VIRIAL PROBLEMS RELATED TO SIMPLE WING PROFILES.

by

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1--Introduction.

The well-known equations due to Blasius

$$\frac{\rho}{2} i \oint w'^2 dz = F_x - i F_y \qquad (1)$$

$$-\frac{\rho}{2} \oint w'^2 z\, dz = M + iN \qquad (2)$$

give the force and moment exerted by a perfect,incompressible,irrotational fluid upon material bodies immersed in it,in the case of steady motion in two dimensions.The real part of (2) gives the moment,and the imaginary part, the quantity N which is called the virial of the forces plays an important ~~ro~~ role when we study the changes that occur when the forces are rotated by a certain amount about their points of application without change of magnitude. For such a change the new resultant has a line of action also turned by the same angle but without change of magnitude.As the angle of rotation is changed from 0 to 2π we will get a family of straight lines having a certain curve as its envelope,and the point where a line of action touches the envelope or the point at which the resultant force rotates for an infinitesimal rotation of all the components is called the Hamilton Centre.In particular for a system of forces $\vec{F}_\nu(X_\nu, Y_\nu)$ acting at (x_ν, y_ν) in the plane,the moment about the origin is $M = \sum_\nu (x_\nu Y_\nu - y_\nu X_\nu)$ and $N = \sum_\nu (x_\nu X_\nu + y_\nu Y_\nu)$ is the virial referred to the origin.It is well-known that the Hamilton Centre is the intersection of the resultant

$$M = xY - yX \qquad (3)$$

where $X = \sum X_\nu$, $Y = \sum Y_\nu$, and the line perpendicular to (3) given by

$$N = xX + yY \qquad (4)$$

If the Hamilton Centre coincides with the origin we get $M = 0$,and $N = 0$.In other words,the Hamilton Centre C is the point ~~where~~ for which both the moment and virial vanish.

The point C has a significance similar to that of the metacentre P. This point is the point of intersection of the original resultant, and the new line of action obtained when we consider not only a change in the direction of the pressure but also the consequent simultaneous change of magnitude in accordance with the hydrodynamical equations. The point C is obtained when the magnitude of the pressure force at any element does not change. It is well-known that for stability the centre of gravity G should lie below P. In the case of a real disturbance under flying conditions, it may be that at the very moment a small angular disturbance occurs the pressure at a point does not change in any way. In such a case we should consider the point C instead of P. Thus in general, one has to ensure for stability that G must lie below both P and C.

The general theory sketched above is due to von Mises (1941, pp. 43-47), but nowhere in the literature does one find problems relating to the Hamilton Centre dealt with in any specific case. Thus in the case of the simple wing profile i.e. a cylindrical wing of infinite span with a cross-section containing one singular point, expressions for the lift force and moment have been derived, and we have the well-known theorem of von Mises that the first axis is the directrix of the metacentric parabola. But no results relating to the virial have however been obtained.

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It is the purpose of the present paper to derive some theorems relating to the virial and the Hamilton Centre in the case of the simple wing profile and particular cases of this like the Joukowski and Karman-Trefftz profiles, and the S-profile due von Mises. We also derive corresponding results relating to stability in these special cases. A parabola termed the virial parabola is introduced, and a theorem analogous to that of von Mises has been obtained. Further the relationship ~~of~~ is established between the Hamilton Centre ~~to the three axes of the profile~~, and the centroid of circulation (Milne-Thomson, 1952, pp. 120-22.).

2. Review of results in the general case.

Considering the effect of uniform flow specified by the velocity vector $\vec{V}$ (or $-Ve^{-i\alpha}$) at $z = \infty$ upon immersed bodies, let $W(z)$ be the complex potential of the two-dimensional (vortex free) motion of the liquid of density ρ. $W'(z)$ is given by its Laurent expansion

$$W'(z) = A + \frac{B}{z} + \frac{C}{z^2} + \cdots \qquad (5)$$

where it can be shown that A, B, C, are linear in V, and that $A = -Ve^{i\alpha}$, $B = \Gamma/2\pi i$ where Γ is the circulation.

Writing $C = V(c_1 e^{i\alpha} + c_2 e^{-i\alpha})$; $B = Vi(-b_1\cos\alpha + b_2\sin\alpha)$, $2a = |\sqrt{(b_1^2+b_2^2)}|$; $\cos\beta = -\frac{b_2}{2a}$, $\sin\beta = \frac{b_1}{2a}$, we get $B = -2aVi\sin(\alpha+\beta)$. Using Blasius' formula (1), we get the lift force

$$L = \rho\Gamma V = 4\pi a\rho V^2 \sin(\alpha+\beta) \equiv L_0 \sin(\alpha+\beta) \qquad (6)$$

in a direction normal to the direction of flow. Also from (2), taking the moment and virial relative to a point z_0, we have

$$\begin{aligned} M_0 + iN_0 &= -\frac{\rho}{2}\oint (z-z_0)\, w'^2 dz \\ &= -\frac{\rho}{2}(B^2+2AC)2\pi i + \frac{\rho}{2} z_0 \cdot 2AB \cdot 2\pi i \end{aligned} \qquad (7)$$

Using the properties of A, B, and C, it can be shown from this that if z_0 be taken to be the point F given by $z_0 = -\frac{c_1}{a}e^{-i\beta}$, the moment M_0 is independent of the angle of attack. The point F is called the focus, and referred to F, we get

$$M_0 = -2\pi\rho V^2 \,\mathrm{Im}\,(c_2 + c_1 e^{-2i\beta}) \qquad (8)$$

Introducing $h_0 = M_0/N_0$

one can draw a line l_0 in the β-direction (called the direction of the first axis) at a distance h_0 from the focus F. If a parallel to V through this point

[Fig. 1.]

meets l_0 at S, it can be easily shown that the normal to $\vec{V}$ through S gives the direction of the lift force L and the the correct magnitude follows from (8). For varying α, S moves along l and L envelopes a parabola called the metacentric parabola, and the point of contact of L is the metacentre P. In the above figure where $h_0 > 0$, the metacentric parabola opens upwards, and if $h_0 < 0$, it opens downwards. Thus the condition for stability viz. the centre of gravity G lying above P is easier to reach in the case $h_0 > 0$ than in the case $h_0 < 0$.

We can find from (7) the value of N_0 and show that if we choose the point G_0 given by $z_0 = -\frac{c_1}{a}e^{-i\beta} + ae^{i\beta}$, then N is independent of α. The point G_0 may be called the virial focus, and referred to it we get

$$N_0' = 2\pi\rho V^2 \mathcal{R}(c_2 + c_1 e^{-2i\beta}) \qquad (10)$$

Introducing $k_0 = N_0 / L_0$, (11)

[Fig.2.]

we plot the straight line n_0 perpendicular to l_0 at a distance k_0 from G_0. Then as in the case of the moment, it can be shown that we get the line $N_0 = 0$ by drawing a normal to $\vec{V}$ through G_0 to meet n_0 at T and taking the line through T parallel to $\vec{V}$. The intersection of $M_0 = 0$, and $N_0 = 0$ gives the Hamilton Centre C.

3. Results in the case of the simple wing profiles.

In the case of such a profile with one singular point B, the problem amounts to finding a transformation $z' = f(z)$ transforming the profile into a circle of radius $\underline{a}$ and centre M, leaving unchanged the region at infinity, and making the point B' corresponding to B a stagnation point. If $w(z)$ be the complex potential satisfying the required conditions, it can be shown that

$$w'(z) = -Ve^{i\alpha} + \frac{\Gamma}{2\pi i}\cdot\frac{1}{z} + \left(Va^2e^{-i\alpha} - Ve^{i\alpha}\kappa\right)\frac{1}{z^2} + \cdots \qquad (12)$$

Comparing this with (5), we have $A = -Ve^{i\alpha}$, and $B = \Gamma/2\pi i$ as obtained previously. Also we have

$$c_1 = -\kappa, \qquad c_2 = a^2 \qquad (13)$$

The first axis is in this case the line B'M i.e. the line joining the mapping point of B and the centre of the profile. Putting k in (13) equal to $c^2e^{2i\gamma}$ the inverse transformation $z = g(z')$ for large $|z'|$ transforms the circle

$|z'| = R$ into an ellipse whose major axis makes an angle γ with the real axis, and this major axis is called the second axis of the profile. The lift force is again given by (6), and to find M_0 we note that the point z_0 is now $= \frac{c^2}{a} e^{i(2\gamma-\beta)}$ This shows that that the second axis bisects the angle between the first axis and the line MF joining M to the focus F (z_0), and also gives a construction to find F if we note that $MF = c^2/a$.

[Fig.3]

M_0 given by (8) now becomes $M_0 = 2\pi\rho V^2 \mathcal{R}\left(-ike^{-2i\beta}\right)$, and since $k = c^2 e^{2i\gamma}$, we have

$$M_0 = 2\pi\rho V^2 c^2 \sin 2(\gamma-\beta) \tag{14}$$

and

$$h_0 = M_0/L_0 = \frac{c^2}{2a} \sin 2(\gamma-\beta) \tag{15}$$

From (15) easily follows the theorem of von Mises that the first axis is the directrix of the metacentric parabola. Finally we can find the moment M knowing it about F, and we get

$$M = 2\pi\rho V^2 c^2 \sin 2(\alpha+\gamma) \tag{16}$$

It might also be noticed that axis II is a tangent to the metacentric parabola for if FRJ be drawn perpendicular from the focus F to axix II meeting it in R, and the first axis in J, we then have FR = RJ since axis II bisects the angle between FM and MJ. Hence R lies on the tangent at the vertex to the metacentric parabola, and this shows that axis II is a tangent to the parabola (Fig.3).

4. Theorems relating to the virial.

We now come to the consideration of the virial in the case ofh the simplw wing profile. Analogous to the metacentric parabola we can think of the envelope of $N_0 = 0$ as the virial parabola, and the point of contact of Q of $N_0 = 0$ with this envelope as the virial centre. (Fig.2). Using (10) and putting here $C_1 = -K = -c^2 e^{2i\gamma}$ and $C_2 = a^2$ we get from (11) that

$$K_0 = \frac{a}{2} - \frac{c^2}{2a}\cos 2(\gamma - \beta) \qquad (17)$$

Let us find the distance of the directrix of the virial parabola from the centre of the profile. This is obviously equal to FB + the resolved part of MF along axis I i.e. equal to $a + 2K_0 + \frac{c^2}{a}\cos 2(\gamma-\beta)$,and using (17) this is equal to 2a. Hence we have

Theorem 1: The directrix of the virial parabola is at a distance from the centre of the profile equal to the diameter of the profile.

If P and Q be corresponding positions of the metacentre and the virial centre, the tangents to the two parabolas at P and Q are invariably at right angles intersecting at the Hamilton Centre C. Also from the properties of the parabola [see Fig.4], triangle FSP is similar to FOS, and the triangle G_0TQ is similar to G_0KT. Hence $FPS = OSF = \alpha+\beta$,and $G_0QT = G_0TK = \alpha+\beta$. Thus FP and G_0Q make equal angles with SP and QT which are perpendicular directions Hence FP and G_0Q are at right angles, and we have

Theorem 2: The lines joining the metacentric focus to the metacentre, and the virial focus to the virial centre are at right angles.

Next, we shall consider the centroid of the circulation given by

$$z_C = \frac{i}{2a\sin(\alpha+\beta)}\left\{a^2 e^{-i\alpha} - c^2 e^{i(\alpha+2\gamma)}\right\} \qquad (18)$$

(Milne-Thomson, p.121). Let K_1 be the point diametrically opposite to B on the circle which transforms ~~the~~ into the profile i.e. K_1 is a point on the first axis such that $MK_1 = a$. (Fig.4). It is known that the locus of the centroid of circulation is the perpendicular bisector of FK_1 ,and is called the third axis of the profile. Also the line of action of the aerodynamic force passes through the centroid of circulation. This point is therefore the intersection of axis III with the tangent to the metacentric parabola at P, and is denoted by C' in Fig.4. Since K_1 is a point on axis I which is the directrix of the metacentric parabola whose focus is F, the perpendicular bisector must be a tangent to the parabola. Thus the third axis is a tangent to the metacentric parabola, and the point of contact is denoted by C'' in Fig.4.

The points S,C',P and C all lie on the tangent at P to the metacentric parabola, and we will now find the distance CC'. Taking O for origin (Fig.4), l_0 for x-axis, and the axis of the parabola for y-axis, the equation to that parabola becomes

$$x^2 = 4h_0 y \qquad (19)$$

The tangent at P is perpendicular to the wind direction, and its slope is therefore equal to cot($\alpha+\beta$). The angle $\alpha+\beta$ which is the angle between the direction of the wind and the first axis is called the absolute incidence, and we will denote it by β_0. Putting cotβ_0 = t, the point P becomes $(2h_0t, h_0t^2)$, and the equation to the tangent at P is

$$xt - y = h_0 t^2 \qquad (20)$$

It is easy to show that Q is the point $(a+k_0-k_0t^2,\ h_0+2k_0t)$, and the tangent at Q to the virial parabola has the equation

$$x + yt = a + k_0 + h_0 t + k_0 t^2 \qquad (21)$$

The intersection of (20), and (21) gives the Hamilton Centre C as

$$C\left(\frac{a}{1+t^2} + h_0 t + k_0,\ \frac{at}{1+t^2} + k_0 t\right) \qquad (22)$$

We see from Fig.4. that $AK_1 = MK_1 - AM = a - \frac{c^2}{a}\cos 2(\gamma-\beta) = 2k_0$ from (17). Hence K_1 is the point $(2k_0, -h_0)$ and axis III which is the perpendicular bisector of FK_1 has the simple equation

$$xk_0 - yh_0 = k_0^2 \qquad (23)$$

It is easily seen that this is a tangent to the metacentric parabola the point of contact being $(2k_0, k_0^2/h_0)$. The centroid of the circulation is given as the intersection of (20) and (23), and has the co-ordinates

$$C'(k_0 + h_0 t,\ k_0 t) \qquad (24)$$

From (22), and (24) we calculate CC', and we have

$$CC' = a/\sqrt{(1+t^2)} = a \sin\beta_0 \qquad (25)$$

leading to

Theorem 3: The distance of the centroid of circulation from the Hamilton Centre is equal to the perpendicular from the stagnation point on the direction of the wind through the centre of the profile.

This theorem gives an alternative construction for the Hamilton centre. We mark MK_1 on the first axis such that $MK_1 = a$, then draw the third axis to meet $M_0 = 0$ at C', and then make C'C = equal to the perpendicular from K_1 on the direction of the wind through M.

By drawing suitable figures we can find the relative positions of C and P for the cases $h_0 \gtrless 0$, $k_0 \gtrless 0$. Summarising these cases we have

Theorem 4: If $h_0 > 0$ (or $\beta < \gamma$), the Hamilton centre is above or below the metacentre according as $k_0 \gtrless 0$, and

Theorem 5: If $h_0 < 0$ (or $\beta > \gamma$), the Hamilton centre is above or below the metacentre according as $k_0 \lesseqgtr 0$, provided in the latter case $k_0 > a\sin^2\beta_0$ otherwise C lies above P in the case $k_0 > 0$ also.

These theorems enable us to make statements about stability based on the relative positions of C and one of the points C or P.

Eliminating t in the co-ordinates of C given by (22) we can derive the locus of C, and since both the co-ordinates are rational functions of a parameter, the locus will be an unicursal curve, a cubic as can easily be shown.

The distance between the metacentre and the Hamilton centre can be found from (22) and using the fact that P is $(2h_0 t, h_0 t^2)$, and we get

$$CP = a\sin\beta_0 + k_0 \operatorname{cosec}\beta_0 - h_0 \operatorname{cosec}\beta_0 \cot\beta_0 \qquad (26)$$

as can also be deduced geometrically by taking (See Fig. 4)

$$CP = G_0T + G_0D - SP$$

Considering the question as to when the Hamilton centre lies respectively on one of the three axes of the profile, it is obvious that since axes II and III are both tangents to the metacentric parabola, the tangents wil also be the directions of the lift forces for suitable α. Thus C will lie on these axes when the wind directions are perpendicular to them. As regards axis I the result is trivial since ~~the~~ when the lift force is along the tangent at th vertex of the metacentric parabola ,C will lie on axis I at infinity.

5.Special profiles.

We will now consider the special profiles like the Joukowski, Karman-Trefftz and S-form profiles.For the Joukowski profiles we have $\gamma = 0$ and $k_0 = \frac{1}{2a}(a^2 - c^2\cos 2\beta)$ and if the camber be c/a_1 we have $c < a_1 < a$ i.e. $a > c$ and therefore $a > c\cos\beta$ i.e $k_0 > 0$ .Also since $\gamma = 0$ we have $\beta > \gamma$ i.e $h_0 < 0$. Thus in the general Joukowski profile $h_0 < 0$ and $k_0 > 0$ and it can be shown that C lies above P.Although in this case the metacentric parabola opens downwards,and it is harder to make P lie above G for all angles of attack,the situation is more favourable with regard to C than with regard to P.

Thes results are ,of course,true for the particular cases of the symmetric Joukowski profile (β = 0),and the still more particular case of the straight slit profile with $c = a$. In the latter case k_0 = 0,and the foot of the perpendicular from the virial focus G_0 on the lift force is the Hamilton centre C.We have in this case CP = $a\sin\alpha$,and the co-ordinates of C are ($a\sin^2\alpha$, $a\sin\alpha\cos\alpha$)i.e we have

Theorem 6: For the straight slit profile,the locus of the Hamilton centre is a circle having as its diameter the line joining the foci of the metacentric and virial parabolas.

In the former case $k_0 = (a^2-c^2)/2a$ and we easily get $CP = a\sin\alpha + \frac{k_0}{\sin\alpha}$ and the co-ordinates of C are given by

$$x_c = k_0 + a\sin^2\alpha; \quad y_c = k_0\cot\alpha + a\sin\alpha\cos\alpha \qquad (27)$$

Elimination of α gives the locus of C as a cubic with symmetry about an axis perpendicular to the forst axis ,and touching the first axis at the centre of the profile.We can also trace this curve by using polar co-ordinates with PC = r,and $\theta = 90^0 - \alpha$,getting

$$r = a\cos\theta + k_0\sec\theta \qquad (28)$$

which can be traced in a manner analogous to Rankine's method of superposition of stremlines by using the two curves $r_1 = a\cos\theta$ which is a circle and $r_2 = K_0 \sec\theta$ which is a straight line, and taking $r = r_1 + r_2$

In the present case we can also find a minimum value for CP. Here $CP = a\sin\alpha + K_0 \operatorname{cosec}\alpha$ and elementary analysis shows that this is a minimum for $\sin^2\alpha = K_0/a$ and that the minimum is given by

$$(PC)_{min} = \sqrt{2(a^2-c^2)} \qquad (29)$$

e/ For the Karman-~~Tr~~efftz profiles obtained by the transformation

$$\left(\frac{z+c_1}{z-c_1}\right) = \left(\frac{z'+c_2}{z'-c_2}\right)^n \qquad (30)$$

where c_1, c_2, $\underline{n}$ are constants, and $\underline{n}$ is nearly equal to 2 (von Mises, 1941, p.62), so that we can write $n = 2+\epsilon$, where ϵ is small, we have as in the case of the Joukowski profile $\gamma = 0$, and

$$K_0 = \frac{1}{2a}\left(a^2 - c^2\cos 2\beta\right)$$

Here we can further write

$$K_0 = \frac{1}{2a}\left\{a^2 - \frac{c_2^2(n^2-1)}{3}\cos 2\beta\right\}$$

Hence $K_0 > 0$ if $\frac{n^2-1}{3} a_1^2 \cos^2\beta \cos 2\beta < a^2$, since $\cos\beta = c_2/a_1$

i.e if $\frac{n^2-1}{3} a_1^2 < a^2$, or $\frac{a^2}{a_1^2} > \frac{n^2-1}{3}$

Since $a > a_1$, this condition can always be satisfied by suitably choosing ϵ, in terms of which $\frac{1}{3}(n^2-1) = 1 + \frac{4}{3}\epsilon + \frac{1}{3}\epsilon^2$. Thus the same conclusions regarding the relative positions of C and P whcih ~~wer~~ were derived in the case of the Joukowski profile hold in this case also.

Finally we come to the S-profiles given by von Mises (1941, p.66.). Confining ourselves to the case where there are only three singular points situated inside the circle at the ~~pints~~ points $a_1 = c'$, $a_2 = c'' e^{i\theta}$, $a_3 = -c'' e^{i\theta}$, we derive the transformation

$$z = z' + \frac{c'^2 + c''^2 e^{2i\theta}}{z'} - \frac{(c'c''e^{i\theta})^2}{3z'^3} \qquad (31)$$

which has the property that $\beta = \gamma$ i.e. a profile having a centre of lift, or where the metacentric parabola reduces to a point. To find the sign of K_0 in this case, we notice that [Fig.5], in the usual notation, $B'G'^2 = c^4/c'^2$, where

$$OG' = c''/c', \text{ and angle } XOG' = \text{twice angle } XOa_2,$$

since

$$c^2 e^{2i\gamma} = c'^2 + c''^2 e^{2i\theta}$$

Hence $B'G' = c^2/c'$. Since for the S-profile $\beta = \gamma$, we shall have $K_0 > 0$ if $a > c$ i.e $a^2 > c^2$ or $c' \cdot B'G' < a^2$. Now, we can choose a_1, a_2, a_3 and the circle of sufficient radius to pass through $-c'$, and include the singular points. Having made this choice, we can further alter it, preserving the original conditions, and so choose the centre M that $\frac{a^2}{c'} > B'G'$ i.e. such that $K_0 > 0$. Thus for an S-profile we can make the Hamilton centre lie above the metacentre as in the case the symmetric Joukowski profile. Hence for the S-profile the situation regarding stability is more favourable than in the Joukowski and Karman-Trefftz profiles. Although the situation is the same in the cases of the symmetric and S-form profiles, we have this difference that in the former case $\beta = \gamma = 0$, and the lift force is very small.

In the above S-form profile we had $h_0 = 0$ and $K_0 > 0$ but we can show that it is possible to find more general profiles of this type with $h_0 > 0$ and $K_0 > 0$ i.e. where the metacentric parabola opens upwards and C lies above P or the situation regarding stability is more favourable than even in the case of the S-form profile with a centre of lift. For this purpose let us write in (32) $b^2 = c''^2 e^{2i\theta}$ and let us take the centre of the circle viz. M(Fig.5) as given by the complex number $m = x_0 + iy_0$ such that $a_3 a_2$ passes through M i.e. we take here

$$m = x_0 + iy_0 = \kappa b = \kappa c'' e^{i\theta} \tag{32}$$

where κ is a real constant. The first axis is the line joining the singular point $-c'$ to the centre. Hence

$$\tan\beta = \frac{y_0}{x_0 + c'}, \text{ or } \tan 2\beta = \frac{2y_0(x + c')}{(x_0 + c')^2 - y_0^2}$$

$$\text{Also, } \tan 2\gamma = \frac{c''^2 \sin 2\theta}{c'^2 + c''^2 \cos 2\theta} = \frac{2x_0 y_0}{k^2 c'^2 + x_0^2 - y_0^2}, \text{ using } (32)$$

Hence the condition for $h_0 \gtreqless 0$ viz. $\gamma \gtreqless \beta$, or $\tan\gamma \gtreqless \tan\beta$ reduces after a slight simplification to the condition

$$x_0^2 + y_0^2 + x_0(1-k^2)c' \gtreqless k^2 c'^2 \tag{33}$$

The equality gives the usual S-profile with the centre of lift. The first sign of inequality gives $h_0 > 0$. The last sign of inequality gives $h_0 < 0$ like the Joukowski profile.

Further regarding $k_0 \gtrless 0$, this is given by

$$a^2 \gtrless c^2 \cos 2(\gamma - \beta) \tag{34}$$

This condition becomes rather complicated when expressed in a form analogous to (33). We therefore make a further simplification of (32) by taking

$$x_0 = y_0 = \mu c' \qquad (\mu > 0) \tag{32'}$$

With this condition (34) reduces after some simplification to

$$k^2 \gtrless \frac{2\mu^2(1+\mu)}{1+4\mu+4\mu^2+2\mu^3}. \tag{34'}$$

and since $\mu > 0$, and both k and μ are left arbitrary a choice is possible such that the upper sign of the inequality in (34') holds i.e. $k_0 > 0$. Hence (33) and (34') show that it is possible to choose a profile similar to the S-profile with $h_0 > 0$ and $k_0 > 0$.

Summary.

Problems of the virial associated with a simple wing profile are studied by considering the relative positions of the Hamilton centre and metacentre. In particular, questions of stability are emphasised. Several theorems are derived relating to the virial parabola, the Hamilton centre and the axes of the profile. These theorems are specialised to the case of the Joukowski profiles, particularly the symmetric and straight slit cases. The S-form profiles are generalised to the case where the metacentric parabola opens upwards, and

the Hamilton Centre is above the metacentre, which are the conditions most favourable for stability.

References

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