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CONCEPT OF TIME*

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1. PRELIMINARIES

I feel honoured because IMS invited me to deliver this year's P.L. Bhatnagar Memorial Lecture. Professor Bhatnagar was a friend, philosopher and guide for many young research workers in the fifties, sixties and seventies of the present century, and I was one such young worker. So this Eklavya pays his respects to his modern Guru, who unlike Vasishtha did not ask for any Guru Dakshina. With due regards for Prof. Bhatnagar's search for correspondence between basic mathematical concepts and axioms of their applications, I have chosen the subject of the present talk as the mathematical and physical aspects of the Concept of Time.

Mathematically, time is a real number measured on the real number line. But looking that way, distance is also a real number measured on the real number line. However, intuitively we recognise time as distinct from distance because we can see distances changing while we do not see but feel the passage of time. The intuitive difference between distance and time has been the subject of study and analysis for mathematicians, physicists and philosophers all along the history of science. In this talk it is our aim to share some of these discussions.

2. NEWTONION CONCEPT OF TIME

It was Newton who explicitly wrote down his basic assumption regarding time. In his book *Principia Mathematica* published in 1689, he wrote:

"There is a true even flowing time, the same for all observers."

We have been so much used to this Newtonion concept of time that we hardly realize that it is an assumption. Take for example the motion of a projectile in the gravitational field of the Earth. We write the equations of motion as

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$$\frac{d^2x}{dt^2} = 0; \quad \frac{d^2y}{dt^2} = -g. \tag{1}$$

And for the motion of a planet in the gravitational field of the Sun, we write

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{\mu}{r^2}; \quad \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0. \tag{2}$$

In (1) we know that (x, y) are the coordinates of the projectile as measured by an observer on the Earth and in (2) (r, θ) are the coordinates of the planet as measured by an observer on the Sun. But equations (1) and (2) also contain t. But we never mention that t in (1) is measured by an observer on the Earth and t in (2) is measured by an observer on the Sun. This is because space coordinates may change from observer to observer, but by Newton's assumption, the time coordinate is "the same for all observers".

There is another reason why we do not realize that this "true even flowing time, the same for all observers" is an assumption. It has worked so well. With this assumption we can work out the time of high tides and low tides on the sea coast and check that the calculations agree with observations. As a matter of fact calculations regarding motions of planets, satellites, space-ships carried out on the basis of this Newtonion concept of time lead to results which entirely agree with observations. So this assumption of Newton works so well and so we hardly realise that it is an asumption.

But even in the days of Newton, several philosophers had objected to this Newtonion Concept of Time. The mathematician and philosopher Leibnitz was the first to criticise publicly this assumption of Newton. A true even flowing time implies that the passage of time can be compared to a flowing river, which can be idealized to a real line—the time axis. So true even flowing time implies that there is a time axis along which time "flows" to indicate the passage of time. Different points on this time axis represent different epochs. The part of the assumption "the same for all observers" implies that there is only one such time axis, which every observer uses. Thus the history of the entire universe is depicted on this single time axis.

Now we pick up the arguments of Leibnitz. Consider the event of the creation of this universe. According to Newton there is a definite point *P* on the time axis which represents this epoch of Creation of the Universe. According to Leibnitz this implies that God Almightly must have created the universe at the precise moment *P*, not a second earlier not a second later! How can we put such a restriction on the Creator of the Universe? Newton, by his assumption of true even flowing time, the same for all observers, is essentially dictating God to create the universe at the precise moment *P*. Or, in other words, Newton's assumption puts a restriction on God Almighty. How can one agree to such an assumption? So argued Leibnitz.

Today we realize that Leibnitz' arguments had seeds of the present theory of Relativity. But in those days, the saying "Proof of the pudding is in the eating" worked. Newton's theory worked so well: the entire edifice of civil and mechanical engineering stood on Newtonian foundation, navigation and astronomical observations confirmed all predictions made by Newtonian theory based on his concept of time. So for about 2 centuries not much attention was drawn to Leibnitz' criticism of Newtonian assumption of time. But by the end of the nineteenth century several observations were made which could not be straightaway explained by Newtonian Concept of Time. So some searching analysis was made about the basic notions of space-time and motion. We go to discuss this analysis now.

3. Propagation of Light

Sound and Light are the two massengers of Nature. Many attempts have been made to understand the mechanism of propagation of light in all directions from its source. Newton's corpuscular theory which postulated that light travels like bullets fired from a gun, was disproved by experiments and by the middle of the nineteenth century Huygens' theory of wave propagation of light was well established. According to this theory a source of light produces light waves which move outwards from the source in a way similar to water waves propagating in still water. This wave theory of light had the backing of experimental evidence.

But then the question arose that a wave is produced and propagated in some medium. In what medium are the light waves produced and propagated? This medium cannot be air or any material medium because there is no such material medium in interstellar space and light from distant stars does reach us! Whenever a scientist feels that some object or agency must exist, but he does not know what it can be, he begins in his time-honoured way to formulate a problem. "Suppose the object or agency exists. Call it x. Now let us find this x." Physicists assumed that there is an all pervading medium in the universe and light propagates as waves in that medium. This medium was not called x but the name 'ether' was given to it. The problem was to find x i.e. to think out and set up experiments to find the properties of this postulated 'ether'.

4. The Michelson-Morley Experiment

The vast interstellar space can be looked upon as an ocean of ether. And in this vast ether-ocean the earth and other planets move like fish in the sea. This comparison leads one to a number of questions like the following:

(1) The sea-water offers resistance to the motion of the fish. Does ether offer any resistance to the motion of planets? (2) Motion of fish and rapidly moving steamships produce temporary local disturbances in the sea. Will the motion of planets through the ether ocean produce similar disturbances in ether? It is obvious that the first step in trying to answer these and several other such questions is to measure the velocity of some 'fish' moving through the ether-ocean, and what other fish is easily assessible to us than our own Earth gliding through ether!

Michelson and Morley thought out an experiment to measure the velocity of earth through ether. It is easy to understand the principles underlying their experiment. Assume that the earth moves in ether with a velocity v. A ray of light has velocity c in ether. What will be its velocity as seen from the earth? Of course this will depend on the direction of light ray vis-a-vis the direction of earth's motion. If we call this velocity of light as seen from the earth as u, the simple, commonsense principle of relative velocities can be used to find u. When the light ray is moving in the same direction as the earth it is clear from Fig. 1 that

$$c = v + u \quad \text{or} \quad u = c - v,$$

Similarly when the direction of light ray is opposite to the direction of earth's motion, a reference to Fig. 2 shows that

$$c = u - v \quad \text{or} \quad u = c + v.$$

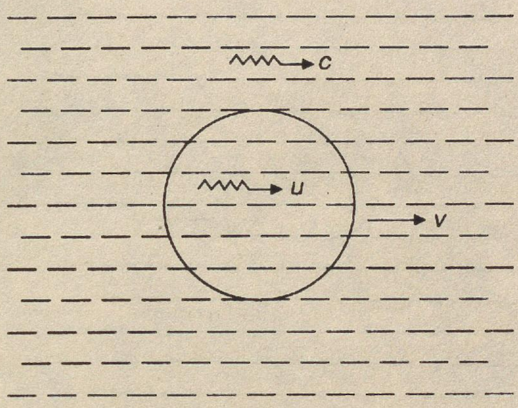

Fig. 1

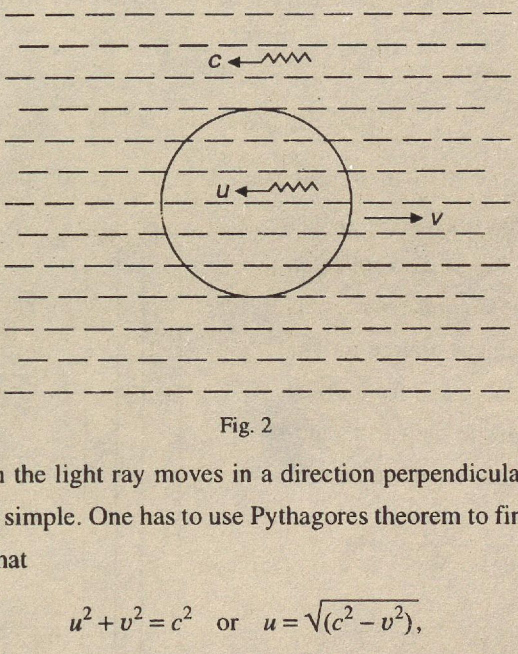

Fig. 2

The case when the light ray moves in a direction perpendicular to the earth's motion is not that simple. One has to use Pythagores theorem to find u. Reference to Fig. 3 shows that

$$u^2 + v^2 = c^2 \quad \text{or} \quad u = \sqrt{(c^2 - v^2)},$$

The simple mathematics given above is sufficient to understand the principles behind the M.M. Experiment. In this experiment a ray of light from a source S is sent in the direction of earth's motion through a distance l, is reflected by a mirror M_1 and returns to the starting position S. The time taken by this ray for its onward journey SM_1 will be $l/(c - v)$ and the time for its return journey M_1S will

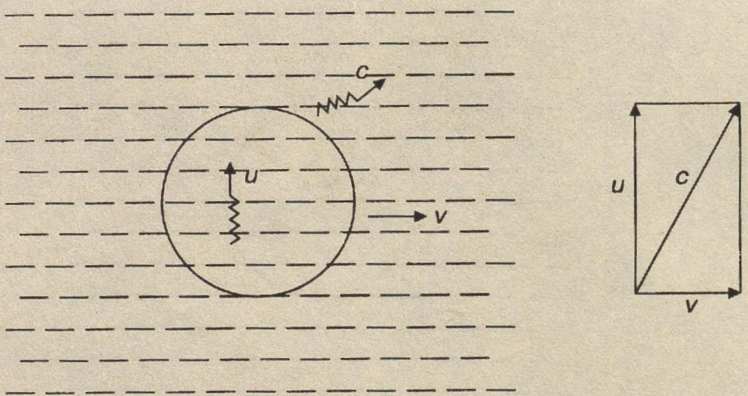

Fig. 3

be $l/(c+v)$. Thus the total time t_1 for to and fro journey in the direction of earth's motion will be

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2 - v^2} = \frac{2l/c}{(1 - v^2/c^2)}. \tag{1}$$

Another ray of light is sent from S in a direction perpendicular to earth's motion. After traversing a distance l this ray is also reflected by a mirror M_2 and retraces its path to reach the source S. The time taken for to and fro journey at right angles to the earth's motion can be calculated to be

$$t_2 = \frac{l}{\sqrt{(c^2 - v^2)}} + \frac{l}{\sqrt{(c^2 - v^2)}}$$

$$= \frac{2l}{\sqrt{(c^2 - v^2)}} = \frac{2l/c}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{2}$$

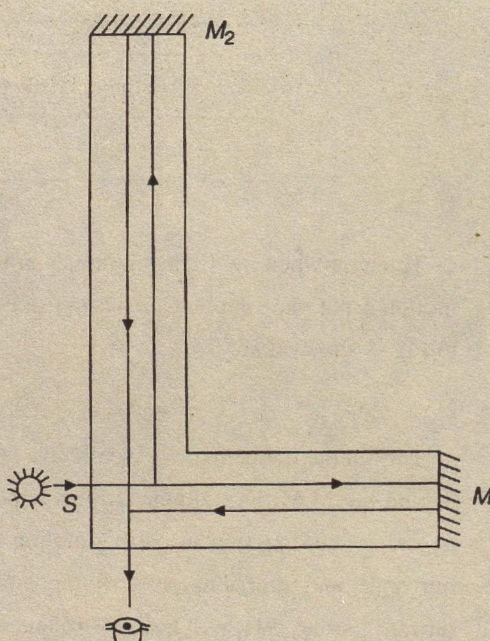

Fig. 4

It is seen from (1) and (2) that $t_1 \neq t_2$. Aim of the M.M. experiment was to measure this time-difference and thus get an equation to determine v.

A stone was floated in a pond a mercury. Two mutually perpendicular arms, each of length l, were fixed on the stone. The floating stone was arranged in such a position that one of the fixed arms will point in the direction of the earth's motion. The problem then reduced to sending a beam of light along one arm and getting it back by reflection from M_1, at the same time sending another beam along the other arm and getting it back and measuring the difference in the time taken by the two. Well it is easy to give such a prescription but it is not so easy to carry it out. Firstly, in order to observe these light beams, the whole experiment must be performed in a dark room. And again the velocity of light is so large (3×10^{10} cm/sec) that in order to get a measurable time difference the length of each arm would have to be enormously large (larger than the diameter of the earth!). But then there is a simplifying circumstance. Light is propagated in waves. In waves we have crests and troughs. Now if the times taken by light to travel along the two arms are different, it is possible to arrange matters in such a way that when a crest of the first beam reaches us the second beam presents its trough to us and so when the two beams mix together they give zero oscillation to ether! The two beams together will then lead to darkness!! Michelson and Morley decided to measure the time difference by such a method of interference of the two beams of light.

They performed the experiment. What was the result of this experiment? They found that the two beams took the same time to travel along the two perpendicular arms! In terms of our formula they found that

$$t_1 = t_2$$

or

$$\frac{2l/c}{1 - v^2/c^2} = \frac{2l/c}{\sqrt{(1 - v^2/c^2)}} \tag{3}$$

But how can equation (3) hold?

5. The Negative Result

The above equation (3) implies that $1 - v^2/c^2 = \sqrt{(1 - v^2/c^2)}$. Which number can be equal to its own square root? 0 and 1 are two such numbers. But in (3)

$1 - v^2/c^2$ comes in the denominator, so it cannot be 0. Therefore it must be 1. Thus $1 - v^2/c^2 = 1$ or $v = 0$. The velocity of the earth is zero. The earth does not at all move in ether. But nature should not show any preference to earth and so we expect that the velocity in ether of any planet will be zero. This is really funny. All planets are at rest in ether!

Many attempts were made to explain this negative result of M.M. experiment. We shall note here one such attempt by Lorentz. (He obtained a correct result in an incorrect way.) He argued that light took the same time to travel along the two arms because though we took the lengths of the two arms to be equal they were not really equal. The arm in the direction of earth's motion had actually become a little shorter. Suppose that its length was l'; then the equality of t_1 and t_2 would give

$$\frac{2l'/c}{1 - v^2/c^2} = \frac{2l/c}{\sqrt{(1 - v^2/c^2)}} \quad \text{i.e.} \quad l' = l\sqrt{(1 - v^2/c^2)}\ .$$

This last equation gives the length of the contracted arm. But why should the arm in the direction of earth's motion contract in length? Well, because it is only on that assumption that one can explain Michelson-Morley's experiment.

It is clear that such assumptions and explanations are not satisfying. It was Einstein who first realised that the reasons for the negative result of this experiment were deep-rooted. They were to be found in our faulty concepts of space, time and motion.

We had written down the following three simple relations between c, u and v by looking at Figs. 1, 2 and 3.

$$c = u + v, \quad c = u - v, \quad c^2 = u^2 + v^2.$$

They (at least the first two of them) follow from 'common sense' notion or the Newtonian notion of space, time and velocities. Einstein analysed these 'common sense' notions and found that they were logically faulty and then he set out to revise them.

6. Measurement of Time

We shall try to follow Einstein. The first step is to forget about all 'common sense' notions of space, time and motion and begin with a clean slate. One thing is certain. The basic concepts are those of space and time and the concept of motion or velocity is derived from these and so depends on them. Thus the concepts of distance-measurements and time-measurements should be first formulated in an independent manner.

As we have seen earlier, the Newtonion concept of time was that of a 'true even flowing time', the same for all observers. According to this concept to measure time difference between any two events is similar to measuring space-distance between any two places on a high-way. We have mile-stones fixed on a highway. If a place A is at the mile-stone 6.7 and a place B at 7.9 then the distance AB is $7.9 - 6.7 = 1.2$ miles. Well, similarly, according to Newton there is a highway of time along which time-stones are fixed. An event X happened at 1946.82 and an event Y happened at 1967.93. Then the time-interval between the two events is 11.11 years.

Einstein tells us to forget about the above analysis and try to think out a logical way of measuring this time interval. We know that time is measured by clocks. If an event X happens at A at clock-time t_1 and another event Y happens at A at time t_2, then the time interval between the two events is $t_2 - t_1$. This seems to be quite sound.

Now suppose the two events X and Y happen at two different places A and B. Then we must have a clock at A and a clock at B. Suppose the two events are again at t_1 at A and t_2 at B. Then is the time-interval between the two events $t_2 - t_1$? Well, it will be so provided the two clocks keep the 'same time'. But how can one assume that the two clocks at different places keep the same time? Well, one can take the clock at B (say) and move it to A and check by oneself whether the two clocks keep the same time or not. But this requires 'moving' one clock. Concept of motion is to be derived from the concept of time measurement, but we need 'motion' in order to define time at two different places. This is something like moving in a circle.

One can think of several methods of comparing the time kept by two clocks at different places but in all these methods one will always have either to 'move' one of the clocks or to 'move' an agent from one place to another. In order to define motion you need to measure time-difference between events at different places and in order to measure this time-difference, you need the concept of motion.

Einstein realised that the clue to the negative result of Michelson-Morley experiment lay hidden in this vicious circle. To break this vicious circle Einstein suggested a very simple way out. In order to make the measurement of time-interval between two events at two different places logically meaningful, we must preassume (i.e. assume in advance) a fundamental agent moving with a fundamental velocity. All motion except the motion of this fundamental agent (F.A.) is derived motion. Then one can use this F.A. for comparing clocks at two different places and the method of measuring time becomes logically consistent.

Using this additional concept of a F.A. Einstein built up a logically consistent dynamics and found that the result of Michelson-Morely experiment leads one to conclude that this F.A. is light and that its velocity c in ether is the fundamental velocity. In order to see this more clearly we recall our Fig. 1. There we wrote down the following: "It is clear from Fig. 1 that $c = v + u$ or $u = c - v$."

Our argument was the 'common sense' argument. But according to Einstein's analysis of the logic of these concepts we would get from the same figure $c = \frac{v+u}{1+uv/c^2}$. Now this is not at all 'clear from Fig. 1'. When our equation was $c = u + v$ we got $u = c - v$ very casily. Let us now solve Einstein's equation $c = \frac{u+v}{1+uv/c^2}$. to find u. If we do it (it is simple algebra!) we shall find that $u = c$. So here lies the funniness of the result $v = 0$ of the M.M. experiment. The illogical Newtonian dynamics led to the result $u = c - v$ but the logical Einsteinian dynamics requires $u = c$. No wonder therefore that the interpretation of Michelson-Morley experiment on the logically faulty Newtonian dynamics will lead to the funny conclusion $v = 0$,—the planets are at rest.

7. Concluding Remarks

The added sharpness of our instruments of observation as represented by the interferometer technique of Michelson and Morley necessitated one more attempt to revise our concepts of space, time and motion. The deep probing by Einstein into the logical consistency of the then prevalent ideas of time-measurement revealed great crevices and thus he started on a course of reformulating these concepts. As a result of this reformulation, amongst other things, he found that Lorentz was right in saying that a moving rod gets shorter in length in the direction of motion. Einstein proved from his logically consistent dynamics that

1. lengths of objects are not absolute, but depend on the motion of the observer measuring them.
2. Time interval between two events also depends on the motion of the observer measuring this interval.

He went much beyond this and deduced the now famous equation $E = mc^2$, but this does not come under the subject of the present talk.

The advance in experimental techniques led to a probing into the foundations of the theoretical structure. This in its turn gave rise to a series of conclusions which need experimental verification. This would naturally lead to a further sharpening of our instruments of observation and so the caravan of science moves on and on, its speed of growth developing in an exponential manner.

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