~~Neutrino theory of light~~

Examples from

Ramsey's Hydrodynamics.

Chapter I · p. 16.

(1) Using polar coordinates, we have here $q_r = 0$, $q_\theta = 0$, $q_\phi = \omega r$

Also $\theta = \pi/2$ & the eqn of continuity in polar coordinates viz

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho q_r r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho q_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho q_\phi) = 0$$

becomes $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \phi}(\rho\omega) = 0$ since r is constant.

(2) The equation of continuity in cylindrical coordinates r, θ, z viz

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho v_r r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

is here reduced to the given expression by the second term dropping out because $v_r = 0$ for every cylinder the motion being on the surface.

(3) Here we have $q_\theta = q_\phi = 0$ using polar coordinates & the equation of continuity reduces to

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u) = 0$$

ie $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho u) + \frac{\rho u}{r^2}\frac{\partial}{\partial r}(r^2) = 0$$

$$\text{ie } \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r^2}\frac{\partial r^2}{\partial r} = 0$$

$$\text{ie } \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0$$

$$\text{ie } \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial}{\partial r}(ur^2) = 0$$

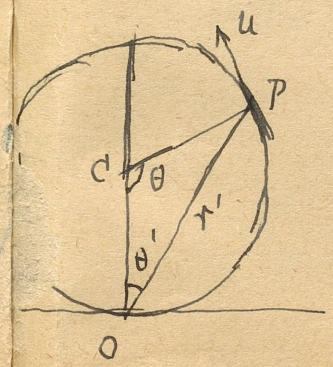
Using polar coordinates we have here $q_\phi = 0$ & hence the eqn of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho q_r r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho q_\theta \sin\theta) = 0.$$

Here $q_\theta = 0$ & the eqn is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho q_r r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho q_\phi) = 0$$

$$\text{ie } \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho q_r) + \frac{2\rho q_r}{r} + \frac{\operatorname{cosec}\theta}{r}\frac{\partial}{\partial \phi}(\rho q_\phi) = 0.$$



Let r', θ', ϕ be polar coords referred to O the origin. The velocity in the plane PCO is u $\perp^r$ to CP & v $\perp^r$ to the plane.

Hence $q_{r'} = u\sin\theta'$

$$q_{\theta'} = -u\cos\theta'$$

and $\theta' = 90 - \theta/2 \quad \therefore q_{r'} = u\cos\frac{1}{2}\theta$

$$q_{\theta'} = -u\sin\tfrac{1}{2}\theta$$

Also $r' = 2r\cos\theta' = 2r\sin\frac{1}{2}\theta$.

The continuity eqn referred to (r', θ', ϕ) is

$$\frac{\partial\rho}{\partial t} + \frac{1}{r'^2}\frac{\partial}{\partial r'}\left(\rho q_{r'} r'^2\right) + \frac{1}{r'\sin\theta'}\frac{\partial}{\partial\theta'}\left(\rho q_{\theta'}\sin\theta'\right) + \frac{1}{r'\sin\theta'}\frac{\partial(\rho q_\phi)}{\partial\phi} = 0$$

ie $$\frac{\partial\rho}{\partial t} + \frac{1}{r'^2}\frac{\partial}{\partial r'}\left(\rho u\cos\tfrac{1}{2}\theta\cdot r'^2\right) + \frac{1}{r'\sin\theta'}\frac{\partial}{\partial\theta'}\left(-\rho u\sin\tfrac{1}{2}\theta\sin\theta'\right) + \text{''} = 0$$

$$r'\sin\theta' = r\sin\theta \ \&$$

$$r\sin\theta\frac{\partial\rho}{\partial t} + \frac{r\sin\theta}{r'^2}\frac{\partial}{\partial r'}\left(\rho u\cos\tfrac{1}{2}\theta\cdot r'^2\right) + \frac{\partial}{\partial\theta'}\left(-\rho u\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta\right) + \frac{\partial(\rho v)}{\partial\phi} = 0$$

~~$r\sin\theta\frac{\partial\rho}{\partial t}+$~~ Since $r' = 2r\sin\frac{1}{2}\theta$, we treat θ as the single independent variable & hence $\underline{u}$ also as a function.

Thus the second term $= \dfrac{r\sin\theta}{r'^2}\cdot\rho u\cos\frac{1}{2}\theta\cdot 2r'$

$$= \frac{2r\sin\theta\cdot\rho u\cos\frac{1}{2}\theta}{r'} = 2\rho u\cos^2\tfrac{1}{2}\theta$$

& the third term $= \dfrac{\partial}{\partial\theta}\left(-\rho u\cdot\frac{1}{2}\sin\theta\right)\left(\dfrac{\partial\theta}{\partial\theta'}\right) = \dfrac{\partial}{\partial\theta}\left(\rho u\sin\theta\right)$

$$= \sin\theta\frac{\partial(\rho u)}{\partial\theta} + \rho u\cos\theta$$

Hence the eqn becomes

$$r\sin\theta\frac{\partial\rho}{\partial t}+2\rho u\cos^2\tfrac{1}{2}\theta+\sin\theta\frac{\partial(\rho w)}{\partial\theta}+\rho u\cos\theta+\frac{\partial(\rho v)}{\partial\phi}=0$$

or $$r\sin\theta\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial\phi}+\sin\theta\frac{\partial(\rho w)}{\partial\theta}+\rho u(1+2\cos\theta)=0,$$ [putting $2\cos^2\frac{1}{2}\theta=1+\cos\theta$

(7) See Ex. 3, p. 91 of Milne-Thomson.

(8) The equation of continuity is in general

$$\frac{\partial\rho}{\partial t}+\nabla(\rho\vec{q})=0$$

and in orthogonal coordinates

$$\nabla\vec{a}=h_1h_2h_3\left\{\frac{\partial}{\partial\xi}\left(\frac{\rho a_1}{h_2h_3}\right)+\cdots+-\right\}$$

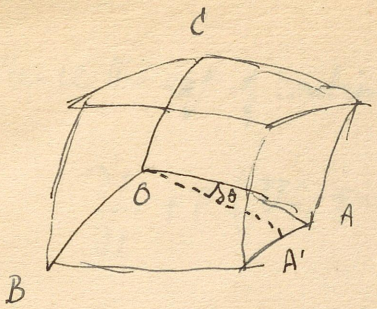
[See Milne-Thomson, p. 57, where h_1, h_2, h_3 are to be replaced by $\frac{1}{h_1}$ etc]

Hence eqn becomes

$$\frac{\partial\rho}{\partial t}+h_1h_2h_3\left\{\frac{\partial}{\partial\xi}\left(\frac{\rho U}{h_2h_3}\right)+\cdots\right\}=0$$

$$\frac{\partial\rho}{\partial t}+h_1\frac{\partial}{\partial\xi}(\rho U)+\cdots+\cdots+\rho U\cdot h_1h_2h_3\frac{\partial}{\partial\xi}\left(\frac{1}{h_2h_3}\right)+-+-=0.$$

Let us find the principal curvatures of the surface $\xi=$ const at the point O of the intersection of the three surfaces.



$OA = \frac{1}{h_1}\delta\xi$, etc..

Let $\delta\theta$ = ~~nor~~ angle between the normals sm to the surface BOC at O & B & Let OA' be ⊥ to the normal at B. Then $AA' = OA \cdot \delta\theta$

and $AA' = \frac{\partial}{\partial\xi}(OB)\,\delta\xi = \frac{\partial}{\partial\xi}\left(\frac{\delta\eta}{h_2}\right)\delta\xi$

$$\therefore \delta\theta = \frac{\partial}{\partial\xi}\left(\frac{\delta\eta}{h_2}\right)\delta\xi \Big/ \frac{\delta\xi}{h_1}$$

If ρ be a principal radius of curvature

$$\frac{1}{\rho} = \frac{\delta\theta}{OB} = \frac{\frac{\partial}{\partial\xi}\left(\frac{\delta\eta}{h_2}\right)\delta\xi}{\frac{\delta\xi}{h_1}\cdot\frac{\delta\eta}{h_2}} = h_1 h_2 \frac{\partial}{\partial\xi}\left(\frac{1}{h_2}\right)$$

Similarly $\frac{1}{\rho'} = h_1 h_3 \frac{\partial}{\partial\xi}\left(\frac{1}{h_3}\right)$

$$\therefore S_1 = \frac{1}{\rho} + \frac{1}{\rho'} = h_1 h_2 \frac{\partial}{\partial\xi}\left(\frac{1}{h_2}\right) + h_1 h_3 \frac{\partial}{\partial\xi}\left(\frac{1}{h_3}\right)$$

$$S_1 = h_1 h_2 h_3 \frac{\partial}{\partial\xi}\left(\frac{1}{h_2 h_3}\right)$$

Hence the eqn of continuity reduces to

$$\frac{\partial \rho}{\partial t} + \rho\left(\bar{v}S_1 + \cdots + \cdots\right) + h_1 \frac{\partial}{\partial f}(\rho \bar{v}) + \cdots = 0.$$

(9) The eqn to be satisfied for the surface $F = 0$ to be a boundary is

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + \cdots + \cdots = 0$$

ie in this case

$$\frac{2x^2}{a^2}\tan t \cdot \sec^2 t - \frac{2y^2}{b^2}\cot t \cdot \operatorname{cosec}^2 t + u \cdot \frac{2x}{a^2}\tan^2 t + v \cdot \frac{2y}{b^2}\cot^2 t = 0$$

This, as well as the continuity eqn

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \text{ is satisfied by}$$

$$u = -x \cot t \sec^2 t = -2x \operatorname{cosec} 2t$$

$$v = y \tan t \operatorname{cosec} 2t = 2y \operatorname{cosec} 2t$$

Hence the result. The normal velocity

$$= \frac{\frac{\partial F}{\partial x}}{\sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}} u + \frac{\frac{\partial F}{\partial y}}{\sqrt{\quad}} v = \text{etc}$$

(10) Since the motion is steady, u, v and w are not functions of t. Let $u = \phi_1(x, y, z)$, $v = \phi_2(x, y, z)$, $w = \phi_3(x, y, z)$.

The stream lines are given

$$\frac{dx}{\phi_1} = \frac{dy}{\phi_2} = \frac{dz}{\phi_3}$$

The solutions of these are given by

$$f_1 = a_1, \; f_2 = a_2.$$

From these solutions

$$\frac{\partial f_1}{\partial x}dx + \frac{\partial f_1}{\partial y}dy + \frac{\partial f_1}{\partial z}dz = 0$$

$$\frac{\partial f_2}{\partial x}dx + \frac{\partial f_2}{\partial y}dy + \frac{\partial f_2}{\partial z}dz = 0$$

which give $\dfrac{dx}{\dfrac{\partial(f_1, f_2)}{\partial(y, z)}} = \cdots = \cdots$

Hence $\dfrac{\phi_1}{\dfrac{\partial(f_1, f_2)}{\partial(y, z)}} = \cdots = \cdots$

i.e. $\phi_1 = F\dfrac{\partial(f_1, f_2)}{\partial(y, z)}$, etc.

We have now to prove that in virtue of the eqn of continuity

F is a function only of f_1 & f_2.

$\frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial y} + \frac{\partial \phi_3}{\partial z} = 0$ gives

$$\sum \frac{\partial F}{\partial x} \cdot \frac{\partial (f_1, f_2)}{\partial (y, z)} + F \sum \frac{\partial}{\partial x} \cdot \frac{\partial (f_1, f_2)}{\partial (y, z)} = 0.$$

$$\sum \frac{\partial}{\partial x} \cdot \frac{\partial (f_1, f_2)}{\partial (y, z)} = 0 \text{ identically}$$

$$\therefore \quad \frac{\partial F}{\partial x} \cdot \frac{\partial (f_1, f_2)}{\partial (y, z)} + \cdots + - = 0$$

ie $\phi_1 \frac{\partial F}{\partial x} + \cdots + - = 0$

In virtue of $dx/\phi_1 = -$, this gives $\frac{\partial F}{\partial x} dx + - = 0$

ie $F = \text{const}$ is a soln of $\frac{dx}{\phi_1} = \cdots$

and hence $F = F(f_1, f_2)$.

(11) $u = -\frac{\partial \phi}{\partial x} = -2\alpha x, \quad v = -2\beta y, \quad w = -2\gamma z$

The continuity eqn requires $\alpha + \beta + \gamma = 0$.

If the given surface be the boundary, the condn to be satisfied is

$$-x'(t) - 2\alpha x \cdot 4ax^3 - 2\beta y \cdot 4by^3 - 2\gamma z \cdot 4cz^3 = 0$$

ie $8(a\alpha x^4 + b\beta y^4 + c\gamma z^4) + x'(t) = 0.$

(noting that a, b, c are fns of the time)

$$\frac{\partial a}{\partial t} x^4 + \cdots - x'(t) - 2\alpha x \cdot 4ax^3 - \cdots = 0$$

ie $$x^4 \left(\frac{da}{dt} - 8a\alpha \right) + \cdots - x'(t) = 0$$

Comparing this with

$$ax^4 + \cdots + x(t) = 0$$

we can choose a, b, c so as to satisfy

$$\left(\frac{da}{dt} - 8a\alpha \right) x(t) = a x'(t), \text{ etc} \cdots$$

These are linear differential eqns for a, b, c if α, β, γ & $x(t)$ are given.

Also adding the given three linear diff eqns after diving them by a, b, c resp^ly

$$\frac{d}{dt}(a+b+c)x(t) = (a+b+c)x$$

$$x(t) \left\{ \frac{1}{a} \frac{da}{dt} + \cdots + - \right\} = 3x'(t) \text{ since } \alpha + \beta + \gamma = 0.$$

$$\frac{d}{dt}(\log abc) = 3 \frac{d}{dt} \log x(t).$$

ie $$abc = x^3(t).$$

The solns of the diff eqns for a, b, c enable suitable choice of arbitrary constants so as to satisfy this relation.

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(3) The motion is supposed steady & putting $\partial\phi/\partial t = 0$ and also external force = 0, the pressure eqn is

$$\int \frac{dp}{\rho} + \frac{1}{2}q^2 = C$$

Here $\frac{p}{\rho} = K$ ie $p = K\rho$

$$\therefore \quad K \log p + \frac{1}{2}q^2 = C \qquad (1)$$

The eqn of continuity gives

$$\frac{d}{dt}(\rho \cdot \pi r^2 v) = 0$$

ie $\quad \rho r^2 v = \text{const} = K_1$

$$\rho_1 d^2 v = K_1$$

$$\rho_2 D^2 V = K_1$$

$$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{D^2 V}{d^2 v}$$

$$K \log p_1 + \frac{1}{2}v^2 = K \log p_2 + \frac{1}{2}V^2$$

$$\therefore K \log \frac{p_1}{p_2} = \frac{1}{2}(V^2 - v^2) \text{ ie } \frac{p_2}{p_1} = e^{\frac{V^2 - v^2}{2K}}$$

$$\therefore \frac{d^2 v}{D^2 V} = e^{(V^2 - v^2)/2K} \text{ ie } \frac{v}{V} = \frac{D^2}{d^2} e^{\frac{V^2 - v^2}{2K}}$$

(4) The eqn of motion is

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{k}{\rho}\frac{\partial \rho}{\partial r} \quad (p = k\rho)$$

ie $$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -k\frac{\partial z}{\partial r} \quad \text{--- (1)}$$

where $z = \log\rho$.

The eqn of continuity is

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\frac{\partial v}{\partial r} = 0$$

or $$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{v}{\rho}\cdot\frac{\partial \rho}{\partial r} + \frac{\partial v}{\partial r} = 0$$

ie $$\frac{\partial z}{\partial t} + v\frac{\partial z}{\partial r} + \frac{\partial v}{\partial r} = 0. \quad \text{--- (2)}$$

ie $$-k\frac{\partial z}{\partial t} = k\frac{\partial v}{\partial r} + kv\frac{\partial z}{\partial r}$$

$$= -v\left(\frac{dv}{dt} + v\frac{\partial v}{\partial r}\right) + k\frac{\partial v}{\partial r} \quad \text{using (1)} \quad \text{--- (3)}$$

Differentiating (1) w.r.t t, & (3) w.r.t r & equating

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) + \frac{\partial}{\partial r}\left(v\frac{\partial v}{\partial t} + v^2\frac{\partial v}{\partial r}\right) = k\frac{\partial^2 v}{\partial r^2}$$

$$\frac{\partial^2 v}{\partial t^2} + \frac{\partial v}{\partial t}\frac{\partial v}{\partial r} + v\frac{\partial^2 v}{\partial r\partial t} + \quad " \quad = "$$

$$\frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial r}\left(v\frac{\partial v}{\partial t}\right) + \frac{\partial}{\partial r}\left(\quad " \quad\right) = "$$

ie $$\frac{\partial^2 v}{\partial t^2} + \frac{\partial}{\partial r}\left(2v\frac{\partial v}{\partial t} + v^2\frac{\partial v}{\partial r}\right) = k\frac{\partial^2 v}{\partial r^2}.$$

(5) Equation of motion is

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{k}{\rho}\frac{d\rho}{\partial x} \quad \text{as in previous example } (p = k\rho) \quad \text{--- (1)}$$

& eqn of continuity is

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0 \quad \text{or} \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \quad \text{--- (2)}$$

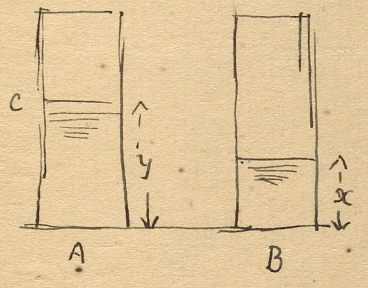
(2) ~~This~~ gives $\frac{\partial^2 \rho}{\partial t^2} = -\frac{\partial^2}{\partial x \partial t}(\rho v) = -\frac{\partial}{\partial x}\left(v\frac{\partial \rho}{\partial t} + \rho\frac{\partial v}{\partial t}\right)$

$$= \frac{\partial}{\partial x}\left\{v\frac{\partial}{\partial x}(\rho v) + \rho v\frac{\partial v}{\partial x} + k\frac{\partial \rho}{\partial x}\right\} \quad \text{using (1) \& (2)}$$

$$= \frac{\partial}{\partial x}\left\{\frac{\partial}{\partial x}(\rho v^2) + k\frac{\partial \rho}{\partial x}\right\}$$

$$= \frac{\partial^2}{\partial x^2}\left\{(v^2 + k)\rho\right\}.$$

(6)



A is initially full of water & B of air.

Initial pressure of air = gh

if $\rho = 1$ for water.

After communication is obtained let y & x be heights of water in A & B at any time t.

<u>motion in A</u>: Continuity eqn is $\frac{\partial u}{\partial y} = 0$

& eqn of motion is $\frac{\partial u}{\partial t} = -g - \frac{\partial p}{\partial y}$ $\quad (\because \rho = 1 \text{ \& } \frac{\partial u}{\partial y} = 0)$

Integrating w.r.t. y since u does not explicitly depend on y $\therefore \frac{\partial u}{\partial y} = 0$,

$$y\frac{\partial u}{\partial t} = -gy - p + f(t) \qquad \text{——(1)}$$

Also throughout $x + y = c$. Hence if x be final height reached in B,

then in (1), $p = 0$ for $y = c - x$

and $p = p_0$ for $y = 0$

Hence substituting these values of p in (1)

$$(c-x)\frac{\partial u}{\partial t} = -g(c-x) + f(t)$$

$$0 = 0 \quad - p_0 + f(t)$$

and by subtraction, $p_0 = g(c-x) + (c-x)\frac{\partial u}{\partial t}$ (2)

Motion in B: Let u' be the velocity

$\frac{\partial u'}{\partial x} = 0$ is the eqn of continuity

and $\frac{\partial u'}{\partial t} = -g - \frac{\partial p}{\partial x}$ is the eqn of motion

$$\therefore \quad x\frac{\partial u'}{\partial t} = -gx - p + \phi(t) \qquad (3)$$

Here for $x = 0$, $p = p_0$ and for $x = x$, $p = \frac{gkc}{c-x}$ since the initial pressure is increased gh is increased due to diminution of vol from $A.c$ to $A(c-x)$

(A = cross section of cylinder)

Hence substituting in (3)

$$0 = -p_0 + \phi t$$

$$x\frac{\partial u'}{\partial t} = -gx - \frac{ghc}{c-x} + \phi(t)$$

& subtracting $x\frac{\partial u'}{\partial t} + gx + \frac{ghc}{c-x} = p_0$ (4)

Equating p_0 from (2) & (4) gives

$$(c-x)\frac{\partial u}{\partial t} + g(c-x) = x\frac{\partial u'}{\partial t} + gx + \frac{ghc}{c-x}$$

Now $\frac{\partial u}{\partial t} = \ddot{y} = -\ddot{x}$ and $\frac{\partial u'}{\partial t} = \ddot{x}$

$$\therefore \quad c\frac{\partial u}{\partial t} = g(2x-c) + \frac{ghc}{c-x}$$

$$\text{ie} \quad -c\ddot{x} = g(2x-c) + \frac{ghc}{c-x}$$

which gives the first integral

$$-\tfrac{1}{2}c\dot{x}^2 = g(x^2 - cx) - gch\log(c-x) + A.$$

& with $\dot{x} = 0$, for $x = 0$ gives $A = gch\log c$

$$\therefore \tfrac{1}{2}c\dot{x}^2 = g(cx - x^2) + gch\log\frac{c-x}{c}$$

& $\dot{x} = 0$ gives the final height $\underline{x}$ of water rising in B as the soln of the eqn

$$cx - x^2 + ch\log\frac{c-x}{c} = 0.$$

Ex. 8. of old edition: The pressure eqn gives

$$\int \frac{dp}{\rho} + \tfrac{1}{2} q^2 + V = C$$

i.e. $K \log p + \tfrac{1}{2} q^2 + V = C$

i.e. $K \log p + \frac{1}{2} C_1 \{ (y+z)^2 + (z+x)^2 + (x+y)^2 \} - C = 0$

Surfaces of equal pressure are given by

$$\Sigma (y+z)^2 = \text{Const.}$$

i.e. $x^2 + y^2 + z^2 + yz + zx + xy = \text{Const.}$

The discriminating cubic is

$$\lambda^3 - 3\lambda^2 + \frac{9\lambda}{4} - \frac{1}{2} = 0$$

i.e. $(\lambda - 2)(2\lambda - 1)^2 = 0$

Hence the surface is one of revolution & can be reduced to

$$2x^2 + \frac{y^2}{2} + \frac{z^2}{2} = \text{const}$$

i.e. an oblate spheroid of eccentricity $= \sqrt{\frac{2 - 1/2}{2}} = \frac{\sqrt{3}}{2}$.

(7) Ex. 3., p. 24, Milne-Thomson.

(8) The first part follows from Ex. 27 by putting $x = a$ in the ratio viz

$$\{3x^2r^4 + (a^3 - 4x^3)r^3 - (a^3 - x^3)x^3\} / 3x^2r^4$$

which becomes $(3a^2r^4 - 3a^3r^3)/3a^2r^4 = \left(1 - \frac{a}{r}\right)$.

As regards the second part, referring to Example 3 worked in the text on p. 32, we have the impulsive pressure for $r' = r$ given by [$\frac{\varpi}{\rho} = r^2 v \left(\frac{1}{r} - \frac{1}{R}\right)$ — See p. 33]

$$p = \rho r v \quad (\text{putting } R = \infty)$$

and $v^2 = \frac{2}{3}\frac{\varpi}{\rho} \cdot \frac{a^3 - r^3}{r^3}$ $(R = \infty$ on p. 33)

Putting $r = a/2$, $v^2 = \frac{14\varpi}{3\rho}$

$$\therefore p = \rho r v = \sqrt{\frac{7\varpi\rho a^2}{6}}.$$

(9) ——————

(10) The pressure equation is given in general by

$$\frac{p}{\rho} = \frac{\partial \phi}{\partial t} - \tfrac{1}{2}q^2 - \Omega + C$$

Putting $\phi = -txyz$, we have $-\frac{\partial \phi}{\partial x} = tyz$, etc.

so that $q^2 = t^2(y^2x^2 + z^2x^2 + x^2y^2)$

and this leads to the given pressure equation with $C = 0$.

Also q is proportional to t. Thus $\phi = -txyz$ is the correct expression for the velocity potential, and gives for Ω

$$\Omega = -(\mu - 1)xyz \qquad \left[\frac{\partial \phi}{\partial t} = -xyz\right]$$

the force components $-\frac{\partial \Omega}{\partial x}$, .. are $(\mu - 1)yz$, ..

ie the force is conservative & independent of the time.

The stream lines are given by

$$\frac{dx}{q_x} = -$$

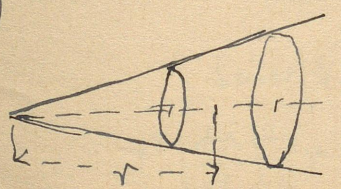
ie $$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dx}{xy}$$

of the or of $x\,dx - y\,dy = 0$ & $y\,dy - z\,dz = 0$

ie $$x^2 - y^2 = K_1 \text{ and } y^2 - z^2 = K_2$$

ie stream lines are intersections of two hyperbolic cylinders.

(11)



Since the volume of the liquid is constant

$$\frac{1}{3}\pi r'^3 \tan^2\alpha - \frac{1}{3}\pi r^3 \tan^2\alpha = \text{const}$$

ie $$r'^3 - r^3 = c^3 \text{ (say)} \qquad (1)$$

Since there are no external forces & no variation of pressure along the direction of the axis, the K.E. of the liquid is constant. Since the cross-section is small the velocity on any cross-section can be taken as the velocity at the centre of the section. If v_1 be the velocity at distance r_1, the eqn of continuity is

$$\pi r_1^2 \tan^2\alpha \cdot v_1 = \pi r^2 V \tan^2\alpha = \pi r'^2 V' \tan^2\alpha$$

ie $$v_1 r_1^2 = V r^2 = V' r'^2. \qquad (2)$$

The K.E. $= \int_r^{r'} \pi r_1^2 \tan^2\alpha \cdot \rho v_1^2 \, dr_1 \qquad \left[v_1^2 = \frac{V^2 r^4}{r_1^4} \right.$

$$= \pi \tan^2\alpha \, \rho r^4 V^2 \int_r^{r'} \frac{dr_1}{r_1^2}$$

$$= \pi \tan^2\alpha \, \rho r^4 V^2 \left(\frac{1}{r} - \frac{1}{r'} \right)$$

Since this is constant

$$r^3 V^2 (r' - r)/r' = \text{const} = c$$

ie $$V^2 = \frac{c r'}{r^3 (r' - r)}$$

which is the second part of the question.

Since $V = dr/dt$

$$r^3 \left(\frac{dr}{dt}\right)^2 (r' - r)/r' = \text{const} \quad \text{ie} \quad \frac{d}{dt}\{\text{L.H.S.}\} = 0$$

or

Taking logarithms & differentiating

$$\frac{3}{r}\frac{dr}{dt} + \frac{2\,d^2r/dt^2}{dr/dt} + \frac{dr'/dt - dr/dt}{r' - r} - \frac{1}{r'}\frac{dr'}{dt} = 0$$

$r^2 v = r'^2 v'$ gives $r^2 \frac{dr}{dt} = r'^2 \frac{dr'}{dt}$

Substituting $\frac{dr'}{dt} = \frac{r^2}{r'^2}\frac{dr}{dt}$, the above eqn gives

$$\frac{3}{r}\frac{dr}{dt} + \frac{2\,d^2r/dt^2}{dr/dt} - \frac{dr}{dt}\cdot\frac{r'+r}{r'^2} - \frac{dr}{dt}\cdot\frac{r^2}{r'^3} = 0$$

ie $$2r\frac{d^2r}{dt^2} + \left(\frac{dr}{dt}\right)^2\left\{3 - \frac{r(r'+r)}{r'^2} - \frac{r^3}{r'^3}\right\} = 0$$

ie $$2r\frac{d^2r}{dt^2} + \left(\frac{dr}{dt}\right)^2\left\{3 - \frac{r}{r'} - \frac{r^2}{r'^2} - \frac{r^3}{r'^3}\right\} = 0.$$

(12) If v' be the velocity at distance r', the equation of motion is

$$\frac{\partial v'}{\partial t} + v'\frac{\partial v'}{\partial r'} = -\frac{\mu}{r'^2} - \frac{1}{\rho}\frac{\partial p}{\partial r'} \qquad (1)$$

The eqn of Continuity is

$$r'^2 v' = F(t) \qquad (2)$$

$\therefore r'^2 \frac{\partial v'}{\partial t} = F'(t)$ & (1) can be written as

$$\frac{F'(t)}{r'^2} + v'\frac{\partial v'}{\partial r'} = -\frac{\mu}{r'^2} - \frac{1}{\rho}\frac{\partial p}{\partial r'}$$

Integrating this from $r' = r$ to R, and noting that the pressure at both ends is zero, (20)

$$-F'(t)\left(\frac{1}{r}-\frac{1}{R}\right)+\frac{1}{2}(v^2-V^2)=\mu\left(\frac{1}{r}-\frac{1}{R}\right)$$

$$F(t) = r^2 v = R^2 V$$

$$\therefore F'(t) = 2r\dot{r}v + r^2\dot{v}$$

$$= 2r\dot{r}v + r^2\frac{dv}{dr}\dot{r} \quad , \quad v=\dot{r}$$

$$= 2rv^2 + r^2 v\frac{dv}{dr}$$

$$\therefore -\left(2rv^2+r^2v\frac{dv}{dr}\right)\left(\frac{1}{r}-\frac{1}{R}\right)+\frac{1}{2}v^2\left(1-\frac{r^4}{R^4}\right)=\mu\left(\frac{1}{r}-\frac{1}{R}\right)$$

Put $v^2 = z$, and multiply by $2r^2$ and noting

$$R^3 - r^3 = A^3 - a^3 = c^3$$

$$2r^2\mu\left(\frac{1}{r}-\frac{1}{R}\right) = -\left\{\frac{1}{r}-\frac{1}{(r^3+c^3)^{1/3}}\right\}\frac{d}{dr}(zr^4)$$

$$+ zr^4\left\{\frac{1}{r^2}-\frac{r^2}{(r^3+c^3)^{4/3}}\right\}$$

Integrating the R.H.S $= 2\mu\left\{r-\frac{r^2}{(A^3-a^3+r^3)^{\frac{1}{3}}}\right\}$

Hence integrating

$$2\mu\left\{\frac{1}{2}(r^2-a^2)-\frac{1}{2}(R^2-A^2)\right\}$$

$$= -\left[zr^4\left\{\frac{1}{r}-\frac{1}{(r^3+c^3)^{\frac{1}{3}}}\right\}\right]_a^r = -v^2r^4\left(\frac{1}{r}-\frac{1}{R}\right)$$

$$\therefore \mu\left(R^2 - r^2 + a^2 - A^2\right) = v^2 r^4 \left(\frac{1}{r} - \frac{1}{R}\right)$$

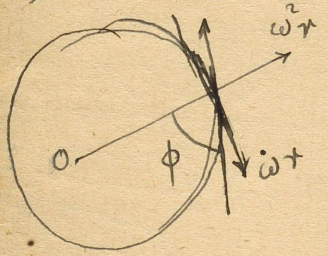
If ϖ be the impulsive pressure at distance r', $\varpi = 0$ when $r' = R$ and as in Ex.(3) p. 32 of the text

$$\frac{\varpi}{\rho} = r^2 v \left(\frac{1}{r} - \frac{1}{R}\right)$$

and substituting the above value of v,

$$\varpi = \rho\sqrt{\mu}\,\sqrt{(a^2 - r^2 - A^2 + R^2)\left(\frac{1}{r} - \frac{1}{R}\right)}.$$

(13)



Measure the time from the instant the ice melts. Reducing the curve to rest we have for the equation of motion

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \omega^2 r\frac{dr}{ds} - r\dot{\omega} r\frac{d\theta}{ds}$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial s} + \omega^2 r\frac{dr}{ds} - \dot{\omega} r^2\frac{d\theta}{ds}$$

The eqn of continuity is $Kv = F(t)$

$$\therefore \frac{\partial v}{\partial t} = \frac{F'(t)}{K}$$

and eqn of motion is

$$\frac{F'(t)}{K} + v\frac{\partial v}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \omega^2 r\frac{dr}{ds} - \dot{\omega} r^2\frac{d\theta}{ds}$$

Integrating $\omega \cdot r$ to s once round the curve

$$F'(t)\int \frac{ds}{K} + \left[\tfrac{1}{2}v^2\right] = -\left[\tfrac{p}{\rho}\right] + \left[\tfrac{1}{2}\omega^2 r^2\right] - 2\dot{\omega}A$$

Since integration is once round the curve, the second expr on the L.H.S and the 1st & 2nd on the R.H.S all vanish

$$\therefore\ F'(t)\int \frac{ds}{K} = -2A\dot{\omega}$$

Integrating with respect to t between the limits 0 and t

$$\{F(t) - F(0)\}\int \frac{ds}{K} = -2A(\omega - \Omega)$$

$$F(t) = Kv,\quad F(0) = 0$$

$$\therefore\ Kv\int \frac{ds}{K} = 2A(\Omega - \omega)$$

or $$v = \frac{2A(\Omega-\omega)}{K\int ds/K}.$$

(14) Proceeding as in Ex (31), p. 32 of the text with $R = \infty$, and the additional term $-\mu/r^2$ in the equation of motion we get for the velocity the expression

$$-\mu(r^2 - a^2) + \frac{2\pi}{3\rho}(a^3 - r^3) = r^3 v^2 \qquad \text{[replacing } \pi \text{ by } \omega\text{]}$$

$$v^2 = \mu\left(\frac{a^2}{r^3} - \frac{1}{r}\right) + \frac{2\pi}{3\rho}\left(\frac{a^3}{r^3} - 1\right) \qquad \left(v = \frac{dr}{dt}\right)$$

ie $$\frac{r^{3/2}\,dr}{\sqrt{\mu(a^2-r^2)+\frac{2\varpi}{3\rho}(a^3-r^3)}} = -dt$$

From the conditions given $\varpi = \frac{1}{2}\frac{\mu}{a}\rho$ or $\mu = 2a\varpi/\rho$

$$\therefore \sqrt{\frac{3\rho}{2\varpi}}\cdot\frac{r^{3/2}\,dr}{\sqrt{3a(a^2-r^2)+(a^3-r^3)}} = -dt$$

ie $$\sqrt{\frac{3\rho}{2\varpi}}\cdot\frac{r^{3/2}\,dr}{\sqrt{a-r}\cdot(2a+r)} = -dt$$

Putting $r = a\sin^2\theta$ & integrating from $\theta = \frac{\pi}{2}$ to 0, $t = 0$ to T, we have

$$T = \int_0^{\frac{\pi}{2}} \sqrt{\frac{3\rho}{2\varpi}}\cdot\frac{a^{3/2}\sin^3\theta\cdot 2a\sin\theta\cos\theta\cdot d\theta}{a^{\frac{1}{2}}\cos\theta\cdot a(2+\sin^2\theta)}$$

$$= 2a\sqrt{\frac{3\rho}{2\varpi}}\int_0^{\frac{\pi}{2}}\left(\sin^2\theta - 2 + \frac{4}{2+\sin^2\theta}\right)d\theta$$

$$= 2a\sqrt{\frac{3\rho}{2\varpi}}\int_0^{\frac{\pi}{2}}\left(\sin^2\theta - 2 + \frac{8}{5-\cos 2\theta}\right)d\theta$$

$$\int_0^{\frac{\pi}{2}}\frac{d\theta}{5-\cos 2\theta} = \frac{1}{6}\int_0^{\infty}\frac{du}{u^2+2/3} \quad (\text{with } u = \tan\theta)$$

$$= \frac{\pi}{12}\sqrt{\frac{3}{2}}.$$

$$\therefore \quad T = 2a\sqrt{\frac{3\rho}{2\varpi}}\left\{\frac{\pi}{4} - \pi + \frac{4\pi}{6}\sqrt{\frac{3}{2}}\right\}$$

$$= 2a\pi\sqrt{\frac{3\rho}{2\varpi}}\left\{\frac{4}{6}\sqrt{\frac{3}{2}} - \frac{3}{4}\right\} = \pi a\sqrt{\frac{\rho}{\varpi}}\left\{\frac{8}{6}\cdot\frac{3}{2} - \left(\frac{3}{2}\right)^{3/2}\right\}$$

$$= \pi a\sqrt{\frac{\rho}{\varpi}}\left\{2 - \left(\frac{3}{2}\right)^{3/2}\right\}.$$

(15). Proceeding as in Ex (3), p. 32 with the additional external force $\phi(r)$ we get, as a generalisation of the first part of Ex. 8, the pressure at distance r immediately after the collapse being given by

$$p = \pi\left(1 - \frac{a}{r}\right) + \rho\left\{\frac{a}{r}\psi(a) - \psi(r)\right\}$$

Initially at distance r the pressure is given by

$$dp = -\rho\,\phi(r)$$

~~& since for r =~~

$$\text{ie} \quad p_0 = -\rho\,\psi(r) + C$$

& since for $r = \infty$, $p_0 = \pi$ & $\psi(r) = 0$

$$p_0 = -\rho\,\psi(r) + \pi.$$

$$\therefore \quad p_0 - p = \pi\frac{a}{r} - \frac{a\rho}{r}\psi(a).$$

If no external force be present, $p = \pi\left(1 - \frac{a}{r}\right)$. Hence $\psi(r)$ is given, so that it might satisfy the stated cond^{ns} of the second

part of the problem, by

$$\psi(r) = a\psi(a)\cdot\frac{1}{r} = \mu/r$$

$$\therefore \quad \phi(r) = -\frac{\mu}{r^2}$$

The velocity of the boundary is given by

$$\rho v^2 r^3 = \int_a^r \rho\left\{2r^2\psi(r) + \frac{2r^2\Pi}{\rho}\right\}dr$$

$$= \int_a^r \left(2\mu r + \frac{2r^2\Pi}{\rho}\right)dr.$$

(16) See Milne-Thomson, Ex. 4, p. 24

(17) We shall follow here the simple method given in §91, a, p. 122 of Lamb's sixth edition.

If R_0 be the radius of the sphere initially & R the radius at time t, we take for the velocity potential

$$\phi = \frac{R^2\dot{R}}{r}$$

because $-\frac{\partial\phi}{\partial r} = \frac{R^2\dot{R}}{r^2}$ becomes $\dot{R}$ for $r = R$.

For the pressure $\frac{p}{\rho} = \frac{\partial\phi}{\partial t} - \frac{1}{2}q^2 + C = \frac{\partial\varphi}{\partial t} - \frac{1}{2}\left(\frac{\partial\varphi}{\partial r}\right)^2 + C$

If p_0 be pressure at $r = \infty$, $C = p_0/\rho$

Ex. 17. old edition:

$$\frac{p}{\rho} = \frac{\partial \phi}{\partial t} - \tfrac{1}{2} q^2 - V + C$$

$$-V = (xy + yz + zx) + k \log (x+y+z)$$

$$\tfrac{1}{2} q^2 = \tfrac{1}{2} t^2 \{ (x+y)^2 + - \}$$

$\therefore$ Vel. components are $t(y+z)$, etc

or $\phi = -t(xy + yz + zx)$

$$\therefore \frac{p}{\rho} = -(xy + yz + zx) + (xy + yz + zx) + k \log (x+y+z) - t^2 (x^2 + y^2 + z^2 + xy + yz + zx) + f(t) + C'$$

where $f(t) = 0$ when $t = 0$ & C' is independent of t

$\therefore$ when $t = 0$, $\frac{p}{\rho} = C' + k \log (x+y+z) = k \log \left(\frac{x+y+z}{h} \right)$

where $C' = -k \log h$.

$\therefore$ $p = k\rho \log \frac{x+y+z}{h} - \rho t^2 (x^2 + y^2 + z^2 + xy + yz + zx) - \rho f(t)$.

Hence the first part. (18

The stream lines are given by

$$\frac{dx}{t(y+z)} = \cdots = \frac{dx+dy+dz}{2t(x+y+z)} = \frac{dx-dy}{t(y-x)} = \cdots$$

$\therefore$ $\frac{1}{2} \log C_1 (x+y+z) = -\log C_2 (x-y) = -\log C_3 (y-z)$ & since (x_0, y_0, z_0) are

$$\therefore \quad \frac{p-p_0}{\rho} = \frac{\partial \phi}{\partial t} - \frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2$$

$$= \frac{R^2\ddot{R} + 2R\dot{R}^2}{r} - \frac{R^4\dot{R}^2}{2r^4}.$$

Putting r

$$\text{ie } p - p_0 = \frac{\rho R}{r}\cdot\left\{R\ddot{R} + 2\dot{R}^2 - \frac{R^3\dot{R}^2}{2r^3}\cdot\right\}$$

Putting $R = b + a\cos nt$, $\dot{R} = -an\sin nt$, $\ddot{R} = -an^2\cos nt$

$$R\ddot{R} + 2\dot{R}^2 = -an^2\cos nt\,(b + a\cos nt) + 2a^2n^2\sin^2 nt$$

$$= -an^2\left\{b\cos nt + a\cos^2 nt - 2a\sin^2 nt\right\}$$

$$= -an^2\left\{b\cos nt + a(1 - 3\sin^2 nt)\right\}$$

&

$$\frac{R^3\dot{R}^2}{2r^3} = \frac{(b + a\cos nt)^3\cdot a^2n^2\sin^2 nt}{2r^3}$$

$$\therefore \quad p_0 - p = \frac{\rho an^2}{r}(b + a\cos nt)\left\{a(1 - 3\sin^2 nt) + b\cos nt + \frac{a\sin^2 nt}{2r^3}\cdot R^3\right\}.$$

(18) In the above example, the pressure at the surface of the sphere is given by putting $r = R = b + a\cos nt$.
(ie $a + b\cos nt$ in this example a & b are interchange)

If this pressure be p, then

$$p = p_0 - \rho bn^2\left\{b(1 - 3\sin^2 nt) + a\cos nt + \frac{1}{2}b\sin^2 nt\right\}$$

initial values

$$\frac{1}{2}\log\frac{x+y+z}{x_0+y_0+z_0} = -\log\frac{x-y}{x_0-y_0} = -\log\frac{y-z}{y_0-z_0}$$

$$\therefore \quad \frac{x_0+y_0+z_0}{x+y+z} = \left(\frac{x-y}{x_0-y_0}\right)^2 = \left(\frac{y-z}{y_0-z_0}\right)^2.$$

(19

$$= \Pi - \rho b n^2 \left\{ b\left(1 - \frac{5}{2}\sin^2 nt\right) + a\cos nt \right\} \quad (\Pi = p_0)$$

The least value of p is attained when the expn $\{\ \}$ has its maximum value which is attained when $\sin nt = 0$ & $\cos nt = 1$ ie

$$p_{min} = \Pi - \rho b n^2 (a+b).$$

(19) The ~~rate of doing work~~ work done is given by the same expn as that of work done by a gas in expanding is

$$\int p\, d(1/\rho)$$

unit mass of gas being considered (Milne-Thomson §1.62)

Since ρ is a fn of x, y, z, t. the rate of doing work on element of mass $\rho\, dx\, dy\, dz$ is given $p \frac{d}{dt}\left(\frac{1}{\rho}\right) \rho\, dx\, dy\, dz$

where $\frac{d}{dt} = \frac{D}{Dt}$ in the usual notation

$\therefore$ for the whole fluid, the rate

$$= \iiint p \frac{d}{dt}\left(\frac{1}{\rho}\right) \cdot \rho\, d\tau$$

$$= -\iiint p \cdot \frac{1}{\rho} \frac{d\rho}{dt} \cdot d\tau$$

From continuity eqn $\frac{d\rho}{dt} + \rho \nabla \vec{q} = 0$

$$\therefore \text{ Rate} = \iiint \rho \, \nabla \vec{q} \cdot d\tau.$$

$$= \iiint \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \right) dx\,dy\,dx.$$

(20) Using the notation of Ex 17 (Vide Lamb. l.c)

$$p - p_0 = \frac{\rho R}{r} \left\{ R\ddot{R} + 2\dot{R}^2 - R^3 \dot{R}^2 / 2r^3 \right\}$$

Since pressure p is considered at the surface of the sphere putting $r = R$

$$p = p_0 + \rho \left\{ R\ddot{R} + 2\dot{R}^2 - \frac{1}{2}\dot{R}^2 \right\}$$

$$= p_0 + \rho \left\{ R\ddot{R} + \frac{3}{2}\dot{R}^2 \right\} = p_0 + \frac{1}{2}\rho \left\{ 2R\ddot{R} + 3\dot{R}^2 \right\}$$

$$= p_0 + \frac{1}{2}\rho \left\{ \frac{d^2}{dt^2}(R^2) + \left(\frac{dR}{dt} \right)^2 \right\}$$

(21) With the above equation

$$p - \Pi = \frac{\rho R}{r} \left\{ R\ddot{R} + 2\dot{R}^2 - R^3 \dot{R}^2 / 2r^3 \right\} \qquad (1)$$

Since the gas in the cavity satisfies Boyle's law

$$\frac{4}{3}\pi a^3 \cdot m\Pi = \frac{4}{3}\pi R^3 \cdot p_R \quad \text{ie } p_R = \frac{m\Pi a^3}{R^3}.$$

Putting $r = R$ in (1)

$$\frac{\Pi}{\rho}\left(\frac{ma^3}{R^3} - 1 \right) = R\ddot{R} + \frac{3}{2}\dot{R}^2$$

Multiplying by $2\dot{R}R^2$

$$\frac{2\pi}{\rho}\left(\frac{ma^3\dot{R}}{R} - \dot{R}R^2\right) = 2R^3\dot{R}\ddot{R} + 3R^2\dot{R}^3$$
$$= \frac{d}{dt}\left(R^3\dot{R}^2\right)$$

Integrating

$$\frac{2\pi}{\rho}\left(ma^3 \log R - \tfrac{1}{3}R^3\right) = R^3\dot{R}^2 + C$$

when $R = a$, $\dot{R} = 0$ & $\therefore C = \frac{2\pi}{\rho}\left(ma^3 \log a - \tfrac{1}{3}a^3\right)$

the cavity expands until $\dot{R} = 0$ again ie when

$$ma^3 \log R - \tfrac{1}{3}R^3 = ma^3 \log a - \tfrac{1}{3}a^3$$

$$ma^3 \log\left(\frac{R}{a}\right) - \tfrac{1}{3}R^3 + \tfrac{1}{3}a^3 = 0$$

Putting $R = na$ gives

$$1 + 3m \log n - n^3 = 0.$$

To prove the second part we substitute for $\dot{R}^2$ in

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{\pi}{\rho}\left(\frac{ma^3}{R^3} - 1\right)$$

which gives $R\ddot{R} = \frac{\pi}{\rho}\left\{\frac{a^3(m-1)}{R^3} - \frac{3ma^3}{R^3}\log\frac{R}{a}\right\}$

& with $m \simeq 1$, $\ddot{R} = -\frac{\pi}{\rho}\cdot\frac{3a^3}{R^4}\log\frac{R}{a}$

Putting $R = a(1+\xi)$ where ξ is small

$$a\ddot{\xi} = -\frac{\Pi}{\rho}\cdot\frac{3a^3}{a^4(1+\xi)^4}\log(1+\xi)$$

$$\ddot{\xi} = -\frac{3\Pi}{a^2\rho}\left(1-4\xi\right)\left(\xi-\frac{\xi^2}{2}+\cdots\right)$$

$$= -\frac{3\Pi}{\rho a^2}\xi$$

$\therefore$ Period of oscillation $= 2\Pi\sqrt{\dfrac{a^2\rho}{3\Pi}}$.

(22) As in Ex. 3, p. 32,

$$v^2 = \frac{2}{3}\frac{\Pi}{\rho}\cdot\frac{(a^3-r^3)/r^4}{\left(\frac{1}{r}-\frac{1}{R}\right)}$$

$a = 2c$ & when $r = c$, $\frac{4}{3}\Pi R^3 = \frac{4}{3}\Pi r^3 + \frac{4}{3}\Pi c^3$

i.e. $R^3 = 2c^3$ i.e. $R = c2^{1/3}$.

$$v^2 = \frac{2}{3}\frac{\Pi}{\rho}\cdot\frac{(8c^3-c^3)/c^4}{\frac{1}{c}-\frac{1}{c2^{\frac{1}{3}}}} = \frac{14\Pi}{3\rho}\cdot\frac{2^{1/3}}{2^{\frac{1}{3}}-1}.$$

(23) 

Let $\vec{v}$ be the velocity of the centre, and $\vec{v}'+\vec{v}$ that of a fluid element $d\tau$ at P; ρ the density at O & ρ' at P.

*

$$(\eta \nabla)\vec{v} = \vec{i}\left(\eta_x \frac{\partial u}{\partial x} + \eta_y \frac{h_e}{h_e}\frac{\partial u}{\partial y} + \eta_z \frac{2e}{h_e}\right)$$

$$+\vec{j}\left(\eta_x \frac{\partial v}{\partial x} + \; - \; + \; -\right.$$

$$+\vec{k}\left(\eta_x \frac{\partial w}{\partial x} + \; - \; + \; -\right.$$

[Note:- @ The excess ~~be~~ taken as

$$\int (u' + u)\rho' d\tau - \int u\rho' d\tau.$$

and not $\int (u' + u)\rho' d\tau - \int u\rho\, d\tau$]

Considering the x-component of the velocity.

$$u' = (x\frac{\partial u}{\partial x} + \cdot) + \frac{1}{2}(x^2\frac{\partial^2 u}{\partial x^2} + \cdot)$$

and $\rho' = \rho + (x\frac{\partial \rho}{\partial x} + \cdots)$

and the excess of momentum considered

$$= \iiint u'\rho' \, dx\,dy\,dz$$

Since radius of sphere is small, let us retain terms up to x^2, etc, and treat the derivatives of u & ρ as constant in the region.

(i) we have $\iiint \rho(x\frac{\partial u}{\partial x} + \cdots)\,dx\,dy\,dz$

$$= \frac{\partial u}{\partial x}\iiint \rho x \, dx\,dy\,dz + \cdots = 0$$

(ii) $$\iiint(\rho x^2\frac{\partial^2 u}{\partial x^2} + \cdot + \rho \cdot 2yz\frac{\partial^2 u}{\partial y\partial z} + \cdots)\,dx\,dy\,dz$$

$$= \frac{\partial^2 u}{\partial x^2}\iiint \rho x^2\,dx\,dy\,dz + \cdots + \cdots$$

$$+ 2\frac{\partial^2 u}{\partial y\partial z}\iiint \rho yz\,dx\,dy\,dz + \cdots$$

$$= \frac{\partial^2 u}{\partial x^2}\iiint \rho x^2\,dx\,dy\,dz + \cdots \quad \text{since } \iiint \rho yz\,d\tau = 0.$$

and $\int \rho x^2 d\tau = \int \rho y^2 d\tau = \int \rho z^2 d\tau = \frac{1}{2}\int \rho(y^2+z^2)d\tau$

$= \frac{\rho}{2} \cdot \frac{2}{5} Ma^2 = \frac{1}{5} Ma^2.$

(iii) $\iiint \left(x\frac{\partial u}{\partial x} + \cdots\right)\left(x\frac{\partial \rho}{\partial x} + \cdots\right) dx\,dy\,dz$

$= \iiint \left(x^2 \frac{\partial u}{\partial x}\frac{\partial \rho}{\partial x} + \cdots\right) dx\,dy\,dz$, since $\iiint x\rho\left(\frac{\partial u}{\partial x}\frac{\partial \rho}{\partial y} + \cdots\right) = 0$

$= \frac{\partial u}{\partial x} \cdot \frac{\partial \rho}{\partial x} \iiint x^2\,dx\,dy\,dz + \cdots = \frac{\partial u}{\partial x}\frac{\partial \rho}{\partial x} \cdot \frac{1}{5}\frac{Ma^2}{\rho} + \cdots$

$\therefore$ Excess of momentum

$= \frac{1}{5}\frac{Ma^2}{\rho}\left\{\left(\frac{\partial \rho}{\partial x}\frac{\partial u}{\partial x} + \cdots\right) + \frac{1}{2}\rho\left(\frac{\partial^2 u}{\partial x^2} + \cdots\right)\right\}.$

(24) As in Ex. 3, p. 32, $v^2 = \frac{2\Pi}{3\rho}\frac{a^3 - r^3}{r^3}$ (Also Lamb, p. 122, eqn (4))

Putting $r = a x^{1/3}$, $\frac{dr}{dt} = \frac{1}{3} a x^{-2/3} \cdot \frac{dx}{dt}$.

$\frac{1}{9} a^2 x^{-4/3}\left(\frac{dx}{dt}\right)^2 = \frac{2\Pi}{3\rho} \cdot \frac{1-x}{x}$.

$\left(\frac{dx}{dt}\right)^2 = \frac{6\Pi}{a^2\rho}(1-x)x^{-1/3}$ i.e. $\frac{t}{a} = \sqrt{\frac{\rho}{6\Pi}}\int_0^1 x^{-\frac{1}{6}}(1-x)^{-\frac{1}{2}}dx$

$$t = a\sqrt{\frac{\rho}{6\Pi}}\;\frac{\Gamma(5/6)\,\Gamma(1/2)}{\Gamma(4/3)}.$$

From $\Gamma(z+1) = z\,\Gamma(z)$, $\Gamma(4/3) = \frac{1}{3}\Gamma(1/3)$.

From the duplication formula [Whittaker-Watson, 1915, p. 234, Cor)

$$2^{2z-1}\Gamma(z)\,\Gamma(z+\tfrac{1}{2}) = \sqrt{\pi}\,\Gamma(2z)$$

we have putting $z = \frac{1}{3}$, $2^{-\frac{1}{3}}\Gamma(\frac{1}{3})\Gamma(5/6) = \sqrt{\pi}\,\Gamma(2/3)$

and from $\Gamma(z)\,\Gamma(1-z) = \dfrac{\pi}{\sin \pi z}$

$$\Gamma(\tfrac{1}{3})\,\Gamma(2/3) = \frac{\pi}{\sin \pi/3} = \frac{2\pi}{\sqrt{3}}.$$

$$\text{or } \Gamma(2/3) = \frac{2\pi}{\sqrt{3}\,\Gamma(1/3)}$$

$$\therefore\ \Gamma(5/6) = 2^{\frac{1}{3}}\sqrt{\pi}\cdot\frac{2\pi}{\sqrt{3}\,\{\Gamma(1/3)\}^2} = \frac{2^{\frac{4}{3}}\,\pi^{3/2}}{3^{\frac{1}{2}}\{\Gamma(1/3)\}^2}$$

Also $\Gamma(1/2) = \sqrt{\pi}$.

$$\therefore\ t = a\left(\frac{\rho}{\Pi}\right)^{\frac{1}{2}}\cdot 6^{-\frac{1}{2}}\cdot\frac{2^{\frac{4}{3}}\,\pi^{3/2}}{3^{\frac{1}{2}}\{\Gamma(1/3)^2\}}\cdot\frac{\pi^{\frac{1}{2}}}{3^{-1}\Gamma(1/3)}$$

$$= \pi^2 a\left(\frac{\rho}{\Pi}\right)^{\frac{1}{2}}\cdot\frac{2^{5/6}}{\{\Gamma(1/3)\}^3}$$

carrying current (ordered motion). When lead ball strikes the ground ordered (103)
movement of molecules changes on impact to disordered (random) movement
The ordered (K.E) → disordered (heat). The reverse procedure is so unlikely
to be, to all intents & purposes, impossible as molecules moving at random
are hardly likely to arrange themselves so that they all act in one direction
causing the stationary (hot) ball suddenly to jump up in the air and
become cold! Left to themselves, then, things tend to become more & more
disordered never the reverse.

(F) The language of modern physics by E. H. Hutten.

(25) Proceeding as in Ex. 3, p. 32, the eqn of motion is

$$\frac{\partial v'}{\partial t} + v' \frac{\partial v'}{\partial r'} = -\frac{1}{\rho}\frac{\partial p}{\partial r'} - \mu r'^2$$

with the eqn of continuity as $r'^2 v' = F(t)$,

$$\frac{F'(t)}{r'^2} + v' \frac{\partial v'}{\partial r'} = -\frac{1}{\rho}\frac{\partial p}{\partial r'} - \mu r'^2$$

Integrating from r to R

$$-F'(t)\left(\frac{1}{r} - \frac{1}{R}\right) + \frac{1}{2}(v^2 - V^2) = \frac{\Pi}{\rho} - \frac{1}{3}\mu(r^3 - R^3)$$

which gives as usual

$$-\left(2rv^2 + r^2 v \frac{dv}{dr}\right)\left(\frac{1}{r} - \frac{1}{R}\right) + \frac{1}{2}\left(v^2 - \frac{v^2 r^4}{R^4}\right) = \frac{\Pi}{\rho} - \frac{1}{3}\mu(r^3 - R^3) \quad \text{--- (1)}$$

If initial radius of outershell be b, then $\frac{4}{3}\pi b^3 = \frac{4}{3}\pi a^3 + \frac{4}{3}\pi c^3$

i.e. $b^3 = a^3 + c^3$

and $R^3 - r^3 = b^3 - a^3 = c^3$

$\therefore$ Thus the right hand side of (1) becomes $\frac{\Pi}{\rho} + \frac{1}{3}\mu c^3$.

The integration of (1) gives as usual

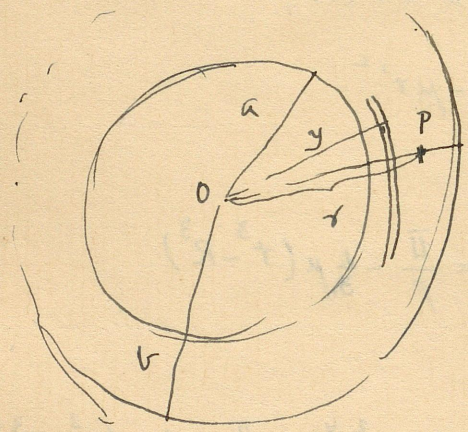
$$\frac{2}{3}\left(\frac{\Pi}{\rho} + \frac{1}{3}\mu c^3\right)\frac{a^3 - r^3}{r^4} = v^2\left(\frac{1}{r} - \frac{1}{R}\right)$$

Putting $r = x$ & $R = (c^3 + x^3)^{1/3}$, $v = \dot{x}$

$$\frac{2}{3}\left(\frac{\Pi}{\rho} + \frac{1}{3}\mu c^3\right)(a^3 - x^3) = x^4 \dot{x}^2 \left\{\frac{1}{x} - \frac{1}{(c^3 + x^3)^{\frac{1}{3}}}\right\}$$

2.1.87

(26) attraction at an internal point P of a spherical shell ($OP = r$)



For shells of radius $> r$, P is an internal point & hence consider shells of radius $\leq r$. The attraction is given by

$$\int_a^r \frac{4\pi y^2 dy \cdot \rho r}{r^2}$$

$$= \frac{4}{3}\pi r\rho \left(\frac{r^3 - a^3}{r^2} \right).$$

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ie $\left(\frac{2\pi}{3\rho}+\frac{2}{9}\mu c^3\right)\left(a^3-x^3\right)\left(c^3+x^3\right)^{\frac{1}{3}} = x^3\dot{x}^2\left\{\left(x^3+c^3\right)^{\frac{1}{3}}-x\right\}.$ (34)

(26) Since the external force is the gravitational attraction at a point P ($OP = r'$), this force on unit mass is given by

$\frac{4}{3}\pi\rho\left(\frac{r'^3-r^3}{r'^2}\right)$ where r = radius of internal boundary at time t. Hence the eqn of motion is (Eq. 3, p. 32 of book)

$$\frac{F'(t)}{r'^2} = -\frac{4}{3}\pi\rho\left(r'-\frac{r^3}{r'^2}\right)-\frac{1}{\rho}\frac{\partial p'}{\partial r'}$$ (p' = pressure at dist r')

which gives, integrating from r' to R

$$-F'(t)\left(\frac{1}{r'}-\frac{1}{R}\right) = -\frac{4}{3}\pi\rho\left\{\frac{1}{2}\left(r'^2-R^2\right)+r^3\left(\frac{1}{r'}-\frac{1}{R}\right)\right\}-\frac{p'}{\rho}$$

since the outer surface is free.

with $r' = a$, $R = b$, $p' = 0$ so that with initial values, ($r = r' = a$)

$$-F'(t)\left(\frac{1}{a}-\frac{1}{b}\right) = -\frac{4}{3}\pi\rho\left\{\frac{1}{2}\left(a^2-b^2\right)+a^3\left(\frac{1}{a}-\frac{1}{b}\right)\right\}$$

which gives $F'(t) = -\frac{2}{3}\pi\rho\left\{ab(a+b)-2a^3\right\}$

$$\therefore \frac{2}{3}\pi\rho\left\{ab(a+b)-2a^3\right\}\left(\frac{1}{r'}-\frac{1}{R}\right) = -\frac{4}{3}\pi\rho\left\{\frac{1}{2}\left(r'^2-R^2\right)+r^3\left(\frac{1}{r'}-\frac{1}{R}\right)\right\}-p'/\rho$$

Putting $r = a$ & $R = b$, we get

Lecturer – one with his hand in your pocket, his tongue in your ear and his faith in your patience.

(E) Physics is fun by Gardner – Book Three.

The laws of thermodynamics summarised as follows:

Law 1 – Heat & mechanical energy are equivalent interchangeable at a fixed rate of exchange (1 kilocalorie = 4.2 to joules)

Law 2 – Heat does not flow of its own accord flow from a cold to a hot substance.

Law 3 – It is impossible to reach absolute zero

Molecular chaos – The second law of thermodynamics can be considered from the molecular point of view. To do this we really need the help of the mathematicians & specially the statisticians.

5 photographs of 75 black balls poured into a glass jar & then 75 white balls on top of them. Jar photographed, shaken for a few seconds & photographed again, etc. Can you say in what order the photographs were taken? If the jar were shaken another 1000 times what would happen? How long would you have to shake it to obtain arrangement like (c) (normal)?

Scientists call original arrangement order & the final (c) disorder

Left to themselves, things become always become more and more disordered

(c)

Ex. of lead ball rolling freely (ordered motion) – Electrons in a wire

$$\bullet p' = -\frac{2}{3}\pi\rho^2\left[\left\{ab(a+b) - 2a^3\right\}\left(\frac{1}{r'} - \frac{1}{b}\right) + (r'^2 - b^2) + 2a^3\left(\frac{1}{r'} - \frac{1}{b}\right)\right]$$

$$= -\frac{2}{3}\pi\rho^2\left[ab(a+b)\left(\frac{1}{r'} - \frac{1}{b}\right) + r'^2 - b^2\right]$$

$$= -\frac{2}{3}\pi\rho^2(b - r')\left[\frac{a(a+b)}{r'} - r' - b\right]$$

$$p' = \frac{2}{3}\pi\rho^2(b - r')\left[r' + b - \frac{a(a+b)}{r'}\right]$$

$$= \frac{2}{3}\pi\rho^2(b - r')(r' - a)\left(\frac{a+b}{r'} + 1\right)$$

(27) Using the notation of Lamb, § 91. a. p. 122, we have

$$\frac{p - p_0}{\rho} = \frac{R^2\ddot{R} + 2R\dot{R}^2}{r} - \frac{R^4\dot{R}^2}{2r^4} \qquad (1)$$

and $$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -p_0/\rho \qquad (2)$$

$$R^3\dot{R}^2 = \frac{2p_0}{3\rho}(R_0^3 - R^3) \qquad (3)$$

Using (3) eqn (2) gives

$$R\ddot{R} = -\frac{p_0}{\rho} - \frac{3}{2}\cdot\frac{2p_0(R_0^3 - R^3)}{3\rho R^3}.$$

$$\therefore R^2\ddot{R} + 2R\dot{R}^2 = -\frac{p_0 R}{\rho} - \frac{p_0(R_0^3 - R^3)}{\rho R^2} + \frac{4p_0(R_0^3 - R^3)}{3\rho R^2}.$$

$$= -\frac{p_0 R}{\rho} + \frac{p_0(R_0^3 - R^3)}{3\rho R^2} = \frac{p_0(R_0^3 - 4R^3)}{3\rho R^2}$$

and respect that it has both demanded & earned — Graphs, tables & general principles — Descriptive statistics (The bristatistician must be sure that his statistical gangplank is anchored in solid mathematics at one end and solid subject matter at the other) — Probability (special application of Bayes's rule) — etc etc — last chapter on Demography & Vital statistics

[* mention here "printers error of Sudeney" & $\frac{16}{64} = \frac{1}{4}$ as illegal cancellation] & other mathematical tricks]

(C) Readings in Statistics for the behavioral sciences by Freeman & Brasksamp;

Very advanced book of not much use

Ref in (A) to G. S. Brown's book on Probability & scientific inference (Longmans Green & Co)

"a delightful little essay on randomness, miracles and science" [not in the Library at 9.9.Sc]

(D) Devil's Dictionary by Ambrose Bierce.

Decide - To succumb to the ~~pressure~~ preponderance of one set of influences over another set.

Handkerchief - A small square of silk or linen, used in various ignoble offices about the face and especially serviceable at funerals to conceal the lack of tears

Insurance - An ingenious modern game of chance in which the player is permitted to enjoy the comfortable conviction that he is beating the man who keeps the table

Hence (1) can be written as

$$\frac{p-p_0}{\rho} = \frac{p_0(R_0^3-4R^3)}{3\rho R^2 \cdot r} - \frac{R^4}{2r^4}\cdot\frac{2p_0(R_0^3-R^3)}{3\rho R^3}.$$

ie $$p = p_0\left\{1 + \frac{R_0^3-4R^3}{3R^2r} - \frac{R(R_0^3-R^3)}{3r^4}\right\}$$

$$p/p_0 = \frac{3R^2r^4 + r^3(R_0^3-4R^3) - R^3(R_0^3-R^3)}{3R^2r^4}$$

In the notation of the book, $R_0 = a$, $R = x$, and the answer is obtained in the required form.

(28) Proceeding as in Ex 3, p. 32

$$-F'(t)\left(\frac{1}{r}-\frac{1}{R}\right) + \tfrac{1}{2}(v^2-V^2) = -\frac{p_r}{\rho}$$

p_r being pressure at inner boundary when its radius is r.

From Boyle's law $\frac{4}{3}\pi a^3 \cdot p = \frac{4}{3}\pi r^3 \cdot p_r$ ie $p_r = \frac{pa^3}{r^3}$

$$\therefore\ -F'(t)\left(\frac{1}{r}-\frac{1}{R}\right) + \frac{1}{2}(v^2-V^2) = -\frac{pa^3}{\rho r^3}$$

Multiplying by $2r^2$ and integrating gives, as in the text,

$$\frac{2pa^3}{\rho}\log(r/a) = \left[v^2r^4\left(\frac{1}{r}-\frac{1}{R}\right)\right]_{r=a}^{r}$$

(v) Analysis of variance & sample sizes - Large samples

C-C bond lengths - Xe-O bond lengths

(vi) Method of least squares - matrix methods - Set of data to fit a model with n parameters

(vii) Propagation of error - Experimental design

(viii) Correlation - Data for O-H-O bonds

Question typical: Suppose that a variance-covariance matrix for ten quantities of physical interest is derived from the least squares variance-covariance matrix for a five-parameter problem. What is the rank r of this matrix. (i.e at least one sub-det of order $r \neq 0$, while all of order $r+1$ vanish) Describe a surface of constant probability

(B) Statistics with applications to the biological & health sciences by Remington & Schork

Biostatistics - useful to students of medicine, public health, dentistry, veterinary medicine, nursing, pharmacy & other health sciences - Ranges from basic cellular physiology to community surveys

Chaps - General intro - Some popular concepts & misuses of statistics

Statement originally made by Disraeli: "There are three kinds of lies: lies, damned lies and statistics" or the statement "figures don't lie, but liars figure or "Statistics can prove anything" - need for not hurrying data collection & rushing to print. Careful work requires time-consuming thought. The grey-flannel-suit approach to research may eventually rob science (statistics specially) of the dignity

ie $\frac{2\rho a^3}{\rho}\log\left(\frac{r}{a}\right) = v^2 r^4\left(\frac{1}{r}-\frac{1}{R}\right) - v_0^2 a^4\left(\frac{1}{a}-\frac{1}{b}\right)$. -(1)

We have now to find v_0, the initial velocity of the inner boundary under the impulsive pressure ϖ on the outer boundary.

If ϖ' be impulsive pressure at distance r'

$$\rho v' = -\frac{\partial \varpi'}{\partial r'}$$

From eqn of continuity $r'^2 v' = a^2 v_0$

$$\frac{a^2 v_0}{r'^2} = -\frac{1}{\rho}\frac{\partial \varpi'}{\partial r'}$$

Integrating from a to b

$$a^2 v_0\left(\frac{1}{a}-\frac{1}{b}\right) = -\frac{1}{\rho}\varpi \qquad \left[\varpi' = 0 \text{ for } r = a\right.$$

$$\text{ie } v_0 = -\frac{\varpi b}{a\rho(b-a)} \qquad (2)$$

The liquid comes to rest if $v = 0$ in (1) ie when

$$\frac{2\rho a^3}{\rho}\log\left(\frac{r}{a}\right) = \frac{\varpi^2 b^2 a^4}{a^2\rho^2(b-a)^2}\cdot\frac{b-a}{ab} = \frac{\varpi^2 ba}{\rho^2(b-a)}$$

$$\text{ie } \log\left(\frac{r}{a}\right) = -\frac{\varpi^2 b}{2p\rho a^2(b-a)}.$$

$$\text{ie } r = a\exp\left\{-\varpi^2 b/2p\rho a^2(b-a)\right\}.$$

6/3/72 – At Institute after quite a long time

Notes for lecture on Statistics on 9/3/72 at CEB

(A) Statistics in Physical Science by W. C. Hamilton

More & more problems involve the estimation of a relatively small number of parameters from a large number of expt. observations, each of which is subject to error. In other problems, only a few observations may be available for the test of a complex physical theory. In either case, it is the purpose of statistical analysis to determine the best value for the parameters & to assess the probability of error when one makes statements or tests hypotheses concerning the parameter values

use of multivariate probability distns & the method of least squares –

Sample spaces & probability

Examples (i) counting rate from a radioactive source is observed to the nearest integral number of counts per minute, at one-minute intervals for 1000 minutes. Each pt. in sample space is an ordered sequence of 1000 integers

(ii) Crystal exposed to x-rays in a camera used by crystallographers. Intensities of the resulting 10,000 spots on the film are produced & recorded. Each pt. in sample space is a list of 10,000 numbers

(iii) Test of hypotheses – A rocket which is intended to hit the moon is fired. Assume that such an experiment has been carried out twice previously, once with success & once with failure, would you be willing to say that the probability of success of the present shot is 0.5

(iv) use of normal distn very often in natural sciences

(30) This is given most simply from the equation of energy. The Kinetic energy $= 2\pi\rho r^4 v^2\left(\frac{1}{r} - \frac{1}{R}\right)$, and the work done is found by knowing the gravitational attraction at any point inside the shell. This is given in Ex. 26 by

$$\frac{4}{3}\pi\rho\left(\frac{r'^3 - r^3}{r'^2}\right)$$

and integrating from r to R this gives

$$\frac{4}{3}\pi\rho\left\{\frac{1}{2}(r^2 - R^2) + r^3\left(\frac{1}{r} - \frac{1}{R}\right)\right.$$

Hence the total work done

$$= \int_{\infty}^{r} 4\pi r^2 \rho\, dr \cdot \left[\frac{4}{3}\pi\rho\left\{\frac{1}{2}(r^2 - R^2) + r^3\left(\frac{1}{r} - \frac{1}{R}\right)\right\}\right] dr$$

$$\therefore v^2 r^4\left(\frac{1}{r} - \frac{1}{R}\right) = \frac{8\pi\rho r}{3}\int_{\infty}^{r}\left\{\frac{3}{2}r^4 - r^2\left(\frac{1}{2}R^2 + \frac{r^3}{R}\right)\right\} dr$$

Using $R^3 = r^3 + c^3$ and integrating the indefinite integral becomes $= \frac{3}{10}r^5 + \frac{1}{5}R^5 - \frac{1}{2}r^3R^2$

At ∞, putting $R = r$ the integral vanishes & hence

(8b)

$$\frac{1}{R} = \frac{1}{(r^3+c^3)^{\frac{1}{3}}} = \frac{1}{c\left(1+r^3/c^3\right)^{\frac{1}{3}}} = \frac{1}{c}\left\{1+\frac{a^3(1+\xi)^3}{c^3}\right\}^{-\frac{1}{3}}$$

$$= \frac{1}{c}\left\{\left(1+\frac{a^3}{c^3}\right)+\frac{a^3}{c^3}\left(3\xi+3\xi^2+\xi^3\right)\right\}^{-\frac{1}{3}}$$

$$= \frac{1}{c}\left(1+\frac{a^3}{c^3}\right)^{-\frac{1}{3}}\left\{1+\frac{a^3}{c^3+a^3}\left(3\xi+\cdots\right)\right\}^{-\frac{1}{3}}$$

$$= \frac{1}{b}\left\{1+\frac{a^3}{b^3}\left(3\xi+\cdots\right)\right\}^{-\frac{1}{3}} = \frac{1}{b}\left(1-\frac{a^3\xi}{b^3}-\cdots\right)$$

$$\frac{1}{r} = \frac{1}{a}(1-\xi+\xi^2) \; ; \; \frac{1}{r}-\frac{1}{R} = \frac{1}{a}(1-\xi)-\frac{1}{b}\left(1-\frac{a^3\xi}{b^3}-\frac{a^3\xi^2}{b^3}\right)$$

$$v^2 r^4 \left(\frac{1}{r} - \frac{1}{R}\right) = \frac{6\pi\rho}{3}\left\{\frac{3}{10} r^5 + \frac{1}{5} R^5 - \frac{1}{2} r^3 R^2\right\}$$

$$\frac{v^2 r^3 (R-r)}{R} = \frac{4\pi\rho}{15}\left(3r^5 + 2R^5 - 5r^3 R^2\right)$$

$$= \frac{4\pi\rho}{15}\left[3r^3(r^2 - R^2) + 2R^2(R^3 - r^3)\right]$$

$$\text{ie } v^2 = \frac{4\pi\rho R}{15 r^3}\left[-3r^3(R+r) + 2R^2(R^2 + Rr + r^2)\right]$$

Replacing r by x and R by z gives the required answer.

(31) The equation of motion can be obtained, as usual, as

$$2\frac{a^3 \Pi}{\rho}\log\left(\frac{r}{a}\right) - \frac{2}{3}\frac{\Pi}{\rho}(r^3 - a^3) = v^2 r^4\left(\frac{1}{r} - \frac{1}{R}\right)$$

with $R^3 - r^3 = b^3 - a^3 = c^3$.

Putting $r = a(1+\xi)$ where ξ is small,

$$\frac{2a^3\Pi}{\rho}\log(1+\xi) - \frac{2}{3}\frac{\Pi}{\rho}a^3\left\{(1+\xi)^3 - 1\right\} = \left(\frac{d\xi}{dt}\right)^2 a^4 (1+\xi)^4$$

$$\times\left\{\frac{1}{a}(1 - \xi + \xi^2 - \ldots) - \frac{1}{b}\left(1 - \frac{a^3}{b^3}\xi - \frac{a^3}{b^3}\xi^2 - \ldots\right)\right\}$$

Retaining terms up to ξ^2 only

this gives

(2) Spin of the top is a constant.

(3) Sleeping top. —axis→ Top spins with its axis vertical & axis remains stationary and there is no apparent motion of the top as a whole. A small disturbance of a sleeping top produces only a small oscillation when the spin is great; when the spin has been considerably reduced by frictional resistance, the top begins to wobble & ultimately falls down. What is the critical value of the spin below which the motion of a sleeping top is unstable?

motion of sleeping top stable if $s^2 \geq \frac{4Amga}{C^2}$

Consider $f(x) = (1-x)^2\left[\frac{2mga}{A}(1+x) - \frac{C^2s^2}{A^2}\right]$ the cubic

reduced cubic eqn for a sleeping top in which $\theta = \dot\theta = 0$, and

$$\alpha = \beta = Cs, \quad E = \tfrac{1}{2}Cs^2 + mga$$

$$f''(1) = 2\left(\frac{4mga}{A} - \frac{C^2s^2}{A^2}\right) \quad \begin{matrix} <0 & \text{stable} \\ >0 & \text{unstable} \\ =0 & \text{stable} \end{matrix}$$

[P. T. O

$$\frac{\rho a^2}{3\Pi b}(b-a)\left(\frac{d\xi}{dt}\right)^2 = -\xi^2$$

ie $$\frac{\rho a^2}{3\Pi b}(b-a)\ddot{\xi} = -\xi$$

Hence the period required is given by $2\pi a\{\rho(b-a)/3\Pi b\}^{1/2}$.

(32) As in the previous example

$$\frac{2c_1^3 p_1}{\rho}\log\left(\frac{r}{c_1}\right) - \frac{2}{3}\frac{\Pi}{\rho}(r^3 - c_1^3) = v^2 r^4\left(\frac{1}{r} - \frac{1}{R}\right)$$

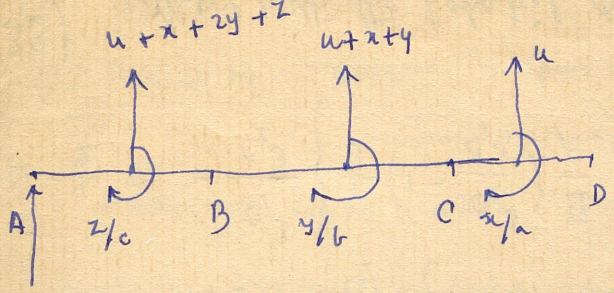
The final configuration is given by putting $r = c_2$ & $R = R_2$ in this ie

$$\frac{2c_1^3 p_1}{\rho}\log\left(\frac{c_2}{c_1}\right) - \frac{2}{3}\frac{\Pi}{\rho}(c_2^3 - c_1^3) = v^2 c_2^4\left(\frac{1}{c_2} - \frac{1}{R_2}\right)$$
$$= v^2 c_2^3(1 - c_2/R_2)$$

ie $$v^2\left(1 - \frac{c_2}{R_2}\right) = \frac{2c_1^3}{c_2^3}\left\{-\frac{1}{3}\frac{\Pi}{\rho}\left(\frac{c_2^3}{c_1^3} - 1\right) + \frac{p_1}{\rho}\log\left(\frac{c_2}{c_1}\right)\right\}$$

ie $$\frac{1}{2}v^2 = \frac{c_1^3}{c_2^3}\left\{\frac{1}{3}\left(1 - \frac{c_2^3}{c_1^3}\right)\frac{\Pi}{\rho} - \frac{p_1}{\rho}\log\left(\frac{c_1}{c_2}\right)\right\}\Big/\left(1 - \frac{c_2}{R_2}\right).$$

(33) Working with the potential $\phi = \frac{R^2\dot{R}}{r^2}$ (Lamb, §91, a, p. 122)



Taking moments about C for the rod CD, about B for the rods BC, CD, and about A for the three rods.

Also resolve whole system at rt ∠s to the rods.

on simplification we get $\frac{u}{3b} = \frac{x}{b} = \frac{y}{-(2b+a)} = \frac{cz}{2ab+3ac+2b^2+4bc}$

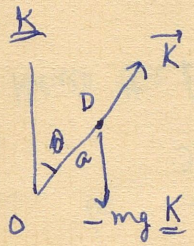
$$= \frac{P}{K\left(\frac{4}{3}ab + ac + \frac{4}{3}bc + \frac{4}{3}b^2\right)}$$

masses = Ka, Kb, Kc

Synge & Griffith

(1) Steady motion takes place under gravity only if

$$p\,(Cs - Ap\cos\theta) = mga \qquad \text{——(1)}$$



C = axial moment of inertia about O.

~~If precession or~~ spin of top = s

precession p = angular velocity with which OD rotates about $\underline{K}$

$$s = \frac{mag}{Cp} + \frac{Ap\cos\theta}{C}$$

If p is small, s is great & $s \cong \frac{mga}{Cp}$.

& using $p = \frac{\partial \phi}{\partial t} - \Omega - \frac{1}{2} q^2 + F(t)$ $\quad \left(q = -\frac{\partial \phi}{\partial r}\right)$

we have $-\frac{\partial \Omega}{\partial r} = -\mu r^{-3/2}$ ie $\Omega = -2\mu r^{-\frac{1}{2}}$

Also the pressure p_0 at $\infty = 0$, hence

$$p = \frac{R^2 \ddot{R} + 2R\dot{R}^2}{r} - \frac{R^4 \dot{R}^2}{2r^4} + \frac{2\mu}{r^{1/2}}$$

Putting $r = R$ & neglecting the internal pressure

$$0 = R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{2\mu}{R^{1/2}}$$

Multiplying by $2\dot{R}R^2$ and integrating

$$R^3 \dot{R}^2 = -\frac{8\mu}{5} R^{5/2} + C$$

for $R = R_0$, $\dot{R} = 0$, hence $C = \frac{8\mu}{5} R_0^{5/2}$

$$\therefore \quad R^3 \dot{R}^2 = \frac{8\mu}{5}\left(R_0^{5/2} - R^{5/2}\right)$$

and time required is therefore given by

$$t = \sqrt{\frac{5}{8\mu}} \int_0^{R_0} \frac{R^{3/2} \cdot dR}{\sqrt{R_0^{5/2} - R^{5/2}}}.$$

Putting $R = R_0 x^{2/5}$ we have

Ex. 35. old edition:

The eqn of energy gives

$$\frac{2\mu}{(n-1)(n-4)}\left\{\frac{1}{r^{n-4}}-\frac{1}{a^{n-4}}\right\}=r^3v^2$$

ie $$dt\sqrt{\frac{2\mu}{(n-1)(4-n)}}=\frac{r^{3/2}\,dr}{\sqrt{a^{4-n}-r^{4-n}}}$$

Putting $r^{4-n}=a^{4-n}z$ we get the integral from $z=0$ to $z=1$

$$\text{R.H.S.}=T\sqrt{\frac{2\mu}{(n-1)(4-n)}}=\frac{a^{\frac{1}{2}(n+1)}}{4-n}\int_0^1 z^{(-3+2n)/(8-2n)}(1-z)^{-\frac{1}{2}}\,dz \qquad (34)$$

$$=\frac{a^{\frac{1}{2}(n+1)}}{4-n}\cdot\sqrt{\pi}\cdot\frac{\Gamma\left(5/8-2n\right)}{\Gamma\left(\frac{9-n}{8-2n}\right)}\quad\text{etc.} \qquad (35)$$

Hence the result.

$$t = \sqrt{\frac{5}{8\mu}} \int_0^{R_0} \frac{R_0^{3/2} x^{3/5} \cdot \frac{2}{5} R_0 x^{-3/5} \cdot dx}{R_0^{5/4} \sqrt{1-x}}$$

$$= \sqrt{\frac{1}{10\mu}} \cdot R_0^{5/4} \int_0^1 \frac{dx}{\sqrt{1-x}} = 2 R_0^{5/4} \frac{1}{\sqrt{10\mu}}.$$

$$= \sqrt{\frac{2}{5\mu}} R_0^{5/4}$$

In the example we have only to replace R_0 by $\underline{c}$.

(34) A complete discussion of this is to be found in Lamb § 138, p. 191.

(35) The equation of continuity is

$$r'v' = rv = RV$$

The Kinetic energy $= \frac{1}{2} \int_r^R 2\pi r' \rho v'^2 dr' = \pi\rho \int_r^R r' \cdot \frac{r^2 v^2}{r'^2} dr'.$

$$= \pi\rho r^2 v^2 \log(R/r)$$

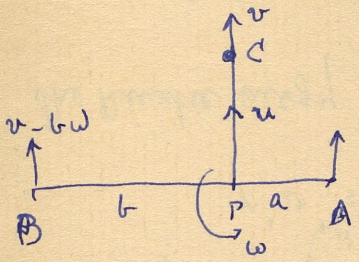
Work done by the pressure

$$= \int_a^R 2\pi R \Pi (-dR) = \pi \Pi (a^2 - R^2)$$

Also $a^2 - b^2 = R^2 - r^2$

(B) Theoretical Mechanics by Love

Ex. (1) Two particles A, B of equal mass are connected by a rigid rod of negligible mass, and a third equal particle C is tied to a point P of the rod at distances a & b from the two ends. C is projected with velocity u ⊥r to AB. Find velocity of C immediately



Let ω = angular vel. with which the rod begins to turn

Eqn of momentum ⊥l to the string is

$$mv + m(v + a\omega) + m(v - b\omega) = mu$$

Eqn of moment of momentum about P is

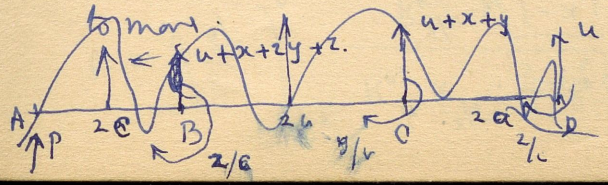
$$ma(v + a\omega) - mb(v - b\omega) = 0$$

$$\text{or } \omega = v(b-a)/a^2 + b^2$$

Eliminating ω we have $3v - \dfrac{(a-b)^2}{a^2+b^2} v = u$

$$\text{or } v = \frac{1}{2} \cdot \frac{a^2+b^2}{a^2+ab+b^2}.$$

Ex. 2. Three uniform rods of masses ∝ to their lengths are freely jointed together and lain out straight, and one of the end rods is struck at the free end at rt ∠s to its length. Find how they begin to move.



Let x/a, $-z/c$ be the angular velocities. u = vel of C.G of first rod

The equation of energy therefore gives

$$\rho r^2 v^2 \log\left(\frac{R}{r}\right) = \Pi\left(a^2 - R^2\right) = \Pi\left(b^2 - r^2\right).$$

ie $$v^2 = \frac{2\Pi\left(b^2 - r^2\right)}{\rho r^2 \log\left(\frac{R}{r}\right)^2} = \frac{2\Pi\left(b^2 - r^2\right)}{\rho r^2 \log\left\{\left(r^2 + a^2 - b^2\right)/r^2\right\}}.$$

(36) The eqn of motion of Ex 3, p. 32 gives on integration from r' to R

$$-F'(t)\log\left(\frac{r'}{R}\right) + \tfrac{1}{2}\left(v'^2 - V^2\right) = -\frac{p'}{\rho} + \frac{\varpi}{\rho}$$

where $r'v' = F(t)$ is the eqn of continuity

Initially putting $R = a$ (in this problem a is outer radius) & $t = 0$

$$-F'(0)\log\frac{r'}{a} = -\frac{p'}{\rho} + \frac{\varpi}{\rho} \quad (1) \qquad (v' = V = 0 \text{ at } t = 0$$

Also with $t = 0$, $r' = b$, $R = b$, $p' = 0$

$$\therefore \quad -F'(0)\log\frac{b}{a} = \frac{\varpi}{\rho} \qquad (2)$$

$\therefore$ From (1) & (2), $\quad \dfrac{p'}{\rho} = \dfrac{\varpi}{\rho} + F'(0)\log\dfrac{r'}{a}$

$$= \frac{\varpi}{\rho} - \frac{\varpi}{\rho}\frac{\log(r'/a)}{\log(b/a)}$$

$$= \frac{\varpi}{\rho}\left\{\frac{\log(b/a) - \log(r'/a)}{\log(b/a)}\right\} = \frac{\varpi}{\rho}\cdot\frac{\log(b/r')}{\log(b/a)} = \frac{\varpi}{\rho}\frac{\log(r'/b)}{\log(a/b)}$$

Ex. 38. old edition:

Change of velocity in r-direction $= \delta u - v\delta\theta$

and since $u = u(r, \theta, t)$

$$\delta u = \frac{\partial u}{\partial r}\delta r + \frac{\partial u}{\partial \theta}\delta\theta + \frac{\partial u}{\partial t}\delta t$$

also $u\delta t = \delta r$; $v\delta t = r\delta\theta$.

$$\therefore \quad \delta u = \left(u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial t}\right)\delta t.$$

$\therefore$ Radial acceleration

$$= \frac{\delta u}{\delta t} - v\frac{\delta\theta}{\delta t} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r}$$

Similarly change of velocity in θ-direction $= \delta v + u\delta\theta$.

and this similarly gives the required expn for the transverse acceleration

---

(37

in

(38

(nb)

$$\therefore p' = \frac{\omega}{\omega} \cdot \frac{\log r' - \log b}{\log a - \log b}.$$

(37) If U, V, W be the velocities referred to fixed axes

$$U = u + \omega_2 z - \omega_3 y, \text{ etc}$$

$$\frac{DU}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + z\frac{d\omega_2}{dt} + w\omega_2 - y\frac{d\omega_3}{dt} - v\omega_3$$

acceleration along the x-axis

$$= \frac{DU}{Dt} + W\omega_2 - V\omega_3$$

$$= \frac{DU}{Dt} + \omega_2(w + \omega_1 y - \omega_2 x) - \omega_3(v + \omega_3 x - \omega_1 z)$$

$$= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - 2v\omega_3 + 2w\omega_2 - x(\omega_2^2 + \omega_3^2) - y\left(\frac{d\omega_3}{dt} - \omega_1\omega_2\right) + z\left(\frac{d\omega_2}{dt} + \omega_3\omega_1\right)$$

Equating this to $-\frac{1}{\rho}\frac{\partial p}{\partial x} + X$ gives the eqn of motion in the required form

(38) The first part is to be found in §2.4.

p. 1.d

$$Q^2 = u^2 + v^2 + w^2 - (\omega_1^2 + \omega_2^2 + \omega_3^2)(x^2 + y^2 + z^2) + (\omega_1 x + \omega_2 y + \omega_3 z)^2$$

$$\frac{1}{2}\frac{\partial Q^2}{\partial x} = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x} - x(\omega_1^2 + \omega_2^2 + \omega_3^2) + \omega_1(\omega_1 x + \omega_2 y + \omega_3 z)$$

$$= u\frac{\partial u}{\partial x} + \cdots - x(\omega_2^2 + \omega_3^2) + \omega_1\omega_2 y + \omega_3\omega_1 z$$

(93)

To prove the second part, we have

$$2\zeta = \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} = \frac{\partial w}{\partial y} + 2\omega_1 - \frac{\partial v}{\partial z}, \text{ etc}$$

using eqns U, V, W of the previous example.

$$2\eta = \frac{\partial u}{\partial z} + 2\omega_2 - \frac{\partial w}{\partial x}$$

$$2\zeta = \frac{\partial v}{\partial x} + 2\omega_3 - \frac{\partial u}{\partial y}$$

Substituting for $\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial z}$ from these last two eqns in the eqns of motion of the previous example we have

$$\frac{1}{\rho}\frac{\partial p}{\partial x} - X + \frac{\partial u}{\partial t} - 2v\omega_3 + 2w\omega_2 + u\frac{\partial u}{\partial x} + v\left(-2\zeta + \frac{\partial v}{\partial x} + 2\omega_3\right)$$

$$+ w\left(2\eta - 2\omega_2 + \frac{\partial w}{\partial x}\right) - (\omega_2^2 + \omega_3^2)x - \left(\frac{d\omega_3}{dt} - \omega_1\omega_2\right)y$$

$$+ \left(\frac{d\omega_2}{dt} + \omega_3\omega_1\right)z = 0$$

Also $\omega_1, \omega_2, \omega_3$ being constants $\frac{d\omega_3}{dt} = \frac{d\omega_2}{dt} = 0$, & the above becomes

$$\frac{1}{\rho}\frac{\partial p}{\partial x} - X + \frac{\partial u}{\partial t} - 2v\zeta + 2w\eta + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}$$

$$- x(\omega_2^2 + \omega_3^2) + \omega_1\omega_2 y + \omega_3\omega_1 z$$

The last six terms $= \frac{1}{2}\frac{\partial}{\partial x}\{u^2 + v^2 + w^2\} - x(\omega_2^2 + \omega_3^2) + \omega_1(\omega_1 x + \omega_2 y + \omega_3 z)$

$$= \frac{1}{2}\frac{\partial Q^2}{\partial x} - \text{Hence the form required.}$$

1/3/72 At C.C.B for Anthra Fluid Mechanics Compressible flow

(A) Fluid Mechanics by V. L. Streeter — Chapter 6 (Frictionless Compressible flows).

(1) Speed of sound wave - Mach number:

Conti in a channel, Continuity eqn is $\rho V A = (\rho+d\rho)(V+dV)A$ [A = Cross section]

or $\rho\, dV + V\, d\rho = 0$ — (1)

The momentum eqn is $pA - (p+dp)A = \rho V A (V+dV-V)$

or $dp = -\rho V dV$ — (2)

Eliminating dV between (1) & (2) gives $V^2 = \frac{dp}{d\rho}$ — (3)

Eqn of speed of sound $= c = \sqrt{dp/d\rho}$.

If K be bulk modulus, $K = \frac{\rho\, dp}{d\rho}$

or $c = \sqrt{K/\rho}$

For sound waves motion is isentropic; then $p\rho^{-k} = \text{const}$, $\frac{dp}{d\rho} = \frac{kp}{\rho}$

& $c = \sqrt{kp/\rho}$

From ideal gas law, $p = \rho g R T$, $c = \sqrt{k g R T}$

ie speed of sound in an ideal gas is a fn of absolute temp. only.

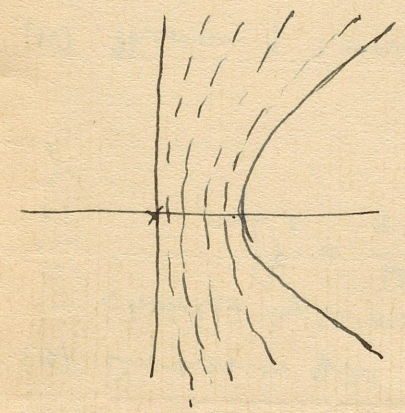
Mach number $M = \frac{V}{c}$, $M^2 = V^2/c^2$ = ratio of K.E of fluid to its thermal energy. For incompressible fluid $K = \infty$ & $M = 0$

For ideal gases $K = kp$ when the compression is isentropic [Correct this]

(39) This equation is the same as eqn (1) of § 2·41.

Chapter III, p. 60

(1)



$z = c\cosh\omega$

lies the stream lines

$$\frac{x^2}{\cos^2\psi} - \frac{y^2}{\sin^2\psi} = c^2$$

of the given hyperbola be

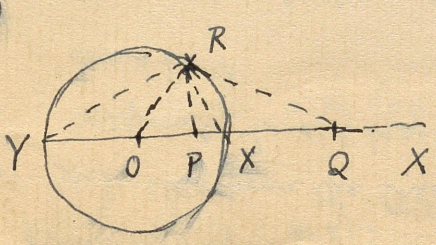
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

we have $c^2 = a^2 + b^2$

and the value of ψ is given by $\tan^{-1}(b/a)$.

The minor axis is given by $\psi = \pi/2$. Thus the stream lines are confocal hyperbolas with foci at $(\pm\sqrt{a^2+b^2}, 0)$.

(2)



Let Q be the point inverse to P w.r.t the circle. Place at Q a source of strength m

It can be shown that this constitutes the

flexural rigidity of the beam.

(6) State and prove the theorem of three moments relating to a uniformly loaded beam supported at a number of points at the same level

(7) Find the general stress - strain relations in an isotropic substance

(8) If O_x, O_y, O_z be the principal axes of stress, prove that the normal and tangential components of the stress (per unit area) on a plane whose normal makes an angle θ with Ox in the x-y plane are

$$p_1 \cos^2\theta + p_2 \sin^2\theta, \text{ and } (p_1 - p_2) \cos\theta \sin\theta$$

(9) Discuss the torsion of a bar of circular section.

(10) Discussion of the case of a bar hanging vertically & stretched by its own weight.

$$\left[\frac{F}{l} = \frac{W}{E\omega} l + \frac{1}{2} \cdot \frac{\rho l^2}{E} \right.$$

ρ = density
E = Young's modulus
l = length of bar
ω = uniform cross-sectional area $\left.\right]$

(11) Theorem of 3 moments applied to case of beam resting on three supports A, B, C of which two are at the ends.

[Here $M_A = 0, M_C = 0$ & hence from $a M_A + 2(a+b) M_B + b M_C = \frac{1}{2} w (a^2 + b^2)$

$$M_B = \frac{1}{2} w (a^2 - ab + b^2)$$

& pressure on supports $= \frac{1}{2} w (a+b) + M_B \left(\frac{1}{a} + \frac{1}{b}\right) - \frac{M_A}{a} - \frac{M_C}{b}$

$$= \frac{1}{8} w (a+b)(a^2 + 3ab + b^2) / ab.$$

image w.r.t the $\circ^{le}$ of the given system of sources & sinks, for let R be any point on the circle and let L^s RYX', RPX', RXX' and RQX' be $\theta_1, \theta_2, \theta_3, \theta_4$, then (since there are sinks $-m$ at X & Y)

~~$-\theta_1+\theta_2-\theta_3+\theta_4 = -\theta_1+\theta_2-(\theta_2+P\hat{R}X)$~~ –

Since Δ^s ORP and OQR are similar, $O\hat{R}P = O\hat{Q}R$

ie ~~$\pi-\theta_4 =$~~ $\theta_2 - 2\theta_1 = \pi - \theta_4$ – (1)

Also XY being a diameter $X\hat{R}Y = \pi/2$

$\therefore \quad \theta_3 = \frac{\pi}{2} + \theta_1$ (2)

From (1) & (2) we have

$$-\theta_1 + \theta_2 - \theta_3 + \theta_4 = \pi/2$$

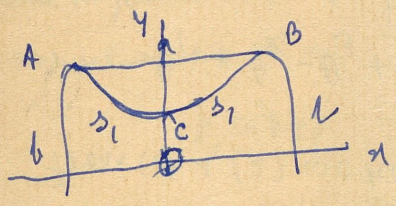
so that the circle is a stream line & this verifies the arrangements of sources & sinks so that there is no flow across the boundary. Hence the Complex potential is given by

$$w = -m \log \frac{(z-b)(z-a^2/b)}{(z^2-a^2)}$$

and $\phi = -m \log \frac{r_2 r_4}{r_1 r_3}$; $\psi = -m(\theta_2 + \theta_4 - \theta_1 - \theta_3)$.

(4) A uniform chain hangs freely suspended from two fixed point. Find the intrinsic ~~equation~~ and cartesian equations to the curve in which the chain hangs.

A heavy uniform strings hangs over two smooth pegs at the same level at a distance $\underline{a}$ apart. Show that equilibrium is impossible unless the length of the string is equal to or greater than $\underline{ae}$ (e = base of natural logs).



[Total length of string is $2s_1 + 2b$. Parameter of catenary is c. For eqm at point B we must have $T_m = b\omega = \omega c \cosh\left(\frac{a}{2c}\right)$

Also $b = c\cosh\left(\frac{a}{2c}\right)$ and $s_1 = c\sinh\left(a/2c\right)$

Hence total length of string $L = 2c\left(\cosh\frac{a}{2c} + \sinh\frac{a}{2c}\right) = 2ce^{a/2c}$

Putting ~~Computing derivative~~ $\frac{dL}{dc} = 0$, $2e^{a/2c} - 2c\cdot\frac{a}{2c^2}e^{a/2c} = 0$

ie $a = 2c$ hence $L = ae$

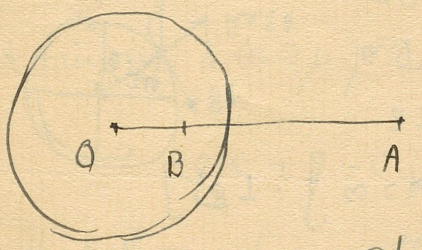
It is easily found that if $a = 2c$, $\frac{d^2L}{dc^2} = \frac{2e}{c} = +^{ve}$

ie ae is the minimum length.

(5) A light beam of length $\underline{l}$ is clamped horizontally at one end and carries a load W concentrated at the other end. Show that the deflection at the end point is given by $y = \frac{1}{3}Wl^2/B$, B being the

(3) Let the source m be at $A(f, 0)$. If B be the inverse of A w.r.t the circle of radius a

$$OB = f' = a^2/f \quad (\text{ie } ff' = a^2)$$



The image consists of a source m at B and a sink $-m$ at O. Hence the complex potential is given by

$$w = -m' \log \frac{(z-f)(z-f')}{z}$$

($z=0$ being the origin & OA the real axis)

$\therefore$ ψ = stream function $= -m(\phi_1 + \phi_2 - \phi)$

$$= -m\left\{\tan^{-1}\frac{y}{x-f} + \tan^{-1}\frac{y}{x+f} - \tan^{-1}\frac{y}{x}\right\}$$

$$= -m\tan^{-1}\left\{\frac{y(x^2+y^2-a^2)}{x(x-f)(x-f') + y^2(x-f-f')}\right\}.$$

Putting $c = \tan(\psi/m)$, the stream line corresponding to c is given by

$$y(x^2+y^2-a^2) + cx(x-f)(x-f') + cy^2(x-f-f') = 0$$

ie $(x^2+y^2)\{cx+y-c(f+f')\} + a^2(cx-y) = 0 \quad (1)$

18/2/72 At Institute – Qns for Madras Mechanics I.

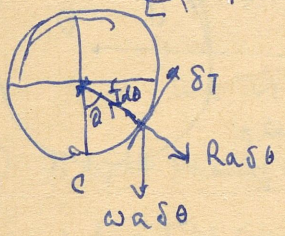
(1) A system of forces on a rigid body are reduced to a force $\vec{F}$ and a couple $\vec{M}$, Define Explain the notion of null plane of a point $\vec{\theta}$ with respect to the system, and find its equation in the form

$$\left(\vec{M} - \vec{r}' \times \vec{F}\right) \cdot \vec{r} = 0.$$

(2) Show that a system of forces can in general be reduced to two forces one of which acts along a given line.

(3) A string is in contact with a smooth plane curve under the action of no forces but the reaction of the surface and the tensions applied at its ends. Show that the reaction at any point varies as the curvature at that point (Ramsey p. 231, §12.11 – Statics)

A heavy uniform string passes round a circular cylinder whose axis is horizontal. Find the tension of the string at the points where it is vertical if the pressure on the cylinder vanishes at the lowest point (Ramsey, Works, p. 234)



[T · [ω = wt per unit length

$\delta T = \omega a \delta\theta \sin\theta$

$\therefore T = C - \omega a \cos\theta$ ——(1)

$T\delta\theta = Ra\delta\theta + \omega a \delta\theta \cos\theta$

$\therefore T = Ra + \omega a \cos\theta$

If $R = 0$, when $\theta = 0$, $T = \omega a$ & from (1) (putting $\theta = 0$), $C = 2\omega a$

$\therefore T = 2\omega a - \omega a \cos\theta$ $\therefore$ for string vertical $\theta = \pi/2$, $T = 2\omega a$

of which (~~type~~ as can be seen putting highest degree terms in x equal to zero after substituting accg to $y = \lambda x + \mu$ & finding λ and μ) the asymptote is

$$cx + y - c(f + f') = 0 \qquad (2)$$

This always passes through the fixed point $(f+f', 0)$.

Changing the origin to $A(f, 0)$ eqn (1) transforms to

$$\{(x+f)^2 + y^2\}\{y + c(x+f) - c(f+f')\} + a^2\{c(x+f) - y\} = 0$$

ie $(x^2+y^2+2fx+f^2)(y + cx - cf') + a^2(cx - y + cf) = 0$

& however since $f^2 cf' = a^2 cf$ this reduces to

$$(x^2+y^2+2fx)(y + cx - cf') + f^2(y + cx) + a^2(cx - y) = 0$$

and the lowest degree terms equated to zero give

$$-2fx \cdot cf' + f^2(y + cx) + a^2(cx - y) = 0$$

ie $(f^2 - a^2)(cx + y) = 0$ ie $cx + y = 0$

which is parallel to the asymptote (2)

See Milne-Thomson, p. 226, Exs. 26-27, and

[P.T.O.]

* The curve passes through the sources $P_1 \cdots P_n$.

Proof:- If the lines $P_K P_1$, $P_K P_2$, $\cdots P_K P_{K-1}$, $P_K P_{K+1}$, $\cdots P_K P_n$ makes angles α_{K1}, $\cdots \alpha_{Kn}$ with OX, then we can draw a line $P_K P$ through P making an angle α with OX such that $\alpha + \alpha_{K1} + \cdots + \alpha_{Kn} =$ given constant angle

$\therefore$ hence by defn P is a point on the stelloid.

for the figure of the stream lines, ibid p. 212, fig 8.61.

(4) This example is a particular case of the general equation of stream lines due to a number of equal sources. The curves that thus arise are called Stelloids (See Loria I, §163, p. 447).

Let the sources be at $P_1, P_2 \cdots P_n$ all of equal strength and let P_K be (a_K, b_K) and $c_K = a_K + ib_K$. The stream lines are given by

$$\sum_{K=1}^{n} \arg (z - c_K) = \text{const}$$

or

$$\arg \prod_{K=1}^{n} (z - c_K) = \text{const}$$

Putting $f(z) = (z-c_1) \cdots (z-c_n) = P(x,y) + i\,Q(x,y)$

the stream lines are $\dfrac{P(x,y)}{Q(x,y)} = \text{const} = c$, the parameter being c. — — (1)

Some properties of stelloids *

(1) The curve is of degree n – This is obvious from the fact that P & Q are of degree n.

(2) The first polar of a point on the line at infinity with respect to a stelloid of degree n is a stelloid of degree $n-1$.

Case p = odd - quite complicated, result being 2 cases

(i) $a^{p^2}=1,\ b^p=1,\ b^{-1}ab=a^{1+p}$

(ii) $a^p=1,\ b^p=1,\ c^p=1,\ ab=bac,\ ca=ac,\ cb=bc$

For the three cases of Abelian groups we have

(i) $a^{p^3}=1$, (ii) $a^{p^2}=1,\ b^p=1,\ ba=ab$

(iii) $a^p=b^p=c^p=1,\ ba=ab,\ ca=ac,\ cb=bc$

(9|2|72) AtCCB Books to be borrowed

(1) Sokolnikoff – Tensor Analysis — 516.

(2) Kemble – Principles of Quantum Mech — 530.12 (K.E.M) — Quant Mech

(3) Milne-Thomson – Hydrodynamics (2nd Edn).

(4) Banach – Mechanics ×

(5) Synge & Griffith. — 531.

(6) Besant & Ramsey – Part I – Hydrostatics — Mechanics

(7) Milne – Vectorial Mechanics ×

(8) Goldstein – Classical Mechanics — 530.15

(9) H.C. Plummer – Principles of Mechanics. ×

P.T.O

Proof:- For a curve to be a stelloid it is necessary & sufficient that its equation be brought to the form

$$a\,P(x,y) + b\,Q(x,y) = 0$$

a & b being constants, and $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$ & $\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x} = 0$.

The first polar of a point on the line at infinity w.r.t the ~~st~~ above has the form

$$a\left(\xi\frac{\partial P}{\partial x} + \eta\frac{\partial P}{\partial y}\right) + b\left(\xi\frac{\partial Q}{\partial x} + \eta\frac{\partial Q}{\partial y}\right) = 0$$

which is equivalent to $a\left(\xi\frac{\partial P}{\partial x} + \eta\frac{\partial P}{\partial y}\right) + b\left(-\xi\frac{\partial P}{\partial y} + \eta\frac{\partial P}{\partial x}\right) = 0$

Hence
$$\left.\begin{aligned} &\frac{\partial}{\partial x}\left(\xi\frac{\partial P}{\partial x} + \eta\frac{\partial P}{\partial y}\right) = \frac{\partial}{\partial y}\left(-\xi\frac{\partial P}{\partial y} + \eta\frac{\partial P}{\partial x}\right) \\ &\frac{\partial}{\partial y}\left(\xi\frac{\partial P}{\partial x} + \eta\frac{\partial P}{\partial y}\right) + \frac{\partial}{\partial x}\left(-\xi\frac{\partial P}{\partial y} + \eta\frac{\partial P}{\partial x}\right) = 0 \end{aligned}\right\}$$

since $\frac{\partial P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0$. These eqns prove the theorem.

(3) For varying c, the eqn (1) represents a pencil of stelloids:-

Proof:- (1) can be written as

$$\frac{\frac{1}{2}\{f(x) + \overline{f(x)}\}}{\frac{1}{2i}\{f(x) - \overline{f(x)}\}} = c, \text{ or } \frac{f(x)}{\overline{f(x)}} = \frac{c+i}{c-i}$$

(17)

[If A (a finite Abelian p-group) = A (b) = direct product of cyclic groups of orders $p^{e_1}, .. p^{e_r}$, (87)

then these numbers are called the invariants. In the special case when are the invariants are $p, .. p$ we say that A is an elementary Abelian group' – Structure theorem :- If a finite Abelian p-group A is the direct product of cyclic groups in two ways $A = A_1 \times A_2 \times .. \times A_r = B_1 \times .. \times B_s$, then $r = s$ and the orders of $A_1 .. A_r$ are the same as those of $B_1 .. B_s$ in some arrangement.

An abelian group of order n contains an element of order p if p is a prime dividing n]

respectively (p^3), (p^2, p), and (p, p, p). In finding non-Abelian groups we handle cases $p = 2$ & p = odd separately.

Case $p = 2$ – ~~Here~~ Result is that there are two non-abelian groups viz the dihedral group with defining relations

$$a^4 = 1, \quad b^2 = 1, \quad b^{-1}ab = a^3$$

and the quaternion group with defining relations

$$a^4 = 1, \quad b^2 = a^2, \quad b^{-1}ab = a^3$$

Easily verified that these define 2-groups of order $2^3 = 8$ & they are not isomorphic

or in the extended form

$$\frac{(x+iy-c_1)\cdots(x+iy-c_n)}{(x-iy-\bar{c}_1)\cdots(x-iy-\bar{c}_n)} = \frac{c+i}{c-i}$$

This is satisfied by putting $x+iy = c_k$, $x-iy = \bar{c}_k$

as also $x+iy = c_h$, $x-iy = \bar{c}_h$

Thus if the points P_k & P_h be denoted as $(c_k, \bar{c}_k)$ and $(c_h, \bar{c}_h)$ in Darboux's notation of imaginary symmetric coordinates the points $(c_h, \bar{c}_k)$ and $(\bar{c}_h, c_k)$ which are the diagonal points of the complete quadrangle formed by P_h, P_k and the circular points at infinity also satisfy the equation. ~~There~~ Thus to the pair P_h, P_k we have another pair also lying on the stelloid which passes therefore through $\frac{1}{2}n(n-1)$ point pairs & the given n points making in all $\frac{1}{2}n(n-1)+n = n^2$ points. Therefore the eqn represents a pencil of stelloids, ~~whose common points~~ Thus the first polars of all points on the line at ∞ form such an example.

(4) To find the polar equation of the stelloid:

Let $f(z) = p_0 z^n + p_1 z^{n-1} + \cdots + p_n$ where $p_0 = 1$

(i) Let G be a finite group, and let α_i = no. of elements in the conjugate class A_i of i. If $a \in A_i$, then $\alpha_i = [G : C_G(a)]$ = index of $C_G(a)$ in G

(ii) For G with above class equation, then $\alpha_i \mid |G|$ for $i = 1, 2, \ldots k$

$|G| = \alpha_1 + \cdots + \alpha_k$.

(iii) Let G be a finite group, $|G| = p^n$, $n > 0$, p a prime, then G has a non trivial center, of order at least p.

(iv) Let G be a group of order p^2, p a prime. Then G is abelian

Proof :- Since $|G| = p^2$, then above theorem has a non-trivial centre $C(G)$. Since $C(G) \lhd G$, consider $G/C(G)$. Since $C(G)$ is non-trivial $|G/C(G)| = p$ or 1. In either case $G/C(G)$ is cyclic. Hence G is abelian [Ex. 9, p. 125]

(7) Th. 4.13.7, p. 133 on direct products & Ex. 14 on same page.

This book to be borrowed

(B) Problems in group theory by John. D. Dixon (Blaisdell Pub. Co., 1967) (with solutions). — Also to be borrowed

(C) Theory of Groups by Marshall Hall

p. 50: For ~~non-abelian~~ groups of order p^3 (p = prime) there are three abelian types with invariants

and let $z = re^{i\theta}$, $p_k = q_k e^{ir_k}$ $(k = 0, \dots n)$, $c = \tan\gamma$

so that $q_0 = 1$, $r_0 = 0$.

Then $P(x,y) = \sum_{k=0}^{n} q_k r^{n-k} \cos\left(r_k + \overline{n-k}\,\theta\right)$

$$Q(x,y) = \sum_{k=0}^{n} q_k r^{n-k} \sin\left(r_k + \overline{n-k}\,\theta\right)$$

and the eqn to the stelloid is therefore

$$\sum_{k=0}^{n} q_k r^{n-k} \cos\left(\gamma + r_k + \overline{n-k}\,\theta\right) = 0. \qquad (2)$$

(5) The stelloid of order n has n real asymptotes which pass through the centroid of the sources, and also the points of intersection with the line at infinity of the first polar of the centroid:

Proof:- Taking the centroid as the pole

$$c_1 + c_2 + \dots + c_n = 0, \quad p_1 = 0, \quad q_1 = 0$$

For $n\theta + \gamma = (2r+1)\frac{\pi}{2}$ $\quad (r = 0, 1, 2, \dots \overline{n-1})$

we have n-directions making r infinite & also for these values eqn (2) is of degree $(n-2)$ in r ie we get the asymptotes.

(4) Let G be Ex: Let G be an abelian group of order pq, $p \neq q$ (85)
being distinct primes. Show that G is cyclic.

Hint: Find an element of order p or q, say. Then consider $G/(a)$ [(a) = cyclic group generated by a]. Then recall the example: If G be an abelian group of order pq with $(p, q) = 1$ [ie $G \cdot C \cdot D = 1$], then if G contains elements of order a & b of orders p & q resply, then G is cyclic

(5) Defns: (i) Let G be a finite group of order α. Let the conjugate classes of G (ie equivalence classes defined by $a = g^{-1}bg$) be $\{A_i\}$, $i = 1, \cdots n$ & let A_i have α_i elements. Then the expn $\alpha = |G| = \alpha_1 + \alpha_2 + \cdots + \alpha_n$ is called the class equation of G

(ii) The set $C_G(g) = \{x \mid x^{-1}gx = g, x \in G\}$ is called the centraliser of g in G.

(iii) If A is a subgroup of G, then set $N_G(A)$
$= \{x \mid x \in G, x^{-1}Ax = A\}$ is called the normaliser of A in G.

Note: An element $g \in G$ is central iff $C_G(g) = G$, and that a subgroup $A \subseteq G$ is normal in G iff $N_G(A) = G$.

(6) Theorems:

In the present case, where the sources are at the vertices of an equilateral triangle, taking the pole at the centre of the triangle the sources are at $z = 1, e^{2\pi i/3}, e^{4\pi i/3}$ ie $z = 1, \omega, \omega^2$ the cube roots of unity. Hence

$$f(z) = (z-1)(z-\omega)(z-\omega^2) = z^3 - 1$$

$$\therefore \quad P(x,y) = r^3\cos 3\theta - 1 \; ; \; Q(x,y) = r^3 \sin 3\theta$$

$\therefore$ Polar equation of the curve is

$$r^3 \cos(3\theta + \gamma) = \cos\gamma$$

$$(g_1K \cdot g_2K)f = [(g_1g_2)K]f = (g_1g_2)H = (g_1H)(g_2H)$$

$$= (g_1K)f \cdot (g_2K)f,$$

f is a homomorphism. The kernel of f is the set $\{gK \mid gH = H\}$

i.e., $\ker f = \{gK \mid g \in H\}$. But since K is a normal subgroup of H

K of $\ker f = H/K$ thus $H/K \triangleleft G/K$ ($\triangleleft$ = n.d)

& hence $G/H \cong (G/K)/(H/K)$

(2) Exc. If $H \triangleleft G$ and $K \triangleleft H$, show that K need not necessarily be normal in G.

Hint: Consider A_4 or D_4 (group of symmetries of a square)

(3) Exc: Let A & B be finite subgroups of a group G. Show that

$$|AB| = \frac{|A||B|}{|A \cap B|} \qquad \left[|AB| = \text{no. of elements in } AB. \right.$$

Hint: observe that $C = A \cap B$ is a subgroup of B. Thus there exist $b_1, b_2, \ldots b_n$ in B such that $B = Cb_1 \cup \cdots \cup Cb_n$ where the Cb_i are the n $|B|/|A \cap B|$ distinct right cosets of C in B.

Then $AB = ACb_1 \cup \cdots \cup ACb_n$. Since $C = A \cap B$, $AC = A$.

Thus $AB = Ab_1 \cup \cdots \cup Ab_n$. If $Ab_i \cap Ab_j \neq \phi$, $i \neq j$,

then $ab_i = a'b_j$ for some a, a' in A. Then $b_j b_i^{-1} = (a')^{-1}a \in A \cap B = C$,

whence $|AB| = |Ab_1| + |Ab_2| + \cdots + |Ab_n|$. Now complete the proof. [Article in Amer. Math. Monthly, 71 (1964), p. 793]

$N \lhd G \longrightarrow N$ is normal in G

$G \rhd N \rightarrow$ "

Abelian. If H is not cyclic, let S be a self-conjugate element of order p and let T be an element of H which is not in $\{S\}$. Then $T^{-1}ST = S$, so that S and T are permutable. It is obvious that they generate a group H

Ex 3: Every group whose order is a multiple of a prime p contains an element of order p

Soln: This is an immediate consequence of Sylow's theorem & the 5th fund. Theorem above.

Unworked Ex: If a group G of order p^3 contains more than one self-conjugate subgroup of order p, then G is Abelian & non-cyclic.

(14|2|72) – At Institute – To work on Madras paper on Algebra

(1) Third Isomorphism Theorem (A) (From Paley & Weichsel – 1st course in Abstract Algebra, 1964

If H & K be normal divisors of G and K a subgroup of H, then H/K is a normal divisor of G/K and $(G/K)/(H/K) \cong G/H$.

Proof: Let $f: G/K \to G/H$ be defined by

$$(gK)f = gH \text{ for all } g \in G$$

Let $g_1K = g_2K$. Then $(g_1K)f = g_1H$ and $(g_2K)f = g_2H$.

But since $g_1K = g_2K$, $g_2^{-1}g_1 \in K$ whence $g_2^{-1}g_1 \in H$ and so $g_1H = g_2H$. Clearly f is onto G/H and since

then G contains at least one subgroup of order p^α. All its subgroups of order p^α form a single complete conjugate set, and their number is $1+kp$, where k is an integer ($+^{ve}$ or zero) — such a subgroup p^α is called a Sylow subgroup.

— $(1+kp)$ is a factor of the order of G [Ref. to Miller, Blichfeldt, & Dickson pp. 23, 30, 64]

Theorem: An abelian group whose order is not a power of a prime number is the direct product of all its Sylow subgroups

Theorem: (A most important theorem relating to Abelian groups accdg to G. A. Miller, p. 89 — 4th fund. theorem accdg to Carmichael)

A non-cyclic Abelian group G whose order is a prime-power p^m is the direct product of cyclic groups no two of which have any element in common except the identity.

Theorem (Fund. 5th-re. prime-power groups)

A prime power group G of order p^m contains a self-conjugate element of order p.

Examples (1) The number of self-conjugate elements in G is a power of p.

Soln: For the self-conjugate elements in G obviously form a group (which is Abelian) and this is a subgroup of G

Ex 2. A group H of order p^2 is Abelian.

Soln:- If H contains an element of order p^2, it is cyclic and hence

~~Elements S and T are conjugate~~

If S & T be elements of G, S & $T^{-1}ST$ are conjugate elements

If every conjugate of S in G is equal to S ($S^{-1}SS = S$)

S is <u>self conjugate</u>

Moore's elementary general topology

Ex 1. Let X be the set of all real nos. and let $\mathcal{T}$ be the family generated by all "half-open" intervals of the form $\{x : a \le x < b\}$ where $a, b \in R$ with $a < b$ then $\{X, \mathcal{T}\}$ is a Hausdorff space.

If $Y = X \times X$, then Y not a Lindelof space for it can be shown that $A = \{(x,y) : x+y=0\}$ is closed & fine an open cover for A which has no countable subcover]. Hence product of Lindelof spaces need not be a Lindelof space

Ex 2 – Product of two normal spaces need not be normal. since X in Ex 1. is normal.

12/2/72 At C.C.B.

(1) Groups of finite order by Carmichael.

p. 55 - Every group G of finite order n can be represented as a regular permutation group on n symbols, the latter being $\cong G$

p. 58 – Sylow's theorem : Let p^α be the highest power of a prime p contained in n (order of G) as a factor, α being a $+^{ve}$ integer

Lemma 2 - A metric space is compact-in-itself iff it is complete and totally bounded.

Theorem - Hilbert space is complete.

Examples: (1) Prove that Frechet space F is complete.

[Let $p_n = (x_1^{(n)}, x_2^{(n)}, \dots)$, $n = 1, 2, \dots$ be an infinite sequence of points of F which satisfies Cauchy's condition; then there exists, for $\epsilon > 0$ and every natural number i, an index μ (from the defn of distance in F) such that

$$\frac{|x_i^{(n+k)} - x_i^{(n)}|}{i!\left(1 + |x_i^{(n+k)} - x_i^{(n)}|\right)} < \epsilon, \text{ for } n > \mu, k = 1, 2, \dots$$

Thus for $\epsilon < 1/2$ we have $|x_i^{(n+k)} - x_i^{(n)}| < 2i!\,\epsilon$, for $n > \mu$, $k = 1, 2, \dots$

Hence there exists a finite limit

$$x_i = \lim_{n \to \infty} x_i^{(n)} \quad (\text{for } i = 1, 2, \dots$$

Setting $p = (x_1, x_2, \dots)$, we have $\lim_{n \to \infty} p^{(n)} = p$. Hence the proof]

Ex 2. Show that the 0-dimnl Baire space is Complete

[The 0-dimnl Baire space is a closed subset of F & since F is complete, the result follows]

Uspensky — Intro to Math. Probability — Rabi Lodaya 10/2/72

10/2/72 At C.C.B. Library.

(1) State and prove the fundamental theorem of homomorphism for
(a) groups —

Any factorgroup of G is a homomorphic image of G and conversely if G' is a homomorphic image of G, then G' is isomorphic to a factor group of G. [Jacobson, Vol I, p.44

(b) Find all possible homomorphic images of the four-group

[If G is is $C_2 \times C_2$, only imgs are (e), C_2 and $C_2 \times C_2$
Hall, 2, Ex 4. p.175 (workout)

(c) Prove that a subgroup H of a group G is invariant (normal) iff it is unchanged (as a whole) by any inner automorphism of G.

11/2/72 At Institute.

Topology (from Sierpinski)

(1) In order that a metric space M be complete it is necessary & sufficient that every totally bounded set contained in M be compact in M.

Cor 1. A metric space which is compact-in-itself is complete.

Books for Madras Setting.

X (1) Fundamental structures of Algebra by G.D. Mostow.

(2) Van der Waerden.

(3) Advanced Calculus for applications by F.B. Hildebrand (517-HIL)

(4) Synge & Griffiths - Mechanics. (531)

(5) Milne - Thomson.

X (6) Fluid Dynamics by Rutherford.

Ex. p. 373 (unsolved) (15-7)

A string with two fixed ends is plucked at the centre. Assume that the string is of length l and is at rest at $t = 0$ and that the initial profile is triangular:

$$y(x,0) = \begin{cases} \frac{2h}{l} \cdot x & x \le l/2 \\ \frac{2h}{l}(l-x), & x > l/2 \end{cases}$$

Show that the even harmonics will be missing and that the expansion coefficients are $a_n = 0$, and

$$b_n = (-1)^m \frac{8h}{(2m+1)^2 \pi^2} \quad \text{for } n = 2m+1.$$

Ex. 15-13, p. 374.

Let $\Delta\nu$ be the frequency spread in cycles per second of a sinusoidal wave train of finite extent. Let Δt be the length of time required for the wave train to pass by a fixed point of observation. Show that the relation [$\omega_n = n(\pi c/l) = n\omega_1$ for the nth eigenfrequency or $K_n = n(\pi/l) = nK_1$ for the associated eigenfunction] between spatial and frequency widths is equivalent to the statement

$$\Delta\nu \cdot \Delta t \sim 1.$$

Notions of group velocity & phase velocity in the case of de Broglie matter waves, and find the relations between them

[phase vel = $\frac{1}{2}$ group vel & ph. vel × gr. vel = c^2

p. 390. Vibrations of a freely suspended rope [length 1, weight 1]

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(x \frac{\partial u}{\partial x}\right).$$ Putting $u = q(t)\,\varphi(x)$

△ assuming aux. condn $\varphi(1) = 0$

we get $\varphi(x) = C J_0(2\sqrt{\lambda x})$ $\quad |\varphi(0)| < \infty$

△ condn $J_0(2\nu) = 0$ determines a sequence of eigenfrequencies $\nu = \sqrt{\lambda}$

Chr. VI — Application of Calculus of variations to eigenvalue problems

p. 416. State the maximum-minimum property of eigen-values of a variation problem and show that in the case of the Sturm-Liouville problem this property makes it possible both to determine the order of magnitude of the nth-eigen-value as to find its asymptotic value

p. 429. Asymptotic distn of eigen-values for the eqn $\Delta u + \lambda u = 0$ for a rectangle: $\lim_{n\to\infty} \frac{\lambda_n}{n} = \frac{4\pi}{ab}$.

p. 462. Ex: If a string is fixed at $(n-1)$ points, and we wish to choose these points in such a way that the fundamental tone of the resulting system is as high as possible, we find that the solution is given by an eigenfunction of the original system having $(n-1)$ zeros at inner points.

Wave phenomena by Towne (Addison-Wesley)

9h. The Schrödinger equation

$$\Delta u + \frac{c}{r} u + \lambda u = 0$$ (c a given $+^{ve}$ const, r, θ, ϕ polar coords

(with conds that eigenfunctions u be continuous at zero & remain finite as $r \to \infty$)

has all positive real numbers λ as eigen-values ie it has a continuous spectrum consisting of the non-negative real numbers.

p. 388 (e) The plucked string (Strings)

$$u(x,t) = \frac{2h}{b(\pi - b)} \sum_{n=1}^{\infty} \frac{\sin nb \sin nx}{n^2} \cos nt$$

[string plucked at $t=0$ by being pulled a lateral distance h from x-axis at point $x = b$]

(b) Impulsive excitation [impulse applied in vert position at $x = b$

$$u(x,t) = 2V \sum_{n=1}^{\infty} \frac{\sin nx \sin nb}{\pi n} \sin nt \quad \left[\int_0^{\pi} u_t(x,0)\,dx = V \text{ (a const)}\right]$$

(c) Forced vibrations [Diff eqn is $u_{tt} - u_{xx} = f(x) \cos nt$]

$$u_n(t) = -\sum_{\nu=1}^{\infty} \sin \nu x \frac{c_\nu}{n^2 - \nu^2} (\cos nt - \cos \nu t)$$

[$u(x,0) = u_t(x,0) = 0$

Explanation of phenomenon of beats

p. 292. The Sturm-Liouville eigenvalue problem

$$(pv')' - qv + \lambda \rho v = 0 \quad \text{---} (1)$$

p. 293

The eigen-values λ of the diff. eqn (1) for given boundary conditions, ordered w.r.t magnitude, form a denumerable sequence $\lambda_1, \lambda_2, \lambda_3, \cdots$, and the corresponding system of eigen-functions is a complete orthogonal system.

p. 214 - Vibration of 3-dimensional Continua.

Consider the vibration equation for the spherical domain $x^2+y^2+z^2 \le 1$ of radius 1. If the boundary condition is given by $u = 0$, & if the α's of the roots of $J_{n+\frac{1}{2}}(\sqrt{\lambda}) = 0$ be denoted by $\alpha_{n,1}, \cdots$, we obtain the soln of the form

$$u = Y_n(\theta, \varphi)\, S_n(\sqrt{\lambda_{n,n}}\, r)$$ for the boundary value problem.

$$\left[Y_n(\theta, \varphi) \text{ are sph. harmonics \& } S_n(\sqrt{\lambda}\, r) = \frac{J_{n+\frac{1}{2}}(\sqrt{\lambda_0}\, r)}{\sqrt{r}} \right]$$

p. 334. Theorem: The absolute values of the solutions of the Sturm-Liouville equation (with continuous r)

$$u'' - ru + \lambda u = 0$$

in the interval $0 \le x \le 1$ remain less than some bound independent of λ and x provided the the solutions are normalised by the condition $\int_0^1 u^2\, dx = 1$

and satisfy the boundary conditions $u(0) = u(1) = 0$

and wave the other wave is longitudinal (along x-axis)

(Subscripts $\underline{l}$ and $\underline{t}$ denote longitudinal & transverse waves)

(b) Eqns (D') & (E') have same form as general wave eqns & accordingly coeffts of R.H.S. are squares of the propagation velocity of the waves, C_t and C_l. Both velocities may be expressed in terms of Lamé constants or practical coeffts E and ν (as on top of p. 72)

In general, there are two superposed and independent acoustic fields

—o—

8/2/72 – At the Central College Library

Notes from Courant-Hilbert

Vol I → p. 277 (Ch. V) – Homogeneous diff. eqns with non-homogeneous boundary conditions are essentially equivalent to non-homogeneous diff. eqns with homogeneous boundary conditions.

p. 286 : General properties of vibrating systems

Th. 3 – As the inertia increases, the pitch of the fundamental tone and every overtone decreases (or remains the same)

Th. 4. If the system stiffens, the pitch of the fundamental tone and every overtone increases (or remains the same)

This is general eqn of motion for an isotropic solid & is sometimes called Lame's eqn

In the case of pure shape deformation (dilatation $\vartheta = 0$) by adding eqns (A), differentiating w.r.t time & denoting the acoustic velocity by $\underline{\underline{v}}$

$$\underline{\underline{v}} = \frac{d\underline{\underline{u}}}{dt} \qquad \text{——— (C)}$$

we obtain the eqn characterising the wave of shearing strains which is called the wave of distortion

$$\rho \frac{\partial^2 \underline{\underline{v}}}{\partial t^2} = \mu \nabla^2 \underline{\underline{v}} \qquad \text{(D)}$$

For a wave corresponding to pure longitudinal strains without rotational motions ($\overline{\omega_x} = \overline{\omega_y} = \overline{\omega_z} = 0$, $\partial \vartheta / \partial x_i = \nabla^2 u$)

$$\rho \frac{\partial^2 \underline{\underline{v}}}{\partial t^2} = (\lambda + 2\mu) \nabla^2 \underline{\underline{v}} \qquad \text{——— (E)}$$

If waves given by (D) & (E) are assumed plane progressing along x-axis, we get

$$\frac{\partial^2 v_y}{\partial t^2} = \frac{\mu}{\rho} \cdot \frac{\partial^2 v_y}{\partial x^2} \qquad \text{——— (D')}$$

$$\frac{\partial^2 v_x}{\partial t^2} = \left(\frac{\lambda + 2\mu}{\rho} \right) \frac{\partial^2 v_x}{\partial x^2} \qquad \text{——— (E')}$$

Physical conclusions (a) The wave of distortion is transverse

[where $c_l^2 = \mu/\rho = \frac{E}{2\rho}\cdot\frac{1}{1+\nu}$; $c_t^2 = \frac{\lambda+2\mu}{\rho} = \frac{E}{\rho}\cdot\frac{1-\nu}{(1+\nu)(1-2\nu)}$]

and actual substitution in (A) gives $c_x = 4480$ m/sec

pp. 78 onwards – Eqns of motion in solids

To establish wave eqn we shall use the eqn of dynamic eq^m of forces relating stresses to the medium

To the forces $\underline{F}$ acting inside the medium, it is necessary to add the inertia force $\rho \frac{\partial^2 \underline{u}}{\partial t^2}$, and neglecting static mass forces, we get

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}$$

for an isotropic body, $\sigma_{kk} = \lambda\vartheta + 2\mu\epsilon_{kk}$, $\sigma_{ij} = 2\mu\epsilon_{ij}$

$i, j, k = x, y, z \quad (i \neq j)$

and we have

$$\left.\begin{aligned} \rho \frac{\partial^2 u_x}{\partial t^2} &= (\lambda+\mu)\frac{\partial \vartheta}{\partial x} + \mu \nabla^2 u_x, \\ &- - - \\ &- - - \end{aligned}\right\} \quad (A)$$

Adding these eqns termwise, taking into consideration that the sum of the first terms at R.H.S. multiplied by versors ($\underline{i}$) is the gradient of ϑ and $\vartheta = \operatorname{div} \underline{u}$, we get

$$\rho \frac{\partial^2 \underline{u}}{\partial t^2} = (\lambda+\mu)\operatorname{grad}\operatorname{div}\underline{u} + \mu \nabla^2 \underline{u} \quad (B)$$

both ends, (b) closed at one end.

Soln: The phase velocity in the air $c_p = 343$ m/sec. The correction for the action of the open end of the pipe is calculated from Rayleigh's formula

$$\Delta l'' = (\pi/4)\, a = 1.17\, a$$

[From eqns: (a) Pipe with both ends closed

$$f = nc/2l$$

[c = vel of wave propagation

(b) Pipe with one end closed & the other open

$$f = \frac{nc}{4(l+\Delta l)}$$

(Δl = correction for the co-vibrating mass of the open end)

(c) Pipe with both ends open

$$f = \frac{nc}{2(l+2\Delta l)}$$

$\{n = 1, 2, 3, \ldots\}$]

Using (2) & (3) we have from (c), (a) $f = 140\, n$ c/sec

(b) $f = 70.5\, n$ c/sec

($n = 1, 2, 3, \ldots$

Exc. p. 540. Calculate the propagation velocity of a longitudinal wave in a thin ($\lambda \leq D$) steel plate for which

$$c_l = 4880 \text{ m/sec}, \quad c_t = 2680 \text{ m/sec}$$

Soln:- An elementary solution gives $c_x = 2(c_l^2 - c_t^2)^{3/2} \frac{c_t}{c_l}$ — (A)

pp. 114–116 of Vibrations and waves by R. V. Sharman

Eqn for transverse vibrations of a rod can be written as

$$\partial^2 s/\partial t^2 + (k^2 E/\rho)\,\partial^4 s/\partial x^4 = 0 \qquad \text{—(1)}$$

[for long. vibrations as in Coulson, $\frac{\partial^2 s}{\partial t^2} = c^2 \partial^2 s/\partial t^2$]

where E = Young's modulus, k = rad. of gyration for a cross-section about the ~~rod~~ neutral filament.

Using vel. of long. vibrations ($c = \sqrt{(E/\rho)}$), (1) can be written

$$\frac{\partial^2 s}{\partial t^2} + k^2 c^2 \frac{\partial^4 s}{\partial x^4} = 0 \qquad \text{— (2)}$$

By assuming a soln $s = A\cos\left[2\pi(x \pm vt)/\lambda\right]$, A = Const.

then $\partial^2 s/\partial t^2 = -4\pi^2 v^2 s/\lambda^2$; $\partial^4 s/\partial x^4 = 16\pi^4 s/\lambda^4$

so that by substitution in (2), $v = 2\pi k c/\lambda$.

Thus transverse waves are dispersed, since short waves will travel faster than long ones.

Example from Physical foundations of technical acoustics by Malecki

Eq (Vide § 63 of Coulson)

Ex: find the resonance frequencies of a pipe with diameter $2a = 30$ m, and length $l = 1.2$ m. The pipe is (a) open at

Now let us suppose that we have an "infinite" soap film formed between parallel wires of indefinite length separated by a distance a. The frequencies of the normal modes in this case are given by

$$f = \frac{n}{2a}\sqrt{\frac{2T}{m}}$$

where n is any +ve integer & m = mass per unit area of the film. We obtain this result, in the limit, from eqn (208) viz

$$\left[f^2 = \frac{T}{4m}\left(\frac{k^2}{a^2} + \frac{l^2}{b^2}\right) \right] \quad \text{Vide eqn (4), p. 45 of Coulson}$$

writing $T = 2\gamma$, $a = a$, $b \to \infty$. Clearly the normal mode of vibration of such a film can be described as standing plane waves, of wave length λ given by $\lambda = 2a/n$. For this membrane-type vibration, plane waves are propagated over the film with phase velocity $\sqrt{2\gamma/m}$. The phase velocity being independent of the wave length, there is no dispersion for plane waves of this type. Indeed, the two-dimensional wave equation (204)

$$\left[\frac{\partial^2 z}{\partial t^2} = \frac{T}{m}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \right] \quad \text{— Vide eqn (1), Coulson, p. 44}$$

is characterised by the more general solution, that, for distances from a point source significantly greater than λ, a membrane will transmit a disturbance of any wave-form, equally in all directions over the surface, with velocity $\sqrt{T/m}$.

[A^* = vector potential]

$$E_y = E_0 \cos\omega t, \quad E_x = E_0 \sin\omega t$$

$$A_y = -\frac{cE_0}{\omega}\sin\omega t, \quad A_z = \frac{cE_0}{\omega}\cos\omega t$$

The motion is given by the formula

$$x = 0, \quad y = -\frac{ecE_0}{r\omega^2}\cos\omega t, \quad z = -\frac{ecE_0}{r\omega^2}\sin\omega t$$

$$p_x = 0, \quad p_y = \frac{eE_0}{\omega}\sin\omega t, \quad p_z = -\frac{eE_0}{\omega}\cos\omega t$$

$$r^2 = m^2c^2 + \frac{c^2E_0^2}{\omega^2}.$$

Thus the charge moves in the y-z plane along a circle of radius $ecE_0/r\omega^2$ with a momentum having the constant amplitude $p = eE_0/\omega$; at each instant the direction of momentum $\underline{p}$ coincides with the direction of the magnetic field $\underline{H}$ of the wave.

(4) Question regarding skin effect (p. 434 of Adler, Chu & Fano, Electromagnetic energy transmission and radiation

That is, the actual interface gives rise to a standing-wave pattern in the air which looks as though it arises from incidence on a purely resistive medium whose surface lies one-half of a skin depth ($\delta/2$) inside the real metal.

(5) Question from Norman Feather's book "Intro to the Physics of vibrations and waves" p. 315

$$\left[W = \frac{1}{8\pi} \left(E^2 + H^2 \right) = \frac{E^2}{4\pi} \right.$$

where $\underline{N}$ is the vector normal to the surface of the wall, and $\sigma_{\alpha\beta}$ & $\sigma'_{\alpha\beta}$ are the components of the energy-momentum tensors for the incident and reflected waves. Using ($\sigma_{xx} = W$, σ_{xn} is the only non-zero component of $\sigma_{\alpha\beta}$) $\underline{f} = W\underline{n}(\underline{N}\cdot\underline{n}) + W'\underline{n}'(\underline{N}\cdot\underline{n}')$

From defn of reflection coefficient, we have $W' = RW$. Also introducing the angle of incidence θ (equal to the reflection angle) & writing out components we find the normal force ("light pressure")

$$f_N = W(1+R)\cos^2\theta$$

& tangential force $f_t = W(1-R)\sin\theta\cos\theta$.

Ex 2: Determine the motion of a charge in a field of a plane monochromatic linearly polarised wave.

Soln: Choosing the direction of the field $\underline{E}$ of the wave as the y-axis, we write $E_y = E = E_0\cos\omega\xi$, $A_y = A = -\frac{cE_0}{\omega}\sin\omega\xi$ ($\xi = t - x/c$). From the formulas (3) & (4) of Ex 2, §47 (very complicated example)

——— omit this example -

Ex. 3. Determine the motion of a charge in the field of a circularly polarised wave.

Soln: For the field of the wave, we have

in state $\underline{a}$, non-relativistically

$$\epsilon_{a,q} = \frac{q^2}{2m} + \delta\epsilon_a \qquad (\delta\epsilon_a = \text{internal energy})$$

Relativistically $\epsilon_{a,q} = c\left(q^2 + M_a^2 c^2\right)^{1/2}$ (A) (M_a = rest mass in state $\underline{a}$)

For applies in solid state or nuclear physics, we may wish to generalise $\epsilon_{a,q}$ to include a portion of the energy of interaction of a particle with the medium through which it is travelling. In this case, we often refer to the system as a "quasi-particle" & there is a more general relationship than (A) between $\epsilon_{a,q}$ and $\underline{q}$

7/2/52 Landau & Lifshitz = Course of Th. Phys. Vol 2.

Chap. 6 – Electromagnetic waves.

Ex 1: Determine the force exerted on a wall from which an incident plane electromagnetic wave is reflected (with reflection coeffit R).

Soln: The force $\underline{f}$ acting on unit area of the wall is given by the flux of the momentum through this area i.e. it is the vector with components

$$f_\alpha = \sigma_{\alpha\beta} N_\beta + \sigma'_{\alpha\beta} N_\beta$$

(2) Bergmann's book on "The Riddle of gravitation"

p. 197 - Quantum theory of gravitation § 22

p. 199 Geometry of space & time cannot be free of uncertainty if a quantum theory of gravitation is to be built up) (if the gravitational field is to be subjected to Heisenberg's uncertainty principle. The precise relationships between world points must be abandoned in favour of some probabilistic formulations. For this reason the quantum theory of the gravitational field calls for novel conceptual approaches ~~to~~ not required in the past but which, once established, are bound to affect the whole of physics".

3/2/72 - finished off the damned CSIR business - One one reprint of paper, a copy of Technical Report to the clerk in charge.

Quasi-particle.

(1) Description of scattering processes - constn of a wave packet (large cf. to particle wavelength $\hbar/p$) & its spreading

$\epsilon_{q,q}$ = K.E + internal energy of ~~state~~ the system in

p. 225 – Heat transfer research in Japan by J. P. Hartnett.

Conducted in the laboratories of the major universities and in a few government laboratories – Recently progressive industries initiating programmes – Institutes of Industrial Science, Aeronautical Research Institutes, Mech. Eng. Depts – Universities of Tokyo, Tohoku, Kyushu, Yokohama, Nagoya, Osaka, Hokkaido – Atomic Energy installation at Tokai-mura –

Workers to be commended for their research accomplishments in laboratories which, for the most part, are modestly equipped and staffed. In accordance with tradition, the Japanese investigator receives no financial reward for research contributions as a supplement to his modest academic stipend, nor will research accomplishment materially accelerate promotion in rank. The primary reward for the Jap. investigator is the sense of satisfaction that comes from the increased understanding of physical processes & a sense of achievement. – References to 48 publications from 1900–57

29|1|72

Ivanenko's article in the Gamow Commemoration Volume

(1) Physics, Logic & History

Formal mathematical point arising from the fact that the interaction –

Borrow this book today

(3) Modern developments in heat transfer. (edited by Warren Ibele, Academic Press, 1963) — Number of articles by different authors, read at the Int. Heat Transfer Conference in August, 1961.

Preface — Rapid developments in heat transfer represent, in microcosm, a phenomenon encountered throughout since and technology. Two simple aspects emerge. One, cited so frequently as to be commonplace, is the active involvements of physicists, chemists and mathematicians in activities usually considered to be the ~~realm~~ responsibility of engineers and vice versa, with the result that the lines once separating them have blurred to the vanishing point. A second aspect is the increased involvement of scientists whose previous connection with the field of heat transfer was at most tenuous. This has grown to the extent that journals of biology, physiology, and zoology have become channels for the reporting of heat transfer investigations, in addition to the more familiar journals of chemistry, physics and engineering. The problems range from blood flow to desalinization of sea water, the temp. range extending from the vicinity of absolute zero to that of the stars, the environment ranging from the earth's interior to outer space. — Ref. to Dr Eckert's Contributions (Heat Transfer Lab. of Univ. of Minnesota) — Vol. dedicated to Dr Eckert.

On p. 123 — Thermal radiation properties of solids by Thomas. F. Irwine. Jr (State Univ. of N.Y., Long Island Center, Stony Brook, N.Y (1, 213) — the external examiner of the thesis

Also p. 1 — by Do — Noncircular duct convective heat transfer

digital methods, the differential analyzer and an electric analogue method called the thermal analyzer have all been used. Systematic use of numerical methods has a great advantage. Applications of these results are of the greatest practical importance. (eg. in geophysics, ice manufacture, solidification of castings, geology,

Ch. 12 – on Laplace transformation – 12.1 - Introduction

Ch. 13 – on Laplace Hyp. methods

Ch 14 – on the use of Green's functions in the solution of the equation of Conduction

Ch. 15 – Further applns of L.T.

Ch. 16 – Steady temperature – Sources & sinks – use of complex variable – Schwarz–Christoffel theorem

Ch. 17 – Integral transforms. – Complex fourier, fourier series, fourier cosine, Hankel transforms. – Applns to variable flow of heat

Ch. 18 – Numerical methods – finite differences – Boundary conditions – Relaxation methods

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Ref. in p. 346 V.R.T. and Ramakrishna's paper in "Quart. Appl. Math. 10 (1952), 255–62

(2) Book on Heat Transfer by Isachenko, Osipova & Sukomel (MIR Publishers, Moscow, 1969)

mathematical work.. Explicit solutions of many problems of practical interest are included and much numerical information in the form of tables and text-figures are given. The discussion of the theory of systems used in exptl. work has been greatly extended, and other subjects of practical importance are briefly noticed such as the theory of automatic temp. control, which have hitherto not appeared in mathematical text-books --- impossible to give a complete bibliography.

p. 35 – Uniqueness of the solution of the problem. (of heat conduction in a finite closed region with prescribed initial & surface temperatures) – usual method of classical proofs reproduced in many places and is not quite satisfactory [Doetsch, Math. Z. 22 (1925) 293, & 25 (1926), 608]. But few attempts have been made to establish broad broad conditions under which the soln is unique & tendency has been to prove uniqueness of solution of each special problem.

To prove that the eqns have a solution is an even more difficult matter than that of proving uniqueness. The physical interpretation of the eqns requires that there be a solution; the math. demonstration of such existence theorems belongs to pure analysis.

p. 283. Ch. 11 on Change of State – Apart from the few exact solutions, all problems have to be attacked by numerical methods: &

29/12/71 Scientific American, Jan, 1971

p. 48 – The nature of pulsars by Jeremiah P. Ostriker –
The leading hypothesis is that they are neutron stars
rotating at a rapid rate

31/12/71 – Went to Office, saw Seshagiri Rao, repended Rs 200/-
out of contingency & retained Rs 100/- & odd for typewriting work,
gave all this in writing – Also receiving cheque for ~~Rs 600~~ Rs 2,000/-
being salary for Aug, Sept, Oct & Nov.

Math. Reviews – May, 1971 – looked at the articles on Relativity
and found them all quite tough.
Sept, 1971 —— Do ——

19/1/72 To refer to

(1) Proc. Relativity Conf. in Midwest held at Cincinnati, Ohio, June 2-6
1969, Edited by Carmeli, Fickler & Witten (Plenum Press, 1970)

(2)

24/1/72 Notes re. value of Kerala Thesis

(1) Conduction of heat in solids by Carslaw & Jaeger (1959)

In this new edition we have tried to make it as useful as possible
to physicists & engineers without altering its character as a

left unadjusted. By most writers the invention of gunpowder is ascribed to the Chinese, but not upon very convincing evidence.

(26) Habit – A shackle for the free.

(27) Hand – A singular instrument worn at the end of human arm and commonly thrust into somebody's pocket.

(28) Happiness – An agreeable sensation arising from contemplating the misery of another

(29) Telephone – An invention of the devil which abrogates some of the advantages of making a disagreeable person keep his distance.

(30) Sorcery – One ancient prototype and forerunner of political influence.

(31) Christian – One who believes that the New Testament is a divinely inspired book admirably suited to the spiritual needs of his neighbour. One who follows the teaching of Christ in so far as they are not inconsistent with a life of sin.

(32) Capital – The seat of mis-government.

(33) Cannon – An instrument employed for the rectification of national boundaries.

(15) Diplomacy – The patriotic art of lying for one's country.

(16) Disobedience – The silver lining to the cloud of servitude.

(17) Dictator – The chief of a nation that prefers the pestilence of despotism to the plague of anarcy. (Bhutto ?)

(18) Abscond – To "move in a mysterious way" commonly with the property of another.

(19) Academy – (from academe) – A modern school where football is taught.

(20) Creditor – One a tribe of savages dwelling beyond the Financial Straits and dreaded for their desolating incursions.

(21) Daring – one of the most conspicuous qualities of a man in security.

(22) Corporation – An ingenious device for obtaining individual profit without individual responsibility.

(23) Condole – To show that bereavement is a smaller evil than sympathy.

(24) Grave – A place in which the dead are laid to await the coming of the medical student.

(25) Gunpowder – An agency employed by civilised nations for the settlement of disputes which might become troublesome if

pretence is not altogether false; Character can really be read very accurately in this way, for the wrinkles in every hand submitted plainly spell the word "dupe!" The imposture consists in not reading it aloud.

(6) April fool – The March fool with another month added to his folly

(7) Poetry – A form of expression peculiar to the land beyond the Magazines.

(8) Ass – A public singer with a good voice but no ear

(9) Pourtugese – A species of geese indigenous to Portugal. They are mostly without feathers and imperfectly edible, even when stuffed with garlic

(10) Apologize – To lay the foundation for a future offence

(11) Mausoleum – The final and funniest folly of the rich.

(12) Man – An animal so lost in rapturous contemplation of what he thinks he is as to overlook what he indubitably ought to be. His chief occupation is extermination of other animals and his own species, which, however, multiplies with such insistent rapidity as to infest the whole habitable earth and Canada.

(13) Impunity – Wealth

(14) Impostor – A rival aspirant to public honours.

Contains some of the "most gorgeous witticisms of the English language -

✓✓ Quite a beautiful book - worth getting parts translated into Kannada. - makes fun of American politics, business, religion, literature & arts

while reading through list of books published by Dover

✓ found one on spinning tops and gyroscopic motion by John Perry.

and also lots of other humorous books.

Examples. (1) Contempt - The feeling of a prudent man for an enemy who is too formidable safely to be opposed

(2) Hatred - A sentiment appropriate to the occasion of another man's superiority

(3) Harbor - A place where ships taking shelter from storms are exposed to the fury of the customs.

(4) Absentee - A person with an income who has had the forethought to remove himself from the sphere of exaction.

(5) Palmistry - The 947th method (according to Mumblestraw's classification) of obtaining money by false pretensions. It consists in "reading character" in the wrinkles made by the hand. The

Preface

Book designed to present a course in pure mathematics for the non-mathematicians - - - - - L. S. Venkataraman prepared the answers to the questions.

Chap 1 - Preliminaries

" 2 - Logic

" 3 - Calculus

" 4 - Probability

" 5 - Linear algebra

" 6 - Linear programming

" 7 - Abstract algebraic systems (Groups, rings, fields etc)

" 8 - Computers.

A lot of advanced mathematics - I wonder if any non-mathematician will really understand the topics dealt with. Thus Chap. 8 on computers which I read through made no sense at all except catching the words CPU (central processing unit), IO (input - out) device & Fortran (formula translation).

(2) The Devil's dictionary by Ambrose Bierce - Dover Publications, Inc. New York (1911 & 1958)

(7) A body ~~placed on a rough plane inclined to the horizon at an angle of 30° is just which~~ the mass of which is 4 lbs, rests in limiting equilibrium when the inclination of the plane ~~is 30°~~ to horizon is 30°. Find the force, which, acting ∥ to the plane, will support the body when the inclination of the plane to the horizon is 60°

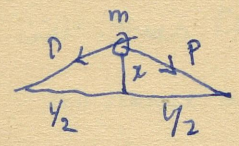
[Ans $\frac{4}{3}\sqrt{3}$ lbs]

(8) A steel ~~ball~~ bearing falls from a height h and bounces vertically and continuously until motion ceases. If coeff of restitution be e show that the distance traversed by the bearing is

$$\frac{1+e^2}{1-e^2}\,h.$$

(9) A particle m, attached to the middle point of a wire which is tightly stretched between two points with a tension P, find the period of small lateral oscillation

$$m\ddot{x} = -4Px/l \quad \left(= -2Px/\tfrac{1}{2}l\right)$$



$$\therefore T = \frac{2\pi}{n} \quad \text{where } n^2 = \frac{4P}{ml}.$$

$$= \pi\sqrt{\frac{ml}{P}}. \qquad [\text{Lamb}]$$

27/12/71 New Books Section

(1) Ideas in Mathematics by M. Evans Munroe
Durham, New Hampshire
(Addison Wesley, 1968)

Eqns of direct central impact

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad -- (1)$$

For perfectly elastic bodies

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad ---(2)$$

From (1) & (2), $v_1 - v_2 = -(u_1 - u_2)$ --- (3)

(1) & (3) are sufficient to find v_1 & v_2 given u_1, u_2 ($\&$ m_1, m_2)

For perfectly inelastic bodies

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \quad -- (4)$$

(4) is sufficient to find v given u_1 & u_2

For cases where e appears

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad --- (5)$$

$$v_2 - v_1 = -e(u_2 - u_1) \quad --- (6)$$

(5) & (6) give v_1 & v_2 given u_1, u_2 & e.

For case $e = 0$ (inelastic impact), loss of K.E $= \dfrac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$

(3) A billiard ball A at rest is struck by a second billiard ball B moving with a speed of 4 ft/sec. Assuming that the masses are equal, $e = 1$, and there is direct central impact, determine the speeds of the balls after impact [$v_A = 0$, $v_B = 4$ ft/sec]

[This is special case of (2) above]

(4) Show that when ~~their~~ there is direct central impact of two inelastic bodies moving in opposite directions with a speed ratio equal to the inverse of their mass ratio, the final speed is zero

(5) Two spheres A & B whose weights are 3 lb & 6 lb, having initial velocities +4 ft. sec & −3 ft. sec. Assuming d. C. impact and $e = 0.5$, determine velocities after impact

(6) A sphere is dropped from a certain height onto a floor which can be considered immovable. The sphere rebounds to a height equal to 0.64 times its original height. What is the coefficient of restitution

[$m_1 u_1 + M \cdot 0 = -m_1 v_1 + M \cdot 0$ i.e. $u_1 = -v_1$

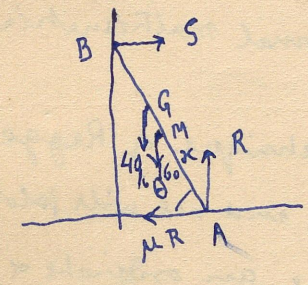
$0 - v_1 = -e(0 - u_1)$ $\quad v_1 = -e u_1$

i.e. $u_1 = e u_1$

$u_1^2 = 2gh_1$

$v_1^2 = 2gh_2$

$\frac{v_1^2}{u_1^2} = \frac{h_2}{h_1} = e^2$ $\quad e^2 = 0.64$ $\quad$ i.e. $e = 0.8$]



$\theta = 60°$, $AM = x$

$\mu = 0.5$

$40 + 160 = R$. vertical

$S = \mu R$. horizontal

$40 \cdot 10 \cos 60° + 160 \cdot x \cos 60° = S \cdot 20 \sin 60°$

$R = 200$

$S = 100$

$200 + 80x = 10\sqrt{3} \cdot 100$

$80x = 1000\sqrt{3} - 200$

$20 + 8x = 100\sqrt{3}$

$8x = 100\sqrt{3} - 20$

$= 173.2 - 20$

$= 153.2$

$x = 19.15$ ft

60 × 1.732

103.920

20

83.92

two points determine a straight line and in Regge theory, once a st. line, always a straight line. Therefore, as I grow older, Prof. P. is growing younger. I am very much looking forward to the next half billion seconds to test my theory by meeting Prof. P. again in another happy situation.

I think you will all join ~~with~~ me in expressing my ~~appreciation~~ appreciation of Prof. P's outstanding achievements, both in the past and in the future. May we all express our best wishes to Prof. P. on this very happy occasion"

The Appendix contains some kind of a "No play, with unrecognisable voices"

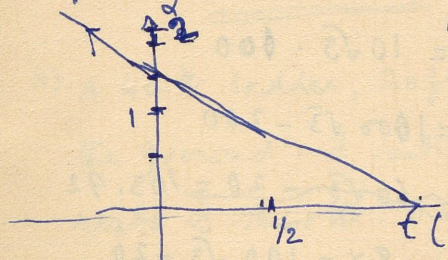
22/12/71 Questions on Statics & Dynamics for Andhra Extra two papers.

(1) A 40-lb ladder, 20 ft. long, makes an angle of 60° with the ground. A vertical wall against which the ladder rests is smooth. The coefficient of friction between the ladder and the ground is 0.5. How far up the ladder can a 160-lb man climb before motion impends?

(2) Show that if two perfectly elastic spheres with equal masses have direct central impact, they will interchange velocities

"Firstly, we know that this Conference covers the entire spectrum of theoretical physics. Prof. P. has worked in all these fields, has made brilliant, remarkable contributions, and Prof. P. has always been right. How can a person who works in all these fields make tremendous contributions and never a single mistake? The reason, of course, is that he knows what the answer is. Prof P. also remembers very well the past, so I believe that $\Delta = \Delta_R + \Delta_A$ is the correct Green's function. Do not be disturbed by the weight function being two (ie the coeffts of Δ_R & Δ_A are both unity). This is where the formal mathematical difference lies.

For a second proof, we use the new technique of Regge trajectory. We will consider the t-channel and we will plot in the adjoining figure an ordinate α. The t-channel now refers to time, and α is a ratio; the numerator is the age of Peierls and the



(I will just use myself as a guinea pig) denominator is my own age. I well remember when I first met Prof. P. — That was about half a billion seconds ago — we were both at the Institute for Advanced Study. At that time, this ratio α was two. At this time, I again meet Prof P. and the ratio is three-halves. These

* All fun, merely in a humorous vein!
My own excellent fun

equations and then to search for the different varieties of solutions. This variety of solns of the same eqn represents, then, the ~~multitude~~ multitude of varieties of different physical phenomena. I will give you an example; take the Navier-Stokes eqn in hydrodynamics. In one case you get laminar flow; change the initial or boundary conditions and you get turbulent flow; change them again, you get shock waves ... = The main activity of today's theoretical physics in which we, through our ingenuity, manage to do the opposite. — we fix the solution and, instead, change the problem. — gives some examples of talks in the Conference where this is done. — Re. the question raised in the first paper of Prof. Peirls's paper re. $t \to -t$, (Lee uses retarded & advanced potentials) says that it is possible, though under extremely rare circumstances, to have a highly intelligent being operating with $-t$ ie one who operates with an advanced ~~and~~ potential or a mentime $\Delta = \Delta_R + \Delta_A$? retarded & advanced potentials. Second it is possible, though again only under extremely rare circumstances, that such a highly intelligent being able to communicate with us. In fact, I strongly feel that the solution is a unique one. I will give you two proofs* to support my conclusion

a behaviour which could be accounted for by being due to a cause in the past. These two statements account for all the known irreversibility. The question whether they are independent or connected, and whether they can be reduced to something more fundamental, remains for the present unanswered.

(d) contains questions which Lee himself asks for.

✓(a) S. Mandelstam – S-matrix Theory

(b) About Regge poles by x, (c) Singularities of the S-matrix

(d) Scattering of unstable particles by Peierls (e) —

Section IV – Nuclear models and many-body theory — 4 papers

Section V – Symmetries, groups & algebras

✓(a) R. H. Dalitz – Some problems with the quark model for elementary particles – discusses non-relativistic quark model.

✓✓(b) Group theory & theoretical physics by H. J. Lipkin

✓(c) Recent progress in current algebra by S. Fubini

✓(d)* on the algebra of fields by T. D. Lee

Concluding lecture by T. D. Lee – It appears as if I have seen this type of lecture using ~ billions of seconds etc" somewhere else – Also gives something of a solution of Peierls problem about past & future

Contrasts main activity of the theoretical physicists of the old days with those of the present day. In the old days, in classical physics, one used to think that the main activity of theoretical physics was to search for a fundamental set of

(*) Problem about intuitive notion about causality that the cause always precedes the effect in time. This very deeply ingrained prejudice is not a necessary part of causality as it appears in classical or quantum physics. One may ask whether there is some fundamental principle involved here which should be added to the laws of physics, so that our intuition would indeed be an important truth or whether our intuition – as so often – merely reflects our experience so that the prejudice about causality would simply be the result of the two statements† I have made above.

(†) The situation is then that in any non-equilibrium situation created by our own interference we are always concerned with the results of an ~~our~~ interference which has taken place in the past. In the non-equilibrium state of the universe around us we see

[See 49(a)]

---- To avoid misunderstanding, it should be clear from the outset that my comments & remarks have no connection with the recent doubts about time-reversal invariance in weak interactions; or if there is any such connection I am not aware of it ----. I have seen that Feynman in his 1964 Cornell lectures (The Character of Physical Law by R. P. Feynman, Lecture 5, (BBC Publications, 1965) discusses the same problem (*), with very similar conclusions

A popular type of ~~book~~ paper (or rather a philosophical type) - talks of the way our mind works & existence of intelligent beings on other planets whose mind works in the opposite direction. This idea would lead to contradictions, at least if we were able to communicate with such beings, because they would remember what happens in the future and could decide today what to do at some time in the past. ---

The three other papers in this Section relate to statistical physics and the last one is

Solid State astrophysics by E.E. Salpeter

Section II - Excitations and reactions - 6 papers

Section III - Elementary particles & field theoretical methods

* Should read this & make a ~~non~~ model

✓ || Also show K.V's regular polyhedra at a public lecture

(*) dealing with Statistical Physics & changes of State.

(2) A souvenir from Paris* by H. Martyn Cundy, p. 310

Many souvenirs for the New Year were polyhedra. I was particularly impressed by a lamp shade which was a feature of many of the decorations. It was in the form of an ~~co~~ icosidodecahedron, the pentagonal faces being dimpled, and with "petals" added so that the whole resembled the flower of a Golden Chalice (Solandra grandiflorai). Eventually I found a little shop by Notre Dame which would sell me one. I found myself the purchaser of a small box containing 20 strips of card, about 35 cm by 7 cm, which I was assured would make up into the lampshade I required.

20/12/71 New Books Section

(1) Methods and problems of Theoretical Physics in honour of R. E. Peierls on his 60th birthday (Edited by J. E. Bowcock)
Papers read during Conference organised by Dept. of Math. Phys. Birmingham Univ — North Holland, 1970
(June, 1967)

Section I* - Time reversal in the second law of thermodynamics by R. E. Peierls.

that the rational view would be to take the mean of the two values ie $\frac{1}{2}$.

But consider on the same reasoning

$$1-2+3-4+\cdots = (1-2)+(3-4)+(5-6)+\cdots$$
$$= -1-1-\cdots$$

and $1-2+3-4+\cdots = 1-(2-3)-(4-5)-(6-7)-\cdots$
$$= 1+1+1+\cdots$$

It is hard to see how any argument about averaging could give the answer $\frac{1}{4}$ for this series.

On the other hand, Consider G.P with C.R $= 2$

$$1+2+2^2+2^3+\cdots = (1+2)+(2^2+2^3)+(2^4+2^5)+\cdots$$
$$= (1+2)(1+2^2+2^4+\cdots)$$

and "
$$= 1+(2+2^2)+(2^3+2^4)+\cdots$$
$$= 1+(2+2^2)(1+2^2+2^4+\cdots)$$

using $1+2^2+2^4+\cdots = \frac{1}{1-2^2} = -\frac{1}{3}$.

we get in both the cases the previous result $1+2+2^2+\cdots = -1$

The odd result concerning sum of negative numbers by positive series requires a sophisticated theory. The appropriate theory is based on the analytic continuation of fns of a complex variable — — deals with notions of summability — —

* $\frac{1}{2}(2+2^2) = \frac{1}{2}(1+2)\cdot 2$

$\frac{1}{2}(2^2+2^3) = \frac{1}{2}(1+2)2^2$, ...

Nevertheless

$$S = 1+2+2^2+2^3+2^4+\cdots = 1+2(1+2+2^2+2^3+\cdots)$$
$$= 1+2S$$

ie $S = -1$

The Euler transformation applied to this series gives

$$S = \tfrac{1}{2} + \tfrac{1}{2}(1+2) + \tfrac{1}{2}(2+2^2) + \tfrac{1}{2}(2^2+2^3) + \cdots \quad (*)$$
$$= \tfrac{1}{2} + \tfrac{1}{2}(1+2)(1+2+2^2+2^3+\cdots)$$
$$= \tfrac{1}{2} + \tfrac{3}{2}S \quad \text{ie } S = -1$$

Also formula for G.P gives sum $= \frac{1}{1-r} = \frac{1}{1-2} = -1$

In spite of its peculiarity, it displays same kind of consistency as the others. Thus the Cauchy product

$$(1+2+2^2+2^3+\cdots)(1+2+2^2+2^3+\cdots)$$
$$= 1+2\cdot 2+3\cdot 2^2+4\cdot 2^3+\cdots$$

L.H.S $= (-1)(-1) = 1$, R.H.S $=$ A-G progression with $a=d=1$, $r=2$

$$\text{hence} = \frac{1}{-1} + \frac{1\cdot 2}{(-1)^2} = 1$$

The above calculations give consistent results. On the other hand the following calculations lead to other results

$$1-1+1-1+\cdots = (1-1)+(1-1)+(1-1)+\cdots = 0$$

and $$1-1+1-1+\cdots = 1-(1-1)-(1-1)-\cdots = 1$$

ie assuming <u>even</u> & <u>odd</u> number of terms. 18C. mathematicians thought

$(a_0 + a_1 + a_2 + \cdots)(b_0 + b_1 + b_2 + \cdots) = a_0 b_0 + (a_0 b_1 + a_1 b_0) + (a_0 b_2 + a_1 b_1 + a_2 b_0) + \cdots$

In accordance with this rule

$$(1 - 1 + 1 - 1 + \cdots)(1 - 1 + 1 - 1 + \cdots)$$

$$= 1 - 2 + 3 - 4 + 5 - 6 + \cdots$$

Which is consistent since L.H.S $= \frac{1}{2} \cdot \frac{1}{2} =$ R.H.S $= 1/4$.

Again $(1 - 2 + 2^2 - 2^3 + \cdots)(1 - 2 + 2^2 - 2^3 + \cdots)$

$$= 1 - 2 \cdot 2 + 3 \cdot 2^2 - 4 \cdot 2^3 + \cdots$$

i.e. $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ showing consistency

Again $(1 - 1 + 1 - 1 + \cdots)(1 - 2 + 2^2 - 2^3 + \cdots)$

$$= 1 - 3 + 7 - 15 + 31 - 65 + \cdots$$

The R.H.S. can be shown by using Euler's transformation method to be equal to $1/6$ & L.H.S $= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$.

The above consistency dulls the ~~result~~ impression that each individual repn. of a number by such a series is distinctly peculiar. The sense of unease returns when the techniques used above are applied to G.P's with C.R $= 2$. Is it not ridiculous to suppose that a series of $+^{ve}$ terms can represent a negative number?

$S = \frac{1}{2} + \frac{1}{2}(-S)$ $\quad S = \frac{1}{2} + \frac{1}{2}kS$ $\quad S\left(1 - \frac{1}{2}k\right) = 1/2$

$S = \frac{1 \cdot 2}{2(2-k)} = \frac{1}{4}$ $\quad \frac{2-k}{2}$

$4 = 2 - k$

$k = -2$

(*) Using Euler's transformation

$S = 1 - 2 + 3 - 4 + \cdots = \frac{1}{2} + \frac{1}{2}(1-2) - \frac{1}{2}(2-3)$

$+ \frac{1}{2}(3-4) - \cdots$

$= \frac{1}{2} + \frac{1}{2}\{1 - 2 - \cdots$

(*) (*).

The formula for a G.P. with common ratio $= -2$ gives $s = \frac{1}{1-(-2)} = 1/3$

Again the Euler transformation here takes the form

$$s = 1 - 2 + 2^2 - 2^3 + 2^4 - \cdots$$
$$= \tfrac{1}{2} + \tfrac{1}{2}(1-2) - \tfrac{1}{2}(2-2^2) + \tfrac{1}{2}(2^2 - 2^3) - \cdots$$
$$= \tfrac{1}{2} + \tfrac{1}{2}(-1 + 2 - 2^2 + 2^3 - \cdots) = \tfrac{1}{2} + \tfrac{1}{2}(-s)$$

again giving $s = 1/3$.

If arithmetic-geometric progressions are studied by the method which was used above for geometric series, one obtains the formula

$$a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \cdots = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

For the progression for which $a = d = 1$, $\& r = -1$, we get

$$1 - 2 + 3 - 4 + \cdots = 1/4 \quad \circledast$$

while for $a = d = 1$ $\& r = -2$, we get

$$1 - 2\cdot 2 + 3\cdot 2^2 - 4\cdot 2^3 + \cdots = 1/9 \quad \circledast\circledast$$

These results can be confirmed by use of the Euler transformation. What is interesting is that these are consistent with the previous results on G.P's

To show consistency between results for the four series considered above we must look at products of series. Ordinary process of multiplying polynomials

$$(a_0 + a_1 x + a_2 x^2 + \cdots)(b_0 + b_1 x + b_2 x^2 + \cdots)$$
$$= a_0 b_0 + (a_0 b_1 + a_1 b_0)x + (a_0 b_2 + a_1 b_1 + a_2 b_0)x^2 + \cdots$$

This rise to Cauchy's rule for multiplication of series

15/12/71. Mathematical Gazette Vol LV, No. 393, June 1971

p. 298. $1-1+1-1+\cdots = 1/2$? by F. Rhodes.

Good for a popular lecture.

(1) Different Cavalier calculations will all give the same answer

$$s = 1-1+1-1+\cdots = 1-(1-1+1-1+\cdots) = 1-s$$

of which only soln is $s = 1/2$

(2) Consider G.P. $s = 1 + r + r^2 + r^3 + \cdots$

Then $rs = r + r^2 + r^3 + r^4 + \cdots$

∴ by subtraction $s - rs = 1$

∴ so long as $r \neq 1$, this gives $s = 1 + r + r^2 + \cdots = \dfrac{1}{1-r}$

In this case $r = -1$ ∴ hence $s = 1/2$

(3) Using ~~another~~ transformation due to Euler

$$v_1 - v_2 + v_3 - v_4 + \cdots = \tfrac{1}{2}v_1 + \tfrac{1}{2}(v_1 - v_2) - \tfrac{1}{2}(v_2 - v_3) + \tfrac{1}{2}(v_3 - v_4) - \cdots$$

This transformation can be applied to some slowly converging series to obtain series which converge more rapidly. In our example all $v_i = 1$ ∴ so transformation gives

$$1-1+1-1+\cdots = \tfrac{1}{2} + \tfrac{1}{2}(1-1) + \tfrac{1}{2}(1-1) + \cdots = 1/2$$

Techniques (1) – (3) above can be applied to other G.P's.

For eg: $s = 1 - 2 + 2^2 - 2^3 + 2^4 - \cdots$

$= 1 - 2(1 - 2 + 2^2 - \cdots) = 1 - 2s$ i.e. $s = 1/3$

(4) Communications Physico-Mathematical – Looked at the several numbers available.

(a) Linear theory of gravitation without potential
by Paul Kustaanheimo, Vol. 32, No. 10, 1966

CSIR ‖ (Might be useful when sending C. S. I. R. Report

(b) Studies in the Principles of Superposition in quantum mechanics
do. ‖ by Matts Roos. Vol 33, No. 1, 1966
– Looks quite an elementary paper

(c) Classification of Lorentz transformations by J. Segercrantz.
Vol 33, No. 5, 1966

(d) On the use of a ruby laser for exciting Raman scattering from coloured powders by F. Stenman, Vol. 33, No. 8, 1967

(e) Displacements of a rigid body by D. Bottema. Vol 33, No. 10, 1968

(f) Hamilton-jacobische Theorie und Systeme partieller Diffgl.
Vol 34, No. 14, 1969

(g) – Also Vol 34, No. 15, 1969

(5) Proc. Camb. Phil. Soc. Vol 70, Part 2, Sept, 1971
p. 343 – Article by D. S. Chandrasekhariah of the CEB Math. Dept on some types of m-t-v plane waves

an indication of hurry in getting the volume near

(8) Singular interactions in non-relativistic quantum theory by H. H. Aly, W. Guttinger & H. J. W. Muller.

(9) — do — in quantum field theory by — do —.

(10) Weak interactions in the nuclear shadow by J. S. Bell

(11) On the composite structure of leptons and the existence of the μ-meson by A. O. Barut (Univ. of Colorado), pp. 341-45

✓✓✓ Discusses the electron-muon puzzle, a subject in which I am greatly interested and want to work

Read this through completely.

Reference to a paper by B. Kursunoglu, Phys. Rev. 167, 1452 (1968)

where an example is given using the 6-dimensional rep^n of the group SU(3,1). His equations also predict two new spin 3/2 leptons — Connection with my 3/2 spin?

Mentions that the consistent quantisation of the second-order Gellmann-Feynman equation implies that every fermion must have its own neutrino. [fermion? is this correct]

(12) Phenomenology of K_{l_3} decay by C. Ryan — Also on weak interactions.

(3) Reliability Mathematics by B. L. Amstadter

— Statistics, management, confidence, probability etc

(*) Hadi Hussain Aly (Pakistani or Bangla Deshi ?)

[B.S & M.S. at Univ. of Calif, Berkeley & his Ph.D at Univ. of Bristol, England – So obviously the former alternative]

a book certainly worth reading through – certainly useful for writing a popular text-book on relativity.

(2) Lectures on particles and fields – Edited by H.H. Aly (Southern Illinois Univ, Edwardsville) (Gordon & Breach, 1970)

(1) Photoabsorption sum rules and non-relativistic quark model by G. Barton.

(2) Strong-interaction sum rules – Intro to recent developments by G. Costa and Graham Shaw.

(3) Asymptotic symmetry, Sum Rules and renormalisation constants by H.H. Aly and P. Narayanaswamy (same Univ)

(4) Some aspects of spectral function sum rules by ~~Fayyazuddin and Riazuddin (Univ of Islamabad, Rawalpindi, Pakistan)~~ by R. Acharya (Univ. of Berne, Switzerland)

(5) The Veneziano model for meson systems and its connection with Chiral symmetry by

(6) Phenomenological hadron couplings by A. N. Mitra (Univ. of Delhi, India) – Long paper

(7) Field theories with indefinite metric by J. G. Taylor

Note – Matter of (6) & (7) have been mixed up in printing, perhaps

(f) Projective Geometry & the 4-color problem by W. T. Tutte

3rd Waterloo Conf. on Comb. pp. 193-199-207

vide his paper in J. Comb. Theory 1 (1966), 15-50 - Lecture notes based on this paper.

Dec. 1970

a The achromatic number of a graph by F. Harary & S. Hedetniemi

J. Comb. N. 8 (1970) pp. 154-61

Introduces a new invariant called achromatic number.

Ref. to paper by R. P. Gupta (Waterloo Conf. pp. 229-35)

13|12|71 - Stopped collecting literature on 4-color problem in view of G. N. R. cancelling his engagement with me.

New Books Section

(1) talking about relativity by J. L. Synge (North-Holland, 1970)

Elementary book for educated people - Treats general relativity first & then goes to special relativity (rather unusual) - A long and ununderstandable preface - 15 Chapters - The last Chapter on particles in collision appears the most interesting - Anyway

Two faces of a planar map are "contiguous" if they have at least one boundary vertex in common. A planar map is "angle colored" if its faces are partitioned into classes such that no two members of the same class are contigous. Dually, a "cyclic (vertex) coloration" of the 1-skeleton of ~~classes~~ a planar map is a partition of its vertices into (color) classes such that no two vertices in the same class lie in the boundary of any one face; the "(cyclic) coloration" number is the minimum number of color classes. Bounds for this number are obtained.

(c) Remarks on the Heawood Conjecture by G. Ringel and J. Youngs

Vide their paper in Proc. Nat. Acad. - Improvement of proof

(d) Sur le problème des quatres couleurs by Pierre Robert

C. R. Acad. A-B. 269 (1969) A937-A939

The author states a conjecture which, if true, would imply the truth of the 4-color problem. Some other well-known observations are made.

(e) Even & odd 4-colorings by W. T. Tutte

Ann Arbor Conf. on Graph Theory, 1969, pp. 161-69

* $\chi(G)$ = Chromatic number of a graph G

S_n $(n = 0, 1, \cdots)$ = an orientable 2-dimensional manifold of characteristic n and $\chi(S_n)$ = the maximum of $\chi(G)$ for all possible finite graphs G of S_n.

Nov. 1970. on dividing a square into triangles by P. Monsky

Amer. Math. Monthly 77 (1970), 161–64

The author gives a slick, easy (but not elementary) proof that a square S cannot be partitioned into an odd number of triangles of equal area.

ist, während die entsprechende Frage $\chi(S_0^*) = 4$ (36)
(Vierfarben problem) noch ungelöst ist).

November, 1970.

(a) Higher dimensional analogues of the 4-color problem and some inequalities for simplicial complexes by B. Grünbaum

J. Combinatorial Theory 8 (1970), 147-53

Theorem). For $d \geq 1$ each simplicial d-complex imbeddable in the Euclidean $(d+1)$ space E^{d+1} is properly colorable by $6d$ colours.

For $d = 1$, this reduces to the assertion that the vertices of each planar graph are colorable by 6 colors, which is weaker than the well-known theorem about the 5-colorability of every such graph.

Cor: Th. 2: The 2-faces of every simple $(d+2)$-polytope P may be colored by $6d$ colors in such a manner that 2-faces with a common edge have different colors

(b) On the coloration of plane graphs by O. Ore and D. M. Plummer

Recent Progress in Combinatorics (Proc. 3rd Waterloo Conf. on this topic, 1968) [Acad. Press, 1969], pp. 287-93.

October, 1970.

(a) Sur le problème des quatres couleurs by G. Benneton

C. R. Acad. Sci A–B. 266 (1968), A 862 – A 863

The two papers contain a summary (with no proofs or references) of some known facts concerning the structure of planar maps and the 4-color conjecture.

(b) On the chromatic number of permutation graphs. by G. Chartrand and J. B. Frechen [Proof techniques in Graph Theory (Proc. Second Ann Arbor Graph Theory Conf. Ann Arbor Mich. 1968, pp. 21–24) – Acad. Press, N.Y. 1969]

(c) An analogue to the Heawood map-colouring problem by H. V. Kronk [J. Lond. Math. Soc (2) 1 (1969, 750–52]

– Ref. to work of G. Ringel & J. W. T. Youngs [Proc. Nat. Acad. Sci. U.S.A, 60 (1968), 438–455; MR. 37 # 3959] re. the formula $\chi(S_n) = \left[\frac{1}{2}(7+\sqrt{(1+48n)})\right]$ and proves in the note an analogous eq^n $p(S_n)$ [a variant of $\chi(S_n)$] $= \left[\frac{1}{4}(9+\sqrt{(1+48n)})\right]$ for all $n > 1$.

Der Autor weist darauf hin, dass $p(S_0) = 3$ bewiesen

10/12/71 Had a letter from G.N.R. today cancelling the engagement re. the 4-color problem on 17/12/71 (Thank God!) – went to Inst. Office & saw Asstt. Regr & concerned clerk to send a reminder to CSIR in suitable terms – Also walked a long distance to E.C. Deptt. Saw B.S. Ramakrishna – got photographed there to get a permit to enter the Inst. from tomorrow onwards – Also borrowed Ore's book – Decided to drop work on 4-color problem, but thought I had better finishing references in Math. Reviews from July 70 – Dec 70.

e) Math. Reviews – August 1970 (contd)

(e) Konstruktion von Graphen mit gewissen Färbungseigen-eigenschaften by A. Sachs & M. Schauble.

Same From Colloquium on Graph Theory, Mannebach, 1967 pp. 131–36 – Teubner, Leipzig, 1968.

Complicated paper – Reference made to "Beiträge zur Graphentheorie" by L. Lovasz in same colloquium pp. 99–106

f) Bemerkungen zu einem Kantenfärbungsproblem by M. Schauble – Same Colloquium pp. 137–142

– Shows connection of 4-color problem with other types of coloring problems.

Ibid – September 1970. – nil.

Mannheim - Vienna - Zurich, 1969, 290pp, D.M. 9.90.

A reducible configuration is one which cannot occur in a non-4-colorable map with a minimal number of regions. The author classifies and presents the previously known reductions and adds many new ones. These are used to prove the following difficult theorem: A map in which no more than 3 regions meet at a point and which contains no hexagons and no three mutually adjacent 7-gons is reducible

The four-colour problem would be proved if we had a set of reducible configurations and a proof that every map has to contain at least one of them. — classification of maps based on value of ρth-curvature (rational no. assigned to each region so that the curvature summed over the whole ~~region~~ sphere is 2 for each ρ. It is assumed, as usual, that no more than 3 regions meet at a point. — — — The book provides a starting point for those who want to push the theory of reductions ~~towards~~ forward by the use of computers. It also happily complements the book of the late O. Ore on the 4-color problem [The four-color problem, Acad. Press, N.Y. 1967; M. R. 36 #74]* in which broader horizons were explored but reduction theory was left essentially where it was 30 years ago.

(2) August, 1970.

(a) R. Halin - A theorem on n-connected graphs

(b) D. Kotzig - Eulerian lines in finite 4-valent graphs and their transformations.

(c) K. Hauschild - Beiträge zur Graphentheorie (Kolloquium Manebach 1967), pp. 61-670, Teubner, Leipzig, 1968

- Über ein Färbungsproblem auf der Kugel

Let M be a finite connected map on the sphere without loops or isthmuses. Then the following theorem holds: If M is vertex-regular of degree 4 and if the number of vertices of every region is a multiple of three, then there exists a 4-coloring of the regions in such a way that two regions already receive a different color if they have a vertex in common (in general, six colors are needed for a coloring of this type, as was shown by Ringel). The author gives several related results, the most interesting of which is:

Let M be a normal map on the sphere and F a 1-factor of M. If the number of edges outside F of every region of M is even, then M admits a 4-coloring of the regions.

(d) * Untersuchungen zum Vierfarbenproblem by Heinrich Heesch

B.I. Hochschulskripten, 810/810a/810b - Bibliographisches Inst.

(*) Brooks' Theorem as in Camb. Phil. Soc (which I looked up)

Let N be a network (or linear graph) such that at each node not more than n lines meet (where $n > 2$), and no line has both ends at the same node. Suppose also that no connected component of N is an n-simplex. Then it is possible to colour the nodes of N with n colours so that no two nodes of the same colour are joined (Proofs for finite & infinite networks)

(11) Notes on the Schwarzschild line element by Florides & Synge.

– Quite an instructive article., 1961

(12) Introduction to the group theory of elementary particles by J. Mc Connell, 1965

– I did not know of the existence of this communication – Shall be read quite soon as a preliminary study

(13) The orthogonal and symplectic groups by F.D. Murnaghan

– To be read later.

Some results on 4-color problem from Math. Reviews – Section on Combinatorial theory & graph theory :-

(1) July, 1970

(a) Finck & Sachs – Math. Nachr. 39 (1969), 373-380 – It is shown that a planar graph G can be 4-coloured if neither G nor any of the graphs obtained by successively removing nodes of minimal degree contains more than 19 triangular faces.

(b) J. Ponstein – J. Comb. Theory 7 (1969), 255-57

A new clear proof is presented of the theorem of R. L. Brooks (Proc. Camb. Philos. Soc. 37, 1941, 194-197)

appears mysterious & not very clear. — Dalitz on p. 594 of the "top quantum number of the Universe as $2 \cdot 607 \times 10^{26}$"

(3) Development in quantum electrodynamics by P. A. M. Dirac, 1946.

(4) Cosmic rays by by L. Janossy — There are two references to the papers by Bhabha & Chakrabarty & Bhabha & Heitler.

(5) Defn of C. G. of an arbitrary closed system in the Theory of relativity by C. Møller, 1949

Quite an interesting paper.

(6) Studies in the non-symmetric generalisation of the Theory of Gravitation by E. Schrödinger, 1951.

(7) Jump conditions at discontinuities in general relativity by O'Brien & Synge, 1952

(8) Papers read at the Cosmic ray Colloquium, Sept. 1951

(9) On relativistically rigid surfaces of revolution by J. P. Pownder, 1954

(10) Geometrical Optics in moving dispersive media by J. L. Synge, 1956.

Theory, (5) Hilbert Space (6) Brief survey of

Quantum Mechanics

The topics of the last chapter are

(i) The fundamental postulates of quantum mechanics

(ii) Expectation values.

(iii) The uncertainty principle; compatible & incompatible observables.

(iv) Projections and the propositions of a physical system.

(v) Quantum Dynamics. (Schrödinger & Heisenberg methods)

~~Quite~~ A good elementary mathematical textbook.

✓✓ May be used when I write a text-book on the subject (God knows when!)

8|12|71 — Writing with the new pen

Some issues of the Dublin Instt. of Advanced Studies:

(1) Quantum Electrodynamics by P. A. M. Dirac, 1943

(2) The combination of relativity theory and quantum theory by Sir Arthur Eddington, 1943.

In Part IV deals with general relativity, but the whole approach

* Hence no mention of Raman Effect or Compton effect.

translated by V. Barocas (Heinemann, London).

– No reference to modern ideas* including relativity and quantum mechanics – At the end gives some ideas of the physiology of vision (a subject dealt with by Raman)

(E) Mathematical methods of Quantum Mechanics by Gaetano Fano (Univ. of Bologna) (Editor of English translation Leon E. Landovitz (Belfer Graduate School of Science, Yeshiva Univ) [McGraw Hill, 1971]

In the preface to the Italian Edition gives von Neumann's remark on Dirac's treatise on quantum mechanics: "Dirac . . . has given a representation of quantum mechanics which is scarcely to be surpassed in brevity & elegance. . .. The method of Dirac (and this is overlooked today in a great part of ~~mathematical~~ quantum mechanical literature, because of the clarity & elegance of the theory) in no way satisfies the requirements of mathematical rigor . . ."

Chapters on (1) finite dimensional linear spaces and matrices, (2) Metric properties of finite dimensional linear spaces, (3) Linear spaces & topological spaces, (4) Measure

(14) M. Kugler – Duality.

(15) D. Hamm – Some remarks on the Quark model.

(16) W. Kuhnhur – Anomalies in Ward identities?

(17) H. Pletschmann – Structure effects in weak interactions

(18) Doebner & Palev – About Lie Algebras [No. ref. to Harish Chandra]

(19) J. Tolar – About SU(3) – symmetry.

(20) D. Atkinson – Summary – First week.

(21) H. Pletschmann – " – Second week.

A large number of interesting papers

Also. references to Acta Physica Austriaca Publications in the back cover of the page. ©

(1) Weak interactions & higher symmetries – 1964

(2) Quantum Electrodynamics – 1965

(3) Elementary Particle Theory – 1966

(4) Special Problems in High Energy Physics – 1967

(5) Particles, Currents & Symmetries – 1968

(6) Particle Physics, 1969

(D) The nature of light – A historical survey by. Vasco

Ranchi

(*) Looks quite an elementary type of paper, quite interesting
– to be read soon

P. Urban (Springer-Verlag, 1970)

Contents:

(1) P. Urban — Introduction

(2) A. Salam — Non-polynomial Lagrangian Theories

(3) D. Atkinson — Intn to the use of non-linear techniques in S-matrix theory.

(4) A. Martin — Some exact results on $\pi\pi$ scattering.

(5) B. Renner — Aspects of Chiral Symmetry.

(6) P. V. Landshoff — Some recent work in Regge Theory: Regge cuts, the absorption model & Glauber Theory.

(7) P. V. Landshoff — Venziano-like model for a two-current amplitude.

(8) G. E. Hite — Absorption ~~coefficients~~ corrections and cuts in the Regge Pole model.

(9) R. J. N. Phillips — Lectures on Regge phenomenology.

(10) E. Hlawka — Differentiable manifolds.

(11) R. V. Seed — General Relativity & gravitational collapse (p. 308) *

(12) L. Streit — The construction of physical states in quantum field theory

(13) W. Ruhl — The $O(3,1)$ Analysis of Currents & Current Commutators.

(19) Inequalities for generalised convex functions by Zvi Ziegler.
pp. 409 - 422

(20) On certain lemmas of Marcinkiewicz & Carlson by A. Zygmund
pp. 423 - 431

on p. (xi) are given the contents of Vol I of Inequalities edited by same author & published by Acad. Pres. - Altogether 14 papers - ~~Contents of the~~ Names of articles not given but only names of authors.

(B) Almost Periodic functions and functional equations
by Amerio & Prouse.. (Van Nostrand Reinhold Co., 1971)

One the books of the University Series in Higher Maths. ~~published by~~ Editorial Board : M. H. Stone - Chairman, L. Nirenberg & S. S. Chern

[The series consists of books by Halmos, the 3 vols of Jacobson and the two ~~three~~ vols of Varadarajan on the Geometry of Quantum Theory]

(C) Developments in High Energy Physics - Proc. Int. Conf. 1970 in Austria 23/2 to 7/3/70. edited by

(10) Another paper by same author pp. 251-254

(11) Singularities of the n-body problem by Harry Pollard and Donald G. Saari, pp. 255-259

(12) Integral inequalities with boundary terms by Ray Redheffer pp. 261-290

(13) Perturbation of zeros of analytic functions I } by
(14) " II } by Paul C. Rosenbloom, pp. 293-308 & 309-334

(15) Cardinal interpolation and spline functions by I.J. Schoenberg pp. 335-374

(16) On two complementary variational characterisations of eigenvalues by William Stenger, pp. 375-386

(17) +ve definite matrices & their role in the study of the characteristic roots of general matrices by Olga Taussky pp. 389-399

[Appears to be the only partly understandable paper in the whole series]

(18) Inequalities expressing degree of convergence of rational functions by J.L. Walsh, pp. 401-407

be small. Perhaps the best known statement of it is the inequality

$$\|f'\|_\infty^2 \leq 4 \|f''\|_\infty \cdot \|f\|_\infty$$

proved by Landau & given in Hardy & Wright. Another gives a generalisation of this inequality

(6) Metric inequalities and symmetric differences by John. B. Kelly pp. 193 – 212

(7) Dependence of some classical inequalities on the index set by H.W. McLaughlin & F. T. Metcalf. pp. 213 – 222

Mention the relation of inequalities proved in the paper as special cases of inequalities relating to convex functions. for eg. if f is convex ie

$$f\left(\frac{x_1+x_2}{2}\right) \leq \frac{f(x_1)+f(x_2)}{2}$$

for all $x_1, x_2 \in E$. Then [Hardy – Littlewood – Polya (14), pp. 71–72], for any rational number λ, such that $0 \leq \lambda \leq 1$, one has

$$f(\lambda x_1 + (1-\lambda) x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$$

for any $x_1, x_2 \in E$.

(8) Inequalities for submatrices by Marvin Marcus pp. 223 – 239

(9) Difference and ratio inequalities in Hilbert Space by T. S. Motzkin pp. 241 – 248

citizens working together, or for evil to set in motion the destroying juggernaut of the totalitarian state. But one thing cannot be done with the spirit of the age – and that is to ignore it, to repress it or pretend it isn't there"!

6|12|71 New Books Section

(A) Inequalities – Edited by Oved Shisha – Proc. 2nd. Symp. on Inequalities held at the U.S. Air Force Academy, Colorado, August 14–22, 1967 (Acad. Press, 1970) Vol II.

(1) Some integral inequalities by N. Aronszajn, pp. 2–148.

(2) Meromorphic plane maps viewed from space by E.F. Beckenbach pp. 149–162.

(3) A metric space connected with generalised means by G.T. Cargo and O. Shisha (pp. 163–177)

(4) Asymptotic cones and duality of linear relations by Ky Fan pp. 179–186.

(5) On the inequality $\|f'\|^2 \leq 4\|f\| \cdot \|f''\|$ by Robert R. Kallmann and Gian-Carlo Rota.

Introduction: If a function is small, and its second derivative is small, then its first derivative must also

1,000,000

100 million

$\frac{120 \times 8}{960}$ = 960,000,000

[ie 96 crores per year]

"that I will not participate in war research or weapons production. I further pledge to counsel my students and colleagues to do the same".

Since World War II, MIT has been a major centre of military and space research. Its affiliated "special laboratories", the Instrumentation Laboratory and the Lincoln Laboratories had reached a budget of $120 million a year or half the total MIT budget.

pp. 144 - 152 - The Federal Government and the Scientific Community by Philip Handler

- A very interesting article dealing with questions like - Political commitment to Education & Research - Decline of Scientific Enterprise - Public attitudes and public support - Science & the Academic Community - Federal Research Support - Is Science "Relevant"? - The environment - A new profession - Multidisciplinary research on Campus - Biomedical research & Public Health - Public understanding & the scientific endeavour.

p. 144. Starts with a quotation from Lady Jackson - Barbara Ward saying that "the forces of change unleashed on the world in the last century can be used for good or evil. For good to create a dynamic society of free

pp(43) (a) Herbicides in Vietnam: AAAS. Study finds
pp. 43-49 widespread devastation. by Philip. M. Bowey.

(b) AAAS Convention: Radicals harass the Establishment
also by Philip M. Bowey.

✓ Most interesting articles showing the brutality of the American Army's crop destruction program - Gives full details of this and also other

p. 48 under (b) in italics at the beginning)

or Science, Technology. We declare its use a sham. And subject all who use it ill to the witches' damn - ~~Aton~~ A hex pronounced on the AAAS Convention by the Women's International Terrorist Conspiracy from Hell (WITCHES)."

- Read both (a) & (b) through & found it a sordid tale of American cruelty.

15/1/71

p. 156 - Visitors ask M.I.T. Faculty to renounce military research

The SESPA (Scientists and Engineers for social and political action) demonstrators (of a national organisation devoted to ending military research projects) went to the MIT on 8/1/71 & passed out leaflets asking the MIT students & faculty to sign a pledge

p. 289. "Intelligence without integrity will fail"

"The means rather than the ends of life are in our command."

"Man requires a challenge and a quest if he is avoid boredom. The Golden Age toward which we move will soon look tawdry if we no longer see endless horizons. We must then, seek a change within man himself. As he acquires more fully the power to control his own genotype and to direct the course of his own evolution, he must produce a Man who can transcend his present nature".

pp. 36 – 42 - Magnetic resonances and waves in simple metals by W. M. Walsh. Jr.

Transparencies of alkali metals near cyclotron and spin resonances reveal many-body interactions

Intn – Simple metals – Wave propagation and reflection – The helicon

{ The case where the electric (or magnetic) field vector of the solid-state plasma waves traces a spiral path into the metal, it is known as the helicon – cyclotron waves – Explicit many-body interactions – Correlation dependent modes

Nice technical paper.

Michigan U.S.A – He told me he has a portable 6" telescope & is now doing theoretical work on the planet Mars, is a good friend of Feynman; he also mentioned my rejoinder to G.H.R. about Feynman at some previous lecture.

Extremely interesting article which I read through completely, where the author supports the view that we are heading towards an age where further progress is not possible ie we may reach a Golden Age of no progress. Hopes that this might only be a passing phase & history may ~~resume~~ resume &

p. 27 – " Let me suggest, at the risk of grave misunderstanding, that in future histories of the world, the decade of the 1960's may be known not significantly for the miserable Vietnam war but as the time when man, with unbridled lust for power over nature and for a so-called high standard of living measured by the consumption of the products of an industrial civilisation, set in motion the final speedy inexorable rush toward the end of progress".

p. 27 " Storage & retrieval of knowledge not undertaken by using computers. " Scientific information has increased a hundredfold since 1900, but most of it is already forgotten. The next age of scholarship will no doubt professors and analysts who need only to delve in the mountains of extant scientific and technological literature for forgotten and uncomprehended items of knowledge"

we have $y' = \lambda \cdot x^{\lambda-1}$, $y'' = \lambda(\lambda-1)x^{\lambda-2}$

leading to $\lambda(\lambda-1)x^{\lambda} + \lambda x^{\lambda} - x^{\lambda} = 0$

i.e. $\lambda^2 - 1 = 0$ i.e. $\lambda = \pm 1$

Giving the C.F, $y = Ax + Bx^{-1}$ or $Ax + \frac{B}{x}$.

---

(2) India - Studies in social and political development 1947-67 [Edited by A. Appadorai - Pandit Hriday Nath Kunzru 80th Birthday Commemorative Volume [Asia Publishing House, Delhi

p. 256 - Sino-Indian relations by P. C. Chakravarti is quite interesting.

3/12/71 - Current Periodicals Room

(1). Science

8/1/71 - pp. 23 & 29 - Science: Endless horizons or Golden Age? by Bentley Glass.

[Is this the fellow mentioned by Shri Vatsa the son of L. S. Purushotham (?) an old student of mine, who gave me a lift in his car to the Inst. this afternoon? This boy Shri Vatsa living in 64, South Cross Street, is an Astronomer working in

~~we~~ one gets

$$u = \left(\frac{1}{x} - \frac{1}{x^2}\right)e^x - \frac{c_1}{2x^2} + c_2$$

$$y = ux = \left(1 - \frac{1}{x}\right)e^x - \frac{c_1}{2x} + c_2 x$$

$$= c_2 x + \frac{c}{x} + \left(1 - \frac{1}{x}\right)e^x, \text{ where } c = -\frac{c_1}{2}.$$

giving the other ~~integral~~ expn c/x of the C.F.

[Note :- This example is rather inappropriate, for

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots a_0 y = X$$

is a standard form which can be ~~solved by~~ reduced to a form with constant coeffts by putting $x = e^t$

& also the form with $X = 0$ can be solved by assuming $y = x^\lambda$ and getting an nth order algebraic eqn in λ with the roots $\lambda_1, \lambda_2 \cdots \lambda_n$ & hence soln being $y = B_n x^{\lambda_n} + B_{n-1} x^{\lambda_{n-1}} +$

[Vide Benny's book]

In fact this form of the eqn should have been treated in Ch. 2 and is a lacuna]

Using the method of putting $y = x^\lambda$ in

~~$x^2 y'' + xy' - y$~~ $x^2 y'' + xy' - y = 0$

(*) $x^2y'' + xy' - y = x^2e^x$

$x^2(2u' + u''x) + x(u + u'x) - ux = x^2e^x.$

$x^2(2u' + u''x) + x^2u' = x^2e^x$

$3u' + u''x = e^x.$

Hence the solution

(6) $x^2y'' + xy' - y = x^2e^x$

The C.F is $y = Ax + \frac{B}{x}$. – Reduced eqn $x^2y'' - xy' - y = 0$

choosing $y = ux + v/x$, $y' = u - v/x^2 + \boxed{u'x + v'/x}$

Proceeding as in Ex. 5 gives soln

Talks of possible difficulties in finding C.F. in cases where the ~~d.e~~ L.H.S. of d.e does not have constant coefficients. ~~Dead~~ Gives a method for the case when only $y = Ax$ [but $y = B/x$ is not known] using variation of one parameter.

Putting $y = ux$, $y' = u + u'x$, $y'' = 2u' + u''x$

This substitution is the d.e gives $u''x + 3u' = e^x$ (*) —— (1)

Putting $p = u'$ in (1) leads to first order eqn

$$xp' + 3p = e^x$$

$$\text{ie } p' + \frac{3}{x}p = \frac{1}{x}\cdot e^x$$

Using the integrating factor $e^{3\log x} = x^3$

$$\frac{d}{dx}(x^3p) = x^2e^x$$

$$\text{ie } x^3p = x^2e^x - \int 2xe^x\cdot dx = x^2e^x - 2xe^x + 2e^x + c_1$$

$$\text{or } p = \frac{1}{x}e^x - \frac{2}{x^2}e^x + \frac{2}{x^3}e^x + \frac{c_1}{x^3} = u'$$

Solving [by noting $\left(\frac{1}{x} - \frac{1}{x^2}\right)e^x + \left(-\frac{1}{x^2} + \frac{2}{x^3}\right)e^x$] $= \frac{d}{dx}\left\{\left(\frac{1}{x} - \frac{1}{x^2}\right)e^x\right\}$

As in Ex. 3, choosing the simplest eqn by putting

expn within rectangles equal to 0 ie

$$u' \cos x + v' \sin x = 0 \quad \text{---} \quad (1)$$

then $y'' = -u \cos x - v \sin x - u' \sin x + v' \cos x$

& substituting this & $y' = -u \sin x + v \cos x$, the original eqn becomes

$$-u' \sin x + v' \cos x = \operatorname{cosec} x \quad \text{---} \quad (2)$$

Solving (1) & (2) algebraically, we get

$$u' = -1 \text{ and } v' = \frac{\cos x}{\sin x}$$

ie $u = -x + C_1$, $v = \log \sin x + C_2$

& hence reqd soln is

$$y = (C_1 - x) \cos x + (C_2 + \log \sin x) \sin x.$$

$$= C_1 \cos x + C_2 \sin x - x \cos x + \sin x \log_e \sin x.$$

(5) non-trivial. $y'' - y = \dfrac{2}{1+e^x}$.

Soln of reduced eqn is $y = Ae^x + Be^{-x}$

So taking $y = ue^x + ve^{-x}$

$$y' = ue^x - ve^{-x} + \boxed{u'e^x + v'e^{-x}}$$

and proceeding as in Ex. (4), we get

$$v' = -\frac{e^x}{1+e^x}, \quad u' = \frac{1}{e^x(1+e^x)} = \frac{1}{e^x} - \frac{1}{e^x+1}$$

$$= e^{-x} - \frac{e^{-x}}{1+e^{-x}}$$

Giving $v = -\log(1+e^x) + C_1$, $u = -e^{-x} + \log(1+e^{-x}) + C_2$.

$u'e^x + v'e^{4x} = 0$ ——— (1)

using (1) we have $y' = ue^x + 4ve^{4x}$

$y'' = ue^x + 16ve^{4x} + u'e^x + 4v'e^{4x}$

• & substitution in the original diff eqn gives

$u'e^x + 4v'e^{4x} = 2e^{2x}$ (terms in u & v cancel out)

——— (2)

(1) & (2) are algebraic eqns in u' & v' and solving we get

$v' = \frac{2}{3}e^{-2x}$ and $u' = -\frac{2}{3}e^x$

leading to $v = c_1 - \frac{1}{3}e^{-2x}$ and $u = c_2 - \frac{2}{3}e^x$

Hence $y = ue^x + ve^{4x}$ becomes $y = (c_2 - \frac{2}{3}e^x)e^x + (c_1 - \frac{1}{3}e^{-2x})e^{4x}$

$y = c_2e^x + c_1e^{4x} - e^{2x}$

———

(4) A non-trivial example:

$y'' + y = \operatorname{cosec} x$.

Here the finding of P.I by trial soln or D-operator method is too difficult, but the method of variation of parameters is easily applicable as follows:—

The reduced eqn (for C.F) is $y'' + y = 0$

Soln $y = A\cos x + B\sin x$.

Assume $y = u\cos x + v\sin x$

$y' = -u\sin x + v\cos x + u'\cos x + v'\sin x$

5892782.

linear eqns is given (for eg. like the given to me by a student of the Biophysics Deptt some time back).

Some trivial examples:

(1) $y' - 2y = e^{3x}$ – known soln is $y = e^{2x}(e^x + c)$ ie of form $y = ue^{2x}$

So starting with $y = ue^{2x}$ [$y' - 2y = 0$ the reduced eqn has the soln $y = Ae^{2x}$] u being a fn of x, we have

$$y' - 2y = e^{3x} \longrightarrow 2ue^{2x} + u'e^{2x} - 2ue^{2x} = e^{3x}$$

$$\text{ie } u' = e^x \text{ or } u = e^x + c$$

ie $y = e^{2x}(e^x + c)$ same as in standard soln

(2) $y' - \frac{3}{x}y = x^2$ [$y' - \frac{3}{x}y = 0 \rightarrow y = cx^3$ & so with $y = ux^3$ leads to $u = \log_e x + c$ & soln is $y = x^3(\log_e x + c)$

(3) $y'' - 5y' + 4y = 2e^{2x}$ [Soln given by C.F + P.I is $y = c_1e^{4x} + c_2e^x - e^{2x}$]

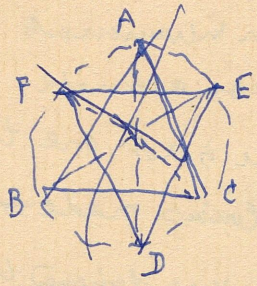
Here reduced eqn $y'' - 5y' + 4y = 0$ has soln (C.F) $y = c_1e^{4x} + c_2e^x$

~~For~~ Here we introduce two parameters u & v and write

$$y = \cancel{ue^{4x}}\ ue^x + ve^{4x} \qquad (u \text{ \& } v \text{ fns of } x)$$

$$y' = ue^x + 4ve^{4x} + u'e^x + v'e^{4x}$$

Here u and v are to be determined & substitution in the original eqn gives only one eqn between u & v. The second eqn is chosen to be the simplest possible one for working & is chosen as


A
F
E
B
C
D

(3) Introduction to Statics by Irving H. Shames

Prentice-Hall Series in Engineering Science

– Quite an elementary book – perhaps useful for Engineers in the beginning courses.

1/12/71 – New Books Section

(1) Ordinary Differential eqns – A programmed course for students of Science & ~~Biology~~ Technology (A. C. Bajpai, I. M. Calus, J. Hyslop) [Wiley Interscience

Chap. 3 – ~~Diff. eqn with~~ (2nd order) with const. coeffts using D-operator methods

Chap 4 – Do – Soln using Laplace transform method

Chap 5 – Simultaneous differential eqns – Soln by D-operator and L.T. methods

Chap. 6 – Linear Diff. eqns (not necessarily with constant coefficients) – Variation of parameters & solution in series.

~~Those Chapters~~ 3-5 contain lots of examples which would be useful from an examination point of view for setting questions

Ch. 6 is also useful using the variation of parameters method.

method by solution in series rather complicates

~~no method~~ use of the above methods for solving simultaneous

The evolution of stars, Galaxies and the Universe – Constellations and sky maps – Appendix – Glossary – Index.

[Nanosecond = 10^{-9} Sec – Modern measurements up to this & still more accurate.

p. 23 – As for Astrology itself, there is not one iota of evidence to support the claims made for it.

p. 29 – The Hindu concept of the Universe designed by Brahmin (A mistake should be Brahman) with a picture of tortoise, ~~xxx~~ serpent etc (Ref. to Bettmann Archive.

✓ (one should really find out what the old ideas of the Hindus were about the Universe & the date)

p. 47 – Discovery of precession shocking the astrologers

pp. 53-5~~6~~ – A profile of Galileo – Inquisition – His death in 1642 the year in which Newton was born.

✓ Chap 5 – giving a panoramic view of the Universe is very interesting.

Chap. 6 – Atoms & radiation is also fundamental p. 83 about neutrinos & anti-neutrinos.

Star maps also most interesting – Read the whole book through for nearly two hours.

(4) ~~Mean~~ Mean-square convergence (5) 1st order linear eqns with constant coeffts (6) Fourier Integral (7) Generalised Fourier Series (8) Sturm-Liouville problem (9) Bessel functions

Appendices. A – Continuum Dynamics
B – Electromagnetic Theory
C – The heat equation (from which Fourier Series started)

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(2) Astronomy by Donald H. Menzel (Paine Prof. of Practical Astronomy at Harvard Univ) – Sky Maps & illustrations by Chuji-Sung Yü Prof. Emeritus of Astronomy, Hood College. – Thames & Hudson, London. 1971.

A most beautiful book with beautiful illustrations, wonderful printing

Contents. Preface, – Introduction – The birth of Astronomy – Early Astronomy – The Mechanistic Universe – The rebirth of Science – The Universe – A panoramic view – Atoms and radiation – The instruments of Astronomy – The Earth as a planet – The moon as a planet – The Sun, our nearest star – Eclipses – The terrestrial planets: Mercury, Venus, Mars & Pluto – The giant planets: Jupiter, Saturn, Uranus & Neptune – Asteroids, Comets and Meteors – The origin of the solar system – Stars and nebulae – Variable stars, Star clusters, Galaxies and Quasars –

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18533

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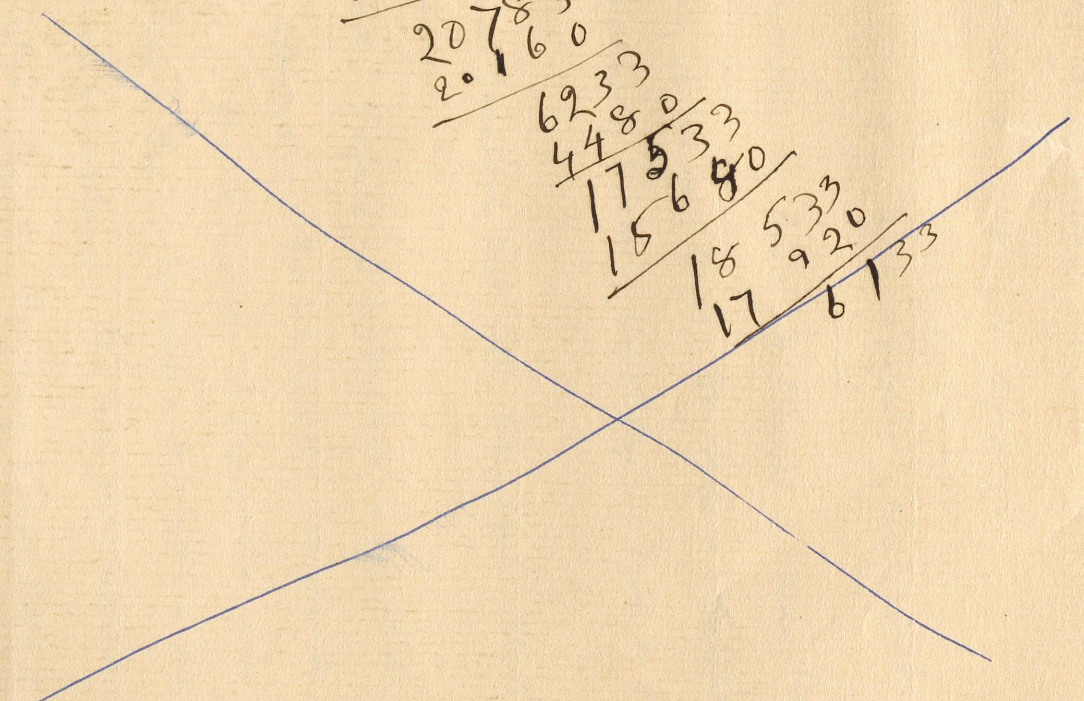

Part I – Fundamental concepts and methods of mathematics

(1) The axiomatic method (2) Analysis of the axiomatic method (3) Theory of Sets (4) Infinite sets (5) Well ordered sets; ordinal numbers (6) Linear Continuum & real number system, (7) Groups and their significance for the foundations

Part II – Development of various viewpoints on foundations.

(8) The early developments, (9) The Frege-Russell Thesis: Maths. an extension of Logic. (10) Intuitionism, (11) Formal systems; Mathematical Logic (includes Gödel's incompleteness theorem – on p. 271 mentions ~~Gotell~~ Gödel number of a formula) (12) The cultural setting of mathematics.

Not an impressive book. In Chaps (12) makes hardly any mention of early Indian contributions to Mathematics.

29/11/71 New Books Section

(1) Fourier Series by Robert K. Pitt (Mc Graw Hill - International Series in Pure & Appl. Math), 1970.

- Book at elementary level, but quite modern

Chaps (1) Periodic fns (2) The Cesaro sum (3) The general question

Ibid – 13/5/71 – p. 382 – The intermediate vector boson (7)
– particle corresponding to the pion but in weak interaction by Prof. Englert
Commins

ibid – 21/1/71 – p. 146 – Lasers – a coherent approach to
art – Lasers providing many new possibilities for artists
and craftsmen, who are beginning to explore and use laser
techniques, just as they have experimented with light for
centuries – Article by Dr. Leon Goldman, Director of Laser
laboratory at the Children's Hospital Research Foundation at the
Medical Centre of the Univ. of Cincinnati

8/7/71, p. 65 – Time-reversal survives another test
p. 67 – Sir Lawrence Bragg (Died last week at age 81)

1/4/71 – p. 39 – In the roots of memory.

31/12/70 – p. 604 – University Research Grants (Limited)

12/11/71 – Merely looked at ~~no~~ journals in the Periodicals Room –
Also helped Raghavendra Rao in transferring rarely used books on
Mathematics & Physics to the fourth Tier above.

15/11/71 New Books Section.

(1) Introduction to the Foundations of Mathematics (2nd Edn)
by Raymond L. Wilder (John Wiley – Wiley International, 1965)

(8) Personnel management by M.J. Jucius, 1971 Edn

(Richard D. Irwin, Inc. Homewood, Illinois)

p. 440 - fig. 23-4 - Illustration re. "The value of Safety Glasses"

(9) Book on liquid Crystals 2 - Sectional Library - Edited by G.H. Brown [Gordon & Breach, Proc. Liquid Crystal Conference 2nd held at Kent State Univ, August 12-16, 1968]

p. 291 - S. Chandrasekhar, D. Krishnamurthy and N.V. Madhusudhana: Theory of Birefringence of Nematic liquid crystals (pp. 291-315) - Dept. of Physics, Univ. of Mysore, Mysore.

New Periodicals

(1) New Scientist 26/8/71 → 30/8/1871 to 19/10/1937

29
37
66

p. 464 - Rutherford the Great by J.G. Crowther -

Rutherford's father had 12 children & Rutherford was his fourth child & second son. The father lived up to 89 & his wife the mother up to 92 - Rutherford's education was started by the experienced hand of his mother (a school teacher) - not fond of money; when he died he left only £ 7000/- - Lot more details - A very interesting article

– Peculiar sort of book with all kinds of topics – Failed to see any coherence in the several topics

(7) Mathematical Physics – An advanced course by S. G. Mikhlin (Leningrad Univ.) – North-Holland Publ. Co. (1970)

Part I – Mean functions & generalised derivatives. (2 Chapters)

Part II – Elements of the Calculus of variations (4 Chapters)

Part III – Elements of the Theory of Integral Equations (2 Chapters)

Part IV – General Aspects of partial diff. eqns (2 Chapters)

Part V – Eqns of elliptic type (9 Chapters)

Part VI – Time-dependent equations (5 Chapters)

Part VII – The proper posing of Problems in Mathematical Physics (1 Chapter)

~~A book~~ A book of a purely mathematical nature re. partial diff. eqns.

✓✓ Part VI – Chaps. 22 & 23 of the heat conduction equation would be useful to value the Kerala Thesis – Also in

✓✓ ~~Chap VII~~ Chap 25 of Part VII, § 6, p. 504 – Heat Conduction problems would also be useful in this connection.

✓✓ Also Chap. 20 (Pt. VI) devoted entirely to the Heat conduction equation is useful.

Quite a comprehensive collection. Reading this suggests that I might write a book on Classical mechanics taking many topics out of this book.

(5) Entropy by J. D. Fast (Macmillan) – 2nd Edn (Revised & Enlarged) – Philips Technical Library (1970)

Intn – Entropy concept plays a role as important as that of the Energy concept in every field of Physics, Chemistry & Technology, Usually presents many difficulties. – The book has been translated from Dutch (Original Edn in Dutch) into English, French, German, Spanish & Polish – – The use (and frequent abuse) of the entropy concept in information theory – Chaps 1 to 3 form a complete & coherent whole which may be studied by readers lacking time to read the complete book – Quantum Statistics simpler than classical statistics – The treatment is not extremely rigorous

(6) Field Theory (Vol I) (Classical Theory) by Jan Rzewuski

Chap I – Spinors & Tensors
" II – Variational Principles
" III – Examples
" IV – Basic solns of field Eqns

(3) An introduction to Lasers & Masers by A.E. Siegman (Mc Graw Hill Series in the Fundamentals of Electronics Science) (Copyright 1971 & latest book) – Front cover shows a brilliant red star of the type shown at the Academy meeting by somebody on 8/11/71 at Raman Research Institute – Meant to be used as a text

(4) Concepts and methods of Theoretical Physics by Robert Bruce Lindsay (Paperback Edn) – Dover Books on Phys. & Math. Phys. (Brown Univ) – A beautiful illustration on the cover. – Writing of the book grew out of a graduate course given for several years –

17 Chapters: Nature of Th. Phys (Ch. I) – Part I – Particle Physics – Ch. II. (Nonrelativ. particles)

Kinematics, Geometry of motion – Concepts of Theory of Mechanics – Principles of Dynamics – Motion of a particle in a St: line, S.H.M – Motion of particles in a plane – Motion of collection of particles – Dynamics of a Rigid body – Minimal Principles, Lagrange's equations – Transformation theory of Mechanics. Atomic Structure (Bohr Theory) – Statistical Mechanics & Thermodynamics – Quantum Theory (pp. 229–275) – Part II – Field Physics – Scalar & Vector fields – Deformable material media and tensor fields – Wave motion in solid and fluid Media – The Electromagnetic field – Electromagnetic radiation – Electron Theory and Relativity.

The book contains nearly sixty to seventy illustrations — Book written, as mentioned on the front cover page, to help the general education of students in Sixth Forms, Colleges of Education, Colleges of Further Education, Technical Colleges & Universities

✓ || Extremely well written readable book, quite elementary — why not try translation of books on ?? || Astronomy into Kannada ?

(2) Topics in Complex Function Theory by C. L. Siegel Vol II — Automorphic Functions and Abelian Integrals (Wiley Interscience, Translated from the original German — Belongs to Interscience Tracts in Pure & Appl. Maths. (Vol I is Elliptic Fns and Uniformisation Theory) (1970)

A book based on Lecture Notes of Siegel prepared by Dr. E. Gottschling at Gottingen.

Preface by Wilhelm Magnus (N. Y. 1969) — A translation of Part III on "Abelian Functions and Modular Functions" in several variables" is in preparation. After its completion an

✓ || English text written by a great mathematician of our time will present one of the most fascinating fields of mathematics

Walker's Quantum Theory of Radiation - notes:

New Book (10/11/71)

New Books Section

(1) Astronomy & Cosmology by A. P. Rowe* (Thames & Hudson, London)

General Studies Library Series, Edited by Percy Wilson C.B [1968]

[* CBE, Hon LLD, B.Sc., D.I.P, ARCS - Lately Vice-Chancellor Univ. of Adelaide, Fellow of the Imperial College]

[Thanks to Wartime Colleagues Sir Bernard Lovell & Sir Martin Ryle]

Contents:

1. The Universe
2. The Solar System
3. Other worlds than ours?
4. The stars.
5. The Galaxies
6. Measuring distances
7. When & How?
8. Life & death of a star
9. The spectroscope - a Master Key.
10. The Radio Universe
11. The Strange world of Relativity
12. The Expanding Universe

[Glossary References Index]

Institute Notes

Started on 10/11/71