

THE 'DIFFUSE' SCATTERING OF ELECTRONS IN METALS AND ALLOYS IN

RELATION TO THEIR RESISTIVITIES

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SYNOPSIS.

In discussions on the 'diffuse' scattering of X-rays in crystals attention is directed mainly to the neighbourhood of the Bragg reflections. Other regions, particularly that inside the first Brillouin Zone, corresponding to wave-lengths too long, or angles too small, to give any Bragg reflections, are also of great interest, and for the following reasons. Let us denote by R_{ϕ} the ratio $S_{\phi}/(\nu\sigma_{\phi})$, where S_{ϕ} is the scattering coefficient of the crystal per unit volume per unit solid angle along a direction making an angle ϕ with the direction of the incident monochromatic beam of X-rays; σ_{ϕ} is the corresponding coefficient of scattering by an isolated atom, and ν is the number of atoms per unit volume. One would like to investigate how, for a given angle of scattering, ϕ , the structure factor of the crystal R_{ϕ} , which depends only on the correlation in phase between the radiations scattered from neighbouring atoms, varies with the wave-length as we pass from the X-ray to the optical region. For wave-lengths much shorter than the interatomic distance, the radiations from neighbouring atoms will be practically of random phases, and R_{ϕ} nearly unity. We shall refer to this value of R_{ϕ} as the Rayleigh value. For long wave-lengths, even a few times the interatomic distance, on the other hand, R_{ϕ} is found to approximate to what we might call the Einstein value $E = \nu K T \beta$, where β is the isothermal compressibility. In other words, the scattering, even in a solid, is due mainly to the fluctuations in density produced by the condensational elastic waves in the thermally agitated crystal, the transverse elastic waves contributing very little to the scattering.

The variation of R_{ϕ} in the intermediate region of wave-lengths is investigated in detail for the alkali metals both in the solid and in the liquid states.

For a given wave-length, the structure factor R_{ϕ} of the crystal, or liquid, will have evidently the same value for electronic scattering too.

Now the finite electrical resistance of a metal arises ultimately from the scattering of the conduction electrons in their passage through the metal. The attenuation ^{coefficient} due to this scattering, i.e. the fraction of the electrons scattered in all directions per unit volume of the medium, gives the reciprocal of the 'mean free path' of the electrons. Now the wave-length of the electrons taking part in conduction will be that corresponding to the Fermi surface, and is known. Hence if the attenuation coefficient can be calculated, the electrical resistivity of the metal becomes known. In other words the calculation of the electrical resistivity of a metal or an alloy reduces ultimately to the calculation of the attenuation coefficient due to scattering.

In the alkali and the noble metals, though the Fermi wave-length is only slightly greater than that needed to give the Bragg reflection in the backward direction, the attenuation coefficient μ approximates to the Einstein value $\mu = \epsilon \cdot \nu \sigma$ where σ is the total scattering from an atom over all directions: $\sigma = \int_0^{\pi} \sigma_{\phi} \cdot 2\pi \sin \phi \, d\phi$. A more detailed ^{precise} calculation on this basis points to an μ ~~The increase in resistance of the alkali metals on melting is calculated on this basis and is found to be reasonable, which is of the proper magnitude.~~

For polyvalent metals, even divalent ones, in the liquid state,

the Fermi wave-length is found to be short enough to make the attenuation coefficient approximate to the Rayleigh value. This accounts for the large resistances of these metals.

In an alloy, the fluctuation not only in density, but also in concentration, will contribute to the scattering. The concentration scattering is readily calculated both in the ordered and the disordered states, since the configurational free energy is known as a function of the concentration. Detailed calculations are given for the electrical resistivities of order-disorder alloys of the β -brass type, over a wide range of temperatures, on this basis, and yield results of some interest.

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