Lectures at the I.I.Sc. under the U.G.C. Scheme

Lecture No. 1 – 5/5/67 – General introduction to Relativity and Quantum Mechanics. :

Revolution in the first 30 to 40 years of the present century – several view points of relativity and complementarity or principle of correspondence or principle of indeterminacy or uncertainty of Relativity known to earlier classical physics ($c = \infty$) ; $x' = \dfrac{x - vt}{\sqrt{1-\beta^2}}$, $y' = y$, $z' = z$, $t' = \dfrac{1 - vx/c^2}{\sqrt{1-\beta^2}}$ ie interrelation of space & time – ($c = 3 \times 10^{10}$ cm/sec)
Four dimensional geometry – notion of groups $[a(bc) = (ab)c,\ ae = ea = a,\ aa^{-1} = a^{-1}a = e]$ – L.T. Characteristic of special relativity in 4-space – $E = mc^2$ – Validity of this in a gravitation field & truth of special relativity only in the absence of gravitational fields – Hence $ds^2 = g_{ik}\, dx^i dx^k$ must contain elements other than $\underline{c}$, since a light ray bends in a gravitational field ($g_{ik} = g_{ki}$ ie ten components) – Minkowskian interval in a Lorentz frame replaced by a generalised metric in a Riemannian space based on (i) principle of general covariance, (ii) Principle of local validity of special relativity ($G = 0$), (iii) Principle of the geodesic line – Creation of 5-dimensional theories involving general relativity & electrodynamics – Successes of Gen. Relativity in Astronomy (3) and in cosmology – ~~no of mechanics in quantum theory at atomic level : Success of 1 : 10^{-2} ; 10^{-14} & 10^{-38}~~.

Quantum mechanics unknown, unlike Relativity in the 19th century & a creation of the genius of Niels Bohr based on Lord Rutherford's expts on nuclei of atoms & surrounding electrons – of 25 years required to embody all atomic phenomena based into a unified structure based on Bohr's Principle of complementarity – Need to generalise classical mechanics to apply in the range of atomic phenomena required a new type of mechanics – Survival of special theory of relativity to the atomic level for eg $E = cp$ for a photon, $E = h\nu$ ($\underline{h}$ appearing in quantum mech. just like $\underline{c}$ in relativity) and de Broglie's relation $p = h/\lambda$ – Uncertainty principle $\Delta x \cdot \Delta p_x = \hbar$, $\Delta t \cdot \Delta E = \hbar$, and the principle of complementarity (mutual exclusiveness of complementary quantities), and the principle of indeterminacy – Complementarity of wave & particle aspects ; Schrödinger's wave equation – Hilbert space formulation

Lecture No. 2 – 8/5/67.

Relativistic quantum mechanics including quantum electrodynamics of Dirac brought in the notion of negative energy states as holes in + ve energy states which could not be discarded in ~~quantum~~ classical mechanics & lead to a whole set of new elementary particles like the positron, antiproton, neutron, pions, neutrinos etc. Also the notion of spin appeared in the theory (mag. mom. of e) leading to antiparticles even for ~~such~~ electrical neutral particles like anti-neutron, anti-pions, anti-neutrinos ($e = 0$, $m = 0$, only spin in reverse direction), and also of photons (light particles) – Colossal amount of work so far done on elementary particles culminating in U-238 – U-235 by introduction of a neutron & the creation of the atom bomb – Present appeal by nuclear powers to non-nuclear powers –

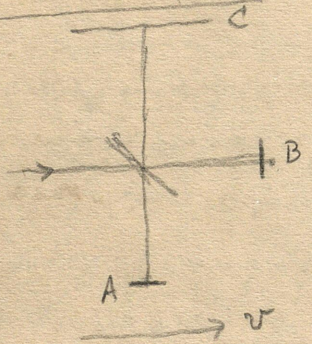
Gen. Rel & Quant. Mech – 3 interactions : (i) short range forces in nuclei between protons & neutrons via pions, (ii) electromagnetic forces between charged particles (iii) & radiation of light by atoms, (iv) weak decay ie β-decay of $n \to p + e^- + $ antineutrino

Principle of special relativity – Haas Vol 2, p. 383

(a) Every physical event which is described by a definite equation in a definite reference system must be described, in any other system moving uniformly & rectilinearly with w.r.t. the first, by the same eqn, provided the spatial & temporal coords of the 1st system be replaced by those of the second.

(b) Conversely, the spatial & temporal components in two systems moving uniformly & rectilinearly w.r.t. each other must be connected quite generally by a universal relation depending only on the mutual relative velocity. The form of this relation is determined by the ~~velocity of light~~ postulate that, in both systems, the velocity of light shall appear to have the same value in all directions, this value being a universal constant.

Michelson – Morley expt



(v = Earth's vel).

t_1 = Time for path SBS in both direcns $= \frac{l}{c+v} + \frac{l}{c-v} = \frac{2lc}{c^2-v^2} \cong \frac{2l}{c} + \frac{2lv^2}{c^3}$.

t_2 = Time for total path SAS $= \frac{2l}{\sqrt{(c^2-v^2)}} \cong \frac{2l}{c} + \frac{lv^2}{c^3}$.

$$\text{or } t_1 = \frac{2l}{c}\left(1 - u^2/c^2\right)^{-1} \; ; \; t_2 = \frac{2l}{c}\left(1 - u^2/c^2\right)^{-1/2}$$

$t_1 \neq t_2$ unless $u/c = 0$, M-M-Expt shows $t_1 = t_2$ without $u = 0$ or $c = 0$

(C) 7/12/6/67.

Taking (i) as unity, the other two are of orders 10^{-2} & 10^{-14} & ~~on~~ small scale gravitational interaction would be of order 10^{-39} – Also difference between linearity & non-linearity – Conversation between Einstein and Bohr.

Plan of lectures & difficulty in covering subjects in full in 40 lectures

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3rd Lecture – 15/5/67 – Classical Mechanics (Holonomic systems) – Lagrangian form of eqns of motion – Lagrangian and Lagrangian system of eqns; ~~their deduction from a variational principle~~; integral of energy $\sum \dot{q}_r \frac{\partial L}{\partial \dot{q}_r}$ ~~–L~~ $= \text{const} = h = T+V$ – Derivation of the Hamiltonian system of eqns and illustration using the case of the simple pendulum

4th Lecture – 22/5/67 – Transformation theory – ∞-group of C.T's – their group nature – P.B's & expn of C.T's in terms of P.B's – $\frac{dF}{dt} = \frac{\partial F}{\partial t} + (F, H)$ – $(A, (B,C)) + (B, (C,A)) + (C, (A,B)) = 0$ – I.C.T's & symbol (f, K) & H eqns as unfolding of an I.C.T – C.T. conserving H-form – Hamilton's partial diff'l eqn with $W = W(q_1 \ldots q_n, Q_1 \ldots Q_n, t)$ Poisson's theorem (proof if time) – Some examples if time permits.

5th Lecture – 29/5/67 – Proof of Poisson's theorem – Maxwell's eqns in $\vec{E}$ & $\vec{H}$ without proof – Derivation of eqn of continuity – Expn of $\vec{E}$ & $\vec{H}$ in terms of potentials $\vec{A}$ & ϕ – Gauge transformations – Transformation of two of Maxwell's eqns into wave eqns for $\vec{E}$ & $\vec{H}$ – Poynting Vector.

6th Lecture – 5/6/67 – Non-covariance of Maxwell's eqns under Galilean trans equivalent to $c' = c+v$ for two relative moving systems – Michelson-Morley Expt. disproving this – Einstein's first postulate – 2nd axiom in electrodynamics of velocity of light being independent of the velocity of the source – use ~~Mention~~ of these two postulates leading to L.T –

7th Lecture – 12/6/67 – Group property of the special L.T. obtained – Addition of velocities in Sp. relativity using the special L.T., and deductions –

From $x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$, $t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}} \longrightarrow$ (i) $u_x = \frac{u_x' + v}{1 + v u_x'/c^2}$; $u_y = \frac{\sqrt{1-\beta^2}\, u_y'}{\text{"}}$; $u_z = \frac{\sqrt{1-\beta^2}\, u_z'}{\text{"}}$ $\left[\begin{array}{l} u_x' = u'\cos\alpha' \\ u_x = u\cos\alpha \end{array}\right.$

(ii) $u = \frac{\left[u'^2 + v^2 + 2u'v\cos\alpha' - (u'v\sin\alpha'/c)^2\right]^{1/2}}{1 + u'v\cos\alpha'/c^2}$ $\left\{\begin{array}{l} u_y^2 + u_z^2 = u^2\sin^2\alpha \\ u_y'^2 + u_z'^2 = u'^2\sin^2\alpha' \end{array}\right.$

$\tan\alpha = \frac{\sqrt{u_y^2 + u_z^2}}{u_x} \longrightarrow$ (iv) $\tan\alpha = \frac{\sqrt{1-\beta^2}\cdot u'\sin\alpha'}{u'\cos\alpha' + v}$

From (ii) $\longrightarrow$ (iii) ~~(iv)~~ $\sqrt{1 - u^2/c^2} = \frac{\sqrt{1 - v^2/c^2}\cdot\sqrt{1 - u'^2/c^2}}{1 + u'v\cos\alpha'/c^2}$

(d) if $u > c$, $v < c$, $u' < 0$ $\left[\begin{array}{l} u_y = u_z = 0 \\ u > c \\ u' = u_x' \end{array}\right.$ no signals with vel $> c$

(a) If $v < c$, $u' < c$, then $u < c$ i.e. combination of velocities $< c \to$ vel $< c$

(c) from (i) if $u_x' = c$, $u_x = c$ $\left[= \frac{u - v}{1 - uv/c^2}\right]$ or if $u_x = c$, $u_x' = c$. or from (iii) if $u' = c$, $u = c$.

(b) From L.T. it follows that $v < c$, otherwise $\sqrt{1-\beta^2}$ is imaginary & L.T $\to$ imaginary coords.

Division Law: If $AB = 0$, either $A = 0$ or $B = 0$, or both A & B are singular.

Commutative property of reciprocals & transverses:

To prove $(A')^{-1} = (A^{-1})'$

Consider $(A^{-1})'A'$ which $= (AA^{-1})' = 1$

$\therefore (A^{-1})' = (A')^{-1}$ reqd result

This is different from the proof given in Ferrar, p. 88

Ex: To show inverse of $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$ & Verify by showing $\begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

<u>8th Lecture</u> – 23/6/67

Elaboration of (d) of last lecture – <u>Scalars</u> in coordsystem $(x^1, .. x^n)$ – <u>Vectors</u> in n-coord system – components of a vector expressed in terms of vector basis – ~~coo~~ contravariant and covariant vectors defined (use of summation convention) – Geometrical illustration by reciprocal coord system (in oblique coords). Scalar product of two vectors – Defn of tensor as generalisation of a vector by the nature of ~~two~~ transformation of the tensor components – mixed, contravariant and covariant tensors, and rank of a tensor – addition, multiplication, and contraction – ~~Stow~~ Symmetric & skew-symmetric tensors w.r.t two contra- or co-variant indices, and symmetric & skew-symmetric tensors – Special case of tensors of rank 2, $(\delta_{ij}, \delta^i_j, \delta^{ij})$; (T^{ij}, T^i_j, T_{ij}) defined; Examples of addition, multiplication & contraction – δ^i_j invariant.

<u>9th Lecture</u> – 26/6/67 – The same as 8th lecture, since only one student appeared thanks to the very heavy rain – Did the lecture 8th up to second class tensors.

<u>10th Lecture</u> – 3/7/67 – Remaining portions of previous lecture about 2nd rank tensors – Surface tensors – Dual surface tensors – metric of special relativity (co- & contra comps same) – metric of general relativity with tensor g_{ik} (symmetric) – (R. Geom – Diff Geom) Formal treatment of putting Maxwell's eqns in tensor form in special relativity – Matrices, defn, conformable for addition, $-A$, $A-B$, 0, rA, associative, commutative and distribution laws.

<u>11th Lecture</u> – 10/7/67 – Remaining portions of previous lecture – Conformality re. multiplication & defn of product – Eg. for $AB \neq BA$ & proof in general (& Exx) – Distribution & associative laws hold – all in summation convention – ~~unit matrix & validity of commutative law of multiplication with I~~ for sq. matrix – Ex. of $AB = 0$ ~~but~~ $BA \neq 0$ – Differences between ordinary numbers & matrices ie $ab \neq ba$, $ab = 0$ without either $a = 0$ or $b = 0$ or both $= 0$. – Square matrices forming an algebra, unit matrix, ~~AA~~ $1.A = A.1$, A^r, $A^r . A^s = A^{r+s} = A^s A^r$ – Transposition $(AB)' = B'A'$ – Determinant of a square matrix $|AB| = |BA| = |A| \times |B| = |B \times A|$, if k = number, $|kA| \neq k|A|$ – Ex. on quaternions from p. 80 of Ferrar.

<u>12th Lecture</u> – 17/7/67 – Square matrices : $1.A = A.1$, – A^r – $A^r A^s = A^s A^r = A^{r+s}$ – Determinant of a square matrix $|A| \times |B| = |B| \times |A| = |AB| = |BA|$, $|kA| \neq k|A|$ – Transpose of ~~a~~ square matrix $(AB)' = B'A'$ – Singular & non-singular matrices – adjoint & reciprocal matrices – Reciprocal as inverse & uniqueness of inverse – Law of ~~ref~~ reversal for reciprocals – Commutative property of reciprocation & transposition – Divn Law – Characteristic eqn & latent roots (or eigen values) of a square matrix – Every sq. matrix satisfies its ~~own~~ characteristic eqn (no proof) (Cayley – Hamilton theorem) – $|g(A)| = g(\lambda_1) g(\lambda_2) .. g(\lambda_n)$ (without proof, g is a polynomial.) – Diagonal,

Law: If $AB = 0$, then either $A = 0$ or $B = 0$, or both A and B are singular – only statement.

Hermitian if $H' = \bar{H}$ or $H^{*'} = H$
unitary if $\bar{U} U^{*'} = 1$ or $U \bar{U}' = 1$ } complex no
or $\bar{U} U' = 1$ A is orthogonal if $A' = A^{-1}$

or if $(*)$ denotes transpose + complex conjugate, $H^* = H$ is hermitian, $U^* = U^{-1}$ is unitary } (A)

Ex: To prove that eigenvalues of a Hermitian matrix are real

Using (A) $SAS^{-1} = B$ reduces (if S is unitary & A Hermitian)

to $SAS^* = B$ & taking Hermitian adjoint of this

$SAS^* = B^*$ $\therefore$ $B^* = B$ ie $\bar{B}_{ki} = B_{ik}$

& for $k = i$, $\bar{B}_{ii} = B_{ii}$ ie real eigenvalue after diagonalisation

Similar matrices have same eigen values

$$|P^{-1}AP - \lambda I| = |P^{-1}AP - \lambda P^{-1} P| = |P^{-1}(A - \lambda I)P| = |P^{-1}| \cdot |A - \lambda \cdot I| \cdot |P|$$
$$= |A - \lambda I| \text{ since } |P^{-1}| = |P|^{-1}$$

Rank - A matrix has rank r when r is the largest integer for which the statement "not all minors of order r are zero" is valid

For a non-singular square matrix A of order n, rank $= n$
" singular " " $< n$
" Zero matrix " $= 0$

Diagonalisation of a matrix: If $SAS^{-1} = B$, $SA = BS$ ie $S_{km} A_{ml} = B_k S_{kl}$ (if B be diagonal)

or $S_{km}(A_{ml} - B_k \delta_{ml}) = 0$

$\therefore$ Hence $|A_{ml} - B_k \delta_{ml}| = 0$ is necessary & sufficient for diagonalisation

Symmetric & Skew-symmetric matrices $(A' = A \text{ \& } A' = -A)$

Permutation and triangular matrices – Hermitian and unitary matrices – Rank of a matrix – The three elementary transformations of a matrix – Equivalent matrices have the same rank – Reduction of a matrix of rank r by elementary transformation to $\left(\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right)$ – Transformation of matrix A to matrix B by a non-singular matrix viz: $SAS^{-1} = B$ or $S^{-1}AS = B$ (A & B are similar matrices) – Diagonalism if B is diagonal leading to problem of eigen-values – Similar matrices have the same characteristic polynomial –

Examples

(1) Prove that the product of a matrix by its transpose is symmetric

(2) If A & B be of same order, and A is symmetrical, prove that $B'AB$ is symmetrical.

(3) Prove that $P^{-1}DP$ is diagonal & describe explicitly the form of $P^{-1}DP$. [For 3 × 3 matrices & generalise]

(4) Prove that a strictly triangular 3 × 3 matrix is nilpotent.

(5) Find the inverses of $E_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $E_3 = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ & $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{pmatrix}$

(6) Show that $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ & $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ have the same eigenvalues.

(7) Show that every matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix

(8) Show that $|kA| \neq k|A|$, where k is a constant $\neq 1$

(9) Defining an orthogonal matrix A by $AA' = 1$, test for orthogonality

(i) $\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}$; (ii) $\begin{pmatrix} 6 & 8 \\ 8 & -6 \end{pmatrix}$

(10) If A and B be orthogonal, prove that $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$ is also so.

(11) Find the ranks of the matrices

(i) $\begin{pmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{pmatrix}$, (ii) $\begin{pmatrix} 2 & 1 & 3 & 4 \\ 5 & 8 & 1 & 4 \\ 6 & 5 & 8 & 1 \\ 3 & 8 & 7 & 2 \end{pmatrix}$, (iii) $\begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$

(12) Given A, B, A^{-1}, B^{-1} & C', find the inverses of the block matrices

(i) $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$; (ii) $\begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$; (iii) $\begin{pmatrix} A & 0 \\ C & B \end{pmatrix}$

(13) Find the eigen values of $A = \begin{pmatrix} -3, & \cancel{-11}\ 0, & 6 \\ 0, & \cancel{-13}\ 3, & 6 \\ 6, & \cancel{-48}\ 6, & \cancel{19}\ 0 \end{pmatrix}^{*}$ and find the matrix S such that $S^{-1}AS$ is diagonal

* $\begin{pmatrix} -3, & 0, & 6 \\ 0, & 3, & 6 \\ 6, & 6, & 0 \end{pmatrix}$

Alternative proof of eigenvalues of a Hermitian matrix being real.

$$HX = \lambda X, \quad \bar{H}\bar{X} = \bar{\lambda}\bar{X}$$

By transpose, $X^* H^* = \bar{\lambda} X^*$ or $X^* H = \bar{\lambda} X^*$ & $X^* H X = \bar{\lambda} X^* X$

Also $X^* H X = \lambda X^* X$ i.e. $\lambda = \bar{\lambda}$.

✓ Q.(14). Defining the <u>spur</u> of a matrix as the sum of its leading diagonal elements, show that the spur is also equivalent to the sum of the eigenvalues of the matrix, and that similar matrices have the same spur.

✓ Q.(15) Prove that for an orthogonal matrix A,

(i) $|A| = \pm 1$; (ii) If λ be an eigen-value of A, so is $1/\lambda$, (iii) every eigen root ~~has unit~~ of a real orthogonal matrix A has unit modulus.

Q.(16) Prove that the eigen roots of a unitary matrix have unit modulus.

Q.(17) If N is a skew-Hermitian matrix, show that $(1-N)(1+N)^{-1}$ is unitary.

13th lecture – 24/7/67 – Solution of examples on matrices – Solved all those marked ✓.

14th lecture (31/7/67) – Remaining ~~Ex. (5) last part, Ex 12 – Ex. 6, Ex. 13~~ – use of relation $AX = \lambda X$ to problem of real eigenvalues of a hermitian matrix, and also Ex. 16 – Ex. 17 – Sets – Theory of sets or topology – Algebraic topology, differential topology, combinatory (geometric) topology – finite sets, enumerable sets, non-enumerable sets, the continuum, subsets* – Sets and associated topological spaces by means of a closure function† corresponding to each point of the set leading to functional topological spaces or function spaces× – Example of a metrical space ie set R is a metrical space if a real valued fn $\rho(x,y)$ $(x, y \in R)$ called <u>distance function</u> is defined for all pairs of elements x, y satisfying (i) $\rho(x,y) = \rho(y,x)$ (ii) $\rho(x,y) > 0$ if $x \neq y$, $\rho(x,x) = 0$, (iii) $\rho(x,y) \leq \rho(x,z) + \rho(z,y)$ $(z \in R)$. – Examples of metrical spaces: (a) Hilbert space $H^{(r)}$ whose points are infinite sequences of real numbers for which Σx_i^2 is convergent & $\rho(x,y) = \left[\sum_1^\infty (x_i - y_i)^2\right]^{1/2}$, (b) complex Hilbert space $H^{(c)}$ whose points are infinite sequences of complex nos. $(x_1, x_2, \ldots)$ such that $\Sigma x_i \bar{x}_i$ is convergent & $\rho(x,y) = \left[\sum_1^\infty (x_i - y_i)(\bar{x}_i - \bar{y}_i)\right]^{1/2}$

(c) Space $\mathscr{C}$ of all continuous functions of a real variable <u>x</u> in the ~~co~~ closed interval $0 \leq x \leq 1$ & $\rho(f,g) =$ upper bound $|f(x) - g(x)|$ – Hausdorff spaces associated with each point of a set R a neighbood or ~~subset~~ subset $R' \subset R$ – Banach spaces which are metric & complete.

* (isomorphism & homeomorphism)

(† Closed set ≡ its closure)

× (a function $\varphi(x)$ is a point of the space)

Groups, rings & fields – Groups* defined by (i) closure property (ii) associative property (multiplicative or additive), (iii) existence of an identity, (iv) existence of an inverse; abelian group if (ii) is commutative – Rings defined by double composition, (i) additively a commutative group, (ii) multiplicatively a semi-group & (iii) distributive laws $a(b+c) = ab + ac$, $(a+b)c = ac + bc$ hold – Fields (i) rings in which non-null elements form a multiplicative abelian group. – Examples of fields are set of all real nos, complex nos, rational numbers.

[* Groupoid if only (i) holds semi-group if (i) & (ii) hold]

* If $g = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_\rho^{\alpha_\rho}$

no. of subgroups including unity $= (\alpha_1+1)(\alpha_2+1) \dots (\alpha_\rho+1) - 1$.

For $g = 2^2$, this no $= (2+1) - 1 = 2$ ie including unity only $(1, s^2)$

† Proof of the general subgroups theorem that if S and T be subgroups of G, then $S \cap T$ (intersection of S & T) is also a subgroup.

(i) If $a \in S \cap T$, $a \in S$, $a \in T$. Similarly if $b \in S \cap T$, $b \in S$, $b \in T$

$\therefore ab \in S$, and $ab \in T$ ie $ab \in S \cap T$

(ii) If $a \in S \cap T$, $a \in S$, $a \in T$, hence $a^{-1} \in S$ & $a^{-1} \in T$ ie $a^{-1} \in S \cap T$

Hence the results

*(i) For $g = 12$ ie $g = 2^2 \cdot 3$, number of subgroups including unity should be $(2+1)(1+1) - 1 = 5$

These are (1), $(1, s^4, s^8)$, $(1, s^3, s^6, s^9)$, $(1, s^6)$ and the subgroup of order $3 \cdot 2 = 6$ viz $(1, s^2, s^4, \dots, s^{10})$ ie 5 altogether

(ii) For $g = 14 = 2 \times 7$, no. of subgroups including unity $= (1+1)(1+1) - 1 = 3$. These are (1), $(1, s^7)$ and $(1, s^2, s^4, \dots, s^{12})$

Groups – Finite and infinite groups – Examples of finite groups (i) symmetric group of order 6, (ii) groups of matrices.

15th Lecture – S_6 and its subgroups – The four groups of matrices (one isomorphic with S_6, 2 cyclic & another non-cyclic of order 4) –

Finite, ∞, & continuous groups} – Simple theorems on groups –

16th Lecture – 14/8/67 – The point raised by a student about $(1, s, s^5)$ in the cyclic group C_6 – Proof that a subgroup of every cyclic group is itself cyclic [for cyclic group C_4, $(1, s^2)$ being the only subgroup] – *cyclic group of order $g = kh$ has only one subgroup of order k since this must be of the form $s^h, s^{2h}, \ldots s^{kh} = 1$ – If $\underline{g}$ is prime, no subgroup except 1 – †If g = product of two relatively prime nos. $(m \times n)$; G is the product of two cyclic subgroups of orders $\underline{m}$ and n; Exs. of $g = 12 = 3 \times 4$, $g = 14 = 2 \times 7$. [If $g_1 = \{1, s^n, s^{2n}, \ldots s^{(m-1)n}\}$, then g_1 is of order $\underline{m}$. Similarly $g_2 = \{1, s^m, s^{2m}, \ldots s^{(n-1)m}\}$ is of order $\underline{n}$. Since $\underline{m}$ & $\underline{n}$ are relatively prime, $g_1 \cap \underline{g_2} = 1$ & hence the result] – Use of (*) for cases $g = 3 \times 4$ & $g = 2 \times 7$.

General group theory (A) Four simple theorems & Ex. of $(ab)^n = a^n b^n$ only if group is abelian.

(B) Isomorphism, (C) Subgroups.

15th Lecture – 21/8/67 – Abstract group being isomorphic with a group of transformations (B & M) – (C) Subgroups; conditions – ~~Four~~ Three theorems. (D) {Coset decomposition of G by a subgroup / – Normal subgroups & ~~Factor group~~.} SL (E) {Conjugate elements & automorphisms / Normal subgroups as self-conj. subgroups}.

16th Lecture – 28/8/67 – Normal Subgroups – (E) Conjugation, automorphism etc. (F) Homomorphisms & Factor groups

Finished general group theory

17th Lecture – 4/9/67 – Permutation groups & Symmetric group – Theory of rep^ns of groups.

18th Lecture – 12/9/67 – Group property of the special L.T's / P, T, PT & L_4 – Lorentz contraction & time dilatation – Rel. Kinematics using ϕ & ψ

19th Lecture – 19/9/67 – Lorentz contraction & time dilatation – Rel. Kinematics using ϕ & ψ; velocity & acc^n vectors –

Electrodynamics: Maxwell's eqns in tensor form – Rel. Mechanics including derivation of $E = mc^2$

20th Lecture – 26/9/67 – Derivation of hyperbolic motion under a constant force in sp. relativity – Rel. mechanics including derivation of $E = mc^2$.

21st Lecture – 5/10/67 – Electrodynamics & special relativity – Eq^n of Continuity $\frac{\partial s^i}{\partial x^i} = 0$. – $de = de_0$ – Sommerfeld's form of eq^n of continuity $\partial(\rho_0 u^i)/\partial x^i = 0$ – 1st set of Maxwell's eqns in tensor form – Vector & scalar potential as ϕ^i & exp^n for field $\vec{E}$ & $\vec{H}$ in terms of $\vec{A}$ & ϕ in tensor form & also Lorentz cond^n – 2nd set of Maxwell's eqns – Lorentz force & eqns of motion

22nd Lecture – 10/10/67 – Covariant & contravariant vectors – Tensors – g_{ij}, $|g_{ij}| = g$, g^{ij}, $|g^{ij}| = \bar{g}$, $g\bar{g} = 1$,

$g_{ij}\, g^{ik} = \delta_j^k$ – Dual tensors – Christoffel 3-index symbols – Contracted symbol Γ^i_{ik}.

23rd Lecture – 17/10/67 – Proof that $\Gamma_{i,jk}$ & Γ^i_{jk} are not tensors & derivation of $\dfrac{\partial^2 x^l}{\partial x'^\mu \partial x'^\nu} = \Gamma'^\lambda_{\mu\nu} \dfrac{\partial x^l}{\partial x'^\lambda} - \Gamma^l_{ij} \dfrac{\partial x^i}{\partial x'^\mu} \cdot \dfrac{\partial x^j}{\partial x'^\nu}$ –

Part of the proof ~~how~~ to show that R^l_{ijk} is a tensor.

24th Lecture – 24/10/67 – Riemannian Geometry

25th Lecture – 31/10/67 –

26th Lecture – 7/11/67 – Introduction to General Relativity – Principle of Equivalence.

27th Lecture – 14/11/67 – Missed this consequent on a lecture by Zaheer at Library Seminar.

28th Lecture – 21/11/67 – Derivation of Einstein's eqns using Principle of covariance.

29th Lecture – 28/11/67 – Missed it by phoning to B.V.R. thanks to Andhra Setty & telling him I would take class on 30th

30th Lecture – 30/11/67 – Went to Inst., but found no students since B.V.R. had not informed him about today's lecture.