~~9/7/79~~

Book. No. 23

9/7/79 at C.C.B.

Math. Reviews –
Vol. 53

No. 1 – Jan. 1977

(1) Erdos: Show that 105 is the largest ~~number~~ integer m such that $m-2^k$ is prime for all $k \geq 1$ for which $m-2^k$ is +ve.

(2) Math. Mag. 48 (1975) no. 5, 263–66.

Peter Hagis Jr: Although about 1100 pairs of amicable nos have been discovered, it is still an open question whether or not a relatively prime pair of amicable numbers exist. Author shows that in such a case mn is divisible by at least 22 different primes – Aid of Computer.

(3) Math. Mag 49 (1976), no. 1, 37–39

N. S. Mendelsohn: It is well known that $\phi(x) = 14$ has no soln & that $\phi(x) = 24$ has ten solns [$\phi(n)$ = Euler's fn]

Proves the theorem: There are ∞^{ly} many primes p such that the eqn

(2)

$\phi(n) = 2^n p$ has no soln for every +ve integer $\underline{n}$.

(4) Peter Bonnar - New Zealand Math Mag. 12 (1975), no. 2, 85-87
Problem of repn by two, three & four consecutive squares & the following eqns are investigated
(i) $(n-1)^2 + x^2 = n^2 + y^2$, (ii) $(n-1)^2 + x^2 = n^2 + y^2 = (n+1)^2 + z^2$
(iii) $(n-1)^2 + w^2 = n^2 + x^2 = (n+1)^2 + y^2 = (n+2)^2 + z^2$ & a complete soln is given

(5) P. Erdos: Repartition des nombres superabundant (Some ...)
S. Ramanujan defined & studied highly composite numbers each of which has more divisors than any smaller number (Ref. Collected Papers, C.U.P. 1927). Let an integer be called superabundant if $\sigma(m)/m < \sigma(n)/n$ for every $m < n$ - ... complicated theorems [Ref. to work of Erdos on highly composite nos: Proc. Lond. Math. Soc. 19 (1944), 130-33]

(6) Math. Reviews Vol. 53 - No 2 - Feb. 1977

(7) Ibid " No. 3 - March 1977

(1) Gauss' 3-square theorem: - A +ve integer is a sum of three squares if & only if $\underline{m}$ is not of the form $4^a(8k+7)$

Math. Reviews. Vol. 53, No. 4, - April 1977

(1) Egyptian fraction

$a/b = \frac{1}{n_1} + \frac{1}{n_2} + \dots + \frac{1}{n_k}$ $(n_1 < n_2 < \dots < n_k)$

(2) on $y^2 = x^3 + k$ by H. Mordell - Review by Chowla

The author gives a nice account of some of the beautiful recent work on this subject (from Thue-Siegel-Roth and Skolem to Baker & Stark)

(3) A conjecture of Schinzel. If x, y, z satisfy $x^x y^y = z^z$ & are different from 1, then they have the same prime factors. - Proof given

(4) Tijdeman R. - on the equation of Catalan

Catalan's Problem. The only soln in integers $p>1, q>1, x>1, y>1$ of the eqn (1) $x^p - y^q = 1$ is $p = y = 2, q = x = 3$.

Theorem 1. The eqn (1) has only finitely many solutions in integers p, q, x, y. (all >1) & effective bounds for the solns p, q, x, y can be given

Review by Cudnovskii (who refers to his own work)

(5) Math. Reviews - Vol. 53, No. 5 - May 1977

(1) Ref. to a paper in Math. Comp 30 (1976) no. 135, 646-48 by S. Brudno - giving a 2-parameter soln of the eqn

(4)

$x^6+y^6+z^6=u^6+v^6+w^6$. The soln obtained is homogeneous of degree four. It also satisfies the eqn $x^3+y^3+z^3=u^3+v^3+w^3$ and $3x+y+z=3u+v+w$.

Ibid – Vol. 53, No. 6, June 1977

(1) Powerful number S. W. Golomb (Amer. Math. Monthly 77 (1970), 848–55) defines a number to be powerful if every prime factor occurs at least to the second degree. Golomb shows how consideration of certain Pell eqns such as $x^2-8y^2=1$ lead to examples of ∞^{ly} many pairs of consecutive numbers that are powerful with one of the numbers a square.
David T. Walker (Fib. Q.S), 14 (1976), no. 3, 111–16 answers Golomb's Qn about ~~pairs~~ existence consecutive nos powerful & non-square in the affirmative.

(2) On the location of frets in a guitar by I. J. Schoenberg (Amer. Math. Monthly 83 (1976), no. 7, 550–52 – Review by Coxeter – food for math. recreations.

1/8/79 At Institute

(1) Math. of Computation – Vol. 32, No. 1, – Oct 1978

P. 1306 — Some primes with interesting digit patterns by H. C. Williams.

Abstract — Several tables of prime numbers whose forms are generalisations of the form $(10^n - 1)/9$ of the repunit numbers are presented. The repunit number $(10^{317} - 1)/9$ is shown to be a prime

Extended forms

$$N_1(n, r) = (10^{n+1} + 9r - 10)/9$$

$$N_2(n, r) = \{(9r+1)10^n - 1\}/9$$

$$N_3(n, r) = [10^{2n+1} + 9(r-1)10^n - 1]/9$$

$$N_4(n, k, r) = 10^r B_k (10^{nk} - 1)/(10^k - 1) + B_r \quad (1 \le r \le k)$$

where $n \ge 1$, $1 \le r \le 9$ and

$$B_k = (10^{k+1} - 9k - 10)/81$$

Each of these is a generalisation of the repunit number ($r = 1$ & $k = 1$)

Digital patterns

$$N_1(n, r) = \underbrace{111\ldots1}_{n \text{ ones}}r$$

$$N_2(n, r) = r\underbrace{111\ldots1}_{n \text{ ones}}$$

(6)

$$N_3(n,r) = \underbrace{111\cdots1}_{n \text{ ones}}\,r\,\underbrace{111\cdots1}_{n \text{ ones}}$$

$$N_4(n,r) = \underbrace{B_k B_k \cdots B_k}_{n\ B_k\text{'s}} B_r$$

where B_k is a block of digits $(1 \le k \le 9)$ with the form

$$123 \cdots k$$

For eg: $N_1(2,3) = 113$, $N_2(2,3) = 311$, $N_3(2,3) = 11311$

$N_4(2,5,3) = 1234512345123$

[Note that $N_3(n,r)$ is a palindrome)

Note that $N_1(n,r)$ cannot be a prime for $r = 2,4,5,6,8$. Also

$N_3(n,2)$ cannot be prime as

$$N_3(n,2) = (10^n+1)(10^{n+1}-1)/9$$

Tables of primes of the form N_1, N_2, N_3, N_4 which have no more than 100 digits were found. This was done on an IBM system 370 model 168 Computer using the prime testing routines described in Brillhart, Lehmer & Selfridge [Math. Comp. 29, 1975 p. 620-647] and Williams & Judd [ibid. 30, 1976, pp. 155-172] and [ibid pp. 867-880]

Table 3 contains primes of the form $N_3(n, r)$ such that $n \leq 50$

r	$n \leq 50$	r	$n \leq 50$
3	1, 2, 19	7	3, 33
4	2, 3, 32, 45	8	1, 4, 6, 7
5	1, 7, 45	9	1, 4, 26
6	10, 14, 40		

The prime $N_4(1,9,1)$ has been previously discovered by Madachy [J.R.M, 4, 1971, p.100] and the prime $N_4(3,9,1)$ by Finkelstein and Leybourne [J.R.M, 6, 1973, pp.204-205]

A new repunit prime $R_{317} = (10^{317}-1)/9$ was proved prime by using Brillhart, Lehmer & Selfridge's method - colossal amount of computer work

[Notation. F from paper of B, L & S.:

$N-1 = F_1 R_1$ where F_1 is the even factored portion

$N-1$, $R_1 > 1$ and $(F_1, R_1) = 1$

$N+1 = F_2 R_2$, $\overline{F}_1 = F_1/2$, $\overline{F}_2 = F_2/2$.

"N is a psp base a" will be used for a ~~prime~~ number N ~~to~~ which satisfies the congruence $a^{N-1} \equiv 1 \pmod N$, $1 < a < N-1$ i.e

N is a "pseudoprime" base $\underline{a}$

p. 637 – Let N be a pseudoprime of the form $(a^{128}+1)/257$, then

$N-1 = (a^{128}-256)/257 = (a^{16}-2)(a^{16}+2)(a^{16}-2a^{8}+2)(a^{16}+2a^{8}+2)(a^{64}+16)/257$

In Tables on p. 638 various factorisation of ~~Fermat~~ Mersenne & Lucas nos are given

N_p (2^p-1) Prime for p = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127

[but 9 (larger Mersenne nos) 23 known]

Ref. H. Yates – J.R.M, $\underline{3}$, 1970, pp. 114 – 119

– " $\underline{8}$, 1975, pp. 33 – 38.

B.V.L (C.C.B) – 5/10/79

Math. Magazine – Vol. 52, No. 1, January 1979

$\underline{p.3}$ – Linearity of exponentiation by John O. Kiltinen

Students' errors. $\sqrt{(a+b)} = \sqrt{a}+\sqrt{b}$, $\frac{1}{a+b} = \frac{1}{a}+\frac{1}{b}$, $(a+b)^2 = a^2+b^2$

$(a+b)^m = a^m+b^m$ – Investigates when such things hold in a ring R.

$\underline{Ex}$ (1) For what +ve integers does $(a+b)^7 = a^7+b^7$ hold in Z_n?

(Z_n = integers modulo n).

Ans; 2, 3, 6, 7, 14, 21 & 42 are the only possible values of n for which Z_n is 7-linear

Ex(2) For what values of m is Z_{115} m-linear?

Ans: It is sufficient for m to be of the form $44k+1$ (k = any +ve integer)

Thus Z_{115} is 45-linear, 89-linear, 133-linear etc.

Quickies. Find all solns to the diophantine eqn

$$1!+2!+\dots+n! = m^2$$

Ans. $n=m=1$ & $n=m=3$

Solns: Find all solns (x,y) of $x^y = y^{x-y}$ (x, y +ve integers)

Ans: only solns are $(1,1)$, $(9,3)$, $(8,2)$

Palindromic sums (p.55)

A decomposition of a +ve integer n is an ordered tuple $(n_1, n_2, \dots, n_k)$ of +ve integers such that $\sum_{i=1}^{k} n_i = n$. Find the total no. of decompositions of n which are palindromes. For eg. for $n=4$, there are four such: (4), $(2,2)$, $(1,1,1,1)$ & $(1,2,1)$ [This eg. due to Michael Capobianco, St John's Univ]

Soln: For each palindromic decomposition of n, we construct two such decompositions of $n+2$ according to the rules:

(a) Append a 1 to each end of the original decomposition

(b) Add 1 to the first & last members of the original decomposition (or add 2 if the original decomposition consists of exactly one number)

That this generates all possible palindromic decompositions of $n+2$ without duplicating any, can be seen by imagining the reverse process, which is a function well-defined on the domain of palindromic decompositions of any $n+2$:

(a') If a palindromic decomposition has 1's at both ends, delete them

(b') If a palindromic decomposition does not have 1's at both ends → subtract 1 from the number at each end (or 2 if there is only one no.)

Since the no. of pal. decomps of 1 and 2 are 1 and 2 resp. we have $2^{[n/2]}$ pal. decomps of each +ve integer n, where $[n/2]$ denotes the greatest integer less than or equal to $n/2$

Marguerite Gerstell

Florida Inst. of Technology

Vol. 52. No. 2. — March 1979.

p.67: Some unconventional problems in number theory by

Paul Erdos (Hungarian Academy of Sciences, Univ. of Colorado, Boulder, CO 80309).

I state some curious, unusual and mostly unsolved problems in various branches of number theory

Large number of problems with 11 references ~~at end~~ at the end mostly his own conjectures more than forty years old

Frey: (1) Does $x^x y^y = z^z$ have any non-trivial solution in integers? (40 years old). – Chao Ko found infinitely many solns. Perhaps he found them all [Ref. J. Chinese Math, $\underline{2}$ (1940), 205–207 Math. Rev. V. 2. p-346]

(2) old conjecture of Straus & Erdos that for every $n > 3$

$$\frac{4}{n} = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}$$

is solvable in integers x_1, x_2, x_3 where $1 \le x_1 < x_2 < x_3$. This conjecture seems surprisingly difficult. A forthcoming paper of Straus & Subbarao deals with some related questions

(3) Cramer's conjecture that

$$\limsup_{n \to \infty} \frac{p_{n+1} - p_n}{(\log n)^2} = 1 \qquad (p_n = n^{th} \text{ prime}$$

This conjecture is completely unattackable by present day methods and I

(12) expect that it will stay in this class for a long time.

Let $Q_1 < Q_2 < \dots$ be the sequence of consecutive squarefree numbers. It is curious that $Q_{n+1} - Q_n$ is almost as difficult as $p_{n+1} - p_n$. The best upper bound is, as far as I know, is still due to Richert & Rankin [Q. J. Math 6 (Ser. 2), 1955, 147–153 & J. Lond. Math. Soc. 29 (1954), 16–20] who proved that for every $\epsilon > 0$, $n > n_0$, $Q_{n+1} - Q_n < n^{2/9+\epsilon}$. There is no doubt that this inequality holds with $2/9+\epsilon$ replaced by ϵ, but the proof is nowhere in sight. Perhaps $Q_{n+1} - Q_n < c \log n$ holds, but I am very doubtful. It is easy to see that

$$\limsup_{n\to\infty} (Q_{n+1} - Q_n) \log\log n \, (\log n)^{-1} \geq \pi^2/6$$

& as far as I know, this has never been improved.

[To be read more carefully later]

Reviewing a book "Mathematical Morsels" by Ross Honsberger

Dolciani Math. Expos, No. 3, MAA, 1978,

Scores of elementary problems culled primarily from the Amer. Math. Monthly.