Die Physik: Siebenter Jahrgang. Heft 1. 1939, p. 1. Quantentheorie und Wellenmechanik. 2 von G. Wentzel.

I Principal Questions.

Deals with the important problems engaging attention today. One cannot however say that much progress has been achieved during the last 3 or 4 years. Only in the case of the so-called "hole theory" of the positron, a formalism has been discovered which at least up to a certain approximation avoids the infinities. The self-energy problem of Quantum Electrodynamics has not been brought nearer to a solution. Mention might be made of Dirac's attempt at a formal modification of the classical Lorentz eqns of motion

Also the hopes raised by the "unitary theories" have proved false. The Mie-Born theory of treating material particles as singularities has been wrecked on the fact that it does not enable electrons to have half-integral spins (Constructed out of light quanta with integral spin. (Pryce: Camb. Phil. Soc. 32, 614 (18, 781). The converse problem based after de Broglie's idea of constructing light-quanta from particles of spin ½ (the neutrino-theory of light of Jordan & Kronig) appears first of all full of promise but later contradicted (Pryce. Proc. Roy. Soc. 165 (1938), 247). There are also other mathematically contradictionfree neutrino-theories (Wentzel. Zs. f. Phys 92, 337 (16, 402)), but it can be hardly be said that particles of these theories are the same as the Pauli-Fermi neutrinos, and the aim of unification has barely been achieved. Finally the discovery of the heavy electron in cosmic rays has created the impression that we do not know as yet all the elementary particles and that it is not worth while attempting any kind of unification, but wait for more data about the new particles & high energies.

besetzen = occupy

II Theory of particle-pairs.

In calculations of pair generation etc., one neglects the electrons of negative spectrum. Recent work has shown that this is justified. When external fields are present we cannot take charge density as sum of density functions of all ext. electrons but subtract a const. infinite density i.e. we have the problem of making $\infty - \infty$ finite and unique. A mathematical idea of doing this was given by Dirac (Proc. Camb. 30, 150) and Heisenberg (Z. Phys, 90, 209) as follows: The charge density is bilinear in ψ & ψ^*. Replace ψ^* at x by ψ^* at x', and thus obtain a density matrix $\rho(x, x')$ which goes to usual limit for $x' \to x$. The Dirac-Heisenberg idea is that one can take out from ρ a matrix $\rho_0(x, x')$ (depending on external field uniquely & not depending on which levels are occupied or not) such that

$$\lim_{x' = x} \left\{ \rho(x, x') - \rho_0(x, x') \right\}$$ is finite, and gives the actual density function.

The simplest application of this theory concerns the case that in vacuum an "external" field can be adiabatically inserted without addition of electron or positrons. An external field induces in "Vakuum" a charge distribution, the so called "Vacuum polarisation. In other words, the external field generates pairs of electrons and positrons whose charge densities have just the concerned expectation values, and the field generated by these charges obey Maxwell's laws.

Heisenberg (ibid), Serber (Phys. Rev. 48, 49) and Uehling (Phys. Rev. 48, 55) have considered the special case where the external field is so weak that in the Störungsrechnung only the first and terms in the field strength & its derivatives can be retained. In this approximation additional terms appear in Maxwell's equations equivalent to saying that Coulomb's law breaks down for lengths less than $\hbar/mc$ distances

angesichts = in the face of.

less than the Compton wavelength $\frac{h}{mc}$. In higher approximations the field equations are no longer linear ie the superposition principle is no longer valid. Also the phenomenon of scattering of light by light (easily understood on pair generation theory, Halpern (Phys. Rev. 44, 855) arises.

Remarkably enough all calculations made hitherto confirm that all variations from linearity can be represented by a universal Lagrange function which is first of all assumed in unitary theories (Mie, Born). If $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \cdots \mathcal{L}_n$ - $\mathcal{L}_n$ being $(2n)^{th}$ order in field components - assuming

$$\mathcal{L}_2 = \alpha (E^2 - H^2) + \beta (EH)^2$$

α and β have been calculated by Euler & Kockel (Naturwiss 23, 246) by comparison with the Dirac-Heisenberg theory) and the result is obtained that

$$\beta = 7\alpha, \quad \alpha = \frac{1}{720\pi^3} \cdot \frac{e^4 h}{m^4 c^7}.$$

The same values are obtained by considering other cases by Heisenberg & Euler (Z. Phys, 98, 714), also Weisskopf & Kemmer (Nature, 137, 659) and Delbrück (Helv. Phys. Acta 10, 112) and Delbrück (Z. Phys. 84 (1933)).

The deviation from the superposition-principle is unimportant as long as the field-strength remains less than $(8\pi\alpha)^{-1/2}$. This critical field strength is remarkably numerically almost equal to the elec field strength on the classical Lorentz theory at the boundary of the electron ie e^2/r_0^2 where $r_0 = e^2/mc^2$. The numerical correspondence depends on the fact that the numerical factor $90\pi^2$ appearing in $1/8\pi\alpha$ is of the same order as the dimensionless natural constant hc/e^2 $(= 2\pi \cdot 137)$ and it is imaginable that this has a deep significance.

In the face of the mathematical complications of the hole theory it is

vernichten = annull, annihilate

natural that one should investigate the relativistic wave mechanics of the electron in such a way that the energy is positive definite. So has Majorana (Nuovo Cim. 14, 171) given a theory for particles of spin ½ in which states of negative energy can be eliminated without contradiction. However, in this theory the conception of electric charge does not enter, since no quantities like charge density or current density satisfying continuity equations have any place in it. ~~Perhaps the case at all it is applicable to~~ It is therefore not applicable to electrons, but perhaps to neutrinos & antineutrinos which are physically indistinguishable (not even to neutrons because sign of magnetic moment distinguishes ~~neutron~~ it from its antiparticle).

That the hole representation is unnecessary under certain circumstances for describing theoretically anti-particles (with charge) was first established by Pauli & Weisskopf (for spin zero) (Helv. Phys. Acta, 7, 709). Starting from Schrödinger's scalar relativistic wave eqn Pauli and Weisskopf, they applied the well known method of double quantisation. They then obtained the energy density as positive but not the charge density. This gives particles & anti-particles (oppositely charged) which can be generated in pairs & also annihilate each other, and all this without applying the hole idea. Even here the subtraction-recipe becomes necessary because of the infinite vacuum polarisation.

From this it at once becomes clear that the wave-mechanics of particles which have spins other than that of an electron is also interesting. Recently the discovery of the heavy electron in cosmic radiation and the development of the field theory of nuclear forces has given numerous ~~papers~~ papers in this direction.

verträglich = consistent

seltsam = curious

anlangen = arrive at.

(* oral communication of Pauli & Fierz)

Dirac has set up wave equations for particles with arbitrary (half or integral) spin (Proc. Roy. Soc, 155, 447). The spin 1 has been in particular investigated by Proca (J. de. Phys 7, 347). In this case the wave equations can be described just like Maxwell's equations. In fact the light quantum has spin 1. The Proca's particles are however more general since they can have rest-mass and charge. The quantising of this wave field (corresponding to Bose statistics) has been done by Kemmer (Proc. Roy. Soc, 166 (1938) 127), Stueckelberg (Helv. Phys. Acta, 11, 225), Bhabha (Proc. Roy. Soc 166, 501), Yukawa, Sakata u Taketani (Proc. Phys. Math. Soc. Japan, 20, 319). A very complete investigation for all spin values (half or integral) is due to Fierz (Helv. Phys. Acta, to appear hereafter). In the case of force-free fields the wave equation is uniquely determined essentially by specifying the mass and spin. The particles with integral spin are quantised according to Bose statistics and those with half-integral spin according to Fermi-Dirac statistics. Any other quantisation would contradict certain general postulates. By integral spin results the energy and by half-integral spin the charge (without subtraction rule) as positive definite. For the density of charge or ~~charge~~ energy there are in general several different assumptions consistent with conservation principles. Only in the cases of spin values 0 and 1 or $1/2$ is the energy and or charge respectively positive definite, which both densities can be uniquely determined. Thus, among the rest, are the smaller spins characterised. For spin values > 1, there are moreover very curious results which result by the action of external forces (for eg. transitions with mass & spin-changes); that such elementary particles which are apparently not found in nature are here connected*. A special case

is that of a particle whose rest mass vanishes (light-quantum !). Here (for spin $\neq 0$) there are two different waves of given direction & frequency corresponding to both the polarisations of a light wave, it being assumed that in Electrodynamics only "gauge-invariant" magnitudes are distinguished for use. For spin 1 the density function can be defined gauge-invariantly (for eg. energy density of Maxwell-field), but not however for spin > 1 which again is a formal characteristic of small spin values.

III Nuclear physics.

Preliminaries about nuclear forces - Breit, Condon & Present's charge independence hypotheses (Phys. Rev. 50, 825). For heavy nuclei the optimum of accuracy is achieved in a work of Euler (Z. Phys. 105, 553). Certain inequalities for the potential parameters can be derived from the fact of stability of certain theoretically constructed nuclei (for eg those built only with neutrons or those with particles having only parallel spins (Breit & Feenberg - Phys. Rev. 50, 850) & (Kemmer. Nature 140 (1937)).

The excited states of nuclei are important for the understanding of transmutation processes which are the transitions between different stationary states. The formal apparatus necessary to describe transmutations is therefore in essence the same as that used in the theory of atomic processes (Dispersion formulae) (Breit & Wigner. (Bethe & Placzek, ibid, 51, 450) Phys. Rev. 49, 519). A quantitative distinction lies in the fact that the resonance phenomenon is more common here because of the energy levels being densely spaced. Bethe (Phys. Rev. 50, 332) has theoretically evaluated the statistical distribution of energy levels, which is in great conformity with the experimental values. A typical case

of resonance is that of a primary particle of suitable energy being absorbed by a nucleus; it gives rise to an intermediate ~~stage~~ state of considerable longevity. The excitation energy is as a rule distributed between the nuclear particles, as for example similar to the stopping of projectiles in a viscous fluid being dependent on the addition of energy due to a rise in temperature (Bohr. Nature 137, 344). Nuclear secondary processes can be paralleled to the damping effect of fluids (Bohr. Naturwiss 24, 241; Frenkel. Phys. Z. Sowj. 9, 533; Weisskopf, Phys. Rev, 52, 295). Detailed treatment by Bethe.

In all the above mentioned problems the phenomenological representation of forces through explicitly specified potential functions is correct in principle because the velocity of all the concerned particles is small compared to c. How are ~~the~~ the laws of force stated when this is no longer the case, as for eg in processes which are set up by cosmic ray particles? For description of electromagnetic phenomena wherein appear relative velocities comparable to c it is no longer admissible to use the famous Coulomb's law, and we have to use field theory ie make use of Maxwell's equations. Analogously it is here necessary to extend the phenomenological description of the "static" nuclear forces to a field theory of nuclear forces. Just as we can determine all electromagnetic interactions either in terms of the Maxwell field or in corpuscular manner by light-quanta, we must similarly be able to determine nuclear forces through a nuclear field or through corresponding corpuscles.

Tamm and Iwanenko (Nature 133, 981) have suggested in 1934 that the nuclear field is the same as the β-field – The difficulties of this (Gamow + Teller

Phys. Rev. 51, 289; Wentzel, Helv. Phys. Acta 10, 107; Kemmer, Phys. Rev 52, 906) are however very great.

Yukawa (Proc. Phys. Math. Soc. Jap, 17, 48; 19, 712) formulated in 1935 a relatively heavy, charged & integral-spin particle which determines the force between neutron and proton. On this picture the collision of a neutron & proton can be so described that a neutron emits a negative Yukawa particle becoming a proton and experiencing a recoil, whereas the original proton further absorbs the Yukawa particle thereby absorbing its impulse & becomes a neutron. By this double transition which is consistent with conservation of charge and spin, both the particles experience a change of impulse under exchange of charge which gives rise to the action of an exchange force. Also positive particles can determine such forces. If the spin of the intermediate particle be zero, as Yukawa assumed in his first article, there is no exchange of spin (for small rel. velocities of the collision partners) & the forces are pure Heisenberg forces which contradicts experience. The next simplest assumption is that the Yukawa particle has spin 1; in this case the proton-neutron force is in general composed of Heisenberg and Majorana forces and the theory, specially in regard to sign (attraction or repulsion), can be shown to be in consonance with the phenomenological theory (Fröhlich, Heitler, Kemmer; Kemmer (Proc, Camb, Phil. Soc 34, 354). In the case of small velocities the potential function of the forces in the first approximation (average over all directions) of the form.

$$V(r) = \text{const.} \frac{1}{r} e^{-r/a}$$

where a is the Compton wavelength of the Yukawa particle (divided by 2π). According to this theory the nuclear forces have a range of width of order a. Identifying the Yukawa particle with the heavy electrons (mass $\sim$ 200m) we get a of the order 10^{-13} cm in consonance with the phenomenological theory. It might also be

burgt (bergen) – save, shelter
conceal

* also Fierz (Z. Phys. 104, 553)

mentioned that the for (p-p), (n-n) and (C-I) hypothesis have been attempted to be explained on the assumption of the neutrinetto. This assumption is however very much obvious since correspondences are to be valid for other sorts of particles (Neutron-proton, Electron-neutrino!) obviously such a quantum theory of nuclear fields hides no less the principal difficulties than the related Quantum Electro-Dynamics and can therefore have for the present only limited possibilities of application. (Wentzel Naturwiss 26, 273).

Yukawa has also remarked in his first paper of the possibility of the application of his theory to the theory of β-decay. Before referring to this we must consider the development of the β-decay theory.

All recent discussions on β-decay are based on Pauli's representation that by β-transformation an electron & neutrino are simultaneously emitted but only the electron is observed. Therefore a nuclear particle which emits a β-particle must transform itself from a neutron to a proton (or conversely) because the total charge is to be conserved. The A.M. Conservation requires the neutrino to have the spin ½. The energy which the nucleus gives off in a determined transition ie the energy difference ΔE between initial & final states distributes itself in some manner between both the emitted particles; the energy spectrum of the only observable electron is consequently a continuous spectrum with an upper limit determined by ΔE.

Fermi (Z. Phys, 88, 161) has given the original theory — Refs Bethe-Bacher about relativistic invariant formulation of this* — Konopinski & Uhlenbeck (Phys. Rev. 48, 7) have modified this, but this again has been contradicted by recent measurements of intensities by Lyman (Phys. Rev, 51, 1), Paxton (Phys. Rev. 51, 170).

Finally we could get away from Fermi's assumption that the β-transformation

is an elementary process. The possibility of denoting it as a double process suggests itself by the Yukawa particle. By the transition of a neutron to the proton state, first a heavy electron is generated which, on account of principle of conservation of energy, is not expelled as a free particle (but only as a virtual intermediate state for the neutron-proton collision). This heavy electron has now the ability of decomposing into an ordinary electron and neutrino so that the end state is the Fermi mechanism of β-decay. In fact the lifetime of β-rays as also the facts of cosmic rays (Euler. Naturwiss 26, 382; Euler & Heisenberg, Erg. ex. Naturwiss 17 (1938), 1) are consistent with the assumption that the heavy electron has in force-free state a mean lifetime of the order of 10^{-6} sec (measured in its rest-system) after which it decomposes itself into an ordinary electron & neutrino. It however appears difficult to so present this theory in a form symmetrical in Electrons & neutrinos (simplest interaction type without derivatives of wave functions), and it further remains to explain the asymmetry of the β-spectrum.

Another possibility which can denote β-transition as a second order process is the assumption of further unstable particles with integral spin which are heavier like protons & neutrons (Wentzel: Z. Phys. 104, 34; 105, 738). The form of the β-spectrum depends on the position of the energy levels and can correspondingly be unsymmetric. Since however no such particles have been observed in nature, this theory is purely hypothetical and also its theoretical possibilities have to be explored.

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I The non-relativistic theory.

II The Dirac theory of the electron.

* Ref. are to Phys. Ber.

Schrödinger's scalar form – Dirac's theory – difficulties of the hole theory – applications of Dirac's theory (Klein-Nishina formula, internal conversion etc, etc, See Heitler's book) –

For the mathematical formulation of the relativistic many body problem there arises, as also for the corresponding classical problem, the question as to how to take into consideration the retardation of electro-magnetic interaction forces of the elementary particles. There is no point of doing this for the non-relativistic case since the outgoing velocity c of the force is very great compared with the velocity v of the material particles. In classical relativity one assumes that all interactions are determined by the electro-magnetic field, and so arrives through a coupling of mechanics and electrodynamics to an invariant formulation of the many body problem. The same aim is pursued in connection with wave mechanics leading to Quantum-Electrodynamics. Here we must mention two limiting cases in which it can be shown that it is possible to eliminate the electro-magnetic field quantities in the first approximation, and achieve a wave-mechanical treatment of the many body problem under the inclusion of the retardation effect. when the particle velocity v is so small that it would be valid to retain $(v/c)^2$ for the retardation effect; Breit (10, 2191; 13, 1291)* has given an eqn for the 2-body problem which gives a correction to the Schrödinger-Pauli formulation of the many body problem The other limiting case is that in which the total interaction of particles (including the Coulomb-force) is so weak that in the development in powers of e^2 the terms with e^4 etc could be neglected. The condition for this is

$$v/c \gg \frac{2\pi e^2}{hc} = \frac{1}{137}$$

ie the velocity v must be suitably near to c. The wave mechanical formulation of the problem for this case has been given by C. Møller (12, 2369; 13, 2151) & others. Associated

Fock:

Schrodinger: Berl. Berichte, 1932, S. 105–128, Nr 11/12.

Schouten: Jour. Math. Phys. 10, 239–271, 1931, Nr. 4, Teil II

Bargmann: Berl. Br. 1932, S. 346–54, Nr. 14

Infeld & van der Waerden: " 1933, S. 2, Nr. 1

Pauli & Solomon: J. de. Phys et le Rad. (7) 3, 452, 582, 1932, Nr. 10 & 12.

Schouten & D. van Dantzig: Z. f. Phys 78, 639, 1932, (9–10).

Pauli: Ann. d. Phys (5), 18, 305, 1933.

" 337, 1933

considerations on the manybody problem and the relation with Quantum-Electrodynamics are given by L. Rosenfeld (13, 500) and also by H. Bethe & E. Fermi (13, 2123)

A general invariant representation of Dirac's wave equation of the electron one-electron problem on the basis of Einstein's general relativity is given by V. Fock (10, 2255). He determines the wave-mechanical behaviour of the motion of an electron in an arbitrarily specified electro-magnetic and gravitation field. This theory is further developed by Schrodinger (13, 1843), Schouten (13, 1104), Bargmann (14, 166), Infeld & Van der Waarden (14, 1680). The next representation of a corresponding wave-equation for Einstein-Mayer's "unitary field theory" has been investigated by Pauli and Solomon (14, 464) and in a different mathematical way by Schouten and van Dantzig (14, 546) and also by Pauli (15, 98 & 99). These theories introduce additional terms in Dirac's equation ~~deviating very slightly~~ the deviations of which from the ordinary equation are so small as to be barely within the range of experimental determination. Also the general theories give no progress in the direction of negative energy problem.

III Quantum Electro-dynamics:

In Dirac's theory of radiation, one makes a principal distinction between the static (Coulomb) interaction forces between elementary particles which are introduced as distant-action forces in the wave equation of mechanical systems and the radiation-forces exerted by quantised light waves. This distinction or division of forces is however dependent on the reference system, and to arrive at a relativistic formulation of the manybody problem it is obvious that, as a consequence of the adoption of the Maxwell-Lorentz standpoint, it should be possible to determine all interactions including the Coulomb forces from the

electro-magnetic field. It is therefore necessary to generalise Dirac's radiation theory torso to a quantum theory of the total field (including the electrostatic). According to Heisenberg, Pauli (11, 498) and Fermi (11, 956), one can go further in the division of the field into plane waves (ie spatial Fourier Analysis) in general transversal and longitudinal, of which the first determine the characteristic light-waves and the second the static interactions. The Schrödinger-function ψ depends on t, on the spatial coordinates of the particles and the ~~Dauplitudes~~ (Fourier-Coefficients) of the partial waves of the field. A mathematical ~~for~~ complication arises here in that the ψ-function must satisfy, besides the wave equation, a further "subsidiary Condition" ~~@~~ that must be satisfied automatically for all t if true for a certain initial time $t = 0$. Consequently we can separate and eliminate the longitudinal waves and thereby arrive at a wave equation ~~which~~ in which the charged particles appear on the one hand coupled by Coulomb forces and on the other hand there also appear transversal light-waves in interaction ie a return in principle to Dirac's theory of radiation. This is nevertheless a progress achieved: ~~As long as one~~ As soon as one can dispose of the relativistic invariant wave equation of charged particles, the Quantum-Electrodynamics leads to the corresp. rel-inv formulation of the many body problem – The uncertainty relations for field-strengths – Bohr & Rosenfeld (Kgl. Danske XII (1933) Nr. 8). This theory of Bohr's contributes essentially to the understanding of the characteristic theory of "exchange relations" for field-strength components (Jordan & Pauli (9, 1032)) and speaks much for the correctness of the theoretical foundations of Quantum-Electrodynamics.

The Heisenberg-Pauli-Fermi formulation of Quant-Electrodynamics has the defect that it assigns to different elementary particles different space-coordinates but only a common time-coordinate. Because of this defect in symmetry the theory

the employment of Minkowski's four-dimensional geometry and makes the proof of relativistic invariance very difficult. This defect has been removed in a more general formulation of Quantum Electrodynamics by Dirac, Fock & Podolsky (14, 1076). Formally every particle has here an individual time-coordinate & also the field has a characteristic time variable, the "field-time". The ψ-fn now depends on the space- & time-coordinates of the particles as well as on the Fourier-coefficients of the field; it satisfies many wave equations (as many as particles present) and a "subsidiary condition"; besides there are also valid certain "field equations" as identities for the field variables. In this formulation the space & time coords (x, y, z, ict) of the particles appear in complete symmetrical form, and the relativistic invariance becomes evident. If besides we put all time-coords (particles & field) equal, we obtain the special ψ-function of the Pauli-Heisenberg-Pauli-Fermi theory. The physical significance of this generalisation has been investigated by Bloch (Phys Z. Sowetj. 5 (1934), 301) — the generalisation appears merely as a mathematical formalism.

The difficulties of quantum-Electrodynamics have been much discussed and appear to be of two sorts. The first can be indicated by considering for the moment the force-free particles of Dirac's wave equation, and by coupling of these electrons with the electromagnetic field ~~gen~~ we have immediately the difficulties connected with negative energy levels which has not yet been removed without exception. The second is that the theory leads to an infinite self-energy of electrons. This difficulty is perhaps not entirely quantumtheoretic in its origin; in fact the classical (Lorentz's) theory of the electron yields, when the electron-radius goes to zero, an infinite quantity "electromagnetic mass"; in other words, a point electron cannot, on account of the infinite inertia of its associated electro-magnetic field, be accelerated by a finite external force. In the Heisenberg-Pauli-Fermi ~~theory~~ Quantum Electrodynamics which, to begin with, starts with point electrons one also obtains for the electrons an infinite inertial mass. (~~Ros~~ Oppenheimer (11, 2510)).

äusseren – manifest

* cf. this with the "unitary" theories of relativity ie geometrisation

in the energy-eigenvalues of an individual electron which is in interaction with its characteristic field, there manifests itself on account of this infinite inertia an additional infinite term (Waller (11, 2184). one could try to avoid these difficulties by considering an extended electron (Born & Rumer (12, 1884)); this would bring a good deal of arbitrariness (in the choice of charge distribution) into the theory, and & contradict relativistic invariance, and finally such a process would also go against the postulate of atomistic constitution of electricity. On this point, the above mentioned Dirac-Fock-Podolsky generalisation of Quantum Electrodynamics has shown unexpectedly a way out. It shows that the boundary values of the field strengths & potentials ~~on the paths of the particles~~ are multiple valued on the paths of the particle, and hence the necessity arises ~~to make the~~ to specify exactly the conditions (Vorschriften relating to ~~me~~ the mode of certain double boundary transitions (Wentzel (15, 179, 242). It is shown in this paper that it is possible to eliminate in principle the "classical part" of the self energy. A further infinite self energy term which contains $\hbar$ as factor and is conditioned by spin has so far not been eliminated. ~~This depends~~ The appearance of these terms is closely connected with the existence of negative energy spectrum, and we can therefore hope to eliminate these infinities also when the ~~whole~~ 'hole theory' is better clarified.

IV New Results & Problems.

Unitary* theory - unifying matter & electro-magnetic field - Born's field theory taking electromagnetic field as fundamental & particles as singularities.

The opposite tendency of taking matter as fundamental is L. de Broglie's "neutrino" theory (14, 467; also C.R. 197, 1377).

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A. March: Naturwiss (1938) p. 649: - The idea of an atomic structure of space.

§1. Divergences appearing in the field theory - infinite self-energy of electron, and infinite energy of cavity radiation ($= \infty \times h\nu/2$).

§2. The boundary of validity of geometry - A point of space P can be limited by a spherical domain G of a determined radius l_0 such that points P' of G are indistinguishable from P.

§3. Statistic significance of the metric - Distance is taken as the mean value of very many measurements. This can also be applied to the case $P_1P_2 < l_0$.

Take $P_1P_2 = P_1P_3 - P_2P_3$ where P_1P_3 & P_2P_3 are ~~as defined~~ measured statistically we can formally use Riemannian geometry with $ds^2 = g_{ik}\, dx_i\, dx_k$, but the geometry would be a statistical one corresponding to relations of classical and quantum mechanics.

§4. Generalisation to the space-time world - Similar definition that all points are equal for which $ds^2 = dx_1^2 + \ldots - dx_4^2$ is +ve and $< l_0^2$ - See Flint (Roy. Soc, 117, 629; 159, 46 / Phyp. Soc. 50, 1, 50, 340)

- Also Mimura & Schuler (J. Sci. Hirosima t. 5, 99 (1935) who treats ds as a matrix.

§5. The spatial nature of the particle:

Heitler: Remarks on nuclear disintegrations by cosmic rays (Phys. Rev, 54 (II), 873):

Problem of nuclear disintegration can be divided into two questions

(i) Which is the process and its probability by which a cosmic ray particle (electron, photon or ~~two~~ mesotron) can transfer a large amount of energy to the nucleus? This is transferred to a nuclear particle (neutron or proton)

(ii) How is energy of nuclear particle distributed among others & how can excited nucleus fall to ground level; which particles are then emitted?

Consider (ii) first: Case (a) the nuclear particle can leave nucleus undisturbed. This happens when particle is at surface & receives a velocity directed outwards (Heisenberg: Sächsische Akad. Wiss, 369 (1937)

(b) if velocity be directed inwards it interacts with other particles. According to Bohr-Kalckar (Vgl. Danske, Vid. Selsk, 14, 10 (1937) collision processes of neutrons with nuclei in region 1-10 M.V ~~show that~~ result in whole energy being transformed into nuclear heat energy & thermal equilibrium reached before a particle leaves the nucleus. It can be shown by simple arguments that the same ~~be~~ is true for cosmic ray energy range (100-1000 M.V).

The case of complete equilibrium has been treated by Weisskopf (Phys. Rev. 52, 295 (1937) whose results are valid ~~&~~ if total excitation energy is less than the total binding energy. For heavy nuclei this last is 2000 M.V & so his results can be applied for cosmic ray energies of 100-1000 M.V. According to Weisskopf the temp. T is given by

$$T = (aE)^{1/2}, \quad a \text{ between } 0.05 \text{ and } 0.2 \text{ M.V} \qquad --- (1)$$

For $E = 100-500$ M.V, $T = 2-10$ M.V & most particles leave nucleus with this energy.

$$\left.\begin{aligned} N + \gamma^{+} &\rightarrow P \\ P + \gamma^{-} &\rightarrow N \end{aligned}\right\} \qquad (2)$$

The "charge" g of mesotrons responsible for (2) is so large that $g^2/\hbar c \simeq 1$, in contrast to β-decay $e^2/\hbar c = 1/137$.

After release of some particles nucleus cools down and slower particles are to be expected. This is in consonance with expts of Blau & Wambacher. In some expts by these workers particles with energy > 10 M.V are noticed & this may be due to particles released from a partially heated nucleus. Total no. of particles is ~ E/T if temp is const. For $E = 100$, then $E/T = 20$ at least half ie 10 would be neutrons and 5–10 protons may be expected.

Next consider (i) a more difficult question and hardly answerable at present viz the question of ~~interaction~~ energy transfer of cosmic particle to a nucleus or question of interaction of cosmic ray particle with a nuclear particle. This can hardly be the long range Coulomb forces. Heisenberg (ibid) & Williams (Nature 142, 341 (1938)) have suggested short range forces as between nuclear particles. This means that cosmic ray particle is itself a proton or neutron & there is no evidence of primary protons in cosmic radiation. The question therefore reduces to the way whether there is a short range interaction between electron, photon or heavy ele mesotron and a nuclear particle.

This is one of the fundamental problems of quantum theory. The interaction between electron and nuclear particle either on Fermi's theory or Yukawa's version of it gives too weak an interaction even if the results of the above theories which are valid at 1 M.V be extrapolated to higher energies. Thus the soft component should not give rise to an appreciable number of disintegrations and ~~this~~ it is hard to explain the increase in the rate of disintegrations with height.

For heavy electrons, it can be understood that they give rise to nuclear disintegrations* and for energies of the order 100 M.V the cross sections for various scattering processes is of a reasonable order of magnitude. For energies high compared with the rest energy of mesotron (50–100 M.V)

we have complicated processes like multiple ones. The very existence of these shows that ordinary methods of quantum theory can no longer be applied in this region. The interaction between heavy electron & nuclear particle is of the form

$$(g/\lambda)\,\mathrm{div}\,\psi(0) + (f/\lambda)\,\sigma\,\mathrm{curl}\,\psi(0) \qquad (3)$$

g & f are constants of the order of fine electron charge, $\lambda = mc/\hbar$ (m = mass of mesotron) $\psi(a)$ = wave fn of mesotron at position of nuclear particle, σ = Spin of latter. (3) explains successfully properties of neutron & proton. (3) however holds only for small energies of mesotron. For higher energies $K > \lambda$, etc it can be shown that the theory leads to divergences even more serious than the well-known divergences of quantum electrodynamics. In all probability the right interaction between a heavy particle & mesotron will be of a <u>non-linear type</u> & this fact marks the limit of the applicability of the present quantum mechanics. The same considerations apply to interactions of β-particles & nuclear ones.

The position is slightly different from that in quantum electrodynamics where the deviations from linear laws are purely of theoretical nature & cannot be verified experimentally. Thus all radiation effects of an electron could very well be treated by a first-order approximation theory & there would be no experiments contradicting such a theory. Here the behaviour of mesotrons in traversing matter provide direct experimental tests for the interaction in question.

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<u>G. Wentzel</u> (Phys. Rev. 54(11), 869). <u>Angular spread of hard Cosmic ray showers</u>:—

* α = parameter in perturbation method or the "charge" g in previous article by Heitler

($\alpha \sim 1/10$; cf. $e^2/\hbar c = 1/137$)

Yukawa's theory like quantum electrodynamics must be assumed to break down in problems involving very large energies

The high energy states are cancelled automatically if the elementary particles are assumed to have non-vanishing spatial extension. In the math. formalism there are appear "form factors" whose introduction may have only the significance that they define the domain in which the unmodified quantum theory holds true.

The matrix element of the interaction of mesotron with proton-neutron is by one degree higher than in the interaction of photon with electron in quantum electro-dynamics (Kemmer: Proc. Roy. Soc, 166, 127 (1938), Bhabha, Ibid, 501)

If a heavy electron having spin 1, endowed with energy $E \gg \mu c^2$ (μ = mass of mesotron) collides with a proton or neutron, the theoretical cross section of process in which n mesotrons are emitted into the solid angles $d\Omega_1, \ldots d\Omega_n$ ($1 \leqq n \ll E/\mu c^2$) has order of magnitude *

$$\alpha^{n+1} \left(\frac{\hbar}{\mu c}\right)^2 \left(\frac{E}{\mu c^2}\right)^{2n} d\Omega_1 d\Omega_2 \ldots d\Omega_n$$

at least for small scattering angles ϑ. Without form factors this holds for any energy as far as $\vartheta \sim \left(Mc^2/E\right)^{1/2}$; the integral cross-section would therefore be of the order

$$\alpha \left(\frac{\hbar}{\mu c}\right)^2 \left(\alpha \frac{M}{\mu} \cdot \frac{E}{\mu c^2}\right)^n$$

and heavy electrons above energies 10^9 eV (1000 M.V) would no longer be penetrating which, of course, can hardly be true. If a form factor be introduced which allows only scattering angles $\vartheta \leqq \left(\hbar c/rE\right)$ the integral cross-section turns out to be of order

$$\alpha \left(\frac{\hbar}{\mu c}\right)^2 \left(\alpha^{\frac{1}{2}} \frac{\hbar}{\mu c} \cdot \frac{1}{r}\right)^{2n}$$

In this case the cross-section will not only become constant with increasing energy but also dependence on n will be comparatively weak so that showers with many particles

($1 \ll n < E/\mu c^2$) can be expected to occur with an accordingly reduced probability. This is in accord with experiments. Thus the "cancelling rule" appears theoretically useful.

Recently Heisenberg has expressed the opinion that quantum-theoretical formulae for transition probabilities should be applicable only to such processes in which the invariant changes of momentum $|(\Delta P)^2 - (\Delta E/c)^2|^{1/2}$ remain below a finite limit $\hbar/r$ for each particle involved in the process (see also Watagh in, Nature 142, 393 & Comptes Rendus 207, 358). According to this if $\hbar/r < \mu c$ the absorption and emission of mesotrons would be entirely beyond the confines of present theories.

The "form-factor" does not remove the infinities associated with self-energies & similar quantities (eg. magnetic moments calculated from self energy in ext. mag. field). But in this respect the rule

$$|(\Delta P)^2 - (\Delta E/c)^2|^{1/2} \lesssim \hbar/r$$

is also of no use since the left-hand side expression vanishes identically in the case of an absorbed or emitted light-quantum.

Thus it appears that infinities of the above type can only be eliminated by means of a subtraction formalism such as has proved indispensable in the "hole theory" of the positron.

E. C. G. Stueckelberg: (Phys. Rev. 54, (11), 889). Rigorous theory of interaction between nuclear particles:

The static interaction between elementary particles can be introduced in the Hamiltonian by way of a contact transformation. In classical theory no serious difficulties arise. However, in quantum theory of radiation divergent terms appear.

O. Laporte (ibid, p. 905. Elastic scattering of yukawa particles I).

Orijinally the mesotron was thought to be of the Pauli-Weisskopf type possessing spin zero (Helv. Phys. Acta, 7, 709 (1934)). This gave wrong sign for nuclear forces. Thus for the first time Dirac's family of solns (Proc. Roy. Soc. 155, 447, 1936) for $S > \frac{1}{2}$ had an application. Fierz's article (in Press, Vide Wentzel's report) shows that certain simplifying facts, for eg, the possibility of the construction of only one stress-energy tensor - are common only to particles of spin $0, \frac{1}{2}$ and 1. Particles with spin 1 have been dealt with by Proca.

In this paper elastic (Heavy Rutherfordian) scattering of mesotron is examined by the relativistically invariant equations developed by Proca & Dirac. According to Mott's calculation of the scattering of electrons a polarisation effect does not occur until the second approximation, such an effect appears here even in the first approximation

References to soln of Poisson-like eqns by successive approximations in the familiar way by means of Green's functions (Born - Wentzel: Z. Phys Hdbuch d. Phys Vol 24, 1, p. 712-21) — also to Weisskopf "destruction-operator" in Dirac's theory of the electron (Z. Phys. 89, 27 (1934)

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C. Eckart (ibid, p. 920) Electrodynamics of material media:

Electrodynamics has never been developed in a manner analogous to analytical mechanics. G. Mie made an important attempt to deduce a theory from a single L. One consequence of this attempt was a formulation of the requirement of gauge invariance by H. Weyl. Mies theory did not satisfy this & Weyl attempt to modify Mie's theory was not successful. In addition to use of a L, classical mechanics is characterised

by the use of a great variety of coordinates

In this paper is obtained the general eqns of a gauge invariant, classical theory of electrodynamics of material media. The gauge invariance is insured by taking

$$\nabla \cdot D = \rho \; ; \quad \nabla \times H - D' = J$$

as constraints auxiliary to the variation principle

$$\delta \iiint \{ L + \sum_n \theta_n [N_n' + \nabla \cdot (N_n V_n)] \, dv \, dt \} = 0 .$$

L depends on D, H, N_n, V_n, θ_n and possibly their derivatives; here N_n is the numerical density of atoms in the state n, V_n their macroscopic or average velocity, and θ_n a variable that functions as velocity potential in some cases and has the dimensions of action. The electromagnetic potentials enter the theory as Lagrangian multipliers only. It is shown that if there is only one state, and

$$L = \tfrac{1}{2} N m V^2 - (\hbar^2/8m)(\nabla N)^2/N + \tfrac{1}{2}(H^2 - D^2)$$

then the Schrödinger wave eqn is obtained on substituting

$$\psi = N^{1/2} \exp(-i\theta/\hbar).$$

If the atoms are stationary

$$L = \sum N_n W_n + D \cdot P + \tfrac{1}{2}(H^2 - D^2)$$

where W_n is energy of the nth state, and

$$P = \sum_{mn} \sum (N_m N_n)^{1/2} P_{mn} \cos[(\theta_m - \theta_n)/\hbar + \alpha_{mn}]$$

is the polarisation of the medium, an adequate theory of dispersion results. However the spontaneous transitions are not correctly accounted for by these equations

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N. Rosen (Phys. Rev. 55 (1), 39, p. 94). A field theory of elementary particles.

Maxwell's eqns have singularity at a charged particle. They make the particle have infinite self-energy and because of the singularity the field eqns are not

valid at the point so that it is necessary to have eqns of motion for the charge in addition to the field equations.

Mie's theory was not gauge-invariant, Born-Infeld's is so, and also gets rid of infinite self-energy but the field still contains a singularity & hence it is necessary to supplement the eqns for the field with eqns of motion for the electron (Frenkel Proc. Roy. Soc 146, 933).

Maxwell's eqns are

$$F^{\mu\nu}{}_{;\nu} = 0 \qquad (1) \qquad (; \text{ denotes covariant diff})$$

$$F_{\mu\nu} = (\partial\varphi_\nu/\partial x_\mu) + (\partial\varphi_\mu/\partial x_\nu) \qquad (2)$$

If $\quad L_f = -F_{\mu\nu}F^{\mu\nu}/8\pi \qquad (3)$

condition $\quad \delta\int L_f(-g)^{1/2}d\tau = 0 \qquad (d\tau = dx_1\,dx_2\,dx_3\,dx_4) \qquad (4)$

leads to Maxwell's eqns. Since L depends on φ's only thro' F's & thus a guage-transform

$$\varphi'_\mu = \varphi_\mu + \partial\lambda/\partial x_\mu \qquad (5)$$

does not change form of L or form of eqns (1).

To derive other systems of eqns in place of Maxwells start with a scalar function of the field variables, L and take the law

$$\delta\int L(-g)^{1/2}d\tau = 0 \qquad (6)$$

with potentials varied as in (4). If eqns be written

$$F^{\mu\nu}{}_{;\nu} = 4\pi S^\mu \qquad (7)$$

S^μ, depending on field variables, can be considered charge-current density vector. If one can find a static spherically symmetrical soln of (7) free from singularities

the solution might be taken to represent an electron at rest in the absence of an external field.

The singularity difficulty in Born's theory can be shown as follows (Pointed out to author by Einstein). If $L = L(F_{\mu\nu})$ we can write (4) as

$$\int D^{\mu\nu} \delta F_{\mu\nu} (-g)^{1/2} d\tau = 0 \qquad (4,a)$$

where $D^{\mu\nu}$ is antisymmetric defined by

$$D^{\mu\nu} = -4\pi \frac{\partial L}{\partial F_{\mu\nu}} \qquad (8)$$

Hence,

$$\int D^{\mu\nu} \delta F_{\mu\nu} (-g)^{1/2} d\tau = 2 \int D^{\mu\nu} \delta \frac{\partial \varphi_\mu}{\partial x_\nu} (-g)^{1/2} d\tau$$

$$= -2 \int D^{\mu\nu}{}_{;\nu} \delta \varphi_\mu (-g)^{1/2} d\tau \qquad (9)$$

after integrating by parts & discarding the (vanishing) surface integral. For (9) to vanish with arbitrary $\delta\varphi_\mu$, one must have

$$D^{\mu\nu}{}_{;\nu} = 0 \qquad (10)$$ which represents field eqns & is therefore equivalent to (7). Since $D^{\mu\nu}$ is anti-symmetric (10) can be written as

$$\frac{\partial}{\partial x_\nu} \left[D^{\mu\nu} (-g)^{1/2} \right] = 0 \qquad (11)$$

In the case of static sph. sym. this reduces to

$$d\left(r^2 D_r\right)/dr = 0 \qquad (12)$$

where D_r is radial component of $D = (D^{41}, D^{42}, D^{43})$. Soln of (12) is

$$D_r = e/r^2 \qquad (13)$$

If L be so chosen that for weak fields the eqns go over to Maxwell's then comparing (8) & (3) one sees that for large r, (13) goes over to

$$E_r = e/r^2 \qquad E_r = \text{radial comp of } \vec{E}\ (F^{41}, F^{42}, F^{43})$$

from that e is to be interpreted charge & hence cannot be put equal to zero. Consequently $D_r = \infty$ for $r = 0$. If D_r is a well-behaved function of E_r, this makes $E_r = 0$ for $r = 0$ & one has a singular point.

Even the modification proposed by Hoffmann & Infeld (Phys. Rev. 51, 765 (1937)) leads to a double-valued action-function which leads to discontinuity in derivatives. Thus the forms of L depending only the $F_{\mu\nu}$ does not appear satisfactory. we have therefore to find an L including the φ_μ so as to avoid singularities and be at the same time guage-invariant.

One of doing this is to introduce φ_μ & other parameters so as to conserve guage invariance - for example in deriving Schroedinger's wave eqn from a L, this depends on the potential and on the wave function, such that φ_μ & ψ are so transformed under a guage trn that L remains invariant. Let

$$L = L_f + L_m \qquad (14)$$

L_m is a fn of φ_μ & new variables. Let this new variable be a scalar fn ψ such that it is transformed under (5) by

$$\psi' = \psi e^{i\epsilon\lambda} \qquad (15) \qquad (\epsilon = \text{real constant of dim } e^{-1})$$

From ψ and φ_μ can be formed two simple invariants for use in variation principle viz $\psi\psi^*$ and $\chi^\mu\chi_\mu^*$ where

$$\chi_\mu = \partial\psi/\partial x_\mu - i\epsilon\varphi_\mu\psi \qquad (16)$$

and take for L_m the linear combination (one constant included under ψ) (using Sp. relativity)

$$L_m = -\chi^\mu\chi_\mu^* + \sigma^2\psi\psi^* \qquad (18) \qquad (\sigma = \text{real const of dim } L^{-1})$$

$$\text{rem } L. = -\frac{1}{8\pi}F^{\mu\nu}F_{\mu\nu} - \chi^\mu\chi_\mu^* + \sigma^2\psi\psi^* \qquad (19)$$

If eqn (6) be taken as the law of the field where φ_μ & ψ are varied independently one

(23) $\epsilon \sim 1/e$, $\sigma \sim 1/a = mc^4/e^2$

Obtains the system of diffl. eqns

$$\chi^{\nu}_{;\nu} - i\epsilon\varphi_\nu \chi^\nu + \sigma^2 \psi = 0 \qquad (20)$$

$$F^{\mu\nu}_{\ ;\nu} = 4\pi S^\mu \qquad (21)$$

with $$S^\mu = \frac{1}{2} i\epsilon (\psi^* \chi^\mu - \psi \chi^{\mu *}) \qquad (22)$$

$(S^1, S^2, S^3, S^4) = (J_1, J_2, J_3, \rho)$, so that

$$\rho = \frac{i\epsilon}{2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right) + \epsilon^2 \varphi \psi \psi^* \qquad (23)$$

Eqn (20) in terms of ψ has practically the same form as the Gordon-Klein ~~eqn~~ rel. wave eqn in quantum-mechanics. In the latter, however, the potentials occurring are those of "outer" field whereas here (is included) fields due to charge & current determined by ψ. The result is that here the system of eqns (20) and (21) is non-linear in ψ, which may not therefore be normalised arbitrarily as done in wave-mechanics.

Also the energy-impulse tensor $T_{\mu\nu}$ given by

$$T_{\mu\nu} = \partial L / \partial g_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} L \quad \text{consists of } (T_{\mu\nu})_f + (T_{\mu\nu})_m$$

Seeing $(T_{44})_f \geq 0$, $(T_{44})_m \leq 0$ ie sign of T_{44} is not definite

Divergence eqn is $\partial T^{\mu\nu}/\partial x_\nu = 0$, and integrating this

$$\frac{d}{dt}\int_V T^{\mu 4} dv = -\int_S T^{\mu n} df$$

giving eqns of motion of a particle corresponding to action of the Lorentz-force

The energy-impulse vector (Weyl. Raum-Zeit-Materie (Berlin 1923) p 204)

$$P^\mu = \int T^{\mu 4} dv \quad \text{satisfies } dP^\mu/dt = 0$$

& energy is given by $W = P^4 = \int T^{44} dv = \int T_{44} dv$

* Author's argument for static sph.-sym. case.

Take $\psi = \theta e^{-i\epsilon\mu t}$

Field Eqns (20) & (21) become

$$\left.\begin{aligned} \frac{d^2(xy)}{dx^2} &= x(1-z^2)y \\ \frac{d^2(xz)}{dx^2} &= -xy^2 z \end{aligned}\right\} \quad \begin{aligned} \theta &= \sigma y / \epsilon\sqrt{4\pi}. \\ \phi + \mu &= \sigma z/\epsilon \\ r &= x/\sigma \end{aligned}$$

charge $e = 4\pi \int_0^\infty \rho r^2 dr = \alpha/\epsilon$, $\alpha = \int_0^\infty y^2 z x^2 dx$

Also $\beta = \epsilon\mu/\sigma$

Find T_{44} and hence

$$W = -\sigma w / 2\epsilon^2 \quad \text{with } w = \alpha\beta + \gamma \text{ , and } \gamma = \int_0^\infty y^2 z^2 x^2 dx$$

of solutions without nodes for given ϵ & σ take the one with minimum energy as representing the electron. This gives $a = 1.6$ ($b = 2.19$, $\alpha = 1.91$, $\beta = 0.015$) by numerical integration $[a = y(0);\ b = z(0)]$. If $\beta \neq 0$, then we have both energy & frequence associated (because $\beta \neq 0$ means $\mu \neq 0$) and calculation of

$$\alpha' = 2\pi e^2 / h'c \quad \text{where } h' = |W/\nu|$$

gives $\alpha' = |2\alpha^2\beta / (\alpha\beta + \gamma)| \lesssim 0.04$

Application to the static spherically symmetrical case leads to eqns admitting electron-like solutions & the electron turns out to have a negative mass, ~~perhaps~~ a difficulty which will perhaps disappear when the equations are quantised. The theory, though classical, offers in principle a possibility of accounting for the Sommerfeld fine structure constant which means that there is a much closer relationship between the classical & quantum theories. The author suggests that a change in the form of L in (19) might lead to better coincidence with the value 1/137.* Also possibilities of soln representing heavy electrons is mentioned.

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F. Zwicky: ~~Highly~~ (Phys. Rev. 55(8) (1939), p. 726. Highly collapsed stars:

The tremendous rate of generation of energy in super-novae requires an explanation along entirely new lines. One suggestion is the transformation of an ordinary star into a collapsed neutron star of density of order 10^{14} gm/cm^3 ie of order of density inside ordinary atomic nuclei. The very properties of space in highly collapsed stars may be radically different from the properties of ordinary space. In this theory it is necessary to introduce general relativistic effects and not merely rely on the current idealised models of stars. Thus the classical theory of gravitation (à la Schwarzschild) even if combined with the fundamental result $E = mc^2$ of the special theory is unsatisfactory in that it does not allows us to formulate clearly how the mass equivalent of the gravitational ~~energy~~ potential energy is to be distributed over space. This difficulty has its real root in the fact that gravitation is a cooperative phenomenon. ~~Systems and~~ (Zwicky, Phys. Rev. 43, 270 (1933)) and the general theory of relativity alone must be used to estimate such effects as the effective masses of large dense stars.

we might mention one other aspect of cosmological aspect of the theory of neutron stars viz the conclusion of Stern (Zs. f. Electrochemie 31, 448 (1925) on the basis of statistical mechanics & thermodynamics that in a state of equilibrium between matter & radiation in the Universe, practically all matter would be found to have been transformed into radiation. The ratio of concentration C_m of matter in the ordinary forms & of the concentration C_R of matter transformed into radiation is determined on Stern's theory by

$$\frac{C_R}{C_m} \sim e^{mc^2/KT} \quad \left(\begin{array}{l} K = \text{Boltzmann's const} = 1.37 \times 10^{-16} \\ T = \text{average abs. temp of Universe} = 100^\circ \text{ abs.} \end{array} \right.$$

$$\sim e^{6 \times 10^7}, \text{ an undefeatable magnitude}$$

i.e. C_m i.e. in a sphere of enormous radius R' (6×10^{39} = rad. of Universe) and not even a single electron could exist in statistical competition with radiation (number N of Eddington is of order 1.2×10^{78}). As an escape from I have suggested elsewhere that it is not permissible to consider only matter in ordinary forms, but matter assuming state of high statistical weight through gravitational scattering. If this clustering leads to configurations, as it does in limiting neutron stars, where the velocity of light becomes zero, the gravitational packing energy of a particle of mass m may here become equal to its original proper energy mc^2 & matter in this form may enter into successful statistical competition with radiation

If the neutron-star hypothesis of origin of supernovae can be proved it will be possible to subject the general theory of relativity to a really crucial test. This theory, although profound and exceedingly satisfactory in its epistemological aspects, has so far practically not lent itself to any very obvious and generally impressive applications. It may thus be possible to remove the unfortunate discrepancy between formal beauty of the general theory of relativity and the meagreness of

its practical applications.

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A. March (Zs. f. Phys. 104 (1937), p. 93): Die Geometrie Kleinster Raume I.

We have became familiar, through Special relativity, with the fact that geometry insofar as it introduces a determination of measure in the space-time world has an essentially empirical character – by the metric $g_{ik}\,dx^i dx^k$. The purely physical ideas of measuring rods + clocks when considered in the microscopic domain of quantum mechanics suggest a similar limitation on the metric. Thus Quantum mechanics does not expressly decline to use geometry, but only makes a sparse use of it because it avoids electrons describing definite orbits. Where however it actually uses the metric, as for example in the self-energy of an electron or the gravitational energy of a photon, it immediately resolves itself into contradictions. The metric here suggested is that the length of the curve-element between two neighbouring points is $\int \sqrt{g_{ik}\,dx^i dx^k} - r$, this being taken in the statistical sense as the result of very many measurement. r is a small positive Scalar invariant of the coordinates and an universal invariant.

This definition has to meet two objections

(i) If $P_1, P_2 \ldots P_n$ be series of points on a line at distances r, then $P_1 P_n = 0$. This contradicts the axioms of Riemannian geometry; this means that we are here dealing with a non-Riemannian geometry where the metric is expressed not in the infepal and not the differential form. A similar example is afforded by quantities measured by sense perceptions conditioned by a certain threshold measure of discrimination.

(ii) If $P_1 P_2 < r$, we can take a P_3 & set $P_1 P_2 = P_1 P_3 - P_2 P_3$ where $P_3 P_1$ & $P_3 P_2 (\text{are}) > \delta$.

Thus the extension would be merely artificial. This difficulty is got over by the statistical definition.

The value of the "metric analyser" r is to be inferred from experience - for eg e^2/mc^2 or h/mc. In the self energy problem the classical Maxwell theory of a point charge the field energy has an infinite value. The difficulty could be got over by the assumption of an extended electron, but the theory becomes non-invariant relativistically. Suppose we introduce a form function $D(x,y,z)$ to give the spatial extension of an electron such that it is ~~zero only~~ different from zero only in the ~~no~~ inside of a small region; then by changing to another coordinate system the time t appears in this function and it becomes impossible to formulate the field laws invariantly. With our metric the field energy depends on $\int \frac{dr'}{(r'+r)^2}$ $(r' = r - r)$, r being section by plane $t = \text{const}$ & independent of direction of t-axis. The limits of integration for r' are 0 & r' & the integral $= \frac{1}{r} - \frac{1}{r'+r} = \frac{r'}{r'+r}$ ie $\frac{1}{r}$ with $r' = 0$. In a similar manner all infinities in Electrodynamics could be avoided.

The simplicity of this solution is quite misleading because the developments from the point of view of the metric alone is so simple. Further developments bring forth its complications and we get in this geometry many contradictions with the old geometry. Thus for eg we have here, circles of zero radius but non-zero circumference. We agree to treat all such theorems as meaningless when extrapolated from the microscopic to the macroscopic region.

Ibid - 2nd paper, p. 161. The introduction of a complex r is shown not to lead to any

fruitful results. The "mathematical" & "physical" points are compared & shown to be alike extensionless, and the latter replaces the ~~last~~ heretofore role played by the former with the addition of the statistical notion. Thus all the formal representation of natural laws through differential equations are not altered, but however field quantities obtain meaning as the mean of a large number of measurements. For ~~wave~~ wave motion, the definitions lose their meaning in the region $\lambda \leqq r$. If a photon with $\lambda > r$ ~~goes~~ goes over from a K to a K' by Coord. trn with $\lambda < r'$, the interpretation is that the photon loses its wave character but appears as a static field

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A. March (Z. f. Phys. 105, 620) - Zur Grundlegung und Anwendung einer statistischen Metrik.

By applying the considerations of the statistical metric to quantum Electrodynamics it is shown that it is necessary to assume the point of view that the path of an electron inside the sphere r is to be considered as undefined.

Ibid (Bd 106, 49). Statistische metrik und Quanten-Elektrodynamik:

It is shown that, by the introduction of the idea that all points of a world region G of definite smallness r are undifferentiable, the Heisenberg-Pauli theory can be developed so as to get rid of all the difficulties connected with infinities and this in a relativistic invariant manner. A consequence of the new metric is that in place of Dirac's δ-function an undetermined function $D(x-x')$ appears in the exchange relations. This function D is zero different from zero inside G and is transformed on a change of coordinates like Dirac's β-matrices. $D(x-x')$ effects, by the development of the potentials Φ_i and the field strengths E, the

appearance of an indeterminacy in orthogonal systems and hence a breaking off of the spectrum at $\lambda = r$; the radiation energy will therefore be finite. Further the electrostatic as well as the magnetic proper energy (appearing as Breit's interaction term) are finite, so that a consequent relativistic invariant solution of the many-electron problem is now possible.

The spectrum of black-body radiation does not reach to arbitrarily small wave-lengths, but breaks off at $\lambda = r$, and consequently the zero-point-energy of radiation is finite.

The proper energy corresponding to terms $s = s'$ (which becomes ∞ in the old theory) is here finite and equal to e times the potential of the space-charge at the centre of the sphere ie equal to $3e^2/2r$. r is not a physical constant, but a magnitude which arises from the world-geometry itself and determines the metric of the space-time continuum. This expression therefore does not denote any special inner condition on the electron, but holds for every elementary particle of charge e. The eigen-energy and hence the mass of the electron is to be considered as a magnitude forced on it by the world geometry. Thus r and not m is to be considered a primary natural constant.

Ibid. p. 291. The gravitational energy of photons:

The application of the statistical metric gives a finite value for the gravitational energy of a photon (L. Rosenfeld, Z. Phys. 65, 589 (1930); and Solomon, ibid 71, 162 (1931)), which for $\nu = 10^{15}\ \text{sec}^{-1}$ (approx) amounts to 1% of $h\nu$. This result has a fundamental significance because the theory is here applied to a case where the electron makes no appearance and thus shows that r is not a physical constant connected with the electron but an universal natural constant.

Ibid, p. 532. Zur Theorie der Kernkräfte:

The universal length r also appears in the theory of nuclear forces and is determined by the value of the neutron-force as almost equal to the classical electron radius. The same value is obtained by considering the measurements of Anderson & Neddermeyer, and Blackett & Wilson on the stopping of fast electrons.

The value of r obtained by taking neutron-proton force as 30×10^{-6} erg and the β-theory factor g as 4×10^{-50} g.cm is 3.1×10^{-13} ie $3r_0$. The self-energy of electron in static field is given by $3e^2/2r$ and also the magnetic self energy as $\frac{e^2\alpha^2}{2r} = \frac{3e^2}{2r}$ & hence the total energy is $3e^2/r = e^2/r_0$ (if $r = 3r_0$). This shows that in the electron mass only has to assume a negative energy which lowers $3e^2/r$ to e^2/r in other words electron mass is not purely electromagnetic in origin.

Ibid Bd. 108, p. 128. Die Frage nach der Existenz einer Kleinsten Wellenlänge:

It is shown that some nuclear phenomena of Blackett are explained by the assumption of a least wavelength $\lambda_0 = 6.2 \times 10^{-13}$ cm ($\pm 10\%$) or $r = 3 \times 10^{-13}$ cm

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Einstein & Infeld: Evolution of Physics p. 256. we have two realities: matter and field. Field laws, electromagnetic and gravitational are structure laws. These break down for very great concentrations of energy or where sources of the field ie electric charges or matter are present. We cannot build physics on the matter-concept alone. Could we not reject concept of matter and build a pure field physics and thus explain all events in nature by structure laws valid always and everywhere. This would mean the modification of field laws in such a way that they would

not break down for regions in which the energy is enormously concentrated. But we have so far not succeeded in fulfilling this programme.

p. 304: The equations of quantum physics determine the probability wave just as Maxwell's eqns determine the electromagnetic field and the gravitational equations determine the gravitational field. The laws of quantum physics are again structure laws. But the meaning of physical concepts determined by the equations of quantum physics is much more abstract than in the case of electromagnetic and gravitational fields.

p. 305: Whereas in classical physics we spoke of positions and velocities of one particle, we must now consider probability waves, in a three dimensional continuum corresponding to this one-particle problem.

p. 306. For n particles the probability waves will be functions in a continuum of 3n dimensions. Thus probability waves are more abstract than the electromagnetic and gravitational field existing and ~~penetrating~~ spreading in our three-dimensional space. The continuum of many dimensions forms the background of probability waves, and only for one particle does the number of dimensions equal that of physical space.

p. 308: In the many particle problem we rise from the classical to the quantum description, from the objective description of events in space and time to probability waves. But how can we describe the interaction between elementary quanta of matter and field? A probability wave ~~of~~ with an infinite number of dimensions would be needed for the quantum description of a field. The transition from the classical field concept to the corresponding problem of probability waves in quantum physics is a very difficult step. Ascending one floor here is no easy

task, and all attempts so far made to solve the problem must be regarded as unsatisfactory.

There is another fundamental problem. In all arguments about the transition from classical to quantum physics, only the pre-relativistic description is used in which space and time are treated differently. If, however, we try to begin from the classical description as proposed by the relativity theory, then our ascent to the quantum problem seems much more complicated. This is another problem tackled by modern physics, but still far from a complete and satisfactory solution.

There is still a further difficulty in forming a consistent physics for heavy particles, constituting the nuclei.

P. But there is also no doubt that quantum physics must still be based on the two concepts: matter and field. It is, in this sense, a dualistic theory and does not bring our old problem of reducing everything to the field concept even one step nearer realisation.

p. 301: It appears that the new quantum physics resembles somewhat the Kinetic theory of matter, since both are of a statistical nature, and both refer to great aggregations. But what are the differences? In the ~~statis~~ Kinetic theory we have statistical laws governing the aggregation, gained on the basis of individual laws. But in quantum physics the state of affairs is entirely different. Here the statistical laws are given immediately, and the individual laws are discarded. In fact, the question of the position and velocity of an electron at any moment has no sense in quantum physics.

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Annals of mathematics 35, 787, 1934. G. A. Hedlund: Metrical transitivity of geodesics on closed surfaces of constant negative curvature.

A dynamical system is regionally transitive if there exists a motion such that the points of the phase-space defined by this motion constitute a set which is everywhere dense in the phase-space. Such a motion is called transitive.

The system is said to be metrically transitive if any set of motions which defines a measurable set of points of the phase-space is either of measure zero or of the measure of the phase-space. An example of a dynamical system of with one degree of freedom which is metrically transitive has been given by Birkhoff and Smith.

It is shown here that the systems of geodesics on a certain set of surfaces which are closed, orientable, of arbitrary genus greater than one, and of constant negative curvature are metrically transitive.

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Dirac: Annals of Math, 37, 429, 1936. Wave equations in conformal space:

In a recent paper it is shown that the equations of physics can be expressed in a tensor form involving indices taking on five values by passing from ordinary space-time to de-Sitter space. By making a further generalisation to 4-dimensional conformal space, eqns have tensor form having 6-valued indices. In this way a still greater symmetry and invariance under a wider group of these equations is shown up.

While all eqns of physics can be generalised to de-Sitter space only a few, viz those that are invariant under conformal transformations of the ordinary space-time, can be generalised to conformal space.

The conformal space dealt with is the four dimensional surface of a hyper-quadric in five-dimensional projective space.

Cunningham & Bateman's (Proc. Lond. math. 8, 17 & 223 (1910) result of

the invariance of Maxwell's eqns under conformal transformations in ordinary space-time follows as a consequence.

Scalars, Vectors, and spinors are introduced in the conformal 6 space, and an attempt is made to set up a spin wave equation equivalent to the usual wave equation for the electron, and it is concluded that there is no simple way of getting such a wave equation in conformal space

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G. Birkhoff and J. v. Neumann: Ann. math. 37, 823 (1936). The logic of quantum mechanics.

Phase-spaces: There is one concept which quantum theory shares alike with classical mechanics and classical electrodynamics. This is the concept of a mathematical "phase space"

According to this concept, any physical system S is ~~at~~ at each instant hypothetically associated with a "point" p in a fixed phase-space Σ; this point is supposed to represent mathematically the "state" of S which is supposed to be ascertainable by "maximal" (Pauling & Wilson: 1935 p. 422), Dirac, 1930, § 4) observations

Furthermore the point p_0 associated with S at a time t_0, together with a prescribed "mathematical law of propagation" fix the point p_t associated with S at any later time t; this assumption is evidently embodies the principle of mathematical Causation. (Heisenberg. Principles, Chicago, 1929). p. 65

Thus in classical mechanics each point of Σ corresponds to a choice of n position and n conjugate momentum coordinates - and the law of propagation may be Newton's inverse square law of attraction. Hence in this case Σ is a region of ordinary 2n-dimensional space. In electrodynamics, the points of Σ can only be specified after certain functions - such as the electro-magnetic and electrostatic potential - are known; hence Σ is a function space of infinitely many dimensions. Similarly in quantum theory the points of Σ correspond to

* In any case the law of propagation may be inquired as inducing a steady fluid motion in the phase-space. It has proved to be a fruitful observation that in many important cases of classical dynamics, this flow conserves volume. It may noted that in quantum mechanics, the flow conserves distances (ie the equations are "unitary").

the so called "wave-functions", and hence Σ is again a function space - usually assumed to be a Hilbert space (Neumann - p. 18). In electrodynamics, the laws of propagation is contained in Maxwell's equations, and in quantum theory, in equations due to Schrödinger. *

Observation-spaces: An "observation" of a physical system S can be described generally as a writing down of the readings from various compatible measurements. Thus if the measurements are denoted by the symbols $\mu_1, \ldots \mu_n$, then an observation of S amounts to specifying numbers $x_1, x_2, \ldots x_n$ corresponding to the different μ_k. The most general form of a prediction concerning S is that the point $(x_1 \ldots x_n)$ determined by actually measuring $\mu_1 \ldots \mu_n$ will lie in a subset T of $(x_1, \ldots x_n)$-space. If the $(x_1 \ldots x_n)$-spaces be called "observation-spaces" we may call the subsets of observation spaces associated with S the "experimental propositions" concerning S.

Propositions as sub-sets of phase-space: Elements and subsets of phase-space must be correlated with experimental propositions before phase-space acquires a reality

This is done in classical mechanics by measuring the position, and velocity (hence momentum) explicitly

In electrodynamics no such simple procedure is possible, and in quantum theory not even this is imagined, and the possibility of predicting in general the readings from measurements on a system S from a knowledge of its state is denied; only statistical predictions are always possible.

In classical mechanics, at least in statistics, it seems best to assume that the it is the Lebesgue measurable subsets of a phase-space which correspond to experimental propositions, two sub-sets being identified if their difference has Lebesgue-measure 0. The experimental propositions of classical mechanics form a "Boolean Algebra".

$$i\beta \begin{pmatrix} iF^{12} & iF^{23}+F^{31} & -iF^{43} & -iF^{41}+F^{42} \\ iF^{23}-F^{31} & -iF^{12} & -iF^{41}+F^{42} & +iF^{43} \\ -iF^{43} & -iF^{41}-F^{42} & iF^{12} & iF^{23}+F^{31} \\ -iF^{41}+F^{42} & -iF^{43} & iF^{23}-F^{31} & -iF^{12} \end{pmatrix}$$

$$\alpha^1\alpha^2 \quad \rho_1\sigma^n\rho_1\alpha^2 = \alpha^1\alpha^2 = i\alpha^3.$$

$$\alpha^{23}. \qquad = i\sigma_1$$

$$\alpha^{31} \qquad \sigma_2$$

$$= \begin{pmatrix} -H^3 & -H^1+iH^2 & iE_3 & iE^1+E^2 \\ -H^1-iH^2 & +H^3 & iE^1-E^2 & iE^3 \quad -iE_3 \\ -iE_3 & -iE^1-E^2 & H^3 & H^1-iH^2 \\ -iE^1+E^2 & -iE^3 & H^1+iH^2 & -H^3. \end{pmatrix}$$

$$+iE_3$$

$$(E^2-H^2)^2$$

The fact that all operators of quantum mechanics are Hermitian, and that all Hermitian-symmetric operators in Hilbert space correspond to observables enables one to build up a propositional calculus for quantum mechanics.

The important conclusion can be derived that the propositional calculus of quantum mechanics has the same structure as an abstract projective geometry

It is suggested that "continuous geometry" may be a more suitable frame of reference for quantum theory than Hilbert space.

The physically significant statements in quantum mechanics actually constitute a sort of projective geometry, while in classical mechanics they constitute a Boolean Algebra.

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J. L. Synge: Ann. math. 36, 650, (1935). Mechanical models of spaces with positive-definite line-element.

The totality of configurations of a dynamical system with N degrees of freedom, holonomic & without moving constraints, constitute the configuration-space V_N whose points are the configurations of the system, and whose topology is therefore determined by the dynamical system. The Kinetic energy is of the form

$$T = \tfrac{1}{2} a_{ij} \dot{x}^i \dot{x}^j .$$

We may assign to V_N the Riemannian line-element (for this geometrical aspect of dynamics see Synge: Phil. Trans, 226, 31–106)

$$ds^2 = a_{ij}\, dx^i dx^j ,$$

and a dynamical system provides us with a Riemannian space, defined topologically and metrically. The interest of this representation is two-fold. In the

first place it gives a dynamical geometrical interpretation to the dynamical problem; secondly it gives a dynamical interpretation to multi-dimensional spaces or to spaces with a topology and metric which cannot be realised in a Euclidean space.

Examples (i) A system consisting of a disc which can turn about an axis perpendicular to its plane, the centre being at the same time free to move along a st. line in the plane. If θ denote angle of rotation and z displacement of centre

$$T = \tfrac{1}{2} m \dot{z}^2 + \tfrac{1}{2} I \dot{\theta}^2$$

Putting $x = z\sqrt{m}$, $y = \theta\sqrt{I}$, $ds^2 = dx^2 + dy^2$

i.e. configuration-space is a flat-space of two dimensions, having the connectivity of a cylinder, since an increase in y of amount $2\pi\sqrt{I}$ restores the configuration

(ii) For two discs rotating independently we have a flat space of 2 dimensions having the connectivity of an anchor-ring, formed by joining the opposite edges of a rectangle.

(iii) Sph. top – manifold of configurations is a 3-space of constant positive curvature $K = 1/4I$.

Dirac: Electron wave equation in de-Sitter space (Ann. Math, 36, p. 657 (1935).

The equations of atomic physics are usually formulated in terms of the space-time of the special theory of relativity. They form a scheme invariant for Lorentz transformations which carry the space-time axis into itself.

Various more general spaces have been proposed. Each of these would necessitate some modifications in the scheme of equations of atomic physics, but these modifications would however be much too small to be of any physical interest, and would therefore be, at most, of mathematical interest.

Nearly all the more general spaces have only trivial groups of operations which carry the spaces over into themselves with the exception of de-Sitter space which is associated with a very interesting group.

Defining de-Sitter space as the surface of a four-dimensional "sphere" (of a hyperbolic character in one direction) embedded in five dimensional space, the principal eqns are expressed in tensor form involving 5-valued indices. In particular it is shown that it is possible satisfactorily to construct the analogue of the electron wave equation.

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Einstein and Bergmann: On a generalisation of Kaluza's theory of electricity

Annals of math, 39, 683 (1938).

So far two fairly simple and natural attempts to connect gravitation and electricity by a unitary field theory have been made, one by Weyl, the other by Kaluza. The theory represented here differs from Kaluza's (O. Klein: Zs. f. Phys, 37, 895, (1926); Einstein: Sitz. d. Preuss. Akad. d. Wiss, 1927, 23-30) in one essential point; we ascribe physical reality to the fifth-dimension. We shall show that it is possible to assign some meaning to the fifth-coordinate without contradicting the four dimensional character of the physical continuum (Vide my abstract for Current Science)

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E. Wigner: Unitary representations of the inhomogeneous Lorentz-group

Ann. math. 40, 149, (1939)

It is perhaps the most fundamental principle of quantum mechanics that the system of states forms a linear manifold in which a unitary scalar

(*v. Neumann pp. 19–24)

product is defined*. The possibility of a future non-linear character of quantum mechanics must be admitted, of course. An indication in this direction is given by the theory of the positron as developed by Dirac, Heisenberg and which does not use wave functions and is a non-linear theory. The states are generally represented by wave functions (Heisenberg picture) in such a way that φ, and constant multiples of φ represent the same physical state. It is therefore possible to normalise the wave function ie to multiply it by a constant factor such that its scalar product with itself becomes 1 (~~Recently~~ The wave fns are complex quantities and the undetermined factors in them are complex also. Recently attempts have been made by Majorana (~~Letter~~ Nuovo Cim. 14, 171) and Dirac (in print ?) toward a theory with real wave functions. Then, only a constant factor of modulus 1, the so called phase, will be left undetermined in the wave function. The linear character of the wave function is called the superposition principle. The square of the modulus of the unitary scalar product (ψ, φ) of two normalised wave functions ψ and φ is called the transition probability from state ψ to φ and conversely. The transition probability has an invariant physical sense.

The wave functions form a description of the physical state, but not however an invariant description ie same state will be described by different wave equations in different coordinate systems $(\varphi_l, \varphi_{l'})$

$$|(\varphi_l, \psi_l)|^2 = |(\varphi_{l'}, \psi_{l'})|^2$$

and it can be shown that

$$\varphi_{l'} = D(l', l)\, \varphi_l$$

where $D(l', l)$ is a linear unitary operator (Wigner's book). Denoting $D(l', l)$ by $D(L)$ we can say that $D(L)$ is determined (apart from constant factor having no physical meaning)

* $D(L)$ forms, up to a factor, a representation of the inhomogeneous Lorentz group by linear unitary operators.

by the Lorentz-transformation L, $l' \to Ll$ and

$$D(L_2)D(L_1) = \omega D(L_2 L_1)^{*}$$

where ω is a number of modulus 1 and can depend on L_2 & L_1.

It may be mentioned finally, that these developments apply not only in quantum mechanics but also to all linear theories eg the Maxwells equations in empty space. The only difference is that there is no arbitrary factor in the description and the ω may be omitted and one is led to real representations instead of representations up to a factor. On the other hand, the unitary character of the representation is not a consequence of basic assumptions.

Einstein, Infeld and Hoffmann: The gravitational equations and the problem of motion (Ann. math. 39, p. 65 (1938)).

The extent to which the relativistic equations of gravitation determine the motion of ponderable bodies is considered here. Previous attacks on this problem [Droste. Wet. Amst. 19, 447 (1916); de Sitter M.N.R.A.S, 67, 155 (1916); Mathisson Z. Phys, 67, 270, 826, 69, 389; Levi-Civita: Am. J. lix, 3, 225 (1937)] have been based upon gravitational equations in which some specific energy-momentum tensor for matter has been assumed. Such tensors must be regarded as purely temporary and more or less phenomenological devices for representing the structure of matter, and their entry into the equations makes it impossible to determine how far the results obtained are independent of the particular assumption made concerning the constitution of matter.

Actually the only equations of gravitation which follow without

[* By comparison with Helmholtz's & Euler's non-linear theories,
Born's non field theory suffers from the fact that the
motion of singularities is not determined by the field eqns alone.
Could not this be modified by a change of action function, since
it is accepted that there is no physically admitted principle so far
restricting the choice of the action-functions]

ambiguity from the fundamental assumptions of the general theory of relativity are the equations for empty space, and it is important to know whether they alone are capable of determining the motion of bodies. The answer to this question is not at all obvious. It is possible to find examples in classical physics leading to either answer, yes or no. For instance in the ordinary Maxwell equations for empty space in which electrical particles are regarded as point singularities of the field, the motion of these singularities is not determined by the linear field equations (Vide Rosen's paper)*. On the other hand, the well known theory of Helmholtz on the motion of vortices in a non-viscous fluid gives an instance where the motion of line singularities is actually determined by partial differential equations alone, which are there non-linear.

We shall show in this paper that the gravitational equations for empty space are in fact sufficient to determine the motion of matter represented as point charges singularities of the field. The gravitational equations are non-linear and because of the necessary freedom of choice of the coordinate system, are such that four differential relations exist between them so that they form an over determined system of equations. The over-determination is responsible for the existence of equations of motion, and the non-linear character for the existence of terms expressing the interaction of moving bodies.

Two essential steps lead to the determination of the motion.

(i) By means of a new method of approximation, specially suited to the treatment of quasi-stationary fields, the gravitational field due to moving particles is determined

(ii) It is shown that for two-dimensional spatial surfaces containing singularities certain surface integral conditions are valid which determine the

motion.

It has not however been proved that higher time derivatives than the second will not ultimately occur in the equations of motion.

For the solution to be unique, non-Galilean values at ∞ and singularities of the type of dipole, quadrupole etc must be excluded.

It is of significance that our equations of motion do not restrict the motion of the singularities more strongly than the Newtonian equations, but this may be due to our simplifying assumption that matter is represented by singularities, and it is possible that it would not be the case if we could represent matter in terms of a field theory from which singularities were excluded (see Rosen's paper again). The representation of matter by means of singularities does not enable the field equations to fix the sign of mass so that, so far as the present theory is concerned, it is only a convention that the interaction of two bodies is always an attraction and not a repulsion. A possible clue as to the why the mass must be positive can be expected only from a theory which gives a representation of matter free from singularities (Einstein & Rosen: Phys. Rev. 48, 73 (1935).

Our method can be applied to the case when the Maxwell energy-momentum tensor is included in the field eqns, and it leads to a derivation of the Lorentz-force.

In the Maxwell-Lorentz Electrodynamics, as also in the earlier approximation method of the solution of gravitational eqns, the problem of determining the field due to moving bodies is solved through the integration of wave equations by retarded potentials. The sign of the flow of time there plays a decisive role since, in a certain sense, the field is expanded in terms of only those waves which proceed towards ∞. In our theory, however, the eqns to be solved at each

Note: the geometry characterised by the line element $ds = \gamma_i dx^i$ is not new

See Iwanenko & Tebrode: Zs. f. Phys, 50, 336, 1928

V. Ambarzumian: " 55, 801, 1929

of approximation are not wave eqns but merely statial potential equations. Since such equations as those of the gravitational and of the electromagnetic field are actually invariant under a reversal of the sign of time, it would seem that the method presented here is the natural one for their solution. Our method in which the time direction is not distinguished corresponds to the introduction of standing waves in the wave equation and cannot lead to the conclusion that, circular in the circular motion of two point masses, energy is radiated to infinity in the form of waves (See Eddington p. 95 §44)

So long as we are dealing with singularities, we have no basis within the theory for excluding negative masses; in other words, for excluding gravitational repulsion between particles. If however we decide always to take mass positive, then the sign with which the Maxwell energy-momentum tensor enters the field equations determines whether like charges shall attract or repel each other. This also reveals the limitations of any theory based upon the existence of singularities

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Va. 7

Hiroscima Journal: Wave Geometry No. 19. Physics & Geometry: Minura and Hosokawa: The theories of Einstein and Weyl have been constructed on a geometrical frame work of space-time assumed a priori, and do not represent a geometry founded on actual physical phenomena. We assume that the domain of existence of physical objects is a 4-dimensional manifold, and that this manifold corresponds to the pure analytical frame (analytical manifolds of topology ?). We introduce the linear operator $ds = \gamma_i dx^i$, where $\frac{1}{2}(\gamma_i \gamma_j + \gamma_j \gamma_i) = g_{ij}$

(* spin-space is the space of ψ-fields)

which is obtained by linearising the metric

$$ds^2 = g_{ij}\, dx^i dx^j$$

with this operater dS we combine the wave function ψ and take up $dS\psi$ as the metric of the manifold, and consider transformations and parallel displacements which make $dS\psi = 0$ invariant. We claim that our wave geometry maybe summed up in the dictum "Geometry is theoretical physics as it stands".

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ibid 7, No. 16. Sibata - Geometry in spin-space*: If we consider a 4-dimensional space in which the 5-dimensional vector-space is attached at each point of the base-space and assume that the fundamental tensor g_{ij} $(i,j = 1, \ldots 4)$ of the base space is related with $g_{\alpha\beta}$ $(\alpha,\beta = 1, \ldots 5)$ in the vector space by

$$g_{ij} = g_{\alpha\beta} r_i^\alpha r_j^\beta$$

then we get a Kind of wave-geometry corresponding to the type of Einstein - Mayer Geometry (Sitz. Berlin (1932), 102).

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ibid · No. 17. Morinaga: General linear displacement making $dS\psi = 0$ invariant:

" No. 18. " Geometrical interpretations of wave geometry:

" No. 21 Mimura & Iwatsuki: New foundation of atomic structure: It is shown that the atomic structure is in itself the construction of the space-time due to the nucleus. Here the electron is treated as a test-particle, and it becomes difficult to construct a wave-geometry of the electron. This theory is applied to the H-atom by Morinaga in No. 22.

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Ibid. Vol 8 : Wave Geom. No. 23. Hosokawa : Many-body problem I. A coordinate space V_{4n} is introduced, and a metric is introduced in this space on the same principle by which ds^2 is taken as the metric in 4-dimensional space.

ibid. No. 24. Iwatsuki, Mimura & Moriuga : Born's Electrodynamics in terms of wave geometry : The following difficulties are mentioned in Born's theory

(i) That f_{ik}'s are relations are not deduced from the theory

(ii) To introduce field eqns, an arbitrary addn of Riemannian curvature scalar density to the original invariant density is necessary

(iii) The theory of complete generalised geometrisation must dispense with variational principles

(iv) A generalisation to general relativity seems very doubtful

It is shown here that wave geometry naturally implies Born's theory as a geometrised form, and consequently the above difficulties are overcome.

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ibid. No. 25. T. Sibata : Wave geometry unifying Einstein's & Born's theories : Here a generalisation of wave geometry is made in the form

$$\nabla_i \psi = \lambda \gamma_i \phi$$

with suitable integrability conditions.

" No. 26 " - A spinor calculus is developed.

" No. 26 : Iwatsuki, Mimura & Sibata : The relativistic theory of gravitation consists of the field eqn

$$K^{ij} - \tfrac{1}{2} K g^{ij} + \lambda g^{ij} = -8\pi T^{ij}$$

and the law of motion of a particle

$$\delta \int ds = 0.$$

These two postulates are completely independent & thus we have not a pure field theory.

there is therefore need to construct a field theory which includes the equation of motion of a particle as part of itself. It is shown in this paper that a vector $u^i = \psi^+ A \gamma^i \psi$ can be constructed and made to satisfy the equation of motion which follows from the field theory itself

Nos 28 – 31 : Cosmology in terms of wave geometry : A cosmology is constructed on the basis of wave geometry and it is concluded that this cosmology admits only an Universe of the de Sitter type with finite concentration of matter, getting rid of the diversity of solutions in other cosmologies

No. 32 : Finslerian wave geometry & Milne's World-structure : Hosokawa : The wave geometry is generalised to the Finslerian type & its relations with Milne's cosmology is investigated

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J. Nuut : Expansionistische Dynamik I & II (Publications of the Tech. Univ of Estonia at Tallinn (Series A. No. 6, Feb & Mar. 39). – Speculations of Milne's Kinematical type.

M. Mathisson : Acta. Phys. Polonica. Vol 6, Fasc. 3, Wilno (1937), p. 163.

Neue Mechanik materieller Systeme : – Based on singularities. The field equations of gravitation and electromagnetism are written in a variational form. The material system is resolved into a sum of multipoles. It is shown that in the case of gravitation, a notion of gravitational skeletons can be set up. In this a simple pole corresponds to mass, the dipole and quadrupole for angular momentum. These notions lead to mechanical equations which contain new terms

~~other than~~ in comparison with the classical equations. Besides the terms which appear only when an external gravitational field is present, we obtain terms of the highest physical significance not connected with the external gravitational field.

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~~J. Pacotte: L'espace Hermetien Quantique (Actualité Series No. 635):~~

Foradori. Z. f. Phys. 111, p. 537. Das Raumbild der Physik: (1938).
(Innsbruck):

Revision of the mathematical foundations of physical space-continuum. The fact that the ~~physical~~ arbitrary division of the physical continuum into arbitrarily small parts on any fundamental principles, makes the geometrical notion of a point and hence also the "mengentheoretische" point-continuum physically meaningless. These difficulties could be avoided if the space continuum be described not through the connection of unextended points but ~~the~~ by the property of divisibility into parts. This is done with the aid of the "Teiltheorie" which makes continua, whose divided parts have a lower limit, understandable mathematically. The basis of this theory is the isomorphy idea, which shows that physical space (represented by its conditioned divisibility) is isomorphous with the geometrical point continuum (of unlimited divisibility); ~~which~~ This result enables the simple description of physical phenomena by means of a shortest length. The indeterminacy resulting from such smallest volume elements condition a notion of overlapping which leads at once to a statistical metrik.

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Proca & Goudsmit: J. de Phys (7). t. 10, May 39, p. ~~20~~ 210. Sur la masse des particules elementaires:

This article restricts itself to a classical theory of free particles subject to the dynamics of special relativity. The fundamental hypothesis consists in defining the motion by geodesics of a configuration space carrying besides the ordinary coordinates two others corresponding to a charge, the other to a spin.

On this theory the mass accessible to experiment depends on the its "quantity of matter", of its charge and of its spin. The theory explains naturally the difference in mass between neutron & proton, furnishes a criterion for distinguishing between light & heavy particles, and completes the relation between electron and photon brought to light by Fermi's theory. Finally the mass of the mesoton (of spin 1) calculated on this theory coincides with that of the Yukawa particle.

Take metric as

$$ds^2 = c^2dt^2 - \sum (dx^i)^2 + \lambda_s\left\{(dx^s)^2 + \lambda_e (dx^e)^2\right.$$

& apply invariance w.r.t. Lorentz-transformations. Put the general eqns in Hamiltonian form. E is given as

$$E = \sqrt{\mu_0^2 c^4 - \frac{c^2 p_s^2}{\lambda_s} - \frac{c^2 p_e^2}{\lambda_e}} \qquad (p_s, p_e \text{ conj to } x^s \& x^e)$$

For light particles $m^2 = -\frac{p_s^2}{c^2\lambda_s} - \frac{p_e^2}{c^2\lambda_e}$.

Proca: C.R. 208. (14), 1074. Fundamental length:

Starts from the above theory & writes mass M as

$$M^2 = \mu_0^2 - s(s+1)\frac{\hbar^2}{\lambda_s c^2} - \frac{p_e^2}{\lambda_e c^2} \qquad \begin{array}{l} s = \text{spin } n\hbar \\ p_e = \text{charge } ne \end{array}$$

Analytical mechanics is associated, in de Broglie's scheme with a geometrical optics, and a metric Riemannian geometry (config. space).
Similarly a metric geometry is associated with a Hamiltonian scheme* and a geometrical optics M. just as classical mechanics → wave mechanics
metric geometry → wave geometry

(* is equivalent to the question: given a space E_n provided with a Riemannian metric is it possible to show that this metric gives in this space a sort of a H-J eq^n which allows the ~~finding~~ determination of a family of surfaces whose orthogonal trajectories are the geodesics of E_n. The answer is in the positive thus: the fixing of the metric means that we are given a family of lines which can be regarded as characteristics, in the sense of Hadamard, of a family of surfaces, solutions of a H-J. eq^n.)

This gives $\sqrt{-\lambda_S} \sim L$ and gives in particular $\sqrt{-\lambda_S} \approx 4{\cdot}4 \times 10^{-13}$ cm.

From the metric for a particle without charge & at rest

$$\frac{\sqrt{\Sigma (dx^s)^2}}{dt} \leqq \frac{c}{\sqrt{-\lambda_S}} \text{ or } \frac{c\,dt}{\sqrt{\quad}} \geqq \sqrt{-\lambda_S}.$$

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J. Mariani: J. de Phys. (7) 10, Jan. 39, p. 32. Macroscopic theory of fields — Cosmological speculations.

Ibid: ibid June, 39, p. 296. Limits of application of metric geometry in nuclear physics I.

The idea is introduced that it is not merely classical mechanics that is invalid for spatial domains of order r_0, but also metric geometry itself. This probably does not mean the abandonment of even the topological axioms of geometry. It is shown that the reasons for limiting the validity of metric geometry are analogous to those which lead to a limitation of the procedure of classical mechanics in the domain of Bohr's atom. For this one puts metric geometry into a Hamiltonian form so as to bring it out in the form of a geometrical optics and showing that the geometry breaks down when the metric studied is a space curve whose ρ is of order r_0. The curvature thus introduced plays the role of a field of force, the "nuclear field" entirely distinct from Einstein's gravitational field.

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G. Fournier: J. de Phys. (7), 9, July 38, p. 273. The regular division of space

and the structure groups : ibid p. 360 — Purely geometrical papers where the author considers the general problem of space filling (i) Space round a point (ii) Space of 3 dimensions with the aid of regular polyhedra,

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J. Mariani : C.R. 208 (21) (22/5/39). Principle of duality & the representation of elementary particles :

In non-Euclidean geometry notions of distance & angle are related in a dual manner. From Cayley's to Laguerre's formula we pass by the substitution

$$u_i = k x^i, \quad u_{n+1} = x^{n+1}$$

A similar thing holds in nuclear physics for conj. variables

For a particle in euclidean space. p (momentum) & r (coord) are

Connected by $pr = h$

meaning? Let there be a br. $p = \lambda r$. Then the two are compatible if $r = \sqrt{\frac{h}{\lambda}}$

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J. Pacotte : L'espace hermitien quantique (Actualité Series. No. 635)

Chap I : The ∞-dim. hermitian space :

(1) abstract ∞-dim. non-complex space : This can be developed first from the abstract, symbolic (non-constructive, non-analytic) point of view & starting with a simple generalisation of Euclidean space to N dimensions. Axioms of affine geometry are first introduced, then the metric notion by the definition of scalar product of two vectors, the notion of orthogonality and finally the idea of linear operator which might be introduced even prior to the metric notion. The transition to the case $N \to \infty$ offers no difficulties from the abstract point of view & it would be otherwise if

the constructive or analytic view-point has been adopted.

(2) The passage to complex space: The passage from real to complex space can be either purely abstract or analytical. In either case the complex geometry would still be abstract since the real geometry has been developped axiomatically. In most cases of such generalisation the method of transition has been purely abstract for eg Eucl geom → non-Eucl geom; Geometries of Riemann & Cartan ie curvature and torsion geometries. In these cases the method of coordinates has not suggested the generalisations. But in this case ~~the~~ of quantum mechanics the passage to Complex space has been purely along analytical lines - in fact based on algebraic analysis viz the ~~reduction~~ question of linear orthogonal transformations of a quadratic form, due to Hermite.

It can be easily shown that this complex geometry can be developed in a manner exactly similar to the real geometry ~~by~~ an axiomatic method.

(3) The reduction to the scalar complex: The last step is the reduction of this abstract geometry to the analytical form by the use of coordinates based on the notion of hermitian symmetry. We thus show that complex metric geometry is essentially hermitian, the operators also being hermitian

Chap II: The statistical character is expressed by introducing a new vector Ω which registers the statistical weights of the eigen-values of a quantum magnitude whatever of a physical system. ~~This~~ ~~we~~ unitary vector Ω can be called the probability axis of the system.

The other chapters appears to be moon-shine.

* In a repn of operators x^k, p_k in the Hilbert space for which x^k are diagonal (δ-fns) the p_k are given by $\frac{\hbar}{i}\frac{\partial}{\partial x_k}$ & vice-versa

$p_k x^k = -\frac{\hbar}{i}\frac{\partial}{\partial p_k}$. Any wave eqn is transformed by

$$\phi(p) = \int \psi(x) \exp\left[\frac{i}{\hbar} p_k x^k\right] dx.$$

P. Jordan: Remarks on Fradoris's paper Zbl. f. Math. 20, (6), (1939), p. 273.

The authors reports first of all on his theory "Teiltheorie" which seeks to characterise the continuum topologically by renunciation of the notion of points. The authors hopes to show by means of this fundamental ideas and on the idea of the non-existence of a (4-dim. lorentz-invariantly measured) length less than $l \approx 10^{-13}$ cm in the actual physical continuum, that it has a structure in which the notion of a point does not play a part. The referee is not aware of the important results from the mathematical side so far deduced from the fundamental ideas of a "teiltheoretic" development of the continuum. From the physical side the most important thing is the statistical representation of the metric, and for this sense the considerations of the author are to be considered as only loosely connected with the deep & fundamental ideas.

M. Born: Proc. Roy. Soc. 165, (1938), 291. Suggestion for unifying quantum theory and relativity:

Principle of reciprocity* leads to the line element

$$d\sigma^2 = \gamma^{kl} dp_k dp_l \quad , \text{ not directly connected with } g_{kl} \text{ in the } x\text{-space.}$$

this metric corresponding to the nuclear world, ds^2 to the molar world. The diff eqn for γ^{kl} is latter

$$P^{kl} - \left(\tfrac{1}{2} P + \lambda'\right) \gamma^{kl} = -K' T^{kl}$$

($\int T^{kl} dp_x dp_y dp_z$ are space & time coords reciprocally).

after limit to momenta – Applications to several fields

1868]

1911 - Inspection section in proof range.

1921 - Capt. as Inspector

1930 - Dum Dum → Kirkee

1939 - moved to T.D.E

1939-45 = War years - Inspection + holding particulars of Ammn Stores & of service maintenance.

- nucleus of Research & design & development.

After WW - No details from U.K & Research Section grew - Advice on imported ammn.

- Formation of I.A.S - 1st & 2nd T.S.O's trained in T.D.E - Quality control.

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M. Born: On Reciprocity: Proc. Ind. Acad. Jubilee No. p. 309.

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K. Stellmacher: Math. Ann. 115 (5), p. 740. Ausbreitungsgesetze für Characteristische Singularitäten der Gravitationsgleichungen:

Hadamards method of characteristics and bicharacteristics is systematically applied to the field eqns of gravitation as well as the Maxwell-Lorentz equations in the domain of general relativity. Also the law according to which singularities are propagated. The physical significance of the results is: In the theory of characteristics it is essential to speak both of gravitational & electrical quanta. A treatment of gravitation quanta alone appears meaningless from this standpoint. There however exists a remarkable relation between electrical and gravitational energy for which specially great stability appears.

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Papers of Pauli and Heisenberg meant for 8th Solvay Congress of 1939

(manuscript lent for 3 days by Dr H. J. Bhabha who was in Bangalore from 15th - 23rd).

Pauli: Bericht über die allgemeinen Eigenschaften der Elementarteilchen:

Kaps I & II p1-74

Develops in full the ideas mentioned in Wentzel's "Physik" report viz work of Fierz and oral communication by Pauli

[P.T.O

Chap I. Transformation properties of field equations and conservation theorems.

§1. Units & notations: $\hbar = 1$, $c = 1$.

E = energy/$\hbar c$. [cm^{-1}]

$\vec{q}$ = momentum in units of $\hbar$ [cm^{-1}]

P = $A \cdot M$ " [dimensionless]

ε = elec-charge in units $\sqrt{\hbar c}$ ["]

$f_{ik} = f_{Ki}$ = electromag fields " $\sqrt{\hbar c}$ [cm^{-1}]

" potential " [cm^{-2}

$\kappa = mc/\hbar$ = recip [cm^{-1}] = length recip. length corresp. to rest mass.

true coordinate $x_0 = ct$, $x_4 = ix_0 = ict$.

Latin indices $i, K,$ - denote tensor indices from 1 to 4.

For magnitudes with index zero conjugate complex is given by means a star ($*$)

" 4 " of U_{ik} is given by U^*_{ik} multiplied

by $(-1)^n$ where n is the number of "4's" in the $ik \ldots$

Dirac's spinors u_p carry Greek indices $\rho = 1 \ldots 4$

Energy-impulse tensor is so defined that the energy density W, and the impulse density G is expressed in natural units by $-T_{44}$, $-iT_{K4}$ with $K = 1, 2, 3$.

§2. Variation principle and energy-impulse tensor. Gauge transformation & current vector:

(a) Absence of external field: All numbers taken c-numbers. L a fn of $U^{(r)}$ and its derivatives $U^{(r)}_K = \partial U^{(r)}/\partial x_K$. U^* conj. of U. V taken as real. L taken (1)

Lorentz-invariant (not U, V necessarily). variation principle

$$\delta \int L \left(U^{(r)}_K, U^{*(r)}_K, V^{(r)}_K, U^{(r)}, U^{*(r)}, V^{(r)} \right) d^4x = 0 \qquad (2)$$

Variation gives field eqns

$$\sum \frac{\partial}{\partial x_k} \frac{\partial L}{\partial u_{ik}^{(r)}} - \frac{\partial L}{\partial u^{(r)}} = 0, \quad \sum \frac{\partial}{\partial x_k} \frac{\partial L}{\partial u_{ik}^{*(r)}} - \frac{\partial L}{\partial u^{*(r)}} = 0$$

$$\sum \frac{\partial}{\partial x_{ik}} \frac{\partial L}{\partial v_k^{(r)}} - \frac{\partial L}{\partial v^{(r)}} = 0. \qquad (3)$$

Energy-impulse tensor $T_{ik} = \sum_r \left(u_i^{*(r)} \frac{\partial L}{\partial u_k^{*(r)}} + \frac{\partial L}{\partial u_{ik}^{(r)}} u_i^{(r)} + \frac{\partial L}{\partial v_k^{(r)}} v_i^{(r)} \right) - L\delta_{ik}$ (4)

which from (3) satisfy continuity eqn

$$\sum \frac{\partial T_{ik}}{\partial x_k} = 0 \qquad (5)$$

T_{ik} is called canonical. In general it is neither symmetrical nor is $-T_{44}$ the energy density in general +ve. In all wellknown cases there exists another energy-impulse tensor Θ_{ik} values of energy-impulse calculated from which corresponds to those calculated from T_{ik}

(Hilbert: Gött. Nachr, 1915, p. 395: Generalisation to spinors by H. Weyl Z. Phys 56, 330 (1929)).

For the total energy the condition is essential that total energy is necessarily +ve in absence of interactions. In many cases in the q-number theory this is yet satisfied.

Let L be unphased invariant for transformations $u^{(r)} \to u^{(r)} e^{i\alpha}$; $u^{*(r)} \to u^{*(r)} e^{i\alpha}$ — (6)

where α is arbitrary. Differentiating L w.r.t phase α

$$\sum \left[u^{*(r)} \frac{\partial L}{\partial u_r^*} + \sum u_k^{*(r)} \frac{\partial L}{\partial u_{ik}^{*(r)}} \right] = \sum_r u^{(r)} \frac{\partial L}{\partial u^{(r)}} + \sum_{ik} u_k^{(r)} \frac{\partial L}{\partial u_k^{(r)}} \qquad (7)$$

This enables definition of a vector S_k (to factor ϵ)

$$S_k = \epsilon i \sum \left(\frac{\partial L}{\partial u_k^{(r)}} u^{(r)} - u^{*(r)} \frac{\partial L}{\partial u_k^{*(r)}} \right) \qquad (8)$$

which as a consequence of (3) + (7) satisfies continuity eqn

$$\sum_k \frac{\partial S_k}{\partial x_k} = 0 \qquad (9)$$

This is the 4-vector of current. For real fields (6) is not set up & real fields cannot represent particles which have electrostatic or magnetostatic properties (ie sources). For real fields we have the electromagnetic field itself which in particle representation denotes the photons.

We have assumed all particles to have same charge; otherwise the invariance of L has to be conditioned ~~by trans~~ for transformations with different values of α proportional to the charges of corresponding to several ~~field~~ particles denoted by the field quantities

(b) Appearance of external ~~fiel~~ electromagnetic field: To introduce an external field we expressly state that $\frac{\partial}{\partial x_k}$ when applied to $\mathcal{U}^{(r)}$ should be replaced by

$$\Pi_k = \frac{\partial}{\partial x_k} - i\epsilon\varphi_k \qquad (10)$$

$$\text{Such,}\quad \Pi_k^* = \frac{\partial}{\partial x_k} + i\epsilon\varphi_k \qquad (10^+)$$

φ_k = potential, ϵ = charge. The field components are

$$f_{ik} = \frac{\partial\varphi_k}{\partial x_i} - \frac{\partial\varphi_i}{\partial x_k} \qquad (11)$$

& its appearance asserts itself in the non-commutability of the operators Π_k according as

$$\Pi_i\Pi_k - \Pi_k\Pi_i = -i\epsilon f_{ik}\ ;\ \Pi_i^*\Pi_k^* - \Pi_k^*\Pi_i^* = +i\epsilon f_{ik} \qquad (11,a)$$

The Lagrangian is taken in the same form as before but

$$\mathcal{U}_k^{(r)} = \Pi_k\mathcal{U}^{(r)}\ ;\ \mathcal{U}_k^{*(r)} = \Pi_k^*\mathcal{U}^{*(r)} \qquad (1')$$

The field eqns become

$$\Pi_k^*\frac{\partial L}{\partial \mathcal{U}_k^{(r)}} - \frac{\partial L}{\partial \mathcal{U}^{(r)}} = 0\ ;\quad \Pi_k\frac{\partial L}{\partial \mathcal{U}_k^{*(r)}} - \frac{\partial L}{\partial \mathcal{U}^{*(r)}} = 0 \qquad (3')$$

This theory is invariant for the guage-transformation

$$\mathcal{U}^{(r)} \to \mathcal{U}^{(r)}e^{i\alpha}\ ;\ \mathcal{U}^{*(r)} \to \mathcal{U}^{*(r)}e^{-i\alpha} \qquad (12a)$$

$$\varphi_k \to \varphi_k - \frac{1}{\epsilon} i \frac{\partial \alpha}{\partial x_k} \qquad (12,b)$$

where α is an arbitrary function of position. From (1') it follows that

$$u_k^{(\gamma)} \to u_k^{(\gamma)} e^{i\alpha} \; ; \; u_k^{*(\gamma)} \to u_k^{*(\gamma)} e^{-i\alpha} \qquad (12c)$$

Further from gauge-invariance of L we have the properties

$$\frac{\partial L}{\partial u_k^{(\gamma)}} \to \frac{\partial L}{\partial u_k^{(\gamma)}} e^{-i\alpha} \; ; \; \frac{\partial L}{\partial u^{(\gamma)}} \to \frac{\partial L}{\partial u^{(\gamma)}} e^{-i\alpha} \qquad (13)$$

$$\frac{\partial L}{\partial u_k^{*(\gamma)}} \to \frac{\partial L}{\partial u_k^{*(\gamma)}} e^{i\alpha} \; ; \; \frac{\partial L}{\partial u^{*(\gamma)}} \to \frac{\partial L}{\partial u^{*(\gamma)}} e^{i\alpha} \qquad (13^T)$$

as follows from (3').

(12,a) and (12,b) are called gauge-transformation of the 1st kind and 2nd kind resply. The distinction will be found to be important for Base particles.

S_k and T_{ik} are defined as before by (8) and (4). In calculation these it is useful to notice that if f and g be two functions of $u^{(\gamma)}$ etc which under a gauge transformation are multiplied by $e^{-i\alpha}$ & $e^{i\alpha}$ resply, then

$$\frac{\partial}{\partial x_k}(f^* g) = (\pi_k^* f^*) g + f^* (\pi_k g)$$

the terms in α_k being annulled. Simly

$$\frac{\partial L}{\partial x_k} = \sum \left[\frac{\partial L}{\partial u_p^{(\gamma)}} \pi_k u^{(\gamma)} + \sum_i \frac{\partial L}{\partial u_i^{(\gamma)}} \pi_k \pi_i u^{(\gamma)} + \text{conj} \right].$$

The continuity eqn (9) for S_k stands as before while that for T_{ik} becomes (using $\frac{\partial L}{\partial x_k}$) and (9) and (8),

$$\frac{\partial T_{ik}}{\partial x_k} = f_{ik} S_k \quad \text{(justifying } S_i\text{'s interpretation)}. \qquad (14)$$

The generation of the electromagnetic field by the u-fields has so far not been considered. This can be attempted to be set up according to Maxwell's eqn $\frac{\partial f_{ik}}{\partial x_k} = S_i$

⊗ f_{ui} here, of course, refers to the electro-magnetic field generated by the U-field.

and the fact that the sum of T_{ik} of the U-fields and the Maxwell energy tensor

$$S_{ik} = f_{ir} f_{kr} - \frac{1}{4} f_{rs} f_{rs} \delta_{ik} \quad \otimes$$

should satisfy the continuity equation

$$\frac{\partial}{\partial x_k} (T_{ik} + S_{ik}) = 0.$$

The affine corpuscular idea & double quantisation of U-field to explain generation of electromagnetic field by it leads to the so far unsolved difficulties of infinities and divergences.

We however try here the possibilities of gauge invariance and add additional terms in L depending explicitly on field strengths f_{ik}. Defn of S_k & continuity eqn for it are still the same, but in place of (14) we have

$$\frac{\partial T_{ik}}{\partial x_k} = f_{ik} s_k - \frac{1}{2} \frac{\partial L}{\partial f_{rs}} \frac{\partial f_{rs}}{\partial x_i} \qquad (14')$$

This gives occasion to introduce additional terms in T_{ik} & s_k viz T'_{ik}, s'_k which satisfy

$$\frac{\partial T'_{ik}}{\partial x_k} = f_{ik} s'_k$$

s'_k is found by variation of q_k

$$\delta \int L \, d^4x = -\int s'_k \delta q_k \, d^4x \qquad (15)$$

i.e. $$s'_k = s_k - \frac{\partial}{\partial x_l} \frac{\partial L}{\partial f_{kl}} \qquad (16)$$

also $$\frac{\partial s'_k}{\partial x_k} = 0 \qquad (9')$$

thus (14') is satisfied by

$$T'_{ik} = T_{ik} - f_{ir} \frac{\partial L}{\partial f_{rk}} \qquad (17)$$

and from (11) we derive Maxwell's eqns

$$\frac{\partial f_{kl}}{\partial x_i} + \frac{\partial f_{li}}{\partial x_k} + \frac{\partial f_{ki}}{\partial x_l} = 0$$

Such additional terms are used later for describing particles with arbitrarily specified magnetic moments.

(3). Derivations from relativistic invariance:

(α) Irreducible tensors. Definition of spins: — Considerations of proper Lorentz-transformations (Ref. B.L.v.d. Waerden) ie continuous linear group leaving invariant the form $\sum_1^4 x_k^2 = x^2 - x_0^2$ and determinant equal to +1 and not reversing time-direction. A tensor or spinor irreducible against this group is characterised by two integers pair (p, q). $U(p, q)$ Characterised by (p, q) has $p \cdot q$ independent components. Thus arise (1,1) the scalar, (2,2) the vector, (3,1) the self dual skew tensor, (3,3) the symmetric tensor with vanishing spur etc. The Dirac's spinor μ_p resolves into two irreducible magnitudes (2,1) and (1,2) both with 2 components. $\mathfrak{U}(p, q)$ transforms accg to

$$\mathfrak{U}'_r = \sum_{s=1}^{p,q} \Lambda_{rs} \mathfrak{U}_s \quad , \text{for } p = q, \quad \Lambda = \Lambda^*$$

In §1 we take $\mathfrak{U}^*$ as those magnitudes which transform equivalently to Λ^* when $\mathfrak{U}$'s transform equivalently to Λ

The most important operation is reduction by multiple cases of

$$\mathfrak{U}_1(p_1, q_1) \cdot \mathfrak{U}_2(p_2, q_2)$$

for which to derive $\mathfrak{U}(p, q)$ the well known rule of A. M gives

$$\left.\begin{aligned} p &= p_1 + p_2 - 1,\ p_1 + p_2 - 3,\ \ldots,\ |p_1 - p_2| + 3,\ |p_1 - p_2) + 1 \\ q &= q_1 + q_2 - 1,\ \ldots \quad \ldots \quad \ldots \end{aligned}\right\} \quad (18)$$

* Zierz H.P.A, 12, 3, (1939)

Limiting consideration to space-rotations difference between p and q vanish and $\mathfrak{U}(p) \cdot \mathfrak{U}(q) = \mathfrak{U}(r)$ (irreducible) with

$$r = p+q-1, \quad \ldots \quad |p-q|+1$$

~~For these transformations representations for $U(r)$ would~~ for these space rotations $U(r)$ transform like two-valued representations for even r, and one-value for odd r. Thus energy magnitude $T(p,q)$ would be one valued (2-valued) for $p+q$ even (odd) for these space rotations.

To define spin of particles belonging to a given field* it is necessary to connection the notion of spin with ~~given values~~ the number of waves (plane) or the values k_i appearing in the phase exp (k_i, x_i). To define spin with a physical meaning let us first consider the case where the rest mass m of all the particles is different from zero. In this case all components of k_i vanish in the rest system and the wave fn. depends only on t. In this system the field components can be divided into $U(r)$ components irreducible for space rotations & hence correspond to particles of spin $\frac{1}{2}(r-1)$. There is only one such group with r irreducible components.

In general $\mathfrak{U}(p,q)$ appearing in the theory gives in general systems a complicated number since these magnitudes must satisfy different conditions with k_i.

In case of vanishing rest mass on the other hand there appears a degeneration, and Fierz has shown that the concerned field satisfies a gauge-transformation of second type. For $m=0$ & definite spin-value there are only two states for given k_i which cannot go into each other by gauge-tr. fn. In these cases definition of spin-values is not unique since intrinsic & orbital A.M. cannot be separately measured. This however

Integral spin:

(b) Eqns $\quad \Sigma k u^+ = \Sigma u^- \; ; \; \Sigma k u^- = \Sigma u^+$ (19)

$$k_i \to -k_i, \; u^+ \to u^+ \; (u^{+*} \to u^{+*}) \; ; \; (u^- \to -u^- \; (u^{-*} \to -u^{-*}) \qquad (20)$$

$$T \sim \Sigma u^+ u^+ + \Sigma u^- u^- + \Sigma u^+ k u^- \qquad (21, a)$$

$$T \to T \qquad (20, a)$$

$$s \sim \Sigma u^+ k u^+ + \Sigma u^- k u^- + \Sigma u^+ \qquad (21, b)$$

$$s \to -s \qquad (20, b)$$

half-integral.

$$\left.\begin{aligned} \Sigma k u^+ + \Sigma k u^{-*} &= \Sigma u^- + \Sigma u^{+*} \\ \Sigma k u^- + \Sigma k u^{+*} &= \Sigma u^+ + \Sigma u^{-*} \end{aligned}\right\} \qquad (22)$$

$$k_i \to -k_i, \; u^+ \to i u^+, \; u^{-*} \to i u^{-*} \; ; \; u^- \to i u^-, \; u^{+*} \to -i u^{+*} \qquad (23)$$

for tensors of geraden Stufe

$$\left.\begin{aligned} T \sim \Sigma u^+ u^+ + \Sigma u^- u^- + \Sigma u^+ k u^- + \Sigma u^+ u^{-*} + \Sigma u^- u^{+*} \\ + \Sigma u^{-*} k u^- + \Sigma u^{+*} k u^+ + \Sigma u^{-*} k u^{+*} \\ + \Sigma u^{-*} u^{-*} + \Sigma u^{+*} u^{+*} \end{aligned}\right\} \qquad (24, a)$$

tensors ungeraden Stufe

$$\left.\begin{aligned} s \sim \Sigma u^+ k u^+ + \Sigma u^- k u^- + \Sigma u^+ u^- + \Sigma u^+ k u^{-*} \\ + \Sigma u^- k u^{+*} + \Sigma u^- u^{-*} + \Sigma u^+ u^{+*} + \Sigma u^{-*} k u^{-*} \\ + \Sigma u^{+*} k u^{+*} + \Sigma u^{-*} u^{+*} \end{aligned}\right\} \qquad (24, b)$$

Result of substitution (23) is the converse of the result of substitution (20): the tensors of even degree change the sign & those of odd degree leave it unchanged

$$T \to -T \qquad (25, a)$$

$$s \to +s. \qquad (25, b)$$

possible in those cases to define spin in a way. Considering a q-theory state for a single type of particle, all eigen-values $j(j+1)$ of total quadratic A.M $\sum P_k^2$ are not possible. But j begins with a determinate minimum value, exactly the spin value & then has only the value $S, S+1, \cdots$ as possible ones & this is true only for rest mass 0. For photons $S=1$ and the value $j=0$ for a photon is not possible (Pauli. Hd Buch artikle, p. 260). For gravitation quanta $S=2$ & values $j=0$, $j=1$ are excluded.

In an arbitrary reference system and arbitrary rest mass only particles with half-integral spins can be described by field theories whose magnitudes U transform according to two-valued (one-valued) representations with odd (even) $p+q$. Whether the theory can represent particles with one or more spins can be decided in each case by special investigation.

(b) Proof of the indefinite character of charge for integral spin, and energy for half-integral spin: As a result of a rather complicated investigation it is shown that (i) the definition of a definite particle density (&current) which transforms like the 4-component of a vector is impossible for integral spin

(ii) A positive definite energy density and also a positive definite total energy is impossible for half-integral spin.

These are not positive statements, since they do not say that for integral spins there is a definite energy-density, and for half-integral spins a definite charge density. In fact Fierz has shown that for spin >1 this is no longer the case for densities the densities, for the half-integral spin (in C-number theory) case for the

Charge, and in the integral case for the energy. Thus the spin 1/2 is characterised by the fact of the possibility of a definite charge density and of spins 0, 1 of definite energy density, from all other higher spins. Nevertheless the present theory allows arbitrary values for the spin quantum numbers, as also arbitrary values for rest-mass, electric charge and magnetic moments of the particles.

(C) Quantisation of the field in absence of interactions. Connection of spin with statistics.

The impossibility of a physically satisfactory definition of particle density for integral spin & energy density for half-integral spin in the c-number theory suggests that the significance of the theory cannot be complete in the domain of the one body problem. In fact all relativistic invariant theories lead to particles which can be absorbed or emitted in external fields and in ~~opposite pairs~~ Oppositely charged pairs for electrical particles and singly for uncharged ones. Therefore the fields must be subjected to a double quantisation. We shall not adopt for this the canonical formalism because here space & time are unnecessarily sharply differentiated but the one originally adopted by Jordan & Pauli. This method is specially useful for the case of no ~~external field~~ interaction where all fields $U^{(\nu)}$ satisfy 2nd order wave eqn

$$\Box U^{(\nu)} - K^2 U^{(\nu)} = 0 \qquad (25)$$

where $\Box = \sum_1^4 \frac{\partial^2}{\partial x_k^2} = \Delta - \frac{\partial^2}{\partial x_0^2}$ & K rest mass $\times$ in unit h/c.

Consider the D-function satisfying (25) & invariant which in the

periodicity volume V of the eigen-function is given by

$$D(\vec{X}, X_0) = \frac{1}{V} \sum \frac{1}{2} \left[\frac{e^{i\{(\vec{K}\vec{X}) + K_0 X_0\}}}{iK_0} - \frac{e^{i\{(\vec{K}\vec{X}) - K_0 X_0\}}}{iK_0} \right] \tag{26}$$

or in the limit $V \to \infty$

$$D(\vec{X}, X_0) = \frac{1}{2\pi^3} \int d^3K \, \frac{1}{2} \left[\frac{e^{i\{(\vec{K}\vec{X}) + K_0 X_0\}}}{iK_0} - \frac{e^{i\{(\vec{K}\vec{X}) - K_0 X_0\}}}{iK_0} \right] \tag{26,a}$$

Here K_0 is the positive root, $K_0 = +\sqrt{\vec{K}^2 + k^2}$ (27)

The D-fn is, besides satisfying the wave eqn

$$\Box D - k^2 D = 0 \tag{28}$$

characterised by the conditions

$$D(\vec{X}, 0) = 0, \quad \left(\frac{\partial D}{\partial X_0}\right)_{X_0 = 0} = \delta(\vec{X}) \tag{29}$$

For $k = 0$, one has

$$D(\vec{X}, X_0) = \frac{1}{4\pi} \frac{1}{r} \left[\delta(r - X_0) - \delta(r + X_0) \right] \tag{30}$$

This expn also determines the singularities of $D(\vec{X}, X_0)$ on the light cone for $k \neq 0$, further in the last case $D \neq 0$ in the inside of light cone. Many times one finds (Dirac, Proc. Camb. Phil. Soc, 30, 150 (1934)]

$$D(\vec{X}, X_0) = \frac{1}{4\pi} \cdot \frac{1}{r} \frac{\partial}{\partial r} F_1(r, X_0) \tag{31}$$

$$F(r, X_0) = \begin{cases} J_0\left[k(X_0^2 - r^2)^{1/2}\right] & \text{for } X_0 > r \\ 0 & r > X_0 > -r \\ -J_0[\quad\quad] & -r > X_0 \end{cases} \tag{32}$$

$D_1(\vec{X}, X_0)$ introduced similarly ~~as $F_1(r, X_0)$ in Neumann~~ by

$$D_1(\vec{X}, X_0) = \frac{1}{(2\pi)^3}\int d^3k\,\frac{1}{2}\left[\frac{e^{\cdots}}{R_0} - \frac{-}{R_0}\right] \qquad (33)$$

R_0 is determined by the fact that $\frac{d^3k}{R_0}$ is invariant on the hyperboloid (27) of the four-dimensional impulse-space $(\vec{k}, R_0)$.

For $\kappa = 0$, one has

$$D_1(\vec{X}, X_0) = \frac{1}{2\pi^2}\cdot\frac{1}{r^2 - X_0^2} \qquad (34)$$

In general

$$D_1(\vec{X}, X_0) = \frac{1}{4\pi}\cdot\frac{1}{r}\cdot\frac{\partial}{\partial r}F_1(r, X_0) \qquad (35)$$

$$F_1(r, X_0) = \left\{\begin{array}{ll} N_0\left[\kappa(X_0^2 - r^2)^{1/2}\right] & \text{for } X_0 > r \\ -i\,H_0^{(1)}\left[i\kappa(r^2 - X_0^2)^{1/2}\right] & r > X_0 > -r \\ N_0\left[\kappa(X_0^2 - r^2)^{1/2}\right] & -r > X_0 \end{array}\right\} \qquad (36)$$

N_0 = Neumann's & $H_0^{(1)}$ the first Hankel's cylinder functions

We shall explicitly postulate that all physical magnitudes are commutable for finite distances outside the light cone (for $|X_0' - X_0''| < |\vec{X}' - \vec{X}''|$ ie the ~~fact~~ P.B of all magnitudes satisfying (25) must allow themselves to be expressed by D and its derivatives (finite order) but not D_1. This is also valid for eventual bracket-symbols with $+^{ve}$ sign, since the ~~bilinearly built~~ gauge-invariant magnitudes bilinearly built from $\psi^{(x)}$ for eg the charge-density gives another case of non-commutability of in the ordinary sense of two points connected by a space-like interval (Pauli: Poincare Inst, 6, 137, 1936, §3). The support for our postulate lies in this that measurements at space-time points with space-like connecting

are never disturbed, since no signal can move with velocity $> c$. Theories which use the D_1-function for quantising are very different in consequences from the well-known theories (For eg $\frac{\partial \mathcal{U}}{\partial x_0} = i[\bar{H}, \mathcal{U}]$ where $\bar{H}$ = energy operator will not hold).

We shall now draw further consequences using ideas of invariance against proper L-T's on the order of derivatives of D-function appearing in the P.B. We denote

For half integral spin:

$$\left[\mathcal{U}^{(r)}[\vec{x}', x_0'), \mathcal{U}^{(r)*}(\vec{x}'', x_0'')\right] = \text{odd number of derivatives of } D(\vec{x}' - \vec{x}'', x_0' - x_0'') \tag{37, a}$$

For integral spin:

$$- \text{do} \quad = \text{even number of derivatives} \tag{37, b}$$

We now consider the expression X symmetric in world pts $(\vec{x}', x_0')$, $(\vec{x}'', x_0'')$

$$X \equiv \left[\mathcal{U}^{(r)}(x', x_0'), \mathcal{U}^{*(r)}(x'', x_0'')\right] + \left[\mathcal{U}^{(r)}(x'', x_0''), \mathcal{U}^{*(r)}(x', x_0')\right]$$

Since the D-fn is even in the space coords & odd in the time-coord — as can be immediately seen from (26) or (31), (32) — follows from the symmetry of X

X = even number spatial time odd number timelike derivatives of D. This is completely in consonance with (37, a) for half-integral spin, but in contradiction with ~~not~~ (37, b) for integral spin — Hence X vanishes. we have the result

$$X \equiv [\quad\quad] + [\quad\quad] = 0 \tag{38}$$

We have so far not differentiated the cases of Bose-statistics and exclusion principle. In the first case, we have the ordinary bracket with $-^{ve}$ sign and in second case according to Jordan & Wigner's formula

$$[A, B]_+ = AB + BA$$

the bracket has a positive sign. By putting in $+^{ve}$ sign in bracket (38) there however arises an algebraic contradiction. We accordingly get the result.

For integral spin, quantisation according to exclusion principle is not possible. On the other hand for half integral spin, quantisation by Bose-statistics is possible formally, but then the energy of the system is not positive. Since this is necessary from the physical point of view, exclusion principle is necessary for half-integral spin.

For the positive result that for higher spins such statistics are valid for two types of spins resp'ly, see Fierz.

The cases of spins 0, ½, 1 will be dealt with at length in the sequel. Case of spin 2 will be taken up last & connection with Einstein's gravitation theory pointed out.

Finally we might say that the explanation of the connection between spin and statistics belong to the most important possibilities of application of the special theory of relativity.

Chapter II : Consideration of special fields

1. Wave fields of particles without spin

(a) Wave eqn, current vector and energy impulse tensor:

$$L = \frac{\partial u^*}{\partial x_i}\frac{\partial u}{\partial x_i} + \kappa^2 u^* u \quad \text{gives by variation } \Box u - \kappa^2 u = 0.$$

T_{ik} is symm - $W = -T_{44}$ is positive definite. Wave eqn can be reduced to 1st order with u_k, u varying independently in L. Wave eqn has form of 1st order

$$\mathcal{U}_k = \frac{\partial \mathcal{U}}{\partial x_{ik}} \; ; \; \frac{\partial \mathcal{U}_k}{\partial x_k} = k^2 \mathcal{U}$$

A dual theory might be mentioned where the scalar $\mathcal{U}$ is replaced by the pseudoscalar $\mathcal{U}_{klmr}$ (skew in all indices) & the vector $\mathcal{U}_k$ by the pseudovector $\mathcal{U}_{klm}$ (skew in all indices). The eqns become

$$\mathcal{U}_{lmn} = \frac{\partial \mathcal{U}_{klmn}}{\partial x_{ik}}$$

$$\frac{\partial \mathcal{U}_{lmn}}{\partial x_{ik}} + \frac{\partial \mathcal{U}_{mkn}}{\partial x_l} + \frac{\partial \mathcal{U}_{klm}}{\partial x_n} + \frac{\partial \mathcal{U}_{kln}}{\partial x_m} = k^2 \mathcal{U}_{klmn}$$

(b) Eigenoscillations in impulse space - charge conjugate solutions.

Splitting up soln of wave eqn into plane waves, we can find expn for E, $\vec{G}$ and e in terms of the wave quantities. We can also show that by exchange of $\mathcal{U}$ and $\mathcal{U}^*$ only s changes sign but not T if $\mathcal{U}^*$ corresponds to "charge-conj" soln. This is also true when ext. field is present, for then wave eqn is

$$\sum \pi_i^2 \mathcal{U} - k^2 \mathcal{U} = 0$$

If $\mathcal{U} \to \mathcal{U}^*$ & $e \to -e$, $\pi_{ik} \rightleftharpoons \bar{\pi}_k^*$ (Pauli-Weisskopf Hel. A, 7. 709)

It will be shown later that for half-integral spin, the connection between conj-comp. and charge-conj. solns is more complicated. For particles treated here without spin only one eigen-vibration exists for given $\vec{k}$ and given sign of k_0.

(c) Quantisation: It is shown that a rel. inv. theory for particles of zero spin quantisation must, on the basis of general postulates, be according to Bose-Einstein statistics.

(c) Real field: $\mathcal{U} = \mathcal{U}^*$, $s = 0$ & particles can never generate an electromag field.

2. Wave fields for particles of spin 1.

(a) The c-number theory in absence of external forces: This case is interesting in view of Yukawa's theory of mesotron interpreting charge dependence of nuclear force between neutron & proton. This theory was earlier developed by Proca. Introduce vector field u_k satisfying $\Box u_k - \kappa^2 u_k = 0$.

Energy-density can be shown positive. From u_k derive u_{ik} by relation satisfying

$$\frac{\partial u_{ik}}{\partial x_k} + \kappa^2 u_i = 0.$$

u_i & u_{ik} are uniquely determined from each other. For u_i ($\kappa \neq 0$) there exists no gauge-transformation of the second kind. Also relation

$$\frac{\partial u_{ik}}{\partial x_l} + \cdot + \cdot = 0 \text{ holds.}$$

The theory can be developed from a Lagrangean

$$T_{ik} = \theta_{ik} + \frac{\partial}{\partial x_k}\left(u^*_{ki} u_i + u_i^* u_{ki}\right)$$

$$\frac{\partial \theta_{ik}}{\partial x_k} = \frac{\partial T_{ik}}{\partial x_k} = 0. \quad \& \int \theta_{4i}\, dV = \int T_{i4}\, dV$$

As in the scalar theory $-T_{44}$ (or $-\theta_{44}$) is $+^{ve}$ definite. Also expn for s_k is not unique. Hereto can be introduced pseudo-vectors & skew-tensors (pseudo) in place of u_i & u_{ik}. Consult

(b) Eigen-vibrations in impulse space:

(c) Quantisation: For non-vanishing rest-mass the exchange relations between the u_i cannot be the same as those between independent scalars.

Quantisation gives numbers $N_+(k)$ and $N_-(k)$ of particles with charge $+1$ and -1 respectively and of impulse k. Also one gets a zero-point energy of half a quantum per eigen-vibration. Also the impossibility of quantisation according to exclusion principle follows as in the scalar theory. The theory as quantised describes only particles with spin 1, and gives longitudinal as well as transverse waves.

(d) The C-number theory in an external electromagnetic field: The discussion of the non-relativistic limiting case shows that it is possible to associate a magnetic moment but this can be arbitrary of form $\frac{e}{2m_0c}\hbar(l-m)$.

(e) Remarks on real fields and the special case of vanishing rest mass: The theory with a real vector makes the current vector & charge vanish. This has recently been used to explain neutrinellos (Proca, J de Phys (7), 9, 61; Kemmer: Proc. Camb. Phil. Soc. 34, 354)

A special case is where $m=0$; this reduces to quantum electrodynamics. L depends only on U_{kk}. $U_{ik} \to U_{ik} + \partial f/\partial x_k$ is gauge-transformation of 2nd Kind. This gauge-transformation of 2nd sort appears to be a fundamental and qualitative characteristic differentiating the case $K \neq 0$. It would be physical unsatisfactory to consider photons as particles of zero mass. Logically it is possible to treat the field with $K=0$ as complex field. But we know already that there is no case of integral spin for $K=0$ which necessitates description through a complex field. We take U_{kk}, U_i as real fields.

The photon spin expresses itself by requiring for total A.M, $j=1$ & not $j=0$. as least eigen-value.

* Considering waves with $(\vec{K}, K_0)$ for given μp. Then $K_0^2 = K^2 + \chi^2$. By given $\vec{K}$ & given sign of K_0 there are two linearly independent solns of the wave eqn of Dirac

$$\gamma_\mu \frac{\partial u}{\partial x_\mu} + \chi u = 0$$

which in the rest-system $\vec{K} = 0$ are transformed into each other by space rotations. The corresponding particle representing the wave field has, according to general considerations of § I, a spin $\frac{1}{2}$.

3. Dirac's theory of positrons:

(a) The c-number theory: The charge density is positive definite while the energy density has two signs in consonance with considerations of § I. (Géhéniau: Electron & Photon, Paris, 1938 has made some considerations which appear to be wrong)

From the wave eqn follows the wellknown method of annihilation operators.* Between the solutions with positive & negative frequencies there is a lorentz-invariant correspondence (Poincaré Int., l.c., p. 130). These solutions are, according to Kramers, called charge-conjugate solutions. (Kramers: Proc. Amst. Ac. 40, 814, 1937)

In presence of external field $\frac{\partial}{\partial x_k} \to \Pi_k$ & of u^+, u^- be solns of wave eqns for $+e$, $-e$, it appears that current vectors does not change sign for charge-conj. solns. This defect is removed in the q-theory.

Introducing, in this connection, analogously to § 2(d) additional terms in L

$$L' - L = l \cdot \frac{1}{2} u^+ \gamma_i \gamma_k u f_{ik} \quad (f_{ik} = \text{ext. field strength}, \; l \text{ of dim cm}^1)$$

without additional term, one has for mag. moment $\frac{1}{2} \cdot l/k$. From the added terms follows in ext. fields the modified wave eqn

$$\gamma_k \Pi_k u + k u + \tfrac{1}{2} l f_{ik} i \gamma_i \gamma_k u = 0$$

which, for small particle-velocities, has an additional mag. moment $-l\sqrt{\hbar c}$. The additional terms give

$$S'_k = \epsilon_{ik} u^* \gamma_k u + l \frac{\partial}{\partial x_k} (u^+ i \gamma_k \gamma_l u)$$

It is remarkable that for electrons the magnetic moment is exactly $\frac{1}{2} l/k$ so that the additional terms are superfluous. It is otherwise for protons & neutrons. For the last the magnetic moment is calculated only by the additional terms since here

on account of $\epsilon = 0$, $\frac{\partial}{\partial x_k} \to \Pi_k$ is unimportant. Further it is important that by transition to charge-conjugate solns corresp. to charge ϵ, also the constant l of the additional term changes its sign; so that these solns are conjugate w.r.t the magnetic moment of the particles (cf. I § 2).

In view of the points of view developed by Heisenberg (Chaps III + IV) it is worthy of note that the additional terms introduced here bring in an additional const of dim. $\frac{n}{\text{cm}}$ (cm) while it is not true of the analogous additional term for spin 1. This gives another distinction between half-integral & integral spins.

(b) The quantisation according to exclusion principle: Dirac's hole theory - not sym. in $+e$ & $-e$ - Heisenberg's formalism (Z. Phys. 90, 209, 92, 692)

(c) Division according to charge-conjugate functions:

Case of uncharged particles of spin ½ (Majorana: Nuovo Cimento, 14, 171, 1937 Racah, Kramers).

§4. Special connections of theories for spin 1 and spin 0:

Wave eqn for particle spin 0 written as

$$\beta_k \frac{\partial u}{\partial x_k} + \varkappa u = 0 \qquad (107)$$

β_k = ~~4-rowed matrices~~ 4 five-rowed matrices, 4 rows related to u_k and one row to scalar u. Field comps taken as $\frac{1}{\sqrt{\varkappa}} u_k$ & $\sqrt{\varkappa}\, u$, and denoted as u_ρ $(\rho = 1, \ldots 5)$. $\beta_k u$ is abbreviation for $\sum_\sigma \beta^k_{\rho\sigma} u_\sigma$.

We can write eqns (37), (38) in same form as (107) for fields of spin 1, where β_k are 4 ten-rowed matrices, four rows related to u_k & six more to u_{kl} tensors u_{kl}

Here again field components are taken $\frac{1}{\sqrt{K}} u_{kik}$, $\sqrt{K}\, u_K$ or as u_ρ ($\rho = 1, \ldots 10$).

Duffin (Phys. Rev. 54 (2), 1114 (1938) has made the interesting remark that in both cases β_K satisfy exchange relations

$$\beta_i \beta_K \beta_l + \beta_l \beta_K \beta_i = \delta_{ik} \beta_l + \delta_{kl} \beta_i \qquad (108)$$

Let us study algebra given by this. The four matrices β_K, the unit matrix & all powers & products of β_K generate from (108), 126 linearly independent magnitudes. We know already a 5-rowed & 10-rowed representation of this algebra. These representations are irreducible in the sense of algebraists and there are no more irreducible representations (The system of hypercomplex numbers β_K; 1 and their powers & products is in the sense of algebraists "semi-simple" and above statement follows from the fact

$$126 = 1^2 + 5^2 + 10^2)$$

In reality (108) contains physically nothing more than the formulation given in § I and 2 of the theory of particles of spins 0 and 1. We shall investigate more fully Duffin's formalism with the view to bring out the connection with de Broglie's "theory of photons", because this formalism is likely to be capable of extension to general spins.

Define in (108)

$$s_{kl} = -s_{lk} = \beta_k \beta_l - \beta_l \beta_k \qquad (109)$$

$$\beta_i s_{kl} - s_{kl} \beta_i = \delta_{ik} \beta_l - \delta_{il} \beta_k \qquad (110)$$

It can be shown that the s_{kl} determine properties of u_ρ for infinitesimal Lorentz-transformations.

For calculation it is important that β_k's have no reciprocals ..... series of long investigations (reference to Kemmer's unpublished work)

A special reducible representation of β_K can be obtained – as Duffin has remarked –

$$\frac{\partial u}{\partial x_i} + \delta_{ik} + \frac{\partial u}{\partial x_k} + u \beta_i u = 0 \qquad (120)$$

by the substitution

$$\beta_k = \frac{1}{2}\left(\gamma_k I' + \gamma_{k'} I\right) \qquad (123)$$

γ_k, $\gamma_{k'}$ are commutable Dirac matrices & I, I' unit matrices of the two systems. The β_k contain therefore 16 rows & 16 columns and the corresp. wave fn $u_{pp'}$ 16 components. When one introduces this special representation (123) in field eqns (107) we obtain exactly de Broglie's eqns of the theory of photons. The eqns can be written

$$\frac{\partial u}{\partial x_0} + \frac{1}{2}\left(\vec{\alpha} I' + I\vec{\alpha}'\right)\frac{\partial u}{\partial \vec{x}} + i\eta \cdot \frac{1}{2}\left(\beta I' + I\beta'\right) u = 0 \qquad (126)$$

where $\alpha_k = i\gamma_4\gamma_k$ for $k = 1,2,3$; $\beta = \gamma_4$. ($\vec{\alpha}$, β will be Hermitian matrices).

Starting from (126) and the further eqns belonging to it through L.T's and from (120) substitute $i = 1,2,3$ through (123) we obtain a repn of the theory which admits constant solutions. (the so-called de Broglie's "solns of annihilation")

The original physical meaning of (126) was that it represents the motion of C.G of two particles which is made of two individual particles of spin 1/2 & equal rest mass. But this fact is not shown in the equation, for the relative motion of the particles is nowhere specified. de Broglie's initial point is purely heuristic and it is no longer contained in the final eqns. The final conclusions re. de Broglie's formalism is: The de Broglie theory does not describe photons, but a particle of spin 0, and a particle of spin 1 with equal non-vanishing rest masses connected with each other in a special, remarkable and interesting manner. Mathematically it does not expressly formulate the assumption that the particle is made up of two smaller particles of spin 1/2.

Also the argument concerning guage-tr. of 2nd kind does not allow photons to have a rest mass:

§5. Remarks on gravitational waves & graivtation quanta (Spin 2).

$$2L = \frac{\partial u^*_{ik}}{\partial x_l}\cdot\frac{\partial u_{ik}}{\partial x_l} - 2\frac{\partial u^*_{rk}}{\partial x_s}\frac{\partial u_{sk}}{\partial x_r} + \frac{\partial u^*_{rk}}{\partial x_r}\cdot\frac{\partial u}{\partial x_k} + \frac{\partial^2 u_{rk}}{\partial x_r}\frac{\partial u^*}{\partial x_k} \qquad (128)$$

$$-\frac{\partial u^*}{\partial x_r}\frac{\partial u}{\partial x_r} + \kappa^2 u^*_{ik}u_{ik} - \kappa^2 u^* u$$

$$u_{ik} = u_{ki}\,, \quad u = \sum u_{rr} \qquad (129)$$

we derive $$u \equiv \sum u_{rr} = 0\,, \quad \frac{\partial u_{ik}}{\partial x_k} = 0. \qquad (133, a, b)$$

$$\Box u_{ik} = \kappa^2 u_{ik} \qquad (134)$$

(133, a, b) and (134) are characteristic for a particle of spin 2. In the rest system where $u_{ik} = u^0_{ik} e^{iK_0 x_0}$; $u^0_{ik} = \text{const}$, we get from (133, b), $u_{i0} = 0$ for $i = 1, \ldots 4$ accordingly $\sum_{i=1}^{3} u_{rr} = 0$ & we have $5 = 2\cdot 2 + 1$ independent eigen-solns which go into each other by space rotations (Fierz: H.P.A. l.c. who has shown that as a consequence of (133, a, b) total energy is $+^{ve}$ definite & has also obtained exchange relations for $\kappa \neq 0$).

The derivation of field eqns is similar to determination of field eqns in an ext. electromagnetic field ie replacing $\partial/\partial x_i$ by π_i & π_i^* in L (Fierz-Pauli: Proc. Roy. Soc. in Press).

Considering the physically interesting case of $\kappa = 0$ & real field (replacing u_{ik} by γ_{ik}

$$2L = \frac{1}{2}\frac{\partial \gamma_{ik}}{\partial x_l}\frac{\partial \gamma_{ik}}{\partial x_l} - \frac{\partial \gamma_{rk}}{\partial x_s}\cdot\frac{\partial \gamma_{sk}}{\partial x_r} + \frac{\partial \gamma_{rk}}{\partial x_r}\cdot\frac{\partial \gamma}{\partial x_k} - \frac{1}{2}\frac{\partial \gamma}{\partial x_l}\cdot\frac{\partial \gamma}{\partial x_l} \qquad (128')$$

where $$\gamma = \sum_r \gamma_{rr} \qquad (129')$$

This is in fact L-fn for general relativity (Einstein - Berl. Ber. 1916, p. 688, 1918, p. 154 Pauli's article, eqn (438).

in the approximation of weak gravitational fields in which

$$g_{ik} = \delta_{ik} + \gamma_{ik}$$

& the field eqns by rejecting of higher order terms in γ_{ik} become linear. We can show that this is the eqn obtained from wave eqn corresp. to (128) goes for $\kappa = 0$ into

$$\Box \gamma_{ik} - \frac{\partial^2 \gamma_{\alpha k}}{\partial x_\gamma \partial x_i} - \frac{\partial^2 \gamma_{\alpha i}}{\partial x_\gamma \partial x_k} + \frac{\partial^2 \gamma}{\partial x_i \partial x_k} + \delta_{ik}\left(\frac{\partial^2 \gamma_{rs}}{\partial x_r \partial x_s} - \Box \gamma\right) = 0 \qquad (135)$$

The canonical energy tensor becomes equivalent to Einstein's energy tensor t_{ik}. From eqns (135) we cannot deduce 2nd order eqns since divergence of this system of eqns is identically zero. The system is invariant for gauge transformations of 2nd kind

$$\gamma'_{ik} = \gamma_{ik} + \frac{\partial \xi_i}{\partial x_k} + \frac{\partial \xi_k}{\partial x_i} \qquad (136)$$

(ξ^i = arbitrary vector field). Also total energy & total impulse are gauge invariant but not densities. This G.T differs from the G.T of Maxwell's theory in that coords (not ϕ's) are transformed infinitesimally

$$x'_i = x_i + \xi_i(x) \text{ & thus alt. } g_{ik}$$

Thus G-I quantities must contain components of curvature tensor $R_{[ik][lm]}$ which can be shown invariant for (136).

The quantisation of γ_{ik}-field follows exactly as in § 2e & the "gravitons" resulting have in the sense of Chap I § 3 the spin 2.

What has been discussed here is only a limit on the quantum side, the general relativity eqns being taken linear. This limitation is connected with the well known divergence difficulties of field theory (J. Solomon: J de Phys (7), 9, 1938 - ibid).

It might be pointed here that the idea of G-T of 2nd Kind can be generalised to fields with arbitrary spins & rest mass 0 (Fierz - l.c) and it is desirable to have a close investigation of these general groups.

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Heitler's report on Meson theory for 1939 Solvay Congress: The Meson from the theoretical point of view:

Introduction:

exp. evidence of cosmic rays – connection with nuclear forces, anomalous mag. moments of protons & neutrons – History to 1935 (Yukawa) – assumption of nuclear force can be described by some sort of linear field (heuristic assumption) – If a nuclear particle surrounded by its static field moves, the field is retarded & assumes nature of propagating waves. These waves when quantised give rise to quanta which, for eg, can be emitted in a collision between 2 nuclear particles. The dispersion law of these waves ie relⁿ between energy & momentum of these quanta can be derived from the range of the forces. If the static field has extⁿ $\Delta x = 1/\lambda$ in the Fourier expⁿ all wave lengths $\lambda > \Delta x$ will occur. For field being static all these waves must be independent of wave lengths ie

$$\nu = \text{const for } \lambda > \Delta x$$

$$\text{or } h\nu = \text{const for } p < \frac{h}{\Delta x} = h\lambda$$

Using $E^2 = c^2p^2 + \mu^2c^4$, we get

$$\mu \simeq h\lambda$$

These quanta have therefore rest energy $\mu c^2 = hc\lambda$ or a rest mass $\mu = \frac{h\lambda}{c}$.

Inserting for $1/\lambda$ electronic radius we have $\mu \simeq 137$ elec. mass.

$\frac{1}{\lambda}$ is a universal constant of dim of length entering the formalism.

Yukawa's wave eqⁿ for nuclear field by scalar field fn was ($\dot\varphi = \frac{1}{c}\partial\varphi/\partial t$)

$$\nabla^2\varphi - \ddot\varphi - \lambda^2\varphi = 0 \qquad (1)$$

Static soln of (1) with a pt-singularity is

$$\varphi = g \cdot e^{-\lambda r}/r \qquad (2)$$

showing range of force is $1/\lambda$ · g is a characteristic of strength of field produced say by a proton or neutron & corresponds to charge in electromagnetic case. To explain exchange forces φ itself must itself carry an electric charge · Quanta arising from quantising field φ will then be $-^{ve}\ell$ $+^{ve}$ charged particles.

On these ideas a "consistent" theory can be built

$$\left.\begin{aligned} P &\rightleftarrows N + Y^{+} \\ N &\rightleftarrows P + Y^{-} \end{aligned}\right\} \qquad (3)$$

shows mesons obey Bose-statistics & have integral spin · Exchange nature does not exclude neutrellos Y^{0} which are suggested by

$$P \rightleftarrows P + Y^{0}\,; \quad N \rightleftarrows N + Y^{0} \qquad (3')$$

Scalar formalism does not explain all details, for eg dependence on spin dirn of proton or neutron. But this is necessary to include magnetic moments. The field is now taken a vector & the field theory is built analogously to Maxwell's theory. There are other (more complicated) cases too (Cf. Pauli's report) by which Bose quanta can be described. The vector case is the most satisfactory however.

§1. The formalism.

(1) Classical fields in vacuo:

$$\chi_{\alpha\beta} = \frac{\partial \varphi_\beta}{\partial x_\alpha} - \frac{\partial \varphi_\alpha}{\partial x_\beta}$$

Field Eqns are $\sum_\alpha \frac{\partial \chi_{\alpha\beta}}{\partial x_\alpha} = \lambda^2 \varphi_\beta$ (In Maxwell's theory $\lambda = 0$)

Here φ & $\chi_{\alpha\beta}$ are on same footing. Lorentz condn

$\sum_\beta \frac{\partial \varphi_\beta}{\partial x_\beta} = 0$ follows as a consequence & need not be assumed. as in Maxwell's theory

All field quantities can be expressed by transverse & longitudinal vectors

($\varphi_i = \vec{\varphi} - \frac{1}{\lambda}\dot{\vec{\psi}}$ (long)

$\chi_{ik} = \operatorname{curl}\vec{\varphi}$

$\chi_{0i} = -\lambda\vec{\psi} - \dot{\vec{\varphi}}$ (trans)

(2) Quantisation: $+^{ve}$ & $-^{ve}$ mesons. Neutrettos: Associate a L, exchange rels as for Bose-particles - Also a H & hence field eqns - In contrast to Maxwell's theory longitudinal part does not reduce to static field only. Pauli-Weisskopf double quantisation leads to + & - mesons, seen easiest in momentum space. Charged character of the field is essentially connected with complex nature of field quantities - Neutrettos can be described by real φ_α, $\chi_{\alpha\beta}$ & quantisation leads to one sort of quanta only

(3) Interaction with nuclear particles: Differences with Maxwell field

(i) Meson field is charged

(ii) φ & $\chi_{\alpha\beta}$ are on the same footing here & interaction can therefore depend either $\chi_{\alpha\beta}$'s or φ_α's. This makes field due to a magnetic point dipole also very simple. Assume that (p) & (n) are characterised by a mesotron charge g & a mesotron dipole moment f/λ, the latter having the direction of spin. Interaction can be satisfactorily treated for slowly moving protons or neutrons. The quantisation is same as in vacuo as long as i_k (s_k) and j_{0k} ($j_{\alpha\beta}$ = rel. expn for density of magnetic dipoles) are neglected. Interaction of neutretto field with heavy particles also can be treated similarly. Matrix elements for emission of mesotrons by a heavy particle can be obtained for cases of: Emission of long. γ^+ by proton, long γ^- by neutron, trans. γ^+ by proton, trans γ^- by neutron in terms of vectors $\vec{f}$ & $\vec{u}$ being unit vectors of polarisation for transverse meson.

§2. (n-p) and (p,p) potentials – Deuteron.

(1) (n-p)-potential: Two ways of processing

(a) Calculate interaction energy classically by using unquantised eqn and interaction of particle with the field

(b) Calculate by 1st order perturbation method in quantised theory. Both give same results

$$W_{PN}(np\text{-}pot) = \frac{e^{-\lambda r}}{r}\left\{g^2 P \pm g_0^2 + (f^2 P \pm f_0^2) R\right\} \qquad (29)$$

P = operator which $N \rightleftarrows P$, $R = (\vec{\sigma}\vec{p})\left(1 + \frac{1}{\lambda r} + \frac{1}{\lambda^2 r^2}\right) - \frac{(\vec{\sigma}\vec{r})(\vec{p}\vec{r})}{r^2}\left\{1 + \frac{3}{\lambda r} + \frac{3}{\lambda^2 r^2}\right\}$

($\vec{\sigma}$ & $\vec{p}$ spin vectors) (g_0, f_0 ref. to neutrettos). (29)

(2) (p-p) potential: Since there is no exchange of charge there is no contribution from charged mesons. Only contribution arises from neutretto field. We get

$$W_{PP} = +\left(g_0^2 + f_0^2 R\right)\frac{e^{-\lambda r}}{r}.$$

W_{PP} & W_{PN} have been found to give same results by two contradictory methods (a) & (b). In (a) $h \to 0$, in (b) first term retained is $\frac{gk}{\lambda}$ ie dimensionless const $g^2R^2/\hbar c d^2$ & shall be valid for large h. Hence we assume that (29) must be exact as shown by Rosenfeld & Moller. (Nature 143, 241, 1939) by a certain canonical transformation.

(3) Deuteron-states: p & n differ in quadrivalent variable viz charge. States of (n-p) system can be classified into 2 parts as eigen wave fn is sym or anti-sym w.r.t. charge variable. The whole wave fn must be antisymmetrical. Thus charge wave fn is

$$\text{sym: for } \left\{\begin{matrix}^1S, & ^1D \\ ^3P, & ^3F\end{matrix}\right\}; \quad \text{anti-sym for } \left\{\begin{matrix}^3S & ^3D \\ ^1P & ^1F\end{matrix}\right\} \qquad (32)$$

$P = \pm$ for (32a) & $= -1$ for (32b). For singlet states $R = -2$ (33)

for triplet states R has matrix elements for transition where orbital quantum number changes by 2. Consider the three lowest $^3S - {}^3D_1$, ^1S, ^1P. Apart from $e^{-\lambda r}/r$, potential for these 3-states is

$$\left.\begin{array}{ll} ^3S - {}^3D_1 & -g^2 \pm g_0^2 - (f^2 \mp f_0^2) R \\ ^1S & +g^2 \pm g_0^2 - 2(f^2 \pm f_0^2) \\ ^1P & -g^2 \pm g_0^2 + 2(f^2 \mp f_0^2) \end{array}\right\} \quad (34)$$

(p-p) potential in

$$^1S \quad \text{is} \quad g_0^2 - 2f_0^2 \quad (34d)$$

For the $^3S - {}^3D_1$ state R does not commute with orbital mom. wave fn & energy for this state have to be calculated from a pair of sim. diff. eqns involving matrix elements of R for the S-D transition. The soln meets the following fund. difficulty: R contains terms $\sim \frac{1}{r^3}$ & diffl. eqn is essentially singular for $r = 0$ & it can be shown that the lowest resulting eigen-value is ∞ (private communication by W. Pauli). "cutting off" at a suitable r enables replacing R by its mean value over ^3S-state

$$\bar{R}_{^3S} = 2/3 \quad (34, e) \quad \text{(valid in rough approximation)}$$

Expt. facts known: (i) A.M of H^2 in ground state $= 1$. It is therefore $^3S - {}^3D_1$ state or ^1P state. It is assumed to be the former because of short range - depth of pot about 20 M.V

(ii) ^1S state is higher & has potential of width $1/2$ ground state.

(iii) (p-p) is attractive & experimentally (pp) = (pn)

(iv) H^2 has in ground state a quadripole moment of $\sim 2 \cdot 10^{-27}$ cm^2

constants f, g, f_0, g_0 can be fitted to explain these facts. Taking case of quadripole moments two such choices are possible accg as quad. mom is + or −

(a) C.I.H, $(np) = (pn)$. From (29)

$$g_0^2 = g^2/2, \quad f_0^2 = f^2/2 \qquad (35)$$

reducing constants to 2 & for potentials

$$(p-n)\left\{\begin{array}{ll} {}^3S - {}^3D_1 & -\tfrac{3}{2}g^2 - f^2 \\ {}^1P & -\tfrac{3}{2}g^2 + 3f^2 \end{array}\right.$$

$$\left.\begin{array}{l} p-n \\ p-p \end{array}\right\} \; {}^1S \qquad -f^2 + \tfrac{1}{2}g^2$$

ie lowest state is ${}^3S - {}^3D_1$ & if $f \cong g$, posn of states is that actually observed. This choice leads to −ve quadripole moment ie "to a pancake-shape".

(b) C.I. Hyp not true, then quad-mom would be +ve ie "cigar-shape" as present indications suggest. Further a choice of constants can be made to explain right posn of ${}^3S - {}^3D_1$ & ${}^1S$ states & equality of (pp) & (pn) for all singlet states only & four constants would be of same order of magnitude & using depth of observed $(p-n)$ potential

$$g^2/\hbar c \simeq \frac{1}{18}; \quad f^2/\hbar c \simeq \frac{1}{7} \qquad (38)$$

(4) Quadripole moment of deuteron: Rabi, Kellog etc (Phys. Rev. 55, 318, 1939) have found this to be $\overline{3z^2 - r^2} \simeq 2 \times 10^{-27}\ \text{cm}^2$ ~~but sign not~~ in ground state but sign not decided. The existence of quad-pole moment in ground state follows easily from the theory & is a great success of the theory. We obtain $\frac{\text{'cigar'}}{\text{'pancake'}}$ shape if $(f^2 \mp f_0^2)$ is $\frac{-\text{ve}}{+\text{ve}}$.

Right order of magnitude is obtained for "cutting off" radius of the order $1/\lambda$

(5) Photo-electric effect of deuteron: Meson theory leads to alterations of the old Bethe-Peirls (Proc. Roy. Soc. 148, 146) theory in which cross-section had a max at $h\nu = 2E_0$ (binding E of H^2) and ~~fell~~ fell off at high frequencies like $1/\nu^{3/2}$. In meson theory ground state is $^3S - {}^3D_1$ combination. ~~The light~~ and there is a departure from the pure S and P distributions as a consequence of existence of quadripole moment. Secondly meson field is charged. The light quantum acts also on mesotron instead of directly on the proton & interaction of a photon is the larger the smaller the mass of particle. owing to smaller mass of Y, cross-section will be rather large. Calculations by Fröhlich, Heitler & Kahn (P.R.S in press) show that for high frequencies cross section decreases like $1/\sqrt{\nu}$. For smaller ν's, ϕ_{mes} increases more slowly than ϕ_{ord} & reaches a max. at very much higher frequencies (~10 M.V). For the Li-γ-rays (17 M.V) ratio $\phi_{mes}/\phi_{ord} \simeq 6$. Also angular distribution is very different & this along with total cross-section appear to provide a crucial test for the theory, because the effect is due to slow mesons & hence independent of divergence difficulties at high energies. It depends however on applicability of perturbation method (see §4 & §5)

§3. Magnetic moments, Coulomb law.

(1) Magnetic moments: Meson theory accounts for anomalous mag. moms of n & p — virtual states of meson emission — among these states are those with A.M about spin axis. Interaction $\Pi f(\vec{\sigma}\ \mathrm{curl}\ \vec{\varphi})$ leads to transitions where n with $S = 1/2$ in the +Z dirn emits a Y with A.M = +1 about +Z dirn whereas $n \to p$ with $S = -1/2$. The A.M is therefore conserved and a neutron with $S = +1/2$ can only emit Y's with A.M = 0 or +1. A Y with A.M = 1 has a large mag. mom because of its small mass. The mag. mom in Bohr units is

(Bohr $m_0 = e\hbar/2Mc$) the ratio $M/\mu = 13$. Again fraction of time meson spends in dissociated state is of order $f^2/\hbar c$ (see below) & thus mag. mom will be of order

$$\frac{m_p}{m_0} \simeq -\frac{m_N}{m_0} \simeq \frac{f^2}{\hbar c}\cdot\frac{M}{\mu} \simeq 1 \qquad (40)$$

Putting $f^2/\hbar c$ value $\frac{1}{7}$ in (38) we obtain for m_p/m_0 the order 1. The signs are also in accordance with expt. The actual calculations meets with a difficulty which is an essential feature of the theory. The time fraction is not $f^2/\hbar c$ actually but $f^2/\hbar c$ multiplied by a diverging integral. The actual calculations give

$$\frac{m_N}{m_0} = \frac{4}{3\pi}\frac{f^2}{\hbar c}\cdot\frac{M}{\mu}\int_0^\infty \frac{k^4\,dk}{\lambda(k^2+\lambda^2)^2} \qquad (41)$$

Divergent integral is of order unity if we cut off at wave numbers λ – same procedure has to be adopted in perturbation theory. Here mag. moments are – in contrast to nuclear forces – a typical quantum effect, $\hbar$ being essentially involved in (41)

(2) The new universal constants: Interpretation of f and g

From (41) $$f^2/\hbar c \simeq \frac{\mu}{M} \qquad (42a)$$

which appears more than accidental. For consider size of deuteron. This depends on strength and mass of proton more than range of nuclear forces. For a rough discussion assume a Coulomb law g^2/r; radius of 1st Bohr orbit would then be $\hbar^2/Mg^2$ in analogy to $\hbar^2/me^2$. Since experimentally size of deuteron is of order of magnitude $e^2/mc^2 \simeq \hbar/\mu c$ is expressed by the eqn

$$\frac{\hbar^2}{Mg^2} \simeq \frac{\hbar}{\mu c} \quad \text{or} \quad \frac{g^2}{\hbar c} \simeq \frac{\mu}{M} \qquad (42,b)$$

Both (42,a,b) cannot be accidental. Again identifying range $1/\lambda$ of nuclear forces with

e^2/mc^2 leads to $\frac{e^2}{\hbar c} \sim \frac{M}{\mu}$ (42,c)

Analogy of (42c) + (42,b) is obvious.

(3) Departure from Coulomb-law: The result of Bohlbeck, Heitler, Kahn (P.R.S, 171, 269, (1939)

is that $V(R) = -\frac{e^2}{R} + \frac{1}{R} \lim_{R\to\infty} (R\,W(R))$ (R = dist bet. proton & point charge)

= finite for all finite R.

V(R) follows Coulomb law down to $\frac{1}{2\lambda}$. For smaller distances attraction becomes less than $-e^2/R$ & goes into a strong repulsion ($\sim \frac{1}{R^3}$ for $R < 0,2/\lambda$). Repulsion is explained by observing that there might be a change in self-energy of ~~proton~~ the proton in the field. This leads to observable effects for eg in H-atom for 1-S & 2s-levels, there are shifts to higher energies by amounts $\sim 3\ cm^{-1}$ & $0.4\ cm^{-1}$ in consonance with observations of R.C. Williams. Secondly effects takes place in a region where the theory just ceases to be valid; hence we cannot be sure whether the effect predicted is real or not.

§ 4. Collision processes.

Processes of large γ – (N or P) interaction – for eg fast mesons thro' matter.

(1) Scattering of γ by P & multiple processes: cross section for solid angle $d\Omega$ is

$$d\phi_1 = \frac{d\Omega}{A^2} \cdot \frac{g^2}{\hbar c}\left(\frac{g^2}{\hbar c} + \frac{2f^2}{\hbar c}\right) \frac{p^4}{\mathcal{E}^2(\mu c^2)^2} \qquad (44)$$

Process is large angle scattering (no dependence on θ). (44) increases rapidly with energy for $\mathcal{E} = 2\mu c^2$ (80 M.V) cross section would correspond to a range of 13 cm in Pb. Another striking feature of theory is occurrence of multiple processes

for eg $Y^+ + N \rightarrow P + Y^+ + Y^-$ (45)

& crosssection for this increases more rapidly for this than 44. Putting $p \sim E$ we find the ratio in the 2 cases $\frac{\phi_2}{\phi_1} \simeq \frac{g^2 + 2f^2}{\hbar c}\left(\frac{\mathcal{E}}{\mu c^2}\right)^2$

which is >1 for $\mathcal{E}$ larger than few times μc^2. If formula be valid in region of $\mathcal{E} \sim \mu c^2$, multiple processes are very probable. There are reasons to believe that this is not so, & perhaps perturbation methods adopted are not valid in this region. Also Born's approximation may not be valid in this region.

Also ϕ for single scattering is independent of mass of scattering particle unlike scattering of light by electron which is $\because^l$ to $(e^2/mc^2)^2$.

Bhabha (in press) has developed a purely classical theory for neutral mesons with the g-interaction only & obtains a cross-section for scattering a formula analogous to Thomson formula (energies $< Mc^2$)

(transverse) $\phi = \frac{8\pi}{3}\left(g^2/Mc^2\right)^2$ (47)

In the quantised theory we obtain a similar result. For charged mesons there is only one inter-stage but for neutrellos two (i) first absorption, then reemission (ii) secondary Y^0 is first emitted & primary one absorbed afterwards. For longitudinal waves in same way

$$\phi \simeq \frac{4N}{3}\left(\frac{g^2}{Mc^2}\right)\left(\frac{\mu c^2}{\mathcal{E}}\right)^4 \qquad (48)$$

also in agreement with classical results. The ϕ's do not increase with $\mathcal{E}$ & are $\because^l$ to $\frac{1}{M^2}$. No multiple processes exist for Y^0's. The behaviours of these is in agreement with corresp. pr. & for tr. waves with quant-electrodyn. The finite rest mass of the

mesotron or universal length $1/\lambda$ does not introduce any essential alterations in these results.

The diff. between Maxwell's theory & Meson theory is not in $1/\lambda$ but in exchange character (also spin dependent). Hence results differ from anything based on corresp. pr or analogy of meson theory to quant-Elec.

Experimentally we have data for energies $2.5\mu c^2$ (200 M.V). For these high energies ϕ_1 or ϕ_2 from (64) or (45) are much larger than expt. values. It would be important to know behavior of mesons in the region μc^2. There is no evidence

(2) Creation of mesons: These in cosmic rays are not primaries but are produced in the atmosphere. Suggested processes are

$$\left.\begin{aligned} &a)\ h\nu + P \rightarrow N + \gamma^+ \\ &b)\ P + N \rightarrow N + N + \gamma^+ \end{aligned}\right\} \qquad (49)$$

nothing decisive

§ 5. Divergences. Self-energy.

(1) Inf. interaction potential, large collision cross-sections: Difficulties here are more serious than in quant-Elec. In the $(p-n)$ potential occur terms $\propto \frac{1}{r^3}$. Inserting these into S-eqn some eigen-values become ∞. Some are finite (all eight terms) but it is not possible to conclude as ours because the finite ones do not form a completely orthogonal system.

Breakdown of theory for dist $< \frac{1}{\lambda}$ has some deep fund. significance

Born's reciprocity paper in Proc. Acad: comprehensive theory of elementary particles should contains only one universal constant a, b, G or m.

There are very few experimental indications of deviations from the accepted linear laws. The rel.-quantum theory of the electron seems to hold up to the highest energies occurring in cosmic rays. The structure of nuclei can be treated even without considering relativistic effects owing to the high mass of the constituent particles. The only deviations seem to occur in the case of fast mesons which do not obey the laws predicted by the linear theory (Heitler)

Therefore we have to rely only on theoretical considerations. The only natural & unique way of introducing non-linearity into a field theory is that used by Einstein in his theory of gravitation namely the postulate of general invariance which leads us to consider space-time as "curved". But Einstein's theory has to do with very small curvatures, imperceptible in region of laboratory dimensions. It is also clear from the smallness of the gravitational constant that cosmological curvature has nothing to do with atomic effects. These latter are bound to be extremely small of order $\underline{a}$. But small $\underline{a}$ means large momentum $\underline{b} = \hbar/a$. I have suggested that the domain of the elementary particle has to be considered from the standpoint of momentum space in which a non-linear geometry with small curvature reigns. The curvature eqn should contain solely a and b (or a & h) but not the gravitational constant.