, 8
- 8, 7, 5, 3, 6, 4, 2, 1
- 1, 2, 4, 6, 3, 5, 7, 8
- 8, 7, 5, 3, 6, 4, 2, 1
- 8, 7, 5, 3, 6, 4, 2, 1
- 1, 2, 4, 6, 3, 5, 7, 8
- 8, 7, 5, 3, 6, 4, 2, 1
- 1, 2, 4, 6, 3, 5, 7, 8

8, 7, 5, 3, 1, 2, 4, 6	56, 0, 56, 0, 56, 0, 56, 0
1, 2, 4, 6, 8, 7, 5, 3	48, 8, 48, 8, 48, 8, 48, 8
8, 7, 5, 3, 1, 2, 4, 6	32, 24, 32, 24, 32, 24, 32, 24
1, 2, 4, 6, 8, 7, 5, 3	16, 40, 16, 40, 16, 40, 16, 40
8, 7, 5, 3, 1, 2, 4, 6	0, 56, 0, 56, 0, 56, 0, 56
1, 2, 4, 6, 8, 7, 5, 3	8, 48, 8, 48, 8, 48, 8, 48
8, 7, 5, 3, 1, 2, 4, 6	24, 32, 24, 32, 24, 32, 24, 32
1, 2, 4, 6, 8, 7, 5, 3	40, 16, 40, 16, 40, 16, 40, 16

57, 2, 60, 6, 59, 5, 63, 8
56, 15, 53, 11, 52, 12, 50, 9
33, 26, 36, 28, 35, 29, 39, 30
24, 47, 21, 43, 22, 44, 18, 41
8, 63, 5, 59,

3, 5, 7, 8, 6, 4, 2, 1	17, 42, 20, 46, 19, 45, 23, 48
6, 4, 2, 1, 3, 5, 7, 8	40, 31, 37, 27, 39, 28, 34, 25
3, 5, 7, 8, 6, 4, 2, 1	49, 10, 52, 14, 51, 13, 55, 16
6, 4, 2, 1, 3, 5, 7, 8	64, 7, 61, 3, 62, 4, 58, 1
3, 5, 7, 8, 6, 4, 2, 1	48, 23, 45, 19, 46, 20, 42, 17
6, 4, 2, 1, 3, 5, 7, 8	25, 32, 29, 38, 27, 37, 31, 40
3, 5, 7, 8, 6, 4, 2, 1	
6, 4, 2, 1, 3, 5, 7, 8	

X rep

Lecture I. Ability with Nature of continuity of functions.

1. What is mathematics? A science dealing in the abstract with the nature of things. Anything can be measured, either as a number (integer), or a quantity (real number). [We shall not go into a priori defs etc, which we shall consider later. Now, assume we know mathematics & physics, & we deal with the relation ^{between} ~~to~~ math & phys, or the logistics of the mathematical representation and analysis of physical phenomena.] Then relations between different quantities are represented by functions and it is our purpose to study the properties of functions of various kinds and their interrelationships. These can be expressed by equations (~~or inequalities more generally, but rarely~~). The equations are of various types - e.g. algebraic equations, differential equations or functional equations etc. They often represent what may be loosely called "laws" of physics, (or any other experimental science), and solutions of particular problems can be obtained by treating mathematically the input data fed along with the relevant equations.

2. The above is, in general, applied mathematics. In fact, all mathematics arose as applied math - e.g. the concept of numbers, arithmetic, geometry, algebra, trigonometry, calculus etc. But soon the abstract mathematics could be generalized to situations beyond those directly connected with physical reality - e.g. complex numbers. We shall consider these a little more to make clear a point