

QUESTION OF INVARIANCE IN THE NEUTRINO THEORY OF LIGHT.

BY B. S. MADHAVA RAO.

(From the Department of Mathematics, University of Mysore, Bangalore.)

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§ 1. Introduction.

THE neutrino theory of light, initiated by Jordan, has been developed in a series of papers by Jordan, Kronig, Born, Nagendra Nath, Pryce and others.¹ The essential point of the theory is the setting up of a relation between the neutrino operators and the photon operators such that while the former satisfy the commutation laws pertaining to the Fermi-Dirac statistics the latter satisfy those of the Einstein-Bose statistics. I have examined in this paper the question of Lorentz-invariance of this fundamental relation connecting the two sets of operators, and, by considering rotations of the frame of reference, shown that the relation itself is not invariant.

§ 2. The Relation between the Operators.

I shall use, here, the form of Jordan's relation as given by Nagendra Nath² wherein he takes the spin also into consideration.

$$b_{k,\rho} = \frac{-1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_i a_{l,i} \gamma_{k-l,i} \quad (1, a)$$

$$b_{k,\lambda} = \frac{-1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{i,j} a_{l,i} \gamma_{k-l,j} \quad (1, b)$$

or, using the relation $\gamma_{-k} = a_k^\dagger$, and the commutation laws for a 's and γ 's, in the form

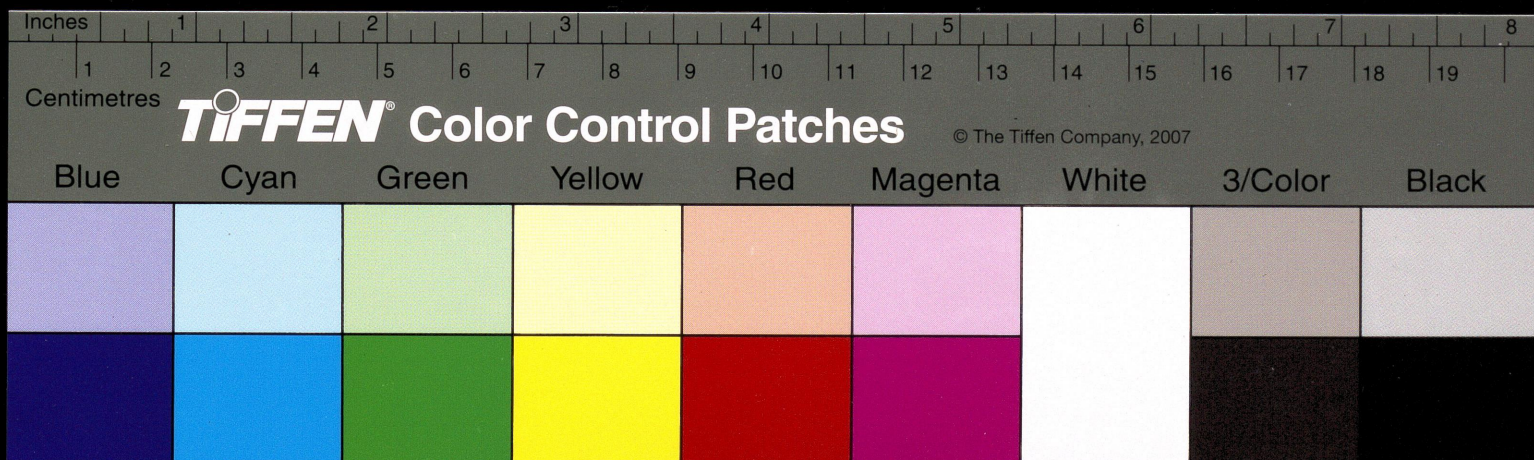
$$b_{k,\rho} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_i a_{l,i}^\dagger a_{l+k,i} \quad (2, a)$$

$$b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{i,j} a_{l,i}^\dagger a_{l+k,j} \quad (2, b)$$

The indices i and j each take two values corresponding to the two spin states of the neutrino denoted by ρ and λ . By the aid of a simple formalism

¹ For a complete bibliography see the report by Kronig in the *Annales de l'Institut Henri Poincaré*, 6.

² *Proc. Ind. Acad. Sci.*, (A), 3 (1936), p. 452, equation (16).



we can get rid of the rather unsymmetrical nature of (2a) and (2b) where we have a summation over i only in the first, and a summation over i and j ($i \neq j$) in the second. Introducing the unit matrix

$$\delta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and one of the Pauli-matrices σ_x (after removing the suffix x) as

$$\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

it can be seen easily that (2, a) — (2, b) are equivalent to the forms

$$b_{k,\rho} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\mu}^\dagger \delta^{\mu\nu} a_{l+k,\nu} \quad (3, a)$$

$$b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\mu}^\dagger \sigma^{\mu\nu} a_{l+k,\nu} \quad (3, b)$$

where the Greek indices μ, ν take values 1 and 2.

§ 3. Transforms of the Operators.

We shall now examine how the operators a , a^\dagger and b are transformed by a rotation of the frame of reference through an angle θ . Since the a 's are spinor magnitudes the transformation matrix for them is given by³

$$T = e^{-\frac{1}{2}i\theta s}; \quad T^\dagger = e^{\frac{1}{2}i\theta s}, \quad (4)$$

where s is the spin-matrix

$$s = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (5)$$

and denoting the transformed quantities by placing a bar over them, we have

$$\left. \begin{aligned} \bar{a}_{l,\mu} &= \sum_{\nu=1}^2 T_{\nu\mu} a_{l\nu} \\ \bar{a}_{l,\mu}^\dagger &= \sum_{\nu=1}^2 a_{l,\nu}^\dagger T_{\nu\mu}^\dagger \end{aligned} \right\} \quad (6)$$

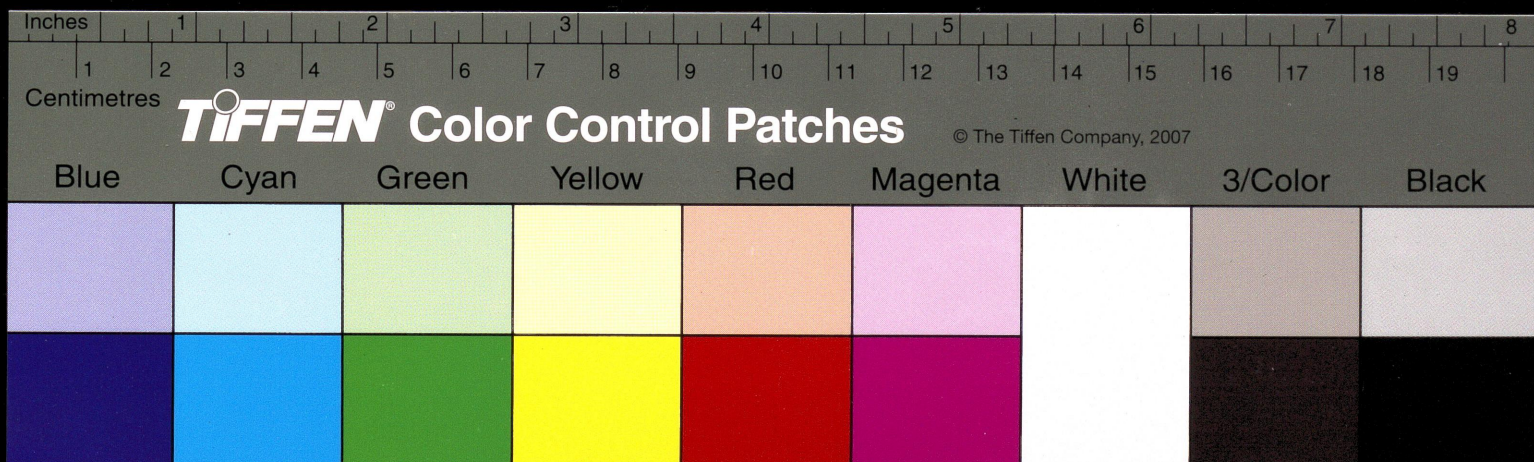
The b -operators undergo the ordinary transformation

$$\left. \begin{aligned} \bar{b}_{k,\rho} &= \cos \theta \cdot b_{k\rho} + \sin \theta \cdot b_{k\lambda} \\ \bar{b}_{k,\lambda} &= -\sin \theta \cdot b_{k\rho} + \cos \theta \cdot b_{k,\lambda} \end{aligned} \right\} \quad (7)$$

§ 4. Non-invariance of the Fundamental Relation.

If the relations (3, a) — (3, b) should be invariant for rotations, they must remain true when a 's and b 's are replaced by \bar{a} 's and \bar{b} 's as given by

³ See Frenkel: *Wave Mechanics (Advanced Theory)*, 1934, pp. 346-62.



(6) and (7). Hence we must have (introducing the necessary dummy indices)

$$\cos \theta \cdot b_{k\rho} + \sin \theta \cdot b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu,\tau,\pi} a_{l,\nu}^\dagger T_{\nu\tau}^\dagger \delta_{\tau\pi} T_{\pi\mu} a_{l+k,\mu} \quad (8, a)$$

$$-\sin \theta \cdot b_{k\rho} + \cos \theta \cdot b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu,\tau,\pi} a_{l,\nu}^\dagger T_{\nu\tau}^\dagger \sigma_{\tau\pi} T_{\mu\pi} a_{l+k,\mu} \quad (8, b)$$

Summing up for the indices τ and π on the right-hand sides, we have them reducing to

$$\left. \begin{aligned} \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\nu}^\dagger (T_{\nu 1}^\dagger T_{1\mu} + T_{\nu 2}^\dagger T_{2\mu}) a_{l+k,\mu} \\ \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\nu}^\dagger (T_{\nu 2}^\dagger T_{1\mu} + T_{\nu 1}^\dagger T_{2\mu}) a_{l+k,\mu} \end{aligned} \right\} \quad (9)$$

In the left-hand sides of (8, a) — (8, b) we can replace $b_{k\rho}$ and $b_{k,\lambda}$ by means of (3, a) — (3, b) and they should then be identically the same as (9). This ought to give us the relations

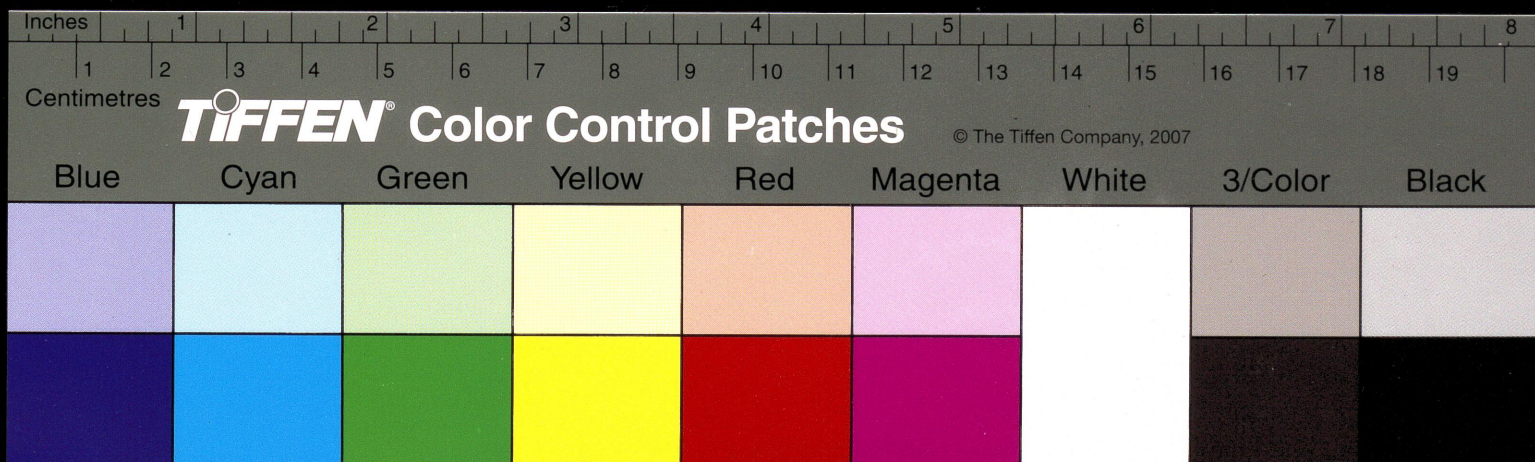
$$\cos \theta \cdot \delta^{\nu\mu} + \sin \theta \cdot \sigma^{\nu\mu} = T_{\nu 1}^\dagger T_{1\mu} + T_{\nu 2}^\dagger T_{2\mu} \quad (10, a)$$

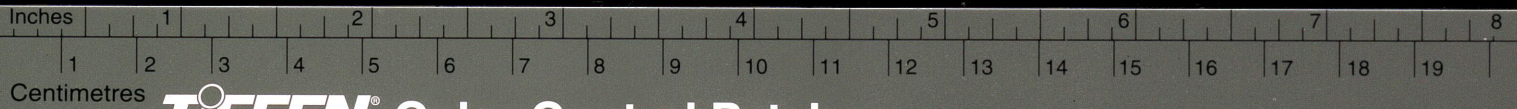
$$-\sin \theta \cdot \delta^{\nu\mu} + \cos \theta \cdot \sigma^{\nu\mu} = T_{\nu 2}^\dagger T_{1\mu} + T_{\nu 1}^\dagger T_{2\mu} \quad (10, b)$$

Equating the sides of (10, a) — (10, b) for the indices $\nu\mu = 11, 12, 21, 22$ and using (4) and (5) we see that these lead to the relations

$$\sin \theta = 0, \quad \cos \theta = 1, \quad \cos \theta = e^{i\theta} = e^{-i\theta}$$

which are not true for any value of θ (except $\theta = 0$). Thus the falsity of (10) establishes the non-invariance of the relation connecting the a - and b -operators.





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