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QUESTION OF INVARIANCE IN THE NEUTRINO THEORY OF LIGHT.

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§ 1. *Introduction.*

The neutrino theory of light, initiated by Jordan, has been developed in a series of papers by Jordan, Kronig, Born, Nagendra Nath, Pryce and others.[1] The essential point of the theory is the setting up of a relation between the neutrino operators and the photon operators such that while the former satisfy the commutation laws pertaining to the Fermi-Dirac statistics the latter satisfy those of the Einstein-Bose statistics. I have examined in this paper the question of Lorentz-invariance of this fundamental relation connecting the two sets of operators, and, by considering rotations of the frame of reference, shown that the relation itself is not invariant.

§ 2. *The Relation between the Operators.*

I shall use, here, the form of Jordan's relation as given by Nagendra Nath[2] wherein he takes the spin also into consideration.

$$b_{k,\rho} = \frac{-1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{i} a_{l,i}\, \gamma_{k-l,i} \qquad (1,\, a)$$

$$b_{k,\lambda} = \frac{-1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{i,j}{}' a_{l,i}\, \gamma_{k-l,j} \qquad (1,\, b)$$

or, using the relation $\gamma_{-k} = a_k^{\dagger}$, and the commutation laws for a's and γ's, in the form

$$b_{k,\rho} = \frac{\;}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{i} a_{l,i}^{\dagger}\, a_{l+k,i} \qquad (2,\, a)$$

$$b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{i,j}{}' a_{l,i}^{\dagger}\, a_{l+k,j} \qquad (2,\, b)$$

The indices i and j each take two values corresponding to the two spin states of the neutrino denoted by ρ and λ. By the aid of a simple formalism

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[1] For a complete bibliography see the report by Kronig in the *Annales de l' Institut Henri Poincare*, 6.

[2] *Proc. Ind. Acad. Sci.*, (A), 3 (1936), p. 452, equation (16).

we can get rid of the rather unsymmetrical nature of (2a) and (2b) where we have a summation over i only in the first, and a summation over i and j $(i \neq j)$ in the second. Introducing the unit matrix

$$\delta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and one of the Pauli-matrices σ_x (after removing the suffix x) as

$$\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

it can be seen easily that (2, a) — (2, b) are equivalent to the forms

$$b_{k,\rho} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\mu}{}^{\dagger}\, \delta^{\mu\nu}\, a_{l+k,\nu} \tag{3, a}$$

$$b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\mu}{}^{\dagger}\, \sigma^{\mu\nu}\, a_{l+k,\nu} \tag{3, b}$$

where the Greek indices μ, ν take values 1 and 2.

§ 3. *Transforms of the Operators.*

We shall now examine how the operators a, $a^{\dagger}$ and b are transformed by a rotation of the frame of reference through an angle θ. Since the a's are spinor magnitudes the transformation matrix for them is given by[3]

$$\mathrm{T} = e^{-\frac{1}{2}i\theta s}\,; \quad \mathrm{T}^{\dagger} = e^{\frac{1}{2}i\theta s}, \tag{4}$$

where s is the spin-matrix

$$s = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{5}$$

and denoting the transformed quantities by placing a bar over them, we have

$$\left.\begin{aligned} \bar{a}_{l,\mu} &= \sum_{\nu=1}^{2} \mathrm{T}_{\nu\mu}\, a_{l\nu} \\ \bar{a}_{l,\mu}{}^{\dagger} &= \sum_{\nu=1}^{2} a_{l,\nu}{}^{\dagger}\, \mathrm{T}^{\dagger}_{\nu\mu} \end{aligned}\right\} \tag{6}$$

The b-operators undergo the ordinary transformation

$$\left.\begin{aligned} \bar{b}_{k,\rho} &= \cos\theta\cdot b_{k\rho} + \sin\theta\cdot b_{k\lambda} \\ \bar{b}_{k,\lambda} &= -\sin\theta\cdot b_{k,\rho} + \cos\theta\cdot b_{k,\lambda} \end{aligned}\right\} \tag{7}$$

§ 4. *Non-invariance of the Fundamental Relation.*

If the relations (3, a) — (3, b) should be invariant for rotations, they must remain true when a's and b's are replaced by $\bar{a}$'s and $\bar{b}$'s as given by

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[3] See Frenkel: *Wave Mechanics* (Advanced Theory), 1934, pp. 346-62.

(6) and (7). Hence we must have (introducing the necessary dummy indices)

$$\cos\theta\cdot b_{k\rho} + \sin\theta\cdot b_{k,\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu,\tau,\pi} a_{l,\nu}^{\dagger}\, T_{\nu\tau}^{\dagger}\, \delta^{\tau\pi}\, T_{\pi\mu}\, a_{l+k,\mu} \qquad (8,\ a)$$

$$-\sin\theta\cdot b_{k\rho} + \cos\theta\cdot b_{k\lambda} = \frac{1}{\sqrt{2k}} \sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu,\tau,\pi} a_{l,\nu}^{\dagger}\, T_{\nu\tau}^{\dagger}\, \sigma^{\tau\pi}\, T_{\mu\pi}\, a_{l+k,\mu} \qquad (8,\ b)$$

Summing up for the indices τ and π on the right-hand sides, we have them reducing to

$$\left.\begin{array}{l} \dfrac{1}{\sqrt{2k}} \displaystyle\sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\nu}^{\dagger} (T_{\nu 1}^{\dagger} T_{1\mu} + T_{\nu 2}^{\dagger} T_{2\mu})\, a_{l+k,\mu} \\ \dfrac{1}{\sqrt{2k}} \displaystyle\sum_{l=-\infty}^{+\infty} \sum_{\mu,\nu} a_{l,\nu}^{\dagger} (T_{\nu 2}^{\dagger} T_{1\mu} + T_{\nu 1}^{\dagger} T_{2\mu})\, a_{l+k,\mu} \end{array}\right\} \qquad (9)$$

In the left-hand sides of (8, *a*) — (8, *b*) we can replace $b_{k\rho}$ and $b_{k\lambda}$ by means of (3, *a*) — (3, *b*) and they should then be identically the same as (9). This ought to give us the relations

$$\left.\begin{array}{l} \cos\theta\cdot\delta^{\nu\mu} + \sin\theta\cdot\sigma^{\nu\mu} = T_{\nu 1}^{\dagger} T_{1\mu} + T_{\nu 2}^{\dagger} T_{2\mu} \\ -\sin\theta\cdot\delta^{\nu\mu} + \cos\theta\cdot\sigma^{\nu\mu} = T_{\nu,2}^{\dagger} T_{1\mu} + T_{\nu 1}^{\dagger} T_{2\mu} \end{array}\right\} \qquad \begin{array}{r}(10,\ a)\\(10,\ b)\end{array}$$

Equating the sides of (10, *a*) — (10, *b*) for the indices $\nu\mu = 11,\ 12,\ 21,\ 22$ and using (4) and (5) we see that these lead to the relations

$$\sin\theta = 0, \quad \cos\theta = 1, \quad \cos\theta = e^{i\theta} = e^{-i\theta}$$

which are not true for any value of θ (except $\theta = 0$). Thus the falsity of (10) establishes the non-invariance of the relation connecting the a- and b-operators.

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