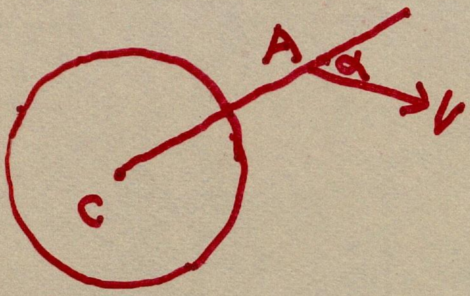
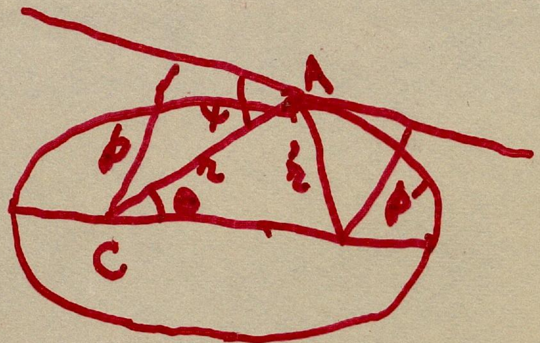


AC = R



$$\frac{p}{r} = \frac{p'}{r'} \qquad pp' = b^2 \qquad r + r' = 2a$$

$$\frac{b^2}{p^2} = \frac{2a}{r} - 1 \qquad \text{But } Vp = h$$

$$\therefore \frac{b^2V^2}{h^2} = \frac{2a}{r} - 1 \qquad v^2 = \frac{h^2}{b^2}\frac{2a}{r} - \frac{h^2}{b^2}$$

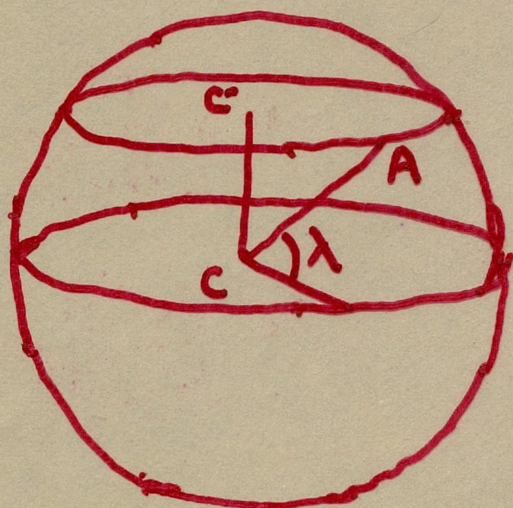
Conservation of Energy ➡ $\frac{1}{2}mV^2 - \frac{m\mu}{r} = mE$

$$\therefore v^2 = \frac{2\mu}{r} + 2E \qquad \therefore \mu = \frac{h^2 a}{b^2} \qquad E = -\frac{1}{2}\frac{h^2}{b^2} = -\frac{1}{2}\frac{\mu}{a}$$

$$\therefore v^2 = \frac{2\mu}{r} - \frac{\mu}{a}$$

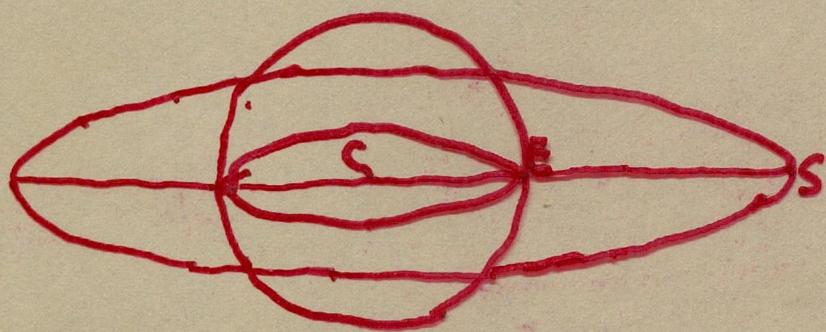
satellite to attain max. heigh H, min. height h and major ~~ax~~ axis of orbit make an $\angle\theta$ with verticle at launch time. The problem is: given H, h, θ find initial conditions

V, R, α



$$\omega = \frac{2\pi}{24(60)^2}$$

$$V = \frac{2\pi R}{24(60)^2} \text{ cm/sec}$$



$CE = a \quad CS = R$

$$\frac{V^2}{R} = \frac{\mu}{R^2} \qquad V^2 = \frac{\mu}{R} \qquad \frac{4\pi^2 R^2}{(24)^2(60)^4} = \frac{\mu}{R}$$

$$R^3 = \frac{\mu(24)^2(60)^4}{4\pi^2} \qquad \mu = ga^2$$

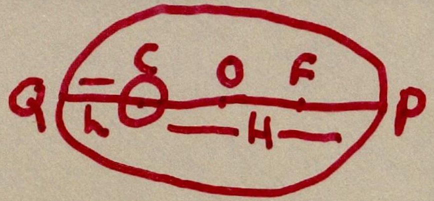
$g = 981 \frac{\text{cm}}{\text{sec}^2}$ $\quad a = 6400\text{km} = 6400 \times 10^5 \text{ cms}$ gives

R in cms. $\quad R \times 10^{-5} - 6400$ = Height of Satellite

Total Energy = K.E. + P.E.

$$K.E. = \frac{1}{2} mV^2 = \frac{1}{2} m\frac{\mu}{R} \qquad P.E. = \int_\infty^R \frac{m\mu}{r^2}\,dr = -\frac{m\mu}{R}$$

$$\therefore \text{ Total Energy} = -\frac{1}{2}\frac{m\mu}{R} \quad -\text{ve!}$$



ρ is Earth's radius

$$\rho + H = CP = a(1+e)$$
$$\rho + h = CQ = a(1-e)$$

$\therefore\ 2a = 2\rho + H + h \qquad 2ae = H - h$

$a = \rho + \frac{H+h}{2} \qquad e = \frac{H-h}{2\rho + H + h}$

From H, h we have geometric parameters a and e

$\frac{v^2}{2\mu} \quad v^2 = \frac{2\mu}{r} - \frac{\mu}{a}$. Put initial values

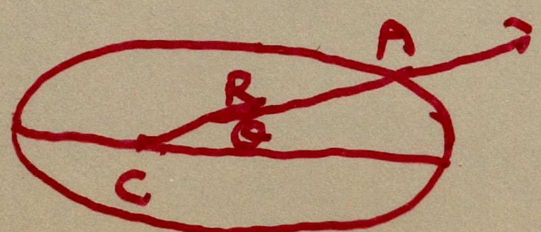
$$\therefore\ V^2 = \frac{2\mu}{R} - \frac{\mu}{a} \qquad (1)$$

Again $\frac{b^2}{p^2} = \frac{2a}{r} - 1$ But $p = r\sin\psi$

$\therefore\ \frac{b^2}{r^2\sin^2\psi} = \frac{2a}{r} - 1$ Put initial values

$$\frac{b^2}{R^2\sin^2\alpha} = \frac{2a}{R} - 1 \qquad (2)$$

Eq. of ellipse give $\frac{l}{R} = 1 - e\cos\theta$ but $l = \frac{b^2}{a} = a(1-e^2)$



$$\therefore\ \frac{a(1-e^2)}{R} = 1 - e\cos\theta \qquad (3)$$

(3) will immediately give

$$R = \frac{a(1-e^2)}{1-e\cos\theta}$$

(1) will now give V. $\frac{V^2}{\mu} = \frac{2}{R} - \frac{1}{a} = \frac{2(1-e\cos\theta)}{a(1-e^2)} - \frac{1}{a}$

$$\frac{V^2}{\mu} = \frac{2(1-e\cos\theta)-(1-e^2)}{a(1-e^2)}, \quad \mu = \rho^2 g$$

(2) now gives $\sin^2\alpha$: $\frac{b^2}{R^2\sin^2\alpha} = \frac{2a}{R} - 1$

Put $b^2 = a^2(1-e^2)$ and the above value of R

and finally get $\sin^2\alpha = \frac{(1-e\cos\theta)^2}{1-2e\cos\theta+e^2}$

or a still simpler expression:

$$\tan^2\alpha = \frac{(1-e\cos\theta)^2}{e^2\sin^2\theta}$$

Thus given the orbit we can calculate the launching data

Example For Rohini satellite $h = 300$ kms

$H = 830$ kms. Show that its period is about 1.5 hours.

We have $T = 2\pi a^{3/2} / \sqrt{\mu}$

Now $a = \rho + \frac{H+h}{2} = 6400 + \frac{830+300}{2} = 6400 + 565$

$= 6965$ kms $= 6965 \times 10^5$ cms

$\mu = g\rho^2 = 980 (6400)^2 \times 10^{10} = 2^{13} \times 7^2 \times 10^{15}$

$$\therefore T = \frac{2 \times (3.14) \times (6965)^{3/2} \times 10^{15/2}}{7 \times 2^{13/2} \times 10^{15/2}}$$

$$= \frac{3.14 \times (6965)^{3/2}}{7 \times 2^{11/2} \times 3600} \text{ hours}$$

which works out to be 1.5 hour