Inaugural Address delivered before

the Mathematical Society of

The St. Joseph's College, Bangalore

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Friday 12th October 1944.

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Subject:- Giant and Dwarf numbers.

Abstract & Summary:

~~Small numbers~~ 1) $32 = g$.

2) Compressibility of water $= 50 \cdot 10^{-6}\ \text{Atm}^{-1}$!

Relativity: (3) $\Delta t = \frac{l}{c} \cdot \frac{v^2}{c^2}$ (difference in time between journeys in Michelson-Morley Expt)

$= 10^{-15}$ sec ($l = 30\,m$, $v = 30\,km/sec$)

(4) $C = 3 \cdot 10^{10}$ km/sec

(5) Energy lost by sun's radiation per year $= 4\pi \cdot 150^2 \cdot 10^{22} \cdot 2 \cdot 4 \cdot 60 \cdot 24 \cdot 365$ erg.

$\simeq 10^{34}$ ergs

$= 10^{34} / (9 \cdot 10^{20} = c^2) = \underline{138 \times 10^{12}\ \text{tons}}$.

(6) Wave length of Cu. K_α-radn $= 1541 \cdot 10^{-11}$ cm.

(7) lattice const $= 3.13 \times 10^{-8}$ cm.

(8) Electrical attraction between ions (separated) of 1 mol of common salt at distance equal to that between the poles of the Earth $= 5.3 \times 10^{10}$ dyn $= 80$ ton weight!

(Refer to Zwicky's article in the College)

(9) Bohr's magneton (mag. moment) $= 9.27 \times 10^{-21}$ el. magn. C.G.S.

(10) $\lambda_{\gamma\text{-rays}} \sim 10^{-11}$ cm

(11) $\Delta E \cdot \Delta t \approx h$. ∴ ~~...~~ time span of atomic transitions $= 10^{-8}$ sec.
If this be divided in 10^7 intervals, error in $E = \frac{h}{10^{-15}} = 6.6 \times 10^{-12}$ erg
which = energy of 5 electron-volts = excitation energy i.e. error is of the same order as the total.

(12) atomic dimn $= 10^{-8}$ cm, nuclear dimensions 10^{-13} to 10^{-12} cm.

(13) Cosmic ray energies above 10^9 e.v.; 1 e.v $= 1.6 \times 10^{-12}$ erg.

1 electron mass $= 5 \cdot 10^5$ e.v.

(14) decay of mesotron $= 10^{-6}$ sec. $10^{-8} / 10^{10} = 10^{-18}$ time to traverse atomic orbit. (?)

(15) $\nu_{\text{Compton}} \sim 10^{25}$. }
$\lambda \sim 10^{-20}$ cm. }

(16) Mass of sun $\sim 10^{33}$ gm.

Imp (17) $R = 2.3 \times 10^{39}$ = ratio between electric & gravitational attraction between an electron and proton.

" (18) In the expanding Universe hyp. $\Delta\nu/\nu = -D/R'$ $R' = 6 \times 10^{39}$.

" (19) N (Eddington's number) $= 1.2 \times 10^{78}$.

" (20) eqm between radn & matter: ratio of concentration of matter in ordinary forms to concentration of matter → radn ie $\frac{C_R}{C_m} = e^{mc^2/KT} \sim e^{6 \times 10^7}$

In a sphere of enormous radius R' not a single electron could exist in statistical competition with radiation

(21) $L_{\text{Star}} \sim 10^{39}$ ergs/sec. & 10^{53} ergs/sec (Supernovae)

(22) ~~10^{60}~~ ergs 10^{55} = total energy of annihilation of whole star.

Astr: (23) masses of stars $\sim 10^{34}$ gm.

(24) Central temp. of stars $\sim 10^8$ degrees }

(25) 1 gm of radium emits 2×10^6 ergs/sec }.

(26) radius of Antares $= 2 \times 10^8$ miles.

(27) diameter of galaxy $= 10^5$ l.y $= 10^5 . 6 . 10^{12}$ miles $= 6 . 10^{17}$
$= 10^5 . 10^{13}$ Km $= 10^5 . 10^{13} . 10^3 . 10^2 = 10^{23}$ cm.

1 l.y $= 10^{13}$ Km
$= 10^{18}$ cm

(28) Rotation of galaxy $= 2 \times 10^8$ years

1 year $\approx 10^7$ secs.

(29) mass of galactic system $= 2 \times 10^5 \times 10^6 \times 10^{33} \sim 10^{44}$ gm.

(30) greatest distance of nebulae $=$ ~~20~~ $2.5 \times 10^8 \times 10^{18} = 2.5 \times 10^{26}$ cm

(31) Total number of stars $= 10^{78}/10^{33} = 10^{45}$.

(32) Age of the Universe $= 10^9$ years.

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Summary of lecture

(1) Introduction – talk about numbers – fundamental notion – Kronecker, Study, Gauss, Dantzig – philosophic aspect – math. aspect (theory of numbers) – nature of theory of numbers – Ramanujan, Littlewood's remark, Hardy's motor car numbr. $1729 = 12^3 + 1^3 = 10^3 + 9^3$ – $158^4 + 59^4 = 134^4 + 133^4 = 635{,}318{,}657$ – Prime number theory, Prime number theorem $\pi(x)$ – Proof all important (no. of protons in Univ. Eddington's proof?) what about $2^{127}-1$ being a prime? Goldbach's theorem any even no > 2 is sum of 2 primes – Pillai & Chowla – other number marvels & freaks – ~~not speaking about these topics – talking about small & big nos – giant & dwarf numbers~~ number cults (mystic nos) – mystery of 1, 2, 3, 4, 5, 6, 7, 8, 9 – ~~each one may be~~ not speaking about these, but about big & small nos ie giant & dwarf numbers – what is there to speak about them? so easy to generate big numbers, 1 followed by ~~sev~~ ciphers ie 10^n, other methods, from big to small $1/10^n = 10^{-n}$ – I shall speak only of those which have some meaning physically or astronomically or otherwise – meaningful giant & dwarf numbers.

(2) Small numbers — microscopic region – smallest nos are m_e & h ~~about~~ $\sim 10^{-27}$ units, quantities specifying pattern & building materials of the Universe – next $m_p \sim 10^{-24}$ – next Bohr's magneton $\sim 10^{-21}$ – $\lambda_{\text{cosmic ray}} \sim 10^{-20}$ cm. – Boltzmann constant $\sim 10^{-16}$ $(E = kT)$ – nuclear dimension $\sim 10^{-13}$ to 10^{-12} cm – ~~atomic dimension 10^{-8}~~ charge on an electron $\sim 10^{-10}$ – atomic dimensions $\sim 10^{-8}$ cm ($r_1 \sim 10^{-8}$) – time span of atomic transitions 10^{-8} sec (~~stellar [illegible] 10~~) – decay of

meson $\sim 10^{-6}$ sec – other derived small quantities eg. time to traverse atomic orbit $\sim 10^{-18}$ sec is however has no meaning since classical pictures cannot be carelessly extended: thus $\Delta E \cdot \Delta t \sim h$ and for $\Delta t \sim 10^{-15}$ secs, $\Delta E \sim 5$ eV $\sim$ excitation energy ie error of same order as the total – another example: M. I of electron as a gyrostat $\sim 10^{-28} \times 10^{-16} \sim 10^{-44}$: this again is a meaningless classical picture – point particles are more satisfactory – So far as meaningful small numbers quantum theory sets a limit, perhaps to $\sim 10^{-27}$ is the ~~least~~ order of the smallest numbers.

(3) Big numbers – start with 10^6 (million) time to count about 10 days at one a second – rotation of galaxy 2×10^8 years $\sim 6 \times 10^{15}$ secs – radius of Antares 2×10^8 miles $\sim 3 \times 10^{13}$ cm – ~~cosmic ray energies 10^9 eV~~ – Central temp of stars 10^8 degrees (Go to a hotter place – cosmic ray energies $\sim 10^9$ eV – age of Universe 2×10^9 years $\sim 6 \times 10^{16}$ secs – velocity of light 3×10^{10} cm/sec (absolute limit – Faraday charge (eL) $\sim 10^{14}$ – N (no of gas atoms in 1 cc) $\sim 10^{19}$ – L (Loschmidt no. = mols in 1 gm-mol) $\sim 10^{23}$ – diameter of galaxy $\sim 10^{23}$ cm – ~~most distant nebulae $\sim 10^{26}$ cm~~ – frequency of cosmic rays $\sim 10^{25}$ – most distant nebulae $\sim 10^{26}$ cm – ~~Radio~~ Mass of stars $\sim 10^{35}$ gm – luminosity of stars $\sim 10^{39}$ erg/sec – ~~Ratio of~~ constant of expansion of Universe $R' \sim 6 \times 10^{39}$ – ratio between elec. & grav. attn between elec & proton $\sim 2 \times 10^{39} = R$ – mass of galactic system 10^{44} gm – Total number of stars 10^{50} – Total no of electrons & protons $\sim 10^{80}$ (Eddington's number) – All big numbers $< 10^{100}$.

1 year $\sim 3 \times 10^7$ secs
1 l. year $\sim 10^{18}$ cm.
1 mile ~ 1.6 km $\sim 1.6 \times 10^5$

(4). Eddington's number ~~or~~ a googol – bigger than any number in every day parlance – grains of sand – rain drops – love of woman –

(5) Kasner's number – $10^{\text{~~googol~~}}$ $10^{\text{Eddington's number}}$ $= 10^{10^{100}}$ – ~~$10^{...}$~~ not $10^{100} \times 10^{100}$ which 10^{200} but $10^{100} \times 10^{100} \times$ - 10^{98} times ie a googol multiplied by itself a hundredth of a googol times or 1 followed by a googol no. of zeros – tour of all stars & nebulae putting zeros – meaning of this – (1) miracle of rising book, molecules of air bombarding etc &(2) heat flowing from cold to hot body – Stern's number – e^{10^8} or $10^{10^{10}}$ same order as Kasner's number

So far as no of elements in the chain are considered. This arises from $\frac{c_R}{c_m} = e^{6\times10^7} \sim e^{10^8}$. eqm between radiation & matter : ratios of concentration of matter in ordinary form & ~~form~~ [illegible] of matter transforming to radn. In a sphere of radius R' ($\sim 10^{39}$ cm) n[illegible] a single electron could exist in statistical competition with radiation. — Chess number $\sim 10^{10^{50}}$ ie number of possible games of chess same order as Kasner's number.

(6) Skewes number — $10^{10^{10^{35}}}$: How this arises

$\pi(x)$ for x large is approx $x/\log x$ $f_1(x)$. (Prime number theorem)

A much better approximation is $\operatorname{li} x = \int_0^x \frac{dt}{\log t} = \phi_2(x)$

A still better one is $\phi_3(x) = \phi(x) - \frac{1}{2}\phi(\sqrt{x}) - \frac{1}{3}\phi(\sqrt[3]{x}) - \ldots$

Naturally one would infer $\pi(x) < \phi(x)$ - Gauss's conjecture & verified by all the evidence ie table of primes up to 10^7 & these no. at intervals up to 10^9. Littlewood proves this conjecture false. Then what is the smallest x ie how ~~far~~ should one go to prove this ie to find $\pi(x) > \phi(x)$. This number is Skewe's number. It is the ~~Also Skewe's number~~ largest number which has ever served any definite purpose in Mathematics.

If Univ. were Chessboard & squares of dimension of [illegible] & protons were Chessmen & ~~no of~~ interchange of two a move, no of possible games would be of order of Skewe's number.

(7) Conclusion — Biggest numbers in Astr & Phys 10^{100} ie one storey high - Biggest even in theories of Astr & Phys, $10^{10^{100}}$ ie 2 storeys high - also in Chess. To go only storey high is to find an appln only in pure Mathematics & that too Theory of numbers. - The Queen ~~of the~~ Queen. ~~In winding up this rambling talk it is my earnest hope that it be no longer possible numbers~~ In winding this

rambling talk, I only want to recall the prayer of the purest of pure mathematicians viz Oh! God let not the purity of my subject be sullied by applications, ~~and~~ but pray Oh! God let numbers greater than the Skewe's number be never constructed & may they have no applications not even in the theory of numbers!

Prel. Math: Numbers:

10^{-27} m_e $\hbar$

10^{-24} m_p

10^{-21} e_M

10^{-20} $\lambda_{cos.}$

10^{-16} κ.

10^{-13} r_n

10^{-8} a_H, t_{at}. A. Unit, G.

10^{-6} mesonde[illegible]

10^{-44} M.Z [illegible]

10^{-5} to 10^{5} passover as terrestrial

10^{6} million.

10^{7} Astr. unit.

10^{8} Tc.

10^{9} E_{cosmic}

10^{10} c

10^{13} $r_{Antares}$

10^{15} Prot. gal.

10^{16} age of Universe

10^{19} (sec atom [illegible])

10^{23} D. galaxie

10^{25} V_{cosmic}

10^{29} dist. nebulae

10^{35} mass [illegible]

10^{39} – luminous [illegible]

10^{44} – mass galaxy

10^{50} – Total no [illegible]

10^{80} – Eddington –

$10^{10^{100}}$ $10^{10^{50}}$

$e^{10^{8}}$

$10^{10^{10^{35}}}$ $f(x) \equiv \Phi(x)$ –

Skewes number [illegible]

Small

1) 10^{-27} — m_e & h

2) 10^{-24} — m_p

3) 10^{-21} — e_M (magneton)

4) 10^{-20} — $\lambda_{cosmic\ ray}$

5) 10^{-16} — k (Boltzmann)

6) $10^{-13}-10^{-12}$ — $r_{nucleus}$

7) 10^{-10} — e

8) 10^{-8} — a_H

9) 10^{-8} — t_{atom}

10) 10^{-7} — ρ_σ

11) 10^{-6} — meson decay

derived $\left\{ \begin{array}{l} 10^{-18} \text{ sec} \\ 14.7 \sim 10^{-44} \end{array} \right\}$ absurd.

C.G.S units :

Big

1) ~~10^6~~ — million ——— (1)

~~$10^{?}$ — rot. of galaxy (6×10^{15} secs).~~

2) 10^8 — $T_{c,\sigma}$ ——— (2)

~~10^9 — cosmic ray energies~~

3) 2×10^8 years — Gal. rot^n — 6×10^{15} ——— (6)

4) 2×10^8 miles — $r_{Antares}$. — 3×10^{13}. ——— (5)

5) 10^9 — Cosmic ray energies ——— (3)

6) 2×10^9 years — Age of Universe — 6×10^{16}. ——— (7)

7) 3×10^{10} — C ——— (4)

8) 10^{19} — N (gas atoms in 1 CC) (8)

9) 10^{23} — L - diam of galaxy ——— 9

10) 10^{25} — $\nu_{cosmic\ rays}$. ——— 10

11) 10^{29} — distant nebulae ——— "

12) 10^{35} — Mass of a star ——— "

13) 10^{39} — luminosity of star — const. of expn of Univ R' "
— elec/grav.

14) 10^{44} — mass of gal. system "

15) 10^{50} — Total no. of stars "

16) 10^{80} — Eddington's number "

Is this a good pencil? Yes it is