THE KADEVI ENGINEERING COMPANY PVT. LTD.
A-1, A-2, ELECTRONIC COMPLEX
HYDERABAD - 500762
(DEFENCE ORIENTED ELECTRONIC INDUSTRY)
GRAMS: "ANTENNA"
HYDERABAD
PH 78450-78457
75824-74803 75791

13/9/83	— Typing Court order ck	17.00
16/9/83	— Postal charges to comm & Rev. officer	8.70
14/12/83	— Xeroxing	5.25
17/12/83	— Auto to Balepet + Phone	9.10
18/12/83	— "	6.00
19 to 21/12/83	— Mysore trip	206.40
22/12/83	— watchman charges to N	5.00
23/12/83	— Auto to Balepet	6.75
7/1/84	— Regd. letter to B.G.R	3.45
12/1/84	— Auto to Balepet	7.00
16/1/84	— Stamp papers from N & Co	11.00
"	— Typing work	7.50
"	— Stationery from F.V	8.35
"	— Auto to Balepet	7.30
17/1/84	— Auto to Notary Public	16.55
"	— Fees to Notary "	40.00
19/1/84	— S.V.R for copies of plaintch	100.00
"	— Auto to Balepet	7.05
20/1/84	— Xeroxing Decree papers	12.00
21/1/84	— Regd. letter to B.G.R	7.95
"	— Cheque to Ramanna	500.00
		992.55

gland — 12 + 4 = 16 + 4 = 20 ✓

w Zealand — 8 + 4 = 12

ustralia — 4 + 4 = 8

Zimbabwe — 4 + 0 = 4

India — 8 + 4 = 12 + 4 = 16 ✓

est Indies — 8 + 4 + 4 = 16 + 4 = 20 ✓

Sri Lanka — 0 + 4 = 4 + 0 = 4.

akistan — 4 + 4 + 4 = 12 ✓

2/6/83 — India vs England
West Indies vs Pakistan.

25/6/63 — Finals
India vs West Indies

India wins by 43 runs
Wonderful !!!

ఎల్లాంచ బంతీ ఇదూ-చదవ

10.40
2.00
5.00
4.50
21.90

3.00
6.00
2.05
11.05
0.35
11.40
0.75
10.65

ANTENNA ARRAYS & AMPLIFIERS

PROFESSIONAL DESIGNS
AND
SYSTEM ENGINEERING

The Kadevi Engineering Company Private Limited

Defence Oriented Electronic Industry

DESIGNERS - ENGINEERS

REGD. OFFICE & WORKS

A1, A2, Electronic Complex,
Kosaiguda,
HYDERABAD-500 762.
Phones : 78450, 78457



ANTENNA DIVISION

Systems Engineering
High Power Communication.

H. F. Range 1 to 30 MHz.

Whip Antenna-Vehicular Mounting.
Centre Fed Dipole Antennae.
Full Wave Cage Dipole Antennae.
Broad Band Dipole Antenna 1.6 to 30 MHz.
Antenna, Omni-Directional 1.6 to 30 MHz.
Delta Antennae.
Inverted Cone.
Monocone Antenna Broad Band 3-30 MHz.
H. F. Log Periodic Antenna.

V. H. F. Range 50 to 200 MHz.

Whip Antenna-Vehicular Mounting.
Yagi Antenna-5 Element, Folded Dipole.
High Gain Yagi Antenna-8 Element.
G. P. Antenna,
T. V. Antenna-High Gain.
Log Periodic Antenna-50 to 100 MHz.
Log Periodic Antenna-100 to 300 MHz.
Air Craft Antenna.
Unipole Antenna.
Strip Aerial.
Hula-Hoop Antenna-Vehicular Mounting.
Omni Directional T. V. Antenna.

U. H. F. Range 200 to 800 MHz.

Biconical Antenna.
Discone Antenna.
Sub Antenna.
Yagi Antenna.
Helical Antecna.
Helicone Antenna.
Corner Reflector.
Log Periodic Antenna.

Development :

Customers designs are developed and we undertake manufacture, supply installation commissioning of the entire system.

AMPLIFIERS DIVISION

1. **O. B. Amplifier for Air & T. V.**
   Frequency response : 30HZ to 15,000HZ less than 1 db.
   Distortion : Less than 1%.
   Equivalent input noise level : Better than minus-120 dbm.
   Designed exclusively for : Broadcast stations, sound recording stations, luxury hotels, floor show halls, discoutheque, cinema theaters & auditoriums fitted with stereophonic sound diffusion system.

2. **Limlter-Compression Amplifier**
   Designed exclusively for Broadcast stations & Recording studios.

3. **HF Amplifier-1.6 to 30 MHz.**

4. **VHE Amplifier-50 to 200 MHz.**

5. **HF Aerial Multi Coupler Amplifier**
   1.6 to 30 MHz.
   To feed signal to 8 receivers without attenuation
   (Approved by Air Force).

6. **VHF Aerial Multi Coupler Amplifier**
   50 to 185 MHz.
   To feed signal to 8 receivers without Attenuation.
   (Under approval of Defence)

7. **Cascode Amplifier :**
   40 db gain 150 to 115 MHz.

8. **On Line SWR Meter :** 50 MHZ to 200 MHz,

9. **Field Strength Meter :** 50 to 200 MHz.

10. **Band Pass Filters :** HF or VHF.

11. **High Pass or Low Pass Filters.**

.12 **Active Filters.**

13. **Passive Filters.**
14. **Power dividers.**
15. **Duplexers, Diplexers, etc.**

BALUNS DIVISION

1. HF wideband matching transformer.
   1.6 to 30 MHz for receing application.
   160 ohms to 50, 60 or 75 ohms made or pot core ferrites.
   VSWR <1 3 : 1
2. HF wide band matching transformer 3 to 30 MHz for transmitting or receiving application upto 2K. Watt Power. VSWR <1 3 : 1
3. V.H.F. range, Coaxial tuned baluns (Spot frequency) both receiving and transmitting for folded dipole antenna 50 MHz to 150 MHz.
   VSWR < 1 05 : 1
4. V.H.F. range Toroidal baluns.
5. Lighting arrestor spark gap type made of tin bronze for receiving antenna.
6. Lighting arrestor, ball type made of Tin bronze for transmitting antennae upto 5K. Watt Power.
7. HF. Broadband Matching transformer 5K Watt Power.
   60 or 75 ohms to 600 ohms.
   60 or 75 ohms to 240 ohms.
   240 ohms to 600 ohms.
8. Coaxial Cable connectors male or female upto 5K. Watt.

ENGINEERING DIVISION

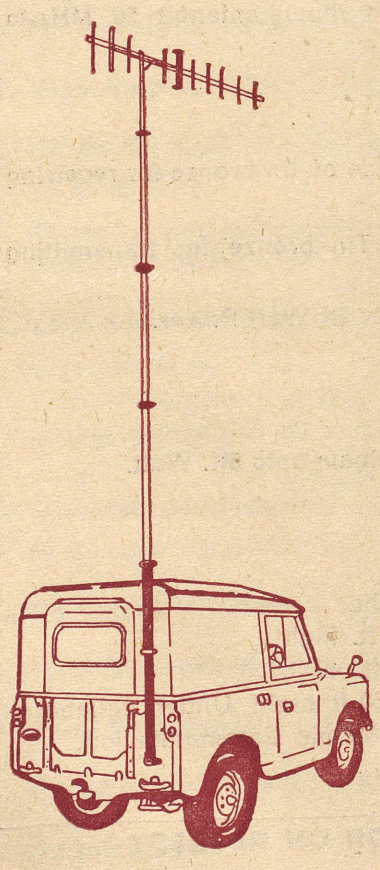
Masts for Aerials-Static & Mobile :

1. Telescopic Masts-30 ft. & 45 ft. hight.
2. Tubular M. S. Mast-100, 80, 60 & 40 ft. hight.
3. Portable Aluminium Alloy Mast. 30 ft. 40 ft.
4. Pneumatic Telescopic Masts up to 60 ft. hight Under collaboration with Hilomast Ltd. London For Mobile Communication.

DETAILED SPECIFICATION ON REQUEST

The TMK series of telescopic Kadevi Masts are pneumatically operated with either a standard vehicle foot pump or a small compressor.

They are intended for light antennas where portability is of prime importance. Easily fitted to a vehicle, they are ideally suited for mobile applications.



Usages :

Mobile Masts are convenient not only to mount antenna, they are useful for temporary lighting, loudspeakers, cameras, instruments mounting etc..

UNDER DEVELOPMENT

The following systems are under active development in our R & D which will be put to commercial use in 4 to 6 months period. We undertake custom built units, depending on customer's requirements.

1. U.H.F. Multicoupler.
2. High Power Duplexers for C.W. Pulsed Radar Systems.
3. Notch type filters upto 600 MHz.
4. Quadrature Hybrids for HF. VHF & UHF.
5. Microwave Dishes.
6. U.H.F. loads low power and High power.
7. V.H.F. Power Sensors.
8. M.I.C. Spiral Antenna.

1982 Diary

Personal Notes

NAME

OFFICE PHONE

RESIDENCE PHONE

CLUB MEETS ON. PHONE

CAR NO. LICENCE NO. EXPIRES ON.

C.D.S. A/C. NO. BANK A/C NO. :

GAS CONNECTION NO. GAS TEL. NO.

IN CASE OF ACCIDENT PLEASE INFORM

PHONE

DOCTOR PHONE

HEALTH BLOOD GROUP WEIGHT

DAYS TO REMEMBER BIRTHDAY'S WED. ANNIVERSARIES

Rates of Income Tax

Income Tax Including surcharge compulsory Deposit for individual

Income Rs.	Income Tax Including S. Charge 1981-82	Income Tax Including S. Charge 1982-83	C.D. 1981-82 and 1982-83
12,000	Nil	Nil	
13,000	330	Nil	
14,000	960	Nil	
15,000	990	Nil	Nil
16,000	1,320	330	720
18,000	1,749	990	810
20,000	2,145	1,650	900
22,000	2,695	2,310	990
24,000	3,245	2,970	1,080
25,000	3,520	3,300	1,125
26,000	3,850	3,674	1,235
28,000	4,510	4,422	1,455
30,000	5,170	5,170	1,675
32,000	6,050	6,050	1,895
34,000	6,930	6,930	2,415
36,000	7,810	7,810	2,350
38,000	8,690	8,690	2,600
40,000	9,570	9,570	2,850
42,000	10,450	10,450	3,100
44,000	11,330	11,330	3,350
46,000	12,210	12,210	3,600
48,000	13,090	13,090	3,850
50,000	13,970	13,970	4,100
55,000	16,720	16,720	4,725
60,000	19,470	19,470	5,350
65,000	22,220	22,220	5,975
70,000	24,970	24,970	6,600
75,000	27,995	27,995	7,350
80,000	31,020	31,020	8,100
85,000	34,045	34,045	8,850
90,000	37,070	37,070	9,600
95,000	40,095	40,095	10,350
1,00,000	43,120	43,120	11,100
1,10,000	49,720	49,720	12,600
1,20,000	56,320	56,320	14,100
1,25,000	59,620	59,620	14,850
1,30,000	62,920	62,920	15,600
1,40,000	69,520	69,520	17,100
1,50,000	76,120	76,120	18,600
1,60,000	82,720	82,720	20,100
1,70,000	89,320	89,320	21,600
1,75,000	92,620	92,620	23,350
1,80,000	95,920	95,920	23,100
1,90,000	1,02,520	1,02,520	24,600
2,00,000	1,09,120	1,09,120	26,100
2,25,000	1,25,620	1,25,620	29,850
2,50,000	1,42,120	1,42,120	33,600
3,00,000	1,75,120	1,75,120	41,100
3,50,000	2,08,120	2,08,120	48,600
4,00,000	2,41,120	2,41,120	56,100
4,50,000	2,74,120	2,74,120	63,600
5,00,000	3,07,120	3,07,120	71,100
10,00,000	6,37,120	6.37,120	1,46,100

Wealth Tax

Income Rs.	Tax 1982-83	Income Rs.	Tax 1982-83
1,00,000	Nil	1,90,000	945
1,10,000	Nil	2,00,000	995
1,20,000		2,50,000	1244
1,30,000		3,00,000	1733
1,40,000		3,50,000	2228
1,50,000	Nil	4,00,000	2723
1,60,000	500	4,50,000	3218
1,70,000	846	5,00,000	3713
1,75,000	871	10,00,000	13,480
1,80,000	896		

Telephone S.T.D. CODES

AHMEDABAD TO

Baroda	...	9265
Bombay	...	022
Delhi	...	011
Mehsana	...	923
Nadiad	...	922
Poona	...	0212
Rajkot	...	9281
Surat	...	0261

Night Service only

Agra	...	0562
Bangalore	...	0812
Chandigarh	...	0172
Coimbatore	...	0422
Jaipur	...	941
Jullundur	...	0181
Madras	...	044
Madurai	...	0452
Nagpur	...	0712

DELHI TO

Agra	...	0562
Ahmedabad	...	975
Amritsar	...	9183
Allahabad	...	0532
Aligarh	...	954
Alwar	...	981
Ambala	...	0171
Bombay	...	022
Bangalore	...	0812
Bhiwani	...	974
Bhopal	...	977
Bhubaneshwar	...	0674
Chandigarh	...	0172
Calcutta	...	033
Dehradun	...	935
Dispur	...	0361
Gurgaon	...	9227
Gandhinagar	...	97559
Gangtok	...	0359
Gauhati	...	0361
Hissar	...	983
Hyderabad	...	0842
Hapur	...	9122
Indore	...	978
Jamshedpur	...	0657
Jodhpur	...	986
Jaipur	...	941
Jammu	...	9191
Jullundur	...	0181
Karnal	...	984
Kharagpur	...	0321
Kohima	...	0386
Kanpur	...	951
Lucknow	...	952
Meerut	...	913
Modinagar	...	9223
Madras	...	044
Moradabad	...	985
Mussoori	...	93585
Panipat	...	933
Patna	...	972
Panjim	...	0832
Pune	...	0212
Rohtak	...	938
Saharanpur	...	982
Shillong	...	0364
Surat	...	0261
Simla	...	9177
Sonepet	...	9226
Srinagar	...	9194
Trivandrum	...	0471
Varanasi	...	0542

Night Service only

Coimbatore	...	0422
Ernakulam	...	0484
Madurai	...	0452
Nagpur	...	0712

BOMBAY TO

Ahmedabad	...	975
Bhopal	...	0755
Calcutta	...	033
Delhi	...	011
Madras	...	044
Nagpur	...	0712
Poona	...	0212
Surat	...	0261

Night Service only

Agra	...	0562
Bangalore	...	0812
Chandigarh	...	0172
Coimbatore	...	0422
Jaipur	...	941
Jullundur	...	0181
Madurai	...	0452

MADRAS TO

Bangalore	...	0812
Bombay	...	022
Bhuvaneshwar	...	0674
Calcutta	...	033
Coimbatore	...	0422
Chingelput	...	947
Ernakulam	...	0484
Kancheepuram	...	942
Madurai	...	0452
New Delhi	...	011
Secunderabad	...	984
Trivandrum	...	0471
Vellore	...	945
Vijayawara	...	986

Night Service only

Ahmedabad	...	975
Agra	...	0562
Chandigarh	...	0172
Jaipur	...	941
Jullundur	...	0181
Nagpur	...	0712
Poona	...	0212
Surat	...	0261
Tirupati	...	981

PUNE TO

Ahmedabad	...	0272
Bombay	...	022
Gandhinagar	...	0271
Surat	...	0261

Night Service only

Agra	...	0562
Bangalore	...	0812
Chandigarh	...	0172
Combaitore	...	0422
Delhi	...	011
Jaipur	...	941
Jullundur	...	0181
Madras	...	044
Madurai	...	0452
Nagpur	...	0712

CALCUTTA TO

Asansol	...	0341
Bombay	...	022
Burnpur	...	03448
Bahula	...	03442
Barakar	...	03446
Bhubneswer	...	0674
Cuttack	...	0671
Choudar	...	067129
Hyderabad	...	0842
Jamuraih	...	03443
Jamshedpur	...	0657
Kharagpur	...	0321
Madras	...	044
Muzaffarpur	...	0621
Neamatpur	...	03445
New Delhi	...	011
Patna	...	0612
Ranigunj	...	03447
Rupanarayanpur	...	03444

BANGALORE TO

Coimbatore	...	0422
Hubli	...	98
Madras	...	044
Madurai	...	0452
Shimoga	...	96
Tumkur	...	95

Night Service only

Agra	...	0562
Ahmedabad	...	975
Bombay	...	022
Chandigarh	...	0172
Delhi	...	011
Jaipur	...	941
Jullundur	...	0181
Nagpur	...	0712
Poona	...	0212
Surat	...	0261

COIMBATORE TO

Madras	...	044
Tirpur	...	0421
Salem	...	0427
Ooty	...	94
Bangalore	...	0812
Ernakulam	...	0484
Quilon	...	0474
Kottayam	...	0481
Alleppey	...	0477
Trivandrum	...	0471

Night Service only

Bombay	...	022
Delhi	...	011

20th Century Calendar

The number opposite each year in the list below indicates which of the calendars on these two pages is the one for that year. Thus the number opposite 1972 is 14, so Calendar 14 can be used for 1972.

Years divisible by four without remainder are leap years with 366 days instead of 365 (29 days in February instead of 28). However the last year of a century is not a leap year except when divisible by 400.

1901 ... 3	1936 ... 11	1971 ... 6
1902 ... 4	1937 ... 6	1972 ... 14
1903 ... 5	1938 ... 7	1973 ... 2
1904 ... 13	1939 ... 1	1974 ... 3
1905 ... 1	1940 ... 9	1975 ... 4
1906 ... 2	1941 ... 4	1976 ... 12
1907 ... 3	1942 ... 5	1977 ... 7
1908 ... 11	1943 ... 6	1978 ... 1
1909 ... 6	1944 ... 14	1979 ... 2
1910 ... 7	1945 ... 2	1980 ... 10
1911 ... 1	1946 ... 3	1981 ... 5
1912 ... 9	1947 ... 4	1982 ... 6
1913 ... 4	1948 ... 12	1983 ... 7
1914 ... 5	1949 ... 7	1984 ... 8
1915 ... 6	1950 ... 1	1985 ... 3
1916 ... 14	1951 ... 2	1986 ... 4
1917 ... 2	1952 ... 10	1987 ... 5
1918 ... 3	1953 ... 5	1988 ... 13
1919 ... 4	1954 ... 6	1989 ... 1
1920 ... 12	1955 ... 7	1990 ... 2
1921 ... 7	1956 ... 8	1991 ... 3
1922 ... 1	1957 ... 3	1992 ... 11
1923 ... 2	1958 ... 4	1993 ... 6
1924 ... 10	1959 ... 5	1994 ... 7
1925 ... 5	1960 ... 13	1995 ... 1
1926 ... 6	1961 ... 1	1996 ... 9
1927 ... 7	1962 ... 2	1997 ... 4
1928 ... 8	1963 ... 3	1998 ... 5
1929 ... 3	1964 ... 11	1999 ... 6
1930 ... 4	1965 ... 6	2000 ... 14
1931 ... 5	1966 ... 7	
1932 ... 13	1967 ... 1	
1933 ... 1	1968 ... 9	
1934 ... 2	1969 ... 4	
1935 ... 3	1970 ... 5	

1

JANUARY						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24	31	
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

FEBRUARY						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22			
2	9	16	23			
3	10	17	24			
4	11	18	25			

MARCH						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			

APRIL						
SUN	30	2	9	16	23	
MON		3	10	17	24	
TUE		4	11	18	25	
WED		5	12	19	26	
THU		6	13	20	27	
FRI		7	14	21	28	
SAT	1	8	15	22	29	

MAY						
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			

JUNE						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

JULY						
SUN	30	2	9	16	23	
MON	31	3	10	17	24	
TUE		4	11	18	25	
WED		5	12	19	26	
THU		6	13	20	27	
FRI		7	14	21	28	
SAT	1	8	15	22	29	

AUGUST						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

SEPTEMBER						
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		

OCTOBER						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24	31	
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

NOVEMBER						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			

DECEMBER						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			

2

JANUARY						
SUN		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24	31	
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

FEBRUARY						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22			
2	9	16	23			
3	10	17	24			

MARCH						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

APRIL						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24		

MAY						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

JUNE						
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		

JULY						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24	31	
SAT	4	11	18	25		

AUGUST						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			

SEPTEMBER						
	2	9	16	23		
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

OCTOBER						
SUN		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24	31	
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

NOVEMBER						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

DECEMBER						
	2	9	16	23	30	
	3	10	17	24	31	
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

3

JANUARY						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

FEBRUARY						
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22			
2	9	16	23			

MARCH						
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		

APRIL						
SUN		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24		
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

MAY						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			

JUNE						
30	2	9	16	23		
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

JULY						
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			

AUGUST						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

SEPTEMBER						
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			
7	14	21	28			

OCTOBER						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

NOVEMBER						
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		

DECEMBER						
	1	8	15	22	29	
	2	9	16	23	30	
	3	10	17	24	31	
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		

4

JANUARY						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24	31	
SAT	4	11	18	25		

FEBRUARY						
	2	9	16	23		
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22			

MARCH						
30	2	9	16	23		
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

APRIL						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24		
FRI	4	11	18	25		
SAT	5	12	19	26		

MAY						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

JUNE						
	1	8	15	22	29	
	2	9	16	23	30	
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		

JULY						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

AUGUST						
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		

SEPTEMBER						
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			

OCTOBER						
30	2	9	16	23		
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

NOVEMBER						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

DECEMBER						
30	2	9	16	23		
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

5

JANUARY						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24	31	

FEBRUARY						
1		8	15	22		
2		9	16	23		
3		10	17	24		
4		11	18	25		
5		12	19	26		
6		13	20	27		
7		14	21	28		

MARCH						
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			
7	14	21	28			

APRIL						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24		
SAT	4	11	18	25		

MAY						
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		

JUNE						
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			

JULY						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24	31	
SAT	4	11	18	25		

AUGUST						
30	2	9	16	23		
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

SEPTEMBER						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			

OCTOBER						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

NOVEMBER						
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			
7	14	21	28			

DECEMBER						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

6

JANUARY						
SUN	31	3	10	17	24	
MON		4	11	18	25	
TUE		5	12	19	26	
WED		6	13	20	27	
THU		7	14	21	28	
FRI	1	8	15	22	29	
SAT	2	9	16	23	30	

FEBRUARY						
	7	14	21	28		
1	8	15	22			
2	9	16	23			
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			

MARCH						
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			

APRIL						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24		

MAY						
30	2	9	16	23		
31	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

JUNE						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			

JULY						
SUN	31	3	10	17	24	
MON		4	11	18	25	
TUE		5	12	19	26	
WED		6	13	20	27	
THU		7	14	21	28	
FRI	1	8	15	22	29	
SAT	2	9	16	23	30	

AUGUST						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			

SEPTEMBER						
	2	9	16	23	30	
	3	10	17	24		
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		

OCTOBER						
SUN	30	2	9	16	23	
MON	31	3	10	17	24	
TUE		4	11	18	25	
WED		5	12	19	26	
THU		6	13	20	27	
FRI		7	14	21	28	
SAT	1	8	15	22	29	

NOVEMBER						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			

DECEMBER						
	4	11	18	25		
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

7

JANUARY						
SUN	30	2	9	16	23	
MON	31	3	10	17	24	
TUE		4	11	18	25	
WED		5	12	19	26	
THU		6	13	20	27	
FRI		7	14	21	28	
SAT	1	8	15	22	29	

FEBRUARY						
	6	13	20	27		
	7	14	21	28		
1	8	15	22			
2	9	16	23			
3	10	17	24			
4	11	18	25			
5	12	19	26			

MARCH						
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

APRIL						
SUN		3	10	17	24	
MON		4	11	18	25	
TUE		5	12	19	26	
WED		6	13	20	27	
THU		7	14	21	28	
FRI	1	8	15	22	29	
SAT	2	9	16	23	30	

MAY						
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			
7	14	21	28			

JUNE						
	5	12	19	26		
	6	13	20	27		
	7	14	21	28		
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

20th Century Calendar (Contd...)

The number opposite each year in the list below indicates which of the calendars on these two pages is the one for that year. Thus the number opposite 1972 is 14, so Calendar 14 can be used for 1972.

Years divisible by four without remainder are leap years with 366 days instead of 365 (29 days in February instead of 28). However the last year of a century is not a leap year except when divisible by 400.

1901 ... 3	1936 ... 11	1971 ... 6
1902 ... 4	1937 ... 6	1972 ... 14
1903 ... 5	1938 ... 7	1973 ... 2
1904 ... 13	1939 ... 1	1974 ... 3
1905 ... 1	1940 ... 9	1975 ... 4
1906 ... 2	1941 ... 4	1976 ... 12
1907 ... 3	1942 ... 5	1977 ... 7
1908 ... 11	1943 ... 6	1978 ... 1
1909 ... 6	1944 ... 14	1979 ... 2
1910 ... 7	1945 ... 2	1980 ... 10
1911 ... 1	1946 ... 3	1981 ... 5
1912 ... 9	1947 ... 4	1982 ... 6
1913 ... 4	1948 ... 12	1983 ... 7
1914 ... 5	1949 ... 7	1984 ... 8
1915 ... 6	1950 ... 1	1985 ... 3
1916 ... 14	1951 ... 2	1986 ... 4
1917 ... 2	1952 ... 10	1987 ... 5
1918 ... 3	1953 ... 5	1988 ... 13
1919 ... 4	1954 ... 6	1989 ... 1
1920 ... 12	1955 ... 7	1990 ... 2
1921 ... 7	1956 ... 8	1991 ... 3
1922 ... 1	1957 ... 3	1992 ... 11
1923 ... 2	1958 ... 4	1993 ... 6
1924 ... 10	1959 ... 5	1994 ... 7
1925 ... 5	1960 ... 13	1995 ... 1
1926 ... 6	1961 ... 1	1996 ... 9
1927 ... 7	1962 ... 2	1997 ... 4
1928 ... 8	1963 ... 3	1998 ... 5
1929 ... 3	1964 ... 11	1999 ... 6
1930 ... 4	1965 ... 6	2000 ... 14
1931 ... 5	1966 ... 7	
1932 ... 13	1967 ... 1	
1933 ... 1	1968 ... 9	
1934 ... 2	1969 ... 4	
1935 ... 3	1970 ... 5	

2

JANUARY						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24	31	
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

FEBRUARY						
						26
						27
						28
1	8	15	22	29		
2	9	16	23			
3	10	17	24			
4	11	18	25			

MARCH						
						25
						26
						27
						28
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

APRIL						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24		
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

MAY						
						27
						28
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			

JUNE						
						24
						25
						26
						27
						28
						29
						30

JULY						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24	31	
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24	31	
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

4

JANUARY						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

FEBRUARY						
						24
						25
						26
						27
						28
1	8	15	22	29		
2	9	16	23			

MARCH						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

APRIL						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24		
FRI	4	11	18	25		
SAT	5	12	19	26		

MAY						
						27
						28
						29
						30
						31
1	8	15	22	29		
2	9	16	23			

JUNE						
						26
						27
						28
						29
						30
1	8	15	22	29		

JULY						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

AUGUST						
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

7

JANUARY						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24	31	

FEBRUARY						
						26
						27
						28
1	8	15	22	29		
2	9	16	23			
3	10	17	24			
4	11	18	25			

MARCH						
						25
						26
						27
						28
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		

APRIL						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24		

MAY						
						27
						28
						29
						30
						31
1	8	15	22	29		

JUNE						
						27
						28
						29
						30
1	8	15	22	29		

JULY						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24	31	

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN	31	3	10	17	24	
MON		4	11	18	25	
TUE		5	12	19	26	
WED		6	13	20	27	
THU		7	14	21	28	
FRI	1	8	15	22	29	
SAT	2	9	16	23	30	

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

8

JANUARY						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24	31	
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

FEBRUARY						
						26
						27
						28
1	8	15	22	29		
2	9	16	23			
3	10	17	24			
4	11	18	25			

MARCH						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

APRIL						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24		
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

MAY						
						27
						28
						29
						30
						31
1	8	15	22	29		

JUNE						
						24
						25
						26
						27
						28
						29
						30

JULY						
SUN	1	8	15	22	29	
MON	2	9	16	23	30	
TUE	3	10	17	24	31	
WED	4	11	18	25		
THU	5	12	19	26		
FRI	6	13	20	27		
SAT	7	14	21	28		

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

9

JANUARY						
SUN		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24	31	
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

FEBRUARY						
						25
						26
						27
						28
1	8	15	22	29		
2	9	16	23			
3	10	17	24			

MARCH						
31	3	10	17	24		
						25
						26
						27
						28
1	8	15	22	29		
2	9	16	23	30		

APRIL						
						28
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			

MAY						
						26
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		

JUNE						
30	2	9	16	23		
						24
						25
						26
						27
						28
						29

JULY						
		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24	31	
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN		7	14	21	28	
MON	1	8	15	22	29	
TUE	2	9	16	23	30	
WED	3	10	17	24	31	
THU	4	11	18	25		
FRI	5	12	19	26		
SAT	6	13	20	27		

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

10

JANUARY						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

FEBRUARY						
						24
						25
						26
						27
						28
1	8	15	22	29		
2	9	16	23			

MARCH						
30	2	9	16	23		
31	3	10	17	24		
						25
						26
						27
						28
						29

APRIL						
						26
						27
						28
						29
						30
1	8	15	22	29		

MAY						
						24
						25
						26
						27
						28
						29
						30

JUNE						
						29
						30
1	8	15	22	29		

JULY						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN		6	13	20	27	
MON		7	14	21	28	
TUE	1	8	15	22	29	
WED	2	9	16	23	30	
THU	3	10	17	24	31	
FRI	4	11	18	25		
SAT	5	12	19	26		

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

1

JANUARY						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24	31	
SAT	4	11	18	25		

FEBRUARY						
						23
						24
						25
						26
						27
						28
1	8	15	22	29		

MARCH						
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			
7	14	21	28			

APRIL						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

MAY						
						27
						28
						29
						30
						31
1	8	15	22	29		

JUNE						
						24
						25
						26
						27
						28
						29
						30

JULY						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24	31	
SAT	4	11	18	25		

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN		5	12	19	26	
MON		6	13	20	27	
TUE		7	14	21	28	
WED	1	8	15	22	29	
THU	2	9	16	23	30	
FRI	3	10	17	24	31	
SAT	4	11	18	25		

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

12

JANUARY						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24	31	

FEBRUARY						
1	8	15	22	29		
2	9	16	23			
3	10	17	24			
4	11	18	25			
5	12	19	26			
6	13	20	27			
7	14	21	28			

MARCH						
						28
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24	31		
4	11	18	25			
5	12	19	26			
6	13	20	27			

APRIL						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24		

MAY						
						27
						28
						29
						30
						31
1	8	15	22	29		

JUNE						
						27
						28
						29
						30
1	8	15	22	29		

JULY						
SUN		4	11	18	25	
MON		5	12	19	26	
TUE		6	13	20	27	
WED		7	14	21	28	
THU	1	8	15	22	29	
FRI	2	9	16	23	30	
SAT	3	10	17	24	31	

AUGUST						
						26
						27
						28
						29
						30
						31
1	8	15	22	29		

SEPTEMBER						
						27
						28
						29
						30
1	8	15	22	29		
2	9	16	23	30		
3	10	17	24			

OCTOBER						
SUN	31	3	10	17	24	
MON		4	11	18	25	
TUE		5	12	19	26	
WED		6	13	20	27	
THU		7	14	21	28	
FRI	1	8	15	22	29	
SAT	2	9	16	23	30	

NOVEMBER						
						25
						26
						27
						28
						29
						30
1	8	15	22	29		

DECEMBER						
						23
						24
						25
						26
						27
						28
						29

13

JANUARY						
SUN	31	3	10

List of Hotels

(a.c.—awaiting classification)

Agra	*****	Clarks Shiraz.
	**	Grand.
	a. c.	Inn, Kiran International, Mughal, Mumtaz, Lauries.
Ahmedabad	***	Cama.
Allahabad	**	Barnetts.
	*	Royal.
Amritsar	**	Airlines, Guest House, (Mrs. Bhandari's)
Aurangabad	**	Aurangabad.
	a. c.	Rama International, Nandanvan.
Bangalore	***	Bangalore International, West End.
	**	Nandi Hotel's (P) Ltd., Shilton.
	a. c.	Barton Court, Ashoka, Cauveri Continental Harsha, East-West.
Bombay	*****	Deluxe Taj Mahal, Sun-N-Sand.
	****	Ambassador, Fariyas, Natraj, Ritz.
	***	West End, Ascot, Grand, Bombay International, Palace Ajanta. Shalimar.
	**	Astoria, Sea Green, Heritage, Sea Palace.
	*	Parkway, Sea Green South.
	a. c.	Horizon, Holiday Inn, Poonam International, Oberoi Sheraton, President Tajmahal Inter-Continental, Airport Plaza, Centaur.
Baroda	a. c.	Utsav, Express.
Bhavnagar	a. c.	Apollo.
Calcutta	*****	Oberoi Grand, Hindustan International.
	****	Park.
	***	Spences.
	**	Fairlawn.
	*	Carlton, Lytton.
	a. c.	Ritz Continental, Great Eastern, Rutt Deen
Chandigarh	a. c.	Mehfil
	***	Oberoi Mount View.
Calicut	*	Alkapuri Guest House, Beach.
Cochin	****	Malabar.
	***	Sealord, Casino.
	**	Grand, International
Coimbatore	*	Alankar, Guru.
Coonoor	***	Ritz.
	**	Hampton.
Dalhousie	**	Aroma-N-Claire's, Grand View.
Darjeeling	***	Oberoi Mount Everest, Windamere.
	**	Central
Dehradun	**	Kwality.
	*	Majestic, White-house, Prince
Delhi/New Delhi	*****	Deluxe Ashoka, Deluxe Oberoi Intercontinental, Claridge, Imperial Taj Mahal, Mayur.
	****	Ambassador, Diplomat, Rajdoot, Oberoi Maidens.
	***	Broadway, Janpath, Vikram, Nirula, President, York's.
	**	Flora, Metro, Lodhi, Manor, Ranjit. New-Delhi YMCA Tourist, Tera.
	*	Bhagirath Palace.
	a. c.	Akbar, Alka, Qutub.
Dhanbad	**	Savoy.
Ernakulam	**	International.
Faridabad	***	Holiday Inn.
Goa	***	Mandovi.
	**	Zuari.
	*	Solmar, Woodland's.
	a. c.	Keni's. La Paz, Fort Aguada.
	**	Oberoi Palm Beach.
Gopalpur-on-Sea	***	Highland's Park.
Gulmarg	**	Nedou's.

Hotels (Contd...)

Gwalior	•	Gujri Mahal.
Hyderabad	•••	Ritz.
	••	Rock castle.
	a.c.	Bluemoon, Firdaus, Nagarjuna, Annapurana Emerald, Parklane, Sarovar.
Hubli	•	Woodlands.
Indore	•••	Lantern.
Jabalpur	••	Jackson's.
Jaipur	••	Kaisar-i-Hind, Khetri House, LMB.
	a.c.	Clark's Amer, Rambagh Palace, State Hotel.
Jammu	a.c.	Jammu Hotel.
Jullundur	••	Skylark.
Khujraho	a.c.	Chandela, Khujraho.
Kodaikanal	••	Carlton.
Kota	••	Navrang.
Kottayam	•	Ambassador.
Kulti	•••	Kulti.
Kanpur	a.c.	Meghdoot.
Lucknow	•••	Carlton.
	a.c.	Clarks Avadh.
Madras	••••	Connemara.
	•••	Dasaprakash, Ashoka, Imperial, New Victoria, Woodland's.
	••	Claridges, Geetha.
	•	Gupta's Ajanta, Swagath, Queen's Tourist Home's (P) Ltd., Tourist.
	a.c.	Palm Grove, Savera, Atalantic, Blue Diamond.
		Maris, Gokula, Chola, Shrilekha, Taj Coromandel.
Madura	•••	Pandyan.
Mangalore	•••	Moti Mahal, Luxury.
Mussoorie	•	Hakman's Grand, Roan Oke.
	a.c.	Savoy.
Mysore	•••	Krishnarajsagar, Metro-Pole.
	•	Dasaprakash.
	a.c.	Highway, Lalitha Mahal, Rajendra Villa.
Manipal	a.c.	Valley View International.
Matheran	••	Lord's Central.
Mount Abu	•••	Palace.
Nainital	••	Grand, Swiss.
Octacamund	•••	Dasaprakash, Savoy.
Patna	•••	Republic.
	a.c.	Patliputra, Satkar.
Poona	••	Gulmohor.
	•	Shalimar.
	a.c.	Blue Diamond Amir & Amer Al Asian.
Puri	••	South Eastern Railway.
Quilon	••	Neela.
Ranchi	••	South Eastern Railway.
Ranikhet	••	West View.
	•	Moon.
Sialem	••	Dwarka.
Slaserh (Alwar)	•	Rajasthan State.
Secunderabad	•	Percy.
Shillong	•	Pinewood.
Simla	•••	Oberoi Cecil, Oberoi Clarkes.
Srinagar	•••	Nedou's.
	a.c.	Oberoi Palace, Broadway.
Tiruchirapalli	•	Aristo.
Trivandrum	••	Mascot.
	a c.	Belair.
Udaipur	••	Garden, State.
	a.c.	Lake Palace, Laxmi Vilas Palace.
Varanasi	•••	Clarks.
	a.c.	Varanasi.
Vijayawada	••	Manorama.
Visakhapatnam	••••	Sun-N-Sea.
	a.c.	Apsara, Vikram.

List of Holidays

				NOTES
New Year's Day	Friday	January	1	
Lohri	Wednesday	January	13	
Republic Day	Tuesday	January	26	
Basant Panchami	Saturday	January	30	
Shiv Ratri	Monday	February	22	
Holi	Monday	March	8	
Dullandhi	Tuesday	March	9	
Ram Navmi	Friday	April	2	
Baisakhi	Tuesday	April	13	
Half-yearly Bank Closing	Wednesday	June	30	
Id-ul-Fittr	Friday	July	23	
Raksha Bandhan	Wednesday	August	4	
Janamashtami	Thursday	August	12	
Independence Day	Sunday	August	15	
Dussehra	Monday	September	27	
Gandhi Jayanti	Saturday	October	2	
Deepawali	Saturday	October	16	
Muharram	Tuesday	October	19	
Guru Nanak Birthday	Tuesday	November	30	
X-mas Day	Saturday	December	25	
Bank Holiday	Friday	December	31	

(Please compare with List of Govt. Holidays)

Lemma

$N = \frac{1}{n^2} \sum_{i,j=1}^{n} A_{ij}$ if $\sum \alpha_x \neq 0$. If $\sum \alpha_x = 0$, N is not determined & may be given the above value

If S_n admits a certain set of configurations so does the square with elements $A'_{ij} = A_{ij} - \frac{1}{n^2} \sum_{i,j=1}^{n} A_{ij}$

& for this square $N = 0$. Hence when we are dealing with a square for which we have assumed merely that it admits certain configurations there is no loss of generality if we assume $N = 0$.

A configuration in S_n is any linear combination of elements $\sum_{x=1}^{r} \alpha_x A(i_x, j_x)$ the i_x, j_x being members of K (any field of characteristic prime to n),

S_n is said to admit the configuration if

$$\sum_{x=1}^{r} \alpha_x A(i + i_x, j + j_x) = N \sum \alpha_x$$

for all i, j, N being independent of i, j

(1)

1 January -Friday

2 January -Saturday

Duke. Math. Journal (1939).

pp. 705 - 728. Article by Rosser and Walker on "The Algebraic theory of diabolic magic squares"

Kindly sent by T.S. Bhanumurthy on 4/3/83

1. Introduction :- No general theory of magic squares has ever been developed, principally because such a theory would have to involve partition problems of a peculiarly difficult character. We have confined our attention to diabolic squares and their generalisations. A group of transformations which carries any diabolic square of order n into a diabolic square is constructed; for n a prime ≥ 7 it is shown that this is the largest group carrying the most general diabolic square into a diabolic square. The latter result is proved by obtaining for a square of prime order the general solution of the $4n$ linear equations which the elements of the square must satisfy.

The latter part is concerned with the application of these results to normal magic squares with elements $1, \ldots n^2$. The principal result here is that there are precisely 28,800 diabolic squares of order 5, all of an easily constructed type which we have called regular. The existence or non-existence of regular and irregular diabolic squares is demonstrated for squares of all orders.

2. Definitions

~~$A_{11} + A_{12} + A_{13} + A_{14} + A_{15} = \sum_{x=1}^{5} A(1+0x, j+x)$ $A_{1j} + A_{2j} + \cdots + A_{5j}$~~

$A_{31} + A_{32} + \cdots = \sum_{x=1}^{n} A(3+0, 0+x)$

~~$A_{31} + A_{32} + \cdots = \sum_{x=1} A(3+0, 0+x)$~~

$A_{32} + A_{42} + \cdots = \sum_{x=1} (2+x, 2+0)$

Rows are paths	$P(0,1)$
cos	$P(1,0)$
u.d's	$P(1,1)$
l.d's	$P(1,-1)$

$\sum_{x=1}^{n} A(i+ax, j+bx)$ where $(a, b, n) = 1$ are called paths $P(a, b)$.

Notes

A square admitting all four paths $P(0,1), P(1,0), P(1,1), P(1,-1)$ are called diabolic squares (d.s).

A square admitting all the configurations $A_{ij} + A_{i+l} - A_{il} - A_{i+j}$ is called a primitive square (p.s)

If d is a factor of n the configuration

$$\sum_{x,y=1}^{n/d} A(i+dx, j+dy)$$ is called a lattice of order n/d and

denoted by $L(i,j;d)$ or merely $L(d)$. Also the word "lattice" is used and the symbol $L(i,j;d)$ is used to refer to the square of order n/d whose elements are

$$B(x,y) = A(i+dx, j+dy)$$

Theorem 2.1 :- A p.s of order n admits all paths $P(a,b)$ with $(ab,n)=1$
This follows readily from defn of p.s.

Th. 2.2 - Each lattice of a p.s is a p.s

Th. 2.3 :- [Quite a complicated theorem]

Th. 2.4 : A d.s of even order admits $L(2)$

Proof :- In a d.s of order 2 with $N=0$

$$2A_{11} = P(1,1;1,1) + P(1,1;0,1) - P(1,2;1,0) = 0$$

So a d.s of order 2 admits A_{ij}. Hence by Th. 2.3, a d.s of order $2m$ admits $L(i,j;2)$.

Th. 2.5 : A d.s of order $3m$ admits $L(3)$

Proof :- In a d.s of order 3 with $N=0$

$$3A_{11} = P(1,1;1,1) + P(1,1;1,-1) - P(1,2;10) - P(1,3;1,0) = 0$$

So a d.s of order 3 admits A_{ij}. Hence by Th. 2.3, Th. 2.5 follows.

Th. 2.6 :- A d.s of order $4m$ admits $L(i,j;4) + L(i+2,j+2;4)$

Proof :- A d.s of order 4 admits $A(i,j) + A(i+2,j+2)$ (*)

[(*) Ref: Rosser & Walker : on the transformation group for diabolic squares of order four; Bull. Amer. Math. Soc. Vol 44 (1938), pp. 416-420 (Th. 2)
This must be referred to either in C.C.B. Lib or I.I.Sc. Lib & if not available in both places refer & write to Bhanumurthy again] [Later got from C.C.B
Then the theorem follows from Th. 2.3.

Th. 2.7 - Suppose that, for a given d.s, each lattice $L(i,j;d)$ of a square A is diabolic, ~~the order of a path~~ the value of a path $L(i,j,d)$ being N_{ij}. If the square of order d with N_{ij} in the ith row & j^{th} col is diabolic, then A is diabolic

Notes

3. Transformations

Let $T = \left\| \begin{matrix} a & c \\ b & d \end{matrix} \right\|$

be a matrix of integers with $|T|$ prime to n. The transformation

6 January - Wednesday

7 January - Thursday

of the square A is defined to be the square B where

$$B(ai+cj, bi+dj) = A_{ij}$$

This clearly amounts to transforming each vector (i,j) into the vector $(ai+cj, bi+dj)$ by means of T and so has the usual characteristic of a transformation of vectors by matrices. In particular:

(a) the unit transformation by T & then by S is the same as the result of transforming by the matrix product ST.

(b) since $|T|$ is prime to n, one can find an ϵ such that

$$\epsilon(ad-bc) \equiv 1 \pmod{n}$$

and so $T^{-1} = \begin{Vmatrix} d\epsilon & -c\epsilon \\ -b\epsilon & a\epsilon \end{Vmatrix}$

(c) $B(ai+cj+(ae+cf)x, bi+dj+(be+df)x) = A(i+ex, j+fx)$

so that T transforms the path $P(e,f)$ into the path $P(ae+cf, be+df)$

(d) $B(ai+cj+(ax+cy)f, bi+dj+(bx+dy)f) = A(i+fx, j+fy)$

so that T transforms the lattice $L(f)$ into a lattice $L(f)$.

9h.3.1 — The transform of a p.s of order n by

$$T = \begin{Vmatrix} a & c \\ b & d \end{Vmatrix}$$

is diabolic if also $(a^2-b^2)(c^2-d^2)$ is prime to n.

A transformation satisfying the conditions of 9h.3.1 shall be called regular.

From its definition a p.s remains permuting under any permutation of rows & cols among themselves: two p.s. obtainable from one another by such permutations are said to be of the same type. There are therefore $(n!)^2$ p.s of order n of a given type if the elements of the square are all different.

We wish to determine the number of d.s obtainable by regular transformations from p.s of a given type whose elements are all different. We first note that two different p.s of the same type may give the same d.s when acted on by two different regular transformations. For if A is a p.s, T a regular transformation and

$$S = \begin{Vmatrix} \alpha & 0 \\ 0 & \beta \end{Vmatrix} \qquad ((\alpha\beta, n) = 1)$$

Notes

Notes

then SA is primitive of the same type as A, TS is regular and $(TS)A = T(SA)$

Conversely, if A and B are p.s of the same type and $T_1 A = T_2 B$, then

$$A = T_1^{-1} T_2 B$$

8 January -Friday

9 January -Saturday

so that $T_1^{-1} T_2$ transforms rows into rows and cols. into cols. But the only matrix transformations which do this are those of the form of S

Hence $T_1^{-1} T_2 = S$ or $T_2 = T_1 S$. Now if T is regular we can find β, δ such that

$b\beta \equiv d\delta \equiv 1 \pmod{n}$, and then

$$T' = T \begin{Vmatrix} \beta & 0 \\ 0 & \delta \end{Vmatrix} = \begin{Vmatrix} a\beta & c\delta \\ b & d \end{Vmatrix}$$

A regular transformation for which $b \equiv d \equiv 1 \pmod{n}$ shall be called normal. Since obviously no two normal trns can be related by $T_1 = T_2 S$ unless S is the identity matrix, the number of d.s which are regular transforms of p.s of a given type with elements all different is $(n!)^2 \theta(n)$, where $\theta(n)$ is the number of normal transforms of order n. We wish to determine the function $\theta(n)$

Lemma (weigh proves!) 3.1 :- Let $F(x_1, \ldots x_r)$ be any function of integers assuming integral values such that for all n

$F(x_1, \ldots x_r) \equiv F(y_1, \ldots y_r) \pmod{n}$

if $x_1 \equiv y_1 \pmod{n}$.

If $\theta(n)$ denotes the number of sets $(a_1, \ldots a_r)$ $(0 \le a_i < n-1)$ such that $F(a_1, \ldots a_r)$ is prime to n, then

10 Sunday

(a) $\theta(mn) = \theta(m)\,\theta(n)$ if $(m, n) = 1$

(b) $\theta(p^s) = p^{r(s-1)}\,\theta(p)$ if p is prime

Theorem 3.2 :- There are $(n!)^2 \theta(n)$ d.s which are regular transforms of p.s of a given type whose elements are all different, where

(a) $\theta(mn) = \theta(m)\,\theta(n)$ if $(m, n) = 1$

(b) $\theta(p^s) = p^{s-2}(p-3)(p-4)$ if p is an odd prime,

(c) $\theta(2^s) = 0$ [Short proof]

Notes

It is well known that any d.s is transformed into a d.s by rotation through a multiple of 90°, reflection in a diagonal or a cyclic permutation of the rows & cols. The following theorem gives a more complete view of the situation

Notes

11 January -Monday

12 January -Tuesday

Theorem 3.3 'The permutation of the elements of a square of order $n \geq 4$ which takes rows, cols and the two sets of diagonals (r, c, d, d') into (r, c, d, d'), not necessarily in the same order, form a group G which is generated by

$$L = \begin{cases} i' = i - 1 \\ j' = j \end{cases}, \quad M = \begin{cases} i' = i \\ j' = j - 1 \end{cases}, \quad S_\alpha = \begin{Vmatrix} \alpha & 0 \\ 0 & \alpha \end{Vmatrix} \quad ((\alpha, n) = 1)$$

$$O = \begin{Vmatrix} 1 & 0 \\ 0 & -1 \end{Vmatrix}, \quad P = \begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix}; \quad Q = \begin{Vmatrix} 1 & 1 \\ 1 & -1 \end{Vmatrix}$$

if n is odd, and by L, M, S_2, O, P is n is even.

[quite a long proof]

Theorem 3.4: The group G is of order $4n^2\varphi(n)$ is n even and $8n^2\varphi(n)$ if n is odd, $\varphi(n)$ being the number of integers less than n and prime to n.

[Quite a long proof]

Theorem 3.5: The set of $(n!)^2 H(n)$ of Th. 3.2 is invariant under G.

~~4 Squares of prime~~ [Proof not too long]

4. Squares of prime order.

The general order of the linear substitutions by the elements of S_n is called the general square of order n admitting these configurations

Lemma 4.1 – The general square of prime order p permitting all paths is $A_{ij} = N$

This is true for squares of any order. The proof is very complicated and is given in the supplement to the Cornell University Library.

Lemma 4.2 – The number of essential independent parameters in the general square S_n of prime order admitting r sets of paths is $p^2 - (p-1)r$

Lemma 4.3 – In a square of prime order p two paths of different sets have exactly one common element.

Notes

Notes

Theorem 4.1 – If A is defined by

$$A_{ij} = \sum_{k=0}^{p} A^k(i, j) \qquad (0 \leq k \leq p+1)$$

13 January - Wednesday

14 January - Thursday

then A is the general square of prime order p admitting all paths $P(a_i, b_c)$ for which $k+1 \leq l \leq p+1$ [Longer proof]

Theorem 4.2 – A square of prime order admitting all paths but rows and columns is primitive

This theorem is true for squares of any order. Vide "The Supplement"

Any transform of a p.s. shall be called a regular square

Corollary 4.1 – A square of prime order admitting all but two paths is regular –

Corollary 4.2 – Every d.s of order ≥ 5 is regular.

Theorem 4.3 – The general d.s. of order 5 admits rows, cols, diagonals, the configuration

$$A(1,1) + A(1,2) + A(1,5) + A(2,1) + A(5,1)$$

and transforms of this latter under the group G of th. 3.3. It admits no other configuration which is the sum of five elements. The general d.s. S_p of prime order ≥ 7 admits no configuration C consisting of the sum of p elements, except rows, cols, and diagonals

Note: The d.s of order 5 admits the configuration named, the d.s of order 4 admits $L(2)$ and $A_{00} + A_{01} + A_{10} + A_{11}$, the d.s. of order 8 admits $L(0, 0; 4) + L(2, 2; 4)$, and the d.s of order 9 admits $L(3)$. So far as we know, these are the only values of n for which the general d.s. of order n admits a configuration of n elements other than a row, column or diagonal

Corollary 4.3 – The only permutations of the elements of a general d.s of prime order ≥ 7 which preserves the diabolicity are those which

Notes

Notes

15 January -Friday

transform the set of rows, cols and diagonals into rows, cols & diagonals

Corollary 4.4 – The group of permutations of the general d.s of prime order ≥ 7 into a d.s is the group G of Theorem 3.3, of order $8p^2(p-1)$

Theorem 4.4 – If H is the group generated by the transform P of Theorem 3.3 and the permutations of the rows among themselves, and if

$$T = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$$

then any transformation of the form $T\,U\,T^{-1}$, where U is in H, will transform interchanges a d.s of order 5 into a d.s of order 5

5. Numerical squares – Any square S_n whose elements are the integers $1, 2, \ldots n^2$ is called a numerical square (n.s).

In a n.s

$$N = \frac{1}{n^2}\sum_{i,j=1}^{n} A_{ij} = \frac{1}{n^2}\sum_{k=1}^{n^2} = \frac{1}{n^2}\cdot\frac{n^2(n^2+1)}{2} = \frac{n^2+1}{2}$$

16 January -Saturday

17 Sunday

Th. 5.1 – There are no n.d.s of order 3

this follows immediately from Lemma 4.1

Th. 5.2 – There are no n.d.s of order n if $n \equiv 2 \pmod 4$

Let $n = 4m+2$ and suppose S_n is an n.d.s. Then S_n admits the lattice $L(2)$ by Th. 2.4. since the elements of the square are integers $L(2)$ is an integer, however $L(2)$ has $(2m+1)^2$ elements whose average value N is $\frac{1}{2}(n^2+1)$. So

$$L(2) = (2m+1)^2\tfrac{1}{2}(n^2+1)$$

which is not an integer.

Notes

Notes

18 January - Monday

19 January - Tuesday

Study of possible structure of n.p.s of orders other than $3 \neq 4m+2$ in order to show that there are n.d's of all other orders. Quite an amplitude study

Particular example - The normalised n.p.s of order 4 may have the structures

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	2	9	10
3	4	11	12
5	6	13	14
7	8	15	16

1	2	5	6
3	4	7	8
9	10	13	14
11	12	15	16

and no others

[There are Knight's schemes leading to algebraic squares]

[In the above three $A_{ij} + A_{kl} - A_{il} - A_{kj} = 0$ is satisfied]

An important property of a n.p.s is the fact that the elements 1 and 2 always occur in the same row or col. This property shall be of use in the construction of irregular squares. The exact number $\Phi(n)$ of normalised n.p.s of order n is the number of ways of choosing $d_1, d_2 \ldots d_s$ and $f_1, f_2, \ldots f_t$ so that $s = t$ or $t+1$; $d_1 \neq 1, d_s = n$; $f_1 \neq 1, f_t = n$; $d_i \neq d_{i+1}$, $f_i \neq f_{i+1}$; d_i divides d_{i+1} and f_i divides f_{i+1}. This is less than or equal to $(n!)^2$

For prime orders there is exactly one normalised n.p.s. Hence there are exactly two types of n.p.s of prime order, namely those of the same type as A, where

$$A_{ij} = (i-1)p + j$$

and those of the same type as PA (P as in $3n-3-3$). If B is got from PA by a regular transformation T, then B is got from A by the regular transformation TP.

Notes

Notes

20 January - Wednesday

21 January - Thursday

A n.d.s cannot be got either from A or PA by an irregular transformation: Such a transformation of any n.p.s of prime order must carry either rows or cols. into rows, cols or diagonals and since we have rows or cols of a n.p.s have the same sum, the transformed square would not be diabolic. Hence a n.d.s of prime order is regular if and only if it can be got from a n.p.s of type A by a regular transformation and so by Th. 3.2, we have

Th. 5.3 - There are exactly $(p!)^2/(p-3)(p-4)$ regular n.d.s of order p if p is an odd prime.

The regular n.d.s of odd prime order obtained from normalised n.p.s are the so called "Step-squares" the regular n.d.s are the natural generalisation of "step-squares". For instance the generalisation "step squares" discussed by A.H. Frost (Q.J. Math, Vol. 15, 1877, pp. 34-49 - on the general properties of Nasik squares).

Corollary 5.1 - There are precisely 28,800 n.d.s of order 5

Corollary 5.2 - Any T which carries the general diabolic square of order 5 into a diabolic square must have the form $T \bar{U} T'$, where T and $\bar{U}$ are as in Th. 4.4

Notes Notes

[Simple proof]

Since n.p.s of all orders exist, the existence of regular n.d.s of ~~all orders~~ any order prime to 6 follows from Th. 3.2. We shall construct regular n.d.s of all

22 January -Friday

23 January -Saturday

orders except those ~~excluded~~ excluded by Ths. 5.1 & 5.2 ; We shall also construct irregular n.d.s of all orders except those excluded by Ths. 5.1, 5.2 and 5.3

Th. 5.4 – Any ~~order~~ d.s of order 1, 4 or 5 must be regular

Theorem is obvious for $n = 1$. For $n = 5$, use Cor 4.2. For a square of order 4, one can find the general square simply by solving the eqns which say that the square contains rows, cols, and diagonals. The general square is then seen to be the transformation by

$$\begin{Vmatrix} 1 & 2 \\ 2 & 1 \end{Vmatrix}$$

of a p.s

Lemma 5.1 – If m and n are both odd integers greater than or equal to three. it is possible to arrange the integers $0, 1, 2, \ldots mn-1$ in a rectangular array of n rows & m cols in such a way that the sum of the n elements in each col. is the same.

[English proof]

24 Sunday

Theorem 5.5 - There are regular n.d.s of all orders except those excluded by theorems 5.1 and 5.2.

English proof. Three cases considered

Case 1 - n prime to 6, use Th 3.2

Case 2 - $n = 4m$ and let

$$T = \begin{Vmatrix} 2 & -1 \\ -3 & 2 \end{Vmatrix}$$

Notes

Notes

25 January - Monday

26 January - Tuesday

T transforms the paths P(1,2), P(2,3), P(3,5), P(1,1) of A

= [$A_{ij} = na(i) + a(j) = n$. where $a(i) = i$, $a(i+2m) = 4m+1-i$ for $1 \leq i \leq 2m$ and part $a(i \pm 4m) = a(i)$]

onto the p.ths P(0,1), P(1,0), P(1,1) and P(1,-1). Proofs are similar for the P(1,2) ch. Gives a proof for P(2,3)

Case 3 :- $n = 3m$, m odd, $m \geq 3$

Proves existence of n.d.s for this case also

Theorem 5.6 :- There are non-regular n.d.s of all orders except those excluded by Theorems 5.1, 5.2 and 5.4.

Case 1 :- n odd and $(n, 6) = 1$

2	47	38	35	24	20	9
26	16	8	6	46	42	31
49	39	33	29	15	12	4
19	11	7	45	41	30	22
37	29	27	17	14	3	48
10	5	44	36	34	25	21
32	28	18	13	1	43	40

$\leq 11 \cdot 66$ (126)

66

is a non-regular n.d.s. One can see from the discussion preceding Th-5.3 that if it were regular it would have to be the transform of a n.p.s in which one columns contains 1, 8, 15, 22, 29, 36, 43. But these do not constitute a path in the given square, and hence such a transformation is impossible

Notes

We now take $n \geq 11$

Let A be defined by $A_{ij} = (i-1)n + a_j$

where $a_1, a_2, \ldots a_{11} = 2, 7, 4, 9, 6, 11, 1, 8, 3, 10, 5$ respectively

Notes

(*) For details of this see p. 25

27 January - Wednesday

28 January - Thursday

and $a_j = j$ for $12 \leq j \leq n$. [Some argument about square being not regular] *

Case 2 - $n = 4m$, $m \geq 2$. Consider the $2m^2$ pairs $(0, 16m^2-4)$, $(4, 16m^2-8)$, $(8, 16m^2-12)$, ... $(8m^2-4, 8m^2)$. If (A, C) and (B, D) are two of the pairs, then

A+1	B+4	C+1	D+4
C+2	D+3	A+2	B+3
A+4	B+1	C+4	D+1
C+3	D+2	A+3	B+2

(4)

is diabolic with $N = \frac{1}{2}(16m^2+1)$. Fill in the $L(m)$'s of S_{4m} with squares like (4) using different sets of pairs for each square. Then the result is a n-d-s by Th. 2.7. Shows that there are at least

$(192)^{m^2}(2m^2)!$ — (5)

for $m = 2$ ie $n = 8$, this no. $(192)^4(8)!$

n-d-s of order $4m$. Shows further that there are at most $\boxed{8^4 \cdot (8!)^2 \psi(8)}$

(6) — $(4m)^4(4m!)^2\psi(4m)$ regular n-d-s of order $4m$

where $\psi(4m)$ is the number of normalised n.p.s of order $4m$.

Shows finally that (5) exceeds (6)

Case 3 - $n = 9$

1	65	48	41	24	58	81	34	17
57	76	33	13	8	70	53	39	20
68	49	44	27	62	75	28	12	4
80	36	16	3	64	47	40	23	60
52	38	19	56	78	32	15	7	72
30	11	6	67	51	43	26	61	74
42	22	59	79	35	18	2	66	46
14	9	71	54	37	21	55	77	31
25	63	73	29	10	5	69	50	45

Shows that this is a non-regular n-d-s

Notes

Notes

Case 4 : $n = 3m$, $(m, 6) = 1$, $m \geq 5$

If $a(x,1), a(x,2), \ldots a(x,m)$ are a set of m distinct integers lying between 0 and $9m-1$ inclusive, and if A_{ij} is defined by

$$A_{ij} = 9mi + (a, x_j) - 9m + 1$$

gives a square which is irregular

Case 5 : $n = 9m$, m odd, $m \geq 3$

Gives a complicated expn for A_{ij} giving a non-regular n.d.s

If A and B are n.d.s of orders m & n, and we define the lattice $L(i, j, m)$ of a square D of order mn to be

$$C_{xy} = B_{xy} + n(A_{ij} - 1)$$

shows that ~~B~~ D is an n.d.s using Th. 27. If B is irregular then D is irregular. If B is regular, then also D is usually irregular

31 Sunday

~~It has been~~ In "the supplement" is described a means of constructing a n.d.s B of order $3m$ from an n.d.s of order m. It is proved that if A is irregular then B is irregular

6. Conclusion. Possibility of applying the above results can be applied to diabolic cubes.

Applm to latin squares. Shown in "the supplement" that (re. d.l.s)

Notes

(a) There are no d.l.s of order divisible by 2 but not by 8, or by 3 but not by 9.

(b) There are regular d.l.s of order n if & only if $(n, 6) = 1$

(c) There are irregular d.l.s of prime order p if & only if $p > 11$

1 February - Monday (14) 2 February - Tuesday

0, 7, 14, 21, 28, 35, 42
1, 2, 3, 4, 5, 6, 7.

2	5	3	7	3	6	2
5	2	1	6	4	7	3
7	4	5	2	1	5	4
5	4	7	3	6	3	1
2	1	6	3	7	3	6
3	5	2	1	6	4	7
4	7	4	6	1	1	5

0	42	35	28	21	14	7
21	14	7	0	42	35	28
42	35	28	21	14	7	0
14	7	0	42	35	28	21
35	28	21	14	7	0	42
7	0	42	35	28	21	14
28	21	14	7	0	42	35

This is an example of the type of Cauchy's sq irregular squares Root square cyclic & Principal square non-latin & non-cyclic

Take the case $n = 11$ where I could not get one of Cauchy's type [See p. 37]

$A_{ij} = (i-1)n + a_j$ where $a_1, a_2, \ldots a_{11} = 2, 7, 4, 9, 6, 11, 1, 8, 3, 10, 5$.

$A_{1j} = a_j$ $A_{11} =$ 1st row: 2, 7, 4, 9, 6, 11, 1, 8, 3, 10, 5

$A_{2j} = 11 + a_j$ 2nd " 13, 18, 15, 20, 17, 22, 12, 19, 14, 21, 16

$A_{3j} = 22 + a_j$ 3rd " 24, 29, 26, 31, 28, 33, 23, 30, 25, 32, 27

$A_{4j} = 33 + a_j$ 4th "

Something wrong somewhere. This is an n.p.s 319

$i = 11$, $110 + a_j$ & not an n.d.s. How to find the d.s

$110 \times 11 = 1210$
80
1276

from the p.s. I think I should have made more detailed notes about this particular case of $n \geq 11$ before returning the Duke Math. Journal

Notes Notes

3 $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ (15) 4 $\frac{1}{2}\cdot 11\cdot \cancel{122}$ 61 (671)

February - Wednesday February - Thursday

$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Vector $(i, j) \to (ai + cj, bi + dj)$

$(i, j) \to (j, i)$ This ~~was~~

2, 7, 4, 9, 6, 11, 1, 8, 3, 10, 5 →

→ 2 + 13 + 24 + 35 + 46 + 57 + 68 + 79 + 90 + 101 + 112 627 ✗

This ~~[illegible]~~ P does not convert the p.s into even a m.s.

$Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $(i, j) \to (i + j, i - j)$

$(1,1) \to (2,0) \equiv (2,11)$ $2 \to 6$.

~~2 → 16~~ $7 \to$

$(1,2) \to (3,10)$

$(1,3) \to (4,9)$

$(1,4) \to (5,8)$

$(1,5) \to (6,7)$ $-10 \equiv 1$.

$(1,6) \to (7,6)$

$(1,7) \to (8,5)$

$(1,8) \to (9,4)$

$(1,9) \to (10,3)$

$(1,10) \to (11,2)$

$(1,11) \to (1,1$

Write to Shannnuntz re. gap after $n \geq 1$.

Bull. Amer. Math. Soc. Vol. 44 (1938) pp. 416 – 420.

On the transformation group for diabolic ~~squares~~ magic squares of order four by Berkeley Rosser & R. J. Walker

(got from C. C. B. on 18/3/83

Notes Notes

This paper concerns only magic squares of order four and all arguments apply only to such squares.

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

(1)

A diabolic magic square clearly remains diabolic if subject to the following transformations

A. Reflection about the a, f, k, p diagonal

B. Rotation through 90° counter-clockwise

C. Putting the first column last

D. Putting the first row last.

For many purposes it is convenient to consider a diabolic square as mapped on a torus. That is the square is bent and stretched until the top and bottom edges coincide and the right and left edges coincide. Then a diagonal like b, g, l, m is continuous and not broken.

7 Sunday

Theorem! - The four elements of any square of order two of a diabolic magic square of order four mapped on a torus add up to 34.

This includes squares like a, d, m, p or e, h, i, l

Proof: Add together the two main diagonals and the second & third row and subtract the first and fourth columns. This gives f + g + j + k = 34. By use of transformation C and D, this proof can be applied to any square of order two.

8 February -Monday

9 February - Tuesday

Theorem 2 - If a diabolic magic square of order four be mapped on a torus, then the sum of any element and the element which is two distant from it along a diagonal is 17.

Note that if one starts at any element and moves two elements along any diagonal, one always stops at the same element regardless of the direction of the diagonal. Two elements so situated work here be called antipodal.

Proof:- Add together the first and the third rows, then add the first and third columns, and the diagonals a, f, k, p and a, n, k, h and subtract the diagonals e, b, o, l; i, f, c, p; m, j, g, d and i, n, c, h. This gives a + k = 17. By use of transformations C and D this proof can be applied to any pair of antipodal elements.

By the use of these two theorems one can readily see that a diabolic magic square remains diabolic if subject to the following transformation E

E. The transforming (1) into

a	d	h	e
b	c	g	f
n	o	k	j
m	p	l	i

Notes

Notes

Theorem 3 - All diabolic magic squares of order four can be derived from

10 February - Wednesday

11 February - Thursday

a single one by successive applications of A, B, C, D and E.

Proof :- By use of C and D, 1 can be brought into the upper left hand corner. Thus there is no loss of generality in taking $a = 1$, $k = 16$. Then c and d can at most be 14 and 15, so that $c + d \leq 29$. Hence $b \geq 4$. Similarly $c \geq 4$, $d \geq 4$, $e \geq 4$, $m \geq 4$. Since $a + b + c + f = 34$, $a + d + e + h = 34$, $a + b + m + n = 34$ and $a + d + m + p = 34$ by theorem 1, one can similarly get $f \geq 4$, $h \geq 4$, $n \geq 4$, $p \geq 4$. Thus 2 and 3 can only occur in the positions g, j, l and o; if 2 occurs in the o or l position, it can be brought into the j position by one or two applications of E. If 2 occurs in the j position, it can be brought into the g position by an application of A. So one can take $g = 2$, $m = 15$ without loss of generality. If 3 occurs in the l or j position, it can be brought to the g position by one or two applications of E. Hence one takes $o = 3$, $e = 14$, $i = 4$, $c = 13$. Now $d = 20 - b$ and by theorem 2, $l = 17 - b$, $j = b - 3$. By theorem 1, $f = 19 - b$ so that $h = b - 1$, $n = 18 - b$, $p = b - 2$. Hence we have to find values of b such that $b - 3$, $b - 2$, $b - 1$, $17 - b$, $18 - b$, $19 - b$ and $20 - b$ are 5, 6, 7, 8, 9, 10, 11 and 12 in some order. Clearly

12 February -Friday

13 February -Saturday

$b = 8$ or $b = 12$, with $b = 8$ we get

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

[We that this is exactly Narayana Pandithas square obtained by the knight-move method Remarkable]

By applying C, B, AND A successively to this we get the square (that would) result from putting $b = 12$. Thus all the diabolic squares can be obtained from the one shown by successive applications of A, B, C, D and E.

Theorem 4 — There are 384 diabolic magic squares of order four.

14 Sunday

Proof :- XY means HM first by Y & then by X. Then

(a) $BA = AB^3$, (b) $CA = AD$, (c) $DA = AC$, (d) $CB = BD$

(e) $DB = BC^3$, (f) $DC = CD$, (g) $EA = AB^2CDE$

(h) $E^2A = B^2CDE^2$, (i) $EB = B^3CDE$, (j) $E^2B = ABCDE^2$

(k) $EC = C^3E^2$, (l) $E^2C = B^2CD^2E$, (m) $ED = ACE^2$

Notes

(n) $E^2D = AB^2DE$, (o) $A^2 = B^4 = C^4 = D^4 = E^3 =$ the identical HM

These relations were obtained by inspection and can be easily verified

By means of (a) – (o) one can, in any product of A, B, C, D, E, get

Notes

15 February - Monday

16 February - Tuesday

all the E's to the right, then all the A's to the left, then all the B's next to the A's, and finally all C's next to the B's; so that any product of A, B, C, D, E is equal to a product of the form $A^\alpha B^\beta C^\gamma D^\delta E^\epsilon$.

Moreover A, B, C and D all carry rows into rows and cols. into cols. or rows into cols. and cols. into rows, so that neither E nor E^2 is a product of A, B, C and D. Moreover, if a square is mapped on a torus, B, C and D all leave the orientation unchanged, so that A is not a product of B, C and D. Clearly B, C and D are independent. Hence $A^\alpha B^\beta C^\gamma D^\delta E^\epsilon = A^a B^b C^c D^d E^e$ if and only if $\alpha \equiv a\ (2)$, $\beta \equiv b\ (4)$, $\gamma \equiv c\ (4)$, $\delta \equiv d\ (4)$ and $\epsilon \equiv e\ (3)$. Hence the order of the group generated by A, B, C, D and E is

$$2 \times 4 \times 4 \times 4 \times 3 = 384$$

As each transformation is a permutation of the 16 elements of the square, different transformations cannot yield the same square. So there are 384 diabolic magic squares.

(Points out some error made by Dehmer who gives the number as 48 in Bull. Amer. Math. Soc., Vol. 39 (1933), p. 981.)

Notes

The present count was checked against Frenicle's list of magic squares. In so doing, we discovered the following startling result (?):—

Notes

17 February - Wednesday

18 February - Thursday

If there are two antipodal elements of a magic square of order four which add up to 17, then the magic square is diabolic

[What is startling about this? Looking at B & J's appendix one finds that in all the 48 magic squares all the antipodal elements add up to 17]

{Note: H.S.M. Coxeter has suggested that the group generated by A, B, C, D & E may be isomorphic with the symmetry group of the Cartesian frame in four dimensions, as the two groups are of the same order. How this is so can be shown as follows. The elements $R_1 = ABCE$, $R_2 = AB^3D$, $R_3 = BCE$, $R_4 = AB$ will generate our group since $A = R_1R_3$, $B = R_1R_3R_4$, $C = R_4R_3R_1R_2R_1R_3$, $D = R_3R_4R_3R_2$, $E = R_1R_3R_2R_3$ as may be verified by the relations (a) - (o). The R_i satisfy the relations

$$R_i^2 = (R_1R_3)^2 = (R_1R_4)^2 = (R_2R_4)^2 = (R_1R_2)^3 = (R_2R_3)^3 = (R_3R_4)^4 = 1,$$

hence our group is a factor group of the above one-to-one symmetry group (Coxeter J. Lond. Math. Soc. Vol. 10 (1935), pp 21-25); the group in question is designated by $[3^2, 4]$. Since the two groups have the same order they are isomorphic}

Notes

The defining conditions of a magic square plus the simple condition that a particular pair of antipodal elements add up to 17 gives ten independent equations in the 16 elements of the square, as contrasted with twelve

Notes

19 February -Friday

20 February -Saturday

independent equations arising from the defining conditions for a diabolic magic square. That the two sets of equations are equivalent is due to the diophantine condition that the elements must be 1, 2, ... 16 in some order.

If we weaken the requirements still further and only ask that a square be magic in the rows and cols., then a pair of antipodal elements can add up to 17 without the square being diabolic. This is illustrated by

1	15	4	14
12	2	9	11
8	7	16	3
13	10	5	6

which is magic on rows and columns, but not in diagonals, and which has $a+K = e+0 = 17$ ie 21 Sunday $1+16 = 12+5 = 17$

An analogous ~~problem~~ treatment of the problem of all diabolic magic squares is given by Kraitchik on p. 167 of his book, La Mathematiques des Jeux, where he shows that all diabolic magic squares can be derived by successive applications of A, B, C and D from three particular ones which he gives

Notes

Notes

22 February - Monday

23 February - Tuesday

Re. The antipodal property for nasik squares I looked into all the 48 cases in B & J's Appendix and found the property holding for all of them. To get this to derive this property: —

From Andrews pp. 136 - 137

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

(1) $f+g+j+k = 34$
(2) $a+d+m+p = 34$
(3) $b+c+n+o = e+h+i+l = 34$
(4) $a+d = n+o$ etc.
(5) $e+h = j+k$, etc
(6) $m+d = f+k$ etc.

$a+n+k+h = 34$ | $d+e+j+o = 34$
$e+b+o+l = 34$ | $c+h+i+n = 34$
$i+f+c+p = 34$ | $b+g+l+m = 34$

from nasik property

1st row + 3rd row + 1st col + 3rd col + diagonals minus 2 other diagonals

$b+c = a+k$

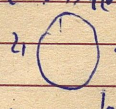
$= a+k+c+d+i+j+k+l+a+l+i+m+c+g+k+o+a+f+k+p$
$+a+n+k+h-e-b-o-l-i-f-l-p-m-j-g-d-l-n-c-k = 68$

$4(a+k) = 68$ or $a+k = 17$ // This is Rosser & Walker's proof.

We must find a simple proof.

$a+f+k+p = a+n+k+h$
$i+f+c+p = a+f+k+p$
$a+k = i+c$ | $a+d = n+o$
$f+p = n+h$ | $a+k = i+c$
$f+p = n+h$
$e+o = m+g$

$a+b+c+d$
$+e+f+g+k+l = 2\mathfrak{S}$
$f+p = n+h$ $(a+k)+(b+l)+(i+c)+(j+d) = 2\mathfrak{S}$
$a+k = i+c$
$a+k = m+g-e$



$a+e+i+m+c+g+k+o = 2\mathfrak{S}$
$(a+k)+(e+o)+(i+c)+(g+m) = 2\mathfrak{S}$
$(a+k)+(i+c)+(c+o)+(g+m) = 2\mathfrak{S}$

Notes

e	f	g	h
i	j	k	l
m	n	o	p
a	b	c	d

k	l	i	j
o	p	m	n
c	d	a	b
g	h	e	f

i	j	k	l
m	n	o	p
a	b	c	d
e	f	g	h

b	c	d	a
f	g	h	e
j	k	l	i
n	o	p	m

Notes

24
February - Wednesday

25
February - Thursday

If we assume Property E of R & W, the antifodal property is an immediate consequence

a	d	h	e
b	c	g	f
n	o	k	i
m	p	l	i

Because a+k = n+h

and n+h+c+i = a+k

a+k = n+h

& from original square n+h = f+p

ie a+k = f+p = ½(a+f+k+p) = 17

Which E is identical from anti-fodal Property & R&W's theorem 1.

i+c = n+h

i+n+c+k = 34.

i+n+c+h = 0

2i+2k = 34

i+c = 17

a+n+k+h = 34

a+f+k+p = 34

n+h = f+p

i+f+c+p = 34.

i+n+c+h = 34.

b+c+n+o = e+i+h+l = 34

b+e+o+l = i+n+c+h = 34

c+n = e+l

b+o = i+h

e+h+i+l = 34

i+c = a+k

2(e+h) = 17

(b+o)+(c+n)+(e+l) = 2(i+h)+c+n = 17.

+(c+l)+(c+n)

b+o+e+l = 34

i+j+m+n = (i+n)+(j+m) = 34-(c+h)+34-(g+d)

= 68-(c+d+g+h)

e+f+i+j = 34-(h+l)

+34-(g+k)

a+f+e+b = 34

f+g+j+k = 34

k+l+o+p = 34

Notes

Notes

26 February -Friday

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

i	j	k	l
m	n	o	p
a	b	c	d
e	f	g	h

27 February -Saturday

k	l	i	j
o	p	m	n
c	d	a	b
g	h	e	f

Bhanumurthy was good enough to send today (24/3/83) a xerox copy of the details re. case $n \equiv 11 \pmod{12}$

Define B as the square with - $B(1,4)$, $B(4,4)$, $B(4,2)$, $B(3,4)$, $B(3,9)$

$B(2,11)$, $B(2,9)$, $B(5,9)$, $B(\frac{1}{2}(n+7),2)$, $B(\frac{1}{2}(n+5),4)$,
$B(\frac{1}{2}(n+11),2)$, $B(\frac{1}{2}(n+3),9)$, $B(\frac{1}{2}(n+9),9)$, $B(\frac{1}{2}(n+9),7)$
$B(\frac{1}{2}(n+5),2)$, $B(\frac{1}{2}(n+7),9)$ equal to $+1$; $B(3,3)$, $B(2,5)$

$B(2,3)$, $B(5,3)$, $B(1,10)$, $B(4,10)$, $B(4,8)$, $B(3,10)$,

$B(\frac{1}{2}(n+3),3)$, $B(\frac{1}{2}(n+9),3)$, $B(\frac{1}{2}(n+9),1)$, $B(\frac{1}{2}(n+7),3)$

$B(\frac{1}{2}(n+7),8)$, $B(\frac{1}{2}(n+5),10)$, $B(\frac{1}{2}(n+5),8)$, and $B(\frac{1}{2}(n+11),8)$

equal to -1, and all other elements equal to zero. Put

28 Sunday

$$T = \begin{Vmatrix} 2 & 1 \\ 1 & 2 \end{Vmatrix}$$

Then TA is a n.d.s, since T is regular and A is a n.p.s. Also it can be readily seen that each row, col. or diagonal of TB contains as many $+1$'s as -1's, so that TB is diabolic. Hence $TA + TB$ is diabolic. Hence it is an n.d.s if it contains each of the elements $1, 2, \ldots n^2$. However $TA + TB = T(A+B)$ and $(A+B)$ can be readily seen to be the result of interchanging pairs of elements A_{ij} and $A_{i,j+6}$ in A.

Notes

Notes

1 March - Monday

2 March - Tuesday

So ~~T(A+B)~~ TA+TB is a n.d.s. If T(A+B) were regular A+B would also be regular. We have $A_{17}+B_{17}=1$, $A_{11}+B_{11}=2$; these elements lie on the unique path $P(1,1;0,6) = P(0,1;0,1)$ since $7-1=6$ is prime to n and so any transform of A+B into a n.p.s would take the entire first row of A+B into a row. Since this row consists of the numbers 1, .. n, the number n+1 must lie in the same column of the transformed square as 1. Since $n+1 = A_{27}+B_{27}$ lies in the same column of A+B as 1, the transformation would take rows into rows and columns into columns so that if the transformed square were primitive, A+B would be primitive. But this is not obviously so.

[So the problem reduces to (i) finding T.A and examining if it be a regular n.d.s
(ii) form T(A+B) which is not a regular n.d.s.]

$$T = \begin{Vmatrix} 2 & 1 \\ 1 & 2 \end{Vmatrix}$$

$A_{ij} \to B(ai+cj,\ bi+dj)$

$\to B(2i+j,\ i+2j)$

$ij \to (2i+j,\ i+2j)$

$(6,1) \to (2,8)$

$4i+2j = 2k$
$i+2j = l$
$2i+j = 1$
$i+2j = 1$
$2i+4j = 2$
$3j = 1$
$j = 4$
$i = 4$
$2i = 8$

$2i+j = k$
$i+2j = l$
$2i+4j = 2l$
$3j = 2l-k$
$3i = 2k-l$

$k=1, l=7$
$3j = 13$
$3i = -5$
$i=2, j=8$

3 March - Wednesday

$(2i+j,\ i+2j)$

$(1,1) \to (3,3)$ ✓
$(1,2) \to (4,5)$
$(1,3) \to (5,7)$
$(1,4) \to (6,9)$
$(1,5) \to (7,11)$
$(1,6) \to (8,13) \equiv (8,2)$
$(1,7) \to (9,15) \equiv (9,4)$
$(1,8) \to (10,17) \equiv (10,6)$
$(1,9) \to (11,19) \equiv (11,8)$
$(1,10) \to (12,21) \equiv (1,10)$ ✓
$(1,11) \to (13,23) \equiv (2,1)$

$(2,1) \to (5,4)$
$(2,2) \to (6,6)$ ✓
$(2,3) \to (7,8)$
$(2,4) \to (8,10)$
$(2,5) \to (9,12) \equiv (9,1)$
$(2,6) \to (10,14) \equiv (10,3)$
$(2,7) \to (11,16) \equiv (11,5)$
$(2,8) \to (12,18) \equiv (1,7)$
$(2,9) \to (13,20) \equiv (2,9)$ ✓
$(2,10) \to (14,22) \equiv (3,11)$
$(2,11) \to (15,24) \equiv (4,2)$

4 March - Thursday

$(3,1) \to (7,5)$
$(3,2) \to (8,7)$
$(3,3) \to (9,9)$ ✓
$(3,4) \to (10,11)$
$(3,5) \to (11,2)$
$(3,6) \to (1,4)$
$(3,7) \to (2,6)$
$(3,8) \to (3,8)$ ✓
$(3,9) \to (4,10)$
$(3,10) \to (5,1)$
$(3,11) \to (6,3)$

$(4,1) \to (9,6)$
$(4,2) \to (10,8)$
$(4,3) \to (11,10)$
$(4,4) \to (1,1)$ ✓
$(4,5) \to (2,3)$
$(4,6) \to (3,5)$
$(4,7) \to (4,7)$ ✓
$(4,8) \to (5,9)$
$(4,9) \to (6,11)$
$(4,10) \to (7,2)$
$(4,11) \to (8,4)$

$(5,1) \to (11,7)$
$(5,2) \to (1,9)$
$(5,3) \to (2,11)$
$(5,4) \to (3,2)$
$(5,5) \to (4,4)$ ✓
$(5,6) \to (5,6)$ ✓
$(5,7) \to (6,8)$
$(5,8) \to (7,10)$
$(5,9) \to (8,1)$
$(5,10) \to (9,3)$
$(5,11) \to (10,5)$

$(6,1) \to (2,8)$
$(6,2) \to (3,10)$
$(6,3) \to (4,1)$
$(6,4) \to (5,3)$
$(6,5) \to (6,5)$ ✓
$(6,6) \to (7,7)$ ✓
$(6,7) \to (8,9)$
$(6,8) \to (9,11)$
$(6,9) \to (10,2)$
$(6,10) \to (11,4)$
$(6,11) \to (1,6)$

$(7,1) \to (4,9)$
$(7,2) \to (5,11)$
$(7,3) \to (6,2)$
$(7,4) \to (7,4)$ ✓
$(7,5) \to (8,6)$
$(7,6) \to (9,8)$
$(7,7) \to (10,10)$ ✓
$(7,8) \to (11,1)$
$(7,9) \to (1,3)$
$(7,10) \to (2,5)$
$(7,11) \to (3,7)$

$(8,1) \to (6,10)$
$(8,2) \to (7,1)$
$(8,3) \to (8,3)$ ✓
$(8,4) \to (9,5)$
$(8,5) \to (10,7)$
$(8,6) \to (11,9)$
$(8,7) \to (1,11)$
$(8,8) \to (2,2)$ ✓
$(8,9) \to (3,4)$
$(8,10) \to (4,6)$
$(8,11) \to (5,8)$

$(9,1) \to (8,11)$
$(9,2) \to (9,2)$ ✓
$(9,3) \to (10,4)$
$(9,4) \to (11,6)$
$(9,5) \to (1,8)$
$(9,6) \to (2,10)$
$(9,7) \to (3,1)$
$(9,8) \to (4,3)$
$(9,9) \to (5,5)$ ✓
$(9,10) \to (6,7)$
$(9,11) \to (7,9)$

$(10,1) \to (10,1)$ ✓
$(10,2) \to (11,3)$
$(10,3) \to (1,5)$
$(10,4) \to (2,7)$
$(10,5) \to (3,9)$
$(10,6) \to (4,11)$
$(10,7) \to (5,2)$
$(10,8) \to (6,4)$
$(10,9) \to (7,6)$
$(10,10) \to (8,8)$
$(10,11) \to (9,10)$

$(11,1) \to (1,2)$
$(11,2) \to (2,4)$
$(11,3) \to (3,6)$
$(11,4) \to (4,8)$
$(11,5) \to (5,10)$
$(11,6) \to (6,1)$
$(11,7) \to (7,3)$
$(11,8) \to (8,5)$
$(11,9) \to (9,7)$
$(11,10) \to (10,9)$
$(11,11) \to (11,11)$ ✓✓

Notes

Notes

5 March - Friday

6 March - Saturday

$A_{11} \to A_{17}$
$A_{12} \to A_{18}$

99
11
110

	1	2	3	4	5	6	7	8	9	10	11
1	2	7	4	9	6	11	1	8	3	10	5
2	13	18	15	20	17	22	12	19	14	21	16
3	24	29	26	31	28	33	23	30	25	32	27
4	35	40	37	42	39	44	34	41	36	43	38
5	46	51	48	53	50	55	45	52	47	54	49
6	57	62	59	64	61	66	56	63	58	65	60
7	68	73	70	75	72	77	67	74	69	76	71
8	79	84	81	86	83	88	78	85	80	87	82
9	90	95	92	97	94	99	89	96	91	98	93
10	101	106	103	108	105	110	100	107	102	109	104
11	112	117	114	119	116	121	111	118	113	120	115

A =

7 Sunday

Principal square of TA is

4, 6, 1, 3, 5, 7, 9, 11, 8, 10, 2
9, 11, 8, 10, 2, 4, 6, 1, 3, 5, 7

Row square of TA is

22, 33, 44, 55, 66, 77, 88, 99, 110, 0, 11
44, 55, 66, 77, 88, 99, 110, 0, 11, 22, 33

Both offer regular with steps (5,6) and (9,2) respectively. So TA is a regular n-d-s.

So I have to find the T(A+B) of trails to find an irregular n-d-s of order 11.

Notes

Notes

8
March - Monday

9
March - Tuesday

26	39	45	58	71	84	97	110	118	10	13	671 ✓
53	66	74	87	90	103	116	1	14 25	27 28	40	671 ✓
72	78	91	104	117	9	22	30	43	46	59	671 ✓
99	107	120	2	15	28	34	47	60	73	86	671 ✓
111	3	16	29	42	55	63	76	79	92	105	671 ✓
19	32	35	48	61	67	80	93	106	119	111	671 ✓
36	49	62	75	88	96	109	112	4	17	23	671 ✓
65	68	81	94	100	113	5	18	31	44	52	671 ✓
82	95	108	121	8	21	24	37	50	56	69	671 ✓
101	114	6	12	25	38	51	64	77	85	98	671 ✓
7	20	33	41	54	57	70	83	89	102	115	671 ✓
671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓	671 ✓

TA ≠

X

To verify

All upper diagonals also sum up to 671

" lower diagonals also " " 671

So this is an n.d.s all right

Next is to find whether it is regular or not

Split up into primefold and squares.

(See previous page)

Notes

Notes

Notes

10 March - Wednesday

$A_{11} \to A_{17}$
$A_{12} \to A_{18}$
$A_{13} \to A_{19}$
$A_{14} \to A_{1,10}$
$A_{15} \to A_{111}$
$A_{16} \to A_{11}$.

11 March - Thursday

$B_{14} = B_{44} = B_{42} = B_{34} = B_{39} = B_{2,11} = B_{29} = B_{59} = B_{92} = B_{84} = B_{82} = B_{99} = 1$

$B_{33} = B_{25} = B_{23} = B_{53} = B_{1,10} = B_{4,10} = B_{48} = B_{3,10} = B_{13} = B_{10,3} = B_{10,1} = B_{93} = B_{98}$

$= B_{8,10} = B_{88} = B_{11,8} = -1$ & all the $= 0$.

$A+B =$

	1	2	3	4	5	6	7	8	9	10	11
1	1	8	3	10	5	11	2	7	4	9	6
2	12	19	14	21	16	22	13	18	15	20	17
3	23	30	25	32	27	33	24	29	26	31	28
4	34	41	36	43	38	44	35	40	37	42	39
5	45	52	47	54	49	55	46	51	48	53	50
6	56	63	58	65	60	66	57	62	59	64	61
7	67	74	69	76	71	77	68	73	70	75	72
8	78	85	80	87	82	88	79	84	81	86	83
9	89	96	91	98	93	99	90	95	92	97	94
10	100	107	102	109	104	110	101	106	103	108	105
11	111	118	113	120	115	121	112	117	114	119	116

Notes

Notes

(31)

12 March - Friday

13 March - Saturday

99
11
110

25	38	46	59	72	85	98	110	117	9	12
54	66	73	86	89	102	115	2	15	28	41
71	79	92	105	118	10	22	29	42	45	58
99	106	119	1	14	27	35	48	61	74	87
112	4	17	30	43	55	62	75	78	91	104
18	31	34	47	60	68	81	94	107	120	11
37	50	63	76	88	95	108	111	3	16	24
64	67	80	93	101	114	6	19	32	44	51
83	96	109	121	7	20	23	36	49	57	70
100	113	5	13	26	39	52	65	77	84	97
8	21	33	40	53	56	69	82	90	103	116

T(A+B) =

14 Sunday

So It is an n.d.s all right. Splitting up into (P) & (R)

3, 5, 2, 4, 6, 8, 10, 11, 7, 9, 1
10, 11, 7, 9, 1, 3, 5, 2, 4, 6, 8
5, 2, 4, 6, 8, 10, 11, 7, 9, 1, 3

22, 33, 44, 55, 66, 77, 88, 99, 110, 0, 11
44, 55, 66, 77, 88, 99, 110, 0, 11, 22, 33
66, 77, 88, 99, 110, 0, 11, 22, 33, 44, 55

So this appears regular just like TA.

Notes

Notes

Notes

Our A + B is primitive, but R & N say that A+B is not primitive

$B_{14} = B_{44} = B_{42} = B_{34} = B_{39} = B_{2,11} = B_{29} = B_{59} = B_{92} = B_{84} = B_{82} = B_{99} = B_{11,2} = B_{79} = B_{10,9} = B_{10,7} = 1$

$B_{33} = B_{25} = B_{23} = B_{53} = B_{1,10} = B_{4,10} = B_{48} = B_{3,10} = B_{73} = B_{10,3} = B_{10,1} = B_{93} = B_{98} = B_{8,10} = B_{88} = B_{11,8} = -1$

15 March - Monday

16 March - Tuesday

32

& all else = 0.

	1	2	3	4	5	6	7	8	9	10	11
1	2	7	4	10	6	11	1	8	3	9	5
2	13	18	14	20	16	22	12	19	15	21	17
3	24	29	25	32	28	33	23	30	26	31	27
4	35	41	37	43	39	44	34	40	36	42	38
5	46	51	47	53	50	55	45	52	48	54	49
6	57	62	59	64	61	66	56	63	58	65	60
7	68	73	69	75	72	77	67	74	70	76	71
8	79	85	81	87	83	88	78	84	80	86	82
9	90	96	91	97	94	99	89	95	92	98	93
10	100	106	102	108	105	110	101	107	103	109	104
11	112	118	114	119	116	121	111	117	113	120	115

A+B =

2-7-4-10-6-11-1-8-3-9-5

This is not an n.p.s for as
85+91 ≠ 96+81
95+103 ≠ 107+92
41+50 ≠ 51+39
etc

$B_{39} = 1$, $B_{34} = 1$, $B_{33} = -1$, $B_{3,10} = -1$, $B_{42} = 1$, $B_{44} = 1$, $B_{4,10} = -1$, $B_{4,8} = -1$

$B_{59} = 1$, $B_{53} = -1$, NO B_{6j}, $B_{79} = 1$, $B_{73} = -1$

$B_{82} = 1$, $B_{84} = 1$, $B_{88} = -1$, $B_{8,10} = -1$

$B_{92} = 1$, $B_{99} = 1$, $B_{98} = -1$, $B_{93} = -1$ || $B_{10,7} = 1$, $B_{10,9} = 1$, $B_{10,1} = -1$, $B_{10,3} = -1$

Notes

$B_{11,2} = 1$, $B_{11,5} = -1$.

The above appears to be the correct & not the one on p. 30. obtaining A+B from A by changing A_{ij} to $A_{i,j+6}$ does not apply to all pairs of elements. only 16 pairs. thus A+B is not a n.p.s.

12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22
~~20, 19, 14, 21~~
13, 18, 15, 21, 17, 22, 12, 19, 14, 20, 16 (33).

9, 6, 1, 3, 5, 8, 9, 11, 7, 9, 2

9, 0, 8, 9, 2, 3, 6, 1, 4, 5, 8

669

17 March - Wednesday

18 March - Thursday

Now to find T(A+B).

	1	2	3	4	5	6	7	8	9	10	11	
1	25	39	45	58	71	85	97	110	117	9	13	669
2	53	66	74	86	98	102	116	1	15	27	41	671
3	72	78	92	104	118	10	22	30	42	46	59	673
4	99	107	120	2	14	28	34	48	60	73	87	672
5	111	3	17	29	43	55	63	76	79	91	105	672
6	19	31	35	47	61	67	80	93	106	119	11	669
7	36	49	62	75	88	95	109	112	4	16	23	669
8	65	68	81	94	101	113	5	18	32	44	52	673
9	82	96	108	121	8	21	24	37	50	56	70	673
10	100	114	6	12	26	38	51	64	77	84	98	670
11	7	20	33	40	54	57	69	83	89	103	115	670

T(A+B) =

There must be some error in R.A.N's assignment of the B's

~~$B_{33} = B_{99} = 0$~~

$B_{12}, B_{44}, B_{42}, B_{34}, B_{39}, B_{2,11}, B_{29}, B_{59}$ to be +1.

$B_{92}, B_{84}, B_{11,2}, B_{79}, B_{10,9}, B_{10,7}, B_{82}, B_{99}$ " +1

$B_{33}, B_{25}, B_{23}, B_{53}, B_{1,10}, B_{4,10}, B_{48}, B_{3,10}$ to be −1

$B_{73}, B_{10,3}, B_{10,1}, B_{93}, B_{98}, B_{8,10}, B_{88}, B_{11,8}$ to be −1

Notes

Notes

~~Either we interchange the letter & B's, then we have B equal to~~ (See p. 36)

$A_{44} + A_{11,1} + A_{1,10} + A_{28} + A_{36} + A_{52} + A_{6,11} + A_{79} + A_{87} + A_{95} + A_{103}$

$43 + 112 + 99 + 109 + 33 + 51 + 60 + 70 + 78 + 94 + 102 = 671$

(3,3), (5,4), (7,5), (9,6), (11,7), (2,8), (4,9), (6,10), (8,11), (10,1), (1,2)

19
March -Friday

(34)

20
March -Saturday

	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	1	0	0	0	0	0	-1	0
2	0	0	-1	0	-1	0	0	0	1	0	1
3	0	0	-1	1	0	0	0	0	1	-1	0
4	0	1	0	1	0	0	0	-1	0	-1	0
5	0	0	-1	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	0	0	0	0	0
7	0	0	-1	0	0	0	0	0	1	0	0
8	0	1	0	1	0	0	0	-1	0	-1	0
9	0	1	-1	0	0	0	0	-1	1	0	0
10	-1	0	-1	0	0	0	1	0	1	0	0
11	0	1	0	0	0	0	0	-1	0	0	0

B =

	1	2	3	4	5	6	7	8	9	10	11	
1	-1	0	0	0	0	1	0	0	-1	-1	0	-2
2	0	0	0	-1	0	0	0	0	1	0	1	0
3	0	0	1	0	1	1	0	0	-1	0	0	2
4	0	0	0	0	-1	0	0	+1	0	0	1	1
5	0	0	1	0	1	0	0	0	0	-1	0	1
6	0	-1	0	-1	0	0	0	0	0	0	0	-2
7	0	0	0	0	0	+1	0	0	0	-1	0	-2
8	0	0	0	0	1	0	0	0	1	0	0	2
9	0	1	0	0	0	0	0	0	0	0	0	2
10	-1	0	0	0	1	0	-1	0	0	0	0	-1
11	0	0	0	-1	0	0	-1	0	0	1	0	-1
	-2	0	2	-3	3	0	-2	1	0	-2	3	1

TB =

20 Sunday

$B_{5,9}$, $B_{8,4}$, $B_{10,1}$, $B_{5,2}$, $B_{4,8}$, $B_{7,3}$

$B_{2,4}$, $B_{2,11}$, $B_{4,4}$ to be zero

$B_{11,8}$, $B_{1,10}$, $B_{3,3}$, $B_{3,10}$, $B_{5,3}$, $B_{1,8}$, $B_{2,7}$ to be 1

$B_{9,9}$, $B_{11,2}$, $B_{1,4}$, $B_{10,7}$, $B_{3,4}$, $B_{9,2}$, $B_{9,11}$ to be -1

$B_{11,8}$ or $B_{3,3}$ should be +1 } 1

$B_{11,8}$ or $B_{3,10}$ should be +1

~~$B_{11,8}$ or $B_{3,10}$ to be +1~~

$B_{9,9}$ or $B_{11,2}$ or $B_{1,4}$ to be -1 } 3

~~$B_{9,9}$~~

$B_{5,9}$ or $B_{8,4}$ to be 0 — 4

$B_{2,11}$ or $B_{4,4}$ to be zero — 5

$B_{3,10}$ or $B_{5,3}$ to be +1 — 6

$B_{1,8}$ or $B_{2,5}$ to be +1 — 7

~~$B_{11,4}$ or $B_{4,5}$ to be +1~~

$B_{10,7}$ or $B_{3,4}$ to be -1 — 8

$B_{9,2}$ or $B_{9,11}$ to be -1 — 9

~~$B_{10,1}$ or $B_{5,2}$~~

$B_{10,1}$ or $B_{5,2}$ to be 0 — 10

$B_{4,8}$ or $B_{7,3}$ to be 0 — 11

Notes

Notes

22
March - Monday

$B_{11,8}$ and $B_{11,2}$ to interchange signs

$B_{1,10}$ and $B_{1,4}$ "

$B_{3,2}$ and $B_{3,10}$ "

B_{98} and B_{92} "

	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	-1	0	0	0	0	0	+1	0
2	0	0	-1	0	-1	0	0	0	1	0	1
3	0	0	-1	-1	0	0	0	1	0	1	0
4	0	1	0	0	0	0	0	0	0	-1	0
5	0	0									
6											
7											
8											
9											
10											
11											

	1	2	3	4	5	6	7	8	9	10	11
1	1	0	0	0	0	1	0	0	-1	-1	0
2	0	0	0	-1	0	-1	0	0	1	0	1
3	0	0	-1	0	1	1	0	0	-1	0	0
4	0	0	0	-1	-1	0	0	1	0	0	1
5	0	0	1	0	0	0	0	0	0	-1	0
6	0	-1	0	1	0	0	0	0	0	0	0
7						1					
8				0	-1	0					
9						0					
10						0					
11				1		0					

Notes

23
March - Tuesday

$B_{11,2}$ & $B_{11,8}$ interchange signs (1 & 3) ✓

$B_{3,2}$ & $B_{3,10}$ " (6, 8) ✓

B_{92} & B_{98} " (7, 9) ✓

~~$B_{10,1}$ & $B_{10,7}$ both to be zero~~

B_{44} and B_{48} to be zero — (5, 11) ✓

B_{59} and $B_{10,1}$ " — (4, 10) ✓

~~B_{33} = +1, 1st col & 1st row of TB~~

If $(TB)_{11}$ be +1, 1st row & 1st col of TB ✓

If $(TB)_{33}$ be −1, 3rd row & 3rd col of TB ✓

If $(TB)_{44}$ be −1 & $(TB)_{64} = (TB)_{11,9} = 0$ +1

then 4th row & 4th col ✓

If $(TB)_{55} = 0$ & $(TB)_{85} = -1$, 5th row & 5th col ✓

If $(TB)_{67} = (TB)_{6,10} = +1$, 6th row & 6th col ✓

If $(TB)_{7,10}$ & $(TB)_{10,7} = +1$, 7th ✓

If $(TB)_{8,8} = 0 = -1$ $(TB)_{8,11} = -1$, 8th ✓

If $(TB)_{9,11} = -1$, 9th ✓

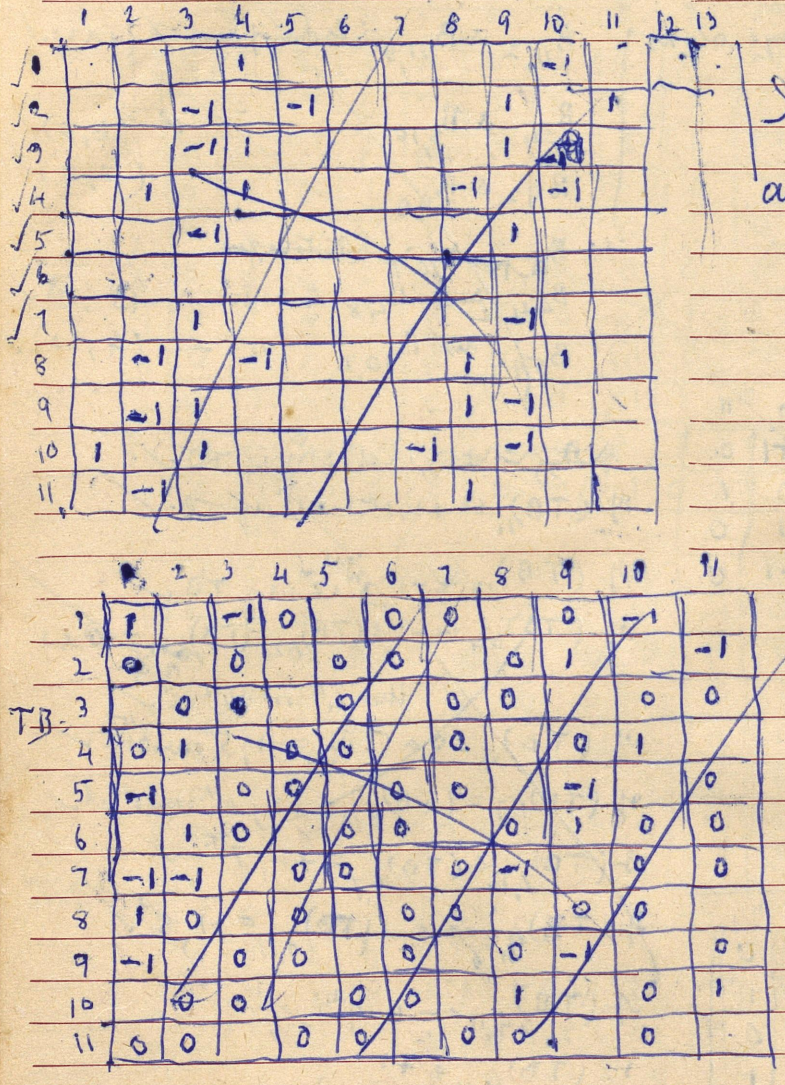
If $(TB)_{10,11} = +1$

Notes

(36)

24
March - Wednesday

25
March - Thursday



I think I have made an error in deriving ~~TA from A~~ T(A+B) from (A+B) on pp. 32 & 33.

For the transformations on p. 27. we have for eg

$A_{11} = (TA)_{33}$.

but I have taken

$(TA)_{11} = A_{33}$

Hence I suspect the error has arisen

So let us take A+B on p. 32 as correct and derive T(A+B) correctly with reasons p. 33

Notes

$(2,1) \to (1,11)$
$(2,2) \to (8,8)$
$(2,3) \to (4,5)$

Notes

26 March - Friday

27 March - Saturday

	1	2	3	4	5	6	7	8	9	10	11	12	
1	43	112	70	33	102	60	19	94	51	9	78	67	✓
2	5	84	39	118	76	23	108	57	15	99	47	67	✓
3	89	53	2	80	44	114	71	30	105	62	21	67	✓
4	59	~~60~~ 67	95	50	7	86	34	119	68	26	110	67	✓
5	31	101	64	13	92	55	4	82	40	116	73	67	✓
6	121	69	27	107	61	18	98	45	10	79	36	67	✓
7	85	42	111	75	24	103	66	14	93	52	6	67	
8	48	11	81	38	117	72	29	109	56	20	90	67	✓
9	16	96	54	1	87	35	113	77	25	104	63	67	✓
10	100	58	22	91	49	8	83	41	120	67	32	67	
11	74	28	106	65	12	~~97~~ 97	46	83	88	~~37~~ 37	115	67	
	671	671	671	671	671	671	671	671	671	671	671	671	

28 Sunday

(1,10) → (1,10)
(1,7) → (2,8)
(1,4) → (3,6)
(1,1) → (4,4)
(1,9) → (5,2)
(1,6) → (6,11)
(1,3) → (7,9)
(1,11) → (8,7)
(1,8) → (9,5)
(1,5) → (10,3)
(1,2) → (11,1)

(2,1) → (1,11)
(2,2) → (8,8)
(2,3) → (2,5)
(2,4) → (11,2)
(2,5) → (7,10)
(2,6) → (3,7)
(2,7) → (10,4)
(2,8) → (6,1)
(2,9) → (2,9)
(2,10) → (9,6)
(2,11) → (5,3)

So get (r, s) & work down in the steps (r+7, s−3).

So the above an n.d.s. (very useful) Next to find whether it is regular or irregular we have to split it into Pure Latin Square & Orthogonal.

Notes

Notes

33	110	66	22	99	55	11	88	44	0	77
0	77	33	110	66	22	99	55	11	88	44
88	44	0	77	33	110	66	22	99	55	11
55	11	88	44	0	77	33	110	66	22	99
22	99	55	11	88	44	0	77	33	110	66
110	66	22	99	55	11	88	44	0	77	33
77	33	110	66	22	99	55	11	88	44	0
44	0	77	33	110	66	22	99	55	11	88
11	88	44	0	77	33	110	66	22	99	55
99	55	11	88	44	0	77	33	110	66	22
66	22	99	55	11	88	44	0	77	33	110

Root square

10	2	4	6	3	5	8	6	7	9	1	✓
5	7	6	8	10	1	9	2	4	11	3	✓
1	9	2	3	11	4	5	8	6	7	10	✓
4	6	7	6	7	9	1	9	2	4	11	×
9	2	9	2	4	11	4	5	7	6	7	×
11	3	5	8	6	7	10	1	10	2	3	×
8	9	1	9	2	4	11	3	5	8	6	×
4	11	4	5	7	6	7	10	1	9	2	×
5	8	10	1	10	2	3	11	3	5	8	×
1	3	11	3	5	8	6	8	10	1	10	×
8	6	7	10	1	9	2	3	11	4	5	✓
×	×	×	×	×	×	✓	×	×	×	✓	

Principal square

Cyclic with step (2, 9)

non-regular

non-latin in 7 rows, 9 cols & some diagonals

~~[illegible]~~

This also appears to be of Cauchy's type

After all I have succeeded in finding a non-regular nasik square of order 11 although I failed in repeated attempts using B & J's method!

Now take B & W's irregular nasik of order 9. B & J considered only prime orders.

Let us split this 9th order one into (P) + (R) [fig on p. 12] (*)

0	63	45	36	18	54	72	27	9
54	72	27	9	0	63	45	36	18
63	45	36	18	54	72	27	9	0
72	27	9	0	63	45	36	18	54
45	36	18	54	72	27	9	0	63
27	9	0	63	45	36	18	54	72
36	18	54	72	27	9	0	63	45
9	0	63	45	36	18	54	72	27
18	54	72	27	9	0	63	45	36

(R)

1	2	3	5	6	4	9	7	8	✓
3	4	6	4	8	7	8	3	2	×
5	4	8	9	8	3	1	3	4	×
8	9	7	3	1	2	4	5	6	✓
7	2	1	2	6	5	6	7	9	×
3	2	6	4	6	7	8	7	2	×
6	4	5	7	8	9	2	3	1	✓
5	9	8	9	1	3	1	5	4	×
7	9	1	2	1	5	6	5	9	×

(P)

Notes

(R) is cyclic with step (4, 5)

(P) has non-latin rows & 9 cols & latin in all the diagonals

(*) The 9th order square on p. 12 can be formed from a partial p.s. by $T = \begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}$

For the case $n = 15$ also there are irregular n.d.s. as shown by R & W on p. 13. ~~But~~ with $m = 5$, but the actual construction ~~would be~~ cannot be carried out since I have not made full notes from R & W's article in Duke Math. Journal.

I might try case ~~n = 13~~ $n = 13$ for which I have ~~all~~ the method. Perhaps one will get an irregular n.d.s. of this order of the Cauchy's type & not Stern's type.

This might be checked by getting ~~one row~~ some one row of this square.

$n = 13$ $\quad A_{ij} = (i-1)n + a_j$

with $a_1, a_2, \ldots a_{11}, a_{12}, a_{13} = 2, 7, 4, 9, 6, 11, 1, 8, 3, 10, 5, 12, 13.$

B(1,4), B(4,4), B(4,2), B(3,4), B(3,9), B(2,11), B(2,9), B(5,9) all +1
B(9,2), B(10,9), B(10,2), B(9,4), B(12,2), B(8,9), B(11,9), B(11,7) all +1.
B(3,3), B(2,5), B(2,3), B(5,3), B(1,10), B(4,10), B(4,8), B(3,10) all −1.
B(8,3), B(11,3), B(11,1), B(10,3), B(10,8), B(9,10), B(9,8), B(12,8) all −1.

Let $A + B = C$ and $TC = D$ $\quad C_{ij} \to D(2i+j, i+2j)$

If $2i + j = k$, $i + 2j = l$ we have ~~3j = 2l − k~~,

$3i = 2k - l$, $3j = 2l - k$

$18 - 1 = 17 \equiv 30$

Let us check up the row $D_{91}, \ldots D_{9,13}$. $\quad 2 - 9) = -7 \equiv 6$

$D(9,1) \to C(10,2)$, $D(9,2) \to C(1,7)$, $D(9,3) \to C(5,12)$ $\quad 16 \quad 4 - 9 = -5 = 8 \equiv 21$

$D(9,4) \to C(9,4)$, $D(9,5) \to C(13,9)$, $D(9,6) \to C(4,1)$ $\quad$ ~~−3 ≡ 36~~ 13

$D(9,7) \to C(8,6)$, $D(9,8) \to C(12,11)$, $D(9,9) \to C(3,3)$ $\quad$ ~~−1 ≡ 12~~ 3

$D(9,10) \to C(7,8)$, $D(9,11) \to C(11,13)$, $D(9,12) \to C(2,5)$ $\quad 13$

$D(9,13) \to C(6,10)$.

Notes Notes

$C(10,2) = A(10,2) + B(10,2) = 124 + 1 = 125$, $C(1,7) = A(1,7) + B(1,7) = 1 + 0 = 1$.

$C(5,12) = A(5,12) + B(5,12) = 64 + 0 = 64$; $C(9,4) = A(9,4) + B(9,4) = 113 + 1 = 114$

$C(13,9) = A(13,9) + B(13,9) = 159 + 0 = 159$, $C(4,1) = A(4,1) + B(4,1) =$ ~~[illegible]~~
$= 41 + 0 = 41$

(261) $\frac{1}{2}$ ~~8~~ 65

304 (40)

-170

$\frac{1}{2}$, 13, 170

$S = 85 \times 13 = 1105$

1145 (40) Epts

2 April - Friday

$C(8,6) = A(8,6) + B(8,6) = 102 + 0 = 102$,

$C(3,3) = A(3,3) + B(3,3) = 30 - 1 = 29$

$C(11,13) = A(11,13) + B(11,13) = 143 + 0 = 143$,

$C(6,10) = A(6,10) + B(6,10) = 75 + 0 = 75$.

3 April - Saturday

$C(12,11) = A(12,11) + B(12,11) = 148 + 0 = 148$.

$C(7,8) = A(7,8) + B(7,8) = 86 + 0 = 86$

~~$C(9,10) = A(9,10) + B(9,10) = 114 - 1 = 113$~~

$C(2,5) = A(2,5) + B(2,5) = 19 - 1 = 18$

So the 9th row of D is 125, 1, 64, 114, 159, ~~54~~ 41, 102, 148, 29, ~~103~~ 86, 143, 18, 75

~~summing up to 1145~~ = 1105

8, 1, 12, 10, 3, 2, 11, 5, 3, 8, 13, 5, 10 ie of Cauchy's type

So there appears no point in deriving R & W's irregular n.d.S of order 13

R & W's case $n = 4m$ - (1) Let us take $m = 2$

(p.12)

consider the 8 pairs (0, 60), (4, 56), (8, 52), (12, 48), (16, 44), (20, 40)

(24, 36), (28, 32)

If (A, C), (B, D) are two of the pairs, say (A 4, C 56) & (B 8, D 52)

A+1	B+4	C+1	D+4
C+2	D+3	A+2	B+3
A+4	B+1	C+4	D+1
C+3	D+2	A+3	B+2

Nasik (16m²+1)

is nasik with $S = \frac{v^2}{4}(16m^2+1) = 0$

4 Sunday

5	12	57	56
58	55	6	11
8	9	60	53
59	54	7	10

(a)

S = 130.

Fill in the 2(m)'s of S_{4m} with squares like above using different sets of pairs for each square. Then the result is a n.d.S.

Let us now choose the other 3 pairs as {(A 0, C 60), (B 28, D 32)}, {(A 12, C 48), (B 24, D 36)}

and {(A 16, C 44), (B 20, D 40)}.

Notes

(a) (5, 125), (70, 60), (120, 10), (130)

(b) (1, 129), (94, 36), (100, 30), (130)

(c) (13, 117), (78, 52), (104, 26), (130)

(d) (17, 113), (70, 60), (108, 22), (130)

5 + 129 + 13 + 113
1 + 125 + 17 + 117

94 + 60 + 70 + 52

5 April - Monday

1	32	61	36
62	35	2	31
4	29	64	33
63	34	3	30

(b)

13	28	49	40
50	39	14	27
16	25	52	37
51	38	15	26

(c)

forming the 8th order square putting in (a), (b), (c) & (d) as quadrants

5	12	57	56	1	32	61	36
58	55	6	11	62	35	2	31
8	9	60	53	4	29	64	33
59	54	7	10	63	34	3	30
13	28	49	40	17	24	45	44
50	39	14	27	46	43	18	23
16	25	52	37	20	21	48	41
51	38	15	26	47	42	19	22

(a) (c)

magic but not nasik

1	32	61	36	5	12	57	56
62	35	2	31	58	55	6	11
4	29	64	33	8	9	60	53
63	34	3	30	59	54	7	10
13	28	49	40	17	24	45	44
50	39	14	27	46	43	18	23
16	25	52	37	20	21	48	41
51	38	15	26	47	42	19	22

(b) (a) (d) (c)

magic but not nasik

1	32	61	36	13	28	49	40
62	35	2	31	50	39	14	27
4	29	64	33	16	25	52	37
63	34	3	30	51	38	15	26
5	12	57	56	17	24	45	44
58	55	6	11	46	43	18	23
8	9	60	53	20	21	48	41
59	54	7	10	47	42	19	22

(b) (a) (c) (d)

not nasik

(1, 129), (94, 36), (100, 30).
(5, 125), (78, 60), (102, 70)
(13, 117), (78, 52), (104, 26)
(17, 113), (78, 60), (108, 22)

1	33	(1, 2, 3, 4, 29, 30, 31, 32, 33, 34, 35, 36, 61, 62, 63, 64)
2	34	(5, 6, 7, 8, 9, 10, 11, 12, 53, 54, 55, 56, 57, 58, 59, 60)
3	35	(13, 14, 15, 16, 25, 26, 27, 28, 37, 38, 39, 40, 49, 50, 51, 52)
4	36	
29	61	(17, 18, 19, 20, 21, 22, 23, 24, 41, 42, 43, 44, 45, 46, 47, 48)
30	62	
31	63	
32	64	

Notes

Notes

6 April - Tuesday

17	24	45	44
46	43	18	23
20	21	48	41
47	42	19	22

(d)

(*) This can be formed from a partial p.s. using $T = \begin{pmatrix} 2, & -1 \\ -3, & 2 \end{pmatrix}$

(49)

7 April - Wednesday

8 April - Thursday

Suppose we choose $\{\overset{A\ C}{(0,60)}, \overset{B\ D}{(28,32)}\}$ (1), $\{\overset{A\ C}{(4,56)}, \overset{B\ D}{(24,36)}\}$ (2), $\{\overset{A\ C}{(8,52)}, \overset{B\ D}{(20,40)}\}$ (3)

and $\{\overset{A\ C}{(12,48)}, \overset{B\ D}{(16,44)}\}$ (4)

1	32	61	36
62	35	2	31
4	29	64	33
63	34	3	30

(1)

5	28	57	40
58	39	6	27
8	25	60	37
59	38	7	26

(2)

9	24	53	44
54	43	10	23
12	21	56	41
55	42	11	22

(3)

13	20	49	48
50	47	14	19
16	17	52	45
51	46	15	18

(4)

(*)

1	32	61	36	5	28	57	40
62	35	2	31	58	39	6	27
4	29	64	33	8	25	60	37
63	34	3	30	59	38	7	26
9	24	53	44	13	20	49	48
54	43	10	23	50	47	14	19
12	21	56	41	16	17	52	45
55	42	11	22	51	46	15	18

This is an n.d.s. allright (5)

So the statement by R & W of taking any two pairs does not appear to work.

non-regular according to R & W

Note: - All the 4th order subsquares are formed by Narayana Pandit's method of Knight moves. In fact the above constn is analogous to the one constructed by me using the b-numbering (Bk. 2, p. 204) where the first numbers of the squares were 1, 2, 3, 4 instead of 1, 5, 9, 13 as in the figure above. In the figure above the numbers on the respective 4x squares (1), (2), (3) and (4) are as in the next page 43 and ~~does not~~ they are not obtained as simply as what we have obtained by the b-numbering method.

Notes

Notes

9 April -Friday

10 April -Saturday

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16
29	25	21	17
30	26	22	18
31	27	23	19
32	28	24	20
33	37	41	45
34	38	42	46
35	39	43	47
36	40	44	48
61	57	53	49
62	58	54	50
63	59	55	51
64	60	56	52

We have to find what is meant by a regular or irregular (non-regular) square of order 4m (in particular of order 8). (5) of the previous page given by R & W is non-regular (?). Let us split it into (P) + (R)

0	24	56	32	0	24	56	32
56	32	0	24	56	32	0	24
0	24	56	32	0	24	56	32
56	32	0	24	56	32	0	24
8	16	48	40	8	16	48	40
48	40	8	16	48	40	8	16
8	16	48	40	8	16	48	40
48	40	8	16	48	40	8	16

(R)

1	8	5	4	5	4	1	8
6	3	2	7	2	7	6	3
4	5	8	1	8	1	4	5
7	2	3	6	3	6	7	2
1	8	5	4	5	4	1	8
6	3	2	7	2	7	6	3
4	5	8	1	8	1	4	5
7	2	3	6	3	6	7	2

(P)

Both (R) & (S) are non-cyclical and non-latin in rows & cols.
Both (R) & (S) are latin in all the diagonals
(R) is cyclical in the first four rows & the last four rows.

Now we have to find a regular n.d.s. of order 8 in order to understand the difference between regular & non-regular.

What about my b-numbering square.

Take Fig on p. 204, Bk. 2

12 April - Monday

Fig (c) on p. 204, Bk 2 (44)
split into (R) + (P)

0	24	48	40	0	24	48	40

(R)

13 April - Tuesday

1	8	1	8	2	7	2	7

(P)

(non-regular)

This will obviously be of the same type as R & W's square obtained on p. 42. I think we have to go to case 2 under Th. 5.5 on p. 10.

$$T = \begin{Vmatrix} 2 & -1 \\ -3 & 2 \end{Vmatrix}$$

$A_{ij} = n\,a(i) + a(j) - n$ where $a(i) = i$, $a(i+2m) = 4m+1-i$ for $1 \leq i \leq 2m$ and put $a(i \pm 4m) = a(i)$.

Here $m = 2$ $A_{ij} = 8\,a(i) + a(j) - n$, where $a(i) = i$, $a(i+4) = 9-i$ for $1 \leq i \leq 4$

$A_{11} = 8+1-8 = 1$, $A_{12} = 8+2-8 = 2$, $A_{13} = 8+3-8 = 3$, $A_{14} = 8+4-8 = 4$

$A_{15} = 8+8-8 = 8$, $A_{16} = 8+7-8 = 7$, $A_{17} = 8+6-8 = 6$, $A_{18} = 8+5-8 = 5$

$A_{21} = 16+1-8 = 9$, $A_{22} = 16+2-8 = 10$, $A_{23} = 16+3-8 = 11$, $A_{24} = 16+4-8 = 12$

Notes

$A_{25} = 16+8-8 = 16$, $A_{26} = 16+7-8 = 15$, ~~$A_{26} = 16$~~, $A_{27} = 14$, $A_{28} = 13$.

$A_{31} = 24+1-8 = 17$, $A_{32} = 18$, $A_{33} = 19$, $A_{34} = 20$

$A_{35} = 24+8-8 = 24$, $A_{36} = 23$, $A_{37} = 22$, $A_{38} = 21$.

14 April - Wednesday

15 April - Thursday

$A_{41} = 32+1-8 = 25$, $A_{42} = 26$, $A_{43} = 27$, $A_{44} = 28$

$A_{45} = 32+8-8 = 32$, $A_{46} = 31$, $A_{47} = 30$, $A_{48} = 29$

$A_{51} = 64+1-8 = 57$, $A_{52} = 58$, $A_{53} = 59$, $A_{54} = 60$

$A_{55} = 64+8-8 = 64$, $A_{56} = 63$, $A_{57} = 62$, $A_{58} = 61$

$A_{61} = 56+1-8 = 49$, $A_{62} = 50$, $A_{63} = 51$, $A_{64} = 52$

$A_{65} = 56+8-8 = 56$, $A_{66} = 55$, $A_{67} = 54$, $A_{68} = 53$

$A_{71} = 48+1-8 = 41$, $A_{72} = 42$, $A_{73} = 43$, $A_{74} = 44$

$A_{75} = 48+8-8 = 48$, $A_{76} = 47$, $A_{77} = 46$, $A_{78} = 45$

$A_{81} = 40+1-8 = 33$, $A_{82} = 34$, $A_{83} = 35$, $A_{84} = 36$

$A_{85} = 40+8-8 = 40$, $A_{86} = 39$, $A_{87} = 38$, $A_{88} = 37$

Having obtained A_{ij} we have to get TA. If $\begin{Vmatrix} a & c \\ b & d \end{Vmatrix} = T$

If $B = TA$

$A_{ij} \Rightarrow B(ai+cj, bi+dj)$

For $T = \begin{Vmatrix} 2 & -1 \\ -3 & 2 \end{Vmatrix}$ $|T| = 1$ which is prime to 8 //

$\therefore A_{ij} \rightarrow B(2i-j, -3i+2j)$

If $2i-j = k$, $-3i+2j = l$

we have only for ~~$4i-2j = 2k$~~ i & j interms of k & l.

~~$-3i+2j = l$~~ readily

$4i-2j = 2k$, $-3i+2j = l$ $\therefore$ adding $i = 2k+l$

$2(2k+l)-j = k$, $j = 4k+2l-k = 3k+2l$.

$i \equiv 2k+l \pmod 8$
$j \equiv 3k+2l \pmod 8$ || working A_{ij} we can get B_{ij}

Notes

Notes

8, 6, 1, 3, 8, 6, 1, 3.

16 April - Friday

$T = \begin{Vmatrix} 2 & -1 \\ -3 & 2 \end{Vmatrix}$ (46)

17 April - Saturday

$A =$

	1	2	3	4	5	6	7	8
1	1	2	3	4	8	7	6	5
2	9	10	11	12	16	15	14	13
3	17	18	19	20	24	23	22	21
4	25	26	27	28	32	31	30	29
5	57	58	59	60	64	63	62	61
6	49	50	51	52	56	55	54	53
7	41	42	43	44	48	47	46	45
8	33	34	35	36	40	39	38	37

this is an n-p.s

$T =$ $i \equiv 2k+l,\ j \equiv 3k+2l \pmod 8$.

$(k,l) = (1,1) \to (i,j) = (3,5)$	$(2,1) \to (5,8)$	$(3,1) \to (7,3)$	$(4,1) \to (1,6)$
$(1,2) \to (4,7)$	$(2,2) \to (6,2)$	$(3,2) \to (8,5)$	
$(1,3) \to (5,1)$	$(2,3) \to (7,4)$	$(3,3) \to (1,7)$	
$(1,4) \to (6,3)$	$(2,4) \to (8,6)$	$(3,4) \to (2,1)$	
$(1,5) \to (7,5)$	$(2,5) \to (1,8)$	–	
$(1,6) \to (8,7)$	$(2,6) \to (2,2)$	–	
$(1,7) \to (1,1)$	$(2,7) \to (3,4)$	–	
$(1,8) \to (2,3)$	$(2,8) \to (4,6)$	–	

$(5,1) \to (3,1)$ | $(6,1) \to (5,4)$ | $(7,1) \to (7,7)$ | $(8,1) \to (1,2)$

Notes

Notes

From $(1,1), (2,1), \ldots (8,1)$ go to next numbers in rows of TA by the Knight-move $(2,-1)$

8,4,1,7,3,5,8
2,6,4,1,7,3,5,8,2
6,4,1,7,3,5,8,2
8,4,3,2,1,5,6,7
56,7,8,4,3,2,1

19 April - Monday

16,8,0,32,40,48,56,24
56,24,16,8,0,32,40,48
0,24,40,8,56,32,16,48
40,8.

20 April - Tuesday

T-A =	1	2	3	4	5	6	7	8
1	24	30	57	51	48	38	1	11
2	61	50	44	39	5	10	20	31
3	43	40	6	9	19	32	62	49
4	7	13	18	28	63	53	42	36
5	17	27	64	54	41	35	8	14
6	60	55	45	34	4	15	21	26
7	46	33	3	16	22	25	59	56
8	2	12	23	29	58	52	47	37

regular according to R & W

this is an n.d.s all right

16	24	56	48	40	32	0	8
56	48	40	32	0	8	16	24
40	32	0	8	16	24	56	48
0	8	16	24	56	48	40	32
16	24	56	48	40	32	0	8
56	48	40	32	0	8	16	24
40	32	0	8	16	24	56	48
0	8	16	24	56	48	40	32

(R)

8	6	1	3	8	6	1	3
5	2	4	7	5	2	4	7
3	8	6	1	3	8	6	1
7	5	2	4	7	5	2	4
1	3	8	6	1	3	8	6
4	7	5	2	4	7	5	2
6	1	3	8	6	1	3	8
2	4	7	5	2	4	7	5

(P)

(R) is Latin in rows & all the diagonals but not in Cols.
(P) is Latin in cols & all the diagonals, but not in rows.
(R) is cyclic with step (6,2) in the rows and in the diagonals with steps (2,6) & (6,2)
(P) is cyclic with step (2,6) in the cols and in the n.d.s diagonals (3,5) & (5,7)

Notes

But the square is irregular according to B & J for it is not fully cyclic, but is regular according to R & W since they define a regular square as one obtained by any transform of a n.d.s (see p. 6. under Th. 4.2)

Notes

21 April - Wednesday

22 April - Thursday

Let us take a prime order say $n = 7$. Use the $T = \begin{Vmatrix} 2, & -1 \\ -3, & 2 \end{Vmatrix}$

	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	8	9	10	11	12	13	14
3	15	16	17	18	19	20	21
4	22	23	24	25	26	27	28
5	29	30	31	32	33	34	35
6	36	37	38	39	40	41	42
7	43	44	45	46	47	48	49

(A)

This is an n.p.s

19	28	30	39	48	1	10
29	38	47	7	9	18	27
46	6	8	17	26	35	37
14	16	25	34	36	45	5
24	33	42	44	4	13	15
41	43	3	12	21	23	32
2	11	20	22	31	40	49

(B)

This is an n.d.s although

$i \equiv 2k + l$, $j \equiv 3k + 2l \pmod 7$.

$(1,1) \to (3,5)$ | $(2,1) \to (5,1)$ | $(3,1) \to (7,4)$ | $(4,1) \to (2,7)$ | $(5,1) \to (4,3)$ | $(6,1) \to (6,6)$

$(1,2) \to (4,7)$ | $(2,2) \to (6,3)$ | $(7,1) \to (1,2)$

$(1,3) \to (5,2)$

Splitting (B) into (R) & (P)

14	21	28	35	42	0	7
28	35	42	0	7	14	21
42	0	7	14	21	28	35
7	14	21	28	35	42	0
21	28	35	42	0	7	14
35	42	0	7	14	21	28
0	7	14	21	28	35	42

(R)

cyclic with step (5,2)

5	7	2	4	6	1	3
1	3	5	7	2	4	6
4	6	1	3	5	7	2
7	2	4	6	1	3	5
3	5	7	2	4	6	1
6	1	3	5	7	2	4
2	4	6	1	3	5	7

(P)

cyclic with step (2,5)

So for a prime order any T^n gives a regular square which is an n.d.s. depending on the transformation.

Suppose we try $T = \begin{Vmatrix} 2 & 1 \\ 1 & 2 \end{Vmatrix}$

$i \equiv \frac{1}{3}(2k - l)$, $j \equiv \frac{1}{3}(2l - k) \pmod 7$

Let $TA = B$. $(1,1) \to (5,5)$ | $(2,1) \to (1,7)$ | $(3,1) \to (4,2)$ | $(4,1) \to (7,4)$

$(1,2) \to (7,1)$ | $(2,2) \to (3,3)$ | $(3,2) \to (6,5)$ | $(5,1) \to (3,6)$

$(1,3) \to (2,4)$ | $(6,1) \to (6,1)$

Add 2, & 3] | $(7,1) \to (2,3)$

Notes

Notes

23
April - Friday

33	43	11	28	38	6	16
7	17	34	44	12	22	39
23	40	1	18	35	45	13
46	14	24	41	2	19	29
20	30	47	8	25	42	3
36	4	21	31	48	9	26
10	27	37	5	15	32	49

24
April - Saturday

33	43	11	28	38	6	16
7	17	34	44	12	22	39
23	40	1	18	35	45	13
46	14	24	41	2	19	29
20	30	47	8	25	42	3
36	4	21	31	48	9	26
10	27	37	5	15	32	49

again an n·1·5

28	42	7	21	35	0	14
0	14	28	42	7	21	35
21	35	0	14	28	42	7
42	7	21	35	0	14	28
14	28	42	7	21	35	0
35	0	14	28	42	7	21
7	21	35	0	14	28	42

(R)

5	1	4	7	3	6	2
7	3	6	2	5	1	4
2	5	1	4	7	3	6
4	7	3	6	2	5	1
6	2	5	1	4	7	3
1	4	7	3	6	2	5
3	6	2	5	1	4	7

(P)

142/12

Again a regular one with (R) having step (2,5) & (P) having step (4,3)

Let us consider the case $n = 42$ i.e. $n = 4m$ with $m = 3$.

Consider the 18 pairs (0, 140), (4, 136), (8, 132), (12, 128), (16, 124)

(20, 120), (24, 116), (28, 112), (32, 108), (36, 104), (40, 100), (44, 96)

(48, 92), (52, 88), (56, 84), (60, 80), (64, 76), (68, 72).

Analogous to what we have done for case $m = 2$, let us take the first & last pairs, 2nd & 17th & so on & form the quadrant

A+1	B+4	C+1	D+4
C+2	D+3	A+2	B+3
A+4	B+1	C+4	D+1
C+3	D+2	A+3	B+2

Notes

Notes

P.T.O

290 × 3
870

145
870

(48)

26 April-Monday

27 April - Tuesday

A C B D
{(0, 140), (68, 72)} || A C B D
{(4, 136), (64, 76)}

{(8, 132), (60, 80)} {(12, 128), (56, 84)}

1	72	141	76
142	73	2	71
4	69	144	73
143	74	3	70

(1)

5	68	137	80
138	79	6	67
8	65	140	77
139	78	7	66

(2)

9	64	133	84
134	83	10	63
12	61	136	81
135	82	11	62

(3)

13	60	129	88
130	87	14	59
16	57	132	85
131	86	15	58

(4)

{(16, 124), (52, 88)}, {(20, 120), (48, 92)}, {(24, 116), (44, 96)}

17	56	125	92
126	91	18	55
20	53	128	89
127	90	19	54

(5)

21	52	121	96
122	95	22	51
24	49	124	93
123	94	23	50

(6)

25	48	117	100
118	99	26	47
28	45	120	97
119	98	27	46

(7)

{(28, 112), (40, 100)}

29	44	113	104
114	103	30	43
32	41	116	101
115	102	31	42

(8)

{(32, 108), (36, 104)}

33	40	109	108
110	107	34	39
36	37	112	105
111	106	35	38

(9)

All these quadrants are best formed by Narayana and their Knight Moves method

(1)	(2)	(3)
(4)	(5)	(6)
(7)	(8)	(9)

Let us put these nine quadrants as on the L.H.S

and get the 12th order square.

Notes

I think there is no need to examine whether it is regular or irregular. Anyway, I don't think I have so far obtained a nasik square of order 12

Notes

$\frac{290 \times 3}{\cancel{1100}}$ 870

(49)

28 April - Wednesday

29 April - Thursday

1	72	141	76	5	68	137	80	9	64	133	84
142	75	2	71	138	79	6	67	134	83	10	63
4	69	144	73	8	65	140	77	12	61	136	81
143	74	3	70	139	78	7	66	135	82	11	62
13	60	129	88	17	56	125	92	21	52	121	96
130	87	14	59	126	91	18	55	122	95	22	51
16	57	132	85	20	53	128	89	24	49	124	93
131	86	15	58	127	90	19	54	123	94	23	50
25	48	117	100	29	44	113	104	33	40	109	108
118	99	26	47	114	103	30	43	110	107	34	39
28	45	120	97	32	41	116	101	36	37	112	105
119	98	27	46	115	102	31	42	111	106	35	38

n.d.s all right with $S = 870$

non-regular according to R & W.

We had such a square for $n = 8$ by b-numbering & also one for $n = 16$. Is there one for $n = 12$ also? Perhaps yes - we have only to write 1 to 144

Notes

Notes

30 April - Friday

1 May - Saturday

1	2	3	4	5	6	7	8	9
18	17	16	15	14	13	12	11	10
19	20	21	22	23	24	25	26	27
36	35	34	33	32	31	30	29	28
37	38	39	40	41	42	43	44	45
54	53	52	51	50	49	48	47	46
55	56	57	58	59	60	61	62	63
72	71	70	69	68	67	66	65	64
73	74	75	76	77	78	79	80	81
90	89	88	87	86	85	84	83	82
91	92	93	94	95	96	97	98	99
108	107	106	105	104	103	102	101	100
109	110	111	112	113	114	115	116	117
126	125	124	123	122	121	120	119	118
127	128	129	130	131	132	133	134	135
144	143	142	141	140	139	138	137	136

Forming the 12x12 square

putting in the quadrants as beginning with

1	2	3
4	5	6
7	8	9

as in ←

we get

1	72	109	108	2	71	110	107	3	70	111	106
126	91	18	55	125	92	17	56	124	93	16	57
36	37	144	73	35	38	143	74	34	39	142	75
127	90	19	54	128	89	20	53	129	88	21	52
4	69	112	105	5	68	113	104	6	67	114	103
123	94	15	58	122	95	14	59	121	96	13	60
33	40	141	76	32	41	140	77	31	42	139	78
130	87	22	51	131	86	23	50	132	85	24	49
7	66	115	102	8	65	116	101	9	64	117	100
120	97	12	61	119	98	11	62	118	99	10	63
30	43	138	79	29	44	137	80	28	45	136	81
133	84	25	48	134	83	26	47	135	82	27	46

which is an n.d.s.

all right with

S = 870

not regular according to R&W

Notes

Notes

3 May - Monday

4 May - Tuesday

Leaving aside the cases of 8, 9, 12, 15 etc. let us now go to the consideration of transformations which retain the nasik property. ie Th. 3.3, p.5.

(1) $L = \left\{ \begin{array}{l} i' = i - 1 \\ j' = j \end{array} \right\}$ $(1,1) \to (0,1) \equiv (5,1)$, etc

ie putting the first row last

(2) $M = \left\{ \begin{array}{l} i' = i \\ j' = j - 1 \end{array} \right\}$ Corresponds to putting the first column last.

(3) $S_\alpha = \left\| \begin{array}{cc} \alpha & 0 \\ 0 & \alpha \end{array} \right\|$ with $(\alpha, n) = 1$

(i) Take $n = 5$, $\alpha = 2$, $k = 2i$, $l = 2j$ $i = \frac{1}{2}k$, $j = \frac{1}{2}l$.

$(1,1) \to (2,2)$ $(1,4) \to (2,3)$
$(1,2) \to (2,4)$ $(1,5) \to (2,5)$
$(1,3) \to (2,1)$

(k,l), (i,j)

$(1,1) \leftrightarrow (3,3)$	$(2,1) \leftrightarrow (1,3)$	$(3,1) \leftrightarrow (4,3)$	$(4,1) \leftrightarrow (2,3)$	$(5,1) \leftrightarrow (5,3)$
$(1,2) \leftrightarrow (3,1)$	$(2,2) \leftrightarrow (1,1)$	$(3,2) \leftrightarrow (4,1)$	$(4,2) \leftrightarrow (2,1)$	$(5,2) \leftrightarrow (5,1)$
$(1,3) \leftrightarrow (3,4)$	$(2,3) \leftrightarrow (1,4)$	$(3,3) \leftrightarrow (4,4)$	$(4,3) \leftrightarrow (2,4)$	$(5,3) \leftrightarrow (5,4)$
$(1,4) \leftrightarrow (3,2)$	$(2,4) \leftrightarrow (1,2)$	$(3,4) \leftrightarrow (4,2)$	$(4,4) \leftrightarrow (2,2)$	$(5,4) \leftrightarrow (5,2)$
$(1,5) \leftrightarrow (3,5)$	$(2,5) \leftrightarrow (1,5)$	$(3,5) \leftrightarrow (4,5)$	$(4,5) \leftrightarrow (2,5)$	$(5,5) \leftrightarrow (5,5)$

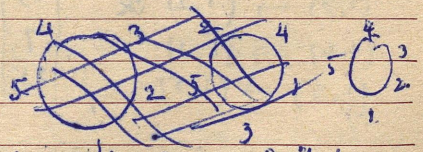
	1	2	3	4	5
1	4	12	21	20	8
2	16	10	3	14	22
3	13	24	17	6	5
4	7	1	15	23	19
5	25	18	9	2	11

(i,j) nasik

$\longrightarrow$

17	13	6	24	5
21	4	20	12	8
15	7	23	1	19
3	16	14	10	22
9	25	2	18	11

(k,l) nasik



Transformation not immediately obvious

21	1	13	24	17	6	5
3	2	7	1	15	23	19
17	3	4	12	21	20	8
15	4	16	10	3	14	22
9	5	25	18	9	2	11

3-1-4-2-5

17	6	13	24	5
15	23	7	1	19
21	20	4	12	8
3	14	16	10	22
9	2	25	18	11

Cols 1-2-3-4-5 $\to$ 2-4-1-3-5 followed by rows. also transformed ie eg 1-2-3-4-5 $\to$ 2-4-1-3-5

(5 2)

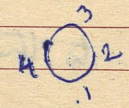
13
2
16
3

5 May - Wednesday

6 May - Thursday

Try with a 4th order square with $\alpha = 3$, $k = 3i$, $l = 3j$ $i = \frac{1}{3}k$, $j = \frac{1}{3}l$

$(1,1) \leftrightarrow (3,3)$	$(2,1) \leftrightarrow (2,3)$	$(3,1) \leftrightarrow (1,3)$	$(4,1) \leftrightarrow (4,3)$
$(1,2) \leftrightarrow (3,2)$	$(2,2) \rightarrow (2,2)$	$(3,2) \leftrightarrow (1,2)$	$(4,2) \leftrightarrow (4,2)$
$(1,3) \leftrightarrow (3,1)$	$(2,3) \rightarrow (2,1)$	$(3,3) \leftrightarrow (1,1)$	$(4,3) \leftrightarrow (4,1)$
$(1,4) \leftrightarrow (3,4)$	$(2,4) \rightarrow (2,4)$	$(3,4) \leftrightarrow (1,4)$	$(4,4) \leftrightarrow (4,4)$



1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

n asik

→

16	5	4	9
2	11	14	7
13	8	1	12
3	10	15	6

n asik

Transformation not immediately obvious → cols 1-2-3-4 → 3-2-1-4. follower by rows 1-2-3-4 → 3-2-1-4 ~~[illegible]~~

$k = 4i$, $j = 4l$

~~[illegible]~~ Returning to $n = 5$, take $\alpha = 4$, $i = \frac{1}{4}k$, $j = \frac{1}{4}l$ $(1,1) \rightarrow (4,4)$

$\alpha = 3$, $k = 3i$, $j = 3l$ $(1,1) \rightarrow (3,3)$

~~$(1,1) \leftrightarrow (2,2)$~~

$(1,1) \leftrightarrow (4,4)$ $(2,2) \rightarrow 4,4$ $i = 2k$, $k = 2i$, $j = 2l$

$(1,2) \leftrightarrow (5,3)$

$(1,3) \leftrightarrow (5,2)$

$(1,4) \leftrightarrow (5,1)$

$(1,5) \leftrightarrow (5,$

Anyway seems a general trn taking (r, c, d, d') into (r, c, d, d') in some order not necessarily the same.

(4) $O = \left\{ \begin{Vmatrix} 1 & 0 \\ 0 & -1 \end{Vmatrix} \right.$ ~~[illegible]~~

$i' = i$, $j' = -j$

n=5

$(1,1) \rightarrow (1,4)$

$(1,2) \rightarrow (1,3)$

$(1,3) \rightarrow (1,2)$ etc

$(1,4) \rightarrow (1,1)$

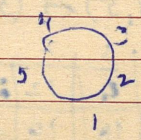
$(1,5) \rightarrow (1,5)$

4	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

→

20	21	12	4	8
14	3	10	16	22
6	17	24	13	5
23	15	1	7	19
2	9	18	25	11

n asik



ie permutation of cols 1-2-3-4-5 → 4-3-2-1-5

n=4

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

n asik

→

13	8	16	12
2	11	14	7
16	5	4	9
3	10	15	6

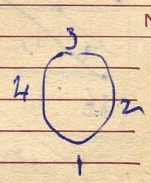
n asik

$(1,1) \rightarrow (1,-1) = (1,3)$

$(1,2) \rightarrow (1,2)$

$(1,3) \rightarrow (1,1)$

$(1,4) \rightarrow (1,4)$



permutation of cols 1-2-3-4 to 3-2-1-4

Notes

Notes

(5) $P = \begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix}$ $K = j,\ l = i$ $i' = j',\ j' = i$

$(1,1) \to (1,1)$
$(1,2) \to (2,1)$
etc

4	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

$\longrightarrow$

4	16	13	7	25
12	10	24	1	18
21	3	17	15	9
20	14	6	23	2
8	22	5	19	11

(*)

This is reflection about the lower diagonal. — Also holds for $n = 4$

(6) $Q = \begin{Vmatrix} 1 & 1 \\ 1 & -1 \end{Vmatrix}$ $K = i + j$, $l = i - j$

$(1,1) \to (2,5)$	$(2,1) \to (3,1)$	$(3,1) \to (4,2)$	$(4,1) \to (5,3)$	$(5,1) \to (1,4)$
$(1,2) \to (3,4)$	$(2,2) \to (4,5)$	$(3,2) \to (5,1)$	$(4,2) \to (1,2)$	$(5,2) \to (2,3)$
$(1,3) \to (4,3)$	$(2,3) \to (5,4)$	$(3,3) \to (1,5)$	$(4,3) \to (2,1)$	$(5,3) \to (3,2)$
$(1,4) \to (5,2)$	$(2,4) \to (1,3)$	$(3,4) \to (2,4)$	$(4,4) \to (3,5)$	$(5,4) \to (4,1)$
$(1,5) \to (1,1)$	$(2,5) \to (2,2)$	$(3,5) \to (3,3)$	$(4,5) = (4,4)$	$(5,5) \to (5,5)$

4	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

$\longrightarrow$

8	1	14	25	17
15	22	18	6	4
16	9	5	12	23
2	13	21	19	10
24	20	7	3	11

This is the case of rows and cols being transferred to upper & lower diagonals a trn given by B & J.

$n=4$

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

$\longrightarrow$

12		2	
	7		1
14		8	
	13		11

n as all

$(3,1)$ & $(1,3)$ both $\to (4,2)$

So this does not appear applicable for n even.

$(1,1) \to (2,4)$	$(2,1) \to (3,1)$	$(3,1) \to (4,2)$
$(1,2) \to (3,3)$	$(2,2) \to (4,4)$	$(3,2) \to (1,1)$
$(1,3) \to (4,2)$	$(2,3) \to (1,3)$	$(3,3) \to (2,4)$
$(1,4) \to (1,1)$	$(2,4) \to (2,2)$	$(3,4) \to (3,3)$

$(4,1) \to (1,3)$
$(4,2) \to (2,2)$
$(4,3) \to (3,1)$
$(4,4) \to (4,4)$

Note that $(3,1)$ & $(1,3)$ lead to same $(4,2)$

Notes

Notes

$K = 2i,\ l = 2j$

$(1,1) \to (2,2)$
$(1,2) \to (2,4)$
$(1,3) \to (2,1)$
$(1,4) \to (2,3)$
$(1,5) \to (2,5$

(54)

10 May - Monday

11 May - Tuesday

~~Now to the article of R & W in Bull. Amer. Math. Soc~~

(1)

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

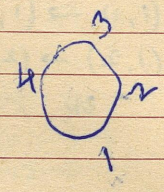
Suppose we take 3-2-1-4 in rows

(1) → (2) still nasik

(2)

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

still nasik



(3)

1	14	4	15
8	11	5	10
13	2	16	3
12	7	9	6

interchange rows & cols in (1)
still nasik

(4)

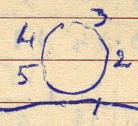
4	14	1	15
5	11	8	10
16	2	13	3
9	7	12	6

3-2-1-4 in (3) → (4)
still nasik

Since (4) ie ⊖ for O holds both for rows & cols, it looks that S_α and O are not different from each other. Let us examine S_α in more

detail by taking $\alpha = 3$, & $\alpha = 4$.

$\alpha = 3,\ K = 3i,\ l = 3j$

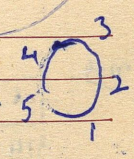


4	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

→

10	14	16	3	22
1	23	7	15	19
12	20	4	21	8
24	6	13	17	5
18	2	25	9	11

rows 1-2-3-4-5 → 3-1-4-2-5
followed by
cols 1-2-3-4-5 → 3-1-4-2-5



13	24	17	6	5
4	12	21	20	8
7	1	15	23	19
16	10	3	14	22
25	18	9	2	11

~~3-1-4-2-5 by rows~~

16	10	3	14	22
7	1	15	23	19
4	12	21	20	8
13	24	17	6	5
25	18	9	2	11

3-1-4-2-5 by rows

10	14	16	3	22
1	23	7	15	19
12	20	4	21	8
24	6	13	17	5
18	2	25	9	11

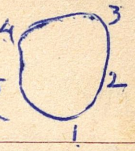
followed by 3-1-4-2-5 by cols

Notes

Notes

12 May - Wednesday

13 May - Thursday



For the case $d = 4$, $1-2-3-4-5 \longrightarrow (4-3-2-1-5)$ by rows

followed by same by columns.

So S_4 gives 3 transformations really for $n = 5$ i.e. $1-2-3-4-5$ going over to $2-4-1-3-5$; $3-1-4-2-5$; $4-3-2-1-5$

by rows followed by same permutations for cols.

while O gives $4-3-2-1-5$ by cols alone.

What about $4-3-2-1-5$ for rows alone

4	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

$\longrightarrow$

7	1	15	23	19
13	24	17	6	5
16	10	3	14	22
4	12	21	20	8
25	18	9	2	11

still remains nasik

So $4-3-2-1-5$ S_4 and O are not different. ✓

Re-even - for the case $n = 4$, only value in S_4 is $\alpha = 3$. what gives for $3-2-1-4$

for rows followed by rows.

For O, we get $3-2-1-4$ by cols. what about rows?

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

$\longrightarrow$

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

still remains nasik

So S_4 and O are identical for $n = 4$. ✓

for higher even values of n like $n = 8$, we can have $\alpha = 3, 5, 7$

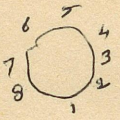
and then perhaps S_4 and O would be different. ✓

Notes

Notes

Take the 8th order on p. 47

(56)



For S_α; $K = \alpha i$, $l = \alpha j$

$\alpha = 3$, $(1,1) \to (3,3)$, $(1,2) \to (3,6)$, $(1,3) \to (3,1)$, $(1,4) \to (3,4)$, $(1,5) \to (3,7)$
$(1,6) \to (3,2)$, $(1,7) \to (3,5)$, $(1,8) \to (3,8)$

$1-2-3-4-5-6-7-8 \to 3-6-1-4-7-2-5-8$ by rows & cols

$\alpha = 5$, $(5,5)$, $(5,2)$, $(5,7)$, $(5,4)$, $(5,1)$, $(5,6)$, $(5,3)$, $(5,8)$

$1-2-\cdots-8 \to 5-2-7-4-1-6-3-8$ by rows & cols.

$\alpha = 7$, $(7,7)$, $(7,6)$, $(7,5)$, $(7,4)$, $(7,3)$, $(7,2)$, $(7,1)$, $(7,8)$

$\to 7-6-5-4-3-2-1-8$ by cols or rows.

Take O: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $K = i$, $l = -j$

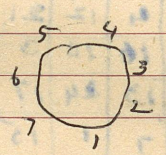
$(i,1) \cdots (1,8) \to (1,7), (1,6), (1,5), \cdots (1,8)$

$i \to 7-6-5-4-3-2-1-8$ by rows or cols.

ie same as S_α for $\alpha = 7$ // just as in the case of $3-2-1-4$ being identical for S_3 & O.

Take $n = 7$, & S_6

$(1,1) \to (6,6), (6,5), (6,4), (6,3), (6,2), (6,1), (6,7)$



ie $1-2-\cdots-6-7 \to 6-5-4-3-2-1-7$

for O: $(1,1) \cdots (1,7) \to (1,6), (1,5) \cdots (1,7)$

ie same as S_6

So again S_6 & O are identical for $n = 7$. In general S_{n-1} & O are identical

---

Let us now consider R & W's paper in Bull. Amer. Math. Soc. relating to $n = 4$ (p. 16) & find out how the transformations therein viz A, B, C, D, E are related to the L, M, S_1, O, P (since Q does not apply to even n).

A is same as P.

C and D are same as M and L respectively.

Notes

In the general theorem 3-3 of The Duke Math. J. paper (p. 6) no transformation corresponding to rotation through a multiple of 90° is given. Is it because it is a trivial one? Try with $n = 5$

Notes

17 May - Monday

18 May - Tuesday

9	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

→

8	22	5	19	11
20	14	6	23	2
21	3	17	15	9
12	10	24	1	18
4	16	13	7	25

by rotation through 90° contraclockwise

$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

$(1,1) \to (5,1)$ $(2,1) \to (5,2)$
$(1,2) \to (4,1)$ $(2,2) \to (4,2)$
$(1,3) \to (3,1)$ $(2,3) \to (3,2)$
$(1,4) \to (2,1)$ $(2,4) \to (2,2)$
$(1,5) \to (1,1)$ $(2,5) \to (1,2)$

$i' = 1 - j$ $\quad i' = ai + cj$
$j' = i$ $\quad j' = bi + dj$ $\quad T_1$

$d = 0,\ b = 1,\ a = 0,$

In fact R & N could have put this as another Ex in addition to the six they have given although they have mentions this transformation

before coming to Th. 3.3 -

for rotation through 180° contra clockwise

$i'' = 1 - j'$ ie $i'' = 1 - i$
$j'' = i'$ $\quad j'' = 1 - j$ $\quad T_2$

for rotation through 270° contra-clockwise

$i''' = 1 - j'' = j$ $\quad i''' = j$
$j''' = i'' = 1 - i$ $\quad j''' = 1 - i$ $\quad T_3$

What about i^{iv}, j^{iv} $\quad i^{iv} = 1 - j''' = i$
$j^{iv} = i''' = j$

$\therefore i^{iv} = i,\ j^{iv} = j$

from this we have for rotation through 90° clockwise same as rotation through 270° contraclockwise

ie $i' = j$
$j' = 1 - i$

for rotation through 180° clockwise is same

$i'' = j'$ and $j'' = 1 - i'$

ie $i'' = 1 - i$ ~~same as 180° contraclockwise~~
$j'' = 1 - j$

for rotation through 270° clockwise

Notes Notes

$$\left.\begin{aligned} i''' &= j'' = 1-j \\ j''' &= 1-i'' = i \end{aligned}\right\}$$

ie same as 90° contra clockwise

In fact we might put in these three trns in addition to the six given by R&W.

What about reflection through a horizontal axis? say contra clockwise

$i+i'=1$

$(1,1) \to (5,1)$	$(2,1) \to (4,1)$	$(3,1) \to (3,1)$
$(1,2) \to (5,2)$	$(2,2) \to (4,2)$	—
—	—	
$(1,5) \to (5,5)$	$(2,5) \to (4,5)$	$(3,5) \to (3,5)$

$$\left.\begin{aligned} i' &= 1-i \\ j' &= j \end{aligned}\right\}$$

T_4

refn about vertical axis:

$$\left.\begin{aligned} (1,1) &\to (1,5) \\ (1,2) &\to (1,4) \\ (1,3) &\to (1,3) \\ (1,4) &\to (1,2) \\ (1,5) &\to (1,1) \end{aligned}\right\} \text{etc.} \quad \text{ie } \left.\begin{aligned} i' &= i \\ j' &= 1-j \end{aligned}\right\}$$

T_5

I think I had better mention these five transformations as trivial ones before going to trns re. Nasik squares.

R & W's trn A for squares of order 4 is the same as the P of the Duke Math. J. Article viz reflection about the lower diagonal.

What about reflection about upper diagonal? Is there a transformation like $P = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$?

Notes

Notes

(59)

21 May - Friday

22 May - Saturday

under reflection about upper diagonal

4	12	21	20	8
16	10	3	14	22
13	24	17	6	5
7	1	15	23	19
25	18	9	2	11

nasik

→

11	19	5	22	8
2	23	6	14	20
9	15	17	3	21
18	1	24	10	12
25	7	13	16	4

nasik

(1,1) → (5,5)
(1,2) → (4,5)
(1,3) → (3,5)
(1,4) → (2,5)
(1,5) → (1,5) ✓

~~(i,j) → (1-j, 1-i)~~

(i,j) → (1-j, 1-i)

~~ai + cj = 1-j~~
~~bi + dj = 1-i~~

$i' + j = n+1 = 1$

$\frac{1-2}{2}$ $\frac{-1}{2} = 2$

$cj = 1-j$
$c = (1-j)/j$

$i' = -j,\ j' = -i$

(2,1) → (5,4)
(2,2) → (4,4)
(2,3) → (3,4)
(2,4) → (2,4) ✓
(2,5) → (1,4)

$i' = 1-j$
$j' = 1-i$

(3,1) → (5,3)
(3,2) → (4,3)
(3,3) → (3,3) ✓
(3,4) → (2,3)
(3,5) → (1,3)

$i' = 1-j$ ✓
$j' = 1-i$ ✓

$1-i = i$
$2i = 1$
$i = 3$

(4,1) → (5,2)
(4,2) → (4,2) ✓
(4,3) → (3,2)
(4,4) → (2,2)
(4,5) → (1,2)

(5,1) → (5,1) ✓
(5,2) → (4,1)
(5,3) → (3,1)
(5,4) → (2,1)
(5,5) → (1,1)

23 Sunday

$i' = 1-j$
$j' = 1-i$ } is correct for reflection about upper diagonal $= T_2P$

Is this correct for $n = 4$

nasik

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

→

nasik

6	9	7	12
3	16	2	13
10	5	11	8
15	4	14	1

Here again $i' = 1-j$
$j' = 1-i$

(1,1) → (4,4)
(1,2) → (3,4)
(1,3) → (2,4)
(1,4) → (1,4)

(2,1) → (4,3)
(2,2) → (3,3)
(2,3) → (2,3)
(2,4) → (1,3)

(3,1) → (4,2)
(3,2) → (3,2)
(3,3) → (2,2)
(3,4) → (1,2)

(4,1) → (4,1)
(4,2) → (3,1)
(4,3) → (2,1)
(4,4) → (1,1)

Notes

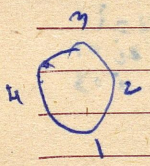
Combine T_2 and P: T_2P makes $i \to j \to 1-j$
$j \to i \to 1-i$ } ie

reflection about upper diagonal is given by $T_2 P$

This is correct since we can see directly by taking the ~~reflection~~ square reflection about the lower diagonal (✱ on p.53) becomes after rotation through 180° the square reflection about the lower diagonal (✱✱ p.59)

~~Notation~~ So A is P
B is T_1
C is M
D is L

So we have only to find E. This must be S_α which is equal to 0 in the case n = 4 & this is permutation of cols or rows in the order 3-2-1-4



a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

(1)

→ (3-2-1-4) by cols

c	b	a	d
g	f	e	h
k	j	i	l
o	n	m	p

→ (by rows)

k	j	i	l
g	f	e	h
c	b	a	d
o	n	m	p

X Try 2-1-4-3
1-4-3-2

a	d	c	b
e	h	g	f
i	l	k	j
m	p	o	n

by rows

e	f	g	h
a	b	c	d
m	n	o	p
i	j	k	l

→ by cols

f	e	h	g
b	a	d	c
n	m	p	o
j	i	l	k

X

a	d	h	e
b	c	g	f
n	o	k	j
m	p	l	i

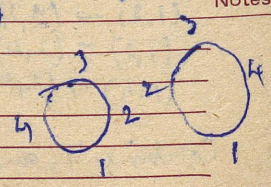
(2)

Notes

For (1) → (2)

(1,1) → (1,1)
(1,2) → (2,1)
(1,3) → (2,2)
(1,4) → (1,2)

Loops very tickling.



$E =$

a	d	h	e
b	c	g	f
n	o	k	j
m	p	l	i

$(1,1) \to (1,1)$ $(2,1) \to (1,4)$ $(3,1) \to (4,4)$ $(4,1) \to (4,1)$
$(1,2) \to (2,1)$ $(2,2) \to (2,4)$ $(3,2) \to (3,4)$ $(4,2) \to (3,1)$
$(1,3) \to (2,2)$ $(2,3) \to (2,3)$ $(3,3) \to (3,3)$ $(4,3) \to (3,2)$
$(1,4) \to (1,2)$ $(2,4) \to (1,3)$ $(3,4) \to (4,3)$ $(4,4) \to (4,2)$

$E^2 =$

a	e	f	b
d	h	g	c
p	l	k	o
m	i	j	n

$E^3 =$

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

$= E$

All the transformations L, M, S_α, O and P takes rows & cols diag into rows, cols and diagonals. So looks futile to expect E to be a product of any of these. Q does not hold for $n = 4$. Hence it appears that E is a new transformation. Let us see if we can find a corresponding T. from the above relations (*)

$i' = ai + cj$
$j' = bi + dj$

$1 = a + c$ | $a + 2c = 2$ | $a + 3c = 2$ | $a + 4c = 1$
$1 = b + d$ | $b + 2d = 1$ | $b + 3d = 2$ | $b + 4d = 4$

$-c = 1$
$a - 4 = 1$

$b + 6 = 2$ $-d = -2$ $d = 2$

$1 = a + c$, $1 = b + d$ | $2 = a + 2c$, $1 = b + 2d$ | $2 = a + 3c$, $2 = b + 3d$ | $1 = a + 4c$, $2 = b + 4d$

$a + c = 1$, $a + 2c = 2$, $c = 1, a = 0$ | $a + 3c = 2$, $a + 4c = 1$, $c = -1, a = 5$ | It looks impossible to get a $T = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

nasik

$\to$ (1-4-3-2 by cols.)

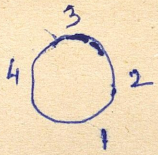
1	12	13	8
14	7	2	11
4	9	16	5
15	6	3	10

nasik

1	12	7	14
8	13	2	11
10	3	16	5
15	6	9	4

nasik

Notes Notes



(62)

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

→ (1-4-3-2 by rows)

1	8	13	12
15	10	3	6
4	5	16	9
14	11	2	7

nasik

& by cols

1	12	13	8
14	7	2	11
4	9	16	5
15	6	3	10

nasik

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

→ (4-3-2-1 by rows)

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

nasik

& by cols

12	13	8	1
7	2	11	14
9	16	5	4
6	3	10	15

nasik

(4-2-3-1) by cols

(3-2-1-4) by ~~cols~~ rows

12	8	13	1
7	11	2	14
9	5	16	4
6	10	3	15

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

nasik

& by cols

13	8	1	12
2	11	14	7
16	5	4	9
3	10	15	6

nasik

(2-1-4-3 by rows) & by cols

14	11	2	7
1	8	13	12
15	10	3	6
4	5	16	9

nasik

8	1	12	13
11	14	7	2
5	4	9	16
10	15	6	3

nasik

From these different results

we have

(1) 1-2-3-4
2-3-4-1
3-4-1-2
4-1-2-3

give L & M for ~~cols~~ rows ~~rows~~ & cols.

30 Sunday

(2) 3-2-1-4 by cols is given by (p. 52)

$$O = \begin{Vmatrix} 1 & 0 \\ 0 & -1 \end{Vmatrix}$$

and successively 2-1-4-3, 1-4-3-2 & 4-3-2-1

are obtained from 3-2-1-4. as MO, M^2O & M^3O

Notes

~~We~~ Similarly from the square obtained by 3-2-1-4 by rows

~~given for~~ those ~~obtained by~~ for 2-1-4-3, 1-4-3-2, & 4-3-2-1

are obtained by multiplying on the left by L, L^2, L^3.

Notes

31 May - Monday

1 June - Tuesday

a	d	h	e
b	c	g	f
n	o	k	j
m	p	l	i

a	d		
b	c	g	f
		k	j
m	p	l	i

So we have to find a tr^n corresponding to O for rows.

Try $O' = \begin{Vmatrix} -1 & 0 \\ 0 & 1 \end{Vmatrix}$ ~~i' = -i~~ $i' = -i,\ j' = j$

Pl. Take 1st col: $(1,1) \to (3,1)$
$(2,1) \to (2,1)$
$(3,1) \to (1,1)$
$(4,1) \to (4,1)$

(3,1)		(1,1)	
(2,1)			
(1,1)		(1,1)	
(4,1)			

i.e. 3 - 2 - 1 - 4 by rows.

Let us call this O'. So the successive ones are given by
LO', L^2O', L^3O'

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

(1)

a	d	h	e
b	c	g	f
n	o	k	j
m	p	l	i

(2)

Spare from the left quadrant $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

The other members in (2) are filled by using antipodal elements of a, b, c, d and using sum of all quadrants = 34.

b, e, o, l This is exactly what R & N say that form (2) can be obtained thus two theorems.

a	d	h	e
b	c	g	f
n	o	k	j
m	p	l	i

with $\begin{vmatrix} a & d \\ b & c \end{vmatrix}$ antipodal elements of c is i (uniquely)
" d is j ("
" b is l (")

~~So that the opposite quadrant is formed~~
~~Next form dgjm — second or first last element as f~~
~~Fourth and 2nd " " p~~

so that the opposite quadrant is uniquely determined as $\begin{vmatrix} k & j \\ l & i \end{vmatrix}$

Next write dgjm. Then everything else is uniquely determined by using sum of quadrants = 34

Suppose I take the first quadrant as $\begin{vmatrix} a & b \\ d & c \end{vmatrix}$ Then opposite quadrant is formed as $\begin{vmatrix} k & l \\ j & i \end{vmatrix}$

a	b	f	e
d	c	g	h
p	o	k	l
m	n	j	i

Then fill up bgfm

~~Another~~ An alternative form

Surely other alternative forms can be obtained by varying a, b, c, d,

Notes

Notes

(64)

№ 16418 606
2N4
K.S.R.T.C.
12-10

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

Suppose a & k are antipodal,

$a + k = 17$

$\therefore f + p = 17$

from diagonal (p, i, f, c)

$i + c = 17$

(from i, n, c, h) $h + n = 17$

b, d
e, f
j, l
m, o

$e + f + i + j = 34$

$p + f + i + c = 34$

$e - p + j - i = 0$ ~~e+j = i+p~~ ~~p+i = e+j~~

m	n	o	p
a	b	c	d
e	f	g	h
i	j	k	l

~~a+n+k+h = 34.~~

$m + g = 17$, $b + l = 17$

$j + d = 17$

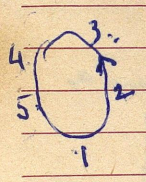
$e + o = 17$

i	j	k	l
m	n	o	p
a	b	c	d
e	f	g	h

It looks rather difficult to prove antipodal property for all diagonals assuming one antipodal pair say $a + k = 17$

Fig. 11 (p. 240, Bk. 2) under $S_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$; $i' = 2i$, $j' = 2j$

(1,1) → (2,2)	(2,1) → (4,2)	(3,1) → (1,2)	(4,1) → (3,2)	(5,1) → (5,2)
(1,2) → (2,4)	(2,2) → (4,4)	(3,2) → (1,4)	(4,2) → (3,4)	(5,2) → (5,4)
(1,3) → (2,1)	(2,3) → (4,1)	(3,3) → (1,1)	(4,3) → (3,1)	(5,3) → (5,1)
(1,4) → (2,3)	(2,4) → (4,3)	(3,4) → (1,3)	(4,4) → (3,3)	(5,4) → (5,3)
(1,5) → (2,5)	(2,5) → (4,5)	(3,5) → (1,5)	(4,5) → (3,5)	(5,5) → (5,5)



11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

under S_2

=

22	6	20	4	13
11	25	9	18	2
5	14	23	7	16
19	3	12	21	10
8	17	1	15	24

(2-4-1-3-5) by cols

+

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

(2-4-1-3-5) by ms.

(not nesik)

2-4-1-3-5
4-1-3-5-2
1-3-5-2-4
3-5-2-1-4
5-2-1-4-3

Notes

Notes

4

June - Friday

under $S_3 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$(1,1) \to (3,3)$
$(1,2) \to (3,1)$
$(1,3) \to (3,4)$
$(1,4) \to (3,2)$
$(1,5) \to (3,5)$

3-1-4-2-5

under S_2 consider the other cases 3-5-2-4-1, 4-1-3-5-2, 5-2-4-1-3, 1-3-5-2-4.

4	13	22	6	20
18	2	11	25	9
7	16	5	14	23
21	10	19	3	12
15	24	8	17	1

(a)

7	16	5	14	23
15	24	8	17	1
18	2	11	25	9
21	10	19	3	12
4	13	22	6	20

(b)

15	24	8	17	1
18	2	11	25	9
21	10	19	3	12
4	13	22	6	20
7	16	5	14	23

(c)

Try 11 under 3-5-2-4-1 by cols. & then by rows

(i) under L

24	8	17	1	15
2	11	25	9	18
10	19	3	12	21
13	22	6	20	4
16	5	14	23	7

(d)

(e) under M

2	11	25	9	18
10	19	3	12	21
13	22	6	20	4
16	5	14	23	7
24	8	17	1	15

(d) under L (e)

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

(f) (d) under M.

If LM = ML = N, then 3-5-2-4-1

$N^2(3-5-1-2-4) = S_2$

$N^2(\quad) = N^3 S_2$

19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24
11	25	9	18	2

LS_2

L	1	3	5	2
4	3	12	21	10	19
1	6	20	4	13	22
3	14	23	7	16	5
5	17	1	15	24	8
2	25	9	18	2	11

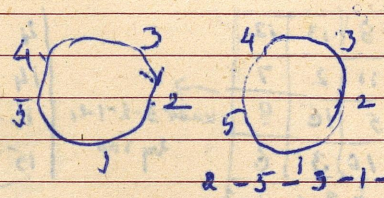
$ML S_2$

Notes

Notes

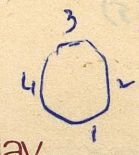
5

June - Saturday



2-5-1-3-4-1-4.

7 June - Monday



1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

→ under 3-2-1-4 by rows

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

→ same by cos

16	5	4	9
2	11	14	7
13	8	1	12
3	10	15	6

↓ 3-4-1-2 by rows

4	5	16	9
15	10	3	6
1	8	13	12
14	11	2	7

→ 3-4-1-2 by cos

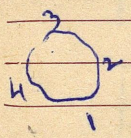
16	9	4	5
3	6	15	10
13	12	1	8
2	7	14	11

Switches interchanging opp. quadrants

$i' = -j,\ j' = -i$

Suppose we take

$$S_3^1 = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$



$(4,1) \to (3,4)$
$(4,2) \to (2,4)$
$(4,3) \to (1,4)$
$(4,4) \to (4,4)$

$(1,1) \to (3,3)$
$(1,2) \to (2,3)$
$(1,3) \to (1,3)$
$(1,4) \to (4,3)$

$(2,1) \to (3,2)$
$(2,2) \to (2,2)$
$(2,3) \to (1,2)$
$(2,4) \to (4,2)$

$(3,1) \to (3,1)$
$(3,2) \to (2,1)$
$(3,3) \to (1,1)$
$(3,4) \to (4,1)$

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

→

16	2	13	3
5	11	8	10
4	14	1	15
9	7	12	6

S_x^1 gives same as 3-2-1-4 by rows + cols but with rows interchanged with columns.

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

16	5	4	9
2	11	14	7
13	8	1	12
3	10	15	6

We have to find the 1st corresponding to

~~4-3-2-1~~
~~3-4-1-2~~ 1-2-3-4
~~4-1-2-3~~ 2-3-4-1
3-4-1-2
4-1-2-3

20	22	4	6	13
9	11	18	25	2
23	5	7	14	16
12	19	21	3	10
1	8	15	17	24

→ under 4-1-3-5-2 by rows

12	19	21	3	10
20	22	4	6	13
23	5	7	14	16
1	8	15	17	24
9	11	18	25	2

→ do by cos

3	12	21	10	19
6	20	4	13	22
14	23	7	16	5
17	1	15	24	8
25	9	18	2	11

(B)

Notes

Fig-11 gp 240

(A)



Notes

(67)

9
June - Wednesday

10
June - Thursday

~~Suppose we apply on Fig II by L.M~~ = N

9	11	18	25	2
23	5	7	14	16
12	19	21	3	10
1	8	15	17	24
20	22	4	6	13

under L

11	18	25	2	9
5	7	14	16	23
19	21	3	10	12
8	15	17	24	1
22	4	6	13	20

under M (A')

23	5	7	14	16
1	8	15	17	24
9	11	18	25	2
12	19	21	3	10
20	22	4	6	13

(A) under 3-5-2-4-1 by rows.

→ do under C

7	6	5	14	23
15	24	8	17	1
18	2	11	25	9
21	10	19	3	12
4	13	22	6	20

do by cols (C)

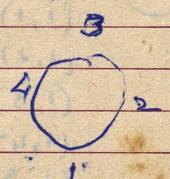
Operate on II(a) by LM = N.

19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24
11	25	9	18	2

→ under M

3	12	21	10	19
6	20	4	13	22
14	23	7	16	5
17	1	15	24	8
25	9	18	2	11

which is exactly (B)



II(a) under L

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

→ under 3-4-1-2 by rows

4	5	16	9
15	10	3	6
1	8	13	12
14	11	2	7

→ 3 by 4 by 2 by 1

16	9	4	5
3	6	15	10
13	12	1	8
2	7	14	11

Notes

Notes

Notes

Latin Conversion Knight's tours interchange of upper diagonal quadrants

for a magic square of order n = 8. Take

11

June - Friday

64	2	3	61	57	7	6	60
9	55	54	12	16	50	51	13
17	47	46	20	24	42	43	21
40	26	27	37	33	31	30	36
8	58	59	5	1	63	62	4
49	15	14	52	56	10	11	53
41	23	22	44	48	18	19	45
32	34	35	29	25	39	38	28

1	63	62	4	8	58	59	5
56	10	11	53	49	15	14	52
48	18	19	45	41	23	22	44
25	39	38	28	32	34	35	29
57	7	6	60	64	2	3	61
16	50	51	13	9	55	54	12
24	42	43	21	17	47	46	20
33	31	30	36	40	26	27	37

12

June - Saturday

8	58	59	5	1	63	62	4
49	15	14	52	56	10	11	53
41	23	22	44	48	18	19	45
32	34	35	29	25	39	38	28
64	2	3	61	57	7	6	60
9	55	54	12	16	50	51	13
17	47	46	20	24	42	43	21
40	26	27	37	33	31	30	36

13 Sunday

20	22	4	6	13
9	11	18	25	2
23	5	7	14	16
12	19	21	3	10
1	8	15	17	24

23	5	7	14	16
20	22	4	6	13
12	19	21	3	10
9	11	18	25	2
1	8	15	17	24

7	23	14	5	16
4	20	6	22	13
21	12	3	19	10
18	9	25	11	2
15	1	17	8	24

Notes

Notes

20	22	4	6	13
12	19	21	3	10
9	11	18	25	2
1	8	15	17	24
23	5	7	14	16

→ by cols to

20	6	22	13	4
12	3	19	10	21
9	25	11	2	18
1	17	8	24	15
23	14	5	16	7

11(e')

Fig. 11 under 1-4-2-5-3 by row

Subject 11(e) to N

$L\{(11(e)\} =$

4	20	6	22	13
21	12	3	19	10
18	9	25	11	2
15	1	17	8	24
7	23	14	5	16

→ under M

20	6	22	13	4
12	3	19	10	21
9	25	11	2	18
1	17	8	24	15
23	14	5	16	7

which is exactly 11(e').

S_4

1	8	15	17	24
12	19	21	3	

Fig. 11 under 4-3-2-1-5 by rows

under S_3 Fig 1 → 1(d)

12	19	21	3	10
23	5	7	14	16
9	11	18	25	2
20	22	4	6	13
1	8	15	17	24

→ by cols

3	21	19	12	10
14	7	5	23	16
25	18	11	9	2
6	4	22	20	13
17	15	8	1	24

Fig 11(f).

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

Fig. 1

→

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

3-2-1-4 by rows

→

16	5	4	9
2	11	14	7
13	8	1	12
3	10	15	6

Do by cols 1(d)

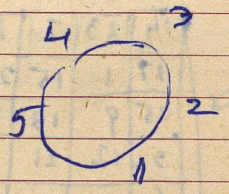


Fig 11(g) of p. 247 of BK. 2. ~~Change~~ Transpose the columns in the order 4, -2-5-3-1, (~~transp to N^2S_3~~) ~~merged~~ & rows in same order (N^2S_3)

1	19	7	25	13
14	2	20	8	21
22	15	3	16	9
10	23	11	4	17
18	6	24	12	5

1	19	7	25	13
10	23	11	4	17
14	2	20	8	21
18	6	24	12	5
22	15	3	16	9

Rows → cols $= T_1 T_4$

8	1	6
3	5	7
4	9	2

T_1

4	3	8
9	5	1
2	7	6

T_4

8	3	4
1	5	9
6	7	2

16 June - Wednesday

6	20	4	22	13
25	9	18	11	2
14	23	7	5	16
3	12	21	19	10
17	1	15	8	24

6	20	4	13	22
25	9	18	2	11
14	23	7	16	5
3	12	21	10	19
17	1	15	24	8

22	20	13	6	4
11	9	2	25	18
5	23	16	14	7
19	12	10	3	21
8	1	24	17	15

14	23	7	16	5
17	1	15	24	8
25	9	18	2	11
3	12	21	10	19
6	20	4	13	22

17 June - Thursday

6	13	20	22	4
18	2	9	11	25
7	16	23	5	14
21	10	12	19	3
15	24	1	8	17

4	22	20	13	6
18	11	9	2	25
7	5	23	16	14
21	19	12	10	3
15	8	1	24	17

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

11	9	2	25	18
5	23	16	14	7
19	12	10	3	21
8	1	24	17	15
22	20	13	6	4

11	25	9	18	2
5	14	23	7	16
19	3	12	21	10
8	17	1	15	24
22	6	20	4	13

5	14	23	7	16
8	17	1	15	24
11	25	9	18	2
19	3	12	21	10
22	6	20	4	13

8	17	1	15	24
11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

12	8	4	25	16
5	21	17	13	9
18	14	10	1	22
6	2	23	19	15
24	20	11	7	3

12	5	18	6	24
8	21	14	2	20
4	17	10	23	11
25	13	1	19	7
16	9	22	15	3

8	5	12	19	24
2	21	8	14	20
23	17	4	10	11
19	13	25	1	7
15	9	16	22	3

Notes

Notes

18 June - Friday

19	13	25	1	7
6	5	12	18	24
23	17	4	10	11
15	9	16	22	3
2	21	8		

5	6	12	18	24
21	2	8	14	20
17	23	4	10	11
13	19	25	1	7
9	15	16	22	3

ie for cols. we have taken the transposition 2-4-1-3-5 of S_2 along with a transposition 4-2-5-3-1 of S_3

4-2-5-3-1 by rows

13	19	25	1	7
21	2	8	14	20
9	15	16	22	3
17	23	4	10	11
5	6	12	18	24

which is 11 (g).

22	6	20	4	13
19	3	12	21	10
11	25	9	18	2
8	17	1	15	24
5	14	23	7	16

Instead of 1-4-2-5-3 try 4-2-5-3-1 for rows

~~Also not nasik in~~ nasik

~~Fig 2 Diagonals~~

Fig 11 with 1-4-2-5-3 for cols.

20	6	22	13	4
9	25	11	2	18
23	14	5	16	7
12	3	19	10	21
1	17	8	24	15

→ (2-4-1-3-5 for rows

19 June - Saturday

Take Fig 11 (e) corresponding to S_3. Apply 2-4-1-3-5 of S_2 for cols and 1-5-4-3-2 of S_4 for rows

23	5	7	14	16
20	22	4	6	13
12	19	21	3	10
9	11	18	25	2
1	8	15	17	24

neither nasik nor magic

→

23	5	7	14	16
1	8	15	17	24
9	11	18	25	2
12	19	21	3	10
20	22	4	6	13

not nasik nor magic

Take Fig. 11 & apply the above two

22	6	20	4	13
11	25	9	18	2
5	14	23	7	16
19	3	12	21	10
8	17	1	15	24

neither magic nor nasik

→

22	6	20	4	13
8	17	1	15	24
19	3	12	21	10
5	14	23	7	16
11	25	9	18	2

neither magic nor nasik

So I think we cannot elect S_2, S_3 & S_4 perhaps only S_2 & S_3. In the above try with 1-4-2-5-3 of S_3 where 1-5-4-3-2 of S_4

20 Sunday

19	3	12	21	10
11	25	9	18	2
8	17	1	15	24
5	14	23	7	16
22	6	20	4	13

nasik.

9	25	11	2	18
12	3	19	10	21
20	6	22	13	4
23	14	5	16	7
1	17	8	24	15

also nasik

Notes

Notes

(72)

Take 11 (f) & use 1-3-5-2-4 for rows & 1-4-2-5-3 for cols
($g S_2$) ($g S_3$)

3	21	19	12	10
25	18	11	9	2
17	15	8	1	24
14	7	5	23	16
6	4	22	20	13

→

3	12	21	10	19
25	9	18	2	11
17	1	15	24	8
14	23	7	16	5
6	20	4	13	22

Nasik

Take 11 (h) & use 1-3-5-2-4 for cols and 1-4-2-5-3 for rows

9	18	2	11	25
23	7	16	5	14
12	21	10	19	3
1	15	24	8	17
20	4	13	22	6

→

9	18	2	11	25
1	15	24	8	17
23	7	16	5	14
20	4	13	22	6
12	21	10	19	3

Do the above for 11 (c)

Nasik

Do the above for 11 (a)

6	22	13	4	20
25	11	2	18	9
14	5	16	7	23
3	19	10	21	12
17	8	24	15	1

→

6	22	13	4	20
3	19	10	21	12
25	11	2	18	9
17	8	24	15	1
14	5	16	7	23

Nasik

11	9	2	25	18
19	12	10	3	21
22	20	13	6	4
5	23	16	14	7
8	1	24	17	15

→

11	9	2	25	18
5	23	16	14	7
19	12	10	3	21
8	1	24	17	15
22	20	13	6	4

Nasik

What is the significance of applying 1-3-5-2-4 (A) by cols followed by applying 1-4-2-5-3 (B) by rows?

20	4	13	22	6
9	18	2	11	25
23	7	16	5	14
12	21	10	19	3
1	15	24	8	17

→

20	4	13	22	6
12	21	10	19	3
9	18	2	11	25
1	15	24	8	17
23	7	16	5	14

20	4	13	22	6
23	7	16	5	14
1	15	24	8	17
9	18	2	11	25
12	21	10	19	3

20	22	4	6	13
12	19	21	3	10
9	11	18	25	2
1	8	15	17	24
23	5	7	14	16

By 11 move 1-3-5-2-4 by cols & 1-4-2-5-3 by rows under (A) by cols & rows ~~under (B) by rows & cols 1-3-5-2-4~~

Notes

20	6	22	13	4
9	25	11	2	18
23	14	5	16	7
12	3	19	10	21
1	17	8	24	15

→

20	6	22	13	4
12	3	19	10	21
9	25	11	2	18
1	17	8	24	15
23	14	5	16	7

under (B) by cols & rows → 1-5-4-3-2 by cols.

1-5-4-3-2 by rows

ie ~~$S_2 S_1$~~ ~~$S_4 S_2$~~ } $O'S_2$

~~$S_4 S_3$~~ } $O S_3$

~~Fig 11 under~~ S_2

$i' = 2i,\ j' = 2j$

(1,1) → (2,2)	(2,1) → (4,2)	(3,1) → (1,2)
(1,2) → (2,4)	(2,2) → (4,4)	
(1,3) → (2,1)	(2,3) → (4,1)	
(1,4) → (2,3)	(2,4) → (4,3)	
(1,5) → (2,5)	(2,5) → (4,5)	

7	23	14	5	16
4	20	6	22	13
21	12	3	19	10
18	9	25	11	2
15	1	17	8	24

22	6	20	4	13
11	25	9	18	2
5	14	23	7	16
19	3	12	21	10
8	17	1	15	24

Fig 14 (2-4-1-3-5) by cols

→

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

Do by rows

9	11	18	25	2
12	19	21	3	10
20	22	4	6	13
23	5	7	14	16
1	8	15	17	24

Fig 11 (2-4-1-3-5) by rows

→

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

Do by cols

20	6	22	13	4
9	25	11	2	18
23	14	5	16	7
12	3	19	10	21
1	17	8	24	15

Fig 11 under 1-4-2-5-3 by cols

↓ again by rows

20	6	22	13	4
12	3	19	10	21
9	25	11	2	18
1	17	8	24	15
23	14	5	16	7

Fig. 11 under 3-1-4-2-5 by cols & rows.

4	20	6	22	13
18	9	25	11	2
7	23	14	5	16
21	12	3	19	10
15	1	17	8	24

by cols

→ by rows

(S_2)

7	23	14	5	16
4	20	6	22	13
21	12	3	19	10
18	9	25	11	2
15	1	17	8	24

↓ this under M

23	14	5	16	7
20	6	22	13	4
12	3	19	10	21
9	25	11	2	18
1	17	8	24	15

→ under L

20	6	22	13	4
12	3	19	10	21
9	25	11	2	18
1	17	8	24	15
23	14	5	16	7

same

ie NS_2

Notes

Notes

25 June - Friday

26 June - Saturday

I have made a mistake in S_2 & S_3. For S_2 we should have transpositions 3-1-4-2-5, 1-4-2-5-3 etc & not 2-4-1-3-5, etc

(S_3) $i' = 3i$, $j' = 3j$

(1,1) → (3,3)	(2,1) → (1,3)	(3,1) → (4,3)	(4,1) → (2,3)	(5,1) → (5,3)
(1,2) → (3,1)	(1,1)			
(1,3) → (3,4)	(1,4)			
(1,4) → (3,2)	(1,2)			
(1,5) → (3,5)	(1,5)			

Fig 11 becomes

18	9	25	11	2
21	12	3	19	10
4	20	6	22	13
7	23	14	5	16
15	1	17	8	24

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

which is 11(e) as corrected

Fig 11 under 2-4-1-3-5 by cols.

(S_3)

22	6	20	4	13
11	25	9	18	2
5	14	23	7	16
19	3	12	21	10
8	17	1	15	24

→

11	25	9	18	2
19	3	12	21	10
22	6	20	4	13
5	14	23	7	16
8	17	1	15	24

by ~~cols~~ rows which is exactly 11(e)

Fig 11 under 4-1-3-5-2 by cols & rows

6	20	4	13	22
25	9	18	2	11
14	23	7	16	5
3	12	21	10	19
17	1	15	24	8

→

3	12	21	10	19
6	20	4	13	22
14	23	7	16	5
17	1	15	24	8
25	9	18	2	11

25	9	18	2	11
3	12	21	10	19
6	20	4	13	22
14	23	7	16	5
17	1	15	24	8

S_3 under M.

→ L

3	12	21	10	19
6	20	4	13	22
14	23	7	16	5
17	1	15	24	8
25	9	18	2	11

Same

ie NS_3

Let us consider the result of transforming Fig. 11 by transposition of cols in the order 1-4-2-5-3 followed by transposition of rows in the order 1-3-5-2-4.

Notes

Notes

28
June - Monday

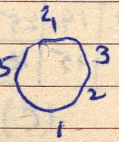
29
June - Tuesday

20	6	22	13	4
9	25	11	2	18
23	14	5	16	7
12	3	19	10	21
1	17	8	24	15

→

20	6	22	13	4
23	14	5	16	7
1	17	8	24	15
9	25	11	2	18
12	3	19	10	21

Interchange rows by 3-1-4-2-5
by cols + 2-4-1-3-5 by rows



Ty·11 under 1-4-2-5-3 by cols + under 1-3-5-2-4 by rows

4	20	6	22	13
18	9	25	11	2
7	23	14	5	16
21	12	3	19	10
15	1	17	8	24

+

18	9	25	11	2
21	12	3	19	10
4	20	6	22	13
7	23	14	5	16
15	1	17	8	24

=

18	9	25	11	2
21	12	3	19	10
4	20	6	22	13
7	23	14	5	16
15	1	17	8	24

$= OS_3$

Ty·11 under 3-1-4-2-5 by cols + 2-4-1-3-5 by rows 11(c) under 4-3-2-1-5 by cols

18	9	25	11	2
21	12	3	19	10
4	20	6	22	13
7	23	14	5	16
15	1	17	8	24

$= O^1S_2$

22	6	20	4	13
11	25	9	18	2
5	14	23	7	16
19	3	12	21	10
8	17	1	15	24

+

5	14	23	7	16
22	6	20	4	13
19	3	12	21	10
11	25	9	18	2
8	17	1	15	24

$= OS_2$
$= O^1S_3$

11(d) under 4-3-2-1-5 by rows Ty·11 under 2-4-1-3-5 by cols + 3-1-4-2-5 by rows

Coming to Q : In Ty·11(g) we have taken the main u.d of Ty·11 in some order & for the square

(1,-1) →
↓
(0,1)

12	8	4	25	16
5	21	17	13	9
18	14	10	1	22
6	2	23	19	15
24	20	11	7	3

(A)

→ rows ↔ cols
$T_4 T_3$

12	5	18	6	24
8	21	17	13	9
18	14	10	1	22
6	2	23	19	15
24	20	11	7	3

12	5	18	6	24
8	21	14	2	20
4	17	10	23	11
25	13	1	19	7
16	9	22	15	3

(B)

Suppose we take in the first row the lower diagonal 12, 8, 4, 25, 16 & numbers obtained from 12 by steps (1,-1). Form the columns by using steps (1,1) starting from numbers in first row we get (A) in which rows are lower diagonals & columns are upper diagonals. In (A) interchange rows & cols which is equivalent to transformation $T_4 T_3$ and get (B).

In (B) transpose cols. in the order 2-4-1-3-5 & get (C) [P.T.O.]

Notes Notes

30 June - Wednesday

4-2-5-3-1

5	6	12	18	24
21	2	8	14	20
17	23	4	10	11
13	19	25	1	7
9	15	16	22	3

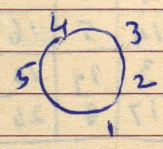
(C) not nasik

Transposing this by rows in order 4-2-5-3-1 yields (D)

1 July - Thursday

13	19	25	1	7
21	2	8	14	20
9	15	16	22	3
17	23	4	10	11
5	6	12	18	24

(D)



∴ (D) is exactly 11(g).

Suppose we take first row in some arbitrarily different order say 8, 25, 4, 16, 12 (steps (2,-2), (-1,1), (2,-2), (1,-1))

To form the cols use steps (2,2), (-1,-1), (2,2), (1,1) and (A')

8	25	4	16	12
14	1	10	22	18
21	13	17	9	5
2	19	23	15	6
20	7	11	3	24

(A') not nasik

$T_4 T_3$

$T_5 T_3$

8	14	21	2	20
25	1	13	19	7
4	10	17	23	11
16	22	9	15	3
12	18	5	6	24

Set (B') not nasik.

Transpose ~~change~~ cols in order 3-4-1-2-5 & get (C')

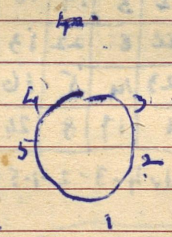
21	2	8	14	20
13	19	25	1	7
17	23	4	10	11
9	15	16	22	3
5	6	12	18	24

(C')

Change rows in order 2-1-4-3-5 & get (D')

13	19	25	1	7
21	2	8	14	20
9	15	16	22	3
17	23	4	10	11
5	6	12	18	24

(D') which is 11(g)



But what are these 3-4-1-2-5 & 2-1-4-3-5? Moreover (A') & (B') are not nasik.

I think we should avoid such choices of the rows.

How can we express the result of the two transpositions 2-4-1-3-5 by cols & 4-2-5-3-1 in rows by means O, O', S_2, S_3?

2-1-5-4-3

5	6	12	18	24
21	2	8	14	20
17	23	4	10	11
13	19	25	1	7
9	15	16	22	3

Fig. 13(a) by 2-4-1-3-5 in cols

+

21	2	8	14	20
13	19	25	1	17
5	6	12	18	24
17	23	4	10	11
9	15	16	22	3

same by rows

If we use transposition 2-1-5-4-3 by rows we get 11(g).

ie by transformation M^2O'.

Notes

Notes

S_2 is 3-1-4-2-5
S_3 is 2-4-1-3-5
S_4 is

19	3	12	21	10
11	25	9	18	2
8	17	1	15	24
5	14	23	7	16
22	6	20	4	13

nasik (A)

8	17	1	15	24
19	3	12	21	10
5	14	23	7	16
11	25	9	18	2
22	6	20	4	13

1	8	15	17	24
12	19	21	3	10
23	5	7	14	16
9	11	18	25	2
20	22	4	6	13

(A) under S_2 (B) nasik

23	5	17	14	16
1	8	15	1	

12	19	21	3	10
9	11	18	25	2
1	8	15	17	24
23	5	7	14	16
20	22	4	6	13

19	3	12	21	10
11	25	9	18	2
8	17	1	15	24
5	14	23	7	16
22	6	20	4	13

(B) under S_3 ie (C') nasik & (C) = (A).

ie $S_3 S_2 = 1 = S_2 S_3$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$$

S_4 is $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

S_4 is 4-3-2-1-5

O is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ & O' is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

P is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, Q is $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$PQ = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$QP = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

4 Sunday

$QP + PQ = S_2$

$S_2^2 = S_4 = S_3^2$, $S_2 S_3 = 1$
$S_4^2 = 1$, $S_2 S_4 = S_3$, $S_3 S_4 = S_2$

Knowing S_2 we find S_4 & from $S_2 S_4 = S_3$ we find S_3 ~~we find S_3 its inverse~~ ~~& next we find $S_4 = S_3^2$~~. Hence only S_2 is enough.

Knowing S_2 & Q we can find P from $PQ + QP = S_2$.

$OQ = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = QP$, $O'Q = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$ & $OQ + O'Q = 0$

This gives 0. Q. L, M, S_2, Q can be taken as fundamental ones

Notes

For $n = 7$, we have S_2, S_3, S_4, S_5, S_6

$S_2^2 = S_4$, $S_3^2 = S_2$, $S_4^2 = S_2$, $S_5^2 = S_4$, $S_6^2 = 1$

$S_2S_3 = S_6$, $S_2S_4 = 1$, $S_2S_5 = S_3$, $S_2S_6 = S_5$

$S_3S_4 = S_5$, $S_3S_5 = 1$, $S_3S_6 = S_4$; $S_4S_5 = S_6$, $S_4S_6 = S_3$, $S_5S_6 = S_2$

Knowing S_2 we find S_4, & S_3 we find S_6, S_5 } use $S_2^2 = S_4$, $S_2S_3 = S_6$, $S_3S_4 = S_5$

ie knowing S_2 & S_3 we find S_4, S_5 and S_6.

$n = 11$. $S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}$

$S_2^2 = S_4$, $S_2S_3 = S_6$, $S_2S_4 = S_8$, $S_2S_5 = S_{10}$, $S_3S_6 = S_7$, $S_4S_5 = S_9$

from S_2, S_3, S_5, S_6 using

we get $S_4, S_7, S_8, S_9, S_{10}$

(13-3)

(7)

18

$S_2S_9 = S_{10}$

ie we found $(n-3)/2$ transpositions

$n = 13$ $S_2^2 = S_4$, $S_3^2 = S_9$, $S_4^2 = S_3$, $S_5^2 = S_{12}$ Assume $S_2, S_3 \to S_4, S_9$.

$S_6^2 = S_{10}$

$S_7^2 = S_{10}$

$S_8^2 = S_{12}$, $S_9^2 = S_3$, S_{10}

$S_2S_3 = S_6$, $S_2S_4 = S_8$, $S_2S_9 = S_5$, $S_3S_4 = S_{12}$, $S_4S_9 = S_{10}$

Assume S_2, S_3 alone we get $S_4, S_5, S_6, S_8, S_9, S_{10}, S_{12}$

S_2, S_3, S_5, S_6, S_7

S_4, S_9, S_{10}, S_{12}

$(13-3)/2 = 5$

81

78

(13-3)/2

(7)

$(13-3)/2$ Assume S_7, S_{11}

S_8, S_{11} $S_2S_4 = S_8$ $S_2S_{12} = S_{11}$ $S_9^2 = S_{13}$

$S_{13}^2 =$

$n = 17$ $S_2^2 = S_4$, $S_3^2 = S_9$, $S_5^2 = S_8$, $S_6^2 = S_2$, $S_7^2 = S_{15}$, $S_9^2 = S_{13}$, $S_{10}^2 = S_{15}$, $S_{11}^2 = S_2$

$S_{13}^2 = S_{16}$

25 36

$S_{12}^2 = S_8$.

2, 3, 5, 7, 9,

4, 9, 8, 15, 13,

Notes

$S_2S_3 = S_6$, $S_3S_9 = S$

$S_5S_{10} = S_{10}$, $S_5S_9 = S_{11}$

$S_7S_9 = S_{12}$

$S_6S_x6x = 12 + 174$

29, 46, 63.

for $n = 7$, $2 + 3$

$n = 11$, $4 + 5$

$n = 13$, $5 + 6$.

ie $q + q + 1 = n - 2$

7 July - Wednesday

8 July - Thursday

~~68, 826, 306~~
~~34, 413, 153~~

Chapters on

(1) Step squares of odd order —— (1)

(2) N-method of combining auxiliary squares —— (2)

(3) Bordered magic squares —— ~~(10)~~ ~~(11)~~ (12)

(4) Composite squares & Heredity —— ~~(9)~~ ~~(10)~~ (11)

(5) Singly-even order squares —— (8)

(6) Cyclical method. [Stern] —— (3)

(7) Irregular (non-latin & non-cyclic) - Also Rosser & Walker —— (4)

(8) Bimagic & trimagic squares —— ~~(11)~~ ~~(12)~~ (13)

(9) Squares of ~~[illegible]~~ whose elements are primes ~~(12)~~ ~~(13)~~ (14)

(10) 3rd order squares —— (5)

(11) 4th order squares —— (6)

(12) 5th order squares —— (7)

(13) Upside-down magic squares Miscellaneous topics & problems —— ~~(15)~~ (16)

(14) ~~[illegible]~~ Squares of orders 8 & 16 —— ~~(8)~~ (9)

(15) Ramanujan's work & anti-magic squares —— ~~(14)~~ (15)

(16) Squares of order 9 and 15 —— (10)

Notes

1	9	16	14
17	13	2	8
4	6	19	11
18	12	3	7

Notes

Chapter I:

Bk. 11, p. 173 - de la Loubère's method from Kraitchik explained in an alternative way - not very simple

Bk. 12 p. 23 - Expln of above method - Perhaps simpler than in Kraitchik

p. 92 - A 7th order nasik square from Rouse-Ball is given

[Here ~~[illegible]~~ primary move is (1, 2) & Break move is (2, -1)]

p. 99 - Terrace method from Domoroyd for n = 5

[Here primary moves (1,1) and breakaway is (2,0).]

Narayana Pandit's squares of order 5 & 7. - ~~Equivalent to normal case~~ - reduced to normal case

11	10	4	23	17
18	12	6	5	24
25	19	13	7	1
2	21	20	14	8
9	3	22	16	15

(1, -1), (-1, 0) - Semi-nasik

~~The~~ associated:

$a + a' = -1$, $b + b' = 0$

1	5	4	3	2
3	2	1	5	4
5	4	3	2	1
2	1	5	4	3
4	3	2	1	5

+

10	5	0	20	15
15	10	5	0	20
20	15	10	5	0
0	20	15	10	5
5	0	20	15	10

Cycle (2,3) Cycle (1, 4). → Hence semi-nasik

$a = 1$, $b = -1$, $a' = -2$, $b' = 1$

$(1 - a - a')2 = 4 = i$

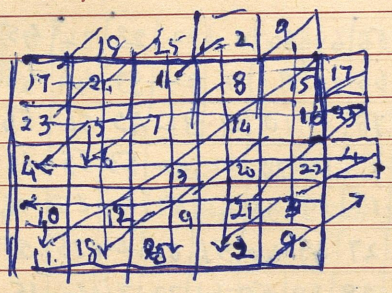
$(1 - b - b')2 = 2 = j$

$(i, j) = (4, 2)$ correctly for the associated case

If these squares are rotated contra-clockwise through 90°, we have exactly the case of the I. D. M with $(i, j) = (2, 4)$ and moves (1, 1) and (0, -1).

Notes Notes

12
July - Monday



17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

$a=1, b=1$ | $i=2$
$a'=-1, b'=-2$ | $j=4$

1st term 6 (1.5+1) (1,2) i.e (2−1, 4−2)
2nd " 11 (2.5+1) $(i+2a', j+2b')$ i.e (0,0)
3rd " 16 (3.5+1) $(i+3a', j+3b')$ i.e (−1,−2)
4th " 21 (4.5+1) $(i+4a', j+4b')$ i.e (−2,−3) (3,1)

5th term in cell (2+4, 4+4) i.e (1,8)

1st D.M $\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}$

Islami $\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$

Wilson $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$a-b=0, a'-b'=1, a+b=2, a'+b'=-3$. ✗
$a-b=0, a'+b'=-3$ ✗ ; $a+b=2, a'-b'=1$

47	58	69	80	1	12	23	34	45
57	68	79	9	11	22	33	44	46
67	78	8	10	21	32	43	54	56
77	7	18	20	31	42	53	55	66
6	17	19	30	41	52	63	65	76
16	27	29	40	51	62	64	75	5
26	28	39	50	61	72	74	1	15
36	38	49	60	71	73	2	14	25
37	48	59	70	81	8	13	24	35

x.4

Fr 4-d: (0,4), (1,4) ... (8,4) $x+4y+1$ fns
(a,4) 37, 38, ... 45 as above

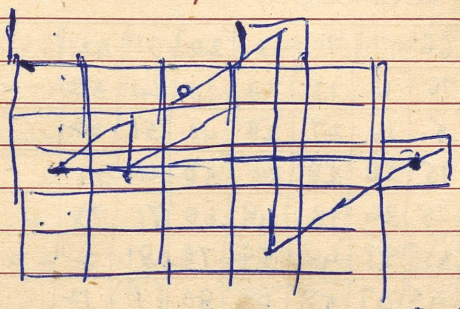
Continuous Dep multiple (Chap I) for 11 composite - Cannon

Notes

(1) $\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}_9$ (2) $\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}_9$ (3) $\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}_{15}$ (4) $\begin{pmatrix} 2 & 8 \\ 7 & 2 \end{pmatrix}_{15}$

Notes

13
July - Tuesday



For (i) three values of $(i,j) = (7,2), (4,8), (1,5)$ 3 cases.

(82)

14

July - Wednesday

53	55	66	77	7	18	20	31	42
63	65	76	6	17	19	30	41	52
64	75	5	16	27	29	40	51	62
74	4	15	26	28	39	50	61	72
3	14	25	36	38	49	60	71	73
13	24	35	37	48	59	70	81	2
23	34	45	47	58	69	80	1	12
33	44	46	57	68	79	9	11	22
43	54	56	67	78	8	10	21	32

$(i,j) = (7,2)$

15

July - Thursday

47	58	69	80	1	12	23	34	45
57	68	79	9	11	22	33	44	46
67	78	8	10	21	32	43	54	56
77	7	18	20	31	42	53	55	66
6	17	19	30	41	52	63	65	76
16	27	29	40	51	62	64	75	5
26	28	39	50	61	72	74	4	15
36	38	49	60	71	73	3	14	25
37	48	59	70	81	2	13	24	35

$(i,j) = (4,9)$ but

not semi-magic ~~not~~ associated

(1) $\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}_{15}$ $(i,j) = (13,5), (10,2), (7,14), (4,11), (7,8)$ 5 cases.

(2) $\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}_9$ $(x,y) = (1,1), (1,4), (1,7)$
$(4,1), (4,4), (4,7)$
$(7,1), (7,4), (7,7)$

If any of these placed in middle cell we have a magic square & corresponding (i,j) is easily determined.

Replacement $(1,4)$
Replacement $(3,-1)$

$i + x + 2y = 4, \quad j + 4x - 5y = 4$

$i = 4 - x - 2y, \quad j = 4 - 4x + 5y$

$(i,j) = (1,5), (4,2), (7,8)$
$(7,2), (1,8), (4,5)$
$(4,8), (7,5), (1,2)$

9 cases

$-16 + 39$
23
-19

$-28 + 9 = -19$

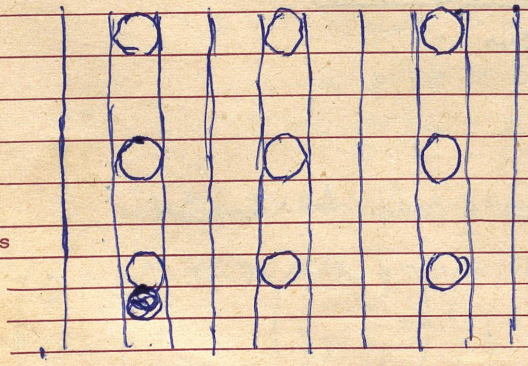
$4 - 7 - 8$
-11

$4 - 28 + 20$

$4 - 7 - 14 = -17$

$4 - 28 + 35$

39
28
11



20	28	45	53	61	69	77	4	12
6	14	22	30	38	46	63	71	79
64	81	8	16	24	32	40	48	56
50	58	66	74	1	18	26	34	42
36	44	52	60	68	76	3	11	19
22	21	29	37	54	62	70	78	5
80	7	15	23	31	39	47	55	72
57	65	73	9	17	25	33	41	49
43	51	59	67	75	2	10	27	35

$(i,j) = (4,5)$

Notes

Notes

$i + 2x + 8y = 7$

$j + 7x + 2y = 7$

$(x, y) = (7, 7)$

$i = 7 - 14 - 56 = -63 \equiv 12$

$j = 7 - 49 - 14 \equiv -56 \equiv 4$

133
7

(837)

(4) $\begin{pmatrix} 2 & 8 \\ 7 & 2 \end{pmatrix}_{15}$

$(x, y) = (7, 7)$ & $i + 2x + 8y = 7$, $j + 7x + 2y = 7$ } gives only one $(i, j) = (12, 4)$ only one case

associated

$(a - b)x + (a' - b')y \equiv c \rightarrow -5x + 6y \equiv c \pmod{15}$

$(a + b) + (a' + b')y \equiv d \rightarrow 9x + 10y \equiv d$

$-5, 6, 9, 10$ are all not prime to $n = 15$

(5) $\begin{pmatrix} 1 & -1 \\ -2 & 4 \end{pmatrix}_{15}$ 5 cases

(6) $\begin{pmatrix} 5 & 2 \\ 2 & -5 \end{pmatrix}_{21}$ only one case $(i, j) = (18, 4)$ ~~[illegible]~~ $(x, y) = (7, 10)$

ie $7 + 210 + 1 = 218$ in central cell

$\frac{1}{2}(n^2 + 1) = \frac{1}{2} \cdot 442 = 221$

$3x + 7y \equiv 16$

$7x - 3y \equiv 19$

$21x + 49y \equiv 112$

$21x - 9y \equiv 57$

$58y \equiv 55$

$21 + 70 - 17$

91
17
74

$210 + 1 + 10$

$30 + 70 - 17$

$100 - 17 = 183$

$3x + 7y \equiv c$

$7x - 3y \equiv d$

$21x + 49y \equiv 112$

$21x - 9y \equiv 57$

$58y \equiv 55$

580
55
525

$21 \quad 100 \quad 10 \quad 40 - 19$
$70 \quad 16 \quad 30 \quad = 21$
$91 \quad 84 \quad 40$
16
75

$(10, 10)$

$9x + 21y = 48$

$49x - 21y = 133$

$58x \equiv 181$

$i + ax + a'y = m$

$j + bx + b'y = m$

$i + 5x + 2y = 10$

$j + 2x - 5y = 10$

$i = 10 - 5x - 2y = -60 = 3$

$j = 10 - 2x + 5y = 40 \equiv 19$ $(3, 19)$

$-60 \equiv x \pmod{21}$

$x + 60 = 0 \pmod{21}$

$x = 3$

$21 = 2 \cdot 10 + 1$

$m = 10$ $(10, 20)$

$-60 \equiv 3$

$40 \equiv 19$

$29y = 41$

$29y - 41$

$i = (1 - 5 - 2)10$, $j = (1 - 2 + 5)10$

$(i, j) = (3,$

$7x + 7y \equiv d$

$58y - 45$

$406 \quad 464 \quad 522$
$45 \quad 45 \quad 45$
$361 \quad 419 \quad 477$

290
41
249

$174 \quad 232$
$45 \quad 45$
$129 \quad 177$

$290 \quad 348$
$45 \quad 45$
$245 \quad 303$

580
45
535

18 Sunday

$525 \quad 25$
42
105

$(1 - 5 - 2)10 = -60$

$(1 - 2 + 5)10 = 40$

580
181
399

$21)399(19$
21
189

$(10, 10)$

$i + 5x + 2y = 10$

$j + 7x - 3y = 10$

$i = 10 - 5x - 2y$

$j = 10 - 7x + 3y$

$i = 10 - 50 - 20 = -60$

$j = 10 - 70 + 30 = -30$

$-60 + x$

Notes

Notes

Also given by

$i = (1 - u - a')m$

$j = (1 - v - b')m$

(84)

19 July - Monday

So that the square must be associated with (i,j) at $(3,19)$

$(i,j) = (3,19)$

20 July - Tuesday

$5x \equiv 1$

$5x - 1$

$i + 5x + 2y$ $\quad (3 + 5x + 2y,\ 19 + 2x - 5y)$

$j + 2x - 5y$ $\quad$ If this be cell $(0,0)$

$3 + 5x + 2y = 0 - 3$

$19 + 2x - 5y = 0 - 19$

$29y - 89$

$29y = -6 + 95 = 89$

This gives $y = 19$

$5x + 2y = -3$

$5x + 38 = -3$, $5x = -41$

At $(0,0)$

$x = 17$

ie number is $14 + 399 + 1$

$(17,19)$

$17 + 399 + 1 = 417$

Let us find out cell complementary to $(0,0)$ ie $(20,20)$

$3 + 5x + 2y = 20$

$19 + 2x - 5y = 20$

$5x + 2y = 17$

$2x - 5y = 1$

$5x = 15$

$29y = 29$, $y = 1$

$x = 3$ $\quad (3,1)$

Number is $3 + 21 + 1 = 25$

$417 + 25 = 442$ & $x^2 + 1 = 441 + 1 = 442$

Let us examine one more complementary pair of cells $(20,0)$ & $(0,20)$

$3 + 5x + 2y = 20$ $\quad$ ~~$5x + 2y = 20$~~ $\quad$ ~~$29y = 135 \equiv 19$~~ $(x = 21)$

$19 + 2x - 5y = 0$ $\quad$ ~~$2x - 5y = -19$~~ $\quad$ ~~$29x \equiv 81 \equiv 18$~~

~~$y = 11 \mid 12 \mid 13 \mid 14 \mid 15 \mid 16 \mid 17 \mid 18$~~

~~$319 \mid 348 \mid 377 \mid 406 \mid 435 \mid 464 \mid 493 \mid 522$~~

~~$x = 18$~~

~~$522 - 18 = 504 = 21 \times 23$~~

~~$90 + 2y =$~~

$5x + 2y = 17$ $\quad$ $29x = 47 = 5$

$2x - 5y = -19$

$493 \mid 522 \mid 551 \mid 580$

$488 \mid 517 \mid 546 \mid 575$

21×26

$x = 19$

$y = 3$

Number is $19 + 84 + 1 = 104$

$19 + 63 + 1 = 83$

$38 - 5y = -19$

$5y = 57$

$y = 3$

$29y = 129$

$= 3$ $\quad y = 3$

Notes

Notes

21

July - Wednesday

In the cell $(0, 20)$, $3 + 5x + 2y = 0$

$19 + 2x - 5y = 20$

$x + 21y = 35$

$x = 1, y = 17$

So I think it is associated although

and $(i, j) = (3, 19)$

50	61	72	74	4	15	26	28	39
60	71	73	3	14	25	36	38	49
70	81	2	13	24	35	37	48	59
80	1	12	23	34	45	47	58	69
9	11	22	33	44	46	57	68	79
10	21	32	43	54	56	67	78	8
20	31	42	53	55	66	77	7	18
30	41	52	63	65	76	6	17	19
40	51	62	64	75	5	16	27	29

for $\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}_9$ the pairs $(x, 4)$ are $(1,1), (1,4), (1,7)$

$(4,1), (4,1), (4,7)$

$(7,1), (7,4), (7,7)$

and corresponding (i, j) given by $i + x + 2y = 4$, $j + 4x - 5y = 4$

i.e. $i = 4 - x - 2y$, $j = 4 - 4x + 5y$

$(i, j) = (1,5), (4,2), (7,8)$

$(1,2) \to (7,7)$

$(1,5) \to (1,4)$

$(1,8) \to (4,4)$

Notes

22

July - Thursday

$5x + 2y = -3$

$2x - 5y = 1$

$29y = -11 = 10$

$29x = -19 = 8$

$x = 1, y = 17$

just major

Notes

23

July - Friday

$\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}_9$ R.M(1,4) B.M(3,-1)

$a(i,j) = (1,8)$

is connected

74	1	18	26	34	42	50	58	66
60	68	76	3	11	19	36	44	52
37	54	62	70	78	5	13	21	29
23	31	39	47	55	72	80	7	15
9	17	25	33	41	49	57	65	73
67	75	2	10	27	35	43	51	59
53	61	69	77	4	12	20	28	45
30	38	46	63	71	79	6	14	22
16	24	32	40	48	56	64	81	8

$\begin{pmatrix} 1 & -1 \\ -3 & 4 \end{pmatrix}_{15}$

$\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}_{15}$

$(i,j) = (13,5), (10,2), (7,14), (4,11), (1,8)$

24

July - Saturday

$2 \cdot 15 \cdot \frac{113}{226}$ 4

1695

$195 \times 2 \cdot 10$

$3x = 1$

If a.d $3x - 5y = 1$

b.d's $-x + 3y = 2, 5, 8, 11, 14$

first d, $(x,y) = ($

648

1695 ✓

128	145	162	179	181	198	215	7	24	41	58	75	77	94	111
144	161	178	195	197	214	6	23	40	57	74	76	93	110	127
160	177	194	196	213	5	22	39	56	73	90	92	109	126	143
176	193	210	212	4	21	38	55	72	89	91	108	125	142	159
192	209	211	3	20	37	54	71	88	105	107	124	141	158	175
208	225	2	19	36	53	70	87	104	106	123	140	157	174	191
224	1	18	35	52	69	86	103	120	122	139	156	173	190	207
15	17	34	51	68	85	102	119	121	138	155	172	189	206	223
16	33	50	67	84	101	118	135	137	154	171	188	205	222	14
32	49	66	83	100	117	134	136	153	170	187	204	221	13	30
48	65	82	99	116	133	150	152	169	186	203	220	12	29	31
64	81	98	115	132	149	151	168	185	202	219	11	28	45	47
80	97	114	131	148	165	167	184	201	218	10	27	44	46	63
96	113	130	147	164	166	183	200	217	9	26	43	60	62	79
112	129	146	163	180	182	199	216	8	25	42	59	61	78	95

Notes

Notes

$(i,j) = (1,8)$

nothing special about this
no diagonals (except the main)
are magic

26
July - Monday

27
July - Tuesday

17	25	6	3	14
8	4	12	20	21
15	16	23	9	2
24	7	5	11	18
1	13	19	22	10

=

2	5	1	3	4
3	4	2	5	1
5	1	3	4	2
4	2	5	1	3
1	3	4	2	5

+

15	20	5	0	10
5	0	10	15	20
10	15	20	5	0
20	5	0	10	15
0	10	15	20	5

By lattice method $(x,y) = (1,1) + r(t,u) + s(v,w)$

$s=0, r=1 \rightarrow (x,y) = 13 = (3,3)$ a) $1+t=3, 1+u=3, t=2, u=2$

$r=0, s=1 \rightarrow (x,y) = 24 = (4,5)$ u $1+v=4, 1+w=5, v=3, w=4.$

$(x,y) = (1,1) + r(2,2) + s(3,4).$

$s=0, r=2 \rightarrow (1,1) + (4,4) = (5,5) = 25$

$r=0, s=2 \rightarrow (1,1) + (6,8) = (7,9) \equiv (2,A) = 17$

Continuation from last page 179 of Note Book No. x. re

Construction of a p.s of order 9 given the corresponding d.s with $T = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$.

63								19
	81			40			2	30
						59	42	
					35	13		
		21		45	70			32
			12	73				
		69	47					
17	76	23	80			46		
52	8							

This is not a p.s

since $69+80 \neq 47+23$

So I think there must be a different T and

2	19	52	76	69	12	45	35	59
70	13	42	80	63	8	23	6	73
60								
81								
32								
46								
40								
21								
17								

d.s is in Bk. 1, p. 78, upper figure

Notes

Let us try $T = \begin{vmatrix} 1 & 1 \\ 7 & 5 \end{vmatrix}$ $k = i+j$

$1 = 5i+7j$

$i = -2$

$A_{11} \rightarrow A_{73}$

$1-5k = 2j$

1, 10, 19, 28, 37, 46, 55, ...

$1-7k = -2j, \quad 2j = 7k-1$

$1-5k = 2i, \quad 2i = 1-5k$

$ai+cj = 1 \quad 7a+3c = 1$

& $bi+dj = 1 \quad 5b+3d = 1$

Notes

$$T = \begin{vmatrix} -1 & -1 \\ 5 & 7 \end{vmatrix}$$

$A_{21} \to A_{66}$ ✗

$i = -12$

$15 = -2i$ (88)

$k = -i + j$

$l = 5i + 7j$

$l - 5k = 2j$ $\quad l - 7k = -3i$

$j = -1$ $\quad i = 3$ $\quad A_{38}$ ✗

Appears difficult to find a suitable T.

$$T = \begin{vmatrix} 1 & 1 \\ -7 & 5 \end{vmatrix}$$

$k = i + j$ $\quad l = 7i + 5j$

$l - 5k = 2i$ $\quad l - 7k = -2j$

$i = -2 \equiv 7$ $\quad j = 3$ $\quad A_{73}$ ✗

Let us see what this T gives when applied to the p.s on p. 178 of the Book X.

$2i = l - 5k, \quad 7k - l = 2j$

$2i = 8 - 3$

$A_{11} \to A_{73}$, $A_{12} \to A_{37}$,

$A_{21} \to A_{99}$, $A_{22} \to A_{56}$

$A_{31} \to A_{21}$

$A_{41} \to A_{49}$

On the first col do by steps $(-1, -2)$ along rows by steps $(4, 4)$

	75	27	56	40	64	51	8	32	16
	65	49	1	33	17	77	25	57	45
	10	78	26	59	43	66	54	2	31
	44	68	52	3	36	11	76	19	60
	34	12	81	20	58	37	69	53	5
	63	38	67	46	6	35	14	79	21
	4	28	15	80	23	61	39	72	47
	24	62	41	70	48	9	29	13	73
	50	7	30	18	74	22	55	42	71

mistake all right

Starting with (P) + (R) the middle rows is

$(7, 3, 9), (2, 4, 1), (6, 8, 5)$

$9, 2, 4, 1, 6, 8, 5, 7, 3$

Cycle $(7, 2)$

$27, 9, 72, 18, 54, 36, 63, 45, 0$

$54, 36, 63, 45, 0, 27, 9, 72, 18$

Cycle $(5, 4)$.

Mistake Notes

Notes

In the corresponding Rome-Bath figure the same cycles, the middle nesik series in (P) is $(2, 1, 7), (4, 6, 3), (9, 8, 5)$ which is different from the above one which is $(7, 3, 9), (2, 4, 1), (6, 8, 5)$

30
July -Friday

31
July -Saturday

Perhaps this means that we should start with a different p.s. (A_{ij})

42	43	38	44	39	40	37	41	45
60	61	56	62	57	58	55	59	63
24	25	20	26	21	22	19	23	27
78	79	74	80	75	76	73	77	81
15	16	11	17	12	13	10	14	18
33	34	29	35	30	31	28	32	36
6	7	2	8	3	4	1	5	9
51	52	47	53	48	49	46	50	54
69	70	65	71	66	67	64	68	72

$(A)_{ij}$

Starting from Rouse-Ball's magic square (not associated) & using $T = \begin{vmatrix} 1 & 1 \\ 7 & 5 \end{vmatrix}$ we first put in A_{33} the number 2 in the first column & transforming by steps $(-1,-2)$ we put the remaining terms of the first column. Starting from these 9 numbers & going by steps $(4,4)$ we put in the numbers in the corresponding rows. Thus we get (A) which is a p.s all right. Starting with this p.s and using $T = \begin{vmatrix} 1 & 1 \\ 7 & 5 \end{vmatrix}$ we get Rouse-Ball's magic square. What is the relation between this $(A)_{ij}$ and the $(A)_{ij}$ on p. 178 of note book ✕? Can this A_{ij} be obtained by using

$$A_{ij} = 9a(i) + a(j) + 1$$

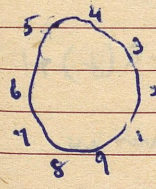
with suitable choice of the $a(i)$'s? In the above A_{ij} we have

$A_{11} = 42 = 9a(1) + a(1) + 1$ i.e. $10 \cdot a(1) = 41$. So not possible

In the above permute the cols as follows:—

$7 \to 1, 3 \to 2, 5 \to 3, 8 \to 4, 1 \to 5, 6 \to 6, 9 \to 7, 2 \to 8, 4 \to 9$

$1 \to 5 \to 3 \to 2 \to 8 \to 4 \to 9 \to 7 \to 1$ & $6 \to 6$ we get $(A)'_{ij}$ which is again a p.s.



37	38	39	41	42	40	45	43	44
55	56	57	59	60	58	63	61	62
19	20	21	23	24	22	27	25	26
73	74	75	77	78	76	81	79	80
10	11	12	14	15	13	18	16	17
28	29	30	32	33	31	36	34	35
1	2	3	5	6	4	9	7	8
46	47	48	50	51	49	54	52	53
64	65	66	68	69	67	72	70	71

$(A)'_{ij}$

Next permute the rows of $(A')_{ij}$ as follows:—

$1 \to 4 \to 7 \to 1$

$2 \to 8 \to 5 \to 2$

$3 \to 3, 6 \to 6, 9 \to 9$

Then we get the p.s on p. 178 of Note Book ✕

Can we get the p.s of p. 178 of Note Book ✕ from the first p.s by permutations?

1 Sunday

Notes

Notes

(90)

2 August - Monday

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81

f.p.s (F)

3 August - Tuesday

Permute cols $5 \to 4 \to 6 \to 5$

$7 \to 8 \to 9 \to 7$

Then we get $(F)'$

Next permuting in $(F)'$ the rows as

$4 \to 6 \to 5 \to 4$ and

$7 \to 8 \to 9 \to 7$

1	2	3	5	6	4	9	7	8
10	11	12	14	15	13	18	16	17
19	20	21	23	24	22	27	25	26
28	29	30	32	33	31	36	34	35
37	38	39	41	42	40	45	43	44
46	47	48	50	51	49	54	52	53
55	56	57	59	60	58	63	61	62
64	65	66	68	69	67	72	70	71
73	74	75	77	78	76	81	79	80

$(F)'$

1	2	3	5	6	4	9	7	8
10	11	12	14	15	13	18	16	17
19	20	21	23	24	22	27	25	26
37	38	39	41	42	40	45	43	44
46	47	48	50	51	49	54	52	53
28	29	30	32	33	31	36	34	35
73	74	75	77	78	76	81	79	80
55	56	57	59	60	58	63	61	62
64	65	66	68	69	67	72	70	71

$(F)'' =$ f.s.m.p. 178 of Book X

So I see no point in R&N's gymnastics about $A_{ij} = 9a(i) + a(j) + 1$ and the choice of the $a(i)$'s starting with $0, 1, 2, 3, 4, 5, 6, 7, 8$.

16	8	81	64	56	48	40	32	24
41	33	25	17	9	73	65	57	49
66	58	50	42	34	26	18	1	74
10	2	75	68	60	49	45	34	26
44	36	29	14	3	76	68	60	52
69	61	53	45	28	20	12	4	77
13	5	78	70	62	54	37	29	21
38	30	22	14	6	79	71	63	46
72	55	47	39	31	23	15	7	80

Notes

Suppose I start with (F) and use

$$T = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

Then start with $F_{11} \to F_{26}$ and getting the first col by steps $(-2, -3)$ and the rows by $(1, 1)$ is diagonal steps.

This is not magic in the cols and also not magic in the diagonals.

Notes

So after all there is a point in R&N's getting $(F)''$ and then applying T to it

It took me getting $(F)''$ from (F) depended on using the f.s.m.p. 178 of Bk. X

4 August - Wednesday

5 August - Thursday

The only thing left over re. $n = 9$ is to find out how one can derive Rouse-Ball's nasik & associated square on p. 78 of Bk. I

Let us start with this square & go backwards to find the corresponding p. s

using $T = \begin{vmatrix} 1 & 1 \\ 3 & 6 \end{vmatrix}$ $k = i + j$, $l = 3i + 6j$ $\quad l - 3k = 3j$, $6k - l = 3i$ $\quad 3i \equiv 5$ for $k = 1, l = 1$. This has no solution.

~~Appro~~ So it looks as if this T is inadmissible. We note here that abcd = 36 which is divisible by 9 unlike the cases $\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$, $\begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix}$ etc. $\begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix}$ $\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$ etc

6	5	4	1	3	2	8	7	9
6	7	2	1	5	9	8	3	4
5	9	8	3	4	6	7	2	1
4	6	7	2	1	5	9	8	3
1	5	9	8	3	4	6	7	2
3	4	6	7	2	1	5	9	8
2	1	5	9	8	3	4	6	7
8	3	4	6	7	2	1	5	9
7	2	1	5	9	8	3	4	6
9	8	3	4	6	7	2	1	5

27	9	72	27	9	72	27	9	72
45	72	36	0	27	54	63	9	18
27	54	63	9	18	45	72	36	0
18	45	72	36	0	27	54	63	9
0	27	54	63	9	18	45	72	36
9	18	45	72	36	0	27	54	63
36	0	27	54	63	9	18	45	72
63	9	18	45	72	36	0	27	54
72	36	0	27	54	63	9	18	45
54	63	9	18	45	72	36	0	27

54	78	44	5	29	58	64	16	21
35	59	65	13	19	52	75	45	6
20	49	73	43	3	36	60	71	14
1	34	57	72	15	26	50	74	40
12	27	51	80	41	2	31	55	70
42	8	32	56	67	10	25	48	81
68	11	22	46	79	39	9	33	62
76	37	7	30	63	69	17	23	47
61	66	18	24	53	77	38	4	28

(B)

On p. 109 top of Note Book I taking steps (7,2) and (5,4) on (P) & (R) we get this square which is nasik & associated

The step (6,3) taken there gives repeated elements in columns.

It is best to avoid the steps (3,6) and (6,3) either in (P) or (R) since ~~it makes~~ they make the corresponding squares non-latin. This also means we avoid $T = \begin{vmatrix} 1 & 1 \\ 3 & 6 \end{vmatrix}$ in the transformation from a p. s.

Notes

Notes

6 August - Friday

7 August - Saturday

(92)

In the above associated nasik square we have taken (7, 2) & (5, 4) in (P) & (R)

Since we have the transformation scheme for $T = \begin{vmatrix} 1 & 1 \\ 7 & 5 \end{vmatrix}$ let us obtain the p.s which gives rise to this associated nasik square.

65	71	72	70	67	68	69	66	64
20	26	27	26	22	23	24	21	19
74	80	81	79	76	77	78	75	73
2	8	9	7	4	5	6	3	1
56	62	63	61	58	59	60	57	55
11	17	18	16	13	14	15	12	10
47	53	54	52	49	50	51	48	46
38	44	45	43	40	41	42	39	37
29	35	36	34	31	32	33	30	28

What is special about this p.s. which gives an associated d.s under $T = \begin{vmatrix} 1 & 1 \\ 7 & 5 \end{vmatrix}$?

$A_{11} \to A_{73}$, $A_{99} \to A_{23,-5} \equiv A_{54}$

$2i = l - 5k$
$2j = 7k - l$

$A_{99} \to A_{99}$

$A_{19} \leftarrow A_{28}$, $A_{91} \leftarrow A_{54}$

A_{39} A_{43} A_{18} A_{64}

A_{45} A_{37}

$k = i + j$
$l = 7i + 5j$ } (A)

A_{98} & A_{12}

A_{65} (13) A_{24}

A_{17} A_{86}

$A_{11} + A_{62} = 82$

$B_{23} + B_{87}$

$A_{11} \to B_{23}$

~~There does not seem to be~~ To go from the p.s (A) to the d.s (B) apply $k = i + j$, $l = 7i + 5j$

& to go from (B) to (A) apply $2i = l - 5k$, $2j = 7k - l$.

There does not seem to be any simple way of deciding whether the p.s (A) gives under $T = \begin{vmatrix} 1 & 1 \\ 7 & 5 \end{vmatrix}$ the ~~d.s (B)~~ associated d.s (B)

only take any two elements of (A) adding to 82 and the corresponding elements of (B) using $k = i + j$, $l = 7i + 5j$ should be associate element

(?) for eg A_{11} + A_{62} add to 82 & corresponding elements of (B) are B_{23} and B_{87} & these are obvious associates since $2 + 8 = 3 + 7 = 10$

From the p.s. property A_{99} & A_{72} & A_{61} add to 82 & corresponding elements are B_{36} and B_{72} & these are associates. Stop for n = 9

Conversely we can start from two associate elements of (B) & verify using $i = l - 5k$, $j = 7k - l$ & the corresponding elements of (A) & verify that they add to 82.

N - method for n = 4. — Relation between usual addition & Chadya-Chadaka addition

2	3	2	3
1	4	1	4
3	2	3	2
4	1	4	1

+

4	0	8	12
8	12	4	0
4	0	8	12
8	12	4	0

=

6	3	10	15
9	16	5	4
7	2	11	14
12	13	8	1

(A)

by usual method (nasik)

14	11	2	7
1	8	13	12
15	10	3	6
4	5	16	9

(B)

by the Chadya-Chadaka method (nasik)

reflection of (A) about central vertical → (A')

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

(A')

& reflection about horizontal axis → (A'')

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

(A'')

which is not (B).

But a cyclic permutation of (A') ~~[illegible]~~ reflect (B). This is valid because (A) & hence (A') are nasik squares. So also will be (B)

Thus it appears that the usual addition method & the Chadya-Chadaka method ~~are~~ do not lead to trivially related squares obtainable by rotation and reflection.

Take a non-nasik square from B & J

1	10	7	16
15	11	6	2
14	8	9	3
4	5	12	13

=

1	2	3	4
3	3	2	2
2	4	1	3
4	1	4	1

0	8	4	12
12	8	4	0
12	4	8	0
0	4	8	12

by ordinary addition

1	2	3	4
3	3	2	2
2	4	1	3
4	1	4	1

+

12	4	8	0
0	4	8	12
0	8	4	12
12	8	4	0

by the Chadya-Chadaka method.

i.e. merely take reflection of the converse (R) about vertical axis and this gives the ordinary (R).

2	3	2	3
1	4	1	4
3	2	3	2
4	1	4	1

+

4	0	8	12
8	12	4	0
4	0	8	12
8	12	4	0

12	8	0	4
0	4	12	8
12	8	0	4
0	4	12	8

True for nasik or non-nasik square

Notes Notes

0, 8, 16, 24, 32, 40, 48, 56

(94)

11
August - Wednesday

Q_1

1	8	1	8
8	1	8	1
8	1	8	1
1	8	1	8

0	24	48	40
48	40	0	24
8	16	56	32
56	32	8	16

Q_2

2	7	2	7
7	2	7	2
7	2	7	2
2	7	2	7

0	24	48	40
48	40	0	24
8	16	56	32
56	32	8	16

Q_3

3	6	3	6
6	3	6	3
6	3	6	3
3	6	3	6

0	24	48	40
48	40	0	24
8	16	56	32
56	32	8	16

Q_4

4	5	4	5
5	4	5	4
5	4	5	4
4	5	4	5

0	24	48	40
48	40	0	24
8	16	56	32
56	32	8	16

4	3	2	1	7	6	5
5	4	3				
7	6	5	4	3	2	1
1	7	6	5	4	3	2
2	1	7	6	5	4	3
3	2	1	7	6	5	4

21, 14, 7, 0, 42, 35, 28

42 35 28 21 14 7 0
0 42 35 28 21 14 7
7 0 42 35 28 21 14
14 7 0 42 35 28 21

Notes

12
August - Thursday

40	48	24	0
24	0	40	48
32	56	16	8
16	8	32	56

0	3	6	5
6	5	0	3
1	2	7	4
7	4	1	2

1	32	49	48
56	41	8	25
16	17	64	33
57	40	9	24

0	1	2
1	2	3
2	0	1

1, 2, 3, 4, 5, 6, 7
2, 3, 4, 5, 6, 7, 1
3, 4, 5, 6, 7, 1, 2

26, 20, 14, 1, 44, 38, 32

21, 28, 35, 42, 0, 7, 14

0, 7, 14, 21, 28, 35, 42
42, 0, 7, 14 21, 28, 35
35, 42, 0, 7, 14, 21, 28
28, 35, 42, 0, 7, 14, 21

26, 20, 14, 1, 44, 38, 32

Notes

(95)

13
August - Friday

2	4	1	3	5
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4

5	1	4	2	3
2	3	5	1	4
1	4	2	3	5
3	5	1	4	2
4	2	3	5	1

+

10	0	15	5	20
5	20	10	0	15
0	15	5	20	10
20	10	0	15	5
15	5	20	10	0

=

15	1	19	7	23
7	23	15	1	19
1	19	7	23	15
23	15	1	19	7
19	7	23	15	1

X

5	1	4	2	3
2	3	5	1	4
1	4	2	3	5
3	5	1	4	2
4	2	3	5	1

+

15	5	10	20	0
10	20	0	15	5
0	15	5	10	20
5	10	20	0	15
20	0	15	5	10

=

20	6	14	22	3
12	23	5	16	9
1	19	7	13	25
8	15	21	4	17
24	2	18	10	11

$$\times \quad \frac{2(n-3)n!\,(n-1)!}{8}$$

(P) $q = 1$

15 Sunday

3	1	5	2	4
2	4	3	1	5
1	5	2	4	3
4	3	1	5	2
5	2	4	3	1

+

15	20	5	10	0
5	10	0	15	20
0	15	20	5	10
20	5	10	0	15
10	0	15	20	5

=

18	21	10	12	4
7	14	3	16	25
1	20	22	9	13
24	8	11	5	17
15	2	19	23	6

(P) $q = 2$

(R) $q = 3$

Notes

0	20	5	15	10
15	10	0	20	5

1	4	5	2	3
5	2	3	1	4

Notes

14
August - Saturday

(96)

16
August - Monday

2	1	3	4	5
4	5	2	1	3
1	3	4	5	2
5	2	1	3	4
3	4	5	2	1

+

0	15	5	10	20
5	10	20	0	15
20	0	15	5	10
15	5	10	20	0
10	20	0	15	5

=

2	16	8	14	23
9	15	22	1	18
21	5	19	10	12
20	7	11	23	4
13	24	5	17	6

(A)

q = 2

5	0	10	15	20
15	20	5	0	10
0	10	15	20	5
20	5	0	10	15
10	15	20	5	0

+

1	4	2	3	5
2	3	5	1	4
5	1	4	2	3
4	2	3	5	1
3	5	1	4	2

q = 3

=

6	4	12	18	25
17	23	10	1	14
5	11	19	22	8
24	6	3	15	16
13	20	21	9	2

(B)

q = 2

25	14	8	16	2
18	1	22	15	9
12	10	19	3	21
4	23	11	6	20
6	17	5	24	13

← T_4 ←

6	17	5	24	13
4	23	11	6	20
12	10	19	3	21
18	1	22	15	9
25	14	8	16	2

↓ M

(A).

q = 2

3	1	2	5	4
5	4	3	1	2
1	2	5	4	3
4	3	1	2	5
2	5	4	3	1

+

15	10	20	5	0
20	5	0	15	10
0	15	10	20	5
10	20	5	0	15
5	0	15	10	20

q = 3

=

18	11	22	10	4
25	9	3	16	12
1	17	15	24	8
14	23	6	2	20
7	5	19	13	21

1-2-3-4-5

4-2-3-1-5

10	0	5	20	15
20	15	10	0	5
0	5	20	15	10
15	10	0	5	20
5	20	15	10	0

+

4	3	5	2	1
5	2	1	4	3
3	4	3	5	2
1	5	2	1	4
2	1	4	3	5

=

24	3	10	22	16
25	17	11	4	8
1	9	23	20	12
18	15	2	6	24
7	21	19	13	5

			10	16
			4	12
1				
	23			
		19		

Notes

17
August - Tuesday

18
August - Wednesday

19
August - Thursday

0, 1, 2, 3, 4
4, 5, 6, 7, 8
4, 0, 1, 2, 3
3, 0, 2, 4, 1
3, 5, 7, 9, 11
3, 0, 2, 4, 1

3	1	5	2	4
4	3	1	5	2
2	4	3	1	5
5	2	4	3	1
1	5	2	4	3

$q=1$

+

10	0	15	20	5
20	5	10	0	15
0	15	20	5	10
5	10	0	15	20
15	20	5	10	0

$q=2$

=

13	1	20	22	9
24	8	11	5	17
2	19	23	6	15
10	12	4	18	21
16	25	7	14	3

10	0	20	5	15
15	10	0	20	5
5	15	10	0	20
20	5	15	10	0
0	20	5	15	10

$q=0$

+

3	1	4	5	2
5	2	3	1	4
1	4	5	2	3
2	3	1	4	5
4	5	2	3	1

$q=2$

=

13	1	24	10	17
20	12	3	21	9
6	19	15	2	23
22	8	16	14	5
4	25	7	18	11

Uniform step squares se cyclic method

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

M. D. M

=

2	4	1	3	5
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4

V $q=2$

+

15	20	0	5	10
20	0	5	10	15
0	5	10	15	20
5	10	15	20	0
10	15	20	0	5

$q=4$ U

4, 1, 3, 5, 2
1, 2, 3, 4, 5

2, 4, 1, 3, 5
4, 5, 1, 2, 3

(3, 4, 0, 1, 2) 1st row of U in natural order
(1, 3, 0, 2, 4) 1st row of V A_2 beginning with 1 is 1, 3, 5, 7, 9 ≡ 1, 3, 0, 2, 4 → 1st row of V
(4, 0, 1, 2, 3) 2nd row of U
(2, 4, 1, 3, 0) 2nd row of V A_2 beginning with 2 is 2, 4, 6, 8, 10 ≡ 2, 4, 1, 3, 0 → 2nd row of V
(0, 1, 2, 3, 4) 3rd row of U
(3, 0, 2, 4, 1) 3rd row of V A_2 beginning with 3 is 3, 5, 7, 9, 11 ≡ 3, 0, 2, 4, 1 → 3rd row of V

Notes

Notes

(1, 2, 3, 4, 0) 4th row of U
(4, 1, 3, 0, 2) 4th row of V A_2 beginning with 4 is 4, 6, 8, 10, 12 ≡ 4, 1, 3, 0, 2 → 4th row of V
(2, 3, 4, 0, 1) 5th row of U
(0, 2, 4, 1, 3) 5th row of V A_2 beginning with 0 is 0, 2, 4, 6, 8 ≡ 0, 2, 4, 1, 3 → 5th row of V

I.D.M — semi-nasik & associates

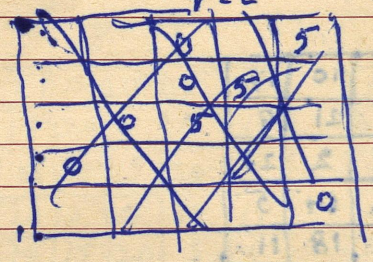
2	4	1	3	5
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4

$q = 2$

5	15	0	10	20
10	20	5	15	0
15	0	10	20	5
20	5	15	0	10
0	10	20	5	15

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

		0		5
		0	5	
	0	5		
0				
				0



15	20	0	5	10
20	0	5	10	15
0	5	10	15	20
5	10	15	20	0
10	15	20	0	5

$q = 4$

(0, 1) $0+1 = 2$
(0, −1) $0-1 = -1 \equiv 4$

▽ U

2, 4, 1, 3, 5 — 2, 4, 6, 8, 10 (A2)
3, 5, 2, 4, 1 — 3, 5, 7, 9, 11 (A2)
4, 1, 3, 5, 2 — 4, 6, 8, 10, 12 "
5, 2, 4, 1, 3 — 5, 7, 9, 11, 13 "
1, 3, 5, 2, 4 — 1, 3, 5, 7, 9 "

$a = 1, b = 1$
$a + a' = 0, b + b' = -1$
$a' = -1, b' = -2$

3, 4, 0, 1, 2 — 3, 4, 5, 6, 7 (A1)
4, 0, 1, 2, 3 — 4, 5, 6, 7, 8
0, 1, 2, 3, 4 —
1, 2, 3, 4, 0 "
2, 3, 4, 0, 1

$a + a' = 0$
$a = 1, b = 2$ — $b + b' = -1$
$a' = -1, b' = -3$

$n(n-1)$

10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

(1, 2)
(0, −1).
(nasik)

~~5, 6, 7, 8, 9~~
22 Sunday
5, 8, 11, 14, 17 (A3)
≡ 5, 3, 1, 4, 2

5, 3, 1, 4, 2
$q = 3$ 1, 4, 2, 5, 3
1, 4, 7, 10, 13
≡ 1, 4, 3, 5, 3

5, 15, 0, 10, 20
10, 20, 5, 15, 0
$q = 2$

1, 3, 0, 2, 4
1, 3, 5, 7, 9 (A2)
2, 4, 1, 3, 0 ≡ 2, 4, 6, 8, 10

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Islamic
(only associates)

~~middle~~
(P) 1, 4, 2, 5, 3 (R1)
4, 2, 5, 3, 1
$q = 4$
1, 4, 7, 10, 13 (A3)
4, 7, 10, 13, 16

10, 20, 5, 15, 0 — 2, 4, 1, 3, 0 ≡ 2, 4, 6, 8, 10 (A2)
0, 10, 20, 5, 15 — 0, 2, 4, 1, 3 ≡ 0, 2, 4, 6, 8
$q = 1$

Notes

Notes

23
August - Monday

Wolfen (Bachelli) (not provided)

23	6	19	2	15
10	18	1	14	22
17	5	13	21	9
4	12	25	8	16
11	24	7	20	3

3, 1, 4, 2, 5
5, 3, 1, 4, 2
$q = 1$
(A_3)

3	2	1	5	4
4	3	2	1	5
5	4	3	2	1
1	5	4	3	2
2	1	5	4	3

(P) $q = 1$ (A_1)

+

5	15	0	10	20
10	20	5	15	0
15	0	10	20	5
20	5	15	0	10
0	10	20	5	15

(R) $q = 2$

$(-2, 1)$ and $(0, -1)$.
$a = -2, b = 1, a' = 2, b' = -2$

8	17	1	15	24
14	23	7	16	5
20	4	13	22	6
21	10	19	3	12
2	11	25	9	18

$(-3, 1)$, $(0, -1)$
semi-main & associated

	(P)	(R)/5	(x, y)	(x', y')
J.D.M	(A_2)	(A_1)	$(1, 1)$, $(0, -1)$, $(2, 4)$.	$q = 2, q = 4$
Islamia	(A_3)	(A_2)	$(1, -1)$, $(0, -2)$, $(2, 1)$	$q = 4, q = 1$.
Bachelli	(A_3)	(A_2)	$(1, 1)$, $(0, 2)$, $(2, 3)$	$q = 1, q = 4$
Mediantica	(A_3)	(A_2)	$(1, 2)$, $(0, -1)$ $(2, 4)$	$q = 3, q = 2$
Mathematica	$(A_{2,1})$	(A_2)	$(-2, 1)$, $(0, -1)$, $(2, 4)$.	$q = 1, q = 2$.

$1 + 2d = 3$
$2 +$

24
August - Tuesday

$a = -2, b = 1$
$a' = 2, b' = -2$

20, 5, 15, 0, 10 | 4, 2, 3, 0, 2
5, 15, 0, 10, 20 | 1, 3, 0, 2, 4
$q = 4$ (A_2)

Difficult an example where (A_4) appears

(R)/n = 1, 3, 0, 2, 4. 1, 3, 5, 7, 9
2, 4, 1, 3, 0 2, 4, 6, 8, 10
3, 0, 2, 4, 1 3, 5, 7, 9, 4
4, 1, 3, 0, 2 4, 6, 8, 10, 12
0, 2, 4, 1, 3 0, 2, 4, 6, 8
(A_2)

Continuation one ant (A_4) for (P)
& (A_2) for (R)/5.

Notes

Notes

Notes

$(-8)(-3)6$ $\quad 24$ $\quad 144$

(100)

6/2/84 Let us take a nasik square of order 5 and find a p.s. from which it can be derived

19	8	2	11	25
12	21	20	9	3
10	4	13	22	16
23	17	6	5	14
1	15	24	18	7

(B)

=

4	3	2	1	5
2	1	5	4	3
5	4	3	2	1
3	2	1	5	4
1	5	4	3	2

$q = 3$

+

15	5	0	10	20
10	20	15	5	0
5	0	10	20	15
20	15	5	0	10
0	10	20	15	5

$q = 2$

$T_7 T = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$. $\quad k = i + j, \quad 2k - l = -i, \quad i = l - 2k$

$l = 3i + 2j \quad 3k - l = j \quad j = 3k - l$

$-1 \quad 2 \quad 6 = 4 \quad -9$

$-7 = 3$

$B_{11} \to A_{42}$	$B_{21} \to A_{25}$	$B_{31} \to A_{53}$	$B_{41} \to A_{31}$	$B_{51} \to A_{14}$
$B_{12} \to A_{51}$	$B_{22} \to A_{34}$	$B_{32} \to A_{12}$	$B_{42} \to A_{45}$	$B_{52} \to A_{23}$
$B_{13} \to A_{15}$	$B_{23} \to A_{43}$	$B_{33} \to A_{21}$	$B_{43} \to A_{54}$	$B_{53} \to A_{32}$
$B_{14} \to A_{24}$	$B_{24} \to A_{52}$	$B_{34} \to A_{35}$	$B_{44} \to A_{13}$	$B_{54} \to A_{41}$
$B_{15} \to A_{33}$	$B_{25} \to A_{11}$	$B_{35} \to A_{44}$	$B_{45} \to A_{22}$	$B_{55} \to A_{55}$

3	4	5	1	2	
13	14	15	11	12	
23	24	25	21	22	
18	19	20	16	17	17
8	9	10	6	7	

(A)

(A) is a p.s all right

~~So (B) can~~ So the n.d.s (B) can be obtained from the n.p.s (A) by $T = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$.

Can we get (B) using the f.p.s?

1	2	3	4	5	3, 4, 5, 1, 2
6	7	8	9	10	8, 9, 10, 6, 7
11	12	13	14	15	13, 14, 15, 11, 12
16	17	18	19	20	18, 19, 20, 16, 17
21	22	23	24	25	23, 24, 25, 21, 22

(A)

~~$T = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $k = i + j$, $l = ai + bj$~~

~~$A_{11} \to B_{2}$, $a + b$ $\quad k = ai + bj$, $l = ci + dj$~~

~~$A_{11} \to B_{51}$~~

~~aim $\begin{bmatrix} a + b = 5 \\ c + d = 1 \end{bmatrix}$~~

permuting cols 1 & 2 as 3 - 4 - 5 - 1 - 2

& then permuting rows as 1 - 3 - 5 - 4 - 2

f.p.s remains f.p.s under permutations

Notes

Notes

27
August - Friday

4	6	3	9	8	5	2	1	7
9	8	5	2	1	7	4	6	3
2	1	7	4	6	3	9	8	5
4	6	3	9	8	5	2	1	7
9	8	5	2	1	7	4	6	3
2	1	7	4	6	3	9	8	5
4	6	3	9	8	5	2	1	7
9	8	5	2	1	7	4	6	3
2	1	7	4	6	3	9	8	5

$q = 6$

28
August - Saturday

18	45	72	63	9	36	27	54	0
27	54	0	18	45	72	63	9	36
63	9	36	27	54	0	18	45	72
18	45	72	63	9	36	27	54	0
27	54	0	18	45	72	63	9	36
63	9	36	27	54	0	18	45	72
18	45	72	63	9	36	27	54	0
27	54	0	18	45	72	63	9	36
63	9	36	27	54	0	18	45	72

$q = 3$

22	51	75	72	17	14	38	28	7
22	51							

repeat ×

1	3	5	2	4
5	2	4	1	3
4	1	3	5	2
3	5	2	4	1
2	4	1	3	5

$q = 2$ by cols

+

15	5	0	10	20
10	20	15	5	0
5	0	10	20	15
20	15	5	0	10
0	10	20	15	5

$q = 2$ by rows.

=

16	8	5	12	24
15	22	19	6	3
9	1	13	25	17
22	20	7	4	11
2	14	21	18	10

4	3	2	1	5
2	1	5	4	3
5	4	3	2	1
3	2	1	5	4
1	5	4	3	2

$q = 2$ by cols
tail q cols + rows

+

15	5	0	10	20
10	20	15	5	0
5	0	10	20	15
20	15	5	0	10
0	10	20	15	5

$q = 2$ by rows

=

19	8	2	11	21

en

Notes
Notes

30
August - Monday

6	8	5	2	4	1	7	3	9
4	1	7	3	9	6	8	5	2
9	6	8	5	2	4	1	7	3
2	4	1	7	3	9	6	8	5
3	9	6	8	5	2	4	1	7
5	2	4	1	7	3	9	6	8
7	3	9	6	8	5	2	4	1
8	5	2	4	1	7	3	9	6
1	7	3	9	6	8	5	2	4

$q = 5$

+

31
August - Tuesday

54	63	9	18	45	72	36	0	27
72	36	0	27	54	63	9	18	45
63	9	18	45	72	36	0	27	54
36	0	27	54	63	9	18	45	72
9	18	45	72	36	0	27	54	63
0	27	54	63	9	18	45	72	36
18	45	72	36	0	27	54	63	9
27	54	63	9	18	45	72	36	0
45	72	36	0	27	54	63	9	18

$q = 4$

60	71	14	20	49	73	43	3	36
76	37	7	30	63	69	17	23	47
72	15	26	50	74	40	1	34	57
38	4	28	61	66	18	24	55	77
12	27	51	80	41	2	31	55	70
5	29	58	64	16	21	54	78	44
25	48	81	42	8	32	56	67	10
35	59	65	13	19	52	75	45	6
46	79	39	9	33	62	68	11	22

(2, -1) naïve + arrondo

$60+7+74+24+70+29+43+52+11 = 369$

$71+30+40+53+12+58+8+75+22 = 369$

$14+63+1+77+27+64+32+45+46 = 369$

(2, 1) $60+39+19+56+44+27+61+40+23 = 369$ ✓

(-2, -1) $60+23+40+61+27+44+56+19+39 = 369$ ✓

(-2, 1) $60+11+52+42+29+70+24+74+7 = 369$ ✓

(1, 2) $60+59+58+61+63+62+56+5\cancel{5}+5\cancel{8} \neq 369$ ×

(-1, 2) $60+6+78+24+69+33+42+51+15 = 378$ ×

(1, -2) $60+15+51+42+33+69+24+78+6 = 369$ ✓

(1, -2) $60+57+55+56+62+65+61+58+\cancel{59}$ ×

Knight-naïve e não (±2, ±1) nem (±1, ±2)

Notes

not that:

(2, -1) ± (-2, 1) same pattern

(2, 1) ± (-2, -1) "

(1, 2) ± (-1, -2) "

(1, -2) ± (-1, 2) "

reverse order

we can call it semi-Knight-naïve

not at all a bad ferz!

not 3×3 game: this is 9×9 ... 41 p. 249

of Rebbok 2n

Notes

1 Rose Ball 74 ie 74 on p. 293 of notebook 2 2 (103)

September - Wednesday ↓ September - Thursday

(2,-1) 2+42+77+4+44+79+9+37+25=369

(2,1) 2+29+11+74+20+56+38+65+47=X

(-2,1) 2+75+37+9+79+44+4+77+42=369

(-2,-1) 47+65+38+56+20 ?

(1,2) 2+54+67+38+63+22+74+18+31=369

(-1,-2) 2+31+18+74+22+63+38+67+54=369

(1,-2) 2+3+1+9+7+8+4+5+6=X

(-1,2) = same as

same terms. same terms. same terms

Now 3+3 same as other

Again same - Right now I'm

at (2,-1), (-2,1), (1,2), (-1,-2).

I had not noted earlier that (±2, ±1) and (±1, ±2) really come to four cases since (a,b) & (-a,-b) have same terms

9	7	8	2	3	1	4	5	6
2	3	1	4	5	6	9	7	8
4	5	6	9	7	8	2	3	1
9	7	8	2	3	1	4	5	6
2	3	1	4	5	6	9	7	8
4	5	6	9	7	8	2	3	1
9	7	8	2	3	1	4	5	6
2	3	1	4	5	6	9	7	8
4	5	6	9	7	8	2	3	1

4	5	6	9	7	8	2	3	1
2	3	1	4	5	6	9	7	8
9	7	8	2	3	1	4	5	6
4	5	6	9	7	8	2	3	1
2	3	1	4	5	6	9	7	8
9	7	8	2	3	1	4	5	6
4	5	6	9	7	8	2	3	1
2	3	1	4	5	6	9	7	8
9	7	8	2	3	1	4	5	6

2	6	7	2	6	7	2	6	7
9	1	5	9	1	5	9	1	5
4	8	3	4	8	3	4	8	3
6	7	2	6	7	2	6	7	2
1	5	9	1	5	9	1	5	9
8	3	4	8	3	4	8	3	4
7	2	6	7	2	6	7	2	6
5	9	1	5	9	1	5	9	1
3	4	8	3	4	8	3	4	8

+

63	0	45	27	72	9	18	36	54
27	72	9	18	36	54	63	0	45
18	36	54	63	0	45	27	72	9
63	0	45	27	72	9	18	36	54
27	72	9	18	36	54	63	0	45
18	36	54	63	0	45	27	72	9
63	0	45	27	72	9	18	36	54
27	72	9	18	36	54	63	0	45
18	36	54	63	0	45	27	72	9

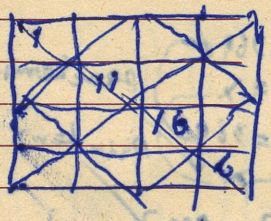
Notes Notes

6,1,8,7,5,3,2,9,4 → 45,0,63,54,36,18,9,72,27.

7,5,3,2,9,4,6,1,8

3 September -Friday

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6



1	2	13	✓
×	11	·	2
✓	✓	16	!
·	2	×	6

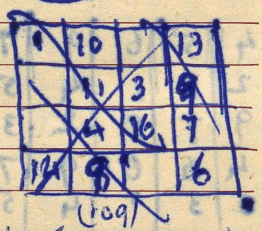
a	c	d	
	b		
		a'	
			b'

(2,15), (3,14), (4,13), (5,12), (7,10), (8,9)

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

(102)

1	8	10	15
12	13	3	6
7	2	16	9
14	11	5	4



(109)

1	8	11	14
15	10	5	4
6	3	16	9
12	13	2	7

(116)

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

(117)

1	8	13	12
15	10	3	6
4	5	16	9
14	11	2	7

(177)

1	12	13	8
14	7	2	11
4	9	16	5
15	6	3	10

(178)

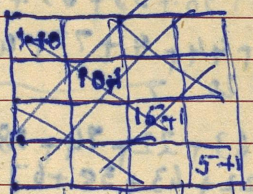
1	12	13	8
15	6	3	10
4	9	16	5
14	7	2	11

Notes

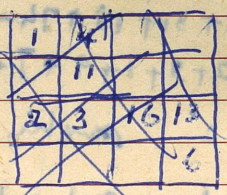
4 September -Saturday

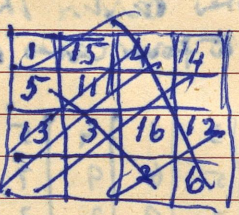
(104)

2+15, 3+14, 4+13, 5+12, 7+10



a	b	c	d
e	f	g	h
a'	d'	a'	b'
g'	h'	e'	f'





(104)

1	8	10	15
14	11	5	4
7	2	16	9
12	13	3	6

(107)

1	8	11	14
12	13	2	7
6	3	16	9
15	10	5	4

5 Sunday

(171)

1	12	6	15
14	7	9	4
11	2	16	5
8	13	3	10

(174)

1	12	7	14
15	6	9	4
10	3	16	5
8	13	2	11

(201)

1	14	7	12
15	4	9	6
10	5	16	3
8	11	2	13

(204)

1	14	11	8
15	4	5	10
6	9	16	3
12	7	2	13

Notes

6

September - Monday

(279)

2	7	9	16
11	14	4	5
8	1	15	10
13	12	6	3

(281)

2	7	9	16
13	12	6	3
8	1	15	10
11	14	4	5

(304)

2	7	14	11
13	12	1	8
3	6	15	10
16	9	4	5

(305)

2	7	14	11
16	9	4	5
3	6	15	10
13	12	1	8

(372)

2	11	14	7
13	8	1	12
3	10	15	6
16	5	4	9

(375)

2	11	14	7
16	5	4	9
3	10	15	6
13	8	1	12

(469)

3	6	9	16
13	12	7	2
8	1	14	11
10	15	4	5

(473)

3	6	12	13
16	9	7	2
5	4	14	11
10	15	1	8

(530)

3	10	5	16
13	8	11	2
12	1	14	7
6	15	4	9

(532)

3	10	8	13
16	5	11	2
9	4	14	7
6	15	1	12

(565)

3	13	12	6
16	2	7	9
5	11	14	4
10	8	1	15

(621)

4	5	10	15
14	11	8	1
7	2	13	12
9	16	3	6

Cont'd p. 109

7

September - Tuesday

(292)

2	7	12	13
11	14	1	8
5	4	15	10
16	9	6	3

(294)

2	7	12	13
16	9	6	3
5	4	15	10
11	14	1	8

(355)

2	11	5	16
13	8	10	3
12	1	15	6
7	14	4	9

(365)

2	11	8	13
16	5	10	3
9	4	15	6
7	14	1	12

(393)

2	13	8	11
16	3	10	5
9	6	15	4
7	12	1	14

(396)

2	13	12	7
16	3	6	9
5	10	15	4
11	8	1	14

(485)

3	6	15	10
13	12	1	8
2	7	14	11
16	9	4	5

(536)

3	6	15	10
16	9	4	5
2	7	14	11
13	12	1	8

(537)

3	10	15	6
13	8	1	12
2	11	14	7
16	5	4	9

(623)

3	10	15	6
16	5	4	9
2	11	14	7
13	8	1	12

(646)

4	5	11	14
15	10	8	1
6	3	13	12
9	16	2	7

(647)

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

Notes

Notes

8 September - Wednesday

n = 7 ½ 7.50 75 = 5

9 September - Thursday

2, 3
4, 5
3, 6, 1, 4

(106)

R & W's 7th order irregular square nasik

2	47	38	35	24	20	9
26	16	8	6	46	42	31
49	39	33	23	15	12	4
19	11	7	45	41	30	22
37	29	27	17	14	3	48
10	5	44	36	34	25	21
32	28	18	13	1	43	40

2	5	3	7	3	6	2	
5	2	1	6	4	7	3	×
7	4	5	2	1	5	4	
5	4	7	3	6	2	1	×
2	1	6	3	7	3	6	(P)
3	5	2	1	6	4	7	×
4	7	4	6	1	1	5	

+

0	42	35	28	21	14	7	
21	14	7	0	42	35	28	
42	35	28	21	14	7	0	
14	7	0	42	35	28	21	(R)
35	28	21	14	7	0	42	
7	0	42	35	28	21	14	
28	21	14	7	0	42	35	

1, 2, 3, 4, 5, 6

2 & 3 a & d c, d
4 & 5 c & b a, b, f
3 & 6 e & d
1 & 4 c & f

a = 2, d = 3, e = 6 c = 4, b = 5, f = 1

a − b = 2 − 5

c − d =

3rd, 5th, 7th | 2nd, 4th, 6th

Accordingly B & J, A = 0, B = 42, C = 35, D = 28, E = 21, F = 14, G = 7

~~A + a = 2, G + a = 9 a = 2~~
~~C + b = 35, E + b = 24 b = 3~~

A + a = 2, G + a = 9 a = 2
C + d = 35, E + d = 24 d = 3
~~C + b = 39, G + b = 14~~
C + b = 39, A + b = 4, b = 4 ✓
D + d = 33, G + d = 12 d = 5
E + = 27, F + = 48, = 6
F + d = 17, A + d = 3 d = 3
D + b = 32, F + b = 18, b = 4
A + = 1, B + = 43 = 1

1, 2, 3, 4, 5, 6

(a, d, e, f)
(b, c)

2, 5, 6, 1
3, 4

(a, d)
(b, e)
(a, f)
(b, c)

a, a, d, d
b, b, c, c
~~e, e, f, f~~
d, d, e, e
~~c, c, f, f~~ c, c, f, f
~~b, b, c, c~~

a = 2, b = 3
c = 4, d = 5
e = 6 (2, 3)
(4, 5)
(3, 6)
(1, 4)

(a, d)
(b, c)
(d, e)
(c, f)

a = 2, d = 3
~~a = 4~~, b = 5, c = 4
d = 3, e = 6
c = 4, f = 1

a = 1, f = 2
b = 5, c = 3
~~d = 4, f = 1~~ e = 6.
~~a = 2, b = 3~~

1, 2, 5, 6

a = 3, b = 4
c = 1, d = 2
e = 5, f = 6

a − b
= c − d
= e − f

(a, 4)
(3, 2)
(4, 5)
(3, 6)
(4, 1)

Notes

Notes

10
September - Friday

2	2	6	3	6	3	5
5	3	7	1	6	4	2
7	4	5	5	4	1	4
5	1	2	7	4	6	4
2	6	3	6	3	7	1
3	7	4	2	5	6	5
4	5	1	4	6	1	7

This must be of B & J's

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

f.p.s with $n = 7$.

Take $T = \begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix}$ $k = i + j$, $l = 4i + 3j$

$a_{11} \to a_{22}$, $a_{21} \to a_{35}$, $a_{31} \to a_{48}$

$a_{12} \to a_{35}$, $a_{22} \to a_{44}$, $a_{32} \to a_{51}$

$a_{13} \to a_{48}$, $a_{23} \to a_{51}$, $a_{33} \to a_{14}$

$a_{14} \to a_{37}$

$a_{24} \to a_{40}$

$a_{34} \to a_{43}$, $a_{44} \to a_{47}$, $a_{54} \to a_{17}$, $a_{64} \to a_{20}$, $a_{74} \to a_{23}$

$a_{15} \to a_{38}$, $a_{25} \to a_{41}$, $a_{35} \to a_{44}$, $a_{45} \to a_{18}$, $a_{55} \to a_{21}$, $a_{65} \to a_{24}$, $a_{75} \to a_{27}$

$a_{16} \to a_{39}$, $a_{26} \to a_{42}$, $a_{36} \to a_{45}$, $a_{46} \to a_{19}$, $a_{56} \to a_{22}$, $a_{66} \to a_{25}$, $a_{76} \to a_{28}$

$a_{17} \to a_{40}$, $a_{27} \to a_{43}$, $a_{37} \to a_{46}$, $a_{47} \to a_{49}$, $a_{57} \to a_{12}$, $a_{67} \to a_{15}$, $a_{77} \to a_{18}$

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

1, 2, 3, 4, 5
6, 7, 8, 9, 10
11, 12, 13, 14, 15
16, 17, 18, 19, 20
21, 22, 23, 24, 25

13	17	21	4	8
22	1	5	9	18
6	10	14	23	2
15	19	3	7	11
24	8	12	16	20

11
September - Saturday

(107)

2	2	6	3	6	3	5
7	4	5	5	4	1	2
5	3	7	1	2	4	6
5	1	2	7	4	6	3
3	7	4	2	5	6	1

Second type with $a = 3$, $b = 4$, $c = 1$, $d = 2$; $e = 5$, $f = 6$

Take $T = \begin{vmatrix} 1 & 1 \\ 6 & 2 \end{vmatrix}$ $k = i + j$, $l = 6i + 2j$ irregular tr.

$a_{11} \to a_{21}$, $a_{21} \to a_{37}$, $a_{31} \to a_{46}$, $a_{41} \to a_{55}$, $a_{61} \to a_{64}$, $a_{71} \to a_{73}$, $a_{12} \to a_{12}$

31	43	20	25	37	14	19
1	27	32	44	21	26	38
34	39	2	28	33	45	8
46	9	35	40	3	15	41
16	42	47	10	22	48	4
49	5	17	29	6	11	23
12	24	36	13	18	30	7

12 Sunday

			43			1
				8		
	15					
					22	
		29				
						36

The irregular transform $\begin{vmatrix} 1 & 1 \\ 6 & 2 \end{vmatrix}$

Order 7 → f.p.s of order 7 with a square which is not even magic

Notes

Notes

In the irregular case of $n = 7$ on p. 11.

$a_{11} \to a_{15}$
$a_{12} \to a_{23}$
$a_{13} \to a_{35}$
$a_{14} \to a_{47}$
$a_{15} \to a_{52}$
$a_{16} \to a_{64}$
$a_{17} \to a_{76}$

7 appears twice in the first indices
second "
For a regular nasik square of order 7.

i.e. Two appear in some row & two in some col

26	31	18	2	8	42	48
7	13	40	45	25	30	15
44	22	35	20	5	10	39
17	4	9	36	49	27	33
41	47	24	32	16	1	14
29	21	6	12	38	46	23
11	37	43	28	34	19	3

$n = 7$, regular (cyclic)
(B)

1 4

1st col of f.p.s of order 7 is
(1, 8, 15, 22, 29, 36, 43
In the regular square ~~these appear~~ each of these appear only once in each row & each col.

$a_{11} \to a_{56}$
$a_{21} \to a_{15}$
$a_{31} \to a_{27}$
$a_{41} \to a_{32}$
$a_{51} \to a_{61}$
$a_{61} \to a_{44}$
$a_{71} \to a_{73}$

$k = i + j$
$l = bi + dj$
$k = ai + cj$, $l = bi + dj$
$5 = a + c$, $6 = b + d$
$1 = 2a + c$, $5 = 2b + d$
$a = -4 \equiv 3$, $c = 2$
$b = -1 \equiv 6$, $d = 7$
$k = 3i + 2j$, $6i + 7j$

$\begin{vmatrix} 3 & 2 \\ 6 & 7 \end{vmatrix}$

Let us go backwards from above & find the corresponding f.p.s.

starting with $T = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}$ $k = i + j$, $l = 2i + 4j$

$l - 2k = 2j$, $j = \frac{1}{2}(l - 2k)$
$l - 4k = -2i$, $i = \frac{1}{2}(4k - l)$

a_{36}

$a_{11} \to a_{53}$	$a_{21} \to a_{72}$	$a_{31} \to a_{21}$
$a_{12} \to a_{17}$	$a_{22} \to a_{36}$	a_{55}
$a_{13} \to a_{44}$	$a_{23} \to a_{63}$	a_{12}
$a_{14} \to a_{71}$	$a_{24} \to a_{27}$	a_{46}
$a_{15} \to a_{35}$	$a_{25} \to a_{54}$	a_{73}
$a_{16} \to a_{62}$	$a_{26} \to a_{11}$	a_{37}
$a_{17} \to a_{26}$	$a_{27} \to a_{45}$	a_{64}

Add 3 & 4 to the indices

30	35	33	32	29	34	31
44	49	47	46	43	48	45
9	14	12	11	8	13	10
16	21	19	18	15	20	17
23	28	26	25	22	27	24
37	42	40	39	36	41	38
2	7	5	4	1	6	3

(A)

Having placed elements of 3 sons of (B) in the f.p.s (A) we can furnish it

Notes

$a_{11}, a_{21}, a_{31}, \ldots$ get from
53, 72, 21, 47, 66, 15, 34

Add 2 & subtract 1 per cross

Permuting — By cols 5 - 1 - 7 - 4 - 3 - 6 - 2
& then by rows 7 - 3 - 4 - 5 - 6 - 1 - 2
we get the f.p.s

Notes

15

September - Wednesday

(647)

4	5	16	9
15	10	3	6
1	8	13	12
14	11	2	7

(646)

4	5	16	9
14	11	2	7
1	8	13	12
15	10	3	6

(702)

4	9	16	5
14	7	2	11
1	12	13	8
15	6	3	10

(704)

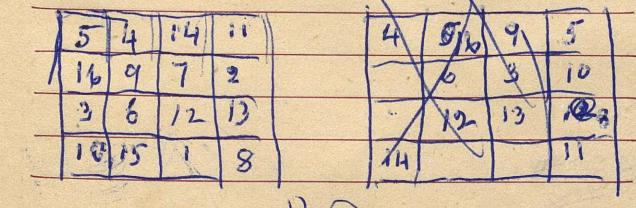
4	9	16	5
15	6	3	10
1	12	13	8
14	7	2	11

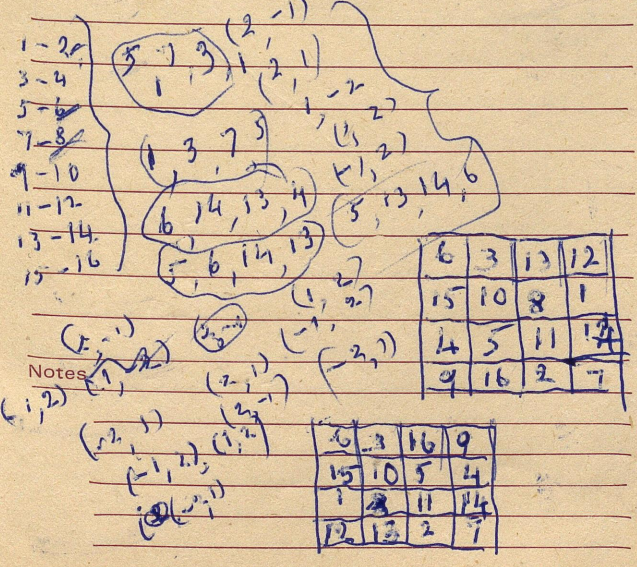
(785)

5	4	14	11
16	9	7	2
3	6	12	13
10	15	1	8

(788)

5	4	15	10
16	9	6	3
2	7	12	13
11	14	1	8





Notes

16

September - Thursday

(690)

4	9	6	15
14	7	12	1
11	2	13	8
5	16	3	10

(691)

4	9	7	14
15	6	12	1
10	3	13	8
5	16	2	11

(744)

4	14	7	9
15	1	12	6
10	8	13	3
5	11	2	16

(748)

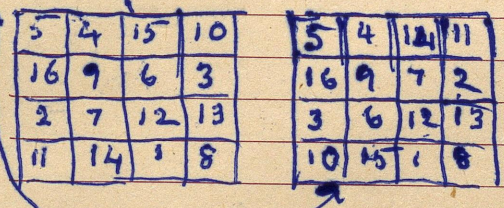
4	14	11	5
15	1	8	10
6	12	13	3
9	7	2	16

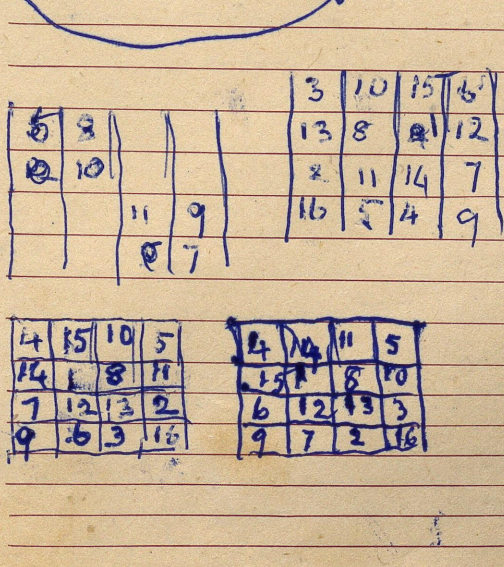
(828)

6	3	13	12
15	10	8	1
4	5	11	14
9	16	2	7

(839)

6	3	16	9
15	10	5	4
1	8	11	14
12	13	2	7

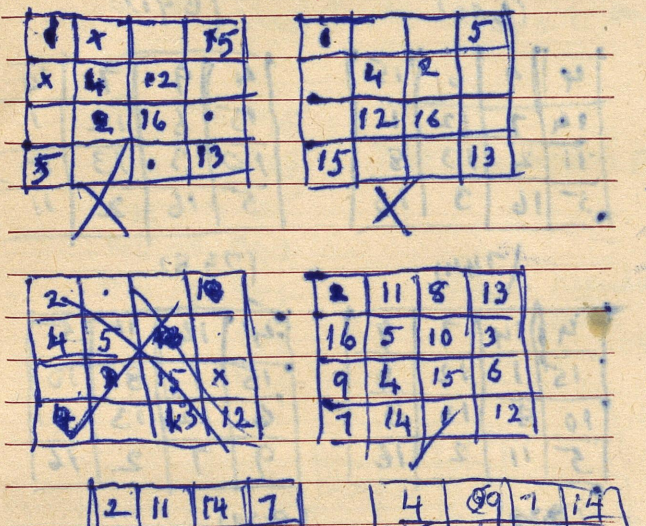


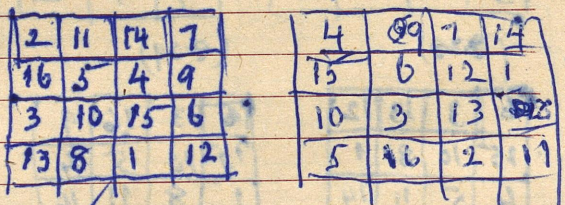


Notes

17

September -Friday





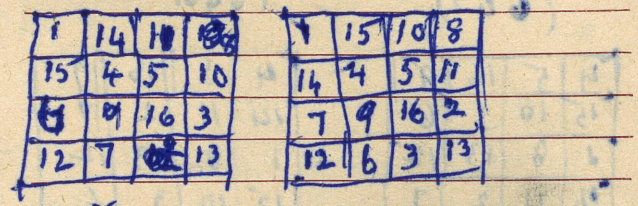
(25) (25)

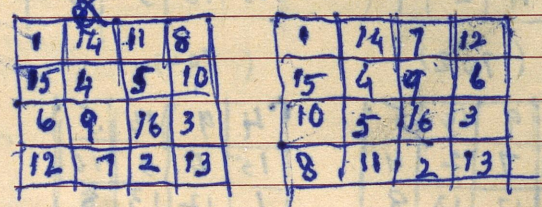
10+15
9+16

Notes

18

September -Saturday





19 Sunday

Notes

20
September - Monday

Notes

21
September - Tuesday

Notes

3	5	2	4	[illegible]
2	4	1	3	5
1	3	5	2	4
5	2	4	[illegible]	3
4	1	3	5	2

10	5	0	20	15
20	15	10	5	0
5	0	20	15	10
15	10	5	0	20
0	20	15	10	5

=

13	10	2	24	16
22	19	11	8	5
6	3	25	17	14
20	12	9	1	23
4	21	18	15	7

A	B	C	D	E
C	D	E	A	B
E	A	B	C	D
B	C	D	E	A
D	E	A	B	C

a	b	c	d	e
d	e	a	b	c
b	c	d	e	a
e	a	b	c	d
c	d	e	a	b

So Mathari Rado problem

Squares r ↔ c.

3	2	1	5	4
5	4	3	2	1
2	1	5	4	3
4	3	2	1	5
1	5	4	3	2

+

10	20	5	15	0
5	15	0	10	20
0	10	20	5	15
20	5	15	0	10
15	0	10	20	5

→

13	22	6	20	4
10	19	3	12	21
2	11	25	9	18
24	8	17	1	15
16	5	14	23	7

umbers

Dr. P. K. Sathyanarayana

973, 9th "A" Main Road,
3rd Block, Jayanagar.
Bangalore - 560011

Res. Phone - 606869
Hospital - 40188
E.S.I. Dispensary, Jayanagar, 560011

23.00
7.65
4.10
34.75

6.25

91.00
76.80
1.00
1.15
109.95

180.00
109.95
70.05
63.05
7.00

~~180.00~~
180.00
70.05

Vy. Bk 9148.52

Div. Bk 11281.85

20430.37

Cancellation 7979.62

Total → 28410.01

UNIVERSITY OF MYSORE

DEPARTMENT OF MATHEMATICS

CENTRAL COLLEGE
BANGALORE

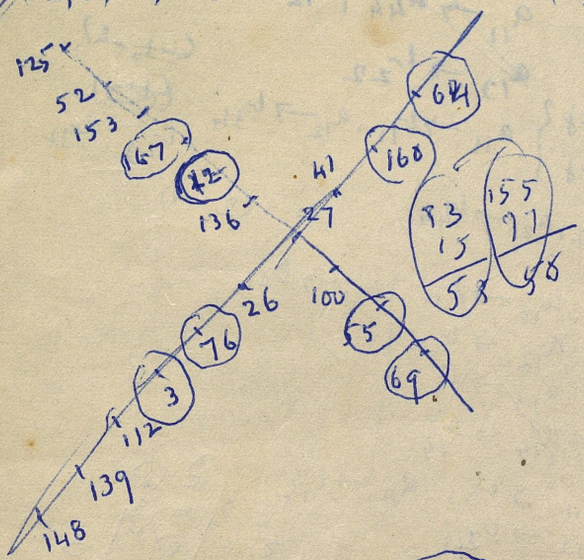
Dated 195

Step) (0, 2)
(2, -2)

$T = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $k = 2i$ $l = 2j$ $a_{11} \to b_{22}, a_{12} \to b_{24}$. $a_{1,13} \to b_{2,13}, a_{21} \to b_{42}$

85	19	122	56	159	93	27	143	77	11	114	48	151
67	1	117	51	154	88	22	125	59	162	96	30	133
108	42	145	79	26	129	63	166	100	34	137	71	5
126	60	163	97	31	134	68	2	105	52	155	89	23
39	142	78	10	115	47	150	84	18	121	55	158	92
21	124	58	161	95	29	132	66	13	116	50	153	87
3	106	40	156	90	24	127	61	164	98	32	135	69
167	101	35	138	72	6	109	43	146	80	14	130	64
149	83	17	120	54	157	104	38	141	75	9	112	46
131	78	12	115	49	152	86	20	123	57	160	94	28
44	147	81	15	116	65	166	102	36	139	73	7	110
62	165	99	33	136	70	4	107	41	144	91	25	128
103	37	140	74	8	111	45	148	82	16	119	53	169

Cycle (157): 7, 6, 5, 4, 3, 2, 1, 13, 12, 11, 10, 9, 8,
2, 1, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3



12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1,

156+6, 78+11, (13+8), 26+9, 39+10, 0+4, 65+12, 130+7, 91+1

143+11, 156+7, (13+8), 117+13, 52+5, 104+3, 91+2, 78+1, 130+12

78+11, 91+1.

78+1, 91+2

6, 11, 9, 10, 4, 12, 7, 1
11, 7, 13, 5, 3, 2, 1, 12

78+11, 65+12, 130+7, 91+1.
143+11, 130+12, 156+7, 78+1

39+12, 130+4, (65+1), 78+2, 91+3, 52+10, 117+5, 13+13, 143+7, 156+8, 0+9, 104+6

26+4, 39+13, (65+1), 0+6, 104+11, 156+9.

4, 13, 6, 11, 9
12, 4, 2, 3, 10, 5, 13, 7, 8, 9, 6

52+8, 156+5, 0+6, 13+7, (143+1), 39+9, 104+4, 65+11, 78+12, 91+13, 26+10, 117+2

13+9, 26+5, 52+6, 156+11, 91+3, (143+1), 130+13, 117+12, 0+10, 104+2

8, (5), (6), 7, (9), 4, (11), (12), (13), (10), (2)
(9), (5), (6), (11), 3, (13), (12), (10), (2)

26+4, 39+13, 0+6, 156+9

156+5, 0+6, 65+11
156+11, 0+10, 26+5

~~1, 5, 12, 10, 8, 13, 2, 4.~~

1, 5, 10, 8, 6, 13, 2, 4, 11, 9, 7, 12, 3
8, 12, 4, 2, 13, 7, 9, 11, 5, 3, 1, 6, 10
7, 11, 3, 1, 12, 6, 8, 10, 4, 2, 13, 5, 9
10, 1, 6, 4, 2, 9, 11, 13, 7, 5, 3, 8, 12
13, 4, 9, 7, 5, 12, 1, 3, 10, 8, 6, 11, 2
9, 13, 5, 3, 1, 8, 10, 12, 6, 4, 2, 7, 11
6, 10, 2, 13, 11, 5, 7, 9, 3, 1, 12, 4, 8
3, 7, 12, 10, 8, 2, 4, 6, 13, 11, 9, 1, 5
12, 3, 8, 6, 4, 11, 13, 2, 9, 7, 5, 10, 1
2, 6, 11, 9, 7, 1, 3, 5, 12, 10, 8, 13, 4
5, 9, 1, 12, 10, 4, 6, 8, 2, 13, 11, 3, 7
4, 8, 13, 11, 9, 3, 5, 7, 1, 12, 10, 2, 6
11, 2, 7, 5, 3, 10, 12, 1, 8, 6, 4, 9, 13

1	96	88	125	162	30	67	117	154	22	59	51	133
147	73	65	102	139	7	44	81	118	168	36	15	110
124	50	29	66	116	153	21	58	95	132	13	161	87
101	14	6	43	80	130	167	35	72	109	146	138	64
78	160	152	20	57	94	131	12	49	86	123	115	28
165	91	70	107	144	25	62	99	136	4	41	33	128
19	114	93	143	11	48	85	122	159	27	77	56	151
42	137	129	166	34	71	108	145	26	63	100	79	5
142	55	47	84	121	158	39	76	113	150	18	10	92
106	32	24	61	98	135	3	40	90	127	164	156	69
83	9	157	38	75	112	149	17	54	104	141	120	46
60	155	134	2	52	89	126	163	31	68	105	97	23
37	119	111	148	16	53	103	140	8	45	82	74	169

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

$T = \begin{pmatrix} i & 2 \\ 2 & 1 \end{pmatrix}$ $K = i + 2j$, $l = 2i + j$

$a_{11} \to b_{33}$, $a_{12} \to b_{54} = b_{14}$
$a_{13} \to b_{31}$, $a_{14} \to b_{12}$, $a_{21} \to b_{41}$
$a_{22} \to b_{22}$, $a_{23} \to b_{43}$

(1, -2)
(3, 3) → (1, 4)

11	4	9	2
13	6	15	8
3	12	1	10
5	14	7	16

13	10	5	2
7	4	15	12
9	14	1	6
3	8	11	16

$T = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$ $K = i + 3j$, $l = 3i + j$

(1, 1)
$a_{11} \to b_{44}$, $a_{12} \to b_{31}$
$a_{13} \to b_{22}$

(-1, -2)

$T = \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}$ $T = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$K = 3i + 2j$, $l = 2i + 3j$
$a_{11} \to b_{11}$, $a_{12} \to b_{34}$
$a_{14} \to b_{32}$, $a_{21} \to b_{44}$

b_{13}, b_{32}
(2, -1)

1	10	3	12
8	13	6	15
11	4	9	2
14	7	16	5

$\begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}$ $K = i + 4j$, $l = 4i + j$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $K = 2i + j$, $l = i + 2j$

$a_{11} \to b_{11}$, $a_{12} \to b_{12}$, $a_{13} \to b_{13}$, $a_{14} \to b_{14}$
$a_{21} \to b_{21}$
$a_{11} \to b_{33}$, $a_{21} \to b_{14}$
$a_{12} \to b_{24}$
$a_{13} \to b_{13}$
$a_{14} \to b_{21}$

28

		3	5
4	6		
9		1	7
2	8		

UNIVERSITY OF MYSORE

DEPARTMENT OF MATHEMATICS

CENTRAL COLLEGE
BANGALORE

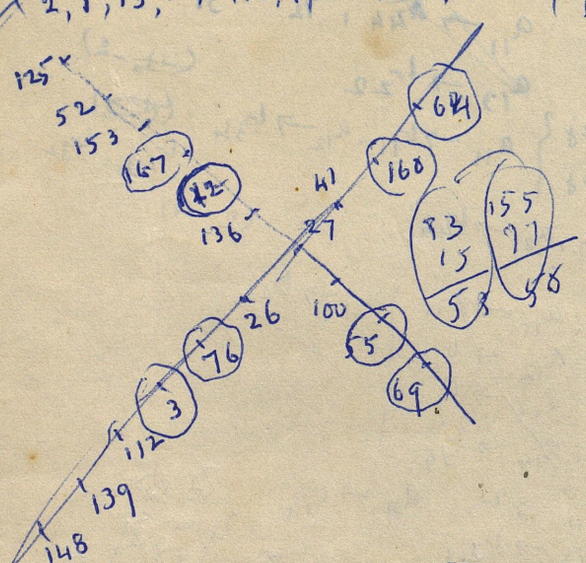
Dated 195

Step) (0, 2)
(2, -2)

$T = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $k = 2i$ $l = 2j$ $a_{11} \to b_{22}$, $a_{12} \to b_{24}$, $a_{1,13} \to b_{2,13}$, $a_{21} \to b_{42}$

85	19	122	56	159	93	27	143	77	11	114	48	151
67	1	117	51	154	88	22	125	59	162	96	30	133
108	42	145	79	26	129	63	166	100	34	137	71	5
126	60	163	97	31	134	68	2	105	52	155	89	23
39	142	76	10	113	47	150	84	18	124	55	158	92
21	124	58	161	95	29	132	66	13	116	50	153	87
3	106	40	156	90	24	127	61	164	98	32	135	69
167	101	35	138	72	6	109	43	146	80	14	130	64
149	83	17	120	54	157	104	38	141	75	9	112	46
131	78	12	115	49	152	86	20	123	57	160	94	28
44	147	81	15	118	65	166	102	36	139	73	7	110
62	165	99	33	136	70	4	107	41	144	91	25	128
103	37	140	74	8	111	45	148	82	16	119	53	169

Cycle: 7, 6, 5, 4, 3, 2, 1, 13, 12, 11, 10, 9, 8,
2, 1, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3



156+6, 78+11, (13+8), 26+9, 39+10, 0+4, 65+12, 130+7, 91+1

143+11, 156+7, (13+8), 117+13, 52+5, 104+3, 91+2, 78+1, 130+12

78+11, 91+1

78+1, 91+2

6, 11, 9, 10, 4, 12, 7, 1
11, 7, 13, 5, 3, 2, 1, 12

78+11, 65+12, 130+7, 91+1
143+11, 130+12, 156+7, 78+1

39+12, 130+4, (65+1), 78+2, 91+3, 52+10, 117+5, 13+13, 143+7, 156+8, 0+9, 104+6

26+4, 39+13, (65+1), 0+6, 104+11, 156+9

4, 13, 6, 11, 9
12, 4, 2, 3, 10, 5, 13, 7, 8, 9, 6

52+8, 156+5, 0+6, 13+7, (143+1), 39+9, 104+4, 65+11, 78+12, 91+13, 26+10, 117+2

26+4, 39+13, 0+6, 156+9

13+9, 26+5, 52+6, 156+11, 91+3, (143+1), 130+13, 117+12, 0+10, 104+2

8, (5), (6), 7, (9), 4, (11), (12), (13), (10), (2)
(9), (5), (6), (11), 3, (13), (12), (10), (2)

156+5, 0+6, 65+11
156+11, 0+10, 26+5

(0,0)	1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12	0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130
(8,5)	11, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9	143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104
(8,5)	9, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4, 6	117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78
(6,5)	5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1, 3	91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52
(5,5)	2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13	65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26
(4,9)	7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5	52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13
(5,8)	10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8	78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39
(2,11)	6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4	156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117
(8,5)	3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1	130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91
(9,5)	13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11	104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65
(2,11)	9, 11, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7	13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143
(5,8)	12, 1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10	39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0
(4,9)	4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2	26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156

$i = \frac{1}{112}(11l - 3K)$, $j = \frac{1}{112}(11K - 3l)$

$b_{11} \to a_{11}$; $b_{12} \to a^{(42)}_{4,12}$, $b_{13} \to a^{(83)}_{7,10}$

$b_{14} \to a_{10,8}(124)$; $b_{15} \to a_{13,6}(165)$

$b_{21} \to a_{12,4}$

$-3, -4$

from $a_{i,1}$ to $a_{i,13}$ jo by steps $(-2,-3)$

from $a_{i,13}$ to $a_{i+1,1}$ $\to (1,-1)$

$\frac{9}{16}$ 3 $\frac{63}{112}$

$b_{61} \to a_{43}$

110 136, 162, 188

$\frac{19}{112}, \frac{5}{112}$ $\frac{32}{112} = \frac{2}{7}$ $\frac{70}{112} = \frac{10}{16} = \frac{5}{8}$

$\frac{30}{112}$ $\frac{15}{2}$ $\frac{2}{112}$ $\frac{1}{56}$ $\frac{1}{4}$

$\frac{41}{112} = \frac{1}{56}$ $\frac{12}{112}$ $\frac{3}{28}$ $\frac{3}{2}$

$\frac{5}{112}, \frac{19}{112}$ $\frac{96}{112} = \frac{6}{7}$

$\frac{32}{112}$ $\frac{2}{7}$

$\frac{6}{112}\left(\frac{3}{56} = \frac{16}{56} = \frac{2}{7}\right.$

288

175 186 37

27 38 251

148 148

Interchange (cols) 5 and 11 & (cols) 6 & 10 as given by B & J

5th row: 22, 63, 104, 132, 4, 45, 86, 127, 168, 27, 68, 109, 150.

6th " 117, 145, 17, 58, 99, 140, 12, 40, 81, 122, 163, 35, 76

10th row: 59, 100, 141, 13, 41, 82, 123, 164, 36, 77, 105, 146, 18

11th row 67, 108, 149, 21, 62, 103, 131, 3, 44, 85, 126, 167, 39

1				76	9	8					127	2
14	26					132	20	19				81
	150	38	37				99	32	31			
		117	50	49					168	43	42	[illegible]
				4	61	60				122	55	54
66	78				140	73	72					22
145	91	90					27	84	83	95		
		45	102	101				163	96	95		
			12	114	113					63	107	106
118					68	125	124				17	119
86	143	142				35	137	136				
	40	155	154					104	148	147		
			109	166	165				58	160	159	

$k = 3i + 11j$ } $a_{11} \to b_{11}$

$l = 11i + 3j$ } $a_{12} \to b_{25,17} = b_{12,4}$

$a_{13} \to b_{36,20} = b_{10,7}$

$a_{1,13} \to b_{46,50} \equiv b_{3,11}$

$a_{21} \to b_{17,25} = b_{4,12}$

not a p. Satall.

$a_{i,1}$ to $a_{i,13}$ step $(3,2)$. from $a_{i,13}$ to $a_{i+1,1}$ step $(1,-1)$

$b_{11}, b_{12,4}, b_{10,7}, b_{8,10}, b_{6,13}, b_{4,3}, b_{2,6}, b_{13,9}, b_{11,12}, b_{9,2}$

$b_{7,5}, b_{5,8}, b_{3,11}$

$T = \begin{pmatrix} 11 & 3 \\ 3 & 11 \end{pmatrix}$ $k = 11i + 3j$ $l = 3i + 11j$ $(3,2)$ $a_{11} \to b_{11}$, $a_{12} \to b_{4,12}$

1, 12
4, 12
$(-2, -3)$

12, 4, 11,
3, 11, 4

$T = \begin{pmatrix} 2 & 12 \\ 12 & 2 \end{pmatrix}$ 5

52+12, 156+4, 91+7, (26+1), 130+6, 65+11, 0+3, 104+8, 39+13, 143+5

117+8, 130+9, 143+10, 156+11, 0+12, 13+13, (26+1), 39+2, 52+3, 65+4

52+12, 156+4, 65+11, 0+3

156+11, 0+12, 52+3, 65+4

281

For (p) cyclic indices

(8,5) 11, 2, 6, 10, 1, 5, 9, 13, 4, 8, 12, 3, 7.
5, 9, 13, 4, 8, 12, 3, 7, 11, 2, 6, 10, 1.

(8,5) 12, 3, 7, 11, 2, 6, 10, 1, 5, 9, 13, 4, 8

After interchange of 2nd & 12th rows & 6th & 8th rows.

11, 2, 6, 10, 1, 5, 9, 13, 4, 8, 12, 3, 7

(10,3) 10, 1, 5, 9, 13, 4, 8, 12, 3, 7, 11, 2, 6

(6,7) 12, 3, 7, 11, 2, 6, 10, 1, 5, 9, 13, 4, 8

(8,5) 6, 10, 1, 5, 9, 13, 4, 8, 12, 3, 7, 11, 2

(8,5) 13, 4, 8, 12, 3, 7, 11, 2, 6, 10, 1, 5, 9

(11,2) 8, 12, 3, 7, 11, 2, 6, 10, 1, 5, 9, 13, 4

(5,8) 1, 5, 9, 13, 4, 8, 12, 3, 7, 11, 2, 6, 10

(5,8) 7, 11, 2, 6, 10, 1, 5, 9, 13, 4, 8, 12, 3

(11,2) 2, 6, 10, 1, 5, 9, 13, 4, 8, 12, 3, 7, 11

(8,5) 9, 13, 4, 8, 12, 3, 7, 11, 2, 6, 10, 1, 5

(8,5) 3, 7, 11, 2, 6, 10, 1, 5, 9, 13, 4, 8, 12

(6,7) 5, 9, 13, 4, 8, 12, 3, 7, 11, 2, 6, 10, 1

(10,3) 4, 8, 12, 3, 7, 11, 2, 6, 10, 1, 5, 9, 13

K =

65+8, 143+2, 13+2, 91+6

13+2, 65+6, 91+8, 143+12

Semi-cyclic

97	73	15
25	145	71
	25	99
	97	155
	340	340

1	51	88	125	162	30	67	117	154	22	59	96	133	
147	15	65	102	139	7	44	81	118	168	36	73	110	
124	161	29	66	116	153	21	58	95	132	13	50	87	
101	138	6	43	80	130	167	35	72	109	146	14	44	
78	115	152	20	57	94	131	12	49	86	123	160	28	
42	79	129	166	34	71	108	145	26	63	100	137	5	
19	56	93	143	11	48	85	122	159	29	77	114	151	X
165	33	70	107	144	25	62	99	136	4	41	91	128	
142	10	47	84	121	158	39	76	113	150	18	55	92	
106	156	24	61	98	135	3	40	90	127	164	32	69	
83	120	157	38	75	112	149	17	54	104	141	9	46	
60	97	134	2	52	89	126	163	31	68	105	155	23	
37	74	111	148	16	53	103	140	8	45	82	119	169	

$T = \begin{pmatrix} 3 & 11 \\ 11 & 3 \end{pmatrix}$ on $\pm p$.

Shift (3,2) & (1,-1)

$a_{11} \to b_{11}$, $a_{12} \to b_{12}$, ...

$a_{13} \to b_{10,7}$

$a_{1,13} \to b_{3,11}$, $a_{21} \to b_{4,12}$

$a_{1,13} \to b_{5,9}$

$a_{2,1} \to b_{19,23} = b_{6,10}$

1					17	8					69	2
35	26					87	20	79				
	92	38	37					40	31			
		51	58	49					110	43	42	
				128	61	60					76	54
66	78					81	72					133
79	99	90					151	84	83			
		159	102	101					117	95		
				122	113					5	107	106
116					23	125	124				120	140
28	143	142					145	136				
		163	154					46	148	147		
			64	166	165					12	159	

$T = \begin{pmatrix} 5 & 9 \\ 9 & 5 \end{pmatrix}$, $k = 5i + 9j$, $l = 9i + 5j$

$a_{11} \to b_{11}$ step $(5, 4)$ // $\therefore (1, -1)$ // 237 224

$a_{12} \to b_{23,19} = b_{10,6}$

1	159	148	137	126	115	104	80	69	58	47	36	25
39	15	4	162	151	140	129	105	94	83	72	61	50
64	40	29	18	7	165	154	143	119	108	97	86	75
89	78	54	43	32	21	10	168	144	133	122	111	100
114	103	79	68	57	46	35	24	13	158	147	136	125
139	128	117	93	82	71	60	49	38	14	3	161	150
164	153	142	118	107	96	85	74	63	52	28	17	6
20	9	167	156	132	121	110	99	88	77	53	42	31
45	34	23	12	157	146	135	124	113	102	91	67	56
70	59	48	37	26	2	160	149	138	127	116	92	81
95	84	73	62	51	27	16	5	163	152	141	130	106
120	109	98	87	76	65	41	30	19	8	166	155	131
145	134	123	112	101	90	66	55	44	33	22	11	169

0+1, 13+13, 26+12, 39+11, 52+10, 65+9, (78+8), 91+7, 104+6 8')
117+5, 130+4, 143+3, 156+2.

130+12, 65+7, 0+2, 104+10, 39+5, 143+13, (78+8), 13+3, 117+11, 52+6,
156+1, 91+9, 26+4

~~26+12, 39+11, 117+5, 65+7~~
~~39+5, 130+12, 26+4, 39+5, 117+11, 65~~

130+12, 65+7, 91+9, 26+4
26+12, 65+9, 91+7, 130+4

0 + 1	130 + 12
13 + 13	65 + 7
26 + 12	0 + 2
✓39 + 11	104 + 10 ✓
✓52 + 10	39 + 5 ✓
65 + 9	143 + 13
91 + 7	13 + 3
✓104 + 6	117 + 11 ✓
✓117 + 5	52 + 6 ✓
130 + 4	156 + 1
143 + 3	91 + 9
156 + 2	26 + 4

1st, 2nd, 12th, 13th
~~11th, 8th, 6th, 3rd~~

~~26+4, 91+9, 65+7, 130+12~~
~~0+1, 13+13, 143+3, 156+2~~

331 344
128, 16, 156, 44
62, 74, 48, 110 ~~224~~
6th, 7th, 9th, 10th
13th, 11th, 5th, 3rd

52+10, 39+11, 104+6, 117+5
~~104+10~~ 39+5, 104+10, 117+11, 52+6
~~11th, 10th, 5th~~
10th, 9th, 5th, 4th
4th, 5th, 9th, 10th

195

6th, 7th ✓, ~~11, 12~~	1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12	0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130
✓(8, 5)	11, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9	143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104
✓(8, 5)	8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4, 6	117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78
✓(4, 9)	13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11	104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65
✓(5, 8)	3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1	130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91
(2, 11)	12, 1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10	39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0
(8, 5)	9, 11, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7	13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143
(8, 5)	6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4	156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117
(2, 11)	2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13	65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26
(5, 8)	5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1, 3	91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52
(4, 9)	10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8	78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39
(8, 5)	7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5	52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13
(8, 5)	4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2	26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 113, 156

182
13
169

273
104
169

299
130
169

260
91
169

286
117
169

39+5, 104+10, 117+11, 52+6
52+10, 39+11, 104+6, 117+5

44	62
114	50
128	110
58	122
344	344

i+0, 1+13, 3+26, 4+39, 5+52, 6+65, (7+78), 8+91, 9+104, 10+117, 11+130, 12+143, 13+156

12+13, 9+52, 6+91, 3+130, 0+13, 10+39, (7+78) 4+117, 1+156, 11+26, 8+65, 5+104, 2+143

3+26, 11+130 | 4+39, 10+117

11+26, 3+130 | 4+117, 10+39

3+26, 4+39, 10+117, 11+130

3+130

39+10, 117+4, 91+6, 65+8

65+6, 91+8, 39+4, 117+10

156+1, 0+13

104+9, 52+5, 156+13, 0+1

0+1, 52+5, 104+9, 156+13

0+1	13+12
13+2	52+9
26+3	91+6
39+4	130+3
52+5	0+13
65+6	39+10
(78+7)	(78+7)
91+8	117+4
104+9	156+1
117+10	26+11
130+11	65+8
143+12	104+5
156+13	143+2

Transformations (n = 13) under $T = \begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}$

$K = i + j$, $l = 8i + 5j$

$a_{1,1} \to b_{2,13}$, $a_{1,2} \to b_{3,18} = b_{3,5}$, $a_{1,3} \to b_{4,10}$

$a_{1,13} \to b_{1,8}$, $a_{2,1} \to b_{3,8}$ steps $(5,-1)$ and $(0,-2)$

26+11	78+7
65+9	52+10
104+7	26+13
143+5	0+3
13+3	143+6
52+1	117+9
(91+12)	(91+12)
130+10	65+2
0+8	39+5
39+6	13+8
78+4	156+11
117+2	130+1
156+13	104+4

65+9 ✓ 52+10 ✓

52+1 ✓ 117+9 ✓

130+10 ✓ 65+2 ✓

117+2 ✓ 130+1 ✓

Interchange 6th & 8th rows.

" 2nd & 12th rows

13, 4, 8, 12, 3, 7

11, 2, 6, 10, 1, 5, 9

5, 9, 13, 4, 8, 12, 3

7, 11, 2, 6, 10, 1

168

173

37	145	97	49	157	109	61	13	121	73	25	133	85
122	74	26	134	86	38	146	98	50	158	110	62	1
51	159	111	63	2	123	75	14	135	87	39	147	99
136	88	27	148	100	52	160	112	64	3	124	76	15
65	4	125	77	16	137	89	28	149	101	40	161	113
150	102	41	162	114	53	5	126	78	17	138	90	29
66	18	139	91	30	151	103	42	163	115	54	6	127
164	116	55	7	128	67	19	140	79	31	152	104	43
80	32	153	92	44	165	117	56	8	129	68	20	141
9	130	69	21	142	81	33	154	93	45	166	105	57
94	46	167	106	58	10	118	70	22	143	82	34	155
23	131	83	35	156	95	47	168	107	59	11	119	71
108	60	12	120	72	24	132	84	36	144	96	48	169

$T = \begin{pmatrix} 1 & 1 \\ 8 & 5 \end{pmatrix}$ m.t.p.s

(8,5)

11, 2, 6, 10, 15, 9, 13, 4, 8, 12, 3, 7

5, 9, 13, 4, 8, 12, 3, 7, 11, 2, 6, 10, 1.

(β)

1+65+116+167+49+100+151+33+84+135+17+68+119 = 1105 (2,-1)

1+111+52+149+90+18+128+56+166+94+35+132+73 = 1105 (2,1)

1+161+152+143+121+112+103+81+72+63+41+32+23 = 1105 ✓ (1,-2)

1+97+24+107+34+130+44+140+54+150+77+160+87 = 1105 ✓ (1,2).

1+134+116+167+49+41+151+79+84+135+17+168+119 = 1261 ✗.

522 169, 110, 46, 69, 92, 5, 151, 128, 28, 64, 87, 23, 133

~~119, 36, 141, 164, 18, 100, 77, 41, 123, 146, 50, 155, 96~~

261
261
290
121
169

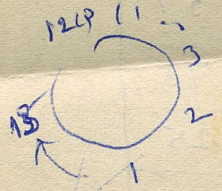
~~1347~~

119, 73, 9, 32, 55, 137, 112, 91, 160, 14, 50, 155, 96

not cyclic { 13, 6, 7, 4, 1, 5, 8, 11, 2, 12, 9, 10, 3
2, 8, 9, 6,

cyclic in cols { 0, 13, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11.
6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 2, 4

1-13-12-11-10-9-8-7-6-5-4-3-2



1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3 } cyclic
11, 9, 7, 5, 3
4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6 cyclic

1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12
8, 10, 12, 1, 3, 5, 7

1, 5, 9, 13, 4, 8, 12, 3, 7, 11, 2, 6, 10
8, 12, 3, 7, 11, 2

52, 156, 0, 13, 39, 104, 65, 78, 91, 26, 117
13, 26, 52, 156, 91, 130, 117, 0, 104

9, 11, 12, 2 — 39, 65, 78, 117
13, 156, 117, 104

156, 0,

39
52, 156, 0, 13, 104 — 5, 6, 7, 9, 4
0

156, 0, 39, 65 — 5, 6, 9, 11.
26, 52, 156,
13, 156.

156 + 5,
156 + 11,

156 + 5, 91 + 13, 0 + 6,
26 + 5, 130 + 13, 0 + 10,

133	71	155	121	13	61	109	157	49	15	99	37	85
108	23	158	50	98	146	38	86	65	136	74	122	1
60	39	87	135	14	75	123	102	4	111	159	51	145
76	124	3	64	112	160	139	41	148	27	88	26	97
161	40	101	149	28	7	91	16	77	125	63	134	113
90	138	17	78	44	128	53	114	162	100	2	150	29
6	54	115	81	165	103	151	30	137	52	18	66	127
104	152	118	33	140	19	67	5	89	55	116	164	43
20	168	70	8	56	117	42	126	92	153	32	80	141
36	107	45	93	154	79	163	142	21	69	130	9	57
144	82	143	22	129	31	10	58	106	167	46	94	73
119	11	59	166	68	47	95	156	35	83	131	110	25
48	96	34	105	84	132	24	72	120	12	147	62	169

n = 13

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \left.\begin{matrix} k = i + j \\ l = i - j \end{matrix}\right\}$$

$a_{11} \to b_{2,13}$, $a_{12} \to b_{3,12}$, $a_{13} \to b_{4,11}$

step $(-1,-1)$

$\left.\begin{matrix} a_{1,13} \to b_{11} \\ a_{21} \to b_{31} \end{matrix}\right\}$ step $(0,-2)$.

X 3, 6, 12, 4, 13, 9, 5, 1, 10, 2, 8, 11, 7
4, 10, 2, 11,

X 3, 4, 8, 11, 5, 12, 6, 13, 7, 10, 1, 2, 9
6, 10, 2,

3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 1
4, 6, 8, 10, 12, 1,

cyclic in u-d
6, 7

8. $\boxed{\cancel{(2,-1),(2,1)}}$ $\cancel{(-2,-1),(-2,1)}$
$\boxed{\cancel{(1,-2),(1,2)}}$ $\cancel{(-1,-2),(-1,2)}$

79
156
43

193
47
146

(3,1) by rows

3	4	1	5	2
1	5	2	3	4
2	3	4	1	5
4	1	5	2	3
5	2	3	4	1

5	10	20	0	15
0	15	5	10	20
10	20	0	15	5
15	5	10	20	0
20	0	15	5	10

(2,3) by n.d

3, 2, 5, 1, 4
1, 4, 3, 2, 5
2, 5, 1, 4, 3
4, 3, 2, 5, 1
5, 4, 2, 1, 3
4, 2, 1, 3, 5
2, 1, 3, 5, 4
1, 3, 5, 4, 2

3, 2, 5, 1, 4
1, 4,

280 272 229
111 103 216
169 169 1443

89
76
13

3	1	2	4	5
4	5	3	1	2
1	2	4	5	3
5	3	1	2	4
2	4	5	3	1

(2,3) by cols

1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3
4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6
7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9
10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12
13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2
3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5
6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8
9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11
12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1,
2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4
5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7
8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10
11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13

(Y) (8,5) by rows & (5,8) by cols

0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130
143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104
117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78
91,

1, 3
11, 13
8, 10
5, 7
2, 4
9, 9
10, 12
6, 8
3, 5
13, 2
9, 11
12, 1
4, 6

4	3	7	6	5	1	2
7	6	5	1	2	4	3
5	1	2	4	3	7	6
2	4	3	7	6	5	1
3	7	6	5	1	2	4
6	5	1	2	4	3	7
1	2	4	3	7	6	5

(5,2) rows
(3,4) cols, (2,5) n.d

1, 74, 134, 38, 98, 158, 62, 122, 26, 86, 146, 50, 110
147, 51, 111, 2, 75, 135, 39, 99, 159, 63, 123, 14, 87. 4, 2, 1, 5, 6, 7, 3
7, 3,

1, 9, 4, 12, 7, 2, 10, 5, 13, 8, 3, 11, 6

11

1, 3, 5, 4, 6, 2, 7
6, 2, 7, 11, 3, 5, 4
(3,4)

4, 12. 11, 6, 1, 9, 4, 12, 7, 2, 10, 5, 13, 8, 3

37, 51, 65, 66, 80, 94, 108, 122, 136, 150, 164, 9, 23. 0, 7, 14, 21, 28, 35, 42
60, 74, 88, 102, 116, 130, 131, 145, 159, 4, 18, 32, 46.

11, 12, 13, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

8

45 72
31 56

25	31	42	13	5	43	16
14	6	47	15	23	32	38
19	22	30	39	10	7	48
37	11	3	49	20	26	29
45	21	27	33	36	9	4
34	40	8	2	46	17	28
1	44	18	24	35	41	12

21	28	35	7	0	42	14
7	0	42	14	21	28	35
14	21	28	35	7	0	42
35	7	0	42	14	21	28
42	14	21	28	35	7	0
28	35	7	0	42	14	21
0	42	14	21	28	35	7

(γ)

0 + 1	130 + 3
13 + 2	65 + 8
26 + 3	0 + 13
39 + 4	104 + 5
52 + 5	39 + 10
65 + 6	143 + 2
78 + 7	78 + 7
91 + 8	13 + 12
104 + 9	117 + 4
117 + 10	56 + 5
130 + 11	156 + 1
143 + 12	91 + 6
156 + 13	26 + 11

13 + 2	65 + 8
65 + 6	143 + 2
91 + 8	13 + 12
143 + 12	91 + 6

6 9 12

a interchange 2nd & 12th rows
and 6th & 8th rows

1	51	88	125	162	30	67	117	154	22	59	96	133
60	97	134	2	52	89	126	163	31	68	105	155	23
124	161	29	66	116	153	21	58	95	132	13	50	87
101	138	6	43	80	130	167	35	72	109	146	14	64
78	115	152	20	57	94	131	12	49	86	123	160	28
165	33	70	107	144	25	62	99	136	4	41	91	128
19	56	93	143	11	48	85	122	159	27	77	114	151
42	79	129	166	34	71	108	145	26	63	100	137	5
142	10	47	84	121	158	39	76	113	150	18	55	92
106	156	24	61	98	135	3	40	90	127	164	32	69
83	120	157	38	75	112	149	117	54	104	141	9	46
147	15	65	102	139	7	44	81	118	168	36	73	110
37	74	111	148	16	53	103	140	8	45	82	119	169

(γ1) with rows 2 & 12 and 6 & 8 interchange

0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130

52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13
117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78
91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52
65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26
156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117
13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104, 143
39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0
130, 0, 39, 78, 117, 156, 26, 65, 104, 143, 13, 52, 91
104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65
78, 117, 156, 26, 65, 104, 143, 13, 52, 91, 130, 0, 39
143, 13, 52, 91, 130, 0, 39, 78, 117, 156, 26, 65, 104
26, 65, 104, 143, 13, 52, 91, 130, 0, 39, 78, 117, 156

(10,3) 1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3
(10,3) 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10
(8,7) 7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9
(8,5) 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12
(8,5) 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2
(11,2) 9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11
(5,8) 6, 4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8
(5,8) 3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5
(11,2) 12, 10, 8, 6, 4, 2, 13, 11, 9, 7, 5, 3, 1
(8,5) 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4
(8,5) 5, 3, 1, 12, 10, 8, 6, 4, 2, 13, 11, 9, 7
(6,7) 4, 2, 13, 11, 9, 7, 5, 3, 1, 12, 10, 8, 6
(10,3) 11, 9, 7, 5, 3, 1, 12, 10, 8, 6, 4, 2, 13

by cols. non-cyclic

INDIAN MATHEMATICAL SOCIETY LIBRARY
RAMANUJAN INSTITUTE FOR ADVANCED STUDY
IN MATHEMATICS
UNIVERSITY OF MADRAS
MADRAS-5 (INDIA)

Prof.Dr.T.S.Bhanu Murthy
Hony.Librarian

Ref: IMSL/334 /82-83.

March 18, 1983.

To
Prof.Dr.B.S.Madhava Rao
4/59, Kanakpura Road
Basavangudi P.O.
Bangalore-560 004.

Dear Respected Sir:

Thank you very much for returning

DUKE MATHEMATICAL JOURNAL VOLUME 5

which you have borrowed from the IMS Library.

With respectful regards,

Yours sincerely

T S Bhanu Murthy

~~Yours Sincerely,~~

Hony.Librarian

INDIAN MATHEMATICAL SOCIETY LIBRARY
RAMANUJAN INSTITUTE FOR ADVANCED STUDY
IN MATHEMATICS
UNIVERSITY OF MADRAS
MADRAS-5 (INDIA)

Received 21/3/83
Wrote asking for the
missing line under the
case $n \equiv 1$ in Roser 4
Walker's paper
BSM
21/3/83

अन्तर्देशीय पत्र कार्ड
INLAND LETTER CARD

नवम एशियाई खेल
IX ASIAN GAMES
DELHI 1982

भारत
35 INDIA 35

Prof.Dr.B.S.Madhava Rao

4/59, Kanakpura Road

Basavanguid P.O.

BANGALORE-4

पिन PIN 5 6 0 0 0 4

पहला मोड़ FIRST FOLD

तीसरा मोड़ THIRD FOLD

दूसरा मोड़ SECOND FOLD

इस पत्र के भीतर कुछ न रखिए NO ENCLOSURES ALLOWED
पते में पिन कोड लिखें WRITE PIN CODE IN ADDRESS
प्रेषक का नाम और पता :— SENDER'S NAME AND ADDRESS :—

INDIAN MATHEMATICAL SOCIETY LIBRARY
RAMANUJAN INSTITUTE FOR ADVANCED STUDY
IN MATHEMATICS
UNIVERSITY OF MADRAS
MADRAS-5 (INDIA)

पिन PIN



अस्पृश्यता ईश्वर और मानवता
के प्रति अपराध है।
UNTOUCHABILITY IS A CRIME
AGAINST GOD AND MAN

यहाँ काट कर खोलिए TO OPEN CUT HERE

बीमा नहीं NOT INSURED

क्रमांक/No. 738

लगाये गये डाक टिकटों का मूल्य रु. पै.

Amount of stamps affixed Rs. 6 P. 20

रजिस्ट्री* Dr. T. S. Bhanumurthy

Received a Registered* Ramanujan Institute for

पानेवाले का नाम

Addressed to Advanced Studies in Mathematics University of Madras Madras - 5

तारीख मोहर

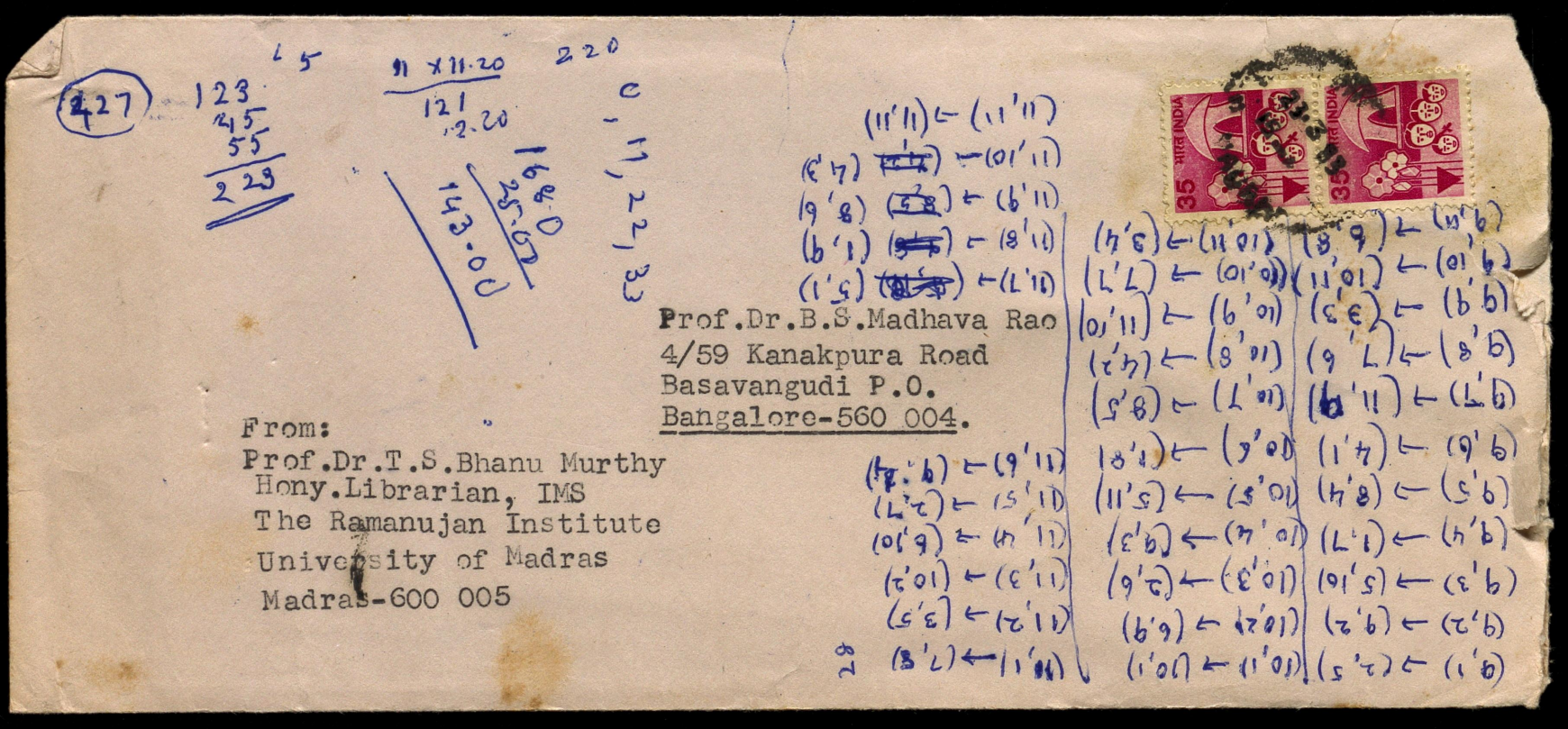
Date Stamp

14/3

पानेवाले अधिकारी के हस्ताक्षर

Signature of Receiving Officer

GENDRA ROAD: 10 BASAVANGUDI
14.3.83



From:
Prof.Dr.T.S.Bhanu Murthy
Hony.Librarian, IMS
The Ramanujan Institute
University of Madras
Madras-600 005

Prof.Dr.B.S.Madhava Rao
4/59 Kanakpura Road
Basavangudi P.O.
Bangalore-560 004.

Prof.Dr.T.S.Bhanu Murthy
Hony.Librarian

The Ramanujan Institute
University of Madras
Madras-600 005

March 23, 1983.

To:
Prof.Dr.B.S.Madhava Rao
4/59 Kanakpura Road
Basavangudi P.O.
Bangalore-560 004.

My dear respected Sir:

Thank you for your letter of 21st/March. I have enclosed a Xerox copy you have referred to in your letter.

My health is fairly all right. Not that I am totally free from the nuisance of a chronic illness. By God's Grace it has not really stood in the way of my life.

My wife works in K.F.I. School (J.Krishnamurthy foundation, India). Our little daughter, now 11 years old, is going to the same school. God has seen to it that I should not complain,

Your firm handwriting, and your scientific enthusiasm, bear witness to the statement:

"Nor hath Age Withered"

With respectful regards.

Yours sincerely,

T.S.Bhanu Murthy
Hony.Librarian

(3,1) → (9,7)
(3,2) → (5,4)
(3,3) → (1,1)
(3,4) → (8,9)
(3,5) → (4,6)
(3,6) → (11,3)
(3,7) → (7,11)
(3,8) → (3,8)
(3,9) → (10,5)
(3,10) → (6,2)
(3,11) → (2,10)

(4,1) → (6,3)
(4,2) → (3,11)
(4,3) → (9,8)
(4,4) → (5,5)
(4,5) → (1,2)
(4,6) → (8,10)
(4,7) → (4,7)
(4,8) → (11,4)
(4,9) → (7,1)
(4,10) → (3,9)
(4,11) → (10,6)

(5,1) → (3,10)
(5,2) → (10,7)
(5,3) → (6,4)
(5,4) → (2,1)
(5,5) → (9,9)
(5,6) → (5,6)
(5,7) → (1,3)
(5,8) → (8,11)
(5,9) → (4,8)
(5,10) → (11,5)
(5,11) → (7,2)

(6,1) → (11,6)
(6,2) → (7,3)
(6,3) → (3,11)
(6,4) → (10,8)
(6,5) → (6,5)
(6,6) → (2,2)
(6,7) → (9,10)
(6,8) → (5,7)
(6,9) → (1,4)
(6,10) → (8,1)
(6,11) → (4,9)

(7,1) → (8,2)
(7,2) → (4,10)
(7,3) → (11,7)
(7,4) → (7,4)
(7,5) → (3,1)
(7,6) → (10,9)
(7,7) → (6,6)
(7,8) → (2,3)
(7,9) → (9,11)
(7,10) → (5,8)
(7,11) → (1,5)

(8,1) → (5,9)
(8,2) → (1,6)
(8,3) → (8,3)
(8,4) → (4,11)
(8,5) → (11,8)
(8,6) → (3,5)
(8,7) → (3,2)
(8,8) → (10,10)
(8,9) → (6,7)
(8,10) → (2,4)
(8,11) → (9,1)

Now

$$\sum_{x=1}^{n} a(i + 4x) = \sum_{x=1}^{n} a(x) = \tfrac{1}{2}n(n-1),$$

and

$$\sum_{x=1}^{n} a(j - 3x) = 3\sum_{x=1}^{m} a(j + 3x) = \tfrac{1}{2}n(n-1).$$

Hence $P(i, j; 4, -3) = nN$.

THEOREM 5.6. *There are non-regular n.d.s. of all orders except those excluded by Theorems* 5.1, 5.2, *and* 5.4.

Case 1. n odd and $(n, 6) = 1$.

2	47	38	35	24	20	9
26	16	8	6	46	42	31
49	39	33	23	15	12	4
19	11	7	45	41	30	22
37	29	27	17	14	3	48
10	5	44	36	34	25	21
32	28	18	13	1	43	40

is a non-regular n.d.s. One can see from the discussion preceding Theorem 5.3 that if it were regular it would have to be the transform of a n.p.s. in which one column contained 1, 8, 15, 22, 29, 36, 43. But these do not constitute a path in the given square, and hence such a transformation is impossible.

We now take $n \geqq 11$. Let A be defined by

$$A_{ij} = (i - 1)n + c_j,$$

where $a_1, a_2, \cdots, a_{11} = 2, 7, 4, 9, 6, 11, 1, 8, 3, 10, 5$ respectively, and $a_j = j$ for $12 \leqq j \leqq n$. Define B as the square with $B(1, 4)$, $B(4, 4)$, $B(4, 2)$, $B(3, 4)$, $B(3, 9)$, $B(2, 11)$, $B(2, 9)$, $B(5, 9)$, $B(\frac{1}{2}(n+7), 2)$, $B(\frac{1}{2}(n+5), 4)$, $B(\frac{1}{2}(n+5), 2)$, $B(\frac{1}{2}(n+11), 2)$, $B(\frac{1}{2}(n+3), 9)$, $B(\frac{1}{2}(n+9), 9)$, $B(\frac{1}{2}(n+9), 7)$, and $B(\frac{1}{2}(n+7), 9)$ equal to $+1$; $B(3, 3)$, $B(2, 5)$, $B(2, 3)$, $B(5, 3)$, $B(1, 10)$, $B(4, 10)$, $B(4, 8)$, $B(3, 10)$, $B(\frac{1}{2}(n+3), 3)$, $B(\frac{1}{2}(n+9), 3)$, $B(\frac{1}{2}(n+9), 1)$, $B(\frac{1}{2}(n+7), 3)$, $B(\frac{1}{2}(n+7), 8)$, $B(\frac{1}{2}(n+5), 10)$, $B(\frac{1}{2}(n+5), 8)$, and $B(\frac{1}{2}(n+11), 8)$ equal to -1; and all other elements equal to zero. Put

$$T = \begin{Vmatrix} 2 & 1 \\ 1 & 2 \end{Vmatrix}.$$

Then TA is a n.d.s., since T is regular and A is a n.p.s. Also it can be readily seen that each row, column, or diagonal of TB contains as many $+1$'s as -1's, so that TB is diabolic. Hence $TA + TB$ is diabolic. Hence it is a n.d.s. if it contains each of the elements $1, 2, \cdots, n^2$. However, $TA + TB = T(A + B)$, and $(A + B)$ is readily seen to be the result of interchanging pairs of elements $A_{i,j}$ and $A_{i,j+6}$ in A. So $TA + TB$ is a n.d.s. If $T(A + B)$ were regular, $A + B$ would also be regular. We have $A_{17} + B_{17} = 1$, $A_{11} + B_{11} = 2$; these elements lie on the unique path $P(1, 1; 0, 6) = P(1, 1; 0, 1)$ since $7 - 1 = 6$ is prime to n, and so any transform of $A + B$ into a p.s. would take the entire first row of $A + B$ into a row. Since this row consists of the numbers $1, \cdots, n$, the number $n + 1$ must lie in the same column of the transformed square as 1. Since $n + 1 = A_{27} + B_{27}$ lies in the same column of $A + B$ as 1, the transformation would take rows into rows and columns into columns, so that if the transformed square were primitive, $A + B$ would be primitive. But this is obviously not so.

Case 2. $n = 4m$, $m \geqq 2$. Consider the $2m^2$ pairs $(0, 16m^2 - 4)$, $(4, 16m^2 - 8)$, $(8, 16m^2 - 12), \cdots, (8m^2 - 4, 8m^2)$. If (A, C) and (B, D) are two of the pairs, then

(4)

$A + 1$	$B + 4$	$C + 1$	$D + 4$
$C + 2$	$D + 3$	$A + 2$	$B + 3$
$A + 4$	$B + 1$	$C + 4$	$D + 1$
$C + 3$	$D + 2$	$A + 3$	$B + 2$

is diabolic with $N = \frac{1}{2}(16m^2 + 1)$. Fill in the $L(m)$'s of S_{4m} with squares like (4), using different sets of pairs for each square. Then the result is a n.d.s. by Theorem 2.7. The number of ways of assigning a pair of pairs to each $L(m)$ is

$$\frac{(2m^2)!}{2^{m^2}}.$$

The square composing any $L(m)$ can be arranged in 384 ways.[6] So there are at least

(5) $$(192)^{m^2}(2m^2)!$$

n.d.s. of order $4m$.[7] We will now show that this exceeds the number of regular n.d.s. of order $4m$. Let $\psi(4m)$ be the number of normalized n.p.s. of order $4m$. Then there are $2((4m)!)^2\psi(4m)$ n.p.s. of order $4m$. However, by an argument

[6] Barkley Rosser and R. J. Walker, loc. cit., Theorem 4.

[7] The squares of order $4m$ constructed by this method satisfy the conditions that each $L(m)$ is a d.s. of order 4. This gives $12m^2$ independent linear conditions on the elements. As the general square of order $4m$ satisfies only $16m - 4$ independent linear conditions, the number of such squares must be immense.