(1)

$$q_{2n+1} = \frac{\alpha^{2n+1} - \beta^{2n+1}}{\alpha - \beta} \quad ; \quad p_{2n+1} = \frac{1}{2}\left(\alpha^{2n+1} + \beta^{2n+1}\right)$$

$(\sqrt2)^4 + (\sqrt2)^4$

$2\cdot 2^2 + \beta^2$

$$= \frac{(1+\sqrt2)^{2n+1} - (1-\sqrt2)^{2n+1}}{2\sqrt2}$$

$$= \frac{1}{2\sqrt2}\Big[\left\{1 + (2n+1)C_1\cdot\sqrt2 + (2n+1)C_2\cdot(\sqrt2)^2 + \cdots + (2n+1)C_{2n}(\sqrt2)^{2n} + (2n+1)C_{2n+1}(\sqrt2)^{2n+1}\right\}$$

$$- \left\{1 + (2n+1)C_1\cdot(-\sqrt2) + (2n+1)C_2(-\sqrt2)^2 + \cdots + (2n+1)C_{2n}(-\sqrt2)^{2n} + \cdots (-\sqrt2)^{2n+1}\right\}$$

$$= \frac{2}{2\sqrt2}\left[(2n+1)C_1\cdot\sqrt2 + (2n+1)C_3\cdot(\sqrt2)^3 + \cdots + (2n+1)C_{2n+1}(\sqrt2)^{2n+1}\right]$$

$$= (2n+1)C_1 + (2n+1)C_3\cdot 2 + (2n+1)C_5\cdot 2^2 + \cdots + (2n+1)C_{2n+1}\cdot 2^n.$$

Similarly $p_{2n+1} = 1 + (2n+1)C_2\cdot 2 + (2n+1)C_4\cdot 2^2 + \cdots + (2n+1)C_{2n}\cdot 2^n.$

$n=0$, $q_1 = 1$ ✓, $p_1 = 1$ ✓

Putting $n=1$, $q_3 = 3C_1 + 3C_3\cdot 2 = 3+2 = 5$ ✓ }

$p_3 = 1 + 3C_2\cdot 2 = 1+6 = 7$ ✓ } as given by Sierpinski

$\frac{5\times4^2}{2}$

$n=2$, $q_5 = 5C_1 + 5C_3\cdot 2 + 5C_5\cdot 2^2 = 5 + 20 + 4 = 29$ ✓ }

$p_5 = 1 + 5C_2\cdot 2 + 5C_4\cdot 2^2 = 1 + 20 + 20 = 41$ ✓ } also given by Sierpinski

$n=3$, $q_7 = 7C_1 + 7C_3\cdot 2 + 7C_5\cdot 2^2 + 7C_7\cdot 2^3 = 7 + 70 + 84 + 8 = 169$ ⊗

$p_7 = 1 + 7C_2\cdot 2 + 7C_4\cdot 2^2 + 7C_6\cdot 2^3 = 1 + 42 + 140 + 56 = 239$ ✓

$\frac{7\cdot6\cdot5}{6}\cdot 4\cdot 8$

$7C_2$

$\frac{7\cdot6}{2} = 21$

239 × 239	169 × 169
2151	1521
717	1014
478	169
57121	28561
57122	
−1 ✓	

$1 + 7(2 + 10 + 12)$

$1 + 7\times24$

$\frac{9\cdot8\cdot7\cdot8^2}{248}$ 126×4 $\frac{9\cdot8^4}{2}$ 36×8

408

36×7

252

$n=4$, $q_9 = 9C_1 + 9C_3\cdot 2 + 9C_5\cdot 2^2 + 9C_7\cdot 2^3 + 9C_9\cdot 2^4$

$= 9 + 168 + 504 + 288 + 16 = \cancel{1321}$ ✓ 985 ⊗ $= 197\times5$

(84)

$p_9 = 1 + 9C_2\cdot 2 + 9C_4\cdot 2^2 + 9C_6\cdot 2^3 + 9C_8\cdot 2^4$

$= 1 + 72 + 504 + \cancel{2016}\ 672 + 144 = \cancel{2737}\ 1393$ ⊗ $= 7\times199$

2378

126

$\frac{9\times8}{2}$ 36×8 84×8 18×7 126×4

$p_9 = 9C_1\cdot 2^4 + 9C_3\cdot 2^3 + 9C_5\cdot 2^2 + 9C_7\cdot 2 + 9C_9$

$p_9 - q_9 = 9C_1(2^4-1) + 9C_3(2^3-2) - 9C_7(2^3-2) - (2^4-1)$

$p_9 - q_9 = p_7 + q_7$

$p_7 - q_7 = p_5 + q_5$

$p_5 - q_5 = p_3 + q_3$

$p_3 - q_3 = p_1 + q_1$

$\cancel{p_9 - q_9 - 2q_7 - 2q_5 - 2q_3}$

$p_n = q_n + q_{n-1}$

$2q_n = p_n + p_{n-1}$

$p_n = 2p_{n-1} + p_{n-2}$

$q_n = 2q_{n-1} + q_{n-2}$

$q_n = q_{n-1} + p_{n-1}$

$2q_n = q_n + q_{n-1} + p_{n-1}$

$q_n = q_{n-1} + p_{n-1}$

$2^{13} - 1 = 8191$

$2^{17} - 1 =$

$$q_{2n} = 2nC_1 + 2nC_3 \cdot 2 + 2nC_5 \cdot 2^2 + \dots + 2nC_{2n-1} \cdot 2^{n-1}$$

$$p_{2n} = 1 + 2nC_2 \cdot 2 + 2nC_4 \cdot 2^2 + \dots + 2nC_{2n} \cdot 2^n$$

[9 − 8 = 1]

$q_0 = 0,\ p_0 = 1$

$q_2 = 2C_1 = 2,\ p_2 = 1 + 2C_2 \cdot 2 = 3$

$q_4 = 4C_1 + 4C_3 \cdot 2 = 4 + 8 = 12$

$p_4 = 1 + 4C_2 \cdot 2 + 4C_4 \cdot 2^2 = 1 + 12 + 4 = 17$

$289 - 288 = 1$

$p_{2n}^2 - q_{2n}^2 = 1$

$p_7 = q_7 + q_6$ $\quad \cancel{2p_7 - 2q_7 + 2q_6 = p_7 + p_6 + p_6 + p_5}$

$q_0 = 0,\ p_0 = 1$

$q_1 = 1,\ p_1 = 1$ → ✓

$q_2 = 2,\ p_2 = 3$

$q_3 = 5,\ p_3 = 7$ ↔ ✓

$q_4 = 12,\ p_4 = 17$

$q_5 = 29,\ p_5 = 41$ ↔ ✓

$q_6 = 70,\ p_6 = 99$

$q_7 = 169,\ p_7 = 239$ → ×

$q_8 = 408,\ p_8 = 577$

$q_9 = 985,\ p_9 = 1393$ → ×
5×197, 7×199

$q_{10} = 2378,\ p_{10} = 3363$

$q_{11} = 5741,\ p_{11} = 8119 = 23 \times 353$ → ×

$q_{12} = 13860,\ p_{12} = 19601 = 17 \times 1153$

$q_{13} = 33461,\ p_{13} = 47321 = 79 \times 599$ → ×

$q_{2n} = 2q_n p_n$ —(1)

$4p_n^2 - 8q_n^2 = 4(-1)^n$

$p_n^2 - 2q_n^2 = (-1)^n$ —(2)

$2q_{m+n} = 2q_m p_n + 2q_n p_m$

$q_{m+n} = q_m p_n + q_n p_m$ —(3)

For n odd, q_n is a divisor of $x^2 + y^2$ —(4)

$q_n^2 - q_{n-1} q_{n+1} = (-1)^{n-1}$ —(5)

If d is the g.c.d. of m & n, then q_d is g.c.d of q_m, q_n —(6)

$8119^2 - 2 \times 5741^2 = -1$

27720	39202
5741	8119
33461	47321

$2q_5 p_5 = 2 \times 29 \times 41 = 58 \times 41$
58
232
$2378 = q_{10}$

11482 + 2378 = 13860

16238 + 3363 = 19601

1154 + 239 = 1393

478 + 99 = 577

198 + 41 = 239

338 + 70 = 408

816 + 169 = 985

1970 + 408 = 2378

2786 + 577 = 3363

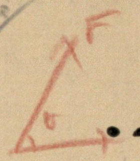
4756 + 985 = 5741

$2^n - 1$ prime for 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 127

Every prime >3 is $6n \pm 1$.

q_{mn} is divisible by q_m, q_n & by their product if m & n are ...

We need consider only q_{2n+1} where $2n+1$ is prime, otherwise if $2n+1$ is composite $= q_{rs}$, then q_{rs} is divisible by q_r & q_s & hence not prime



$$(q_7^2 - 1) = q_6 q_8 \qquad (q_7 - 1)(q_7 + 1) = q_6 q_8 = 408 \times 70$$

$q_7 = 2q_6 + q_5$

$= 2(2q_5 + q_4) + q_5$

$q_8 = 5q_5 + 2q_4$

$q_6 = 10q_4 + 5q_3$

2621

47321

$47322 = 9 \times 5258$

$= 2 \times 9 \times 17 \times 239$

$= 198\, p_7$

$\sqrt{33461}$ (183

1

28 | 234, 224

363 | 1061, 1089

$33462 = 9 \times 3718$

$= 2 \times 9 \times 1859$

$= 2 \times 9 \times 11 \times 169$

$= 2 \times 3^2 \times 11 \times 13^2$

$= 198\, q_7$

$\frac{18 \times 11}{8}$

$c = a + b$

$q\,c = p \cdot d$

$(a+b) = p \cdot d$

23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139.

impossible to test primality

anyway tested till 79 & succeeded.

~~$q_{13} = 6 \cdot 33\, q_7 - 1 = 2p_6 q_7 - 1$~~, $p_{13} = 6 \cdot 33\, p_7 - 1$.

$= 2p_6 p_7 - 1$.

~~$q_7 p_6 + q_6 p_7$~~

~~ie $q_7 p_6 - q_6 p_7 = 1$~~

~~$q_{13} = 239 \times 70 + 169 \times 99$~~

~~Since 239 is prime & 70 is divisible only by 5, 7 but not the second term q_{13} is not divisible by any prime from 1 to 181. ie q_{13} is prime.~~

$p_{13} = 2p_6 p_7 - 1$

$= 2 \cdot 99 \cdot 239 - 1$] ~~Similar argument for $p_{13}+1$ since R.H.S. is not divisible by any prime from 1 to 181, for if such~~

~~2.99.239 is not divisible by any prime except 2, 3, 11 & 239 ie not divisible by any prime from 1 to 181~~

~~$181m$~~

~~if $p_{13} = 181m$~~ $\quad 181m + 1 = 2 \cdot 99 \cdot 239$

$$q_7 = q_4 p_3 + q_3 p_4 = 12 \times 7 + 17 \times 5$$

$$q_7 = 7C_1 + 7C_3 \cdot 2 + 7C_5 \cdot 2^2 + 7C_7 \cdot 2^3$$

$$= 7 + 35 \cdot 2 + 21 \cdot 4 + 8$$

19/3/67 $\quad 47321 = 79 \times 599$. Hence q_{13} & p_{13} is not a prime set.

but $q_{13} = 33461$ is prime

241 296 181 282 276 236 39 13 143 181 259

$q_{14} = \cancel{33461}$

$q_{14} = 80782$, $p_{14} = 114243$

$q_{15} = 195025$, $p_{15} = 275807$

$q_{16} = 470832$, $p_{16} = 665857$

$q_{17} = 1136689$, $p_{17} = 1607521$

$q_{18} = 2744210$, $p_{18} = 3880899$

$q_{19} = 6625109$, $p_{19} = 9369319$

$q_{21} = 15994428$, $p_{21} = 22619537$

$q_{23} = 38613965$, $p_{23} = \cancel{54608393} = 54608393$

$q_{17} = 137 \times 8293$ (both quotients prime)

$p_{17} = 103 \times 15607$ (")

ie q_{17} and p_{17} are both not primes

$q_{19} = 37 \times 179057$ (both quotients prime)

$p_{19} =$?? (prime)

$\left.\begin{array}{l} q_{23} = 5 \times 7722793 \\ p_{23} = 7 \times 7801199 \end{array}\right\}$ both not prime

So it looks as if there are only the first three cases of $n = 0, 1, 2$. This is to be proved

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$$p^2 - 2q^2 = -1 \quad \text{—(2)} \quad \text{(Sierpinski's problem)}$$

Are there infinitude of solns in primes of this eqn?

For $p^2 - 2q^2 = 1$. —(1)

the only soln known ~~are~~ is $p = 3$, $q = 1$. This can be easily proved for solns of (1) are given by the convergents (q_{2n}, p_{2n}) of the continued fraction for $\sqrt{2}$, and the ~~even~~ convergent $q_{2n} = 2q_n p_n$ & hence not prime except for $n = 1$ for which $q_2 = 2$, & $p_2 = 3$.

For (2) solns are given by the ~~even~~ odd convergents (because of single cycle in the contd. fraction for $\sqrt{2}$) by (q_{2n+1}, p_{2n+1}). The cases where $2n+1$ is not a prime can be discarded, for in these cases $q_{2n+1} = q_{rs}$ which is divisible by q_r, q_s & their product if r & s are relatively prime & hence (q_{2n+1}, p_{2n+1}) does not form a prime set. So we need consider only the cases where $(2n+1)$ is prime.

(1) For $n = 0$, $q_1 = 1$, $p_1 = 1$ is a prime set

(2) " $n = 1$, $q_3 = 5$, $p_3 = 7$ "

(3) " $n = 2$, $q_5 = 29$, $p_5 = 41$ "

but for higher cases we have so far found

(4) $n = 5$, $q_{11} = 5741$ (prime), but $p_{11} = 8119 = 23 \times 353$ (not a prime)

(5) $n = 6$, $q_{13} = 33461$ (prime), but $p_{13} = 47321 = 79 \times 599$ (not a prime)

(6) $n = 8$, $q_{17} = 1136689 = 137 \times 8297$ (not prime) & $p_{17} = 1607521 = 103 \times 15607$ (not prime)

(6)

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(7) $n = 9$, $q_{19} = 6625109 = 37 \times 179057$ (not prime, but both quotients prime)

& $p_{19} = 9369319$ which is prime.

(8) $n = 11$, $q_{23} = 38613965 = 5 \times 7722793$
$p_{23} = 54608393 = 7 \times 7801199$ } both not prime.

(a) Angular momentum & spin.

$$\text{orbital A.M} - M = r \times p \rightarrow [M_x, M_y] = i\hbar M_z,\ [M_y, M_z] = i\hbar M_x,\ [M_z, M_x] = i\hbar M_y.$$

Latter may be taken more general – M_x, M_y, M_z commute with M^2 – Representation in which M_z and M^2 are diagonal – Putting $L = M_x \pm i M_y$ & using $M_x = \frac{1}{2}(L^* + L)$, $M_y = \frac{1}{2}i(L^* - L)$ we can find reps of M_x & M_y if we know reps of L – General result that there are an infinite number of reps for M^2, M_z and L, each characterised by ~~0 or half integral value~~ $j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$ and has $2j+1$ rows & columns ie $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$ ie $2j$ is a +ve integer or 0. The eigenvalues of M_z range from j to $-j$ by unit steps. Those of M^2 are given by $j(j+1)\hbar^2$. Those of L are given by elements one step off the diagonal & diagonal elements = 0 – Matrices for different values of j (S. p. 146)

$M_x = \frac{1}{2}\hbar\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $M_y = \frac{1}{2}\hbar\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$, $M_z = \frac{1}{2}\hbar\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

– Pauli matrices correspond to $j = \frac{1}{2}$ & are denoted by $\frac{1}{2}\hbar\sigma_x$, $\frac{1}{2}\hbar\sigma_y$, $\frac{1}{2}\hbar\sigma_z$ & we have $\sigma_x\sigma_y - \sigma_y\sigma_x = 2i\sigma_z$ & also $\sigma_x\sigma_y + \sigma_y\sigma_x = 0$, $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$, $\sigma^2 = 3$. ($M^2 = \frac{3}{4}\hbar^2$) To find eigenvalues of σ_x, $\sigma_x\psi = \alpha\psi$ or $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}c_1\\ c_2\end{pmatrix} = \alpha\begin{pmatrix}c_1\\ c_2\end{pmatrix}$ ie $c_2 = \alpha c_1$, $c_1 = \alpha c_2$

ie $\alpha^2 = 1$ or $\alpha = \pm 1$ & normalised wave fns are $(\psi_+)_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$ & $(\psi_-)_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$ & this is the these are called spinors (column vectors or semi-vectors)

---

b For all integral j correspond orbital A.M & the corresponding matrices satisfy $M = r \times p$. For half-odd integral j this is not true since corresponding matrices satisfy $[M_y, M_z] = i\hbar M_x$, etc, but not $M = r \times p$. These correspond to intrinsic A.M. of particles or spin AM or merely spin. Experimentally it is found that electrons, protons, neutrons & μ-mesons have spin $\frac{1}{2}$, while ~~p~~ π-meson has spin 0.

(b) Identical particles and spin.

(1) $$i\hbar\frac{\partial}{\partial t}\psi(1, 2, \ldots n, t) = H(1, 2, \ldots n)\psi(1, 2, \ldots n, t) \qquad (H \text{ sym in } 1, 2, \ldots n)$$

Satisfied by symmetric & antisymmetric wave fns. Symmetry & antisymmetry invariant with time for ~~$H\psi$~~ if ψ_S be symmetric so is $H\psi_S$ & hence $i\hbar\frac{\partial\psi_S}{\partial t}$; but at time $t + dt$, $\psi_S \rightarrow \psi_S + \frac{\partial\psi_S}{\partial t}dt$ & hence also symmetric. Same argument holds for ψ_A. ∴ ψ_A cannot be constructed from a soln unaltered by interchange of any pair.

(2) Since H is symmetric, $n!$ solns obtained by permutations of any general unsymmetrised soln is also a soln – obtain ψ_S & ψ_A from these permutations – Solns obtained by permutations have exchange degeneracy since $\psi(1, 2, \ldots n; t) = u(1, 2, \ldots n)e^{-iEt/\hbar}$ is the stationary soln with $[H(1, 2, \ldots n) - E]u(1, 2, \ldots n)$ – Cases of $n = 2$ & $n = 3$ giving ψ_S & ψ_A – Case $n = 2$: ψ_S & $\psi_A = u(1,2) \pm u(2,1)$ for $n = 3$, $u(1,2,3)$, $u(2,3,1)$, $u(3,1,2)$ and $u(2,1,3)$, $u(3,2,1)$ and $u(1,3,2)$ leading to $[u(1,2,3) + u(2,3,1) + u(3,1,2)] \pm [u(2,1,3) + u(3,2,1) + u(1,3,2)]$

2^n-1 prime — $2^{n-1}(2^n-1)$ is perfect. (6, 28, 496)

$n=2,\ 3 \to 2\times3 = 6 = 1+2+3$

$n=3,\ 7 \to 4\times7 = 28 = 1+2+4+7+14 = 28$

$n=4,\ \times$

$n=5,\ 31 \to 16\times31 = 496 = 1+2+4+8+16+31+62+124+248 = 496$

$n=6,\ \times$

$n=7,\ 127 \to 64\times127 = 1+2+4+8+16+32+64+127+254+508+1016$

$=8128$ $\qquad +2032+4064+\cancel{8128} = 16256$

$\ 8128$

$\ 8128$ ✓

Do odd perfect numbers exist?

$1+2+2^2+\dots+2^{n-1}+(2^n-1)(1+2+2^2+\dots+2^{n-2})$

$=(2^n-1)+(2^n-1)(2^{n-1}-1) = (2^n-1)(2^{n-1}\cancel{-1+1})$

$=2^{n-1}(2^n-1)$ trivial.

3^n+2 $\qquad 3^m(3^n+2)$.

$1+3+\dots+3^m+(3^n+2)(1+3+\dots+3^{m-1})$

$\frac12(3^{m+1}-1)+(3^n+2)\cdot\frac12(3^m-1)$

$\cancel{\frac12(3^{m+1}-1)}\ \frac12\{(3^{m+1}-1)+(3^n+2)(3^m-1)\}$

$=\frac12\{3^{m+1}-1+3^{m+n}+2\cdot3^m-3^n-2\}$

$=\frac12\{3^{m+1}+3^{m+n}+2\cdot3^m-3\}$

Is 8191 prime?

7	117×
13	63×
17	48×
19	43×
23	35×
29	28×
31	26×
37	22×
41	19×
43	19×
47	17×
53	15×
59	13×
61	13×
67	12×
71	11×
73	11×
79	10×
83	9×
89	9×
97	8×

9)8191(99
81
18) 919

239 352 199 186 131 409 369 401 389 387 21 349 329 201 289 265 241 229 177 521 209 183 261 149 134 157 109 71 381 89 73 161 291 819 747 721 51 819 801 181

Perfect nos

6
28
496
8128
49,932,336.

8192

Yes 8191 is prime ie $2^{13}-1$ is prime

Hence $2^{12}(2^{13}-1)$ is perfect.

2^n-1 is prime for $n=2,3,5,7,13$,

8191×4096

49146
73719
0000
49146

49932336

There are four more independent eigenfunctions that can be formed out the two [ ]'s for in case $n=3$ for eg with one - spin in each & 2 minus spins in each. But they don't appear to describe any particles found in nature.

(3) Probability density associated with ψ_S or ψ_A for two particles is sum of probability densities associated with individual particles if interference effects between exchange degenerate wave fns are neglected

3/2/68

i.e. $|u(1,2) \pm u(2,1)|^2 = |u(1,2)|^2 + |u(2,1)|^2 + 2\,\mathrm{Re}\,[u(1,2)\,u(2,1)]$

$= |u(1,2)|^2 + |u(2,1)|^2$ if last term be neglected

(1) Exclusion principle - Neglecting interactions between the ~~particles of the~~ system (ideal system), unperturbed $H = H_0$ is

2/2/68

$$H_0(1,2,\ldots n) = H_0'(1) + H_0'(2) + - - + H_0'(n)$$

approximate energy eigenfunction

$$u(1,2,\ldots n) = v_\alpha(1)\,v_\beta(2) \ldots v_\nu(n)$$

$$\& \quad E = E_\alpha + E_\beta + \ldots + E_\nu$$

$$H_0'(1)\,v_\alpha(1) = E_\alpha\,v_\alpha(1), \text{ etc} \quad - - - \quad (1)$$

For electrons (Spin 1/2)

$$u_A(1,2,\ldots n) = \begin{vmatrix} v_\alpha(1) & v_\alpha(2) & \ldots & v_\alpha(n) \\ v_\beta(1) & v_\beta(2) & \ldots & v_\beta(n) \\ - & - & - & - \\ v_\nu(1) & v_\nu(2) & \ldots & v_\nu(n) \end{vmatrix}$$

[P.T.O]

& u_A is a soln of the wave eqn $(H_0 - E)\,u_A = 0$ as seen from (1). The determinant vanishes if two or more v's are the same. Thus H_0 has no soln for which there is more than one electron in any one of the states $\alpha, \beta, \ldots \nu$. This is Pauli's exclusion principle (for H-atom, statement is no two electrons ~~have~~ can have the same quantum numbers) $(H_0 - E)\,u_A = 0$.

(2) Connection with Statistical mechanics.

In addition to the above u_A we can form a u_S obtained from u by the sum of all different permutations of $1, 2, \ldots n$ among the one particle eigenfunctions $v_\alpha, v_\beta, \ldots v_\nu$. u_A is characterised ~~by~~ as a system (antisymmetric) in which no. of particles in each state is zero or 1 ~~and~~ is called a system obeying Fermi-Dirac Statistics & the particles are Fermions. Similarly the u_S obtained is characterises a system (symmetric) in which the no of particles in any state can be ~~from~~ 0, 1, 2, -- n & the system is said to be obey Bose-Einstein Statistics & particles are called Bosons.

(3) Spin & ~~Any~~ Statistics

(1) Addition of A.M - ~~one~~ $\vec{M} = \vec{M}_1 + \vec{M}_2$ (i) one repn in which M_1^2, M_2^2, M_{1z} & M_{2z} are diagonal & is not of much physical significance (ii) 2nd repn in which M_1^2, M_2^2, M^2 & M_z are diagonal. In this case, since $M_z = M_{1z} + M_{2z}$, we it can be shown that values of j corresponding to M_2 ranges from $j_1 + j_2$ to $|j_1 - j_2|$. If an aggregate consists of n particles each of

$$H_0 u(1,2,\ldots n) = E u(1,2,\ldots n)$$

$$H_0 v_\alpha v_\beta \cdots v_\gamma = E v_\alpha v_\beta \cdots v_\gamma \qquad \text{——— (1)}$$

$$\{H_0'(1) + H_0'(2) + \cdots + H_0'(n)\} v_\alpha(1) v_\beta(2) \cdots v_\gamma(n)$$

$$= (E_\alpha + E_\beta + \cdots + E_\gamma) v_\alpha(1) v_\beta(2) \cdots v_\gamma(n) \qquad \text{——— (2)}$$

Using $H_0'(1) v_\alpha = E_\alpha v_\alpha$ while $H_0'(1)$ commutes with $v_\beta(2)$, (2) reduces to

$$H_0 u_A = E u_A$$

$$\{H_0'(1) + \cdots + H_0'(n)\} u_A = (E_\alpha + E_\beta + \cdots + E_\gamma) u_A$$

$$H_0'(1) v_\alpha = E_\alpha v_\alpha$$

$$H_0'(2) v_\beta = E_\beta v_\beta$$

$$— \quad — \quad — \quad —$$

$$H_0'(n) v_\gamma = E_\gamma v_\gamma$$

$$u_A = \Sigma\, v_\alpha(1) v_\beta(2) \cdots v_\gamma(n)$$

$$\cancel{H_0'(1) v_\beta(2) = 0}.$$

$$\cancel{E_\alpha v_\alpha + E_\beta}$$

$$E_\alpha v_\alpha v_\beta \cdots v_\gamma + E_\beta v_\alpha v_\beta \cdots +$$

$$= E v_\alpha v_\beta \cdots v_\gamma$$

~~which has spin 1/2 & any no. of particles of spin 0 (ignoring internal orbital A.M), the total spin can be any integer from 0 to $\frac{1}{2}n$ if $\underline{n}$ is even, or can vary by integer steps from $\frac{1}{2}$ to $\frac{1}{2}n$ if $\underline{n}$ is odd. If $\underline{n}$ is odd, no symmetric but only an antisymmetric spin wave fn can be constructed for the system ie particles with half-odd integral spin obey the Fermi-Dirac statistics. For $\underline{n}$ even ~~only~~ symmetric wave fns can be constructed & hence particles of integral spin obey Bose-Einstein statistics~~

Thus all elementary particles (considered as aggregates of particles sufficiently tightly bound) are of integral or of half-odd integral spin (n or $\frac{2n+1}{2}$). Re. Statistics obeyed by particles of either type there are 4 possibilities viz as shown in the figure. Even if a single

experimental evidence of the association be known, the possibilities are easily made definite. Thus electrons of spin $\frac{1}{2} = \frac{1}{2}(0+1)$ satisfy the Exclusion Principle ie the F.D. statistics. Hence we conclude that particles of half-odd integral spin obey the F.D & particles of integral spin the B.E. statistics respectively

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(4) __The Klein-Gordon Equation:__ Relation $E^2 = c^2\vec{p}^2 + m^2c^4$ between momentum & energy in special relativity - According to Pauli they ~~4-vector~~ form a 4-vector given by $(p_1, p_2, p_3) = c\vec{p}$, $p_4 = -iE$ ie $\sum_{i=1}^{4} p_i^2 = c^2p^2 - E^2$; as a special case $(0, 0, 0, -im_0c^2)$ is also a four-vector ie $\sum p_i^2 = -m_0^2c^4$ $\therefore c^2p^2 - E^2 = -m_0^2c^4$ ie $E^2 = c^2p^2 + m_0^2c^4$ -

__Alternatively__: $E = mc^2 = m_0c^2/\sqrt{1-v^2/c^2}$ & $\vec{p} = m\vec{v} = m_0\vec{v}/\sqrt{1-v^2/c^2}$

$$\therefore E^2 - c^2p^2 = \frac{m_0^2c^4}{(1-v^2/c^2)} - \frac{m_0^2v^2c^2}{(1-v^2/c^2)} = \frac{m_0^2c^2}{1-v^2/c^2}(c^2-v^2) = m_0^2c^4.$$

$$\text{ie } E^2 = c^2p^2 + m^2c^4.$$

Making the quantum operators for E & p, $E \to i\hbar\frac{\partial}{\partial t}$, $\vec{p} \to -i\hbar\,\text{grad}$, the wave eqn for a free particle becomes

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + m^2c^4\psi \quad \text{or} \quad \frac{\partial^2\psi}{\partial t^2} = c^2\nabla^2\psi - \frac{m^2c^2}{\hbar^2}\psi \qquad (1)$$

which is the Klein-Gordon eqn of the second order.

ψ is a ~~[illegible]~~ plane wave soln ~~[illegible]~~ given by $\psi = \exp i(\underline{p}\cdot\vec{r} - \underline{E}t)$ if $\qquad (2)$

$$\hbar^2E^2 = \hbar^2c^2p^2 + m^2c^4 \quad \text{or } E^2 = c^2p^2 + m^2c^4/\hbar^2$$

$\hbar E$ and $\hbar\vec{p}$ being the eigenvalues of the eigen fns of the operators E & p being (2).

$$\text{ie } E = \pm\sqrt{(c^2p^2 + m^2c^4/\hbar^2)}^{1/2}$$

ie there are negative energy eigenvalues & there is seen to be no way of ~~removing~~ explaining them in the 2nd order eqn

$M_x = yp_z - zp_y, \quad M_y = zp_x - xp_z, \quad M_z = xp_y - yp_x \qquad [x, p_x] = ik$

$$[M_x, M_y] = (yp_z - zp_y)(zp_x - xp_z) - (zp_x - xp_z)(yp_z - zp_y)$$

$$= yp_z zp_x - \cancel{yp_z xp_z} - \cancel{zp_y zp_x} + zp_y xp_z$$
$$- zp_x yp_z + \cancel{zp_x zp_y} + \cancel{xp_z yp_z} - xp_z zp_y$$

$$= \cancel{zp_z(p_y x - p_x y)}\ yp_x(p_z z - zp_z) + xp_y(zp_z - p_z z)$$

$$= ik(xp_y - yp_x) = ik M_z .$$

$$[M_x, M^2] = M_x(\cancel{M_x^2} + M_y^2 + M_z^2) - (\cancel{M_x^2} + M_y^2 + M_z^2)M_x$$

$$= M_x M_y^2 - M_y^2 M_x + M_x M_z^2 - M_z^2 M_x .$$

$$\cancel{ik(M_y M_z + M_z M_y) = M_y(M_x M_y - M_y M_x) + M_z(M_z M_x - M_x M_z)}$$

$$\cancel{ik(M_z M_y + M_y M_z) = M_z(M_z M_x - M_x M_z) + M_y(M_x M_y - M_y M_x)}$$

Subtracting $\cancel{0 = -M_y^2 M_x + M_z^2 M_x - M_z^2 M_x + M_y^2 M_x}$

$$ik(M_y M_z + M_z M_y) = M_y(M_x M_y - M_y M_x) + M_z(M_z M_x - M_x M_z)$$

$$ik(M_z M_y + M_y M_z) = \cancel{M_z(M_z M_x - M_x M_z) + M_y(M_x M_y - M_y M_x)}$$
$$= (M_x M_y - M_y M_x)M_y + (M_z M_x - M_x M_z)M_z$$

~~Subtracting~~ $\cancel{-M_y^2 M_x}$

Subtracting $0 = -M_y^2 M_x + M_z^2 M_x - M_x M_y^2 + M_x M_z^2$

$$\cancel{ik(M_z M_y + M_y M_z) = ik\, M_z(M_z M_x - M_x M_z) +}$$
$$\cancel{= (M_x M_y - M_y M_x)M_y + (M_z M_x - M_x M_z)M_z}$$

$$\cancel{ik(M_y M_z + M_z M_y) = (M_z M_x - M_x M_z)M_z + (M_x M_y - M_y M_x)M_y}$$
$$\cancel{= M_x M_y^2 - M_z M_z^2 + M_x M_z^2 + M_x M_y^2}$$

$$ik(M_y M_z + M_z M_y) = M_y(M_x M_y - M_y M_x) + M_z(M_z M_x - M_x M_z)$$

$$ik(M_z M_y + M_y M_z) = (M_x M_y - M_y M_x)M_y + (M_z M_x - M_x M_z)M_z$$

Subtracting $-M_y^2 M_x$

$\frac{\partial \psi}{\partial t} = \psi \cdot p_x$

$\frac{\partial^2 \psi}{\partial t^2} = \psi \cdot p_x^2$

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \nabla^2 \psi - \frac{m^2c^4}{k^2}\psi$$

$$E^2 = c^2 p^2 - \frac{m^2 c^2}{k^2}$$

$\psi = e^{(\vec{p}\cdot\vec{r} - Et)}$

$\frac{\partial \psi}{\partial t} = \psi(-E) \qquad \nabla^2 \psi = \psi \cdot p^2$

$\frac{\partial^2 \psi}{\partial t^2} = E^2 \psi$

Moreover, as shown by very general arguments given by Dirac, every wave equation must be linear in $\partial/\partial t$, for only to such a ~~real~~ linear operator acting on ψ can a meaning be attached. But the Klein-Gordon eqn is of second order in $\partial/\partial t$.

Thirdly, this equation cannot represent a particle having a spin other than zero. For e.g. if spin be $1/2$, the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$ cannot be included in the equation, since the Klein-Gordon eqn, being relativistic, can only include four matrices. Also the wave eqns spin-functions in this case are two in number viz the two spinors, while those the K-G eqn is only one.

Hence an alternate wave ~~function~~ equation removing the above three difficulties had to ~~must~~ be found, and this was done by Dirac.

(5) The Dirac wave equation:

$e^{ix} = \cos x + i\sin x$

$e^{-ix} = \cos x - i\sin x$

$\tan i\psi = \dfrac{e^{-\psi} - e^{\psi}}{i(e^{-\psi}+e^{\psi})} = i\tanh\psi$

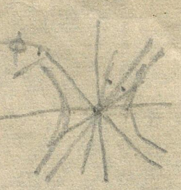
$\tan\phi = i\psi \to \tanh\psi = \beta$

with $\tanh\psi = \beta$, $\quad x_1' = \dfrac{x_1 - c\tanh\psi . t}{\sqrt{1-\tanh^2\psi}} = x_1\cosh\psi - x_4\sinh\psi$

$x_1^2 - x_4^2 = 1$

$x_4' = -x_1\sinh\psi + x_4\cosh\psi.$

$\cos\phi = \dfrac{1}{\sqrt{1-\beta^2}}$ ~~&c. Lorentz-Fitzgerald~~ $l = l_0\cos\phi$, $\tau_0 = \tau\sec\phi$.



or $\cosh\psi = \dfrac{1}{\sqrt{1-\beta^2}}$ $\qquad l = l_0/\cosh\psi$, $\tau = \tau_0\cosh\psi$.

$x^2 - y^2 = 1$

$y = mx$, $y = m'x$ are conjugate if $mm' = 1$

~~Addition theorem for velocities~~

$$v^i = \frac{dx^i}{d\tau} \quad \text{~~defined~~} \quad s = c\tau \text{ or } ic\tau.$$

$ic \cdot \dfrac{dt}{d\tau}$

$$\frac{d\tau}{dt} = \sqrt{1-\beta^2} \qquad v_1 = \frac{v_x}{\sqrt{1-\beta^2}}, \ldots, v_4 = \frac{ic}{\sqrt{1-\beta^2}}$$

$x^4 = m x^1$

$x^4 = m' x^1$

$d\tau = dt\sqrt{\ }$

$dt = \dfrac{d\tau}{\sqrt{1-\beta^2}}$

$$v_1 = \frac{dx_1}{dt}\cdot\frac{dt}{d\tau} = \frac{v_x}{\sqrt{1-\beta^2}} \qquad v_4 = \frac{dx_4}{d\tau} = \frac{d(ict)}{dt}\cdot\frac{dt}{d\tau} = \frac{ic}{\sqrt{1-\beta^2}}$$

$c^2 d\tau^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$

$-c^2 dt^2$

$-c^2\tau^2$

$$w = \frac{v_1 + v_2}{1 + v_1 v_2/c^2}$$

$v_1 = -ic\tan\phi_1$

$v_2 = -ic\tan\phi_2$

$$\frac{-ic(\tan\phi_1 + \tan\phi_2)}{1 - \tan\phi_1\tan\phi_2} = -ic\tan(\phi_1+\phi_2) \text{ resultant velocity}$$

$= w\ldots$

$= w\ldots - c^2 = \dfrac{v^2 - c^2}{1 - v^2/c^2} = -c^2$

$$\frac{dv_i}{d\tau} = \text{accn}_4 \left(\frac{d^2x}{d\tau^2}, \; ic\frac{d^2t}{d\tau^2}\right)$$

$v_i^2 = \dfrac{v_x^2 + v_y^2 + v_z^2 - c^2}{1-\beta^2}$

$v_i\, dv_i = 0$

$\dfrac{x_1^4}{x_1^1}\cdot\dfrac{x_2^4}{x_2^1} = 1.$

$$\boxed{\frac{x_1'^4 x_2'^4}{x_1'^1 x_2'^1} = \frac{(-x_1'\sinh\psi + x_1^4\cosh\psi)(-x_2'\sinh\psi + x_2^4\cosh\psi)}{(x_1'\cosh\psi - x_1^4\sinh\psi)(x_2'\cosh\psi - x_2^4\sinh\psi)}}$$

$x_1^4 x_2^4 = x_1' x_2'$

$(\cosh\psi, \sinh\psi)$

$$= \left\{\begin{array}{l} x'\sinh\psi - x^4\cosh\psi = 0 \\ x'\cosh\psi - x^4\sinh\psi = 0 \end{array}\right\} \text{Conj. Diam.}$$

$\dfrac{x^4}{\sinh\psi} = \dfrac{x'}{\cosh\psi}$

$x\cosh\psi - y\sinh\psi = 0$

$\dfrac{\cosh\psi}{\sinh\psi}\cdot\dfrac{\sinh\psi}{\cosh\psi} = 1$

~~$(-x'\sinh\psi + x^4\cosh\psi)$~~ $\quad (x'\cosh\psi - x^4\sinh\psi)^2 - (-x'\sinh\psi + x^4\cosh\psi)^2$

$= x'^2(\cosh^2\psi - \sinh^2\psi) - x^{4\,2}(\cosh^2\psi - \sinh^2\psi)$

$= (x'^1)^2 - (x'^4)^2$

$x'^1\sinh\psi + x'^4\cosh\psi$

$= x^4$

$x'^1\cosh\psi + x'^4\sinh\psi = x^1$

$$\left.\begin{array}{l} x'^1 \cancel{\sinh\psi'} - x'^4 \cancel{\cosh\psi'} = 0 \\ x'^1 \cancel{\cosh\psi' - x'^4\sinh\psi'} = 0 \end{array}\right\}$$

$(x'^1\cosh\psi + x'^4\sinh\psi)\sinh\psi = (x'^1\sinh\psi + x'^4\cosh\psi)\cosh\psi$

$= 0$

$\therefore x'^4 = 0$ & $x'^1 = 0$?

$$\frac{y}{y_1} = \frac{x}{x_1}$$

$$\left.\begin{array}{l} xx_1 - yy_1 = 1 \\ xy_1 - yx_1 = 0 \end{array}\right\} \quad y = \frac{x_1}{y_1}\cdot x \qquad m\cdot\frac{x_1}{y_1} = 1$$

$$y = m\cdot x \qquad m = \frac{y_1}{x_1}$$

$$x_1^2 - y_1^2 = 1 \qquad y = \frac{y_1}{x_1}\cdot x$$

$$x\cosh\psi - y\sinh\psi = \qquad y = x\coth\psi$$

$$x^2(1 - \coth^2\psi) = 1 \quad -x^2 = \sinh^2\psi$$

$$x = i\sinh\psi,\ y = i\cosh\psi.$$

$$x'^1\cosh\psi - x'^4\sinh\psi$$

$$(x'\cosh\psi - x^4\sinh\psi)\cosh\psi$$

$$-(+x'\sinh\psi + x^4\cosh\psi)\sinh\psi = 0.$$

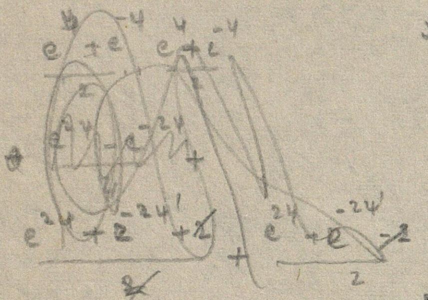
$$x'(\cosh^2\psi + \sinh^2\psi) - 2x^4\sinh\psi\cosh\psi$$

$$(x'\cosh\psi - x^4\sinh\psi)\cosh\psi'$$

$$-(-x'\sinh\psi + x^4\cosh\psi)\sinh\psi' = 0$$

$$x'(\cosh\psi\cosh\psi' + \sinh\psi\sinh\psi') - x^4(\sinh\psi\cosh\psi' + \cosh\psi\sinh\psi')$$

$$x'\cosh(\psi+\psi') - x^4\sinh(\psi+\psi').$$



$$x'^1\cosh\psi' - x'^4\sinh\psi'$$

$$= (x'\cosh\psi - x^4\sinh\psi)\sinh\psi' - (-x'\sinh\psi + x^4\cosh\psi)\sinh\psi$$

$$= x'(\cosh\psi\sinh\psi' + \sinh\psi\cosh\psi')$$

$$- x^4(\sinh\psi\sinh\psi' + \cosh\psi\cosh\psi') = 0$$

$$\frac{e^{\psi}+e^{-\psi'}}{2}\cdot\frac{e^{\psi'}+e^{-\psi'}}{2} + \frac{e^{\psi}-e^{-\psi}}{2}\cdot\frac{e^{\psi'}-e^{-\psi'}}{2}$$

$$\frac{e^{\psi+\psi'} + e^{-(\psi+\psi')} + e^{(\psi-\psi')} + e^{-(\psi-\psi')}}{4}$$

$$x'\sinh(\psi+\psi) - x^4\cosh(\psi+\psi') = 0$$

$$\left.\begin{array}{l} \sinh(x\pm y) = \sinh x\cosh y \pm \cosh x\sinh y \\ \cosh(x\pm y) = \cosh x\cosh y \pm \sinh x\sinh y \end{array}\right\}$$

$$l = l_0/\cosh\psi,\quad \underline{t = \tau\cosh\psi}$$

Velocity

Sygn & defn $\quad u^i = \dfrac{dx^i}{d\tau}$

$$u' = \frac{dx'}{dt}\cdot\frac{dt}{d\tau} = \frac{dx^1/dt}{\sqrt{1-u^2/c^2}},\quad (u', u^2, u^3) = \frac{\vec{u}}{\sqrt{1-u^2/c^2}}$$

$$u^4 = \frac{dx^4}{dt}\cdot\frac{dt}{d\tau} = \frac{ic}{\sqrt{1-u^2/c^2}}$$

Transformation formulae

$$\left\{\begin{array}{l} u'^1 = u'\cos\phi + u^4\sin\phi \\ u'^4 = -u'\sin\phi + u^4\cos\phi \end{array}\right. ;\quad u'^1 = \frac{u' + (v/c)u^4}{\sqrt{1-v^2/c^2}},\quad u'^4 = \ldots$$

$$\left.\begin{array}{l} \tan\phi = i\beta \\ \tanh\psi = \beta \\ \phi = i\psi \end{array}\right]$$

Addition theorem for velocities in same direction:

$$= \frac{v_1 + v_2}{1 + v_1v_2/c^2} \longrightarrow \frac{-ic\tan\phi_1 - ic\tan\phi_2}{1 - \tan\phi_1\tan\phi_2}$$

$$\left.\begin{array}{l} \frac{iv}{c} = \tan\phi \\ v = -ic\tan\phi \\ w = -ic\tan(\phi_1+\phi_2) \end{array}\right]$$

$$= -ic\,\frac{\tan\phi_1 + \tan\phi_2}{1 - \tan\phi_1\tan\phi_2}\ \cancel{-ic\tan(\phi_1+\phi_2)}.$$

$$\text{ie } w = \tan(\phi_1+\phi_2)$$

$\therefore$ 2 rotations ϕ_1 & ϕ_2 can be added & yield $\phi_1 + \phi_2$

(2).

$u = \frac{dx}{dt}$ $u' = \frac{dx'}{dt'}$

$$= \frac{1}{c^2}\left\{c^2\left(1+\frac{vv'}{c^2}\right)^2 - (v+v')^2\right\} = \frac{1}{c^2}\left\{c^2 + \cancel{2vv'} + \frac{v^2v'^2}{c^2} - v^2 - v'^2 - \cancel{2vv'}\right\}$$

$$= \left\{1 - \frac{v^2}{c^2} - \frac{v'^2}{c^2} + \frac{v^2v'^2}{c^4}\right\} = (1-\beta^2)(1-\beta'^2)$$

beautiful given by me

$$t'' = \frac{t' - v'x'/c^2}{\sqrt{1-\beta'^2}} = \frac{t - vx/c^2 - \frac{v'}{c^2}(x - vt)}{\sqrt{1-\beta^2}\cdot\sqrt{1-\beta'^2}} = \frac{t(1 + vv'/c^2) - (v+v')\,x/c^2}{\sqrt{1-\beta^2}\sqrt{1-\beta'^2}}$$

$$v'' = \frac{v+v'}{1+vv'/c^2}, \quad t'' = \frac{(1+vv'/c^2)}{\sqrt{1-\beta^2}\sqrt{1-\beta^2}}\cdot(t - v''x/c^2)$$

$$\left.\begin{aligned} x' &= \frac{x - v_x t}{\sqrt{1-\beta^2}} \\ y' &= \frac{y - v_y t}{\sqrt{1-\beta^2}} \\ z' &= \frac{z - v_z t}{\sqrt{1-\beta^2}} \\ t' &= \frac{t - vx/c^2}{\sqrt{1-\beta^2}} \end{aligned}\right\}$$

(B) Rel. Kinematics:

$x^i (x^1, x^2, x^3, x^4)$ or x_i $\quad ds = c\,d\tau$

vel 4-vector $= \frac{dx^i}{d\tau}$

(B) Lorentz contraction & time dilatation

(C)

$$\left.\begin{aligned} x_2' - x_1' &= l_0 \ (\text{in moving } K') \\ x_2(t) - x_1(t) &= l \ (\text{in stationary system}) \\ \frac{x_2(t) - vt}{\sqrt{1-\beta^2}} &- \frac{x_1(t) - vt}{\sqrt{1-\beta^2}} = l_0 \end{aligned}\right\}$$

$x_2 - x_1 = l_0$

$x_2'(t') - x_1'(t') = l$

$$l_0 = \frac{x_2' + vx'/c^2}{\sqrt{1-\beta^2}} - \frac{x_1' + vx'/c^2}{\sqrt{1-\beta^2}} = \frac{l}{\sqrt{1-\beta^2}}$$

$$\frac{l}{\sqrt{1-\beta^2}} = l_0 \quad \text{or } \underline{l = l_0\sqrt{1-\beta^2}} \text{ contraction}$$

$$\tau_0 = \frac{t_2 - vx}{\tau_0} \ \cancel{\frac{t_2 + vx/c^2}{\sqrt{1-\beta^2}}} - \cancel{\frac{t_1 - vx/c^2}{\sqrt{1-\beta^2}}}$$

$$\left.\begin{aligned} \tau_0 &= t_2' - t_1' \ (\text{in } K') \\ \tau &= t_2 - t_1 \ (\text{in } K) \end{aligned}\right\} \longrightarrow t = \frac{t_0}{\sqrt{1-\beta^2}} \text{ dilatation}$$

$$\tau = \cancel{\frac{t_2' + vx'/c^2}{\sqrt{1-\beta^2}}} - \cancel{\frac{t_1' + vx'/c^2}{\sqrt{1-\beta^2}}} = \cancel{\frac{\tau}{\sqrt{1-\beta^2}}} \quad \tau = \tau_0\sqrt{1-\beta^2}$$

(E) Mostly put $x' = 0$ in $t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}}$

$dx^2 + dy^2$

or $t = \tau_0/\sqrt{1-\beta^2}$ $\quad x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$ $\quad t_0$

$$= \frac{t_2' - t_1'}{\sqrt{1-\beta^2}} = \frac{\tau_0}{\sqrt{1-\beta^2}} \,//$$

(B) (C) Space reflection, time reversal & total inversion P, T, PT are also L.T's, but of no significance since invariance under L_4 is a minimal requirement. Hence consideration of subgroups (L_4, PL_4), (L_4, TL_4), (L_4, PTL_4) of the total L.T. each containing L_4 as a proper subgroup. These signif. in theory of elem. particles.

(D) Rel. Kinematics:

(i) 4 dimnl representation of L.T's in terms of ϕ & ψ ($\phi = i\psi$) $\quad \left.\begin{aligned} x^1 = x, x^2 = y, x^3 = z, x^4 = ict \\ x^4 = ct \end{aligned}\right\}$

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}, \quad t' = \frac{t - vx/c^2}{\sqrt{1-\beta^2}}, \quad \left[\tan\phi = i\beta, \quad v = -ic\tan\phi \quad \tanh\psi = \beta\right.$$

$$x' = \frac{x + ic\tan\phi\cdot t}{\sqrt{1+\tan^2\phi}} = x\cos\phi + ic\sin\phi\, t, \quad \left.\begin{aligned} x_1' &= x_1\cos\phi + x_4\sin\phi \\ x_4' &= -x_1\sin\phi + x_4\cos\phi \end{aligned}\right\} \quad \boxed{x_4 = ict}$$

(A)

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}, \quad t' = \frac{t - vx/c^2}{\sqrt{1-\beta^2}}$$

$v = 0 \to$ identity

$$x'\sqrt{1-\beta^2} = x - vt$$
$$t'\sqrt{1-\beta^2} = t - vx/c^2$$

$$\sqrt{1-\beta^2}\cdot vx'/c^2 + t'\sqrt{1-\beta^2} = t(1-\beta^2)$$

$$\sqrt{1-\beta^2}(x' + vt') = x(1-\beta^2) \text{ or } x = \frac{x' + vt'}{\sqrt{1-\beta^2}}, \quad t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}} \quad \text{inverse}$$

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$

may be proved

$$\cancel{x'' = \frac{x' - vt'}{\sqrt{1-\beta^2}}, \quad t'' = \frac{t' - vx'/c^2}{\sqrt{1-\beta^2}}}$$

$$\cancel{x'' = \frac{(x-vt) - v(t - vx/c^2)}{(1-\beta^2)}} = \qquad x'' = \frac{x' + vt'}{\sqrt{1-\beta^2}}, \quad t'' = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}}$$

$$x'' = \frac{x - vt + v(t - vx/c^2)}{(1-\beta^2)}, \quad t'' = \frac{(t - vx/c^2) + v/c^2(x - vt)}{(1-\beta^2)}$$

$x'' = x, \quad t'' = t$ beautiful given by Einstein

$s = c\tau$ once

$x_4 = ct$

$ds = c\,d\tau$ $\quad u^i = \frac{dx^i}{d\tau}$

$$\frac{dx_i}{d\tau}, \frac{dx_0}{d\tau}$$

$$\cancel{\frac{c\,dt}{d\tau} = \frac{c\,dt}{ds}}$$

$$\text{or } x'' = \frac{x' - v't'}{\sqrt{1-\beta'^2}}, \quad t'' = \frac{t' - v'x'/c^2}{\sqrt{1-\beta'^2}}$$

$$x'' = \frac{x - vt - v'(t - vx/c^2)}{\sqrt{(1-\beta^2)}}$$

$$= \frac{x(1 + vv'/c^2) - t(v + v')}{\sqrt{1-\beta^2}\sqrt{1-\beta'^2}}$$

$$= \frac{1 + vv'/c^2}{\sqrt{1-\beta^2}\sqrt{1-\beta'^2}}(x - v''t)$$

Let $\left(\dfrac{v + v'}{1 + vv'/c^2}\right) = v''$

Addition of velocities

$$\frac{dx}{dt} = v'$$

$$\frac{dx'}{dt'} = \frac{dx' + v\,dt'}{dt' + v\,dx/c^2} = \frac{v + v}{1 + vv'/c^2}.$$

$$1 - v''^2 = 1 - \frac{(v + v')^2}{(1 + vv'/c^2)^2}$$

$$= \frac{(1 + vv'/c^2)^2 - (v + v')^2}{(\quad)^2}$$

$$= \frac{1 + \frac{2vv'}{c^2} + \frac{v^2v'^2}{c^4} - v^2 - 2vv' - v'^2}{(\quad)^2}$$

$$= \frac{(1 + v + v' + vv'/c^2)(1 - v - v' + vv'/c^2)}{(\quad)^2}$$

$$\frac{1 + vv'/c^2}{1-\beta^2} = \frac{1}{1-\beta''^2}$$

$$1 + vv'/c^2 = \frac{1-\beta^2}{\sqrt{1-\beta''^2}}$$

$$\sqrt{(1-\beta''^2)}\,\cancel{(1 + vv'/c^2)} = \sqrt{1-\beta^2}$$

$$1 + \cancel{\frac{vv'}{c^2} - \beta''^2 + vv'\beta}$$

$$1 - \frac{v''^2}{c^2} = \frac{(1 - v^2/c^2)^2}{(1 + vv'/c^2)^2}$$

x'^1, x'^4

$$= -x'^2\cosh\psi\sinh\psi - x^{4\,2}\cosh\psi\sinh\psi$$
$$- 2x'x^4(\cosh^2\psi + \sinh^2\psi)$$

$$1 - \frac{(v + v'/1 + vv'/c^2)^2}{c^2} = 1 - \frac{(v + v')^2}{c^2(1 + vv'/c^2)^2}$$

$$= \frac{c^2(1 + vv'/c^2)^2 - (v + v')^2}{c^2(1 + vv'/c^2)^2} = c^2 + \cancel{2vv'} + vv'/c^2 - v^2 - v'^2 - \cancel{2vv'}$$

$$\frac{\dfrac{1-\beta^2}{1 + vv'/c^2}}{\sqrt{1-\beta^2}\cdot\sqrt{1-\beta'^2}} = \frac{1}{\sqrt{1-\beta''^2}}$$

$$(1 + vv'/c^2)^2(1-\beta''^2)$$
$$= (1-\beta^2)(1-\beta'^2)$$

$$(1 + vv'/c^2)^2\left\{1 - \frac{(v + v')^2}{c^2(1 + vv'/c^2)^2}\right\} = (1 + vv'/c^2)^2\left\{\frac{c^2(1 + vv'/c^2)^2 - (v + v')^2}{c^2(1 + vv'/c^2)^2}\right\}$$

Rel. Kinematics:

(1) $x_2' - x_1' = l_0$ $\qquad \dfrac{x_2(t) - vt}{\sqrt{1-\beta^2}} - \dfrac{x_1(t) - vt}{\sqrt{1-\beta^2}} = l_0$

$x_2(t) - x_1(t) = l$

$$\frac{l}{\sqrt{1-\beta^2}} = l_0 \quad \text{or } l = l_0\sqrt{1-\beta^2} \text{ — Lorentz Contraction}$$

$$t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}} \qquad t' = \tau \text{ (proper time) at } x' = 0, \text{ gives } t = \frac{\tau}{\sqrt{1-\beta^2}} = \text{dilatation in time}$$

(2) Special L.T's in particular forms

(1) $x^1 = x,\ x^2 = y,\ x^3 = z,\ x^4 = ict$

(2) $x^1 = x,\ x^2 = y,\ x^3 = z,\ x^4 = ct$

using $x^i x_i = s^2$, first set has covariant & contravariant vectors the same, the second set has $x_4 = -ct$

using $\tan\phi = i\beta$, the L.T eqns become for (1)

$v = -ic\tan\phi$

$\beta^2 = -\tan^2\phi$

$$x'^1 = \frac{x^1 - vx^4/ic}{\sqrt{1-\beta^2}},\quad x'^2 = x^2,\quad x'^3 = x^3,\quad x'^4 = \frac{x^4 - \cancel{ic}\, vx^1/c^2}{\sqrt{1-\beta^2}}$$

$$= \frac{x^1 + x^4\tan\phi}{\sec\phi}$$

$$= x^1\cos\phi + x^4\sin\phi \quad x'^2 = x^2,\ x'^3 = x^3,\ x'^4 = -x^1\sin\phi + x^4\cos\phi.$$

i.e. a real rotation

$$\sec^2\phi = 1 + \tan^2\phi = 1 + v^2/i^2c^2 = 1 - \beta^2$$

$$\cos\phi = \frac{1}{\sqrt{1-\beta^2}} \qquad \sin\phi = \frac{i\beta}{\sqrt{1-\beta^2}}$$

For (2) use $\tanh\psi = \beta$ $\qquad \tan i\psi = i\tanh\psi$.

$$x'^1 = x_1\cosh\psi - x_4\sinh\psi$$
$$x'^4 = -x_1\sinh\psi + x_4\cosh\psi$$

$$\left[\begin{aligned} l &= l_0/\cosh\psi \\ t &= \tau\cosh\psi \end{aligned}\right]$$

$$(x'^1)^2 - (x'^4)^2 = (x^1)^2 - (x^4)^2$$

Conjugate diams $\rightarrow$ conj. diams. on $(x^1)^2 - (x^4)^2 = 1$.

Addition of velocities $w = \dfrac{v_1 + v_2}{1 + v_1v_2/c^2}$ & using $v_1 = -ic\tan\phi \rightarrow ic\tan\phi = ic\tan(\phi_1 + \phi_2)$

$$= \frac{\tan\phi_1 + \tan\phi_2}{1 - \tan\phi_1\tan\phi_2}$$

Sommerfeld $\qquad$ Add rotations

(3) Velocity $\quad u^i = \dfrac{dx^i}{d\tau},\quad (u^1, u^2, u^3) = \dfrac{\vec{u}}{\sqrt{1-u^2/c^2}},\quad u^4 = \dfrac{ic}{\sqrt{1-u^2/c^2}}$

Transformation tomula $\quad u'^1 = u^1\cos\phi + u^4\sin\phi$
$\qquad u'^4 = -u^1\sin\phi + u^4\cos\phi$

(D) Acc^n $\quad B^i = \frac{du^i}{d\tau} = \frac{d^2x^i}{d\tau^2}$.

$$u^i u_i = -c^2, \quad u_i \frac{du^i}{d\tau} + u^i \frac{du_i}{d\tau} = 0, \quad 2u_i \frac{du^i}{d\tau} = 0, \quad u_i B^i = 0$$

$$(B^1, B^2, B^3) = \frac{d\vec{u}}{d\tau} = \frac{\dot{\vec{u}}}{1-\beta^2} + \frac{\vec{u}\,(\vec{u}\cdot\dot{\vec{u}})}{c^2} \cdot \frac{1}{(1-\beta^2)^2} \qquad [\beta = u/c$$

$$B^4 = i\,\frac{\vec{u}\cdot\dot{\vec{u}}}{c} \cdot \frac{1}{(1-\beta^2)^2}$$

K' moving instantaneously with the medium i.e. in K' $(B'^1, B'^2, B'^3, B'^4 = 0)$ & ~~[illegible]~~

Relative to K matter moves with vel $\vec{u}$ & taking this as x-axis we have

$$B^1 = \frac{\dot{u}_x}{(1-\beta^2)} + \frac{\beta}{i} B^4, \quad B^2 = \frac{\dot{u}_y}{1-\beta^2}, \quad B^3 = \frac{\dot{u}_z}{1-\beta^2}. \qquad \text{—(A)}$$

* Taking inverse of rotation, the corresponding components in K are

$$(XX)\quad B^1 = \frac{B'^1}{\sqrt{1-\beta^2}}; \quad B^2 = B'^2, \quad B^3 = B'^3, \quad B^4 = \frac{i\beta B'^1}{\sqrt{1-\beta^2}} \qquad \text{—(B)}.$$

Comparing (A) & (B)

$$\left.\begin{aligned} \dot{u}_x &= \dot{u}'_x (1-\beta^2)^{3/2} \\ \dot{u}_y &= \dot{u}'_y (1-\beta^2) \\ \dot{u}_z &= \dot{u}'_z (1-\beta^2) \end{aligned}\right\} \text{(C)}$$

$$\frac{\dot{u}_x}{1-\beta^2} + \frac{\beta}{i} \cdot \frac{i\beta B'^1}{\sqrt{1-\beta^2}} = \frac{B'^1}{\sqrt{1-\beta^2}}$$

or $\frac{\dot{u}_x}{1-\beta^2} = \frac{B'^1}{\sqrt{1-\beta^2}}(1-\beta^2) = B'^1\sqrt{1-\beta^2}$

or $\dot{u}_x = \dot{u}'_x (1-\beta^2)^{3/2}$

Constant acceleration $\dot{u}'_x = b$, leads to $x^2 - c^2t^2 = c^4/b^2$ i.e. hyperbolic motion

---

$$B^1 = \frac{\dot{u}_x}{1-\beta^2} + \frac{\beta}{i} \cdot \frac{i\beta B'^1}{\sqrt{1-\beta^2}} \qquad \cancel{\frac{\beta^2 \beta'^1}{\sqrt{1-\beta^2}} = B' - \frac{\dot{u}_x}{1-\beta^2}}$$

$$\frac{\dot{u}_x}{1-\beta^2} = -\frac{\beta^2 B'^1}{\sqrt{1-\beta^2}} + \frac{B'^1}{\sqrt{1-\beta^2}} = \dot{u}'_x / \sqrt{1-\beta^2}.$$

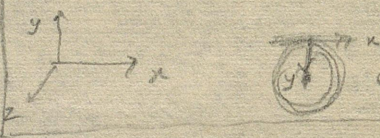
(A)

$$B^1 = \frac{\dot{u}_x}{(1-\beta^2)} + \frac{u^2 \cdot \dot{u}_x}{c^2(1-\beta^2)^2}$$

$$B^4 = i\,\frac{u \cdot \dot{u}_x}{c} \cdot \frac{1}{(1-\beta^2)^2}$$

$$\frac{\beta}{i} B^4 = \frac{u^2}{c^2} \cdot \frac{\dot{u}_x}{(1-\beta^2)^2} \qquad [\beta = u/c]$$

Also $u_y = 0$ does not mean $\dot{u}_y = 0$.



σ^r motion

* For acc^{ns} also

$$\left.\begin{aligned} B'^1 &= B^1\cos\phi + B^4 \sin\phi \\ B'^4 &= -B^1 \sin\phi + B^4\cos\phi \end{aligned}\right\}$$

& inverse is given by

$$B^1 = B'^1\cos\phi - B'^4 \sin\phi = (B'^1 - i\beta B'^4)/\sqrt{1-\beta^2} \qquad \text{using } \tan\phi = i\beta.$$

$$B^4 = B'^1 \sin\phi + B'^4\cos\phi = (i\beta B'^1 + B'^4)/\sqrt{1-\beta^2}$$

(XX) ~~Using~~ with $B'^4 = 0$ in the K' system

$$B^1 = \frac{B'^1}{\sqrt{1-\beta^2}}; \quad B^4 = \frac{i\beta B'^1}{\sqrt{1-\beta^2}}, \quad B^2 = B'^2, \quad B^3 = B'^3.$$

with $\dot{u}_x' = b$, (C) ie Einstein's accn eqns give for the first one

$$\frac{\dot{u}_x}{(1-u_x^2/c^2)^{3/2}} = b \quad \text{ie} \quad \frac{\dot{u}_x u_x}{c^2(1-u_x^2/c^2)} = \frac{b u_x}{c^2}$$

Integrating $(1-u_x^2/c^2)^{-1/2} = b(x-x_0)/c^2$

$\left[-\tfrac{1}{2}(1-u_x^2/c^2)^{-3/2}\cdot -2u_x\dot{u}_x/c^2\right]$ $\quad \left(\frac{c^2-u_x^2}{c^2}\right)^{-1} = b(x-x_0)^2/c^2$

$$\frac{c^2}{c^2-u_x^2} = b^2(x-x_0)^2/c^4 \quad \text{or} \quad \frac{c^2-u_x^2}{c^2} = \frac{c^4}{b^2}\cdot\frac{1}{(x-x_0)^2}$$

$\frac{a-b}{a}$

$$1-\frac{u_x^2}{c^2} = \frac{c^4}{b^2}\cdot\frac{1}{(x-x_0)^2} \qquad \frac{u_x^2}{c^2} = 1-\frac{c^4}{b^2(x-x_0)^2} = \frac{b^2(x-x_0)^2-c^4}{b^2(x-x_0)^2}$$

$$\frac{u_x}{c} = \frac{\sqrt{b^2(x-x_0)^2-c^4}}{b(x-x_0)c} \quad \text{or} \quad \frac{dx\cdot b(x-x_0)}{\sqrt{b^2(x-x_0)^2-c^4}} = c\cdot dt$$

~~Again integrating putting~~ ie $\frac{dx\cdot(x-x_0)}{\sqrt{(x-x_0)^2-c^4/b^2}} = c\,dt$

Again integrating $\sqrt{(x-x_0)^2-c^4/b^2} = c(t-t_0)+k$

$\left[\tfrac{1}{2}\{(x-x_0)^2-c^4/b^2\}^{-1/2}\cdot 2(x-x_0)\cdot dx\right]$ ie $(x-x_0)^2-\frac{c^4}{b^2} = \cancel{\{c(t-t_0)+k\}^2} = c^2(t-t_0)^2$ choosing k properly.

$$(x-x_0)^2-c^2(t-t_0)^2 = \frac{c^4}{b^2} = \text{const} = a^2 \quad \text{—(D)}$$

$\left[\frac{dx}{dt}=0,\ x=x_0\right]$

$2(x-x_0)\frac{dx}{dt} - 2c^2(t-t_0) = 0$

$\frac{dx}{dt}=0,\ t=t_0\ \frac{c^2(t-t_0)}{x-x_0}=0$

$t=0,\ \frac{-c^2t_0}{x-x_0} = 0$ ie $t_0=0$.

If $t_0=0$, $(x-x_0)^2 = \frac{c^4}{b^2}$.

If $x = c^2/b$, $\left(\frac{c^2}{b}-x_0\right)^2 = \frac{c^4}{b^2}$

ie $x_0=0$

ie a suitable choice to bring (D) to form

$$x^2-c^2t^2 = a^2 \quad \text{— (E)}$$

is to choose origin of space & time-coords by

$$t=0,\ \dot{x}=0,\ x=c^2/b$$

Thus (E) shows that "uniformly accelerated motion" in relativity ~~conforms~~ is hyperbolic motion in contrast to parabolic motion in classical mechanics

~~In (E) parametrically $x^1 = a\cos\frac{s}{a},\ x^4 = a\sin\frac{s}{a}$.~~

~~($s = ict$)~~

~~$x^{1\,2} - x^{4\,2} = a^2\cos^2\frac{s}{a} + a^2\sin^2\frac{s}{a}\cdot i$~~

(B) Proof of $E = mc^2$.

using $f^i = \frac{d}{d\tau}(m_0 u^i)$

$$(f^1, f^2, f^3) = \frac{1}{\sqrt{1-u^2/c^2}} \frac{d}{dt}\left(\frac{m_0}{\sqrt{1-u^2/c^2}}\vec{u}\right) = \frac{d}{dt}(m\vec{u}).$$

Introducing the Minkowski force $(f_1, f_2, f_3) = \frac{\vec{K}}{\sqrt{1-\beta^2}}$. (in contrast to Newtonian force)

These eqns can be written $\frac{d}{dt}(m\vec{u}) = \vec{K}$

$$f^4 = \frac{1}{\sqrt{1-u^2/c^2}} \cdot \frac{d}{dt}\left(m_0 \frac{ic}{\sqrt{1-u^2/c^2}}\right) = \frac{i}{c\sqrt{1-m^2/c^2}} \cdot \frac{d}{dt}(mc^2)$$

$$\frac{\cancel{i}(\vec{f}\cdot\vec{u})}{\cancel{c}} = \frac{i}{\cancel{c}\sqrt{1-u^2/c^2}} \cdot \frac{d}{dt}(mc^2)$$

$$(\vec{f}\cdot\vec{u})\sqrt{1-u^2/c^2} = \vec{K}\cdot\vec{u}.$$

$$\frac{d}{dt}(mc^2) = \vec{K}\cdot\vec{u}$$

$\frac{d}{dt}(mc^2) = K \cdot u =$ [illegible]

using $f_K u^K = 0$ follows from $B_K u^K = 0$

$$\because \vec{f}\sqrt{1-u^2/c^2} = \vec{K}.$$

$$E_{kin} = mc^2 + \text{Const}$$
$$= \frac{m_0 c^2}{\sqrt{1-\beta^2}} + \text{Const}.$$

$$f_4 = \frac{i(\vec{f}\cdot\vec{u})}{c}.$$

$$f_i u^i = 0 \qquad \frac{\vec{f}\vec{u}}{\cancel{\sqrt{1-\beta^2}}} + f^4 \cdot \frac{ic}{\cancel{\sqrt{1-\beta^2}}} = 0 \qquad \cancel{f^4 = \frac{(\vec{f}\cdot\vec{u})\sqrt{1-\beta^2}}{ic}} = \frac{i}{c}(\vec{f}\cdot\vec{u})$$

$$f^4 = -\frac{(\vec{f}\cdot\vec{u})}{ic} = \frac{i}{c}(\vec{f}\cdot\vec{u}).$$

Topics for 17/7/67 (Sq. matrices A)

(1) $1.A = A.1$, $A^2, \cdots A^r$, $A^{r+s} = A^{s+r}$.

(2) $|A|\cdot|B| = |AB| = |B|\cdot|A| = \det|BA|$

$|kA| \neq k|A|$

(3) Singular & non-singular matrices - Law of division of matrices.

(4) Transpose, adjoint and reciprocal matrices, - Reciprocal as inverse - Law of reversal for transposes, and reciprocals - Commutative property of transposes & reversal - Ex. for finding an inverse.

(5) Diagonal, permutation and triangular matrices - Symmetric & skew-symmetric matrices - Splitting as sum Hermitian and unitary matrices, & orthogonal.

(6) Characteristic eqn $|A-\lambda.I| = 0$ or $(-1)^n(\lambda^n + p_1\lambda^{n-1} + \cdots + p_n) = 0$ - Cayley-Hamilton Theorem: $A^n + p_1 A^{n-1} + \cdots + p_n = 0$. - $|g(A)| = g(\lambda_1)\, g(\lambda_2) \cdots g(\lambda_n)$ - ~~Eigenvalues~~ of Similar matrices - Diagonalisation leading to eigenvalue problem - reality of eigenvalues of a Hermitian matrix - Similar matrices have same eigenvalues.

(7) Rank of a matrix - The three elementary transformations - Equivalent matrices and their having the same rank - Redn of matrix of rank r to normal form by elementary trns.

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ANNUAL REPORT 1977-78

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$$\mathrm{div}\,\vec{E} = 4\pi\rho\ ; \quad \mathrm{div}\,\vec{H} = 0.$$

$$\left.\mathrm{rot}\,\vec{E} + \frac{1}{c}\frac{\partial \vec{H}}{\partial t} = 0 \quad , \quad \mathrm{rot}\,\vec{H} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}\ \right\}\ \text{Max. eqns}$$

---

(1) Deduction of $\frac{\partial \rho}{\partial t} + \mathrm{div}\,\vec{j} = 0$.

(2) Expn of $\vec{E}$ & $\vec{H}$ in terms of $(\vec{A}, \phi)$ potential.

$$\vec{E} = -\frac{1}{c}\frac{\partial A}{\partial t} - \mathrm{grad}\,\phi\ ; \quad \vec{H} = \mathrm{rot}\,\vec{A}$$

(3) Eqns two & three of Maxwells are identically satisfied by these expns.

(4) Gauge trn of A & ϕ giving same E & H. $\left.\begin{array}{l} \vec{A}' = \vec{A} - \mathrm{grad}\,\psi \\ \phi' = \phi - \frac{1}{c}\frac{\partial \psi}{\partial t} \end{array}\right\}$

(5) Eqns 1 & 4 of Maxwell put in terms of A & ϕ.

(6) Gauge trn such that $\mathrm{div}\,A' + \frac{1}{c}\frac{\partial^2 \phi'}{\partial t^2} = 0$ (L. Cond.)

(7) Eqns in 5 $\rightarrow$

$$\left.\begin{array}{l} \nabla^2 A' - \frac{1}{c^2}\frac{\partial^2 A'}{\partial t^2} = -\frac{4\pi}{c}\,j \\ \nabla^2 \phi' - \frac{1}{c^2}\cdot\frac{\partial^2 \phi'}{\partial t^2} = -4\pi\rho \end{array}\right\}$$

$$\frac{c}{4\pi}(\vec{E} \times \vec{H})$$

Poynting Vector.

5/6/67

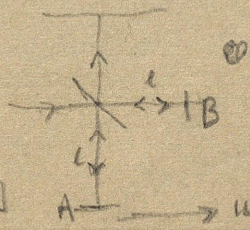
---

(1) Maxwell's eqn not covariant under $x' = x - vt$, $y' = y$, $z' = z$, $t' = t$ i.e. Gal. trns of classical mechanics is proved by showing that the law of addition of velocities in classical mechanics is not valid in Classical Electrodynamics viz

(2) M. M. Expt

$\xrightarrow{v-u}$ u
$\xleftarrow{v+u}$
$\xrightarrow[v]{}$

$\underline{v' = v + u}$. Similarly $c' = c + u$ should hold, but Michelson-Morley expt shows this is not true.



$t_1 = \frac{l}{c-u} + \frac{l}{c+u} = \frac{2lc}{c^2-u^2}$

$t_1 = \frac{2l}{c(1-u^2/c^2)}$

$= 2l/c\,(1-u^2/c^2)^{-1}$

$t_2 = \frac{2l}{\sqrt{c^2-u^2}} = \frac{2l}{c}(1-u^2/c^2)^{-1/2}$

$t_1 \neq t_2$ $t_1 = t_2$ only if $u = 0$ or $c = \infty$

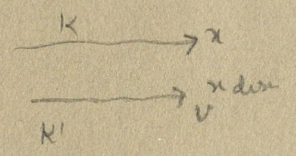
[u may be Earth's motion]

but M.-M. expt shows $t_1 = t_2$ without either $u = 0$ or $c = \infty$. – Attempts at explan of this famous expt by using hypothetical ether as an elastic medium in

(3) which light travels all failed ~~tt~~ – Einstein ~~made~~ created the postulate of relativity viz. phenomena in a system are independent of the uniform translatory motion of the system as a whole. or physical phenomena are the same in all general Galilean reference systems (ie steadily uniform translatory motion)

(4) Velo $\underline{c}$ ind. of vel of source (2nd postulate or axiom?) or relativity of simultaneity [$c(t_1 - t_2) = AB$ has meaning if hands of both clocks at A & B simultaneously are in the same posn] – ~~De~~ de Sitter's observations on double stars showing $\underline{c}$ ind. of vel of source & vel. of observer & ind. of dirn [Light signals are propagated rectilinearly with the same constant velocity $\underline{c}$ at all times in all dirns, in all gen. Gal. frames.

(5) Derivation of L.T. from above two postulates.



(i) K' moves rel to K in x-dirn with vel. v
(ii) trns should be linear since uniform rect. motion in K is also uniform & rect. in K'
(iii) finite coords in K are finite in K'
(iv) validity of Euclidean geometry & homogeneous nature of space & time.

$$x^2+y^2+z^2-c^2t^2=0 \longrightarrow x'^2+y'^2+z'^2-c^2t'^2=0$$

$(x'^2 + \ldots) = K(x^2 + \ldots)$, K depends on $\underline{v}$ ~~since not ... only ...~~ ~~... so after trn~~

$\boxed{\begin{matrix} r = ct \\ r' = ct' \end{matrix}}$

$$x'^2 - c^2t'^2 = K(x^2 - c^2t^2)$$

$$x' = K \cdot \frac{x - vt}{\sqrt{1-\beta^2}}, \quad y' = K \cdot y, \quad z' = K z, \quad t' = K \cdot \frac{t - (v/c^2)x}{\sqrt{1-\beta^2}} \quad [\beta = v/c$$

$$x'' = K(-v) \cdot \frac{x' + vt'}{\sqrt{1-\beta^2}}, \quad \ldots, \quad t' = K(-v)\ t +$$

$$x'' = K(v)K(-v)x, \ldots$$

$$x'' = x, \quad K(v)K(-v) = 1 \quad \text{or } K(v) = K(-v) = 1.$$

hence the L.T. – Inverse of L.T.

<u>12/6/67</u> – (1) <u>Group property of the special L.T.'s</u>, $x' = \dfrac{x - vt}{\sqrt{1-\beta^2}}$, $t' = \dfrac{t - vx/c^2}{\sqrt{1-\beta^2}}$

(i) unit element $\longrightarrow v = 0$

(ii) $x'' = \dfrac{x' - v't'}{\sqrt{1-\beta'^2}}$, $t'' = \dfrac{t' - v'x'/c^2}{\sqrt{1-\beta'^2}}$

$$= \frac{x - vt - v'(t - vx/c^2)}{\sqrt{1-\beta^2} \cdot \sqrt{1-\beta'^2}} =$$

$$\frac{(x-vt)^2}{1-\beta^2} + y^2 + z^2 - c^2 \left\{ \frac{(t - vx/c^2)^2}{1-\beta^2} \right\}$$

$$= \frac{1}{(1-\beta^2)} \left\{ (x-vt)^2 + \cancel{\ldots} - c^2(t - vx/c^2)^2 \right\} + y^2 + z^2$$

$$= \frac{1}{(1-\beta^2)} \left\{ x^2 + v^2t^2 - \cancel{2xvt} + \cancel{\ldots} - c^2\left(t^2 + \frac{v^2x^2}{c^4} - \cancel{\frac{2vtx}{c^2}}\right) \right\} +$$

$$= \frac{1}{(1-\beta^2)^2} \left\{ x^2\left(1 - \frac{v^2}{c^2}\right) + t^2(v^2 - c^2) \right\} = \frac{1}{c^2(1-\beta^2)} \left\{ (c^2 - v^2)(x^2 - c^2t^2) \right.$$

ie (ii) the special L.T. satisfies $x'^2+y'^2+z'^2-c^2t'^2 = x^2+y^2+z^2-c^2t^2$

Hence $x''^2+y''^2+z''^2-c^2t''^2 = x'^2+y'^2+z'^2-c^2t'^2$

$$= x^2+y^2+z^2-c^2t^2$$

proves second postulate of group theory

(iii) For the inverse

$$x' = \frac{x-vt}{\sqrt{1-\beta^2}}, \quad t' = \frac{t-vx/c^2}{\sqrt{1-\beta^2}}$$

$$\left.\begin{aligned} x-vt &= \sqrt{1-\beta^2}\,x' \\ t-vx/c^2 &= \sqrt{1-\beta^2}\cdot t' \end{aligned}\right\} \quad \begin{aligned} x - x\cdot v^2/c^2 &= \sqrt{1-\beta^2}\,(x'+vt') \\ x(1-\beta^2) &= \quad " \end{aligned}$$

$$\text{or } x = \frac{x'+vt'}{\sqrt{1-\beta^2}}$$

Similarly eliminating $x \rightarrow$ $t = \dfrac{t'+vx'/c^2}{\sqrt{1-\beta^2}}$

Amounts to changing v to $-v$ in the L.T.

---

(2) Addition of velocities $\quad \underset{K'}{x' = x'(t')}, \ldots \quad ; \quad \underset{K}{x = x(t)}, \ldots$

$$\frac{dx'}{dt'} = u'_x = u'\cos\alpha', \quad \frac{dy'}{dt'} = u'_y, \quad \frac{dz'}{dt'} = u'_z, \quad u' = \sqrt{u'^2_x + u'^2_y + u'^2_z}$$

ie $u'^2 \sin^2\alpha' = u'^2_y + u'^2_z$.

$$\frac{dx}{dt} = u_x = u\cos\alpha, \quad \frac{dy}{dt} = u_y, \quad \frac{dz}{dt} = u_z, \quad u = \sqrt{u_x^2+u_y^2+u_z^2}$$

$$x = \frac{x'+vt'}{\sqrt{1-\beta^2}}, \quad y = y', \quad z = z', \quad t = \frac{t'+vx'/c^2}{\sqrt{1-\beta^2}}$$

$$dx = \frac{dx'+v\,dt'}{\sqrt{1-\beta^2}}, \quad dy = dy', \quad dz = dz', \quad dt = \frac{dt'+v\,dx'/c^2}{\sqrt{1-\beta^2}}$$ & dividing by the last eqn

(A)

19/12/67 — (1) Critique of Bohr's theory (observable quantities like frequencies of spectral lines should appear and not unobservables ones like pos^n of electron in orbits, period of revolu & so on)

* Hermitian if real ie $q_{nm} = q_{mn}^{*}$

(2) All dynamical variables in Q.M. are matrices.* $\left[q = q_{mn} e^{2\pi i \nu_{mn} t} \right] \left[\begin{matrix} \nu_{mn} \neq 0 \text{ if } m \neq n \\ \nu_{nn} = 0 \end{matrix} \right]$

(3) Results from Classical mechanics about P.B's i.e. $[x, y] = \sum_{r=1}^{n} \left(\frac{\partial x}{\partial q_r} \cdot \frac{\partial y}{\partial p_r} - \frac{\partial x}{\partial p_r} \frac{\partial y}{\partial q_r} \right)$, $H = \sum p_r \dot{q}_r - L$
$= 2T - (T - V)$
$= T + V = E$

$$\left. \begin{aligned} \dot{q}_r &= \frac{\partial H}{\partial p_r} \; ; \; \dot{p}_r = -\frac{\partial H}{\partial q_r} \\ &= -[H, q_r] \quad = -[H, p_r] \end{aligned} \right\}$$ invariance for canonical trns
ie $\sum (P_r dQ_r - p_r dq_r) = dW$.

$[Q_i, Q_j] = [P_i, P_j] = 0, \; [Q_i, P_j] = \delta_{ij}$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H].$$

~~$[x_1 x_2, y] = [x_1, y] + [x_2, y]$~~

~~$[x, y_1 y_2] = [x, y_1]$~~ $+$ $[x_1 x_2, y] = [x_1, y] x_2 + x_1 [x_2, y]$

order to be kept ~~correct~~ for in Q.M since $\leftarrow$ $[x, y_1 y_2] = [x, y_1] y_2 + y_1 [x, y_2]$

for matrices $AB \neq BA$ in general.

(4). In quantum-mechanics of Heisenberg it is taken as an axiom that

$$xy - yx = \frac{ih}{2\pi} [x, y] \qquad [\because \text{ cf } (2)]$$

for $h \to 0$, $xy = yx$ as in classical mechanics

(5) Quantum condns — $q_m q_n - q_n q_m = p_m p_n - p_n p_m = 0$, $q_m p_n - p_m q_n = \frac{ih}{2\pi} \delta_{mn}$

(6) Eqns of motion — Taken over from (3) as $\dot{q}_r = [q_r, H]$, $\dot{p}_r = [p_r, H]$ or

$$\frac{ih}{2\pi} \dot{q}_r = q_r H - H q_r, \quad \frac{ih}{2\pi} \dot{p}_r = p_r H - H p_r.$$

[$(\dot{x})_{mn} = (\dot{x}_{mn})$ for time-derivative of a matrix]

[In $\dot{x} = [x, H]$ replacing x by t, we have $[t, H] = 1$ & ie time & energy are canonical variables]

(7) Equation of energy & Bohr's frequency relation

(i) If x is constant in time, ~~then from (2), $\dot{x} = (2\pi i \nu_{mn} x_{mn} e^{2\pi i \nu_{mn} t})$~~

$\dot{x} = 0$, ~~$xH = Hx$ ie x is a diagonal matrix~~ since in this case all the non-diagonal elements are zero, because $\nu_{mn} \neq 0, m \neq n$ & the diagonal elements are constant $\nu_{nn} = 0$ ~~the def of $\dot{x}$~~, $\dot{x}$ -

Putting $x = H$ in $\dot{x} = [x, H]$, $\dot{H} = 0$ ie $H = $ const ie eq^n of energy.

~~(ii)~~ ~~also $\dot{x} = 0$ gives $xH = Hx$~~ [$\dot{x} = 0$ means x a diagonal matrix)

(ii) $\dot{H} = 0$ means H diagonal & if $\nu_{mn} = T_m - T_n$ (T_{mn} diagonal), then H & T commute & $H - hT$ is also diagonal & commutes with every $x = (x_{mn} e^{2\pi i \nu_{mn} t})$

$$x (H - hT) - (H - hT) x$$

$$\sum_k \left[x_{mk} e^{2\pi i \nu_{mk} t} (H_{kn} - h T_{kn}) - (H_{mk} - h T_{mk}) x_{kn} e^{2\pi i \nu_{kn} t} \right] = 0$$

or $\cancel{x}_{mn}(H_n - hT_n) - (H_m - hT_m)\cancel{x}_{mn} = 0$ taking out $x_{mn}\, e^{2\pi i \nu_{mn} t}$

$$H_m - H_n = h(T_m - T_n) = h\nu_{mn} \quad \text{or} \quad \nu_{mn} = \frac{E_m - E_n}{h}$$ ie Bohr's frequency relⁿ

(8) Linear harmonic oscillator. (particle of mass μ)

$$\mu\ddot{q} = -kq \quad \text{or} \quad p = \mu\dot{q}, \quad T = \tfrac{1}{2}\mu\dot{q}^2 = \frac{p^2}{2\mu}, \quad V = \tfrac{1}{2}kq^2$$

~~$\mu\ddot{q} = -kq$~~

$$H = \frac{p^2}{2\mu} + \tfrac{1}{2}kq^2; \quad \dot{q} = \frac{p}{\mu}, \quad \dot{p} = -\frac{\partial H}{\partial q} = -kq$$

$$\ddot{q} = \frac{\dot{p}}{\mu} = -\frac{k}{\mu}q$$

If particle is electrified, then due to accelerated motion, it will emit radiation of frequency ν_0

& from classical electrodynamics $\frac{k}{\mu} = (2\pi\nu_0)^2$ ie eqⁿ of motion is

$$\ddot{q} + (2\pi\nu_0)^2 q = 0 \quad \text{——— (1)}$$

(a) From (1) we can deduce $q_{mn} = 0$ when $n \neq m \pm 1$ & $q_{mn} \neq 0$, when $n = m \pm 1$

(b) $|q_{n,n-1}|^2 = \dfrac{nh}{8\pi^2\mu\nu_0}$. [long discussion to prove (a) & (b)]

To find energy values associated with the stationary states, we take

$$H = \frac{p^2}{2\mu} + \tfrac{1}{2}kq^2 = \tfrac{1}{2}\mu\left\{\dot{q}^2 + (2\pi\nu_0)^2 q^2\right\}$$

After a long calculation, using matrix expⁿ for q, we can show that H is diagonal. But this also follows if we assume conservation of energy ie $\dot{H} = 0$.

$$H_{nn} = \tfrac{1}{2}\mu\left\{\sum_m \dot{q}_{nm}\dot{q}_{mn} + (2\pi\nu_0)^2 \sum_m q_{nm}q_{mn}\right\}$$

$$= \tfrac{1}{2}\mu\left\{\sum (2\pi i)^2 \nu_{nm} q_{nm} \nu_{mn} q_{mn} + (2\mu\nu_0)^2 \sum q_{nm}q_{mn}\right\}$$

$$= 2\pi^2\mu\left\{\sum_m (\nu_{nm}^2 + \nu_0^2)|q_{nm}|^2\right\}$$ $\nu_{mn} = -\nu_{nm}$ & $q_{nm}q_{mn} = |q_{nm}|^2$ ∵ of hermiticity

Σ reduces to only 2 terms ie where $m = n-1$ & $m = n+1$.

ie $H_{nn} = 4\pi^2\mu\nu_0^2\left\{|q_{n,n-1}|^2 + |q_{n,n+1}|^2\right\}$ using $\nu_{n,n-1} = +\nu_0$, $\nu_{n,n+1} = -\nu_0$

$$= 4\pi^2\mu\nu_0^2\left\{\frac{nh}{8\pi^2\mu\nu_0} + \frac{(n+1)h}{8\pi^2\mu\nu_0}\right\} = \frac{h\nu_0}{2}(n + n + 1)$$

$$= (n + 1/2)h\nu_0$$

While in Bohr's theory stationary states are given by $nh\nu_0$; a brilliant confirmation of Heisenberg's theory

(9) Theorems on matrices

Defⁿˢ (1) ~~adjo~~ ~~Hermitian~~ adjoint $a^{\dagger}_{mn} = a^{*}_{nm}$

(2) Hermitian $a^{\dagger} = a$ (self adjoint)

(3) unitary if $u = (u^{\dagger})^{-1}$ ie $uu^{\dagger} = u^{\dagger}u = 1$.

Theorem (1) If a be Hermitian, then for any S, $b = S^{\dagger}aS$ is also Hermitian

(b) Product of two unitary matrices is unitary

(c) If $b = u^{-1} a u$, then $b^{+} = u^{-1} a^{+} u$. & if $\underline{a}$ is hermitian

(d) If $\underline{a}$ is hermitian & $\underline{u}$ unitary, $b = u^{-1} a u$ is also hermitian

(e) If $\underline{a}$ & $\underline{b}$ be hermitian, and S a matrix such that

$b = S^{-1} a S$, then $c = S S^{+} a$ and $d = S^{+} S b$ are also hermitian

Matrix Analysis — Transformation to diagonal form — (f) Canonical transformation matrices preserving the quantum conditions ~~are unitary matrices~~ and hermitian character are unitary matrices.

(f) (i) Let $Q_r = S^{-1} q_r S$, $P_r = S^{-1} p_r S$, then $f(Q_r, P_r) = S^{-1} f(q_r, p_r) S$

This ensures preservation of quantum condns $q_r q_s - q_s q_r = p_r p_s - p_s p_r = 0$, $q_r p_s - p_r q_s = \delta_{rs}$

(ii) Using $S^{+} q_r S$ is hermitian if q_r is hermitian, & (i) we should have

$$Q = S^{-1} q S$$

and $Q = S^{+} q S$

i.e. S must be such that $S^{+} = S^{-1}$ i.e. unitary

(B) Schrödinger's wave theory

classical wave eqn is $\nabla^2 \phi = \frac{1}{u^2} \frac{\partial^2 \phi}{\partial t^2}$; putting $\phi = \psi(x, y, z) e^{2\pi i \nu t}$

$\nabla^2 \psi + \frac{4\pi^2 \nu^2}{u^2} \psi = 0$ or $\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0$; using de Broglie's reln $\lambda = h/mv$

and also $\frac{1}{2} m v^2 = T = E - V$. & combining $\frac{1}{\lambda^2} = \frac{2m(E-V)}{h^2}$ & substituting for $1/\lambda^2$ we

have $\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$ i.e. Schrödinger's eqn

(1) [illegible]

(2) matrices (hermitian [illegible]) $x = x_{mn} e^{2\pi i \nu_{mn} t}$ $\quad \nu_{mn} = T_m - T_n \neq 0$ if $m \neq n$

$= 0$ if $m = n$.

(3) classical dynamics: H eqn $\frac{dq_r}{dt} = \frac{\partial H}{\partial p_r} = -[H, q_r]$; $\frac{dp_r}{dt} = -\frac{\partial H}{\partial q_r} = -[H, p_r]$.

$[x, y] \equiv$ P.B.

(4) matrices not necessarily commutative, ~~assum~~ $xy - yx = \frac{ih}{2\pi}[x, y] \to 0$ as $h \to 0$

(5) quantum condn ~~$[q_m, q_n] = 0, [p_m, p_n] = 0, [q_m, p_n] = \frac{ih}{2\pi}\delta_{mn}$~~

$q_m q_n - q_n q_m = 0, \; p_m p_n - p_n p_m = 0, \; q_m p_n - p_n q_m = \frac{ih}{2\pi}\delta_{mn} \to 0$ [illegible].

(6) Eqn of motion: taken directly from classical mechanics

$$\dot{q}_r = [q_r, H], \; \dot{p}_r = [p_r, H]$$

ie

$$\frac{ih}{2\pi}\dot{q}_r = q_r H - H q_r; \; \frac{ih}{2\pi}\dot{p}_r = p_r H - H p_r$$

In general $\dot{x} = [x, H]$. [Defn of $\dot{x}$ given by $(\dot{x})_{mn} = (\dot{x}_{mn})$

$[q_r, p_r] = 1$

Put $x = t$, $[t, H] = 1$ ie t & H are canonical variables (energy & time like coord & momentum)

(7) Eqn of energy & Bohr's frequency condn

$\dot{H} = 0$ $\quad$ [If $\dot{x} = 0$, x is a diagonal matrix)

ie H diagonal with entries in diagonal constant ie $H \equiv$ Const.

H & T diagonal $\quad H - hT$ diagonal, commutes with x

$$x(H - hT) - (H - hT)x = 0$$

$$\sum_k x_{mk} e^{2\pi i \nu_{mk} t}(H_{kn} - hT_{kn}) - (H_{mk} - hT_{mk}) x_{kn} e^{2\pi i \nu_{kn} t} = 0.$$

$$x_{mn} e^{2\pi i \nu_{mn}}(H_n - hT_n) - (H_m - hT_m) x_{mn} e^{2\pi i \nu_{mn} t}$$

$$H_n - H_m = h(T_n - T_m) \text{ or } H_m - H_n = h(T_m - T_n)$$

$$\nu_{mn} = \frac{E_m - E_n}{h}.$$

(8) L.H. oscillator:

$\mu\ddot{q} = -kq$: $\quad \left[H = \frac{p^2}{2\mu} + \frac{1}{2}kq^2 \leftarrow T = \frac{1}{2}\mu\dot{q}^2 = \frac{p^2}{2\mu}, V = \frac{1}{2}kq^2\right.$

$\left(-\frac{\partial V}{\partial q} = F = -kq\right)$

$\ddot{q} = -\frac{k}{\mu}q$ $\quad \left.\dot{p} = -\frac{\partial H}{\partial q} = -kq \text{ ie } \mu\ddot{q} = -kq\right]$

For charged particle $\frac{k}{\mu} = (2\pi\nu_0)^2$

$\ddot{q} + (2\pi\nu_0)^2 q = 0$ (q matrix).

$q_{mn} = 0$ if $n \neq m \pm 1$, $\neq 0$ if $n = m \pm 1$

$$|q_{n, n-1}|^2 = \frac{nh}{8\pi^2\mu\nu_0}$$

Subject .... (1)

Lecture on 12/12/67 — 1st Lecture on Quantum Mechanics — Historical review.

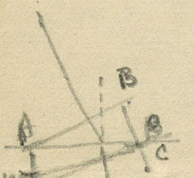
(1) (a). Newton's & Huyghens' corpuscular & wave theories of light ~~[illegible]~~ (17th Century)

~~& diffraction~~ — Explanation of refraction of light (Newton's theory wrong & Huyghen's correct since in latter refractive index (for eg. from Air into Glass) $\nu_{AG} = \frac{v_A}{v_G} = \frac{\text{velocity of light in A}}{\text{" in glass}} = \frac{\sin i}{\sin r} > 1$

while in Newton's theory, forces pulling the light particles into denser materials would add to their velocity ie $v_G > v_A$

(b) Interference explainable only by by wave theory & leading to Huyghen's Principle that "every point along a wave front serves as a source from which new waves spread out."

(c) Diffraction (Young 1803) explainable only by wave theory ie when light passes through an opening whose dimensions are comparable to the wave length of the light ($\simeq 0.001$ mm), the light is scattered (or diffracted) & all that is seen on the screen behind will be a diffused luminous spot composed of concentric light & dark rings (unlike the case, when the opening $> \lambda$, when one has a spot of light of the same shape as the opening on the screen behind).



(d) Maxwell's theory (1864) predicting e.m. waves was proved in (1888) by Hertz & practical [importance realised by Marconi who established radio communication across British Channell in 1899. & in 1901 across Atlantic Channel] — Hertz showed that light waves are e.m. waves & differ from radio waves only by their very short wave lengths.

* Also Zeemann effect.

(h) Discovery of electron by J. J. Thomson in 1897* consisting in showing that it behaves like a Newtonian particle — found e/m — Determ. of e & m by Millikan — discovery of isotopes.

(g) By strange irony, Hertz himself discovered the photo-electric effect in 1887 ie when a beam of ultra-violet rays fall on an uncharged conductor, it comes becomes +vely charged. After discovery in (h), this was ascribed to ejection of electrons from atoms of the conductor, but no correct theory was set up.

(e) Mendeleef's periodic table (1871) — use of Eka & Dvi (Sanskrit words). — atomic number.

(f) Discovery of radio-activity (natural) (1896) by Becquerel & later study by Curie & others — α, β, γ rays.

2 — Birth of quantum theory

(a) Black body radiation or heat radiation ($\lambda >$ red light ie infrared radiation) — ~~Lummer~~ & ~~Pringsheim (1899)~~ Kirchoff (1860) showed that the total energy of radiation & distn. among various λ depended only on temp T — Stefan-Boltzmann law of total energy $W = \sigma T^4$ — Lummer & Pringsheim's law of a particular λ, when E_λ is max & the ~~[illegible]~~ corresp λ becomes shorter as T is increased — To derive these ~~laws~~ Wien's ~~law~~ assumed $E_\lambda = \frac{1}{\lambda^5} f(\lambda, T)$ as derived by Thermodynamics confirming Kirchoff's result — In particular he took $f(\lambda, T) = a e^{-b/\lambda T}$ ie

1 (a) Newton's & Huyghens on refraction (17th century)

(b) Interference & Huyghen's Principle (1801)

(c) Diffraction (1803

(d) Maxwell's laws & Hertz's e.m. waves, light of them (1864, 1868)

(e) Mendeleev's periodic table (1871) Eka-Boron - Atomic mass ~~[illegible]~~

(f) Photoelectric effect by Hertz (1887)

(g) Natural radioactivity α, β, γ rays

(h) Discovery of electron - e/m, e & m — isotopes

2. Old Quant. Theory:

(a) Blackbody radiation - K, P & L, St., Wien, Rayleigh-Jeans — Planck (use of $E = h\nu$ (graphs)

Eq. of 6 ~~[illegible]~~ among 6 people) (14/12/1900)

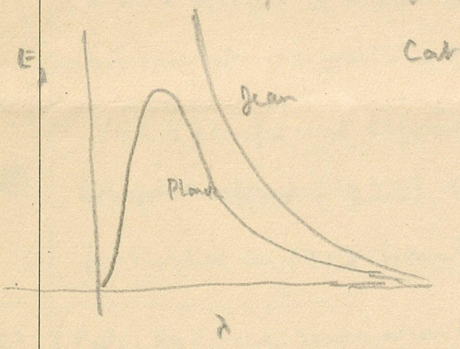
(b) Photoelectric effect explained by ~~N~~ Einstein (1905) $h\nu - W = \frac{1}{2}mv^2$ - $W = h\nu_0$, $\frac{1}{2}mv^2 = h(\nu - \nu_0)$

v max if $W = 0$, i.e. $\nu_0 = 0$, $v_{max}^2 = \frac{2h\nu}{m}$ — Duality

(c) ~~Bohr~~ R-R combination Principle about wave numbers

(d) Rutherford's expts on atom

$E_\lambda = \frac{a}{\lambda^5} \cdot e^{-b/\lambda T}$, a formula giving good results only for short wave lengths, but fails for long wave-lengths – Rayleigh-Jeans formula $f(\lambda, T) = C\lambda . T$ leading to $E_\lambda = \frac{C}{\lambda^5} . \lambda T = \frac{CT}{\lambda^4}$ giving good results for long wave-lengths only but fails for short wave-lengths, leading to the ultraviolet



Catastrophe. – Planck took $f(\lambda, T) = \frac{c_1}{e^{c_2/\lambda T} - 1}$. ie

$$E_\lambda = \frac{c_1}{\lambda^5} \cdot \frac{1}{e^{c_2/\lambda T} - 1}.$$

If λ is small, this $\rightarrow$ Wien's law

λ is large $\longrightarrow$ Rayleigh-Jeans formula.

To derive this Planck broke away from classical optics & on 14/12/67 announced his assumption about light quanta (ie energy quanta just like matter quanta or atoms. It is just as absurd to talk of 3/4ths of a quantum of green light as it is to talk of 3/4ths of an atom of copper) viz the amount of energy of a light quantum is inversely ∷ l to the wave length of the radiation or directly ∷ l to the frequency (vibrations / second) ν ie $E = h\nu$, h being known as Planck's constant $\simeq 6.62 \times 10^{-27}$ erg-sec [h a most fundamental constant of Physics like c & e & m]. [Example of divisibility 6 dollars among 6 persons].

$(\nu = v/\lambda)$

$(\nu = c/\lambda)$

(b) Explanation of the photoelectric effect by Einstein (1905) using corpuscular theory of light & Planck's light quanta theory that interchanges of energy between atom & radiation can only occur discontinuously by quanta & not continuously as assumed in the classical theory (When struck by ultraviolet radiation, a zinc plate loses electrons). Emitted electrons called photo-electrons; no emission if $\nu <$ a certain ν_0; energy of emitted photoelectrons depends only on ν (not on intensity or temperature) & velocity increases with ν. Einstein assumed that light of frequency ν consists of particles having energy $\epsilon = h\nu$ and momentum $p = \frac{h\nu}{c}$. If W be energy required to remove an electron from the substance, & $\frac{1}{2}mv^2$ the K.E of electron as it leaves the surface, then

$$h\nu - W = \frac{1}{2}mv^2.$$

Threshold frequency is given by $\nu = \nu_0 = \frac{W}{h}$. ie $\frac{1}{2}mv^2 = h(\nu - \nu_0)$. v is max when $W = 0$ ie $\nu_0 = 0$. & $v_{max} = \sqrt{\frac{2h\nu}{m}}$. Also in terms of potential difference V, $\frac{1}{2}mv^2 = eV$ so that $\frac{1}{2}mv^2 = h\nu - W = eV$ (Einstein's eqn verified exactly by Millikan later)

Photoelectric effect brings out nature of ~~complementarity~~ duality of wave & corpuscular aspects of light ie both used (if quanta are assumed) as for eg. interference & diffn can be explained only on wave aspect & photoelectric effect only on quantum-corpuscular aspect

(d) Rutherford's expts on atoms using α particles (R knew no mathematics & did not know what a hyperbola was) showing that atom had a nucleus (1911) of very small dimensions with $+^{ve}$ charge Ne (N an integer) and its surrounded by N electrons – N exactly equal to the atomic number ie serial number of the elements when arranged in Mendeleev's table of increasing atomic weight – Rutherford's model of a nucleus with W protons & (N-W / (W-N)) electrons, and N peripheral electrons prevailed almost till 1932.

(e) ~~Compton effect (1923)~~ Bohr's (Bohr a student of Rutherford) epoch-making paper published in 1913 formulating that Planck's h is fundamental not only for radiation but matter

(c) Ritz-Rydberg combination Principle (1908), Einstein's theory of sp. heats (1907) → Debye (1912)

[wave no. of any spectral line is the difference of the wave numbers ($1/\lambda$) of two other lines]

also. He put forward the following postulates

(1) An atomic system has only a discrete set of energy values ie $E_1, E_2, \ldots E_n$ in stationary states & the atom is stable in any one of the states, or emission or absorption of energy can take place in a transition from one stationary state to another.

(2) Classical laws hold as long as the system is one of the stationary states, but not in a transition

(3) $E_2 - E_1 = h\nu$ for transition ie Bohr's frequency relation

(4) For H-like atoms ie an electron rotating round a nucleus, different stationary states are orbits for which $A.M = \frac{nh}{2\pi} = n\hbar$ ($n = 1, 2, 3 \ldots$) or Bohr's quantum condition

Successes of Bohr's theory (i) Hydrogen atom, $\nu = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$ or putting $\nu = c/\lambda$ (n = principal quantum no)

$\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$ proving combination principle & giving correct value for R-R constant.

Putting $n' = 2, n = 3, 4, 5, 6, \ldots$ we get the Balmer lines of Hydrogen & other values to n' & n give other lines in the spectrum (ii) Singly ionised He^+ & doubly ionised Li^{++} & their spectra

(iii) Generalisation of quantum conditions by adiabatic invariants by Ehrenfest (1924),

(iv) Correspondence Principle ie Classical theory as limiting case of quantum theory for large q. n's.

(v) Relativity theory of the fine-structure of spectral lines (each spectrum line divides into several components if observed by spectrographs of high resolving power for eg: the H_α-Balmer line breaks into 3 components) – introduction of another quantum number l (azimuthal)

(vi) Explanation of normal Zeeman effect (due to external mag. field)

(vii) Stark effect (due to external electric field)

} new quantum number m (magnetic quantum no)

(viii) atomic magnetism – $M = m.\frac{eh}{4\pi m_0 c}$ (Bohr magneton) (m_0 = mass of electron, M = magnetic moment)

(ix) Spectra of heavier atoms – Paschen Back effect for strong magnetic fields requiring a fourth quantum number (spin quantum number) s – Uhlenbeck & Goudsmit's spinning electron theory of electron having spin $A.M = s.h/2\pi$ ($s = +\frac{1}{2}$ or $-\frac{1}{2}$) (1925)

$$\frac{\partial H}{\partial p_r} = -[H, q_r)$$

$$-\text{R.H.S} = \sum_{s=1}^{1} \left(\frac{\partial H}{\partial q_s} \cancel{\frac{\partial q_r}{\partial p_s}} - \frac{\partial H}{\partial p_s} \cdot \frac{\partial q_r}{\partial q_s} \right)$$

$$= -\frac{\partial H}{\partial p_r}.$$

Time—3 hours
Max. Marks—100

Mathematics II : ~~Geometries~~

~~NOTE.~~—

(e) Structure of atoms & explanation of periodic table of elements based on Pauli's exclusion Principle (Pauli-Verbot) (1925) i.e. each stationary state of the quantum theory is defined completely & uniquely by a given set of values of ~~s, t~~ n, l, m, s — Shells K, L, M, N, O, P, Q of electrons & nos. in each shell ~~form~~

Thus Bohr's theory reigned supreme till 1925 or so.

(f) Compton effect (1923) — ~~Scattering~~ collision of ~~elect~~ (almost) free electrons by X-rays & in the process the quanta of X-rays deflected by large angles will have a smaller amount of energy and consequently a smaller ~~wavelength~~ frequency - giving additional support to the hypothesis of quantum nature of radiation

(g) de Broglie's (1924) suggestion that matter must also have a dual (particle-like & wave-like) character & if momentum of particle be p, the wave length of corresponding wave length λ is given by $\lambda = h/p$. [Later confirmed by Davisson & Germer (~~1927~~) & independently by G. P. Thomson (1928) by observing the diffraction of electrons by crystals.]

~~(2)~~ 19/12/67

3. ~~New quan~~ Heisenberg's Quantum Mechanics (1925).

(a) Critique of Bohr's theory — Requirement of a rational & unified reformulation i.e. that only experimentally observable quantities like frequencies & intensities of spectra should come in the theory and not position of electron in orbit, period of revolution etc which are not observable — Leading to matrix mechanics

(b) Poisson brackets — Quantum P.B's $(xy - yx) = \frac{ih}{2\pi}[x, y]$ as an axiom

(c) Quantum condns & eqns of motion: $q_m q_n - q_n q_m = 0$, $p_m p_n - p_n p_m = 0$, $q_m p_n - q_n p_m = \frac{ih}{2\pi}[q_m, p_n] = \delta_{mn}$

are q-consts — Eqns of motion are $\frac{ih}{2\pi}\dot{q}_r = [q_r, H]$; $\frac{ih}{2\pi}\dot{p}_r = [p_r, H]$

(d) Linear harmonic oscillator — Bohr's theory gives for energy $H = n(h\nu_0)$ which is not experimentally correct, but Heisenberg's theory gives $H = (n + \frac{1}{2})h\nu_0$ which is correct.

(e) Further successes with A.M, selection & polarisation rules etc.

4. Schrodinger's wave mechanics (Jan 1926) based on de Broglie's concept of matter waves, de

(a) Broglie's wave fn $\psi = \psi_0 \cdot e^{2\pi i/h\,(px - Et)}$ for one dimension — ~~Schrodinger's~~

(b) Schrodinger's wave eqn based on classical wave eqn $\nabla^2\phi = \frac{1}{u^2}\frac{\partial^2\phi}{\partial t^2}$. Putting $\phi = \psi(x, y, z)e^{2\pi i\nu t}$

we get $\nabla^2\psi + \frac{4\pi^2\nu^2}{u^2}\psi = 0$ or $\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0$. $\lambda = h/mv$. Also $\frac{1}{2}mv^2 = T = E - V$.

Combining $\frac{1}{\lambda^2} = \frac{2m(E-V)}{h^2}$ & we get the famous Schrodinger wave eqn

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0.$$

V. *~~M.Sc. Degree Examination (New rules), 193~~*

Time—~~3~~ hours
Max. Marks—100 (5)

~~Mathematics III : Theory of Functions of a Complex Variable~~

NOTE.—

(b) Eigenvalues & eigenfns in wave mechanics — polynomial method of findg eigenvalues

(c) Appln to linear harmonic oscillator — identical results with matrix mech

(d) " to hydrogen like atoms — Superior to Bohr's theory in connectg the continuous spectrum by a uniform analytic process with the line spectrum of Hydrogen

(e) Equivalence of matrix & wave mechanics — Dedn of matrix elements from eigen-fns — Transformation theory.

(~~5~~) ~~Statistical~~

(5) Physical interpretations

(a) Born's interpretation in terms of probability (1927)

(b) Uncertainty relations $\Delta p \cdot \Delta q \gtrsim h/4\pi$, $\Delta E \cdot \Delta t \gtrsim \frac{h}{4\pi}$

(c) Principle of indeterminacy & concept of causality — Bohr, Heisenberg, Born, Eddington & Dirac follow principle of indeterminacy ie physical laws are statistical laws.

(6) Dirac's relativistic quantum theory (1928)

(a) Earlier 2nd-order wave eqns unsatisfactory

(b) Diracs linear eqn with Dirac matrices showing (invariant under a L.T)

(i) existence of magnetic moment of electron, (ii) existence of spin A.M of electron

(iii) Fine structure of spectral lines (iv) Zeemann effect (H-line split up into 3 compns when viewed at rt Ls to an external ~~field~~ mag. field of intensity H & into 2 compns when viewed along the field axis of these frequencies ie ν_0, $\nu_0 + \Delta\nu$ & $\nu_0 - \Delta\nu$, $\Delta\nu = eH/4\pi m_0 c$ (m_0 = mass of electr)

(c) Negative energy states not discarded in a linear theory but particles associated as "holes" in -ve energy states — Proof of existence of positron — Experimentally demonstrated by Carl Anderson in 1932.

(7) ~~Statistical Phys~~ Quantum Statistics — Spin & Statistics — F-D & B-E Statistics.

(8) Elementary Particles — Discovery of Neutron by Chadwick in 1932 modified Rutherford's & Bohr's theories —

[P.T.O]

(9) Nuclear Physics

(10) Recent Quantum Theory

Anti-protons, Anti-neutrons (mutual annihilation with neutrons), neutrino (ν)

eg. ${}_4Be^7 \rightarrow {}_2Li^7 + e^+ + \nu$ – ($p + \nu \rightarrow n + e^+$) – π mesons & μ-mesons.

$\pi^\pm \rightarrow \mu^\pm + \nu$, $\mu^\pm \rightarrow e^\pm + 2\nu$, $\pi^0 \rightarrow \gamma + \gamma'$ (photon)

K-mesons (mass between electrons & nucleons) – Λ, Σ, Ξ particles (hyperons)

Xi $(\Xi)^\pm$

$\Sigma^\pm$

Λ^0

Neutron $n^\pm$

Proton $p^\pm$

K-meson $K^\pm$

" K^1

Pion $\pi^\pm$

" π^0

Muon $\mu^\pm$

electron $e^\pm$

Neutrino ν

(anti-neutrino ?) opp. spin

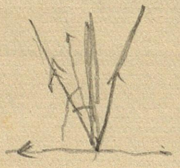
Are all elementary ?

$\nabla^2 \Phi k^2$

$$\frac{h^2}{8\pi^2 m}\nabla^2\psi + (E-V)\psi = 0$$

$$\frac{\partial^2\psi}{\partial x^2} =$$

$$E\psi = -\frac{h^2}{8\pi^2 m}\nabla^2\psi + V\psi$$



$-\frac{h}{\lambda}\sin\epsilon$

$+\frac{h}{\lambda}\sin\epsilon$

Lectures on 4/9/67 (1)

(A) (1) Permutation groups, or substitution groups or Symmetric groups S_n (order $n!$ degree n)

Proof that S_n forms a group - Illustration by $s = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ & $t = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

(2) Even and odd permutations using $\prod_{i<j} (x_i - x_j) = P$. A_n also a normal subgroup. (order $\frac{n!}{2}$, degree n)

(2') Sym. poly - Elem. sym. polys Σx_i, $\Sigma x_i x_j$, .. $\sigma_n = x_1 \cdots x_n$ & theorem

(3) cyclic permutation of n symbols has order n. (length n) $[r^n = 1]$

Eg $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix} = (123)(45)$

(4) Any permutation = product of disjoint cycles $[a_1 \phi = a_2, \ldots$ until $a_n \phi = a_1)$, (ϕ'), (ϕ'') -

(5) order of any permutation = l. c. m of lengths of disjoint cycles

$[\phi = r_1 \cdots r_t$ & $\phi^n = r_1^n \cdots r_t^n$ & $\phi^n = 1$ only if every $r_i^n = 1]$

(6) If ϕ be any permutation & $r = (a_1, \ldots a_m)$, & $r' = (a_1\phi, \ldots a_m\phi)$, then $\phi^{-1} r \phi = r'$

(7) Any permutation = product of transpositions

$a_1 a_2 \cdots a_n = (a_1 a_2)(a_1 a_3) \cdots (a_1 a_n)$ [Illustrate with 2431]

Every +ve permutation = products of abc $[= (ab)(ac)]$

-ve " = " abcd $[(ab)(ac)(ad)]$

Symmetric group S_n generated by $(12)(13)(14) \cdots (1n)$

Alternating group A_n " $(123)(124) \cdots (12n)$

(8) Simple group - Every alternating group A_n $(n \neq 4)$ is simple. [A_3 has no normal subgroup. A_4 contains 1, $(12)\cdot(34)$, $(13)\cdot(24)$, $(14)(23)$ as normal subgroup. i.e. 4-group.

(9) Examples of special permutation groups & their subgroups.

(a) S_3 expressed by (1), (12), (13), $(2,3)$, (123), (132)

Subgroups: A_3 by 1, (123), (132)

S_2 by 1, (12), S_2' by 1, (13), $S_2'' = 1$, (23)

and (1)

S_3 has class no $= 3$. ~~and~~ A_3 also has class number $= 3$.

(b) S_4 has besides itself and unity only the two following normal subgroups

(i) A_4 and (ii) V_4 = Klein's four group $1, (12)(34), (13)(24), (14)(23)$ [S_4/V_4]

S_4 Has class no $= 5$, ~~Klein's 4-group~~ $S_4/V_4 \cong S_3$ has class no. $= 3$. [only upto $n = 16$ have so far subgroups etc. form]

A_4 " $= 4$

(10) Quaternion group Q_8 – Algebra of quaternions $1, i, j, k$ with $i^2 = j^2 = k^2 = -1$

$ij = -ji = k$, etc. – ± 1, $\pm i$, $\pm j$, $\pm k$ form a multiplicative group of order 8.

~~This group has class no = 5~~ – also defined by $j^4 = 1$, $k^2 = j^2$, $kj = j^3 k$. The normal subgroup $(1, j^2)$ has the 4-group as factor group – class no of Q_8 group $= 5$. ~~a matrix repn~~

(B) Representation of groups by matrices: ~~Completely reducible representations not of interest~~

(1) Defn: Repn of a group G by linear trn in a field K is a group homomorphism such that to each element a of G corresponds a linear transformation A with coefficients in K.

Since linear trn ~~lead to matrices~~ >1 ($\because$ of homomorphy), we have repn by matrices

(2) (i) ~~can~~ There are completely reducible repns which means that there are no. matrix repn

(ii) Theorem about irreducible repns (a) No. of irreducible repns = no. of classes of conjugate elements, (b) If $n_1, n_2 \ldots n_s$ be orders of these repns

$$n_1^2 + n_2^2 + \cdots + n_s^2 = h \quad (h = \text{order of group}).$$

(3) The several representations already mentioned

(4) Repns for case (9) & (10) in (A)

9 (a) for S_3, $n_1^2 + n_2^2 + n_3^2 = 6$ ie $n_1 = n_2 = 1$, $n_3 = 2$ & this is the repn in (3) by matrices

A_3, $n_1^2 + n_2^2 + n_3^2 = 3$, $n_1 = n_2 = n_3 = 1$ & hence no additional matrix repns except those in S_3

(b) S_4: $n_1^2 + n_2^2 + n_3^2 + n_4^2 + n_5^2 = 24$

Since ~~4-group~~ S_4/V_4 has 3 irreducible repns of order 1, 1, 2 (See (3) for this). If these be denoted by n_1, n_2, n_3, $n_1^2 + n_2^2 + n_3^2 = 6$, so that $n_4^2 + n_5^2 = 18$

A_4 has class no. 4 & has 4 repns of order 1, 1, 1, 3 so that $1^2+1^2+1^2+3^2 = 12 = (24/2)$

& we might take the last one as the n_4 of S_4. Hence n_5 also $= 3$.

(10) For quaternion group Q_8 class no $= 5$ ie

$$n_1^2+n_2^2+n_3^2+n_4^2+n_5^2 = 8$$

ie $n_1 = n_2 = n_3 = n_4 = 1$, $n_5 = 2$ & this last is given by

$$i = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad j = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad K = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

Groups represented by matrices

(i) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0, & -1 \\ 1, & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix},$

and 1, abc, acb, ab, ac, bc can be put in 1-1 correspondence

ie isomorphic as regards multiplication [mention isomorphism

(ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1, & 1 \\ -1, & 0 \end{pmatrix}, \begin{pmatrix} 0, & -1 \\ 1, & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$

not isomorphic to S_6 but is cyclic.

(iii) $\underset{1}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}, \underset{a}{\begin{pmatrix} -1, & 0 \\ 0, & -1 \end{pmatrix}}, \underset{b}{\begin{pmatrix} 1, & 0 \\ 0 & -1 \end{pmatrix}}, \underset{c}{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}$

constitute a non-cyclic group of order 4 under multiplication ie four-group.

(iv) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$

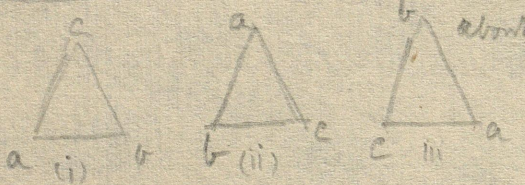
Constitute a cyclic group of order 4.

Re (iii) $a^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$, similarly $b^2 = c^2 = 1$.

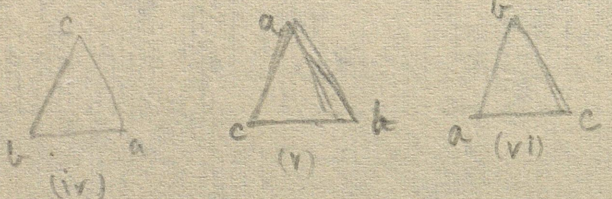
$bc = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = a$, and $ca = b$, $ab = c$

ie (iii) is the 4-group.

∵ Symmetric group of order 6 — For eq. Δ^{a} abc, rotations thr' $0°$, $120°$, $240°$ about centre going in ↻ give (i), (ii) & (iii) or ↻



and rotations of plane about altitudes thr' a, b, c give (iv), (v), (vi)



or denoted by

$$\begin{pmatrix} abc \\ abc \end{pmatrix}, \begin{pmatrix} abc \\ bca \end{pmatrix} \begin{pmatrix} abc \\ cab \end{pmatrix} \begin{pmatrix} abc \\ bac \end{pmatrix} \begin{pmatrix} abc \\ cba \end{pmatrix} \begin{pmatrix} abc \\ acb \end{pmatrix}$$

or 1, abc, acb, ab, ca, bc (all possible permutations 6 in number order 6, degree 3)

multiplication: ab · ac = ~~bc~~ · abc ~~→ bca~~

ie abc → bac → bca = abc ie a→b, b→c, c→a

ac · ab is abc → cba → cab = acb ie a→c, c→b, b→a

ie non-commutative multiplication

abc $\boxed{\begin{matrix} cba \\ bca \\ acb \end{matrix}}$

Subgroups: (1) 1, abc, acb. of order 3 (forming themselves a group of rotation $0°$, $120°$, $240°$)

(2), (3) & (4) of order two are 1, ab; 1, ac; 1, bc.

ie Symmetric group ~~has~~ of order 6 has 4 subgroups besides the identity

(Mention Lagrange's theorem)

(1) <u>General considerations</u>. (a) Modification of Newton's law of gravitation of instantaneous action, which is incompatible with special relativity, by Poincaré, Minkowski, Sommerfeld & Lorentz not satisfactory because not based on Poisson's modifying the fundamental Poisson's <u>field</u> eqn viz

$$\nabla^2 \phi = 4\pi K \rho_0 \text{ \& eqn of motion } \frac{d^2 \vec{r}}{dt^2} = -\operatorname{grad} \phi.$$

(b) Einstein's attempt to extend the relativity principle to reference systems in non-uniform motion ie the postulate of equivalence of physical laws even in systems other than Galilean.

(c) Principle of equivalence – Generalisation of Newton's theory of equivalence of homogeneous gravitational field with a uniformly accelerated reference system viz that all other processes should take place in the same way in both systems – The principle makes it possible to calculate effect of a homogeneous gravitational field on arbitrary processes – eg (i) rate of clocks slower at points of lower ϕ_{gr} than at higher ϕ_{gr}, leading to shift towards the red of spectra emitted by the Sun, (ii) <u>c</u> not constant in a gr. field so that light rays become curved leading to displacement of the fixed stars seen at the edge of the Sun which he calculated as 0.83", (iii) not only an <u>inertial</u> mass, but also a <u>gravitational</u> mass $m = E/c^2$ has to be ascribed to an energy in all cases; (iv) Special relativity not valid in gr. fields & L.T loses all meaning & spec. rel. can only be correct in absence of gr. fields.

(d) Covariant form of physical laws based on principle of equivalence & also valid for non-hom. gr. fields to be formulated – Several unsuccessful attempts – Einstein's 1st attempt to determine the gravitational field by the 10-component tensor g_{ik} in $ds^2 = g_{ik}\, dx^i dx^k$ without setting up generally covariant eqns for the g_{ik}'s also – He did these later making use of Riemann's theory of curvature – This enabled him to explain the perihelion displacement of Mercury & bending of light rays in Sun's gravitational field quantitatively obtaining the size of this deflection as twice ~~its~~ the original value obtained on earlier theory.

(2) <u>General formulation of principle of equivalence – Connection between gravitation & metric.</u>

(a) Transforming away gravitation in an infinitesimally small world region K_0 (X_1, X_2, X_3, X_4) is possible because of equality of gravitational & inertial mass – Result of Eötvös – to each form of energy a wt. has also to be attached. – Special relativity valid in K_0.

(b) Transformation from the X_i to general x_i leads from $ds^2 = dX_1^2 + \quad - dX_4^2$ to $ds^2 = g_{ik}\, dx^i dx^k$ (G-field of the g_{ik}'s) & world line of a particle subject to no force other than gravity becomes a geodesic line $\delta \int ds = 0$ or $\frac{d^2 x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^j}{ds} \cdot \frac{dx^k}{ds} = 0$. ~~Here~~ The ten tensor components in Einstein's theory take the place of the Newtonian potential ϕ & comps of Γ^i_{jk} determine magnitude of the gravitational force. – ~~For~~ Light rays which are st. lines in K_0 now become geodesic null lines in general ie above eqns for particle plus $ds^2 = g_{ik}\, dx^i dx^k = 0$.

(c) Galilean principle of inertia is here replaced by $\delta \int ds = 0$ – Connection between geometry & space, Gen. Relativity shows that geometry of space is not given à priori but ~~deter~~ is only determined by matter.

(3) <u>Postulate of general covariance of physical laws</u>

(a) General physical laws are to be brought to such a form that they must be covariant in arbitrary coord. ~~systems~~ transformations. This covariance is made possible by incorporating the g_{ik} into the physical laws. Covariance by itself does not make any assertion about the content of physical laws, but has only a heuristic aspect. Covariance + principle of equivalence makes up the physical content of general relativity

(4) Simple deductions from principle of equivalence:

(α) Eqns of motion of a point mass for small velocities & weak gravitational fields

Eqn of motion is $\dfrac{d^2x^i}{ds^2} + \Gamma^i_{jk}\dfrac{dx^j}{ds}\cdot\dfrac{dx^k}{ds} = 0$

can be written for small velocities (i.e. neglecting v^2/c^2) as

$$\frac{d^2x^i}{dt^2} + \Gamma^i_{jk}\frac{dx^j}{dt}\cdot\frac{dx^k}{dt} = 0$$

weak gravitational field means $g_{ik} = +1$ (for $i = k = 1, 2, 3$)

$$g_{ik} = 0 \ (i \neq k), \quad g_{44} = -1$$

then $\dfrac{d^2x^i}{dt^2} = -c^2\,\Gamma^i_{44} \quad (x_4 = ct) \quad (i = 1, 2, 3)$

Also neglecting time derivatives of g_{ik} (static field), $\Gamma^i_{44} = \Gamma_{i,44}$ i.e. $-\frac{1}{2}\partial g_{44}/\partial x^i$ & eqns of motion

reduce to Newton's eqns of motion

$$\frac{d^2x^i}{dt^2} = -\frac{\partial\phi}{\partial x^i}, \quad \text{where } \phi = -\tfrac{1}{2}c^2(g_{44}+1) \text{ or } g_{44} = -1 - \frac{2\phi}{c^2}$$

possibility of describing gr. fields approximately by scalar potential.

(β) Red shift of spectral lines

In general $t = x^4/c$ will be different from the normal proper time τ of a clock at rest.

For the world line of a clock at rest is $ds^2 = g_{44}(dx^4)^2$

~~and~~ since $s = ic\tau$, $t = \dfrac{\tau}{\sqrt{(-g_{44})}} = \dfrac{\tau}{\sqrt{\{1 + \frac{2\phi}{c^2}\}}} \simeq \tau\left(1 - \dfrac{\phi}{c^2}\right)$ i.e. $\dfrac{\Delta t}{\tau} = -\dfrac{\phi}{c^2}$

i.e. of two originally synchronous clocks at rest, if one is placed in a gr. field for a certain length of time, it will have lost.

Regarding vibration process of light as a clock, ~~frequency measured by t will be same at two different points P & P', but measured by τ will be different~~ the above can be interpreted as saying that if a spectral line produced ~~on the earth~~ Sun be observed on the Earth, its frequency will be shifted towards the ~~violet~~ red end compared with the corresponding terrestrial frequency & this amount is given by

$$\frac{\Delta\nu}{\nu} = \frac{\phi_E - \phi_S}{c^2}.$$

[wave emitted at S will ~~be of form~~ have time dependence $\exp(2\pi i t/T_S)$
" E " $\exp(2\pi i t/T_E)$

Numerical calculation gives $\dfrac{\Delta\nu}{\nu} = 2{\cdot}12 \times 10^{-6}$ – Recently for the companion of Sirius (a super dense star) $\phi > 30\,\phi_S$ & has been detected by experiment.

$$\left[\; dt = d\tau\,(1 - \phi/c^2) \qquad T_S = T_0\left(1 - \frac{\phi_S}{c^2}\right) \qquad \frac{T_E - T_S}{T_0} = \frac{\phi_S - \phi_E}{c^2} \right.$$

$$\left. T_E = T_0\,(1 - \phi_E/c^2) \qquad \Delta\nu/\nu = \text{"} \;\right]$$